MATHEMATICS

MATERIAL FOR GRADE 12

Trigonometry

QUESTIONS

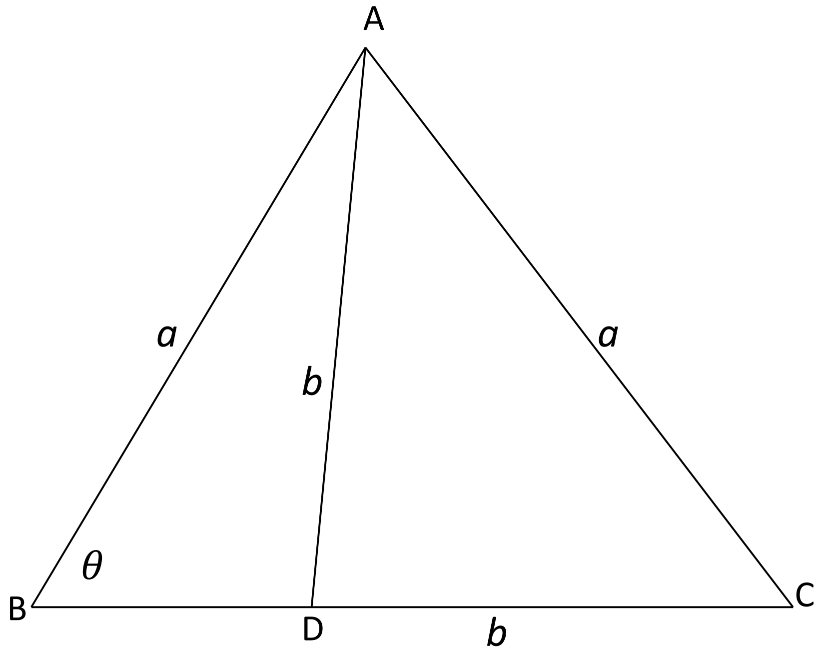
QUESTION 1

In the diagram below, ABC is an isosceles triangle. D lies on BC.

AB= $AC = a\ units$

AD= $DC = b\ units$

$\hat{B} = \theta$.



1.1 Determine, **without** reasons, the size of $A\hat{D}C$ in terms of θ. (2)

1.2 Prove that:

$$\cos 2\theta = \frac{a^2}{2b^2} - 1$$ (4)

1.3 Hence, determine the value of θ if $a = 3$ and $b = 2$

(Rounded off to two decimal digits.) (3)

[9]

QUESTION 2

Simplify the following **without** using a calculator.

2.1 $\cos 56° \cos 26° + \cos 146° \sin(-26°)$ (4)

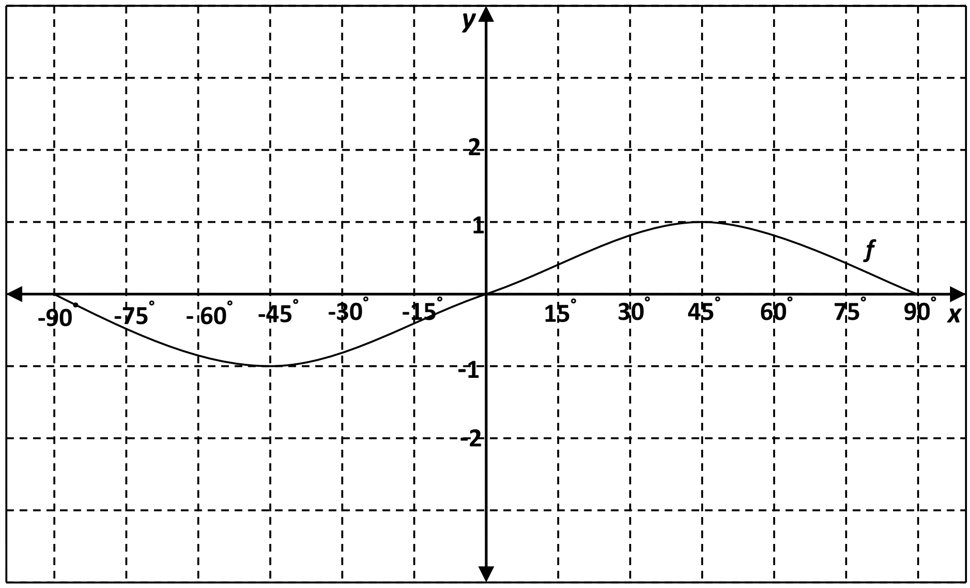
2.2 $$\frac{\tan(180° + x)\cos(360° - x)}{\sin(x - 180°)\cos(90° + x) + \cos(720° + x)\cos(-x)}$$ (6)

2.3 Prove the identity : $\dfrac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} = \dfrac{1}{\cos^2 x}$ (5)

[15]

QUESTION 3

Consider the function $f(x) = \sin 2x$ for $x \in [-90°; \ 90°]$



3.1 Write down the period of *f*. (1)

3.2 Sketch the graph of $g(x) = \cos(x - 15°)$ *for* $x \in [-90° \ ; \ 90°]$ on the diagram sheet provided for this sub-question. (5)

3.3 Solve the equation: $\sin 2x = \cos(x - 15°)$ *for* $x \in [-90° \ ; \ 90°]$ (7)

3.4 Find the values of x for which $f(x) < g(x)$. (3)

[16]

QUESTION 4

4.1.1 Simplify the following expression to a single trigonometric function:

$$\frac{2\sin(180°+x)\sin(90°+x)}{cos^4x - sin^4x}$$ (5)

4.1.2 For which value(s) of x, $x \in [0°; 360°]$ is the expression in 4.1 undefined? (3)

4.2 Evaluate, without using a calculator: $\frac{cos347°.\ sin193°}{tan315°\ .\ cos64°}$ (5)

4.3 Prove the following identity: $\frac{cos3x}{cosx} = 2\cos 2x - 1$ (5)

[18]

QUESTION 5

The graphs of $f(x) = -2\cos x$ and $g(x) = \sin(x + 30°)$ for $x \in [-90°; 180°]$ are drawn in the diagram below.

