basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

EXEMPLAR 2013

MEMORANDUM

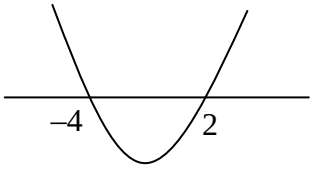
MARKS: 150

This memorandum consists of 13 pages.

QUESTION 1

1.1.1	$(2x-1)(x+5)=0$ $x=\frac{1}{2}$ **OR** $x=-5$	✓✓ answers (2)
1.1.2	$2x^2-4x+1=0$ $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$ $x=\frac{4\pm\sqrt{8}}{4}$ $x=\frac{4\pm\sqrt{4\,.\,2}}{4}$ $x=\frac{4\pm2\sqrt{2}}{4}$ $x=\frac{2\left(2\pm\sqrt{2}\right)}{4}$ $x=\frac{2\pm\sqrt{2}}{2}$	✓ substitution into correct formula ✓✓ answers (3)
1.2.1	$125^{\frac{2}{3}}$ $=\left(5^3\right)^{\frac{2}{3}}$ $=5^2$ $=25$	✓5^3 ✓answer (accept 25 or 5^2) (2)
1.2.2	$\left(3\sqrt{2}-12\right)\left(2\sqrt{2}+1\right)$ $=6.2+3\sqrt{2}-24\sqrt{2}-12$ $=-21\sqrt{2}$	✓ $6.2+3\sqrt{2}$ ✓ $-24\sqrt{2}-12$ ✓answer (3)
1.3.1	$3x-9=0$ $3x=9$ $x=3$	✓$3x-9=0$ ✓ answer (2)
1.3.2	$\frac{x^2-x-6}{3x-9}=\frac{(x-3)(x-2)}{3(x-3)}$ $=\frac{x-2}{3}$	✓ $(x-3)(x-2)$ ✓$3(x-3)$ ✓answer (3) **[15]**

QUESTION 2

2.1.1	$(x+2)(x-3)<-3x+2$ $x^2-x-6+3x-2<0$ $x^2+2x-8<0$ $(x+4)(x-2)<0$ $+ \quad 0 \quad - \quad 0 \quad +$ $-4 \qquad 2$ or  $-4<x<2$	✓ standard form ✓ factors ✓ $-4<x$ ✓ $x<2$ (4)
2.1.2	$x^2+2x-8<0$ $-4<x<2$ Sum of integers $=(-3)+(-2)+(-1)+(0)+(1)$ $=-5$	✓ $-4<x<2$ ✓ $-3,-2,-1,0,1$ ✓answer (3)
2.2.1	$\frac{4^{x-1}+4^{x+1}}{17.12^x}=\frac{4^x.4^{-1}+4^x.4^1}{17.3^x.4^x}$ $=\frac{4^x\left(4^{-1}+4\right)}{17.3^x.4^x}$ $=\frac{4^x\left(\frac{1}{4}+4\right)}{17.3^x.4^x}$ $=\frac{\left(\frac{17}{4}\right)}{17.3^x}$ $=\frac{1}{4}.3^{-x}$ or $\frac{1}{4.3^x}$	✓ factorise numerator ✓ $3^x.4^x$ ✓ simplification of numerator to $\frac{17}{4}$ ✓ answer (4)
2.2.2	$\frac{4^{x-1}+4^{x+1}}{17.12^x}=\frac{1}{4}.3^{-x}$ $=\frac{1}{4}.4t$ $=t$	✓answer (1)

2.3	$3^y = 81^x$ and $y = x^2 - 6x + 9$ $3^y = 3^{4x}$ $y = 4x$ $4x = x^2 - 6x + 9$ $0 = x^2 - 10x + 9$ $0 = (x-9)(x-1)$ $x = 9$ or 1 $y = 4(9)$ or $4(1)$ $= 36$ or 4 $(x;y) = (9\,;\,36)$ or $(1\,;\,4)$	✓ $3^y = 3^{4x}$ ✓ $y = 4x$ ✓ $4x = x^2 - 6x + 9$ ✓standard form ✓factors ✓x-values ✓y-values (7) **[19]**

