



MATHEMATICS

FUNCTIONS

GRADE 10

QUESTION 6

Given: $f(x) = \frac{3}{x} + 1$ and $g(x) = -2x - 4$

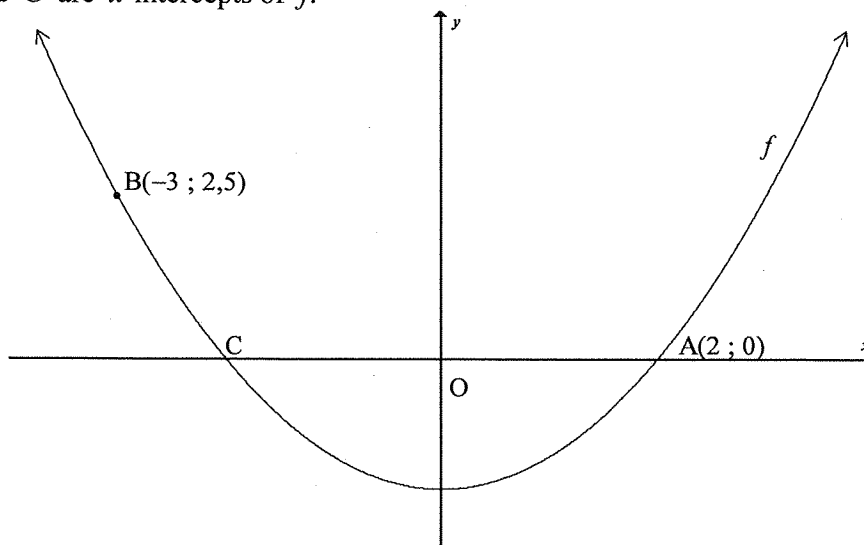
- 6.1 Sketch the graphs of f and g on the same set of axes. (4)
- 6.2 Write down the equations of the asymptotes of f . (2)
- 6.3 Write down the domain of f . (2)
- 6.4 Solve for x if $f(x) = g(x)$. (5)
- 6.5 Determine the values of x for which $-1 \leq g(x) < 3$. (3)
- 6.6 Determine the y -intercept of k if $k(x) = 2g(x)$. (2)
- 6.7 Write down the coordinates of the x - and y -intercepts of h if h is the graph of g reflected about the y -axis. (2)
- [20]**

QUESTION 7

The graph of $f(x) = ax^2 + q$ is sketched below.

Points A(2 ; 0) and B(-3 ; 2,5) lie on the graph of f .

Points A and C are x -intercepts of f .



- 7.1 Write down the coordinates of C. (1)
- 7.2 Determine the equation of f . (3)
- 7.3 Write down the range of f . (1)
- 7.4 Write down the range of h , where $h(x) = -f(x) - 2$. (2)
- 7.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and which passes through the point A. (3)
- [10]**

TOTAL: 100

QUESTION 3

3.1 Given the linear number pattern: 8 ; 3 ; -2 ; ...

3.1.1 Write down the NEXT TWO terms of the pattern. (2)

3.1.2 Determine the n^{th} term of the pattern. (2)

3.1.3 Calculate T_{30} , the thirtieth term of the pattern. (2)

3.1.4 Which term of the pattern is equal to -492? (2)

3.2 The first four terms of PATTERN A and PATTERN B are shown in the table below:

Position of term (n)	1	2	3	4
PATTERN A	1	3	5	7
PATTERN B	1	9	25	49

3.2.1 Determine a general formula for the n^{th} term of PATTERN A. (2)

3.2.2 Hence, or otherwise, determine a general formula for the n^{th} term of PATTERN B. (1)

3.2.3 Hence, determine a general formula for the pattern 0 ; -6 ; -20 ; -42 ...
Simplify your answer as far as possible. (4)
[15]

QUESTION 4

$f(x) = -2x^2 + 2$ and $g(x) = 2^x + 1$ are the defining equations of graphs f and g respectively.

4.1 Write down an equation for the asymptote of g . (1)

4.2 Sketch the graphs of f and g on the same set of axes, clearly showing ALL intercepts with the axes, turning points and asymptotes. (6)

4.3 Write down the range of f . (1)

4.4 Determine the maximum value of h if $h(x) = 3^{f(x)}$. (2)

4.5 What transformation does the graph of $y = f(x)$ undergo in order to obtain the graph of $y = 2x^2 - 2$? (2)
[12]

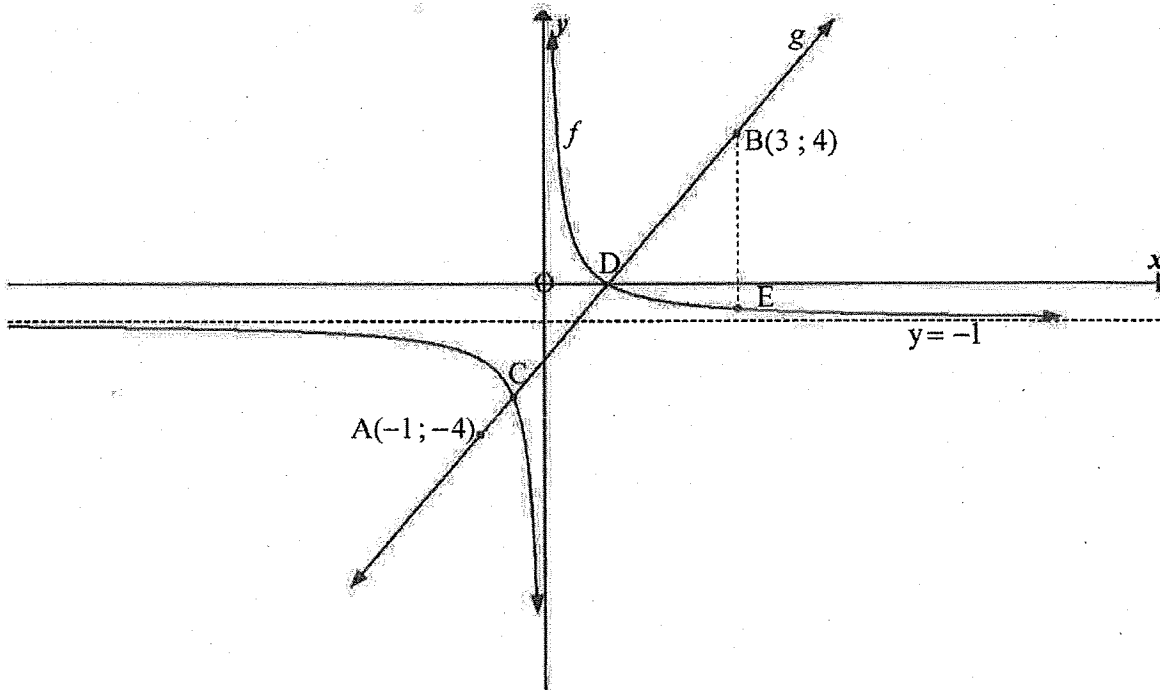
QUESTION 5

The sketch below shows f and g , the graphs of $f(x) = \frac{1}{x} - 1$ and $g(x) = ax + q$ respectively.

Points $A(-1; -4)$ and $B(3; 4)$ lie on the graph g .

The two graphs intersect at points C and D .

Line BE is drawn parallel to the y -axis, with E on f .



- 5.1 Show that $a = 2$ and $q = -2$. (2)
 - 5.2 Determine the values of x for which $f(x) = g(x)$. (4)
 - 5.3 For what values of x is $g(x) \geq f(x)$? (3)
 - 5.4 Calculate the length of BE . (3)
 - 5.5 Write down an equation of h if $h(x) = f(x) + 3$. (1)
- [13]**

QUESTION 6

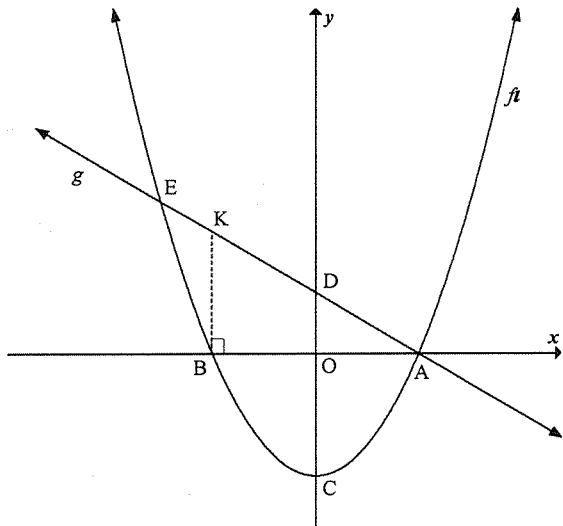
Given: $f(x) = ax^2 + c$

f passes through the x -axis at $(d-5)$ and $(d-1)$, where $d \in R$.

- 6.1 Determine the value of d . (2)
 - 6.2 Determine the values of a and c if it is also given that $f(1) = -9$. (4)
- [6]**

QUESTION 5

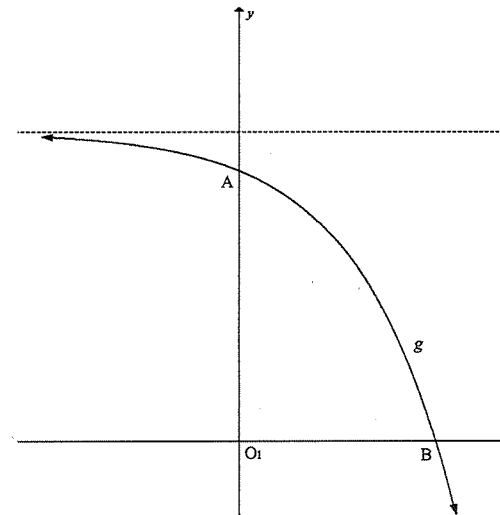
The graphs of $f(x) = x^2 - 4$ and $g(x) = -x + 2$ are sketched below. A and B are the x -intercepts of f . C and D are the y -intercepts of f and g respectively. K is a point on g such that $BK \parallel x$ -axis. f and g intersect at A and E.



