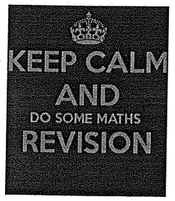
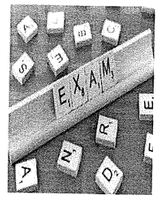




STANMORE SECONDARY SCHOOL

REVISION EXERCISE

MATHEMATICS

GRADE 12

JUNE /JULY 2021

REVISION (FUNCTIONS) (A)

QUESTION 4

Given: $g(x)=\frac{2}{x+3}-1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 the y-intercept of g. (1)

4.2.2 the x-intercept of g. (2)

4.3 Sketch the graph of g, clearly indicating the asymptotes and the intercepts with the axes. (3)

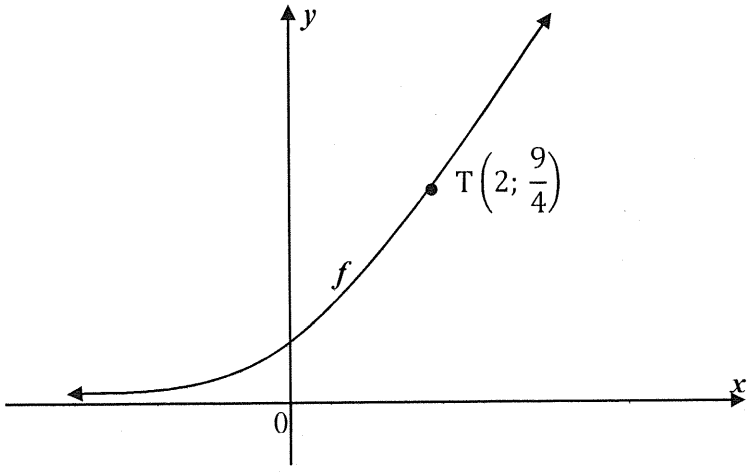
4.4 Determine the equation of the line of symmetry that has a negative gradient in the form $y = \ldots\ldots$ (2)

4.5 Determine the value(s) of x for which $\frac{2}{x+3}-1 \geq -x-4$. (2)

[12]

QUESTION 5

The graph of $f(x) = a^x$ where $a > 1$ is shown below. $T\left(2\,;\frac{9}{4}\right)$ is a point on the graph of f.



5.1 Calculate the value of a. (2)

5.2 If $g(x) = f(-x)$, determine the equation of g. (1)

5.3 Write down the equation of f^{-1}. (2)

5.4 Determine the values of x for which $f^{-1}(x) \geq 2$ (2)

5.5 Is the inverse of f a function? Explain your answer. (2)

[9]

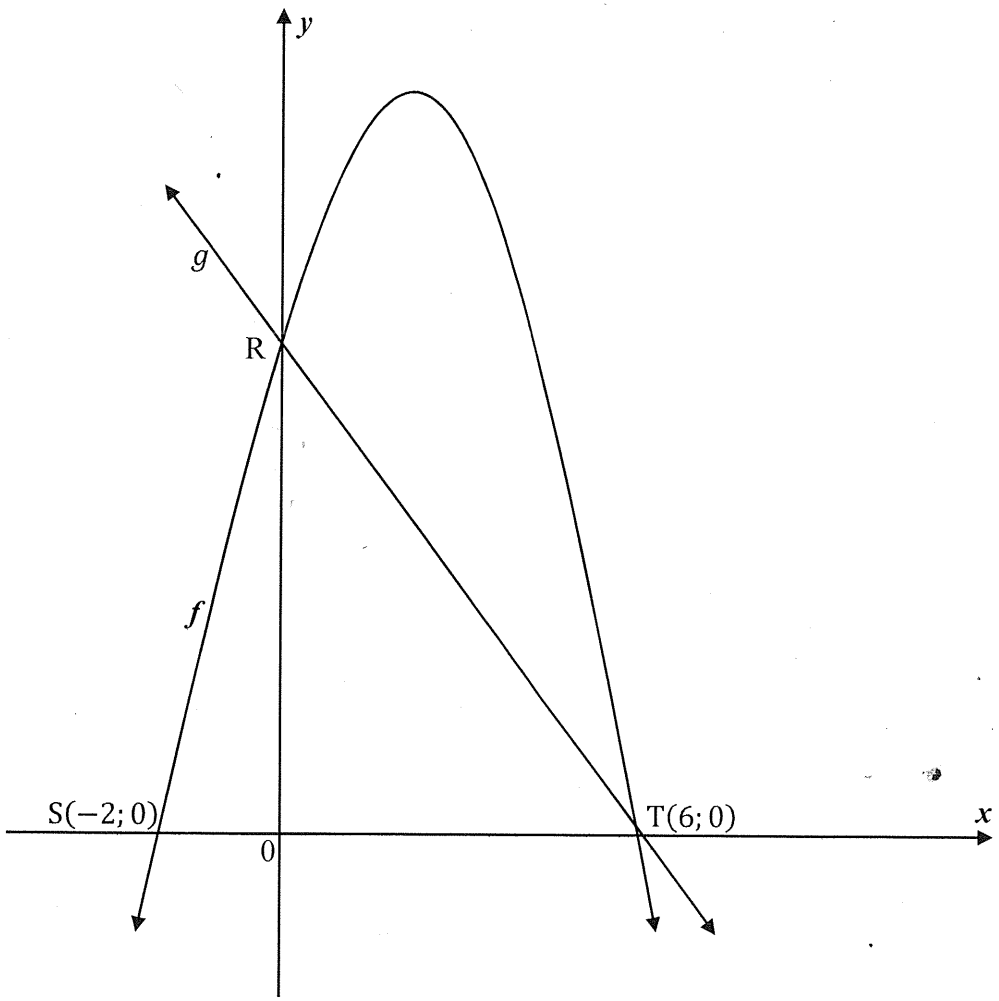
(A)

QUESTION 6

Given: $f(x) = ax^2 + bx + c\,; a \neq 0$ and $g(x) = -2x + d$.

The graphs of f and g are sketched below.

- S$(-2;\ 0)$ and T$(6; 0)$ are the x-intercepts of the graph of f;
- R is the y-intercept of f;
- g passes through R and T.



6.1 Determine the value of d. (2)

6.2 Determine the values of a, b and c. (4)

6.3 If $f(x) = -x^2 + 4x + 12$, calculate the co-ordinates of the turning point of f. (3)

6.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x + 2) - 4$. (2)

6.5 For which value(s) of k will $f(x) = k$ have two distinct roots? (2)

[13]

REVISION (FUNCTIONS)

Given: $f(x) = \frac{3}{x-2} + 1$

Question 4

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine coordinates of B, the x-intercept of f. (2) (B)

4.3 Write down the domain of g if $g(x) = f(x+1)$. (3)

4.4 One of the axes of symmetry of f is an increasing function. Write down the equation of this axis of symmetry. (2) (9)

QUESTION 5

Given $h(x) = 3^{-x}$ and $k^{-1}(x) = 2x^2$ for $x \geq 0$.
A(0,57 ; 0,53) is a point of intersection between h and k.