5.1 Determine the period of g. (1)

5.2 Calculate the x-coordinates of P and Q, the points where f and g intersect. (7)

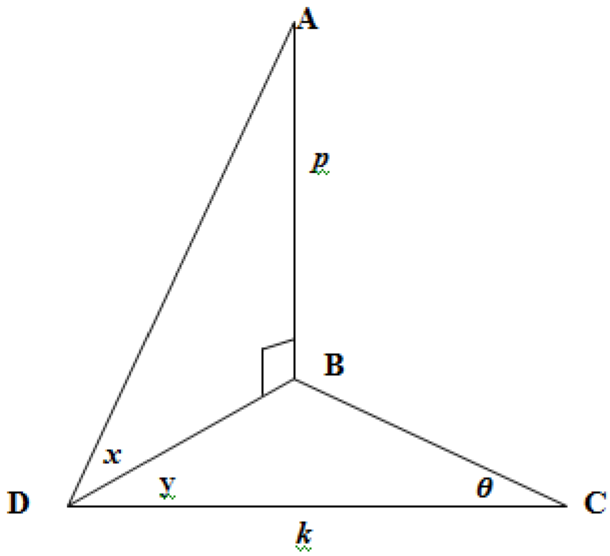
5.3 Determine the x-values, $x \in [-90°; 180°]$, for which:

5.3.1 $g(x) \leq f(x)$ (3)

5.3.2 $f'(x).g(x) > 0$ **[14]**

QUESTION 6

AB is a vertical tower of p units high.
D and C are in the same horizontal plane as B, the foot of the tower.
The angle of elevation of A from D is x. $B\widehat{D}C = y$ and $D\hat{C}B = \theta$.
The distance between D and C is k units.



6.1.1 Express p in terms of DB and x. (2)

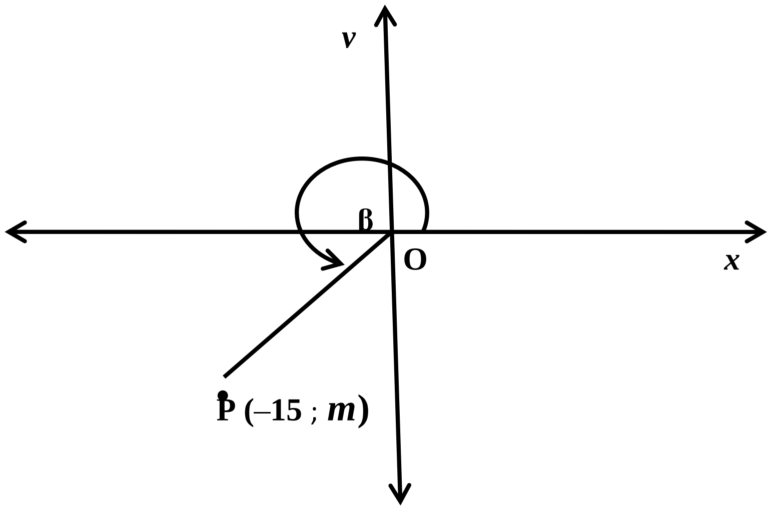
6.1.2 Hence prove that: $p = \dfrac{k\sin\theta\tan x}{\sin y\cos\theta + \cos y\sin\theta}$ (5)

6.2 Find BC to the nearest meter if $x = 51{,}7°$, $y = 62{,}5°$, $p = 80\ m$ and $k = 95\ m$. (4)

[11]

QUESTION 7

In the diagram below, P (–15; *m*) is a point in the third quadrant and 17cos β + 15 = 0.



7.1 WITHOUT USING A CALCULATOR, determine the value of the following:

7.1.1 m (3)

7.1.2 $\sin\beta + \tan\beta$ (3)

7.1.3 $\cos 2\beta$ (3)

7.2 Simplify:

$$\frac{\sin(180^\circ - x).\cos(x-180^\circ).\tan(360^\circ - x)}{\sin(-x).\cos(450^\circ + x)}$$ (7)

7.3 Consider the identity: $\frac{\sin x + \sin 2x}{1+\cos x + \cos 2x} = \tan x$

7.3.1 Prove the identity. (5)

7.3.2 Determine the values of x for which this identity is undefined. (4)

[25]