QUESTION 3

3.1.1	$4 - 8p = 0$ $-8p = -4$ $p = \frac{1}{2}$	✓ $4 - 8p = 0$ ✓ answer (2)
3.1.2	$4 - 8p < 0$ $p > \frac{1}{2}$	✓ $4 - 8p < 0$ ✓ answer (2)
3.2.1	$\sqrt{5-x} = x + 1$ $5 - x \geq 0$ and $x + 1 \geq 0$ $x \leq 5$ and $x \geq -1$ Hence $-1 \leq x \leq 5$	✓ $5 - x \geq 0$ ✓ $x + 1 \geq 0$ ✓and (3)
3.2.2	$5 - x = x^2 + 2x + 1$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ Since $-1 \leq x \leq 5$, $x = 1$ only	✓ square both sides ✓ standard form ✓ factors ✓ answers ✓ selection of 1 (5)
3.2.3	$x = -4$	✓ answer (1) **[13]**

QUESTION 4

4.1	$A = P(1-in)$ $= 145000[1-(0,17)(5)]$ $= \text{R } 21\ 750$	✓ substitution ✓ answer (2)
4.2.1	$\frac{8\%}{4} = 2\%$ per quarter	✓ answer (1)
4.2.2	$A = P(1+i)^n$ After 1 year, $A = P(1+i_{eff})^1$ and $A = P(1+0.02)^4$ Hence $1+i_{eff} = (1+0.02)^4$ $i_{eff} = (1+0.02)^4 - 1$ $= 0,0824$ The effective interest rate is 8,24% p.a.	✓ $1+i_{eff} = (1+0,02)^4$ ✓ answer (2)
4.3	$A = 14000\left(1+\frac{0,09}{2}\right)^3\left(1+\frac{0,075}{12}\right)^{42}$ $= \text{R } 20\ 755,08$	✓ $\frac{0,09}{2}$ ✓ $14000\left(1+\frac{0,09}{2}\right)^3$ ✓ $\frac{0,07}{12}$ ✓ 42 ✓ answer (5) **[10]**

QUESTION 5

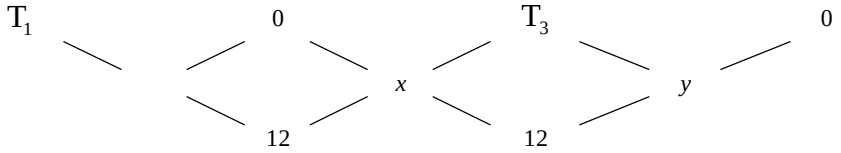
5.1	R 15 000	✓ answer (1)
5.2	Simple interest	✓ answer (1)
5.3	$A = P(1+in)$ $31 = 15(1+6i)$ $\frac{31}{15} = 1+6i$ $i = \left(\frac{31}{15}-1\right) \div 6$ $= \frac{8}{45}$ $= 0{,}1778$ $= 17{,}78\%$	✓ substitution of (6 ; 31) into correct formula ✓answer (2)
5.4	$A = P(1+in)$ $w = 15(1+0{,}1778\times 12)$ $= 47$ $A = P(1+i)^n$ $47 = 15(1+i)^{12}$ $\sqrt[12]{\frac{47}{15}} = 1+i$ $i = \sqrt[12]{\frac{47}{15}} - 1 = 0{,}09985 = 9{,}99\%$	✓w = 47 ✓ substitutes (12 ; w) ✓ $\sqrt[12]{\frac{47}{15}}$ ✓answer (4) **[8]**

QUESTION 6

6.1.1	Multiply $\frac{1}{8}$ by $\frac{1}{2}$	✓ multiply $\frac{1}{8}$ ✓ $\frac{1}{2}$ (2)
6.1.2	$T_n = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$ **OR** $T_n = \left(\frac{1}{2}\right)^{n}$ **OR** $T_n = 2^{-n}$	✓ $a = \frac{1}{2}$ ✓ $\left(\frac{1}{2}\right)^{n-1}$ (2) ✓✓ answer (2) ✓✓ answer (2)
6.1.3	Continuing the pattern: $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{64}; \frac{1}{128}; \frac{1}{256}; \frac{1}{512}; \frac{1}{1024}$ Hence $n = 10$ **OR** $\frac{1}{2^n} = \frac{1}{1024}$ $2^{-n} = 2^{-10}$ $n = 10$	✓ expand sequence ✓n = 10 (2) ✓ $\frac{1}{2^n} = \frac{1}{1024}$ ✓n = 10 (2)
6.2.1	124	✓answer (1)
6.2.2	$T_n = -8n + 164$	✓ $-8n$ ✓+164 (2)
6.2.3	$-8n + 164 < 0$ $164 < 8n$ $20{,}5 < n$ Hence T_{21} is the first term to be negative.	✓ $-8n + 164 < 0$ ✓ $20{,}5 < n$ ✓answer (3)