- 5.1 Write down the coordinates of C. (1)
 - 5.2 Write down the coordinates of D. (1)
 - 5.3 Determine the length of CD. (1)
 - 5.4 Calculate the coordinates of B. (3)
 - 5.5 Determine the coordinates of E, a point of intersection of f and g . (4)
 - 5.6 For which values of x will:
 - 5.6.1 $f(x) < g(x)$ (2)
 - 5.6.2 $f(x) \cdot g(x) \geq 0$ (2)
 - 5.7 Calculate the length of AK. (4)
- [18]**

QUESTION 6

The graph of $g(x) = -2^x + 8$ is sketched below. A and B are the y - and x -intercepts respectively of g .



- 6.1 Write down the range of g . (1)
 - 6.2 Determine the coordinates of B. (3)
 - 6.3 If g is reflected over the x -axis to form a new graph h , determine the equation of h . (2)
 - 6.4 Explain why the x -intercepts of g and h are both at B. (2)
- [8]**

QUESTION 7

A hyperbola, h , is described with the following characteristics:

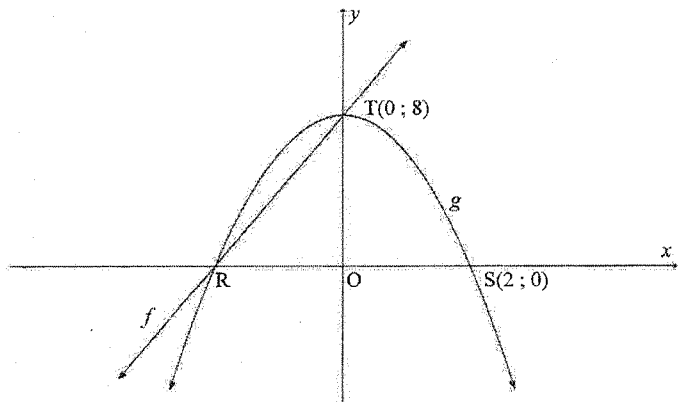
- The equation of the vertical asymptote is $x = 0$
- The range of h is $(-\infty; 3) \cup (3; \infty)$
- The x -intercept of h is $(2; 0)$

Determine the equation of h . (4)



QUESTION 5

The diagram shows the graphs of $g(x) = ax^2 + q$ and $f(x) = mx + c$.
 R and S(2; 0) are the x-intercepts of g and T(0; 8) is the y-intercept of g .
 Graph f passes through R and T.



- 5.1 Write down the range of g . (1)
 - 5.2 Write down the x-coordinate of R. (1)
 - 5.3 Calculate the values of a and q . (3)
 - 5.4 Determine the equation of f . (3)
 - 5.5 Use the graphs to determine the value(s) of x for which:
 - 5.5.1 $f(x) = g(x)$ (2)
 - 5.5.2 $x \cdot g(x) \leq 0$ (3)
 - 5.6 The graph h is obtained when g is reflected along the line $y = 0$.
 Write down the equation of h in the form $h(x) = px^2 + k$. (2)
- [15]



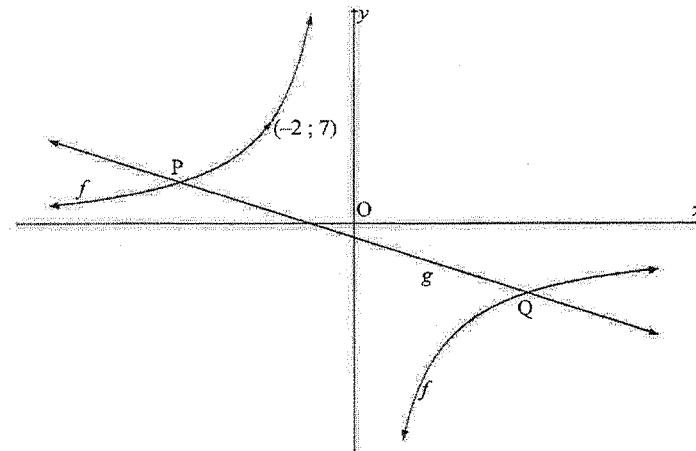
QUESTION 6

6.1 The function $p(x) = k^x + q$ is described by the following properties:

- $k > 0; k \neq 1$
- x-intercept at (2; 0)
- The horizontal asymptote is $y = -9$

- 6.1.1 Write down the range of p . (1)
- 6.1.2 Determine the equation of p . (3)
- 6.1.3 Sketch the graph of p . Show clearly the intercepts with the axes and the asymptote. (3)

6.2 The sketch below shows the graphs of $f(x) = \frac{k}{x} + w$ and $g(x) = -x - 1$.
 The graph g is an axis of symmetry of f . The graphs f and g intersect at P and Q.



- 6.2.1 Write down the value of w . (1)
 - 6.2.2 The point $(-2; 7)$ lies on f . Calculate the value of k . (2)
 - 6.2.3 Calculate the x-coordinates of P and Q. (4)
 - 6.2.4 Write down the values of x for which $\frac{-16}{x} > -x$. (2)
- [16]



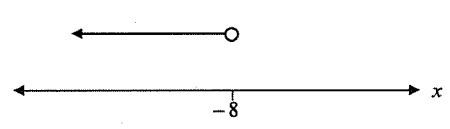
QUESTION 6

6.1		<ul style="list-style-type: none"> ✓ shape of f ✓ x-int of f ✓ x-intercept of g ✓ y-intercept of g <p style="text-align: right;">(4)</p>
6.2	$x = 0$ and $y = 1$	<ul style="list-style-type: none"> ✓ answer ✓ answer <p style="text-align: right;">(2)</p>
6.3	$(-\infty; 0) \cup (0; \infty)$	<ul style="list-style-type: none"> ✓ values ✓ notation <p style="text-align: right;">(2)</p>
6.4	$\frac{3}{x} + 1 = -2x - 4$ $\frac{3}{x} = -2x - 5$ $3 = -2x^2 - 5x$ $2x^2 + 5x + 3 = 0$ $(2x + 3)(x + 1) = 0$ $x = -\frac{3}{2}$ or $x = -1$	<ul style="list-style-type: none"> ✓ $\frac{3}{x} + 1 = -2x - 4$ ✓ standard form ✓ factors ✓ answers <p style="text-align: right;">(5)</p>
6.5	$-1 \leq -2x - 4 < 3$ $3 \leq -2x < 7$ $-1,5 \geq x > -3,5$ $-3,5 < x \leq -1,5$ 'OR' $x \in (-3,5; -1,5]$	<ul style="list-style-type: none"> ✓ $-1 \leq -2x - 4 < 3$ ✓ $3 \leq -2x < 7$ ✓ answer <p style="text-align: right;">(3)</p>
6.6	$k(x) = 2(-2x - 4)$ $= -4x - 8$ y-intercept: $(0; -8)$	<ul style="list-style-type: none"> ✓ equation of $k(x)$ ✓ answer <p style="text-align: right;">(2)</p>
6.7	x-intercept: $(2; 0)$ y-intercept: $(0; -4)$	<ul style="list-style-type: none"> ✓ x-intercept ✓ y-intercept <p style="text-align: right;">(2)</p> <p style="text-align: right;">[20]</p>

QUESTION 7

7.1	$C(-2; 0)$	<ul style="list-style-type: none"> ✓ answer <p style="text-align: right;">(1)</p>
7.2	$f(x) = ax^2 + q$ $f(x) = a(x^2 - 4)$ $2,5 = a((-3)^2 - 4)$ $2,5 = 5a$ $a = \frac{1}{2}$ $f(x) = \frac{1}{2}(x^2 - 4)$	<ul style="list-style-type: none"> ✓ $f(x) = a(x^2 - 16)$ ✓ substitution of $(-5; 2,25)$ ✓ answer <p style="text-align: right;">(3)</p>
7.3	Range of f : $[-2; \infty)$	<ul style="list-style-type: none"> ✓ answer <p style="text-align: right;">(1)</p>
7.4	Range of h : $(-\infty; 0]$	<ul style="list-style-type: none"> ✓ notation ✓ critical values <p style="text-align: right;">(2)</p>
7.5	$g(x) = b^x - 4$ $0 = b^2 - 4$ $4 = b^2$ $b = 2$ $g(x) = 2^x - 4$	<ul style="list-style-type: none"> ✓ $g(x) = b^x - 4$ ✓ substitution ✓ answer <p style="text-align: right;">(3)</p> <p style="text-align: right;">[10]</p>

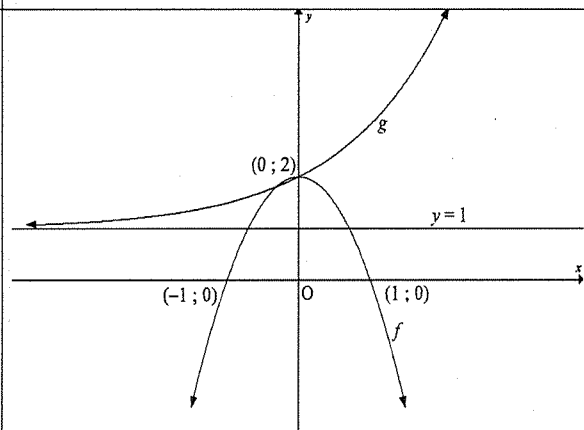
TOTAL: 100

2.2.2		<ul style="list-style-type: none"> ✓ indicating numbers to the left of -8 and -8 not included/ <i>dui getalle links van -8 aan met -8 nie ingesluit</i> <p>(1)</p>
2.3	<p>Let the amount of money Mary had be Rx/Laat die bedrag geld wat Mary gehad het x wees.</p> $\frac{1}{5}x = \frac{1}{3}x - 28$ $3x + 420 = 5x$ $2x = 420$ $x = 210$ <p>Mary had R210/Mary het R210 gehad.</p>	<ul style="list-style-type: none"> ✓ $\frac{1}{3}x - 28$ ✓ $\frac{1}{5}x$ ✓ equation/vergeliking ✓ 210 <p>(4)</p>