5.1 Write down the equation of k in the form $y = \ldots$ (2)

5.2 Sketch the graphs of h and k on the same set of axes, clearly indicating the intercepts with the axes. (5)

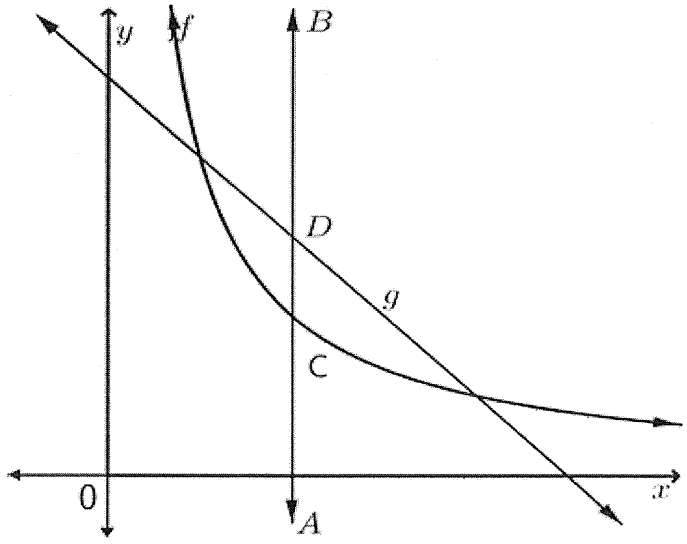
5.3 Write down the range of h. (1)

5.4 For which values of x is $k(x) \leq h(x)$? (2)

5.5 For which values of t will $k(x) + t = h(x)$ have no real roots? (2) [12]

QUESTION 6

The diagram below shows the graphs of $f(x) = \frac{4}{x}$; $x > 0$ and $g(x) = 6 - x$. Line BA intersects f and g respectively at points C and D. BA is perpendicular to the x-axis.



6.1 Write down the y-intercept of g. (1)

6.2 Write down the equation of h if h is a translation of f one unit to the right. (1)

6.3 Calculate the values of x for which $h(x) = g(x)$. (3)

6.4 Write down the length of CD in terms of x. (2)

6.5 Determine the value of x for which CD has a maximum length. (4)

QUESTION 4

Two functions are defined by $f(x) = (x - 4)(x + 2)$ and $g(x) = 2x - 12$.

A is a point on f such that g is a tangent to f at A.

4.1 Write down the gradient of g. (1)

4.2 Calculate the coordinates of A. (5)

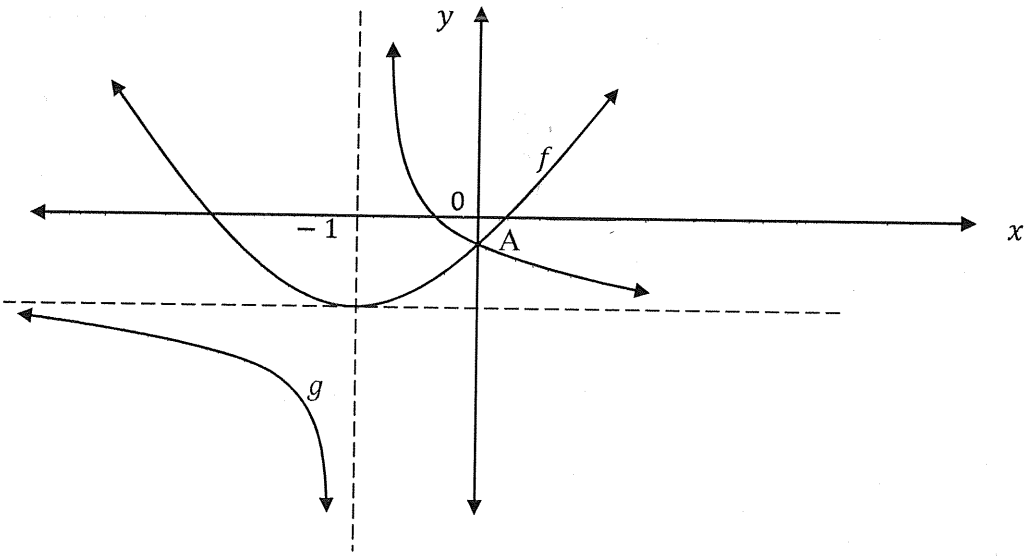
4.3 Determine the equation of the graph h which is the reflection of f about the $y-$axis. (2)

4.4 Determine value(s) of x for which $f(x).g(x) < 0$, given that $x > 0$. (2)

4.5 Determine g^{-1}, the inverse of g, in the form $y = \ldots$. (2)

QUESTION 5

The graphs of $f(x) = x^2 + 2x - 3$ and $g(x) = \frac{a}{x+p} + q$ are drawn below. A is the $y-$intercept of both f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f. The equation of the vertical asymptote of g is $x = -1$.



5.1 Determine the equations of the asymptotes of g. (3)

5.2 Write down the coordinates of A. (1)

5.3 Determine the equation of g. (4)

5.4 Determine points of intersection of g with its axis of symmetry that has a positive gradient. (4)

5.5 Write down the range of $-f(x)$. (2)

QUESTION 6

Given: $f(x) = 3^{-x+1} - 3$

6.1 Draw the graph of f showing all asymptotes and intercepts with the axes. (3)

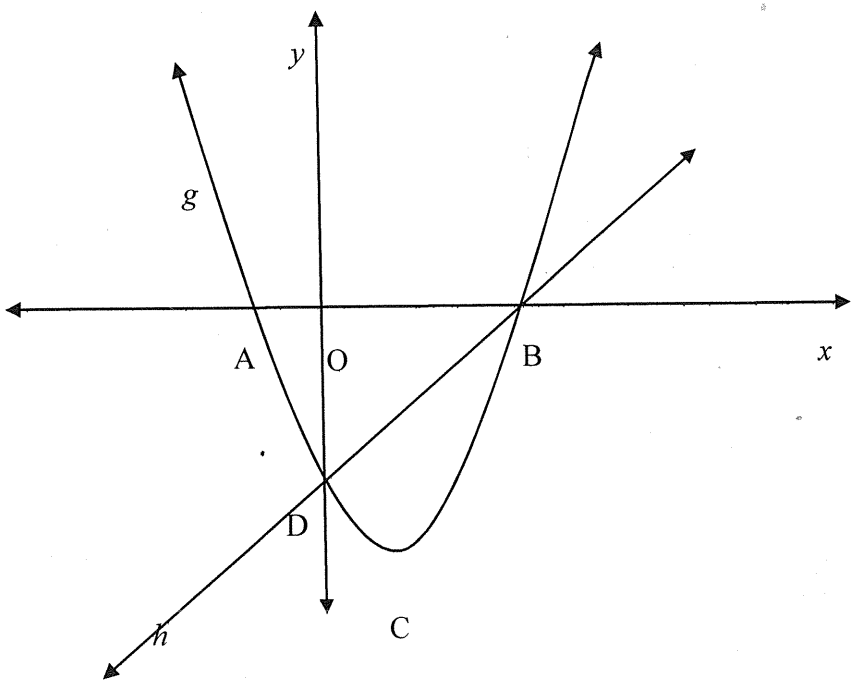
6.2 Calculate the $x-$value when $y = 5$. (3)

