basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

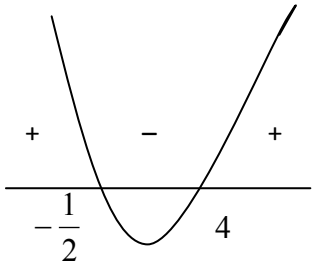
NOVEMBER 2014

MEMORANDUM

MARKS: 150

This memorandum consists of 14 pages.

QUESTION 1

1.1.1	$x = -2$ or $x = \frac{7}{3}$	✓ $x = -2$ ✓ $x = \frac{7}{3}$ (2)
1.1.2	$x^2 - 5x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{5 \pm \sqrt{25 - 4(1)(-2)}}{2}$ $x = \frac{5 \pm \sqrt{33}}{2}$ $x = 5{,}37$ or $x = -0{,}37$ **OR** $x^2 - 5x + \left(\frac{25}{4}\right) = 2 + \left(\frac{25}{4}\right)$ $\left(x - \frac{5}{2}\right)^2 = \frac{33}{4}$ $x - \frac{5}{2} = \pm\frac{\sqrt{33}}{2}$ $x = \frac{5 + \sqrt{33}}{2}$ or $x = \frac{5 - \sqrt{33}}{2}$ $x = 5{,}37$ $x = -0{,}37$	✓standard form ✓correct substitution into correct formula ✓ $x = 5{,}37$ ✓$x = -0{,}37$ (4) ✓ completing the square ✓ $\sqrt{33}$ ✓ $x = 5{,}37$ ✓$x = -0{,}37$ (4)
1.1.3	$\sqrt{x-3} = 5 + 4$ $\left(\sqrt{x-3}\right)^2 = (9)^2$ $x - 3 = 81$ $x = 84$	✓isolating $\sqrt{}$ ✓squaring both sides ✓ simplify ✓ answer (4)
1.1.4	$2x^2 - 7x - 4 \geq 0$ $(2x+1)(x-4) \geq 0$ CV's : $-\frac{1}{2}; 4$ + – + $-\frac{1}{2}$ 4 $x \leq -\frac{1}{2}$ or $x \geq 4$ **OR** $x \in (-\infty; -\frac{1}{2}] \cup [4; \infty)$	 ✓ factors ✓method ✓ notation ✓ critical values (4) ✓ notation ✓ critical values

1.2	$x = 2y + 1$ .....(1) $x^2 - 2y + 3xy = 6$ .....(2) $(2y+1)^2 - 2y + 3y(2y+1) = 6$ $4y^2 + 4y + 1 - 2y + 6y^2 + 3y - 6 = 0$ $10y^2 + 5y - 5 = 0$ $2y^2 + y - 1 = 0$ $(2y-1)(y+1) = 0$ $y = \frac{1}{2}$ or $y = -1$ $x = 2$ $x = -1$ **OR** $y = \frac{x-1}{2}$ $x^2 - 2\left(\frac{x-1}{2}\right) + 3x\left(\frac{x-1}{2}\right) = 6$ $2x^2 - 2x + 2 + 3x^2 - 3x - 12 = 0$ $5x^2 - 5x - 10 = 0$ $x^2 - x - 2 = 0$ $(x+1)(x-2) = 0$ $x = -1$ or $x = 2$ $y = -1$ $y = \frac{1}{2}$	✓ substitution of $x = 2y + 1$ ✓ simplification ✓ standard form ✓ factors ✓ both y values ✓ both x values (6) ✓ substitution of $y = \frac{x-1}{2}$ ✓ simplification ✓ standard form ✓ factors ✓ both x values ✓ both y values (6) **[20]**