QUESTION/VRAAG 3

3.1.1	$-7; -12$	<ul style="list-style-type: none"> ✓ -7 ✓ -12 <p>(2)</p>
3.1.2	$T_n = -5n + 13$	<ul style="list-style-type: none"> ✓ $-5n$ ✓ 13 <p>(2)</p>
3.1.3	$T_n = -5n + 13$ $T_{30} = -5(30) + 13$ $= -137$	<ul style="list-style-type: none"> ✓ substitution of/substitusie van $n = 30$ ✓ answer/antwoord <p>(2)</p>
3.1.4	$-5n + 13 = -492$ $-5n = -505$ $n = 101$	<ul style="list-style-type: none"> ✓ $-5n + 13 = -492$ ✓ answer/antwoord <p>(2)</p>
3.2.1	$T_n = 2n - 1$	<ul style="list-style-type: none"> ✓ $2n$ ✓ -1 <p>(2)</p>
3.2.2	$T_n = (2n - 1)^2$ $= 4n^2 - 4n + 1$	<ul style="list-style-type: none"> ✓ $(2n - 1)^2$ <p>(1)</p>
3.2.3	$T_n = (2n - 1) - (2n - 1)^2$ $= 2n - 1 - (4n^2 - 4n + 1)$ $= 2n - 1 - 4n^2 + 4n - 1$ $= -4n^2 + 6n - 2$	<ul style="list-style-type: none"> ✓ $(2n - 1) - (2n - 1)^2$ ✓ $2n - 1 - (4n^2 - 4n + 1)$ ✓ $2n - 1 - 4n^2 + 4n - 1$ ✓ answer/antwoord <p>(4)</p>

QUESTION/VRAAG 4

4.1	$y = 1$	<ul style="list-style-type: none"> ✓ answer/antwoord <p>(1)</p>
4.2		<p><i>f:</i></p> <ul style="list-style-type: none"> ✓ shape of <i>f/vorm van f</i> ✓ x-intercepts of <i>f/x-afsnitte van f</i> ✓ y-intercept (TP) of <i>f/y-afsnit (DP) van f</i> <p><i>g:</i></p> <ul style="list-style-type: none"> ✓ shape of <i>g/vorm van g</i> ✓ asymptote of <i>g/asimptoot van g</i> ✓ y-intercept of <i>g/y-afsnit van g</i> <p>(6)</p>
4.3	Range of <i>f</i> /Waardeversameling van <i>f</i> : $(-\infty; 2]$	<ul style="list-style-type: none"> ✓ $(-\infty; 2]$ <p>(1)</p> <p>OR/OF</p> <p>Range of <i>f</i>/Waardeversameling van <i>f</i>: $y \leq 2$</p> <ul style="list-style-type: none"> ✓ $y \leq 2$ <p>(1)</p>
4.4	<p>Maximum of $3^{f(x)}$ will be obtained when <i>f</i>(<i>x</i>) is at maximum. Max of <i>f</i>(<i>x</i>) is 2 Max of <i>h</i> will be $3^2 = 9$</p> <p><i>Maksimum van $3^{f(x)}$ sal verkry word wanneer <i>f</i>(<i>x</i>) by maksimum is. Maks van <i>f</i>(<i>x</i>) is 2 Maks van <i>h</i> sal $3^2 = 9$ wees.</i></p>	<ul style="list-style-type: none"> ✓ Max of <i>f</i>(<i>x</i>) is 2/ <i>Maks van <i>f</i>(<i>x</i>) is 2</i> ✓ Max of <i>h</i> = 9/ <i>Maks van <i>h</i> = 9</i> <p>(2)</p>
4.5	<p><i>f</i> would have been reflected in the <i>x</i>-axis <i>f</i> sou in die <i>x</i>-as gereflekteer gewees het</p>	<ul style="list-style-type: none"> ✓ reflected/gereflekteer ✓ in the <i>x</i>-axis/ <i>in die <i>x</i>-as</i> <p>(2)</p>

QUESTION/VRAAG 5

<p>5.1</p> <p>$a = \text{gradient of } g$</p> $= \frac{-4-4}{-1-3}$ $= 2$ $4 = 2(3) + q$ $q = -2$ $g(x) = 2x - 2$ <p>OR/OF</p> <p>$a = \text{gradient of } g$</p> $= \frac{4-(-4)}{3-(-1)}$ $= 2$ $-4 = 2(-1) + q$ $q = -2$ $g(x) = 2x - 2$ <p>OR/OF</p> $g(x) = ax + q$ $4 = 3a + q \dots\dots\dots 1$ $-4 = -a + q \dots\dots\dots 2$ $1 - 2:$ $8 = 4a$ $a = 2$ <p>Substitute in 1/Substitusie in 1:</p> $4 = 3(2) + q$ $q = -2$ $g(x) = 2x - 2$	<p>$\checkmark a = \frac{-4-2}{-1-2}$</p> <p>$\checkmark$ substituting/substitusie B(3 ; 4) (2)</p> <p>$\checkmark a = \frac{4-(-4)}{3-(-1)}$</p> <p>$\checkmark$ substituting/substitusie A(-1 ; -4) (2)</p> <p>\checkmark substituting both points/ substitusie van beide punte</p> <p>\checkmark solving simultaneously/ los gelyktydig op (2)</p>
<p>5.2</p> $\frac{1}{x} - 1 = 2x - 2$ $\frac{1}{x} = 2x - 1$ $1 = 2x^2 - x$ $2x^2 - x - 1 = 0$ $(2x + 1)(x - 1) = 0$ $x = -\frac{1}{2} \quad \text{or} \quad x = 1$	<p>\checkmark equating/gelykstelling</p> <p>\checkmark standard form/ standaardvorm</p> <p>\checkmark factors/faktore \checkmark x-values/-waardes (4)</p>

<p>5.3</p> $-\frac{1}{2} \leq x < 0 \text{ or/of } x \geq 1$ <p>OR/OF</p> $\left[-\frac{1}{2}; 0\right) \cup [1; \infty)$	<p>$\checkmark x \geq -\frac{1}{2}$</p> <p>$\checkmark x < 0$</p> <p>$\checkmark x \geq 1$ (3)</p> <p>$\checkmark [-0,5$</p> <p>$\checkmark 0)$</p> <p>$\checkmark [1; \infty)$ (3)</p>
<p>5.4</p> $f(3) = \frac{1}{3} - 1$ $= -\frac{2}{3}$ <p>Length of BE = $4 - f(3)$</p> $= 4 - \left(-\frac{2}{3}\right)$ $= 4 + \frac{2}{3}$ $= 4\frac{2}{3}$ <p>OR/OF</p> $BE = 2x - 2 - \frac{1}{x} + 1$ $= \frac{2x^2 - x - 1}{x}$ $(x = 3) BE = \frac{2(3)^2 - (3) - 1}{3}$ $= \frac{18 - 4}{3}$ $= 4\frac{2}{3}$	<p>$\checkmark \frac{1}{3} - 1 \text{ or } -\frac{2}{3}$</p> <p>$\checkmark 4 - f(3)$</p> <p>$\checkmark$ answer/antwoord (3)</p> <p>$\checkmark 2x - 2 - \frac{1}{x} + 1$</p> <p>$\checkmark \frac{2(3)^2 - (3) - 1}{3}$</p> <p>$\checkmark$ answer/antwoord (3)</p>
<p>5.5</p> $h(x) = f(x) + 3$ $h(x) = \frac{1}{x} + 2$	<p>\checkmark answer/antwoord (1)</p> <p>[13]</p>

QUESTION/VRAAG 6

6.1	$d - 5 + d - 1 = 0$ $2d = 6$ $d = 3$	$\checkmark d - 5 + d - 1 = 0$ $\checkmark d = 3$ (2)
6.2	$y = a(x - 2)(x + 2)$ $-9 = a(1 - 2)(1 + 2)$ $-9 = a(-1)(3)$ $-3a = -9$ $a = 3$ $f(x) = 3(x^2 - 4)$ $= 3x^2 - 12$ $c = -12$	$\checkmark y = a(x - 2)(x + 2)$ \checkmark subs (1 ; -9) $\checkmark a = 3$ $\checkmark c = -12$ (4) [6]