6.3 If $h(x) = 3^x$; explain what transformations f has undergone to become h. (3)

QUESTION 7

(C)

The sketch below shows the graphs of $g(x) = x^2 - 3x - 10$ and $h(x) = ax + q$. The graphs intersect at B and D. The graph of g intersects the x – axis at A and B and has a turning point at C. The graph of h intersects the y – axis at D and the x – axis at B.



7.1 Write down the coordinates of D. (1)

7.2 Determine the coordinates of A and B. (4)

7.3 Write down the values of a and q. (2)

7.4 Calculate the coordinates of C, the turning point of g. (3)

7.5 Write down the turning point of t, if $t(x) = g(-x) + 3$. (2)

7.6 For which values of x will $g'(x).h'(x) \geq 0$? (2)

[14]

QUESTION 8

Given $p(x) = 3^x$.

8.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

8.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axes. Show clearly all the intercepts with the axes and at least one other point on each graph. (4)

8.3 Determine the values of x for which $p^{-1}(x) \leq 3$ (4)

REVISION (CALCULUS) (A)

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 2x$. (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = (x^2 + 1)(2x - 3)$ (2)

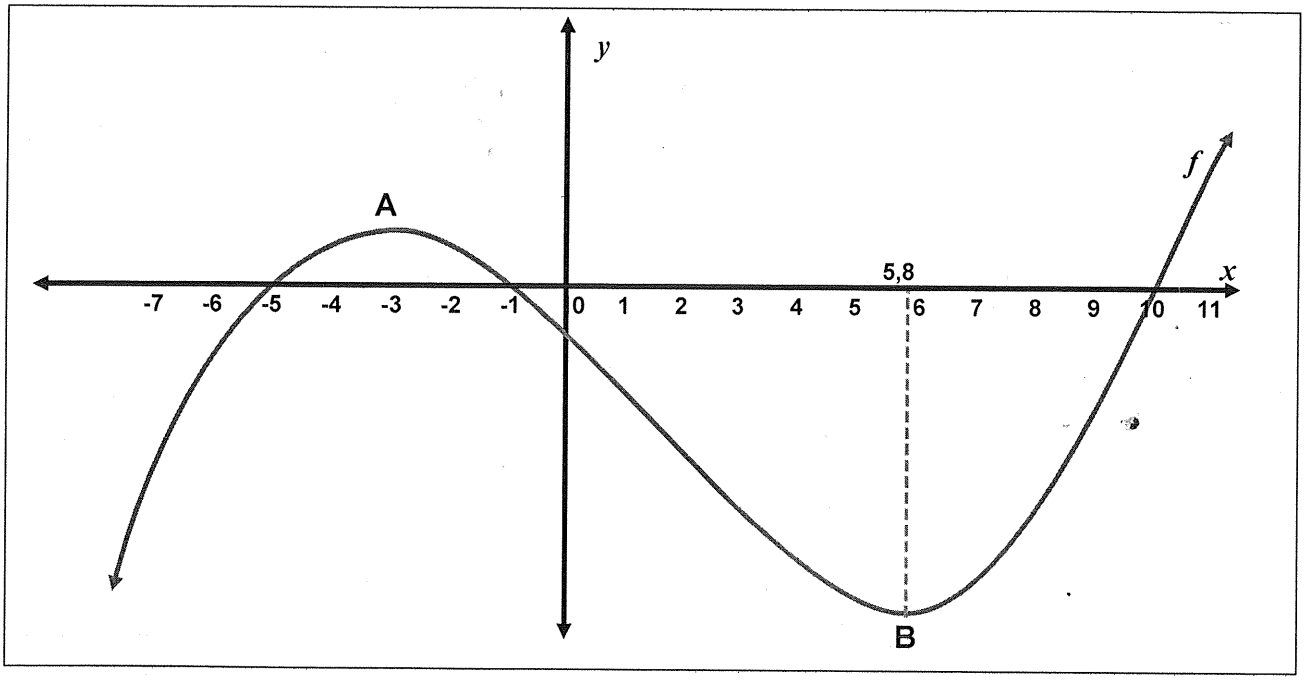
7.2.2 $\frac{dy}{dx}$ if $xy + 2y = x^3 + 2x^2$. State all restrictions. (4)

7.3 For which value(s) of x on the curve $p(x) = 12x^3$ will the tangent to the curve have a $45°$ angle with the positive x-axis? (4)

[15]

QUESTION 8

8.1 The function f is drawn below. A and B are the turning points of f.



Say whether each of the following is positive, negative or zero.

8.1.1 $f(-5)$; 8.1.2. $f'(5;8)$; 8.1.3 $f''(8)$, 8.1.4. $f(1).f'(1)$ (1)

8.2 Consider the function $f(x) = x^3 - 6x^2 + 9x - 4$.

8.2.1 Calculate all the intercepts with the axes. (4)

8.2.2 Calculate the co-ordinates of the stationary points of f. (4)

8.2.3 For which value(s) of x is the graph concave down? (3)

C

8.2.4 Sketch the graph of f in your answer book. (3)

[18]