QUESTION 2

2.1	$\frac{3^x(3-3^{-1})}{2.3^x}$ $=\frac{3-\frac{1}{3}}{2}$ $=\frac{8}{3}\times\frac{1}{2}$ $=\frac{4}{3}$ **OR** $\frac{3^{x-1}(3^2-1)}{2.3^x}$ $=\frac{3^x.3^{-1}(8)}{2.3^x}$ $=\frac{1}{3}\times 4$ $=\frac{4}{3}$	✓ common factor 3^x ✓ $3-3^{-1}$ ✓ answer (3) ✓ common factor 3^{x-1} ✓simplification ✓ answer (3)
2.2	$(x-2)^{-\frac{3}{2}}=64$ $x-2=\left[\left(4^3\right)\right]^{\frac{-2}{3}}$ $x-2=4^{-2}$ $x=2+\frac{1}{16}$ $\therefore x=2\frac{1}{16}$ **OR** $\sqrt{(x-3)^{-3}}=64$ $(x-3)^{-3}=4096$ $(x-2)^3=\frac{1}{4096}$ $x-2=\frac{1}{16}$ $x=2\frac{1}{16}$	✓applying exp. law ✓ 4^3 ✓simplifying ✓answer (4) ✓ squaring ✓ applying exp. law ✓ simplification ✓ answer (4)

2.3	$\frac{x.x^{\frac{1}{2}}.x^{\frac{1}{4}}.x^{\frac{1}{8}}}{\sqrt[8]{x^7}}$ $= \frac{x^{\frac{7}{8}}}{x^{\frac{7}{8}}}$ $= x$	✓applying surd law ✓applying surd law ✓simplifying ✓answer (4) **[11]**

QUESTION 3

3	$AC.(x-2) = x^2 + 2x - 8$ $AC.(x-2) = (x+4)(x-2)$ $AC = (x+4)$ cm $\therefore FD = (x+4)$ cm $\therefore ED = x + 4 - (x-2)$ $ED = 6$ cm	✓ statement ✓ factors ✓ $AC = (x+4)$ cm ✓ method ✓ answer (6) **[6]**

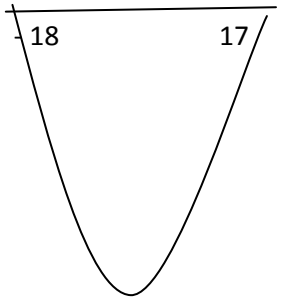
QUESTION 4

4.1	$-7 \quad 0 \quad 9 \quad 20$ $7 \quad 9 \quad 11$ $2 \quad 2$ $2a = 2$ $a = 1$ $3(1) + b = 7$ $b = 4$ $(1) + (4) + c = -7$ $c = -12$ $\therefore T_n = n^2 + 4n - 12$ **OR** $2a = 2$ $a = 1$ $T_2 = 2^2 + b(2) + c = 0$ $2b + c = -4 \quad (1)$ $T_3 = 3^2 + b(3) + c = 9$ $3b + c = 0 \quad (2)$ $(2) - (1) \quad b = 4$ $\therefore c = -4 - 2(4) = -12$ $T_n = n^2 + 4n - 12$	✓ $2a = 2$ ✓ a value ✓ b value ✓ c value (4) ✓ $2a = 2$ ✓ a value $3(1) + b = 7$ $b = 4$ OR $1 + a + c = -7$ $c = -12$ ✓ b value ✓ c value (4)

	OR $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}.d_2$ $= -7 + (n-1).7 + \frac{(n-1)(n-2)}{2}.2$ $= -7 + 7n - 7 + n^2 - 3n + 2$ $= n^2 + 4n - 12$	✓formula ✓✓substitution ✓simplification (4)
4.2	$n^2 + 4n - 12 = 128$ $n^2 + 4n - 140 = 0$ $(n+14)(n-10) = 0$ $n \neq -14$ or $n = 10$ invalid $\therefore n = 10$	✓ equation ✓factors ✓answers for n ✓n = 10 (choice) (4)
4.3	$-7 ; 0 ; 9 ; 20 ; \ldots$ first difference 7 9 11 second difference 2 2 $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$	✓first differences Answer only: Full Marks ✓c = 5 (3)
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ this difference will be between term 297 and term 298	✓ equating ✓297 ✓298(3) **[14]**