QUESTION/VRAAG 7

7.1	$\frac{R5000}{9,518569 \text{ rands per dollar}} = \$525,29$ OR/OF $R5000 \times 0,105058 \text{ dollars per rand} = \$525,29$	\checkmark selects/kies 9,518569 \checkmark answer/antwoord (2) \checkmark selects/kies 0,105058 \checkmark answer/antwoord (2)
7.2.1	$A = P(1 + i)^n$ $= 5000(1 + 0,061)^3$ $= R5\ 971,95$	\checkmark formula/formule $\checkmark 5000(1 + 0,061)^3$ $\checkmark R5\ 971,95$ (3)
7.2.2	Let the amount that Zach invests each year be x /Laat die bedrag wat Zach elke jaar belê, x wees. $x(1 + 0,09)^2 + x(1 + 0,09)^1 = 5980$ $x[1,09^2 + 1,09] = 5980$ $x = \frac{5980}{1,09^2 + 1,09}$ $= R2\ 624,99$ OR/OF Let the amount that Zach invests each year be x /Laat die bedrag wat Zach elke jaar belê, x wees. $[x(1 + 0,09)^1 + x(1 + 0,09)^1] = 5980$ $x(2,09)(1,09) = 5980$ $x = \frac{5980}{(2,09)(1,09)}$ $= R2\ 624,99$	$\checkmark x(1 + 0,09)^2$ $\checkmark x(1 + 0,09)^1$ $\checkmark x$ as common factor/ as gemeenskaplike faktor \checkmark answer/antwoord (4) $\checkmark x(1 + 0,09)^1$ $\checkmark [x(1 + 0,09)^1 + x]$ $\checkmark x$ as common factor/ as gemeenskaplike faktor \checkmark answer/antwoord (4) [9]



4.3	$A = P(1+i)^n$ $2P = P(1+i)^5$ $2 = (1+i)^5$ $\sqrt[5]{2} = 1+i$ $i = \sqrt[5]{2} - 1$ $i = 0,148698 \times 100$ $r = 14,87\% \text{ p.a/per jaar}$	$\checkmark A = P(1+i)^n$ $\checkmark 2P = P(1+i)^5$ $\checkmark r = 14,87\% \text{ p.a /pj}$	(3) [16]
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QUESTION 5/ VRAAG 5			
5.1	C(0 ; -4)	\checkmark ans/ant	(1)
5.2	D(0 ; 2)	\checkmark ans/ant	(1)
5.3	$CD = 2 - (-4)$ $CD = 6 \text{ units/eenhede}$	\checkmark ans/ant	(1)
5.4	$x^2 - 4 = 0$ $(x-2)(x+2) = 0$ $x = 2 \quad x = -2$ $B(-2 ; 0)$	$\checkmark y = 0$ \checkmark factors/faktore \checkmark ans/ant	(3)
5.5	$x^2 - 4 = -x + 2$ $x^2 + x - 6 = 0$ $(x-2)(x+3) = 0$ $x = 2 \quad x = -3$ $E(-3 ; 5)$	\checkmark equating/vergeelyk \checkmark factors/faktore \checkmark x-answer/antwoord \checkmark y-answer/antwoord	(4)
5.6.1	$-3 < x < 2$ OR/OF $(-3 ; 2)$	\checkmark values/waardes \checkmark notation/notasie	(2)
5.6.2	$(-\infty ; -2] \cup \{2\}$	$\checkmark (-\infty ; -2]$ $\checkmark 2$	(2)
5.7	$K(-2 ; 4)$ $BK = 4 \text{ units/eenhede}$ $AB = 4 \text{ units/eenhede}$ $AK = \sqrt{4^2 + 4^2} \text{ (Pythagoras)}$ $= 5,66 \text{ units/eenhede}$	\checkmark BK \checkmark AB \checkmark method/methode \checkmark answer/antwoord	(4) [18]

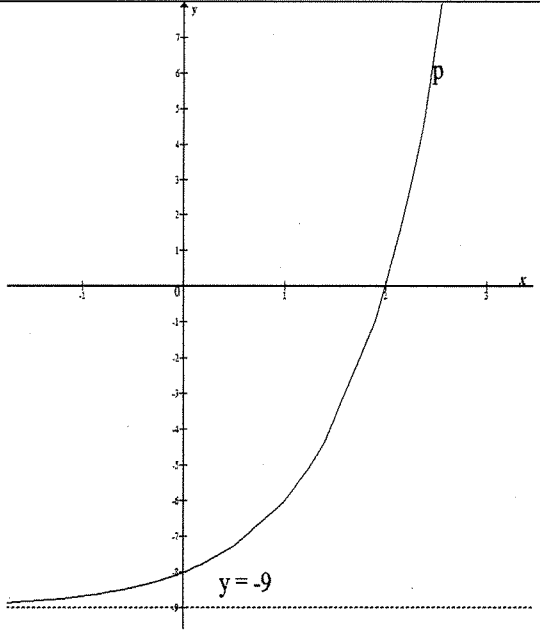
QUESTION 6/VRAAG 6			
6.1	$y < 8$	\checkmark answer/antwoord	(1)
6.2	$-2^x + 8 = 0$ $2^x = 8$ $2^x = 2^3$ $x = 3$ $B(3 ; 0)$	\checkmark equating to 0/vergeelyk met 0 \checkmark simpli/vereenv. \checkmark answer/antwoord	(3)
6.3	$h(x) = 2^x - 8$	$\checkmark\checkmark$ answer/antwoord	(2)
6.4	The graph of g was reflected over the x -axis to form h . This means when $y = 0$, the solution will be the same for both functions. This means that both g and h will have an x -intercept at B. Grafiek g oor die x -as gereflekteer om h te vorm. As $y = 0$, sal die oplossing dieselfde wees vir albei funksies. Beide g en h sal n x -afsnit by B hê.	\checkmark reflection over x -axis/reflek oor x -as \checkmark explanation/verduideliking	(2) [8]

QUESTION 7/VRAAG 7			
	$h(x) = \frac{a}{x} + 3$ $0 = \frac{a}{2} + 3$ $a = -6$ $h(x) = \frac{-6}{x} + 3$	$\checkmark +3$ \checkmark subs of (2 ; 0)/sub van (2 ; 0) \checkmark answer for a / antwoord van a	(4) [4]

QUESTION/VRAAG 5

5.1	Range of/Waardeversameling van $g: y \leq 8$	✓ answ/antw (1)
5.2	$x = -2$	✓ answ/antw (1)
5.3	$g(x) = ax^2 + 8 \Rightarrow q = 8$ $g(2) = a(2)^2 + 8 = 0$ $\Rightarrow a = -2$	✓ $q = 8$ ✓ subst./verv. (2 ; 0) ✓ $a = -2$ (3)
5.4	$f(x) = mx + c \Rightarrow c = 8$ $f(-2) = -2m + 8 = 0$ $\Rightarrow m = 4$ $f(x) = 4x + 8$	✓ $c = 8$ ✓ subst./verv. (-2 ; 0) ✓ $m = 4$ (3)
5.5.1	$x = -2$ or $x = 0$	✓ $x = -2$ ✓ $x = 0$ (2)
5.5.2	$x \cdot g(x) \leq 0$ $-2 \leq x \leq 0$ or $x \geq 2$	✓ $-2 \leq x \leq 0$ or ✓ $x \geq 2$ (3)
5.6	$h(x) = -(-2x^2 + 8)$ $h(x) = 2x^2 - 8$	✓ $\sqrt{2x^2 - 8}$ (2)
		[15]

QUESTION/VRAAG 6

6.1.1	The range/Waardeversameling van is $y > -9$	✓ answ/antw (1)
6.1.2	$p(x) = k^x + q$ $p(x) = k^x - 9$ $0 = k^2 - 9$ $k^2 = 9$ $k = \pm 3$ $k = 3$ since $k > 0$ $p(x) = 3^x - 9$	✓ $q = -9$ ✓ subst./verv. (2 ; 0) ✓ $k = 3$ (3)
6.1.3		✓ asymptote/asimptoot ✓ intercepts/afsnitte ✓ shape/vorm (3)

6.2.1	$w = -1$	✓ answ/antw (1)
6.2.2	$f(x) = \frac{k}{x} - 1$ $7 = \frac{k}{-2} - 1$ $k = -16$	✓ subst./verv. (2 ; -7) ✓ answ/antw (2)
6.2.3	$f(x) = g(x)$ $\frac{-16}{x} - 1 = -x - 1$ $x^2 - 16 = 0$ $(x - 4)(x + 4) = 0$ $x_Q = 4$ or $x_P = -4$	✓ equating/verg. ✓ simpl./vereenv ✓ $x = -4$ at/by P ✓ $x = 4$ at Q (4)
6.2.4	$-4 < x < 0$ or $x > 4$	✓ $-4 < x < 0$ ✓ $x > 4$ (2)
		[16]

QUESTION/VRAAG 5		
5.1.1	D(0; -3)	✓ x value/x-waarde ✓ y value/y-waarde (2)
5.1.2	Range: $y > -4$	✓ answer/antwoord (1)
5.2.1	$0 = \left(\frac{1}{2}\right)^x - 4$ $2^{-x} = 4$ $2^{-x} = 2^2$ $x = -2$ A(-2; 0)	✓ equating to 0/ gelykstelling aan 0 ✓ answer/antwoord (2)
5.2.2	$f(x) = ax^2 + q$ $3 = a(1)^2 + q$ at E(1; 3) $3 = a + q$(1) $0 = a(-2)^2 + q$ at A(-2; 0) $0 = 4a + q$ $q = -4a$(2) $a = -1$ $q = 4$	✓ subst./verv. ✓ subst./verv. ✓ a value/a-waarde ✓ q value/q-waarde (4)
5.3.1	$CD = y_C - y_D$ $= 4 - (-3)$ $= 7$ units/eenhede	✓ subst./verv. ✓ answer/antwoord (2)
5.3.2	$y = mx + c$ $y = -\frac{3}{2}x + c$ $0 = -\frac{3}{2}(-2) + c$ $c = -3$ $y = -\frac{3}{2}x - 3$	✓ m value/m-waarde ✓ subst./verv. ✓ equation/ vergelyking (3)
5.4.1	$-2 < x < 2$ OR $x \in (-2; 2)$	✓ critical values/ kritieke waardes ✓ notation/notasie (2)
5.4.2	$x > 0$ OR $x \in (0; \infty)$	✓ answer/antwoord (1)
		[17]



QUESTION/VRAAG 6		
6.1.1	$g(x) = \frac{a}{x} + q$ $2 = \frac{a}{3} + 1$ $a = 3$ $\therefore g(x) = \frac{3}{x} + 1$	✓ $q = 1$ ✓ subst./verv. ✓ answer/antwoord (3)
6.1.2	$h(x) = x + 1$	✓ positive gradient/positiewe gradiënt ✓ answer/antwoord (2)
6.2		✓ shape of g/ vorm van g ✓ asymptotes/asimptote ✓ positive gradient of h/positiewe gradiënt van h ✓ x intercepts of g and h 6/ x-afsnitte van g en h 6 (4)
6.4	$g(x) = -\left(\frac{3}{x} + 1\right) + 5$ $g(x) = -\frac{3}{x} + 4$ $x = 0$ $y = 4$	✓ equation of g/ vergelyking van g ✓ $x = 0$ ✓ $y = 4$ (3)
		[12]

FUNCTIONS (GRAPHS)

TOPIC DISCUSSIONS

THE LINEAR FUNCTION (STRAIGHT LINE GRAPH)

The equation in **general form**: $ax + by + c = 0$
 The equation in **standard form**: $y = mx + q$ / $y = mx + c$



THE MOST IMPORTANT FEATURE OF THE STRAIGHT LINE IS ITS **GRADIENT**

$$m = \frac{\text{change in } y (\Delta y)}{\text{change in } x (\Delta x)} = \frac{\text{rise}}{\text{run}}$$

m is usually used to denote the gradient.



