basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 3 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. THREE diagram sheets for QUESTION 1.5, QUESTION 6.1, QUESTION 9, QUESTION 10, QUESTION 11.1, QUESTION 11.2 and QUESTION 12 are attached at the end of this question paper. Write your name on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

8. Number the answers correctly according to the numbering system used in this question paper.

9. Write neatly and legibly.

QUESTION 1

The 100th Tour de France took place from 29 June 2013 to 21 July 2013. The race was made up of 21 stages of varying distances. The distance, in kilometres, covered in each stage is given in the table below:

Stage	Distance	Stage	Distance	Stage	Distance
1	213	8	195	15	247
2	156	9	168	16	168
3	145	10	197	17	32
4	25	11	33	18	172
5	228	12	218	19	204
6	176	13	173	20	125
7	205	14	191	21	133

[Source: www.letour.fr.le-tour/2013/us]

1.1 Calculate the mean distance. (3)

1.2 Calculate the standard deviation of the distances. (2)

1.3 Determine the number of stages that lie beyond ONE standard deviation of the mean. (2)

1.4 The distance covered in each stage has been rearranged in ascending order and is shown below. Determine the five-number summary of this data.

25	32	33	125	133	145	156
168	168	172	173	176	191	195
197	204	205	213	218	228	247

(4)

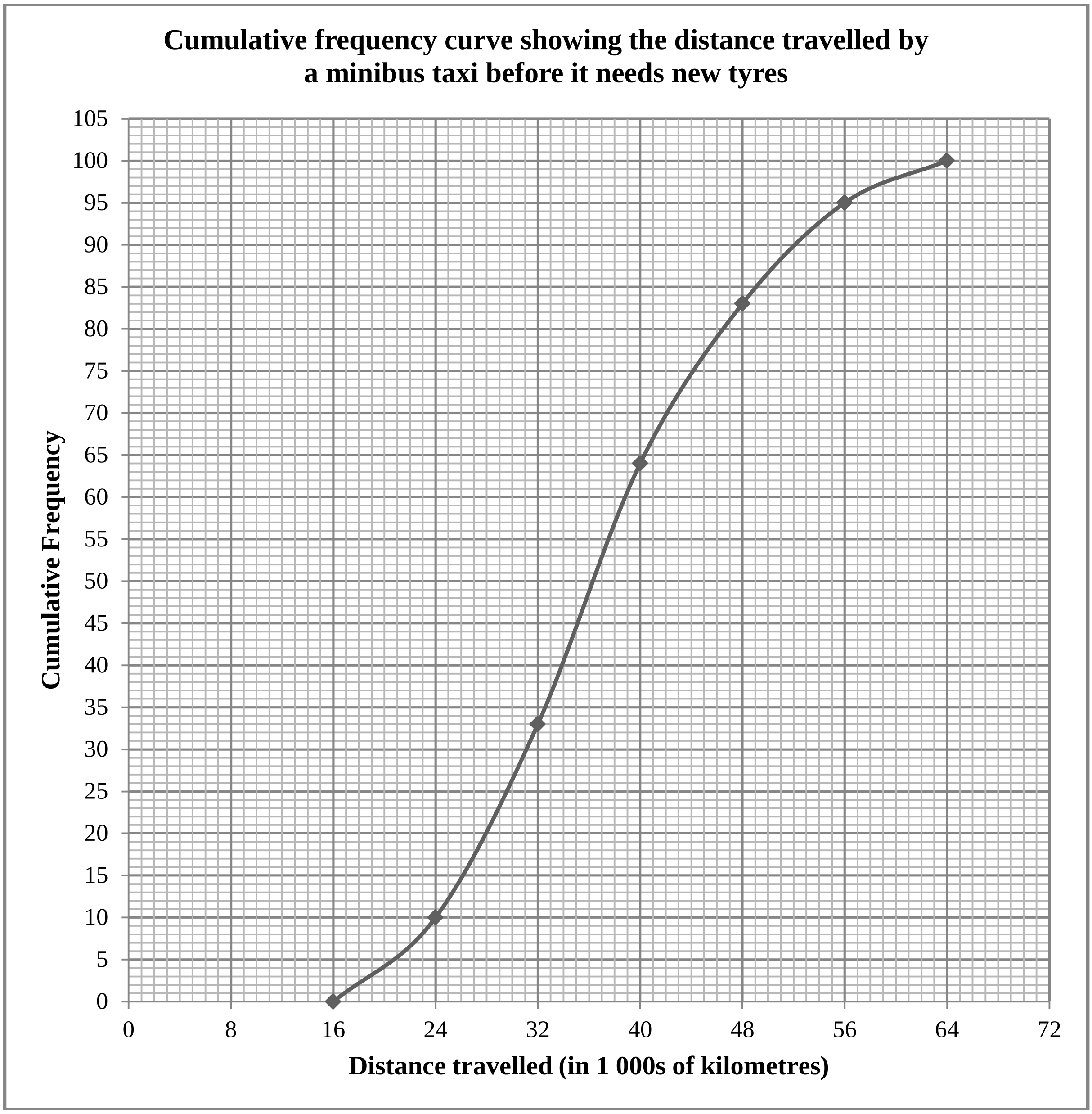
1.5 Use the scaled line provided in DIAGRAM SHEET 1 to draw a box and whisker diagram to represent the distance covered in each stage. (2)