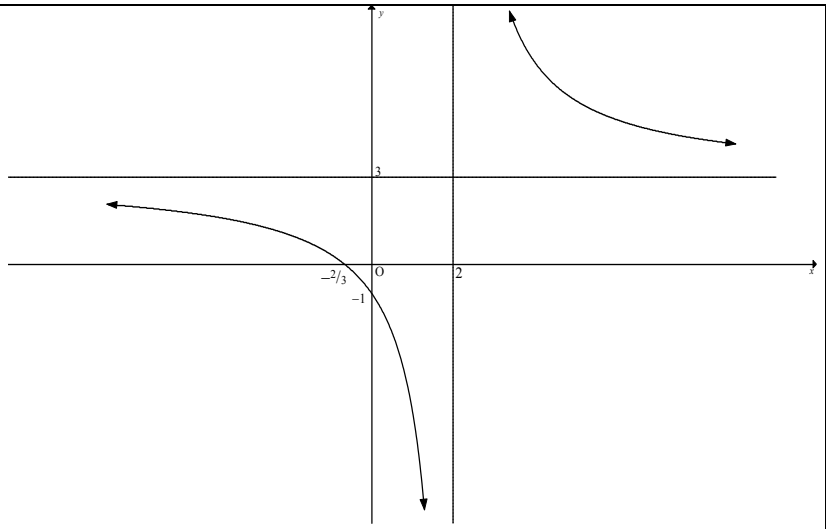
QUESTION 5

5.1	<table><tr><td>Pattern</td><td>1</td><td>2</td><td>3</td></tr><tr><td>White squares</td><td>4</td><td>12</td><td>24</td></tr></table> 40	✓✓answer(2)
5.2	$W_n = 2n^2 + 2n$ $W_{157} = 2(157)^2 + 2(157)$ $= 49612$	✓W_n ✓substitution ✓ answer (3)

5.3	$2n^2+2n+1<613$ $2n^2+2n-612<0$ $n^2+n-306<0$ $(n-17)(n+18)<0$  $\therefore n=16$	✓setting up inequality ✓factors ✓method ✓answer (4)
5.4	$P_n=4n^2+4n+1$ $=(2n)^2+2(2n)+1$ $2n$ is even for all $n\in Z$ $\therefore$ Total squares used in the n^{th} pattern is always odd. **OR** $P_n=4n^2+4n+1$ $=2(2n^2+2n)+1$ $2(2n^2+2n)$ is odd for all $n\in Z$ $2(2n^2+2n)+1$ is odd for all $n\in Z$ $\therefore$ Total squares used in the n^{th} pattern is always odd.	✓ $P_n=4n^2+4n+1$ ✓rewriting P_n ✓conclusion (3) ✓ $P_n=4n^2+4n+1$ ✓ rewriting P_n ✓ conclusion (3) **[12]**

QUESTION 6

6.1	$x=2$ $y=3$	✓ $x=2$ ✓ $y=3$ (2)
6.2	$x.\text{int}: \frac{8}{x-2}+3=0$ $8+3(x-2)=0$ $3x+2=0$ $\therefore x=-\frac{2}{3}$ $\therefore x-\text{int}\left(-\frac{2}{3};0\right)$ $y=\frac{8}{0-2}+3$ $y=-1$ $y.\text{int}: (0;-1)$	✓ $\frac{8}{x-2}+3=0$ ✓ $\left(-\frac{2}{3};0\right)$ ✓ $(0;-1)$ (3)

6.3		✓asymptotes ✓intercepts with axes ✓shape (3)
6.4	$3 = 2 + k$ $k = 1$ **OR** $y = (x-2) + 3$ $y = x + 1$ $\therefore \quad k = 1$	✓ substitute ✓ answer (2) ✓ $y = x + 1$ ✓answer (2) **[10]**