6.2.4	$2a = -8$ $a = -4$ $3a + b = 156$ $3(-4) + b = 156$ $b = 168$ $T_5 = -24$ $-4(5)^2 + 168(5) + c = -24$ $c = -764$ $T_n = -4n^2 + 168n - 764$	✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ substitution ✓ $c = -764$ (5)
	OR	
	$T_5 = -24$ (given) $T_6 = -24 + 124$ $= 100$ $T_n = -4n^2 + bn + c$ $-24 = -4(5)^2 + b(5) + c$ $76 = 5b + c$ ... (1) $100 = -4(6)^2 + b(6) + c$ $244 = 6b + c$ ... (2) $168 = b$ ... (2) – (1) $c = -764$	✓ $T_6 = 100$ ✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ $c = -764$ (5)
	OR	
	$T_5 = -24$ (given) $T_4 = -24 - 132$ $= -156$ $T_n = -4n^2 + bn + c$ $-24 = -4(5)^2 + b(5) + c$ $76 = 5b + c$ ... (1) $-156 = -4(4)^2 + b(4) + c$ $-92 = 4b + c$ ... (2) $168 = b$ ... (1) – (2) $c = -764$	✓ $T_4 = -156$ ✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ $c = -764$ (5) **[17]**

QUESTION 7



$y = x + 12$

$T_3 = 0 + x = x$ **AND** $T_3 + y = 0$

$y = -x$

AND $y = x + 12$

Hence $-x = x + 12$

$-2x = 12$

$x = -6$

✓introduce variables

✓ $T_3 = x$

✓ $y = -x$

✓ $y = x + 12$

✓ $-x = x + 12$

✓answer

[6]

OR

2a = 12

a = 6 $T_n = 6n^2 + bn + c$

$n = 2$ and $n = 4$:

$24 + 2b + c = 0$

$96 + 4b + c = 0$

$72 + 2b = 0$

$2b = -72$

$b = -36$

$24 - 72 + c = 0$

$c = 48$

$T_n = 6n^2 - 36n + 48$

$T_3 = 6(3)^2 - 36(3) + 48$

$= 102 - 108$

$= -6$

✓ $a = 6$

✓ $24 + 2b + c = 0$

✓ $96 + 4b + c = 0$

✓ $b = -36$

✓ $c = 48$

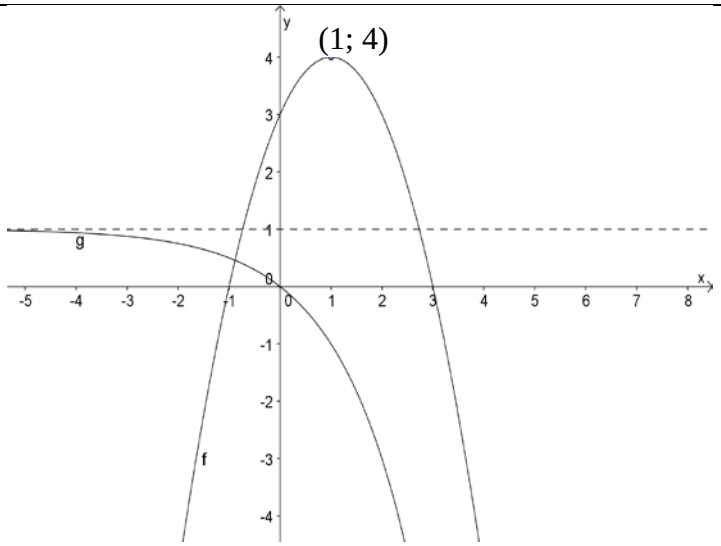
✓answer

[6]