1.6 Are there any outliers in the data set? Explain. (2)

[15]

QUESTION 2

A manufacturer recorded how far a minibus taxi travels before it needs new tyres. He recorded the distances, in 1 000s of kilometres, covered by a number of taxis that travelled the same route. This information is shown in the cumulative frequency graph (ogive) below.



2.1 How many times did they record the distance travelled by a minibus taxi before it needed new tyres? (1)

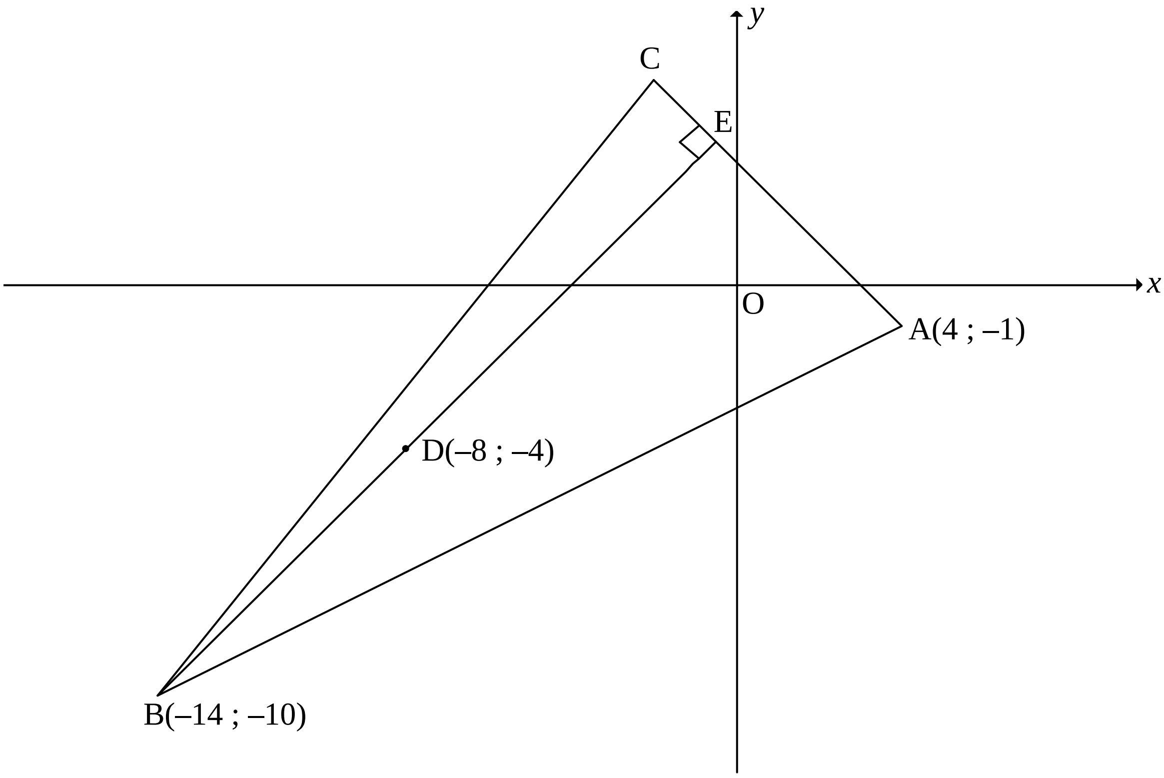
2.2 Write down the modal class of the data. (1)

2.3 Estimate the median distance travelled before new tyres are needed. (1)

2.4 Estimate the inter-quartile range for this data. (3)
[6]

QUESTION 3

In the diagram below, A(4 ; –1), B(–14 ; –10) and C are the vertices of a triangle.
E is a point on AC such that BE ⊥ AC.
The point D(–8 ; –4) lies on BE.
The equation of the line BC is $4y - 5x - 30 = 0$.