QUESTION 8

Consider: $f(x) = \cos 2x$ and $g(x) = \sin(x - 60°)$

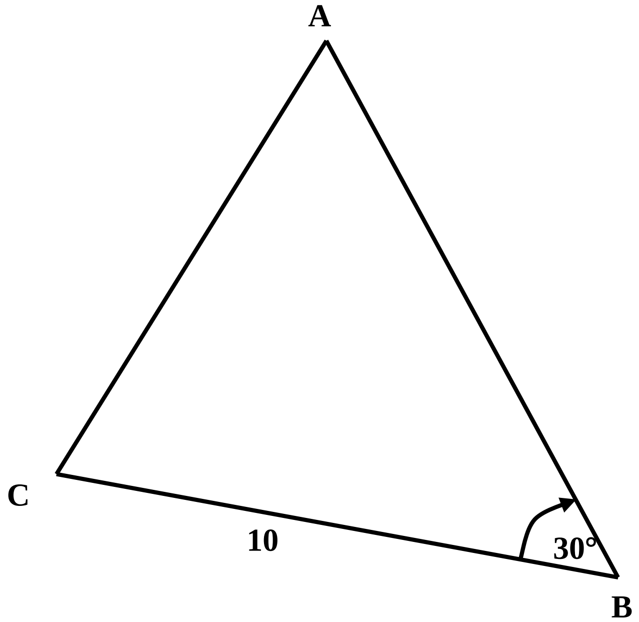
8.1 Use the grid provided to sketch the graphs of f and g for $x \in [-90° ; 180°]$ on the same set of axes. Show clearly all the intercepts on the axes and the coordinates of the turning points. (6)

8.2 Use your graphs to determine the value(s) of x for which $g(x) > 0$. (3)

[9]

QUESTION 9

In the diagram, ΔABC is given with BC = 10 units, $\hat{B} = 30°$ and $\sin(B + C) = 0,8$.



Determine the length of AC, WITHOUT USING A CALCULATOR. **[5]**

QUESTION 10

10.1 If sin 31° = *p*, determine the following, without using a calculator, in terms of *p*:

10.1.1 sin 149° (2)

10.1.2 cos (–59°) (2)

10.1.3 cos 62° (2)

10.2 Simplify the following expression to a single trigonometric ratio:

$$\tan(180° - \theta).\sin^2(90° + \theta) + \cos(\theta - 180°).\sin\theta$$ (6)

10.3 Consider: $\dfrac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

10.3.1 Prove the identity. (5)

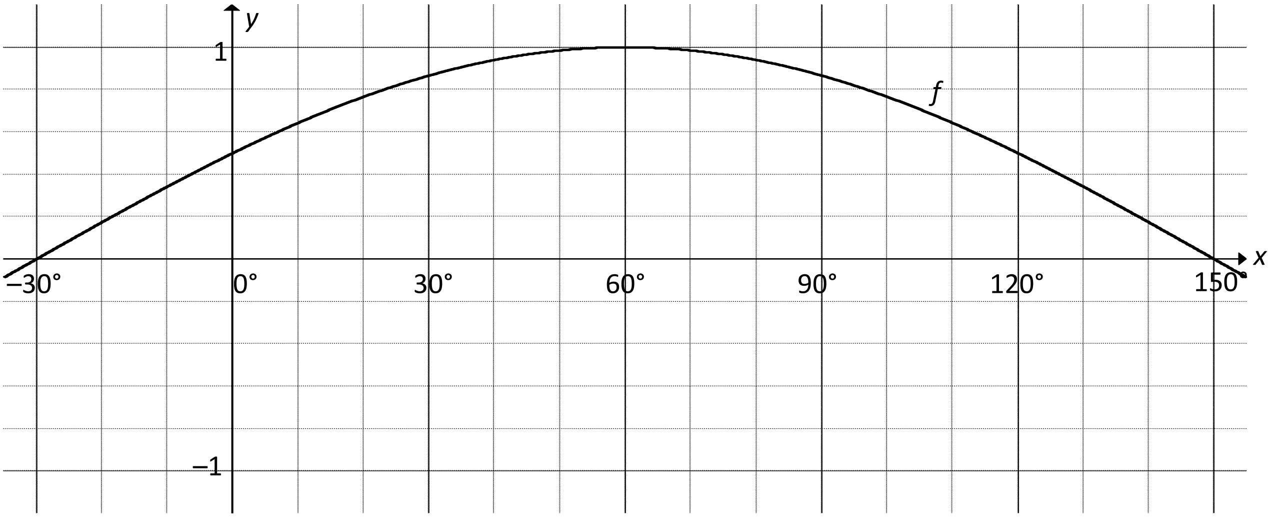
10.3.2 Determine the values of *x*, where $x \in [180° ; 360°]$, for which the above identity will be invalid. (2)

[19]

QUESTION 11

11.1 Determine the general solution of : $\sin(x + 30°) = \cos 3x$. (6)

11.2 In the diagram below, the graph of $f(x) = \sin(x + 30°)$ is drawn for the interval $x \in [-30° ; 150°]$.



11.2.1 On the same system of axes sketch the graph of g, where $g(x) = \cos 3x$, for the interval $x \in [-30° ; 150°]$. (3)

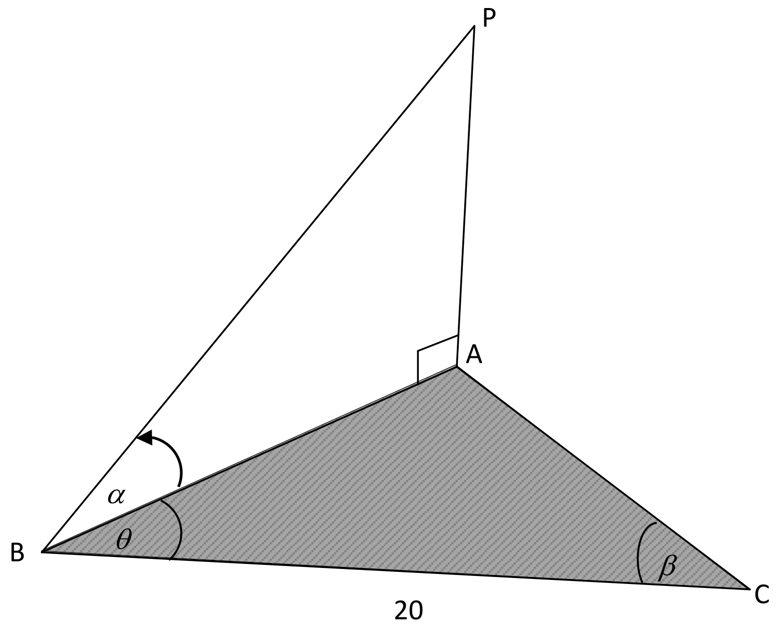
11.2.2 Write down the period of g. (1)