QUESTION 8

8.1 Determining $f'(x)$ from first principles if $f(x) = x^3$ (5)

8.2 Determine:

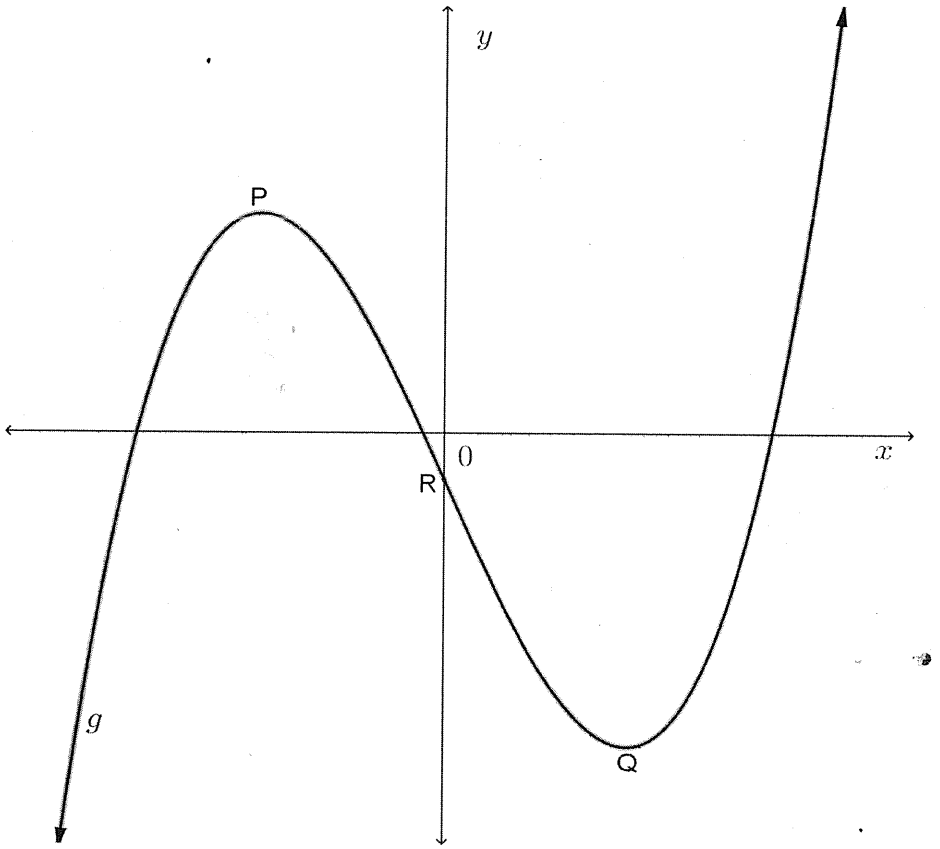
8.2.1 $f'(x)$ if $f(x) = \left(x + \frac{1}{x}\right)^2$ (3)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{3}{x^{\frac{2}{3}}} - \pi x$ (3)

[11]

QUESTION 9

The sketch below shows the graph of $g(x) = x^3 - 6x - 1$. P and Q are the turning points and R the y-intercept of g.



9.1 Determine the coordinates of R. (2)

9.2 Determine the coordinates of the turning points P and Q. (6)

9.3 Calculate the values of x for which g strictly increases as x increases. (2)

9.4 If $h(x) = g'(x)$, determine for which values of x is $h(x) \leq 0$. (2)

9.5 Determine the equation of the tangent to g at R. (4)

9.6 Write down the equation of the line perpendicular to the tangent at P. (2)

[18]

REVISION (CALCULUS) (C)

QUESTION 8

(C)

8.1 Determine the derivative of $f(x) = 2x - x^2$ from first principle. (5)

8.2 Differentiate the following with respect to x:

8.2.1 $f(x) = 6x^4 - \sqrt[3]{x^{-2}}$ (3)

8.2.2 $y = (4x^2)^3 + 2x$ (3)

[11]

QUESTION 9

Given: $f(x) = ax^3 + bx^2 + 3x + 3$ and $g(x) = f''(x)$ where $g(x) = 12x + 4$.

9.1 Show that $a = 2$ and $b = 2$. (4)

9.2 Prove that f will never decrease for any real value of x. (5)

9.3 Determine the minimum gradient of f. (4)

9.4 Explain the concavity of f for all values of x where $g(x) < 0$. (1)

[14]

9.1 Given $f(x) = x^3$. Determine the derivative of f from first principles. (5)

9.2 Determine:

9.2.1 $f'(x)$ if $f(x) = \sqrt{x} - \frac{4}{x^2}$ (4)

9.2.2 $\frac{dy}{dx}$ if $\frac{1}{2}x^2 - 3 = \sqrt{y}$ (4)

QUESTION 10

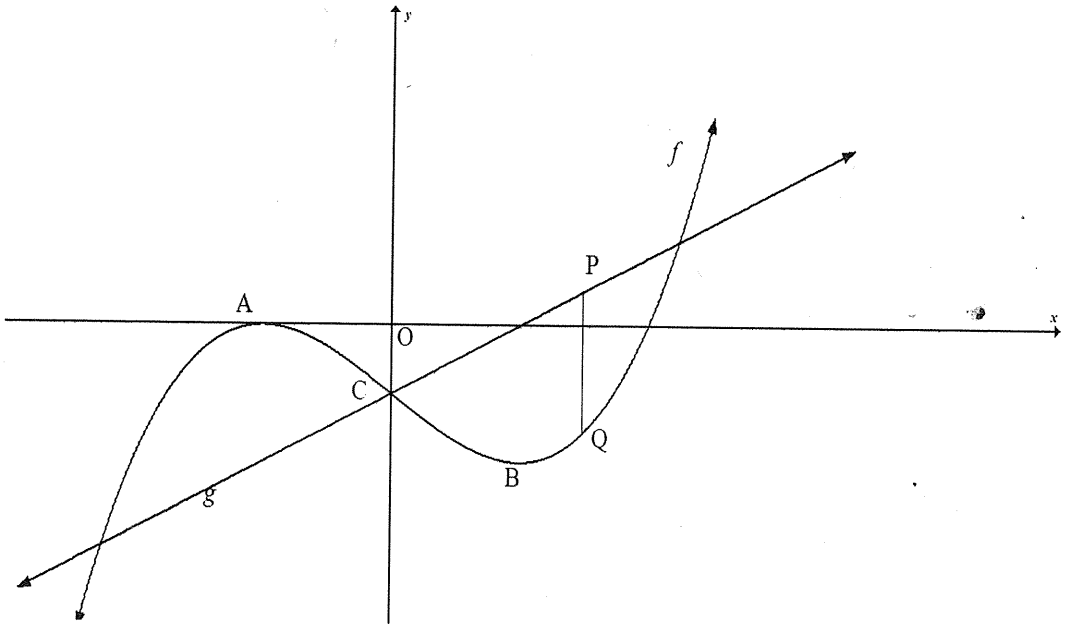
10.1 Given $f(x) = x^2 - 8$

10.1.1 Calculate $f(-3)$. (1)

10.1.2 Calculate $f'(-3)$. (1)

10.1.3 Determine the equation of the tangent to $f(x) = x^2 - 8$ at $x = -3$. (2)

10.2 The graph of a cubic function with equation $f(x) = x^3 - 3x - 2$ and $g(x) = 2x - 2$ is drawn. A and B are the turning points of f. P is a point on g and Q is a point on f such that PQ is perpendicular to the x – axis.