3.1 Calculate the gradient of BD. (2)

3.2 Hence, write down the gradient of AC. (1)

3.3 Determine the equation of AC in the form $y = mx + c$. (2)

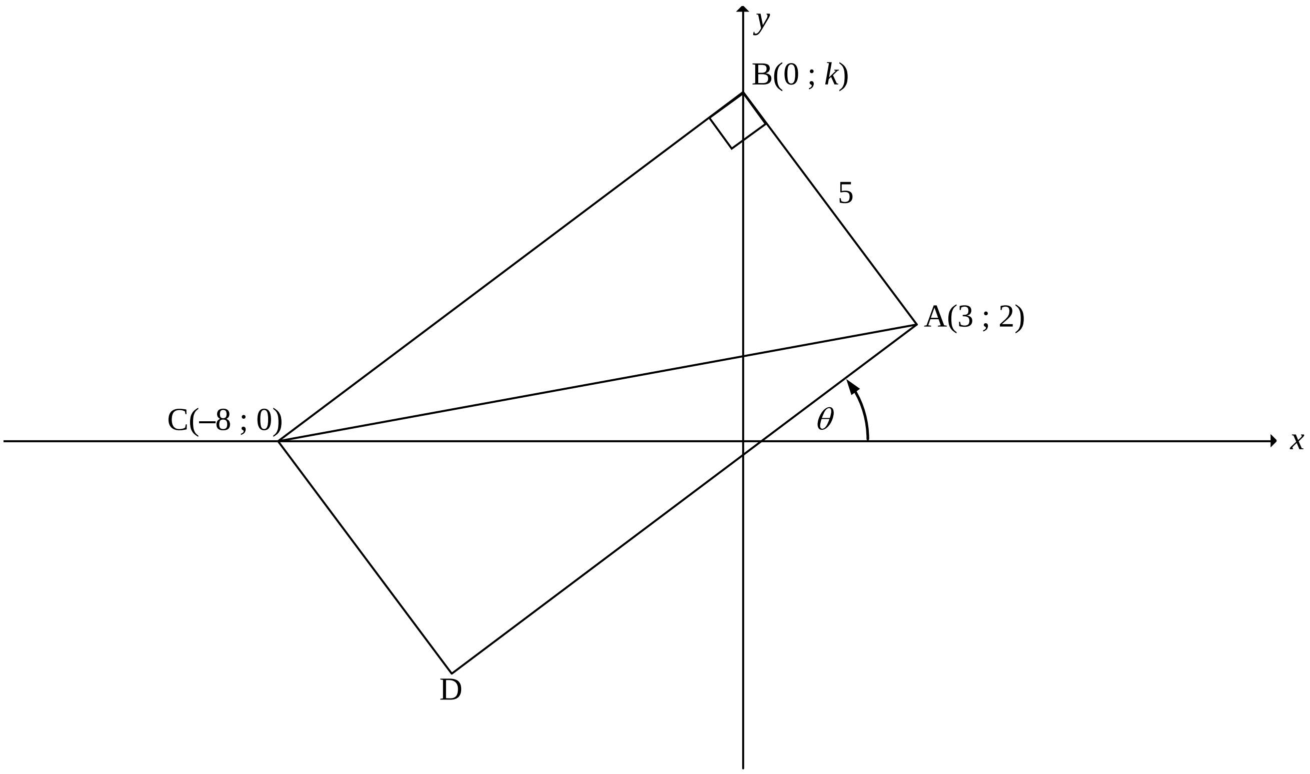
3.4 The point G(p ; –5) lies on AB. Calculate the value of p. (3)

3.5 Calculate the coordinates of C. (4)

[12]

QUESTION 4

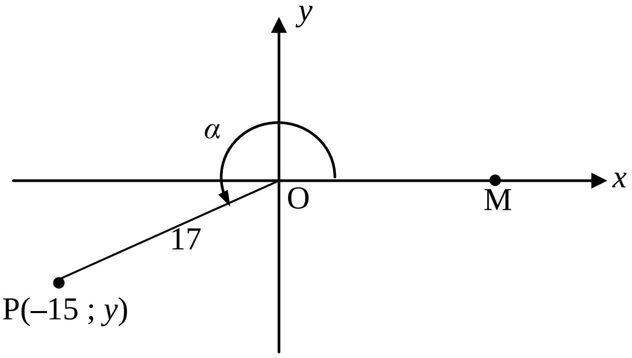
A(3 ; 2), B(0 ; k), C(–8 ; 0) and D are the vertices of a rectangle.
AB = 5 units.
The angle of inclination of AD is θ, as shown in the diagram.