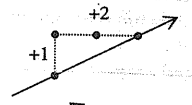
THE GRADIENT DETERMINES THE BASIC SHAPE OF THE GRAPH

I.2.1 Gradient (Slope) - $y = mx + q$

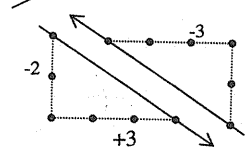
Gradient is the way in which a point **MOVES** on a straight line. This value is the coefficient of x i.e. the number in front of x .
 For a straight line this is a **CONSTANT** rate of change.

EXAMPLES

If $m = \frac{1}{2} = \frac{\text{rise}}{\text{run}}$



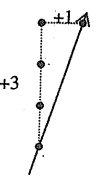
If $m = -\frac{2}{3} = \frac{\text{rise}}{\text{run}}$



Note: The negative can be used for the rise or for the run but NOT for both.

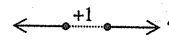
If $m = 3$ (first change this into a proper fraction):

$\therefore m = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$



This is like going up stairs! First lift the foot, then move it across, or else you will trip!

If $m = 0 = \frac{0}{1}$



Rise is zero.

If $m = \text{undefined} = \frac{1}{0}$



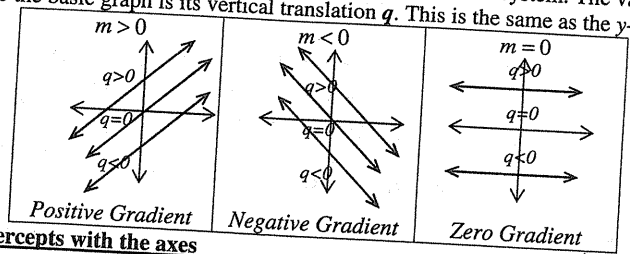
Run is zero.

SUMMARY

$m > 0$	$m < 0$	$m = 0$	$m \text{ undefined}$
Positive Gradient (Increasing function)	Negative Gradient (Decreasing function)	Zero Gradient (constant function)	Undefined Gradient (Not a function)

I.2.2 Vertical translation - $y = mx + q$

Once the gradient of the straight line has been determined we need to determine where to place it on the Cartesian Plane i.e. the x, y coordinate system. The value added to the basic graph is its vertical translation q . This is the same as the y -intercept



I.2.3 Intercepts with the axes

Irrespective of the graph that is being drawn the intercepts are always found as follows: For the y intercept always make $x = 0$ and for the x intercept(s) make $y = 0$

I.2.4 Drawing the Straight Line Graph

Remembering the importance of the shape as well as the position on the axes, the following is needed to draw the graph of the function.

1. The equation in the form $y = mx + q$ (Standard form)
2. The vertical translation, q .
3. The y -intercept - let $x = 0$ (same as the vertical translation)
4. The x -intercept - let $y = 0$
5. The gradient, m (the coefficient of x when the equation is in standard form)

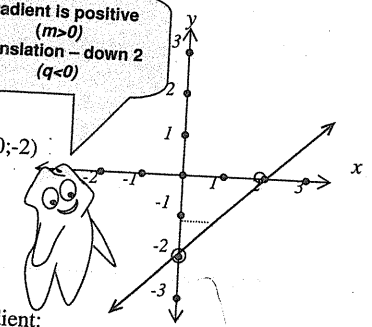
EXAMPLE 1

Sketch the graph of f where $f: x \rightarrow x - 2$

- Write the equation in the form $y = mx + q$:
 $y = x - 2$ (Already in standard form)
- Determine the vertical translation: $q = -2$
- Determine the y -intercept ($x = 0$): as coordinates $(0; -2)$
- Determine the x -intercept ($y = 0$):
 $0 = x - 2 \therefore x = 2$ as coordinates $(2; 0)$
- Determine the gradient m in $y = x - 2$:

$$m = 1 = \frac{1}{1} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

Gradient is positive ($m > 0$)
Translation - down 2 ($q < 0$)



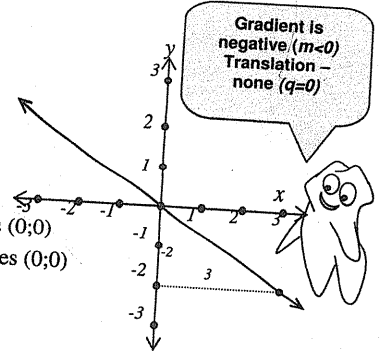
- Draw the axes and plot the points, drawing in the gradient:

EXAMPLE 2

Sketch the graph of $\{(x; y): 2x + 3y = 0\}$

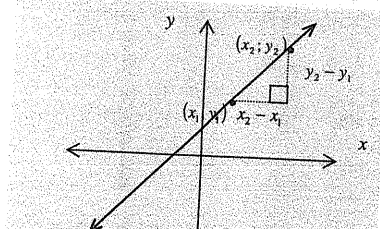
- Write the equation in the form $y = mx + q$:
 $2x + 3y = 0$
 $3y = -2x$
 $y = -\frac{2}{3}x$
- Vertical translation: $q = 0$
- y -intercept ($x = 0$): $y = -\frac{2}{3} \cdot 0 = 0$ coordinates $(0; 0)$
- x -intercept ($y = 0$): $0 = -\frac{2}{3}x \therefore x = 0$ coordinates $(0; 0)$
- gradient m : $m = -\frac{2}{3} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

Gradient is negative ($m < 0$)
Translation - none ($q = 0$)



1.2.5 More About Gradients

1.2.5.1 Gradient of a line passing through two points



The gradient of a line passing through two points $(x_1; y_1)$ and $(x_2; y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE Determine the gradient of the line passing through $(2; -3)$ and $(1; 4)$.

In graphs of curves the **AVERAGE** gradient is found using this formula.

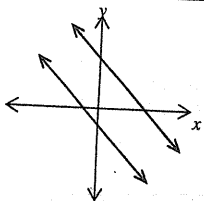
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - (-3)}{1 - 2}$$

$$m = \frac{4 + 3}{-1}$$

$$m = -7$$

1.2.5.2 Gradients of parallel lines



Parallel lines have the same slope, i.e. gradient.

$$\therefore m_1 = m_2$$

EXAMPLE

Determine the gradient of the line parallel to $4x - 7y = 7$ and passing through $(2; 3)$.

For the gradient: the gradient of $4x - 7y = 7$ is needed:

$$-7y = -4x + 7$$

$$y = \frac{-4}{-7}x - \frac{7}{-7}$$

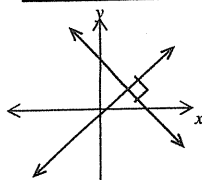
$$y = \frac{4}{7}x - 1$$

Always divide these separately to determine the gradient and translation.

$$\therefore \text{gradient of given line: } m = \frac{4}{7}$$

$$\therefore \text{gradient of new line: } m = \frac{4}{7}$$

1.2.5.3 Gradients of perpendicular lines



The product (multiplication) of the gradients of perpendicular lines is negative one.

$$m_1 \times m_2 = -1$$

EXAMPLE Determine the gradient of the line passing through the origin, perpendicular to $2x + 3y = 6$.

The gradient of $2x + 3y = 6$ is needed.

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

$$\therefore m = -\frac{2}{3}$$

For perpendicular gradients use

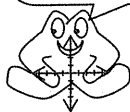
$$m_1 \times m_2 = -1$$

$$-\frac{2}{3} \times m_2 = -1$$

$$-2m_2 = -3$$

$$\therefore m_2 = \frac{3}{2}$$

The origin is the point $(0; 0)$.



PARABOLA / QUADRATIC FUNCTION

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The equation in completed square form is $y = a(x - h)^2 + k$. THE MOST IMPORTANT FEATURE OF THE PARABOLA IS ITS

TURNING POINT

1.4.1 The Turning Point $y = ax^2 + q$

The value of a determines whether the graph has a **minimum** or **maximum** turning point. The turning point lies on its axis of symmetry.

If $a > 0$ (positive) minimum turning point	If $a < 0$ (negative) maximum turning point
First a <i>decreasing</i> then an <i>increasing</i> function	First an <i>increasing</i> then a <i>decreasing</i> function

1.4.2 Vertical translation: $y = ax^2 + q$

If $q = 0$ the graph will turn at the origin. q determines the vertical translation of the graph.