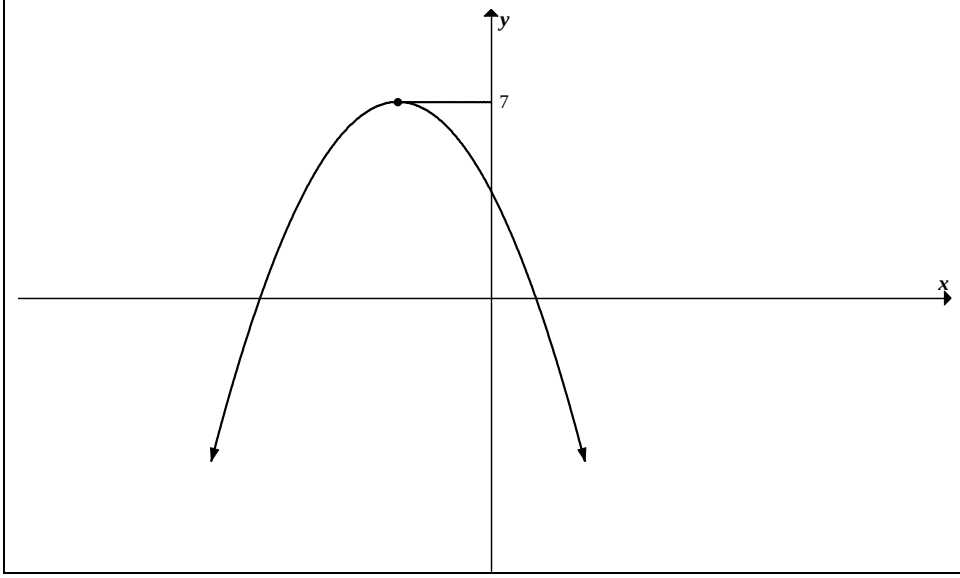
QUESTION 8

8.1	$x = 3$ $y = -1$	✓ answer ✓ answer (2)
8.2	$R\,;\, x \neq 3$ **OR** $(-\infty\,;\,3) \cup (3\,;\,\infty)$	✓ R ✓ $x \neq 3$ (2) ✓ $(-\infty\,;\,3)$ ✓ $(3\,;\,\infty)$ (2)
8.3	$d = \tan 76°$ $d = 4$ $6 = 4(3) + e$ $e = -6$ $g(x) = 4x - 6$	✓ $d = \tan 76°$ ✓ $d = 4$ ✓ $e = -6$ (3)
8.4	$\frac{2}{x-3} - 1 = 4x - 6$ $\frac{2}{x-3} = 4x - 5$ $2 = 4x(x-3) - 5(x-3)$ $2 = 4x^2 - 12x - 5x + 15$ $0 = 4x^2 - 17x + 13$ $0 = (4x - 13)(x - 1)$ $x = \frac{13}{4}$ or $x = 1$ $y = 4\left(\frac{13}{4}\right) - 6$ or $y = 4(1) - 6$ $y = 7$ or $y = -2$ Points of intersection are A $(1\,;-2)$ and C $\left(\frac{13}{4}\,;\,7\right)$	✓ equating ✓ simplification ✓ standard form ✓ factors ✓ x-values ✓ y-values (6)
8.5	$1 \leq x < 3$ or $x \geq \frac{13}{4}$ **OR** $x \in [1\,;\,3) \cup \left[\frac{13}{4}\,;\,\infty\right)$	✓ $1 \leq x$ ✓ $x < 3$ ✓ $x \geq \frac{13}{4}$ (3)
8.6	$y = (x-3) - 1$ $y = x - 4$ **OR** $y = x + c$ Substitute $(3;\,-1)$ $-1 = 3 + c$ $c = -4$ $y = x - 4$	✓ $x - 3$ ✓ -1 ✓answer (3) ✓✓ substitute $(3\,;-1)$ ✓answer (3) **[19]**