4.1	Calculate the length of AC.	(2)
4.2	Calculate the value of k.	(4)
4.3	Determine the equation of BC in the form $y = mx + c$.	(3)
4.4	Calculate the size of θ.	(3)
4.5	Calculate the area of ABCD.	(3)
4.6	Calculate the size of $B\hat{A}C$.	(2)
		[17]

QUESTION 5

5.1 In the diagram, P(–15 ; y) is a point in the Cartesian plane.
OP = 17 units and reflex $\hat{MOP} = \alpha$.



Determine the value of the following **without using a calculator**:

5.1.1 y (2)

5.1.2 $\sin(90° + \alpha)$ (2)

5.1.3 $\tan\beta$, if $\alpha + \beta = 540°$ (3)

5.2 Simplify the following expression to a single trigonometric ratio:

$$\frac{\sin(180° - x) - 2\cos(90° - x)\cos x}{2\cos^2(360° + x) - \cos(-x)}$$ (6)

5.3 5.3.1 Prove that $\frac{1 - \tan x}{1 + \tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ (3)

5.3.2 For which value(s) of x in the interval $0° \leq x \leq 180°$ is the identity in QUESTION 5.3.1 undefined? (2)

5.4 Determine the general solution of the following equation:

$2\tan x = 5\sin x$ (8)

[26]

QUESTION 6

6.1 Use the system of axes provided on DIAGRAM SHEET 1 to draw the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x + 1$ for the interval $-180° \leq x \leq 180°$. Show clearly ALL intercepts with the axes, turning points and end points. (6)

6.2 Write down the period of f. (1)

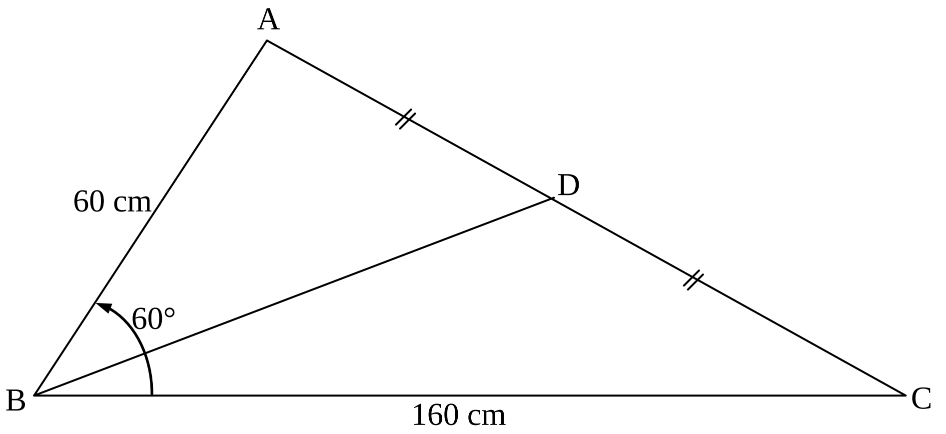
6.3 For which value(s) of x in the interval $-180° \leq x \leq 180°$ will $g(x) - f(x)$ be a maximum? (1)

6.4 The graph f is shifted $45°$ to the right to obtain a new graph h.
Write down the equation of h in its simplest form. (2)
[10]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $c^2 = a^2 + b^2 - 2ab \cos C$. (6)

7.2 In ΔABC, AB = 60 cm, BC = 160 cm and $\hat{ABC} = 60°$.
BD is the bisector of AC with D a point on AC.

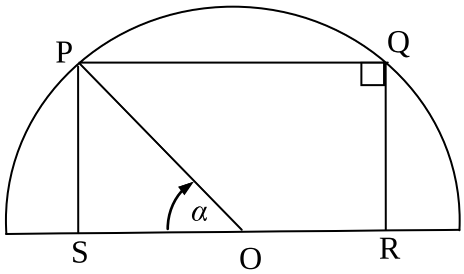


7.2.1 Calculate the length of AC. (3)

7.2.2 Determine the value of sin A. Leave the answer in its simplest surd form. (3)

7.2.3 Calculate the area of ΔABD. Give your answer correct to ONE decimal place. (3)

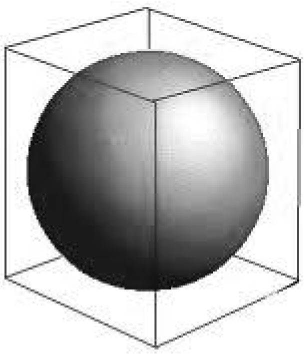
7.3 In the diagram, O is the centre of a semi-circle.
PQRS is a rectangle drawn inside the semi-circle such that O lies on RS.
$P\hat{O}S = \alpha$.