If $a > 0$ (positive)	If $a < 0$ (negative) maximum turning point
$q > 0$ $q = 0$ $q < 0$	$q > 0$ $q = 0$ $q < 0$

1.4.3 Intercepts with the axes

For the y -intercept let $x = 0$. For the x -intercepts let $y = 0$ and solve the quadratic equation. If there are x -intercepts these, as well as the y -intercept (turning point) can be used as the points to draw the graph.

1.4.4 Drawing the graph of the Parabola

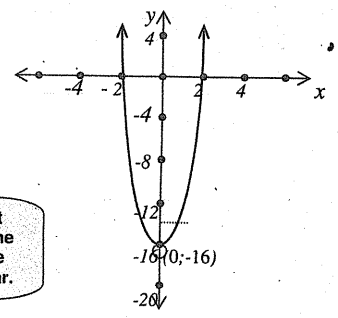
The following is needed for the graph:

1. The shape i.e. if $a > 0$ $a < 0$
2. Vertical translation: q
3. The coordinates of the Turning Point.
4. The y -intercept - let $x = 0$
5. The x -intercept - let $y = 0$ - factorise if possible.
6. If there are no x intercepts choose an x value and find its corresponding y value. Then using symmetry find another ordered pair. (see example 2)

Always try to read:
 $a > 0$ as: a is positive
 $a < 0$ as: a is negative.

EXAMPLE 1 Sketch the graph of $p(x) = 4x^2 - 16$

- Shape: $a = 4 \therefore$ min
- Vertical translation: $q = -16$
- The y-intercept - let $x = 0$:
 $y = 4(0)^2 - 16$
 $y = -16$
- The Turning Point: $(0; -16)$

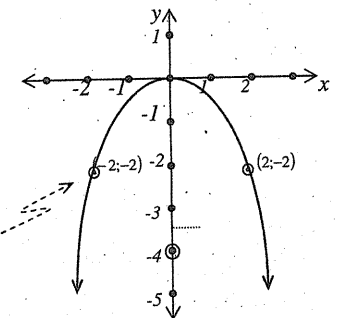
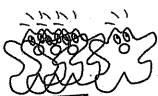


At this stage the y intercept and vertical translation are the same. This is not always the case as you will see next year.

- The x-intercept - let $y = 0$:
 $0 = 4x^2 - 16$
 $0 = 4(x^2 - 4)$
 $0 = 4(x - 2)(x + 2)$
 $\therefore x = 2$ or $x = -2$

EXAMPLE 2 Sketch the graph of $\{(x; y) : y = -\frac{1}{2}x^2\}$

- Shape: $a = -\frac{1}{2} \therefore$ max
- Vertical translation: $q = 0$
- The Turning Point: $(0; 0)$
- The y-intercept - let $x = 0$:
 $y = 0$
- The x-intercept - let $y = 0$:
 $\therefore x^2 = 0$
 $\therefore x = 0$



When so many points coincide choose an x value and find its corresponding y value. Then use symmetry to get another point on the graph.

Using symmetry.

- Other points:
If $x = 2$ then $y = -\frac{1}{2}(2)^2 = -\frac{1}{2} \cdot 4 = -2$
 $\therefore (2; -2)$ lies on the graph

DETERMINING THE EQUATION OF THE PARABOLA

I.4.5.1 Given 2 points on the graph

1. Substitute for x and y in $y = ax^2 + q$ twice.
2. Solve simultaneously, to determine the values of a and q .

I.4.5.2 Given the graph

1. If possible look to see if you can find the vertical translation, substitute this value for q .
2. Find a point on the graph and substitute for x and y .
3. If the vertical translation cannot be found, find two points and substitute as before.

EXAMPLE

Determine the equation of the parabola passing through $(0;3)$ and $(1;1)$

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$$3 = a(0) + q$$

$$3 = q$$

$$1 = a(-1)^2 + q$$

$$1 = a + q$$

$$1 = a + 3$$

$$a = -2$$

Note: If the x coordinate is 0 the translation will be the y value.



- Write down the equation with the values:

$$y = -2x^2 + 3$$

THE HYPERBOLA



The equation in **general form**: $xy = k + qx$
The equation in **standard form**: $y = \frac{k}{x} + q$

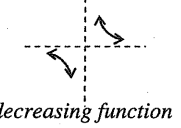
THE MOST IMPORTANT FEATURE OF THE HYPERBOLA IS THE QUADRANTS IN WHICH THE GRAPH OCCURS RELATIVE TO ITS ASYMPTOTES. (shape)
NB: An ASYMPTOTE IS A LINE THAT A GRAPH APPROACHES BUT NEVER REACHES.



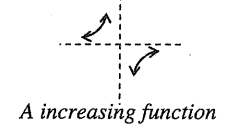
I.3.1 Quadrants: $y = \frac{k}{x} + q$

The value of k determines the quadrants in which the hyperbola will lie.

If $k > 0$ (Positive) the graph lies in quadrants I and III.



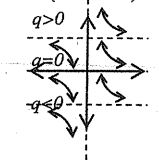
If $k < 0$ (Negative) the graph lies in quadrants II and IV.



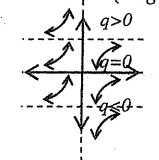
I.3.2 Vertical translation: $y = \frac{k}{x} + q$

Once the shape of the hyperbola has been determined we need to determine where to place it on the Cartesian Plane i.e. the x, y coordinate system. If $q = 0$, the asymptotes will coincide with the axes. The value added to the basic graph is its vertical translation. The equation of this asymptote must be given i.e. $y = q$.

If $k > 0$ (Positive)



If $k < 0$ (Negative)



I.3.3 Intercepts with the axis of symmetry

A critical point on the graph of the hyperbola is where it intersects its axis of symmetry. The axis of symmetry will be either $y = x + q$ or $y = -x + q$.

The coordinates will be $(\pm\sqrt{k}; \pm\sqrt{k} + q)$

I.3.4 Drawing the Graph of the Hyperbolic Function

The following are needed for the graph:

1. The quadrants.
2. The asymptotes must be clearly indicated (usually with a broken line) $x = 0$; $y = q$
3. The axis of symmetry that passes through the graph and its points of intersection with the graph. $(\pm\sqrt{k}; \pm\sqrt{k} + q)$
4. A table of x and y coordinates through which the graph passes. Symmetry can be used here.

I.3.3 To Determine the Equation of the Hyperbola

For the equation the values of k and q must be found, therefore two "bits" of information need to be given.

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1. Substitute for x and y in $y = \frac{k}{x} + q$ or $xy = k + qx$ twice.
2. Solve simultaneously, to determine the values of k and q .

It is easier to use $xy = k + qx$.

I.3.3.2 Given the graph

1. If possible look to see if you can find the vertical translation; substitute this value for q .
2. Find a point on the graph and substitute for x and y .
3. If the vertical translation cannot be found, find two points and substitute as before.

EXAMPLE

Determine the equation of the hyperbola passing through $(2; -3)$ and $(-1; 12)$

- Substitute twice into $xy = k + qx$:

Substitute $(2; -3)$	Substitute $(-1; 12)$
$-3 \times 2 = k + 2q$	$12 \times -1 = k - 1q$
$-6 = k + 2q$	$-12 = k - q$
- Solve by elimination: Write one equation under the other and subtract

$$\begin{array}{r} -6 = k + 2q \\ -(-12 = k - q) \\ \hline 6 = 3q \\ \therefore q = 2 \end{array}$$

This eliminates k .

- Substitute for q in either of the equations:

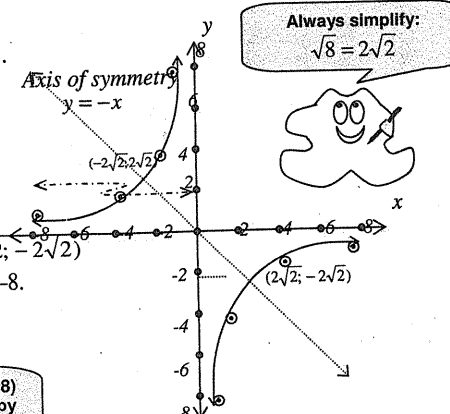
$$\begin{array}{r} -6 = k + 2(2) \\ -6 = k + 4 \\ \therefore k = -10 \end{array}$$

- Write down the equation: $xy = -10 + 2x$ or $y = -\frac{10}{x} + 2$

EXAMPLE 1 Sketch the graph of $h(x) = -\frac{8}{x}$.

- Quadrants: $k = -8$
 $k < 0 \therefore$ quads II and IV
- Vertical translation: $q = 0$
- Asymptotes: $x = 0$; $y = 0$
- Axis of symmetry: $y = -x$
- Intercepts with $y = -x$: $(-2\sqrt{2}; 2\sqrt{2})$ $(2\sqrt{2}; -2\sqrt{2})$
- Table: The product of x and y must equal -8 .

x	2	4	1	8	-4	-2	-8	-1
y	-4	-2	-8	-1	2	4	1	8



Always simplify: $\sqrt{8} = 2\sqrt{2}$

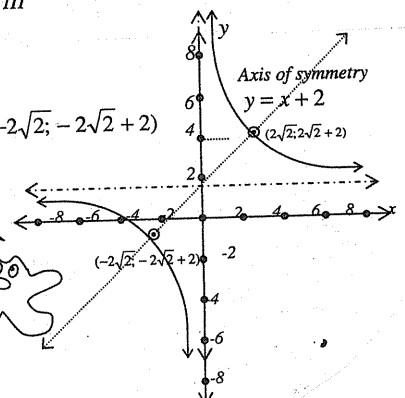


Just using the points $(2; 4)$ and $(1; 8)$ all the other points can be found by using symmetry.