11.2.3 For which values of x will $f(x) \geq g(x)$ in the interval $x \in (-30° ; 150°)$? (3)

[13]

QUESTION 12

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is α.
$A\hat{C}B = \beta$, $A\hat{B}C = \theta$ and BC = 20 units.



12.1 Write AP in terms of AB and α. (2)

12.2 Prove that $AP = \dfrac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$ (3)

12.3 Given that AB = AC, determine AP in terms of α and β in its simplest form. (3)
[8]

QUESTION 13

13.1 If $90^0 < A < 360^0$ and $\tan A = \frac{2}{3}$, determine without the use of a calculator.

13.1.1 $\sin A$ (3)

13.1.2 $\cos 2A - \sin 2A$ (4)

13.2 Given that $\sin x = t$, express the following in terms of t, without the use of calculator.

13.2.1 $\cos (x - 90^0)$ (2)

13.2.2 $\sin 2x$ (3)

[12]

QUESTION 14

14.1 Calculate without the use of a calculator: $\dfrac{\cos^2 208°}{\tan 118°.\sin 124°}$ (6)

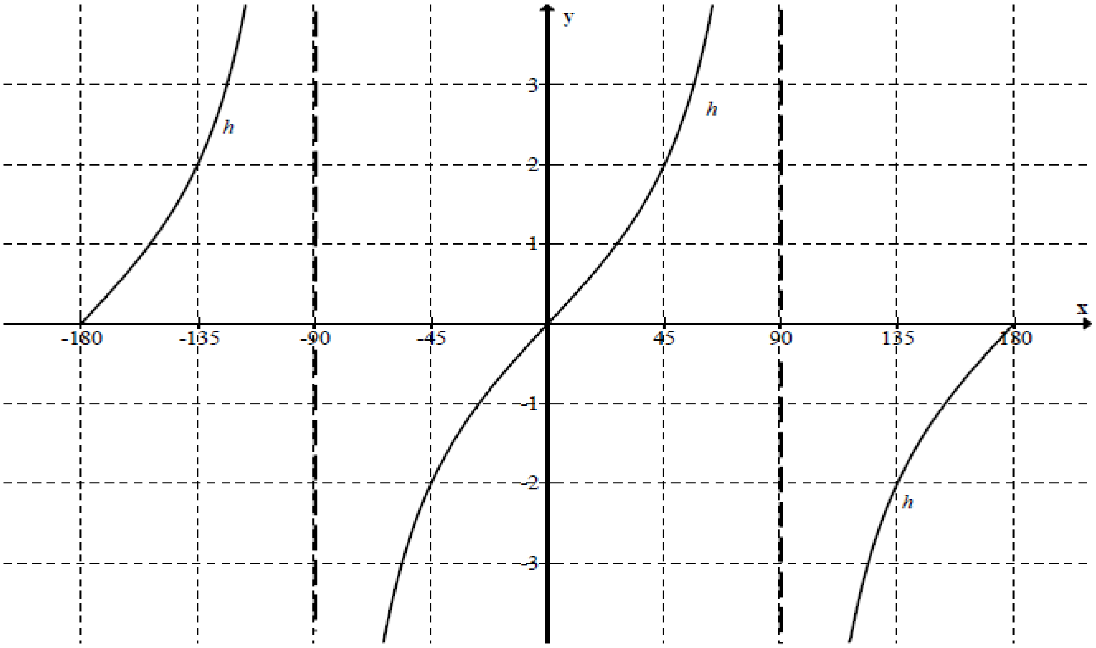
14.2 Calculate the general solution of θ where $\sin\theta \neq 0$ and

$1 - \cos 2\theta = 8\sin\theta.\sin 2\theta$ (6)

[12]

QUESTION 15

The graph of $h(x) = a\tan x$; for $x \in [-180°; 180°]$, $x \neq -90°$, is sketched below.



15.1 Determine the value of *a*. (2)
)

15.2 If $f(x) = \cos(x + 45°)$, sketch the graph of f for $x \in [-180°; 180°]$, on the diagram provided in your ANSWER BOOK. (4)

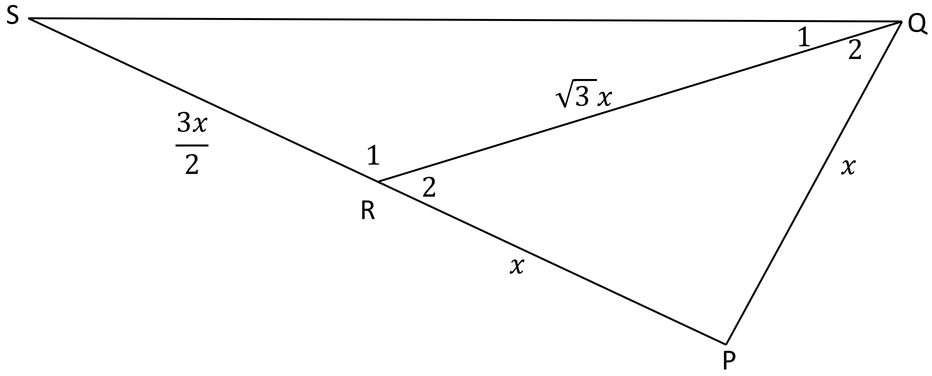
15.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°; 180°]$? (1)

[7]

QUESTION 16

Triangle PQS represents a certain area of a park. R is a point on line PS such that QR divides the area of the park into two triangular parts, as shown below.

PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}x$ units.



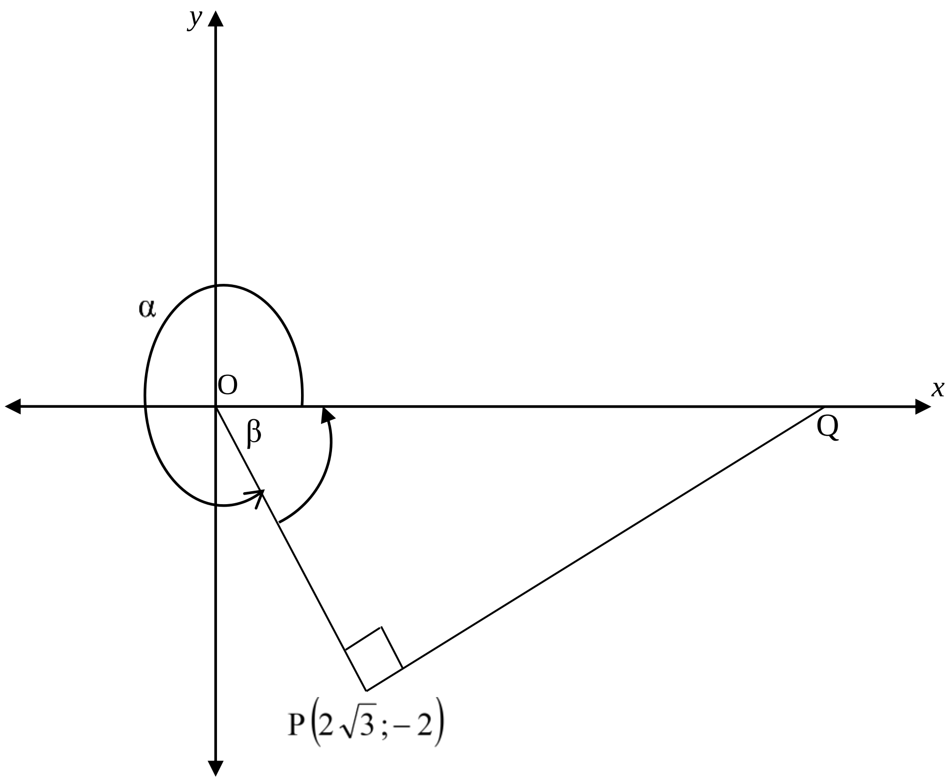
16.1 Calculate the size of $\hat{P}$. (4)

16.2 Determine the area of triangle QRS in terms of x. (5)

[9]

QUESTION 17

17.1 In the diagram below, $P\left(2\sqrt{3};-2\right)$ is a point in the Cartesian plane, with reflex angle $Q\hat{O}P = \alpha$. Q is the point on the x – axis so that $O\hat{P}Q = 90^{\circ}$



Calculate without measuring:

17.1.1 β. (3)

17.1.2 the length of OP. (2)

17.1.3 the co-ordinates of Q. (3)

17.2 If $\cos\alpha + \sqrt{3}\sin\alpha = k\sin\left(\alpha + \beta\right)$.

Calculate the values of k and β. (5)

[13]

QUESTION 18

18.1 On the same system of axes, sketch the graphs of $f(x) = 3\cos x$ and $g(x) = \tan \frac{1}{2}x$ for $-180° \leq x \leq 360°$. Clearly show the intercepts with the axes and all turning points. (5)

Use the graphs in 18.1 to answer the following questions.

18.2 Determine the period of *g*. (1)

18.3 Determine the co-ordinates of the turning points of *f* on the given interval. (2)

18.4 For which values of *x* will both functions increase as *x* increases for $-180° \leq x \leq 360°$? (2)

18.5 If the *y*–axis is moved 45^o to the left, then write down the new equation of *f* in the form *y* = ….. (1)

[11]

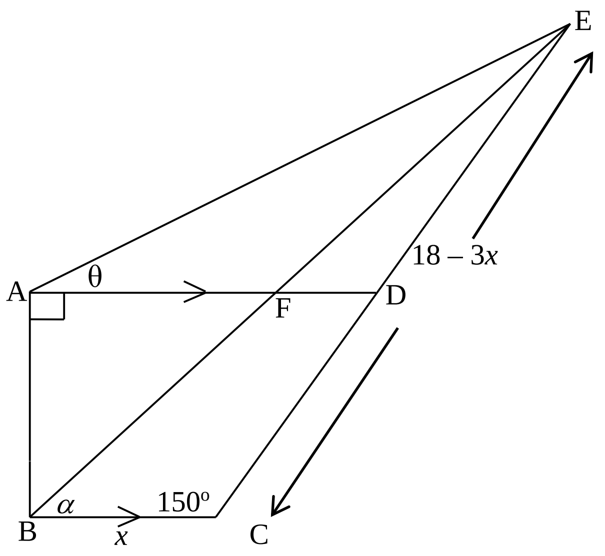
QUESTION 19

19.1 Determine the general solution of:

$\cos 54^{\circ}.\cos x + \sin 54^{\circ}.\sin x = \sin 2x$ (5)

19.2 ABCD is a trapezium with AD || BC, $B\hat{A}D = 90^{\circ}$ and $B\hat{C}D = 150^{\circ}$.
CD is produced to E. F is point on AD such that BFE is a straight line, and $C\hat{B}E = \alpha$.