QUESTION 7

7.1	$q = -6$	✓answer (1)
7.2	$-5\frac{1}{4} = a.2^{-1-1} - 6$ $\frac{3}{4} = \frac{1}{4}a$ $a = 3$	✓substitute x ✓substitute y ✓simplifying ✓answer (4)
7.3	$x\text{int}: 2^{x-1} = 2 \quad \therefore x = 2 \ \therefore (2;0)$ $y\text{int}: y = 3.2^{-1} - 6 = -4\frac{1}{2} \quad \therefore \left(0; -4\frac{1}{2}\right)$ Average Gradient $= \frac{0 + 4\frac{1}{2}}{2 - 0}$ $= \frac{9}{4}$ or $2\frac{1}{4}$	✓ $2^{x-1} = 2$ ✓ $x = 2$ ✓$y = -4\frac{1}{2}$ ✓subst. into gradient formula ✓answer (5)
7.4	$y = 3.2^{x-3} - 6$	✓✓answer (2) **[12]**

QUESTION 8

8.1	C(– 1 ; 0)	✓ C(– 1 ; 0) (1)
8.2	$y=(x-3)(x+1)$ $y=x^2-2x-3$	✓ $(x-3)$ ✓ $(x+1)$ ✓ $y=x^2-2x-3$ (3)
8.3	TP : $y=(1)^2-2(1)-3$ $y=-4$ R: $y\in[-4;\infty)$ **OR** $y\geq -4$	✓ $y=-4$ ✓ $[-4;\infty)$ (2) ✓ $y\geq -4$
8.4	$m=\frac{0+4}{3-1}=2$ $y-0=2(x-3)$ $y=2x-6$	✓substituting into gradient formula ✓ $m=2$ ✓ equation (3)
8.5.1	$x\leq -1$ or $x\geq 3$ **OR** $x\in(-\infty;-1]\cup[3;\infty)$	✓ $x\leq -1$ ✓ $x\geq 3$ (2) ✓ $(-\infty;-1]$ ✓ $[3;\infty)$ (2)
8.5.2	$-1<x<3$ or $x>3$ **OR** $x>-1$; $x\neq 3$ **OR** $(-1;3)\cup(3;\infty)$	✓ critical values ✓ notation (2) ✓ $x>-1$ ✓ $x\neq 3$ (2) ✓ $(-1;3)$ ✓ $(3;\infty)$ (2)
8.5.3	$-1<x<0$ or $x>3$ **OR** $(-1;0)\cup(3;\infty)$	✓ critical values ✓ notation (2) ✓ $(-1;0)$ ✓ $(3;\infty)$ (2)

8.6	$x^2 - 2x - p = 0$ $\Delta = (-2)^2 - 4(1)(-p)$ $= 4 + 4p$ for non - real roots $\Delta < 0$ $4 + 4p < 0$ $4p < -4$ $\therefore p < -1$ **OR** $A(1;-4)$ $x^2 - 2x - 3 = 0$ $x^2 - 2x - p = 0$ $-p > 1$ $\therefore p < -1$	✓ $4 + 4p < 0$ ✓ $p < -1$ (2) ✓ $-p > 1$ ✓ $p < -1$ (2)
8.7	$PM = (2x - 6) - (x^2 - 2x - 3)$ $= -x^2 + 4x - 3$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(-1)} = 2$ $Max.PM = -(2)^2 + 4(2) - 3 = 1$ unit **OR** $PM = (2x - 6) - (x^2 - 2x - 3)$ $= -x^2 + 4x - 3$ $= -(x^2 - 4x + 4 - 4 + 3)$ $= -[(x - 2)^2 - 1]$ $= -(x - 2)^2 + 1$ $Max.PM = 1$ unit	✓subtraction ✓quadratic expression ✓ method ✓maximum value (4) ✓subtraction ✓quadratic expression ✓method ✓maximum value (4) **[21]**