EXAMPLE 2 Sketch the graph of $h(x) = \frac{8}{x} + 2$.

- Quadrants: $k = 8$: $k > 0 \therefore$ quads I and III
- Vertical translation: $q = +2$
- Asymptotes: $x = 0$; $y = 2$
- Axis of symmetry: $y = x + 2$
- Intercepts with $y = x + 2$: $(2\sqrt{2}; 2\sqrt{2} + 2)$ $(-2\sqrt{2}; -2\sqrt{2} + 2)$
- Table:

x	2	4	1	8	-4	-2	-8	-1
y	6	4	10	-1	0	-2	-1	-6



Symmetry can't really be used here.

Certain calculators have a table function that helps you work this out.

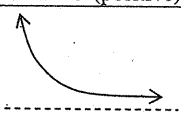
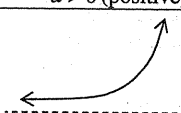
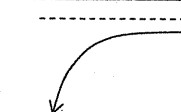
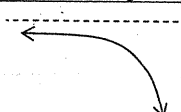
EXPONENTIAL GRAPH

The equation in **standard form**: $y = ab^x + q; b > 0$

THE MOST IMPORTANT FEATURE OF THE EXPONENTIAL FUNCTION IS WHETHER IT IS INCREASING OR DECREASING

I.5.1 Increasing or decreasing function $y = ab^x + q; b > 0$

The value of a determines on which side of the asymptote the graph will lie. The value of b determines whether the function will increase or decrease. It has already been noted that b must be positive, $b > 0$, but if b is a fraction of 1, or if $b > 1$ the function increases or decreases as indicated below.

If $0 < b < 1$ and $a > 0$ (positive)	If $b > 1$ and $a > 0$ (positive)
	
Note the graph is decreasing	Note the graph is increasing
If $0 < b < 1$ and $a < 0$ (negative)	If $b > 1$ and $a < 0$ (negative)
	
Note the graph is now increasing	Note the graph is now decreasing

NOTE:

If $0 < b < 1$ i.e. b is a fraction we can rather write it as a whole number using a negative exponent,

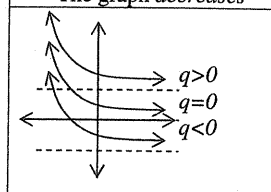
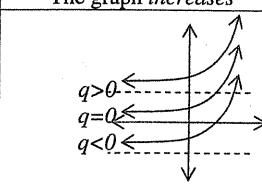
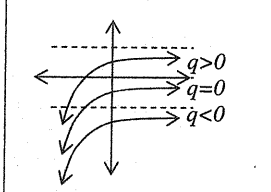
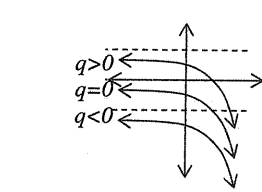
e.g. $\left(\frac{1}{2}\right)^x = 2^{-x}$



I.5.2 Vertical translation/Asymptote - $y = ab^x + q$

Once again the value of q determines the vertical translation of the graph. When $q = 0$ the asymptote coincides with the x -axis. Hence the equation of the asymptote will be $y = q$.

Downloaded from Stanmorephysics.com

If $0 < b < 1; a > 0$ The graph decreases	If $b > 1; a > 0$ The graph increases
	
If $0 < b < 1; a < 0$ The graph increases	If $b > 1; a < 0$ The graph decreases
	

I.5.3 Intercepts with the axes

For the y intercept let $x = 0$. Thus $y = ab^0 + q \therefore y = a + q$. So the coordinates of the y intercept is $(0; a + q)$.

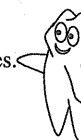
For the x intercept let $y = 0$. Thus $0 = ab^x + q$ and solve.

I.5.4 Drawing the graph of the exponential function

The following are needed for the graph:

1. The shape, increasing or decreasing
2. The asymptote $y = q$
3. The y intercept $(0; a + q)$
4. Two other points through which the graph passes.

The best values to substitute for x are 1 and -1.



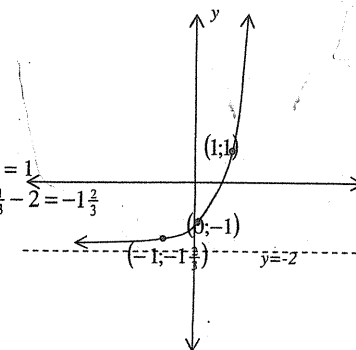
EXAMPLE 1

Sketch the graph of $f(x) = 3^x - 2$

- Increasing: $a > 0$ and $b > 1$
- The asymptote: $y = -2$
- The y intercept: $(0; 1 - 2) = (0; -1)$
- Other points: Let $x = 1$ then $y = 3 - 2 = 1$

Let $x = -1$ then $y = 3^{-1} - 2 = \frac{1}{3} - 2 = -1\frac{2}{3}$

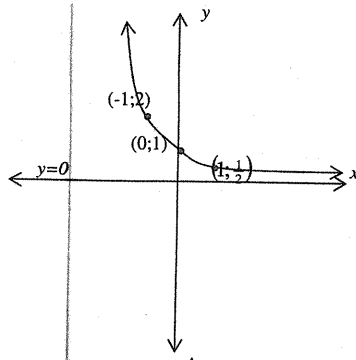
Points: $(1; 1); (-1; -1\frac{2}{3})$



EXAMPLE 2

Sketch the graph of $f(x) = 2^{-x}$

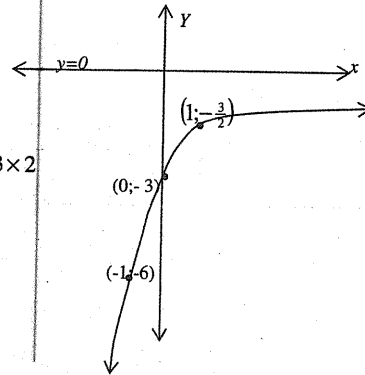
- Decreasing: $a > 0; 0 < b < 1$ ($-x$)
- The asymptote $y = 0$ (x -axis)
- The y intercept $(0;1)$
- Other points: Let $x = -1$ then $y = 2^{-(-1)} = 2$
 $y = 2$
Let $x = 1$ then $y = 2^{-1} = \frac{1}{2}$
Points: $(-1;2); (1;\frac{1}{2})$



EXAMPLE 3

Sketch the graph of $f(x) = -3(\frac{1}{2})^x$

- Increasing: $a < 0; 0 < b < 1$ ($-x$)
- The asymptote $y = 0$ (x -axis)
- The y intercept $(0;-3)$
- Other points: Let $x = -1$ then $y = -3(\frac{1}{2})^{-1} = -3 \times 2$
 $y = -6$
Let $x = 1$ then $y = -3(\frac{1}{2})^1 = -\frac{3}{2}$
Points: $(-1;-6); (1;-\frac{3}{2})$



TO DETERMINE THE EQUATION OF THE EXPONENTIAL GRAPH

The values of a , b and q must be determined. These are then substituted into $y = ab^x + q$. At this stage a will be given as 1 or (-1)

1.5.5.1 Given 2 points on the graph

- Substitute for x and y in $y = b^x + q$ or $y = -b^x + q$ twice.
- Solve simultaneously, to determine the values of b and q .

1.5.5.2 Given the graph

- If possible look to see if you can find the vertical translation, then substitute this value for q .
- Find a point on the graph and substitute for x and y .
- If the vertical translation cannot be found, find two points and substitute as before.

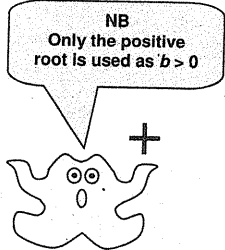
EXAMPLE

Determine the equation of the exponential graph passing through $(0; -3)$ and $(2; -6)$ where $a = -1$

- Substitute into $y = -b^x + q$

$$\begin{aligned} -3 &= -b^0 + q \\ -3 &= -1 + q \\ q &= -2 \end{aligned}$$

$$\begin{aligned} -6 &= -b^2 + q \\ -6 &= -b^2 - 2 \\ -4 &= -b^2 \\ b^2 &= 4 \\ b &= 2 \end{aligned}$$



- Write down the equation: $y = -2^x - 2$

TO DETERMINE A LINEAR / STRAIGHT LINE FUNCTION

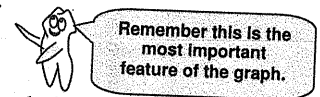
When given information or even the graph, it is often important to be able to determine the equation (mathematical model) of the linear function. For the equation $y = mx + q$ the value of m (the gradient) and q (c) (vertical translation) must be determined. These values are then substituted back into the equation. Since there are two unknowns, two "bits" of information must be given.

To find m , the gradient:

Find m by using the relevant method from Section I.2.5.

To find q :

The last value required in any equation is often found by substituting a given point for x and y into the equation found so far, and hence solve for q . Remember this is the vertical translation, so if it is given, use it.



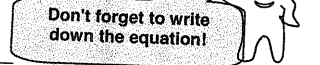
EXAMPLE 1

Determine the equation of the straight line passing through $(-1; -2)$ and $(-5; 6)$.

- Determine m :

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{6 - (-2)}{-5 - (-1)} \\ m &= \frac{6+2}{-5+1} \\ m &= -2 \end{aligned}$$

- Equation so far: $y = -2x + q$
- Substitute either point for x and y :
Using $(-1; -2)$ $x = -1; y = -2$
 $-2 = -2(-1) + q$
 $-2 = 2 + q$
 $-4 = q$
 $\therefore y = -2x - 4$



EXAMPLE 2

Find the equation of the straight line passing through $(4; -1)$ perpendicular to $4 - y = 2x$.