The angle of elevation of E from A is θ, BC = x and CE = $18 - 3x$.



19.2.1 Show that: $BE = \dfrac{AB\cos\theta}{\sin(\alpha - \theta)}$ (5)

19.2.2 Show that the area of Δ BCE = $\dfrac{9}{2}x - \dfrac{3x^2}{4}$ (3)

19.2.3 Determine, without the use of a calculator, the value of x for which the area of ΔBCE will be maximum. (3)

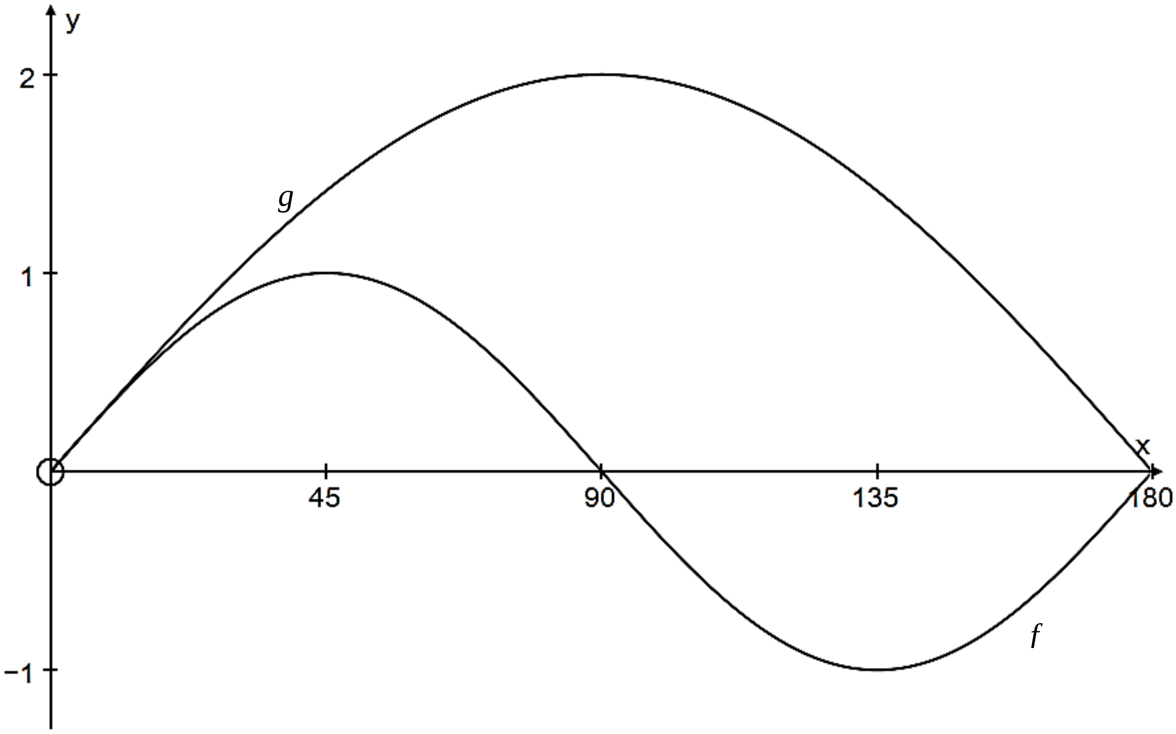
19.2.4 Calculate the length of BE if $x = 3$. (3)

[19]

QUESTION 20

The graphs below represent the functions of f and g.

$f(x) = \sin 2x$ and $g(x) = c\sin dx$, $x \in [0°; 180°]$



20.1 Determine the value(s) of x, for $x \in [0°; 180°]$ where:

20.1.1 $g(x) - f(x) = 2$ (1)

20.1.2 $f(x) \leq 0$ (2)

20.1.3 $g(x).f(x) \geq 0$ (3)

20.2 f in the graph drawn above undergoes transformations to result in g and h as given below. Determine the values of a, b, c and d if

20.2.1 $g(x) = c\sin dx$ (2)

20.2.2 $h(x) = a\cos(x - b)$ (2)

[10]

QUESTION 21

THIS QUESTION HAS TO BE ANSWERED WITHOUT THE USE OF A CALCULATOR:

21.1 Simplify fully: 6.1.1 $\frac{\sin 140°.\tan(-315°)}{\cos 230°.\sin 420°}$ (5)

6.1.2 $\frac{\sin 15°.\cos 15°}{\cos(45°-x)\cos x-\sin(45°-x)\sin x}$ (5)