QUESTION 9

9.1	 $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3 \quad or \quad x=-1$ $x=\frac{-2}{2(-1)}=1$ $y=-(1)^2+2(1)+3=4$ $0=1-2^x$ $2^x=2^0$ $x=0$ $y=1-2^0$ $y=0$	*f:* ✓ shape ✓ *x*-int ✓ *y*-int ✓✓ turning point *g:* ✓ shape ✓ *x*-int ✓ *y*-int ✓ asymptote (9)
9.2	Average gradient $=\frac{f(0)-f(-3)}{0-(-3)}$ $=\frac{3-(-12)}{3}$ $=5$	✓ correct formula ✓ $f(-3)=-12$ ✓ answer (3)
9.3	$-1\le x\le 0$ or $x\ge 3$	✓ $-1<x$ ✓ $x<0$ ✓ $x>3$ (3)
9.4	Given: $f(x)+c=0$ has one solution/equal roots i.e. $f(x)=-c$ has one solution $\Rightarrow -c=f(1)=4$ $\Rightarrow c=-4$ **OR** *h* is *f* translated 4 units down *y*-intercept of *h* will then be at -1 $\therefore 3+c=-1$ $c=-4$	✓ $-c=f(1)$ ✓answer (2) ✓ $3+c=-1$ ✓answer (2)
9.5	(0 ; 1)	✓✓ (0 ; 1) (2)
9.6	$k(x)=1-2^{-x}$	✓ answer (1) **[20]**

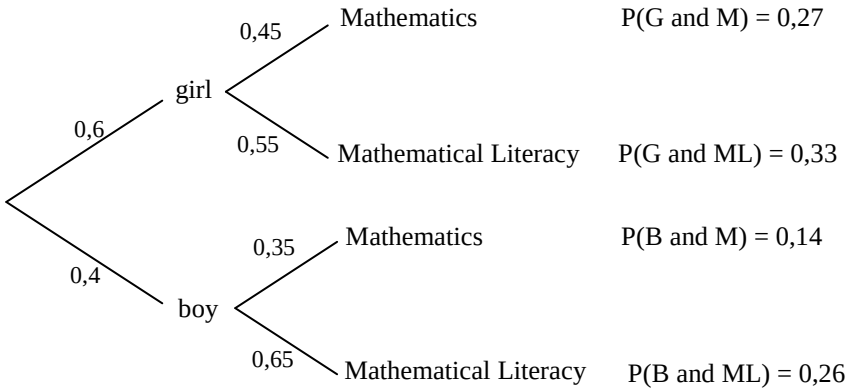
QUESTION 10

Range of $f\left(-\infty;7\right] \Rightarrow$ *y*-part of turning point [Max value of $f(x)$] is 7 a <0 and shape ∩ $b < 0 \Rightarrow b$ negative $\Rightarrow$ axis of symmetry on left of *y*-axis roots real, unequal & opposite signs $\Rightarrow$ *x*-ints on opposite sides of *y*-axis  	✓ shape ✓turning point at $y = 7$ ✓ axis of symmetry on left of *y*-axis ✓ roots are on opposite sides **[4]**

QUESTION 11

11.1	No, W and T are **not** mutually exclusive Because P(W and T) $\neq 0$ **OR** No, W and T are **not** mutually exclusive Because P(W or T) = 0,61 $\neq$ 0,75 = P(W) + P(T)	✓not mutually exclusive ✓ P(W and T) $\neq 0$ (2) ✓not mutually exclusive ✓ P(W or T) $\neq$ P(W) + P(T) (2)
11.2	P(W and T)= 0,14 (given) and P(W)×P(T)= 0,4 × 0,35 =0,14 $\Rightarrow$P(W and T) = P(W)×P(T) Therefore yes, W and T are independent events	✓ P(W)×P(T) = 0,14 ✓ P(W and T) = P(W)×P(T) ✓ conclusion (yes) (3) **[5]**

QUESTION 12

12.1.1	$a = 5$ $b = 4$ $c = 8$ $d = 1$ $e = 6$	✓ $a = 5$ ✓ $b = 4$ ✓ $c = 8$ ✓ $d = 1$ ✓ $e = 6$ (5)
12.1.2	6	✓answer (1)
12.1.3	$\frac{4}{33}$	✓ answer (1)
12.1.4	$\frac{4+3+2+a+b+c}{33} = \frac{26}{33}$	✓ answer (1)
12.2	 $P(\text{Mathematics}) = P(\text{G and M}) + P(\text{B and M})$ $= (0,6)(0,45) + (0,4)(0,35)$ $= 0,27 + 0,14$ $= 0,41$	✓0,4 ✓0,45 ✓0,35 ✓P(G and M) = 0,27 ✓P(B and M) = 0,14 ✓answer (6) **[14]**

TOTAL: 150