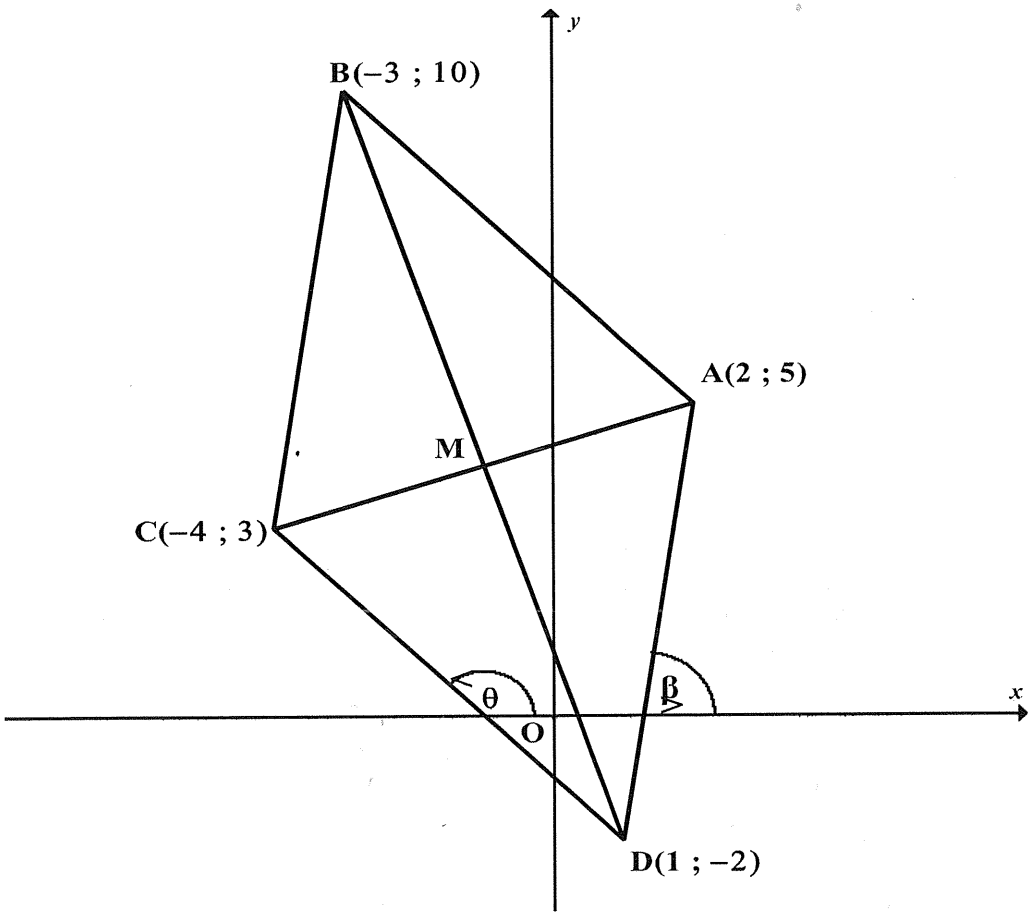


10.2.1 Calculate the coordinates of A and B. (4)

10.2.2 If PQ is perpendicular to the x – axis, calculate the maximum length of PQ, (4)

10.2.3 Determine the values of k for which $f(x) = k$ has only two real roots. (2)

10.2.4 Determine the values of x for which f is concave up. (3)



3.1 Calculate the length of AC. (Leave the answer in simplest surd form.) (2)

3.2 Determine the coordinates of M, the midpoint of AC. (2)

3.3 Show that BD and AC bisect each other perpendicularly. (5)

3.4 Calculate the area of ΔABC. (4)

3.5 Determine the equation of DC. (3)

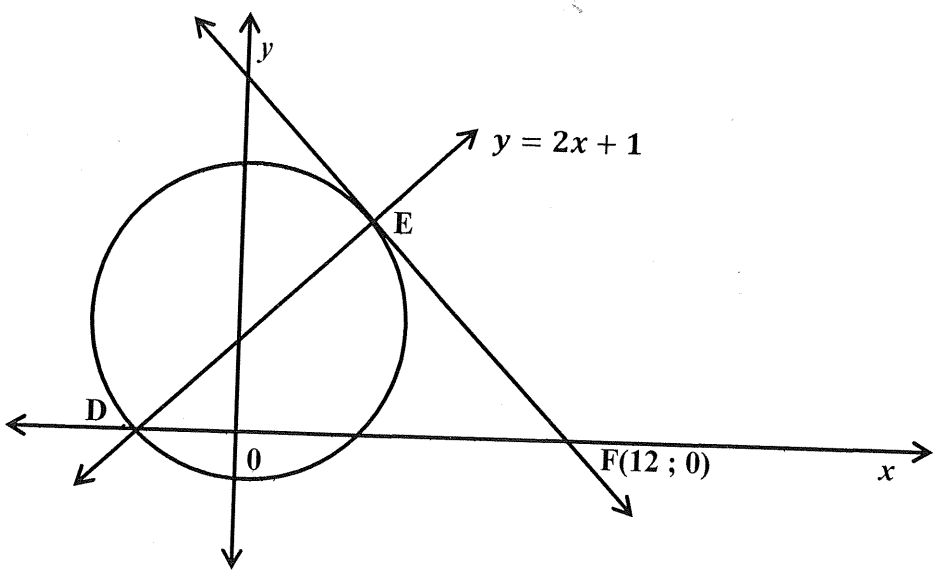
3.6 Determine θ, the angle of inclination of DC. (3)

3.7 Calculate the size of $A\hat{D}C$. (3)

[22]

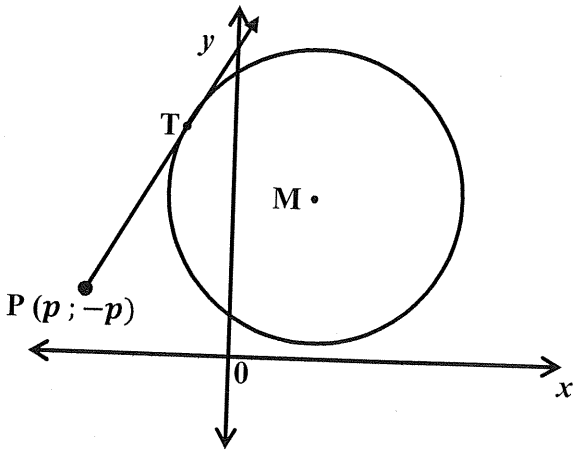
QUESTION 4

4.1 A circle has a diameter with equation $y = 2x + 1$. The tangent to the circle at point E intersects the x-axis at F(12 ; 0).



Determine the coordinates of E. (6)

4.2 M is the centre of the circle defined by $x^2 + y^2 - 2x - 4y + 1 = 0$.
P$(p ; -p)$ is any point on the tangent to the circle at T.