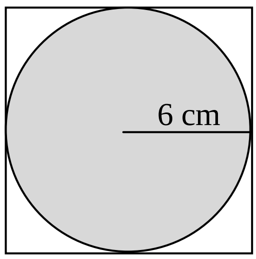


Calculate the size of α for which PQRS will be a square. (3)
[18]

QUESTION 8

A spherical glass ball is tightly packed in a box. The box is in the shape of a cube, as shown in the picture on the LEFT. The radius of the ball is 6 cm. The diagram on the RIGHT shows the cross-section of the glass ball placed in the box.



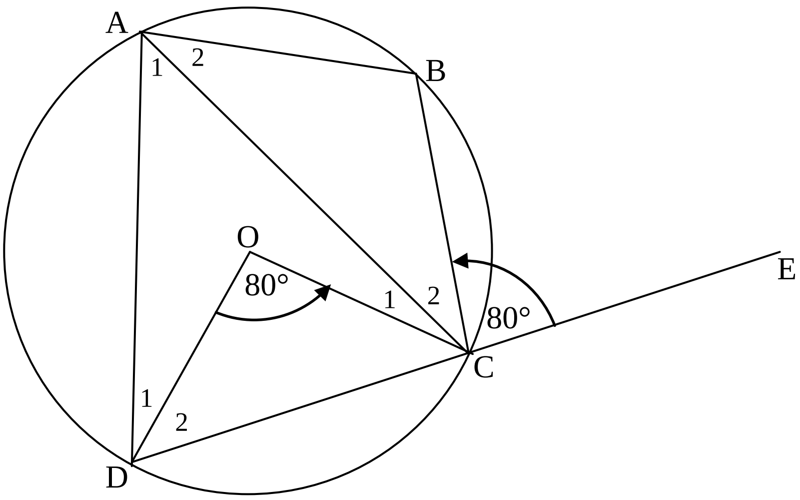


What volume of the box remains after the glass ball is placed in it? **[5]**

Give reasons for your statements in QUESTIONS 9, 10, 11 and 12.

QUESTION 9

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Chord DC is produced to E. AC is drawn. $D\hat{O}C = 80°$ and $B\hat{C}E = 80°$.



9.1 Calculate the size of the following angles:

9.1.1 $D\hat{A}C$ (2)

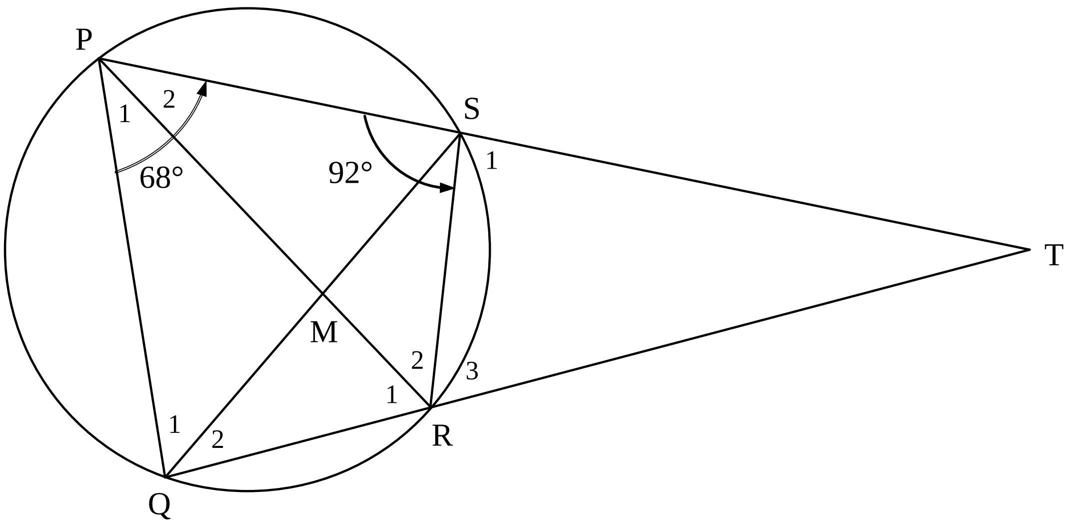
9.1.2 $D\hat{A}B$ (2)

9.1.3 $B\hat{A}C$ (1)

9.2 Hence, or otherwise, prove that DC = BC. (2)
[7]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced and meet at T. PR bisects $Q\hat{P}S$. Also, $P\hat{S}R = 92°$ and $Q\hat{P}S = 68°$.