21.2.1 Express $\cos^2 A$ in terms of $\cos 2A$ (2)

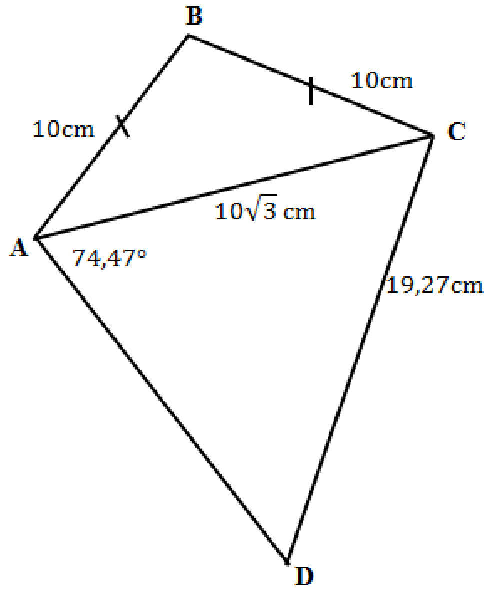
21.2.2 Hence show that $\cos 15° = \frac{\sqrt{\sqrt{3}+2}}{2}$ (4)

21.3 Calculate x when $\sin 2x = \cos(-3x)$ for $x \in [-90°; 90°]$ (6)

[22]

QUESTION 22

Quadrilateral ABCD is drawn with AB = BC = 10cm, AC = $10\sqrt{3}$ cm , CD = 19,27 cm and $C\hat{A}D = 74{,}47°$.



22.1 Calculate the size of $A\hat{B}C$. (3)

22.2 Determine whether ABCD is a cyclic quadrilateral. Justify your answer with the necessary calculations and reasons. (5)

[8]

QUESTION 23

23.1 Determine the value of $\frac{\cos(180°+x).\tan(360°-x).\sin^2(90°-x)}{\sin(180°-x)}+\sin^2 x$ (6)

23.2 23.2.1 Prove the identity: cos (A – B) – cos (A + B) = 2sin A sin B (3)

23.2.2 Hence calculate, without using a calculator, the value of

cos 15° – cos 75° (4)

23.3 Find the value of tan θ, if the distance between A (cos θ; sin θ) and B (6; 7) is $\sqrt{86}$. (4)

[17]

QUESTION 24

Consider : $f(x) = \cos(x - 45°)$ and $g(x) = \tan\frac{1}{2}x$ for $x \in [-180° ; 180°]$

24.1 Use the grid provided to draw sketch graphs of f and g on the same set of axes for $x \in [-180° ; 180°]$. Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)

24.2 Use your graphs to answer the following questions for $x \in [-180° ; 180°]$

24.2.1 Write down the solutions of cos $(x - 45°) = 0$ (2)

24.2.2 Write down the equations of asymptote(s) of g. (2)

24.2.3 Write down the range of f. (1)

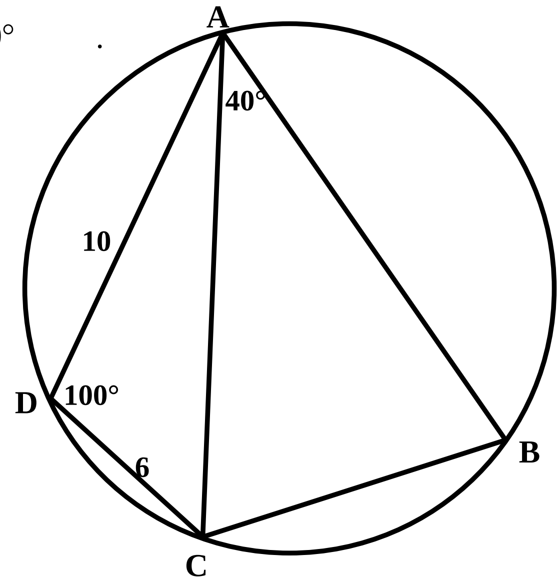
24.2.4 How many solutions exist for the equation $\cos(x - 45°) = \tan\frac{1}{2}x$? (1)

24.2.5 For what value(s) of x *is* $f(x).\,g(x) > 0$ (3)

[15]

QUESTION 25

In the diagram below, ABCD is a cyclic quadrilateral with DC = 6 units, AD = 10 units $A\hat{D}C = 100°$ and $C\hat{A}B = 40°$.



Calculate the following, correct to ONE decimal place:

25.1 The length of BC (6)

25.2 The area of ΔABC (3)

[9]

QUESTION 26

26.1 If $\sin 34^{\circ} = p$, determine the value of each of the following in terms of p, **WITHOUT USING A CALCULATOR.**

26.1.1 $\sin 214°$ (2)

26.1.2 $\cos 34^{\circ}.\cos(-22°) + \cos 56°.\sin 338°$ (4)

26.1.3 $\cos 68°$ (2)