4.2.1 Show, by calculation, that the coordinates of M are (1 ; 2). (3)

4.2.2 Prove that the length of PT = $\sqrt{2p^2 + 2p + 1}$ (3)

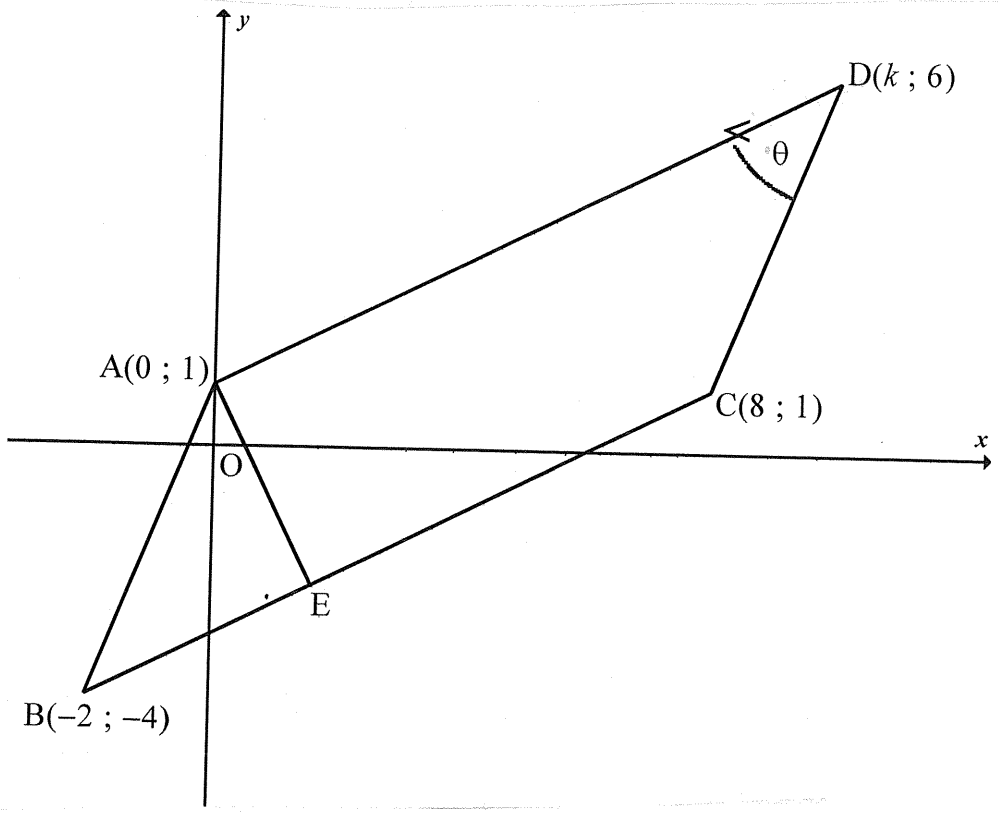
4.2.3 Calculate the coordinates of P where P is as close as possible to T and hence calculate the minimum length of PT. (5)

[17]

QUESTION 3

ANALYTICAL GEOMETRY

In the figure below, ABCD is a parallelogram with vertices: A(0; 1), B(−2; −4), C(8; 1) and D(k; 6). AE is perpendicular to BC. (B)



3.1 Calculate the length of BC. Leave your answer in simplest surd form. (3)

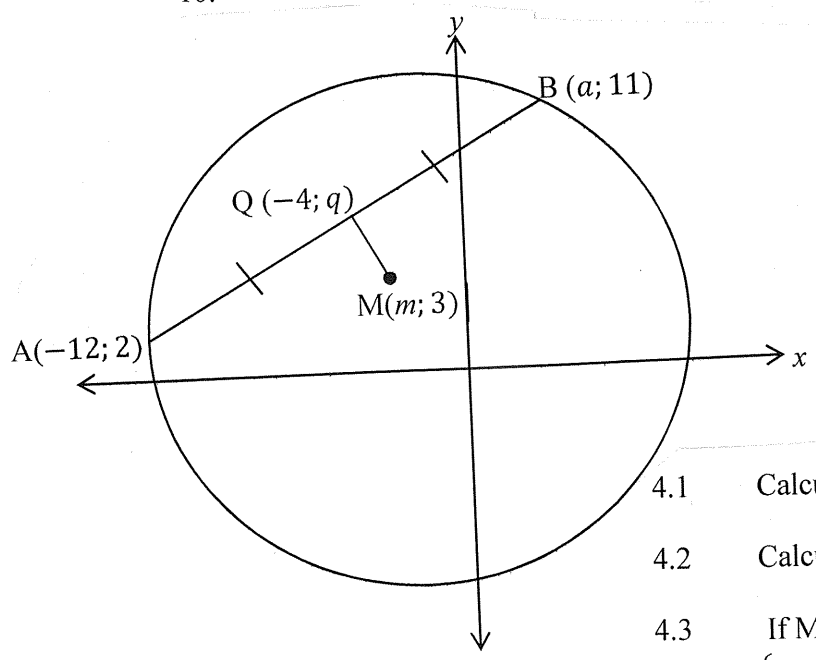
3.2 Determine the value of k. (3)

3.3 Determine the equation of AE. (4)

3.4 Calculate the size of θ rounded off to two decimal places. (6) [16]

QUESTION 4

In the diagram below, M (m; 3) is the centre of the circle. Q (−4; q) is the midpoint of chord AB with A (−12; 2) and B (a; 11). The length of the radius of the circle is 10.



4.1 Calculate the values of a and q. (3)

4.2 Calculate the value of m. (3) [7]

4.3 If M(−2; 3), determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.4 Determine the equation of a tangent to the circle at point A. (3)