- Determine m :

First find the gradient of the given line - $4 - y = 2x$.

$$y = -2x + 4 \quad \therefore m = -2$$

For perpendicular lines:

$$\begin{cases} m_1 \times m_2 = -1 \\ \therefore -2 \times m_2 = -1 \\ m = \frac{-1}{-2} = \frac{1}{2} \end{cases}$$

- Equation so far: $y = \frac{1}{2}x + q$
- Substitute point for x and y :
Using $(4; -1)$ $x = 4; y = -1$
 $-1 = \frac{1}{2}(4) + q$
 $-1 = 2 + q$
 $-3 = q$
 $\therefore y = \frac{1}{2}x - 3$

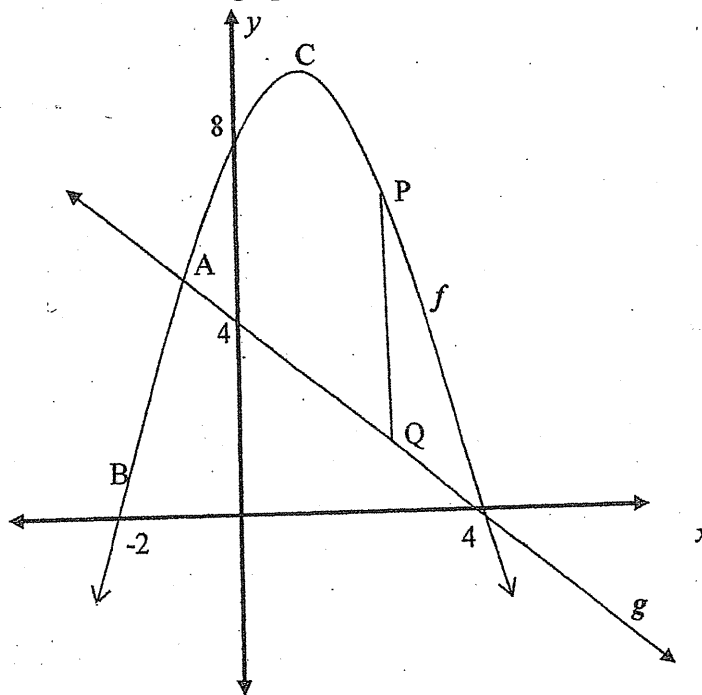
-6-

1. Consider the function $f(x) = \frac{4^x}{x+1} - 1$
- 1.1. Write down the equations of the asymptotes of f . (2)
 - 1.2. Calculate the intercepts of the graph with the axes. (4)
 - 1.3. Sketch the graph of f in your answer book. (3)
 - 1.4. Write down the domain of $y = \frac{4}{x+1} - 1$. (2)
 - 1.5. If the graph of f was moved 2 units to the right to give us $g(x)$, write down the equation of $g(x)$. (2)

[13]

QUESTION TWO

The graph of f cuts the x axis at -2 and 4 and cuts the y axis at 8 . The straight line, g , cuts the y axis at 4 and the x axis at 4 . The graphs of f and g intersect at the points A and B .

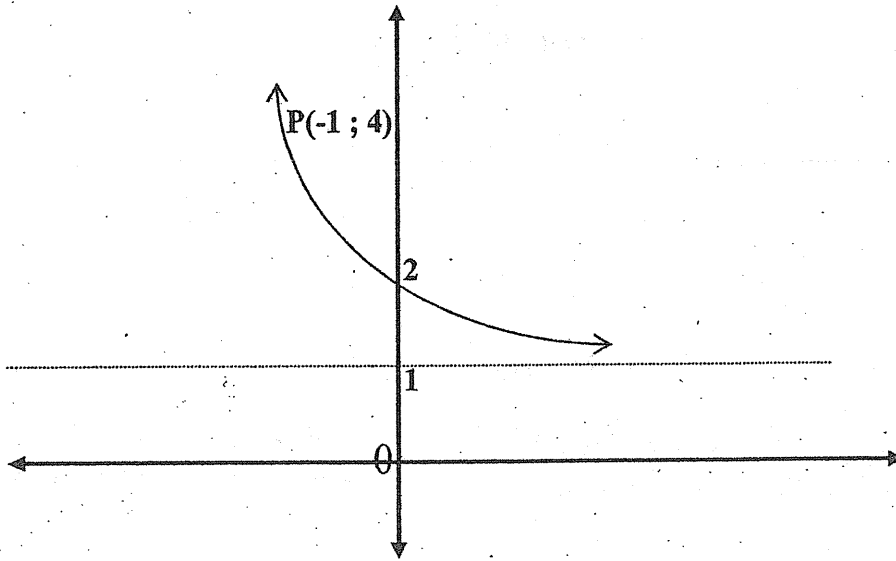


- 2.1. Determine the equation of f in the form $y = ax^2 + bx + c$. (6)
- 2.2. Determine the equation of g . (4)
- 2.3. Determine the coordinates of A . (5)
- 2.4. Determine the coordinates of C , the turning point of f . (4)
- 2.5. Determine the length of PQ if the x value at P is 3 and $PQ \parallel y$ axis. (4)
- 2.6. Determine the value of x if:
 - 2.6.1. $f(x) \geq 0$ (2)
 - 2.6.2. $f(x) \geq g(x)$ (2)

[27]

QUESTION THREE

The figure below show the sketch of $g(x) = a^x + q$. The point $P(-1; 4)$ lies on g .



3.1. Write down the value of q . (2)

3.2. Determine the value of a and hence write down the equation of g . (5)

3.3. Write down the equation of $h(x)$ if it is the reflection of $g(x)$ about the y -axis. (3)

[10]

QUESTION 4

$$g(x) = -2^x + 8$$

- 1) Determine the x and y intercepts
- 2) Draw g graph
- 3) Write down the domain and range.

Grade 10. Revision.

$$5. \quad f(x) = -2x^2 + 18 \quad \text{and} \quad g(x) = -3x + 9$$

5.1. Det the y int of f .5.2. Det the x int of f .5.3. Det the T.P of f .5.4. Sketch the graph of f .5.5. Det the range of f .5.6. Sketch the graph of g .5.7. Det x if

$$(a) \quad f(x) = g(x)$$

$$(b) \quad f(x) < 0$$

$$(c) \quad f(x) \leq g(x)$$

QUESTION 6

$$\text{Given: } f(x) = \frac{3}{x} + 1 \quad \text{and} \quad g(x) = -2x - 4$$

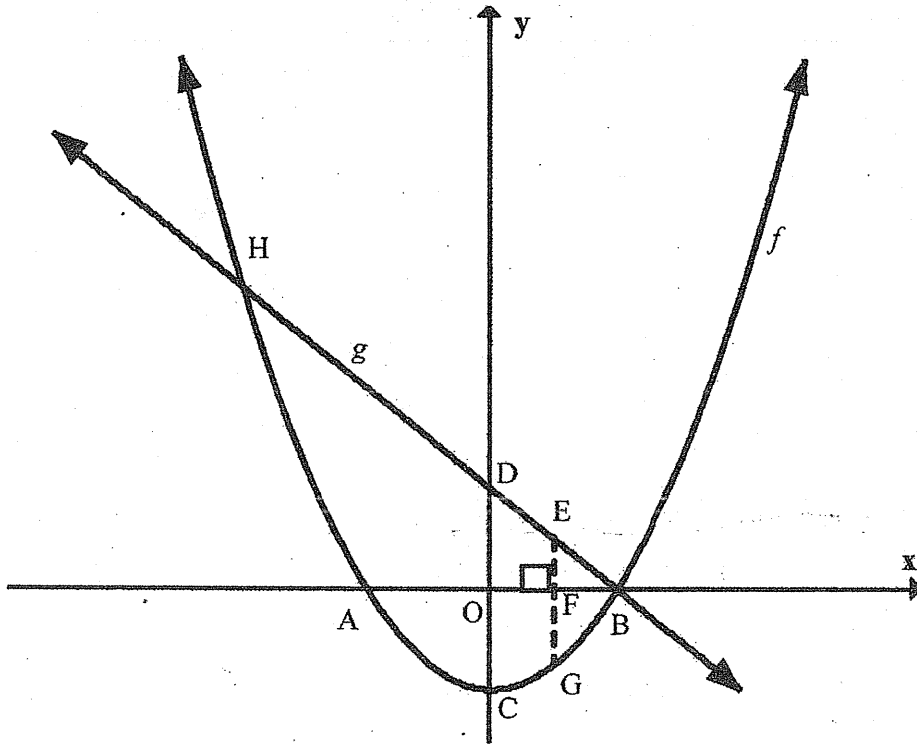
- 6.1 Sketch the graphs of f and g on the same set of axes. (4)
- 6.2 Write down the equations of the asymptotes of f . (2)
- 6.3 Write down the domain of f . (2)
- 6.4 Solve for x if $f(x) = g(x)$. (5)
- 6.5 Determine the values of x for which $-1 \leq g(x) < 3$. (3)
- 6.6 Determine the y -intercept of h if $h(x) = 2g(x)$. (2)
- 6.7 Write down the coordinates of the x - and y -intercepts of h if h is the graph of g reflected about the y -axis. (2)

[20]

QUESTION 7

Sketched below are the graphs of $f(x) = 2x^2 - 2$ and $g(x) = -2x + 2$.

The graph of f intersects the x -axis at A and B and the y -axis at C. The graph of g intersects the x -axis at B and the y -axis at D. f and g intersect at H and B.



Use the graphs and the information above to determine the following:

- 7.1 The coordinates of A and B. (4)
 - 7.2 The coordinates of C. (1)
 - 7.3 The coordinates of D. (1)
 - 7.4 The length of EG if $OF = \frac{1}{2}$ unit and E lies on g and G lies on f . (5)
- [11]**