QUESTION 9

9.1	$A = P(1-i)^n$ $11090,41 = 120000(1-i)^{12}$ $\therefore i = 1 - \sqrt[12]{\frac{11090,41}{120000}}$ Thus $i = 0,179999...$ Rate of Depreciation $= 18\%$	✓substitution ✓making i subject ✓i value as decimal ✓answer (4)

9.2	$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$ $= \left(1+\frac{0,098}{12}\right)^{12} - 1$ $= 0,10252.....$ $\text{rate} = 10,25\%$	✓formula ✓substitution into formula ✓10,25% (3)
9.3	$A = P(1+i_1)^{n_1}(1+i_2)^{n_2}$ $= 80000\left(1+\frac{0.075}{4}\right)^{16}\left(1+\frac{0,092}{12}\right)^{36}$ $= R141768,60$ **OR** $A_1 = 80000\left(1+\frac{0,075}{4}\right)^{16}$ $= 107689,1465..$ $A_2 = 107689,1465\left(1+\frac{0,092}{12}\right)^{36}$ $= \text{R}141768,60$	✓$\left(1+\frac{0.075}{4}\right)^{16}$ ✓$\left(1+\frac{0,092}{12}\right)^{36}$ ✓multiplication ✓answer (4) ✓$\left(1+\frac{0,075}{4}\right)^{16}$ ✓A_1 ✓$\left(1+\frac{0,092}{12}\right)^{36}$ ✓answer (4)
9.4.1	Investment : end of third year : $A = P(1+i)^n$ $= 30\,000\left(1+\frac{0,065}{12}\right)^{96}$ $= R50390,07$	✓$\frac{0,065}{12}$ ✓subst. into correct formula ✓answer (3)
9.4.2	T_0 T_3 T_5 T_8 30000 −10000 +10000 $A = 30000\left(1+\frac{0,65}{12}\right)^{96} - 10000\left(1+\frac{0,65}{12}\right)^{60} + 10000\left(1+\frac{0,65}{12}\right)^{36}$ $A = \text{R}48708,61$ $\therefore \text{difference} = 48708,61 - 50390,07$ $= -\text{R}1681,46$	✓$30000\left(1+\frac{0,65}{12}\right)^{96}$ ✓$-10000\left(1+\frac{0,65}{12}\right)^{60}$ ✓$10000\left(1+\frac{0,65}{12}\right)^{36}$ ✓ R48708,61 ✓subtracting ✓answer (7)

	Investment : end of third year : $A = P(1+i)^n$ $= 30\,000\left(1+\frac{0{,}065}{12}\right)^{36}$ $= \text{R}36\,440{,}14881$ Principal(new) : $R36440{,}14881 - R10000{,}00 = R26\,440{,}14881$ Investment : end of fifth year : $A = P(1+i)^n$ $= 26440.14881\left(1+\frac{0{,}065}{12}\right)^{24}$ $= \text{R}30100{,}2304$ Principal(new) : $\text{R}30100{,}2304 + \text{R}10000{,}00 = R40100{,}2304$ Investment : end of eighth year : $A = P(1+i)^n$ $= 40100{,}2304\left(1+\frac{0{,}065}{12}\right)^{24}$ $= \text{R}48708{,}61$ Tashil had a deficit of R1681,46	✓subst. into formula ✓answer ✓subst. into formula ✓answer ✓subst. into formula ✓answer ✓conclusion(7) **[21]**