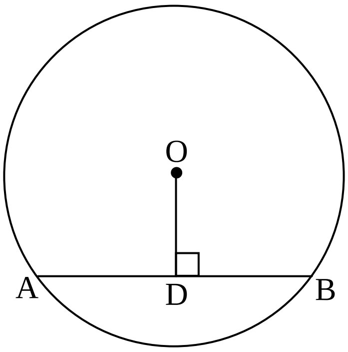


Calculate the size of the following angles:

10.1	$R\hat{P}T$	(1)
10.2	$T\hat{Q}S$	(2)
10.3	$P\hat{Q}S$	(3)
10.4	$\hat{T}$	(4)
		[10]

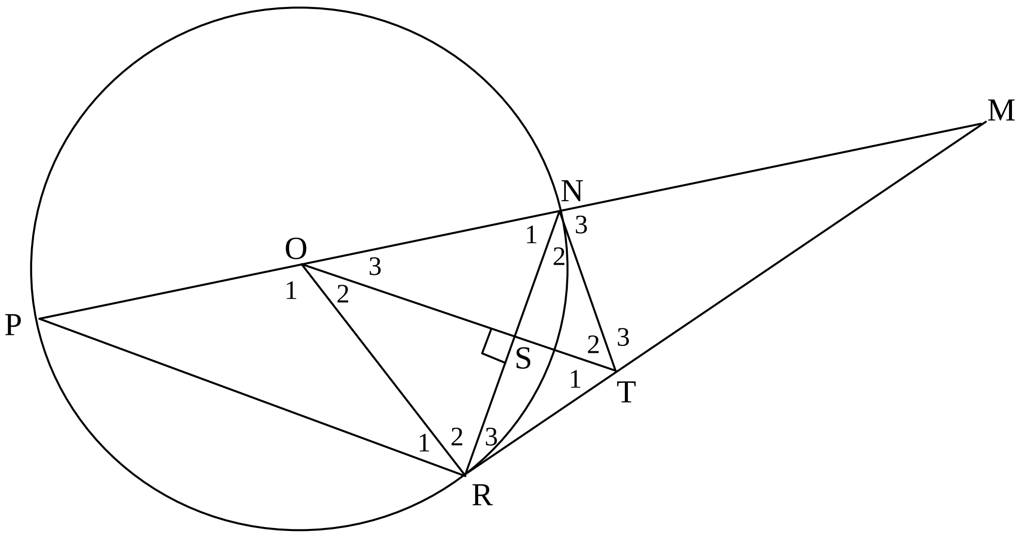
QUESTION 11

11.1 In the diagram, O is the centre of the circle and AB is a chord. D is a point on AB such that OD ⊥ AB. Use Euclidean geometry methods to prove the theorem which states that AD = DB.



(5)

11.2 In the diagram, PN is a diameter of the circle with centre O. RT is a tangent to the circle at R. RT produced and PN produced meet at M. OT is perpendicular to NR. NT and OR are drawn.



11.2.1 Prove that TO || RP. (3)

11.2.2 It is further given that $T\hat{R}N = x$. Name TWO other angles each equal to x. (3)

11.2.3 Prove that NTRO is a cyclic quadrilateral. (2)

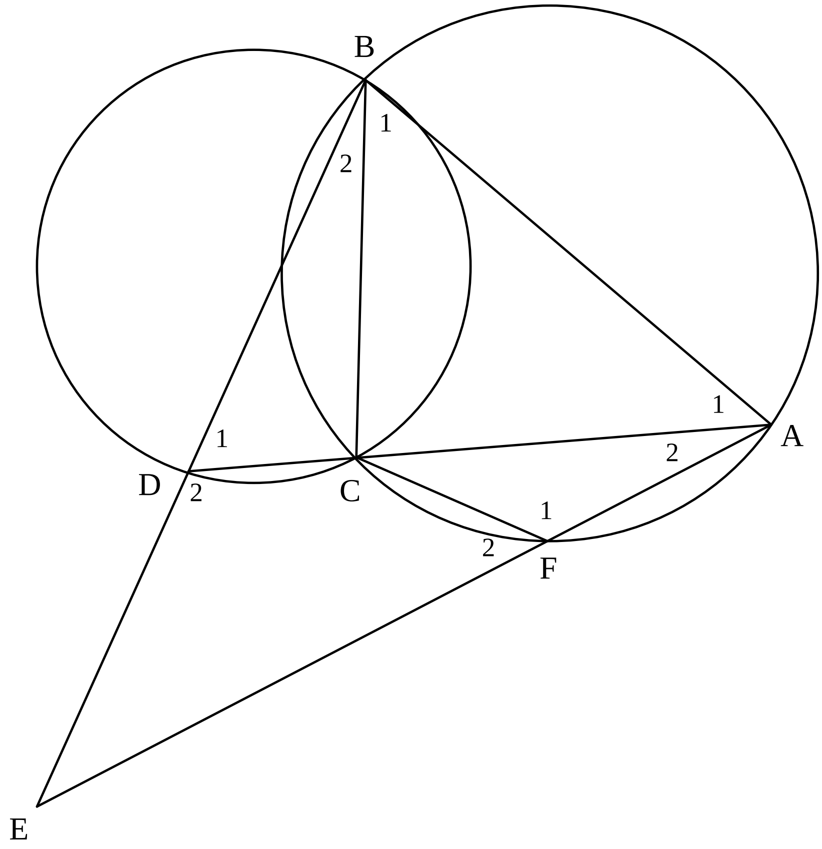
11.2.4 Calculate the size of $\hat{M}$ in terms of x. (3)