QUESTION 5

5.1 Calculate the value of k if the points A(6;5), B(3;2) and C($2k$;$k+4$) are collinear. (3)

5.2 The equation of circle is given: $x^2 + y^2 - 4x + 4y + 3 = 0$.

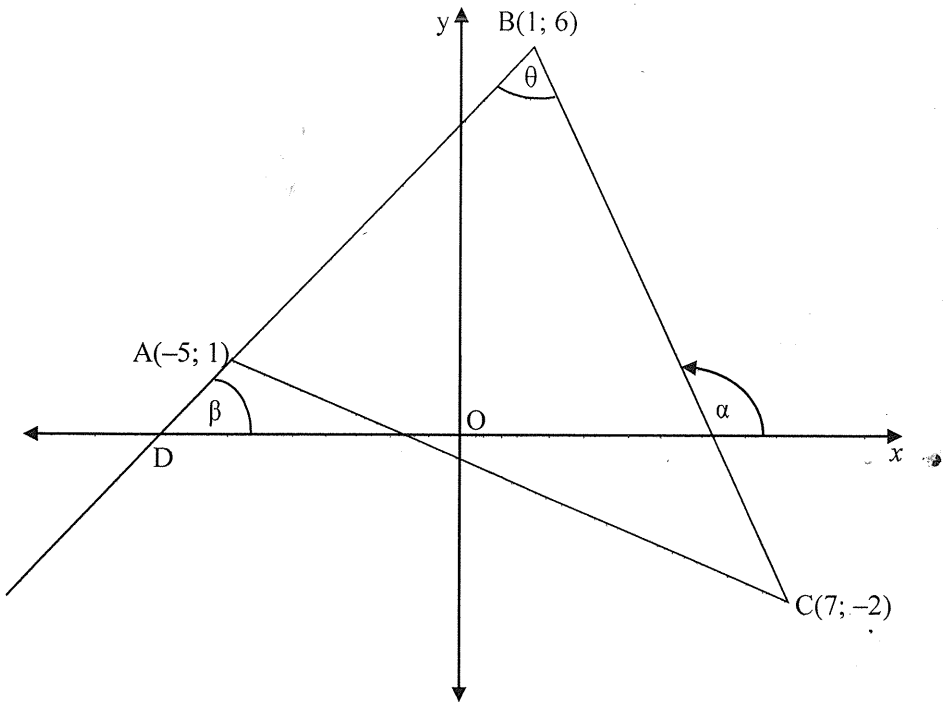
5.2.1 Determine the coordinates of the centre of the circle and the length of the radius. (4)

5.2.2 Determine whether the point T(3;−3) lies inside, outside or on the circle. Show all your calculations. (2)

[9]

QUESTION 3

In the diagram below, A (–5; 1) , B(1; 6) and C(7; –2) are vertices of Δ ABC with AB produced to D. BD forms an angle, β , with the negative x – axis and BC forms an angle, α , with the positive x – axis. $A\hat{B}C = \theta$



Determine:

3.1 the length AC (2)

3.2 the equation of line BC (3)

3.3 $A\hat{B}C$ (5)

3.4 the midpoint P of AB (2)

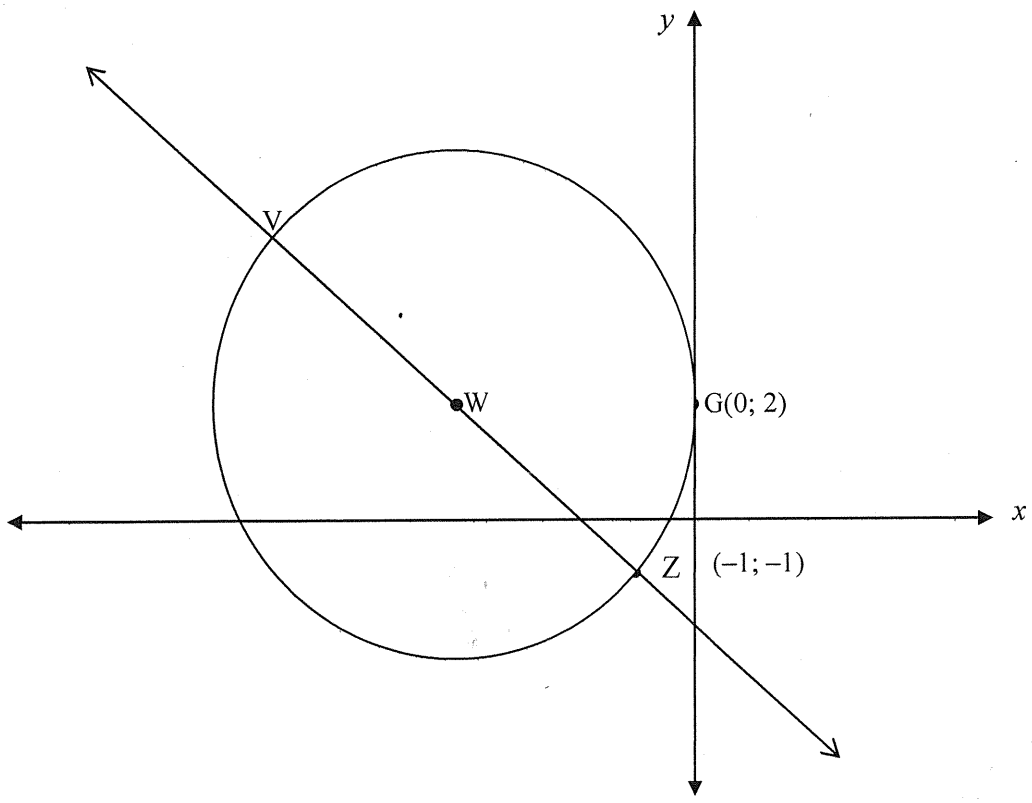
3.5 the equation of the line parallel to AC and passing through the point (–1; 3) (3)

3.6 Show that AB is perpendicular to $6x + 5y = 18$ (3)

[18]

QUESTION 4

In the diagram below, centre W of the circle lies on the straight line $3x + 4y + 7 = 0$ The straight line cuts the circle at V and Z(–1; –1). The circle touches the y-axis at G(0; 2)



4.1.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (5)

4.1.2 Determine the length of diameter VZ (1)

4.1.3 Calculate the gradient of GZ. (2)

4.1.4 Write down the coordinates of the midpoint of GZ. (1)

4.1.5 Determine the equation of the line that is the perpendicular bisector of GZ. (3)

4.1.6 Show that the line in QUESTION 4.1.5 and straight line VZ intersect at W. (2)

4.2 The circle defined by $(x + 2)^2 + (y - 1)^2 = 25$ has centre M, and the circle defined by $(x - 1)^2 + (y - 3)^2 = 9$ has centre N.