QUESTION 10

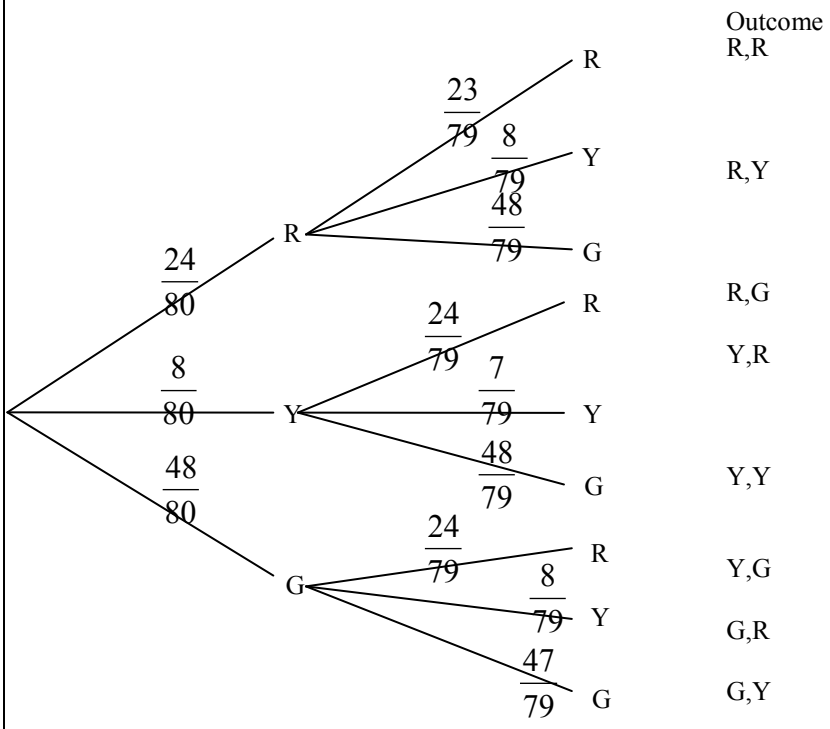
10.1	5 customers	✓answer (1)
10.2	P(C and B) $\neq 0$ Thus events B and C are not mutually exclusive	✓P(C and B) $\neq 0$ ✓ conclusion (2)
10.3.1	P(V only) = $\frac{58}{240} = \frac{29}{120}$	✓ answer (1)
10.3.2	P(C and B) = $\frac{29}{240}$	✓ answer (1)
10.3.3	P(not C) = 1 – P(C) $= 1 - \frac{122}{240} = \frac{59}{120}$ **OR** $\text{P(not C)} = \frac{52+3+58+5}{240}$ $= \frac{118}{240} = \frac{59}{120}$	✓ formula ✓ substitution ✓answer (3) ✓✓numerator and denominator ✓answer (3)

10.3.4	P(B or V) = P(B) + P(V) – P(B and V) $= \frac{84}{240} + \frac{82}{240} - \frac{15}{240}$ $= \frac{151}{240}$ **OR** $P(\text{B or V}) = \frac{17 + 52 + 12 + 3 + 9 + 58}{240}$ $= \frac{151}{240}$	✓ $\frac{84}{240}$ ✓ $\frac{82}{240}$ ✓ $\frac{15}{240}$ ✓ $\frac{151}{240}$ (4) ✓✓numerator and denominator ✓✓answer (4) **[12]**

QUESTION 11

	$P(\text{A or B}) = P(A) + P(B) - P(A \cap B)$ $0,428 = 0,12 + 0,35 - P(A \cap B)$ $P(A \cap B) = 0,042$ $P(A) \times P(B) = 0,12 \times 0,35 = 0,042$ $\therefore P(A \cap B) = P(A) \times P(B)$ Thus A and B are independent events	✓substitution ✓value of $P(A \cap B)$ ✓value of $P(A) \times P(B)$ ✓conclusion (4) **[4]**

QUESTION 12

12.1	There are 100% – 60% – 10% = 30% red marbles $\therefore \frac{30}{100} \times 80 = 24$ red marbles	✓30% ✓24 (2)
12.2		✓first branch ✓second branch ✓values on diagram (3)
12.3	P(G and Y) = P(G, Y) + P(Y, G) $= \frac{48}{80} \times \frac{8}{79} + \frac{8}{80} \times \frac{48}{79}$ $= \frac{48}{395}$	✓ multiplication rule ✓ addition ✓answer (3) **[8]**
		TOTAL: 150