11.2.5 Show that NT is a tangent to the circle at N. (3)

[19]

QUESTION 12

In the diagram, ABCF is a cyclic quadrilateral. AB is a tangent to circle BCD at B.



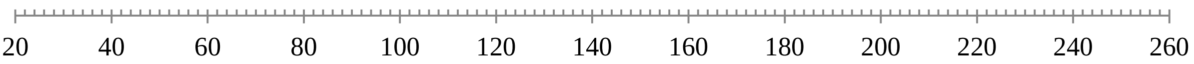
Prove that CDEF is a cyclic quadrilateral. **[5]**

TOTAL: 150

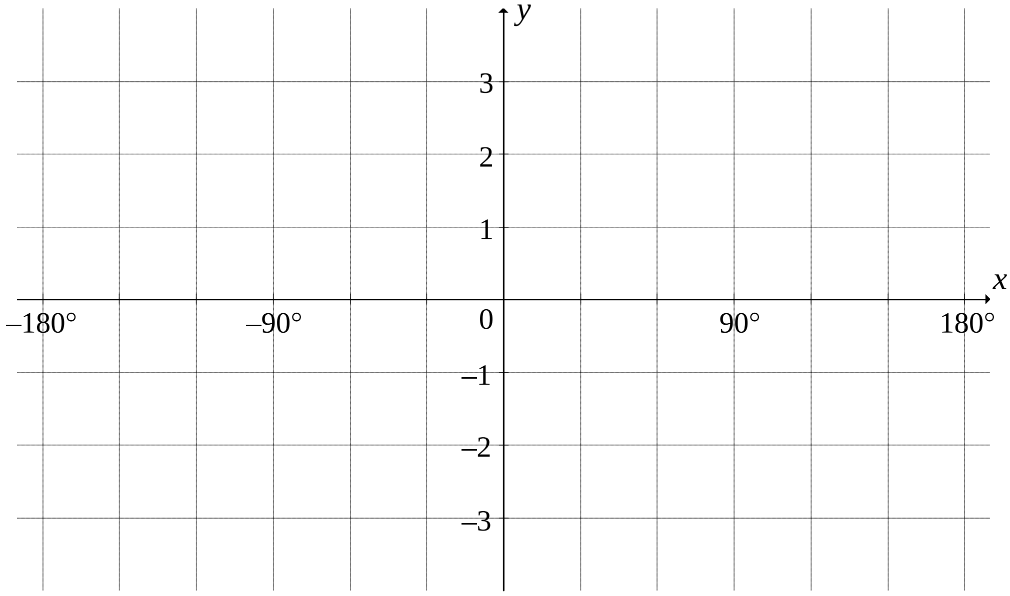
LEARNER'S NAME:

DIAGRAM SHEET 1

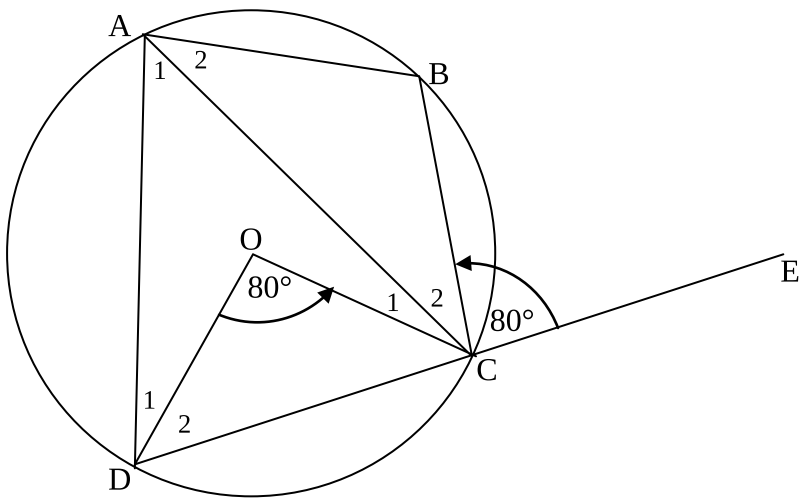
QUESTION 1.5



QUESTION 6.1



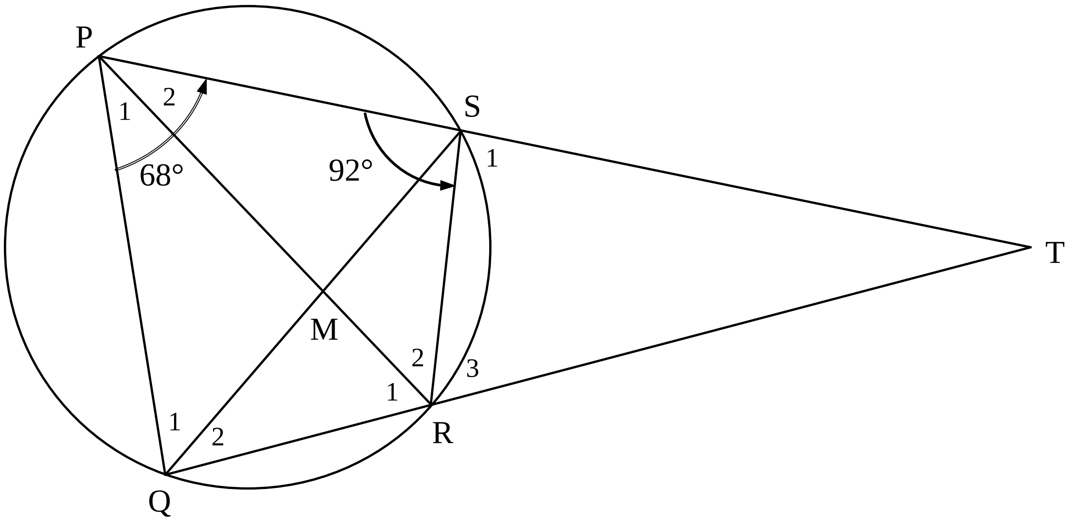
QUESTION 9



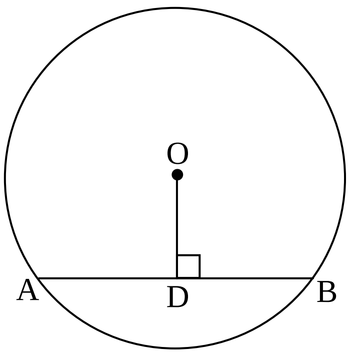
LEARNER'S NAME:

DIAGRAM SHEET 2

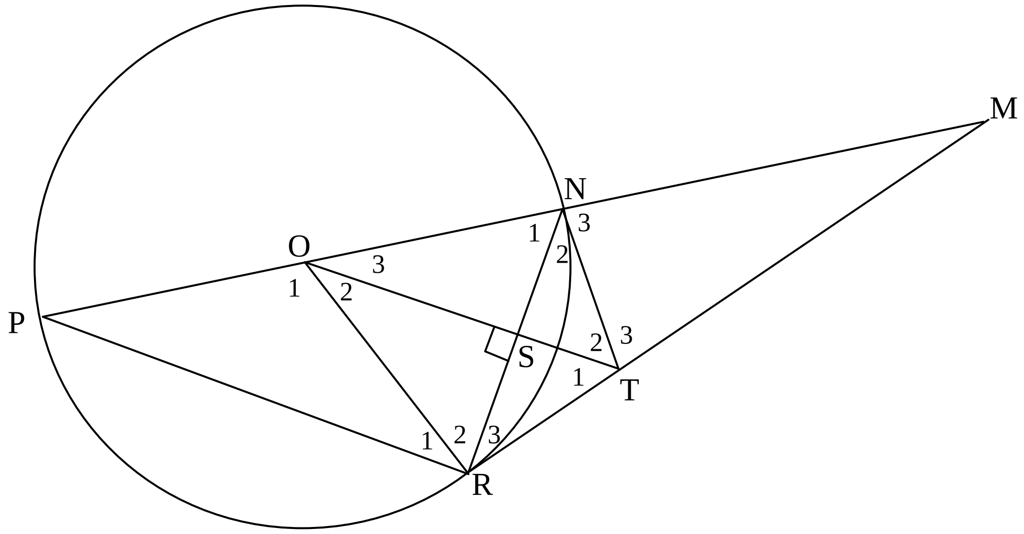
QUESTION 10



QUESTION 11.1



QUESTION 11.2



LEARNER'S NAME:

DIAGRAM SHEET 3

QUESTION 12