26.2 Determine the value of each of the following expressions:

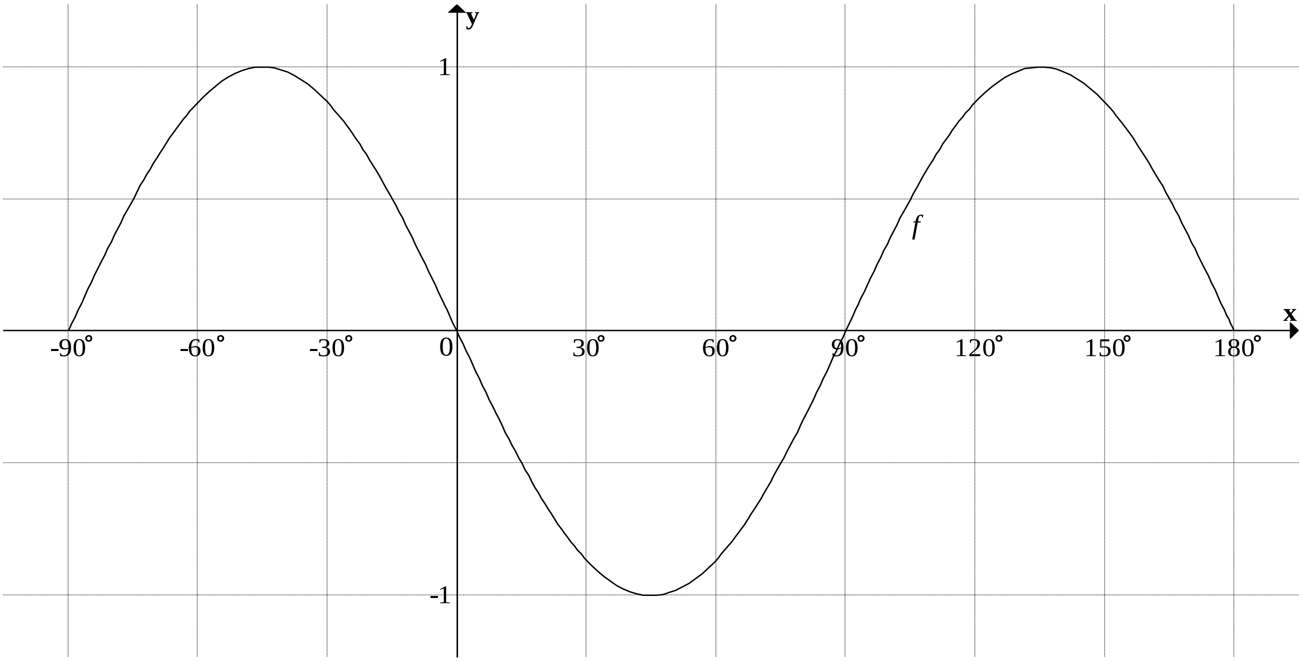
26.2.1 $\dfrac{\cos(90° - 2\theta).\sin\theta}{\sin^2(180° + \theta).\cos(720° + \theta)}$ (6)

26.2.2 $\dfrac{1}{\sin^2 2x} - \dfrac{1}{\tan^2 2x}$ (4)

[18]

QUESTION 27

In the diagram, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \in [-90°; 180°]$.



27.1 Draw the graph of g, where $g(x) = \cos(x - 60°)$, on the same system of axes for the interval $x \in [-90°; 180°]$ in the ANSWER BOOK. (3)

27.2 Determine the general solution of $f(x) = g(x)$. (5)

27.3 Use your graphs to solve x if $f(x) \leq g(x)$ for $x \in [-90°; 180°]$ (3)

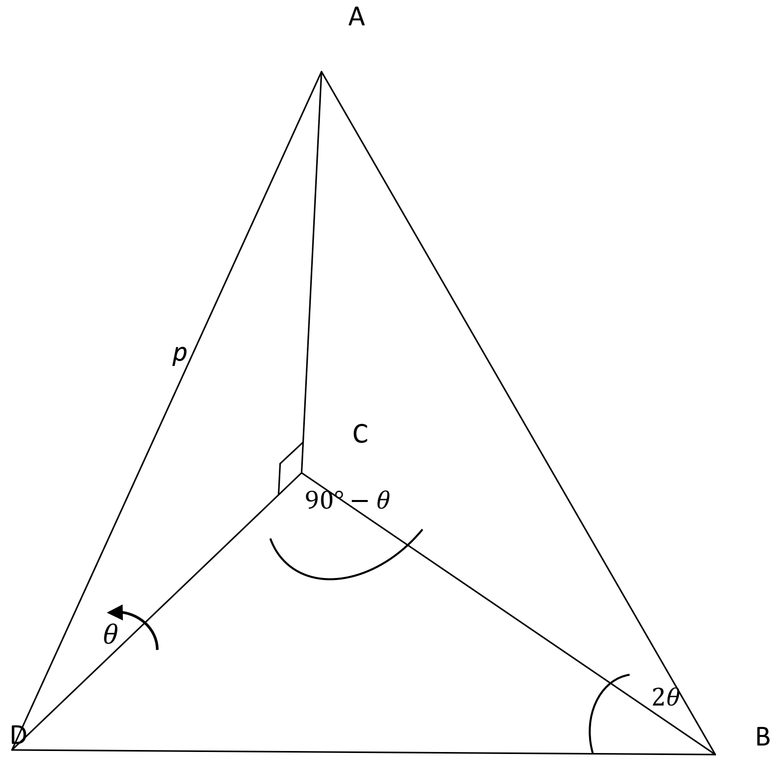
27.4 If the graph of f is shifted 30° left, give the equation of the new graph which is formed. (2)

27.5 What transformation must the graph of g undergo to form the graph of h, where $h(x) = \sin x$? (2)

QUESTION 28

In the diagram below, D, B and C are points in the same horizontal plane. AC is a vertical pole and the length of the cable from D to the top of the pole, A, is p meters. AC ⊥ CD. $A\hat{D}C = \theta$;

$D\hat{C}B = (90° - \theta)$ and $C\hat{B}D = 2\theta$.



28.1 Prove that:

$$BD = \frac{p\cos\theta}{2\sin\theta}$$ (5)

28.2 Calculate the height of the flagpole AC if $\theta = 30°$ and $p = 3$ meters. (2)

28.3 Calculate the length of the cable AB if it is further given that $A\hat{D}B = 70°$ (5)

[12]