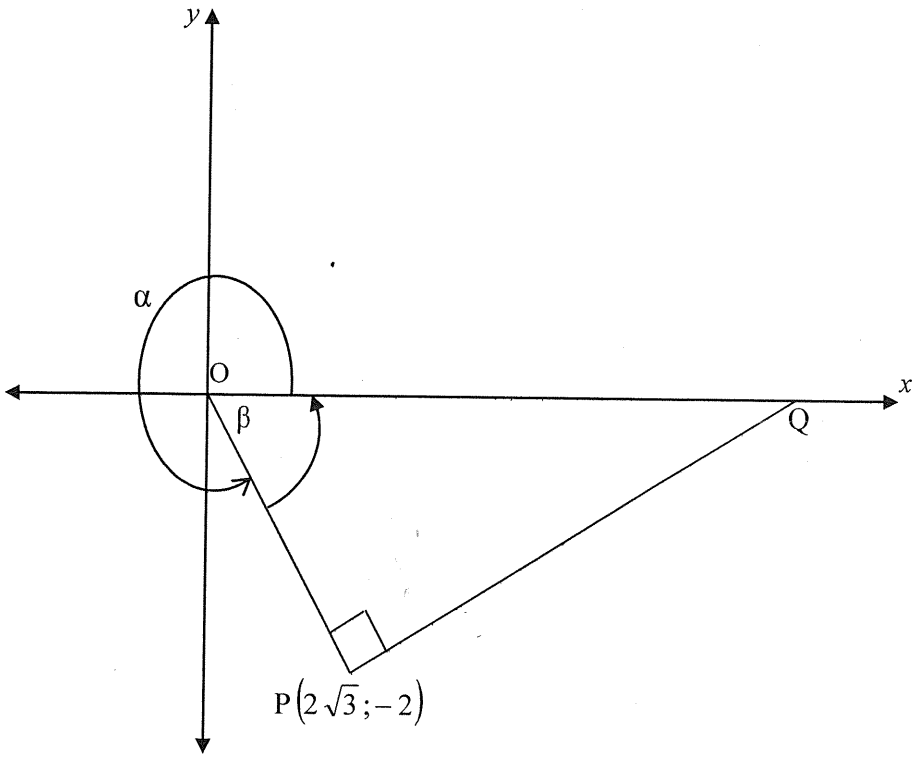
4.2.1 Show that the circles intersect each other at two distinct points. (6)

4.2.2 Determine the equation of the common chord. (3)

[23]

QUESTION 5

5.1 In the diagram below, $P(2\sqrt{3};-2)$ is a point in the cartesian plane, with reflex angle $Q\hat{O}P = \alpha$. Q is the point on the $x-$axis so that $O\hat{P}Q = 90°$



Calculate without measuring:

5.1.1 β. (3)

5.1.2 the length of OP. (2)

5.1.3 the co-ordinates of Q. (3)

5.2 If $\cos\alpha + \sqrt{3}\sin\alpha = k\sin(\alpha+\beta)$.

Calculate the values of k and β. (5)

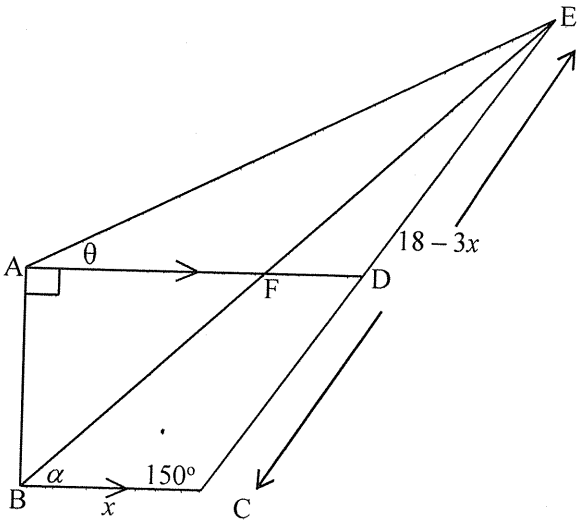
[13]

TRIGONOMETRY REVISION (3D FIGURES)

7.2 ABCD is a trapezium with AD ∥ BC, $\hat{BAD} = 90°$ and $\hat{BCD} = 150°$.

CD is produced to E. F is point on AD such that BFE is a straight line, and $C\hat{B}E = \alpha$.

The angle of elevation of E from A is θ, BC = x and CE = $18 - 3x$.



7.2.1 Show that: $BE = \dfrac{AB\cos\theta}{\sin(\alpha - \theta)}$ (5)

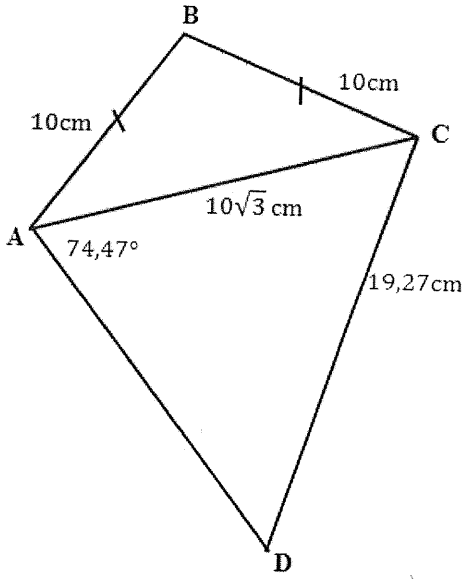
7.2.2 Show that the area of $\Delta BCE = \dfrac{9}{2}x - \dfrac{3x^2}{4}$ (3)

7.2.3 Determine, without the use of a calculator, the value of x for which the area of ΔBCE will be maximum. (3)

7.2.4 Calculate the length of BE if $x = 3$. (3) [19]

QUESTION 8

Quadrilateral ABCD is drawn with AB = BC = 10cm, AC = $10\sqrt{3}$ cm, CD = 19,27 cm and $C\hat{A}D = 74{,}47°$.



8.1 Calculate the size of $A\hat{B}C$. (3)

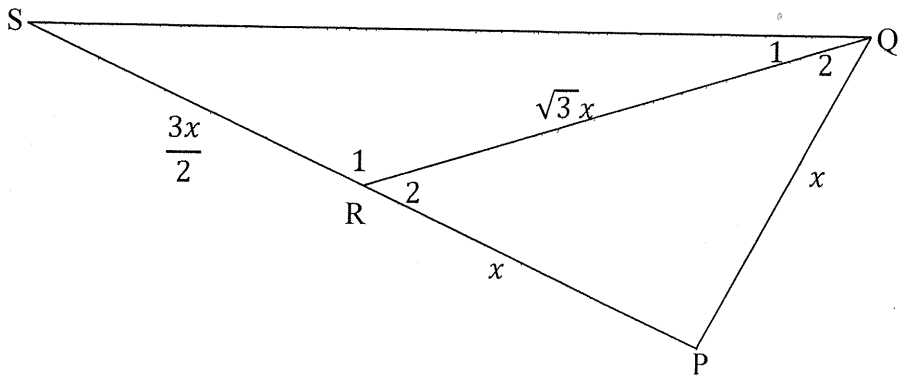
8.2 Determine whether ABCD is a cyclic quadrilateral. Justify your answer with the necessary calculations and reasons. (5) [8]

QUESTION 9 REVISION TRIGONOMETRY 3D FIGURES

(B)

Triangle PQS represents a certain area of a park. R is a point on line PS such that QR divides the area of the park into two triangular parts, as shown below.

PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}x$ units.



9.1 Calculate the size of $\hat{P}$. (4)

9.2 Determine the area of triangle QRS in terms of x. (5)

[9] (B)