PROBABILITY

- **Outcome**: a single observation of an experiment.
- **Sample space** of an experiment: the set of all possible outcomes of the experiment.
- **Event**: a set of outcomes of an experiment.
- **Probability** of an event: a real number between 00 and 11 that describes how likely it is that the event will occur.
- **Relative frequency** of an event: the number of times that the event occurs during experimental trials, divided by the total number of trials conducted.
- **Union** of events: the set of all outcomes that occur in at least one of the events, written as P(A or B)
- **Intersection** of events: the set of all outcomes that occur in all of the events, written as "P(A and B)".
- **Mutually exclusive events**: events with no outcomes in common, that is (A and B)= 0.
- **Complementary events**: two mutually exclusive events that together contain all the outcomes in the sample space. We write the complement as "not A".
- **Independent events**: two events where knowing the outcome of one event does not affect the probability of the other event. Events are independent if and only if P(A and B)=P(A)×P(B)
- Identities:
- A **tree diagram** is a visual tool that helps with computing probabilities for dependent events. The outcomes of each event are shown along with the probability of each outcome. For each event that depends on a previous event, we go one level deeper into the tree. To compute the probability of some combination of outcomes, we

- find all the paths that contain the outcome of interest;
- multiply the probabilities along each path;
- add the probabilities between different paths.

- A **two-way contingency table** is a tool for organising data, especially when we want to determine whether two events, each with only two outcomes, are dependent or independent. The counts for each possible combination of outcomes are entered into the table, along with the totals of each row and column.

- The **addition rule** (also called the sum rule)
  - P(A or B)=P(A)+P(B)−P(A and B)
- This rule relates the probabilities of events with the probabilities of their union and intersection.
- The **addition rule for mutually exclusive events** is
  - P(A or B)=P(A)+P(B) , No P (A and B)
- This rule is a special case of the previous rule. Because the events are mutually exclusive, P(A and B)=0.
- The **complementary rule** is
  - P(not A)=1−P(A)
- This rule is a special case of the previous rule. Since A and (not A) are mutually exclusive, P(A or (not A))=1

1) The product rule for independent events:

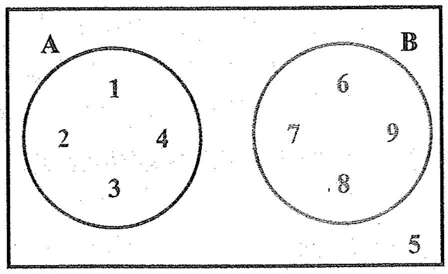
$$P(A \text{ and } B) = P(A) \times P(B)$$

2) The product rule for dependent events:

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

VENN DIAGRAM

MUTUALLY EXCLUSIVE

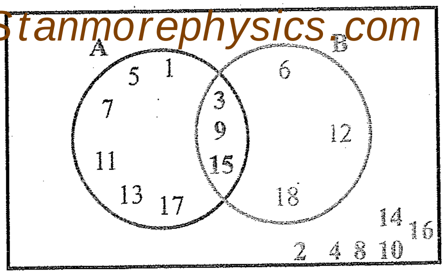


$P(A)=\frac{4}{9}$ and $P(B)=\frac{4}{9}$ and $P(A \text{ or } B)=P(A \cup B)=P(A)+P(B)=\frac{4}{9}+\frac{4}{9}=\frac{8}{9}$

$P(\text{not } A)=P(A')=\frac{4}{9}+\frac{1}{9}=\frac{5}{9}$

For mutually exclusive events: $P(A \text{ or } B)=P(A \cup B)=P(A)+P(B)$.

INCLUSIVE / NON MUTUALLY EXCLUSIVE



Set A and B are **not** mutually exclusive as there are events which are common to both sets. This is the **intersection of the sets** and is written as: A **and** B $= A \cap B = \{3;9;15\}$.

The $P(A \text{ or } B)$ can be read by counting directly from the diagram: $P(A \text{ or } B) = \frac{12}{18}$ or

by using the formula: $P(A \overset{or}{\cup} B) = P(A) + P(B) - P(A \overset{and}{\cap} B) = \frac{9}{18} + \frac{6}{18} - \frac{3}{18} = \frac{12}{18}$

EXAMPLE OF VENN DIAGRAM

In a study 160 Grade 11 learners were asked if they take Biology, Accounting or Science. The results of the study are shown below:

- **80 people said they take Biology**
- **70 people said they take Accounting**
- **80 people said they take Science**
- **42 people said they take Biology and Science**
- **36 people said they take Accounting and Science**
- **136 people said they take at least one of the subjects**
- **14 said they take all 3 subjects.**

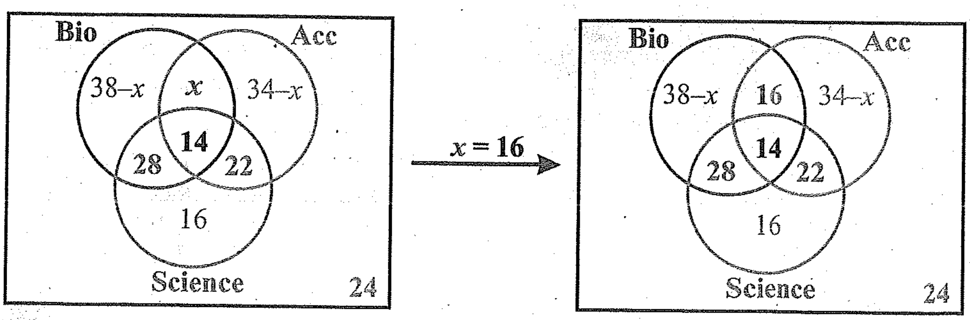
a) Record the above information in a Venn diagram.

b) How many people take none of the subjects?

c) How many people take Biology and Accounting but not Science?

d) What is the probability that a person chosen at random takes at least two of the subjects?

a)



Method:

Step 1: Determine the number of people that take none of the subjects. There are 160 people in the sample space and 136 people who take at least one of the subjects.

$\therefore$ the number of people who take none of the subjects $= 160 - 136 = 24$

Step 2: It is given that **14** people take all of the subjects. Write this down in the intersection of all the subjects.

Step 3: It is given that 42 people take Biology and Science. However, this includes the **14** people that take all of the subjects. The 14 has to be deducted from the 42 people who take Biology and Science. The same must be applied to Accounting and Science.

$\therefore$ Biology and Science $= 42 - 14 = \mathbf{28}$ Accounting and Science $= 36 - 14 = \mathbf{22}$

Step 4: You can now work out the number of people who take only Science.

Number of people who take only Science $= 80 - 28 - 14 - 22 = \mathbf{16}$

Step 5: As you don't know the number of people who take Biology and Accounting let that be x.

You can now work out the number of people that take only Biology and only Accounting.

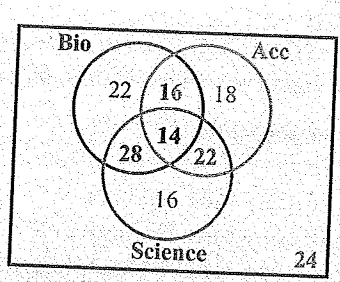
$\therefore$ Biology only $= 80 - 28 - 14 - x$ $\qquad$ Accounting $= 70 - 14 - 22 - x$

$= \mathbf{38 - x}$ $\qquad$ $= \mathbf{34 - x}$

Step 6: You can now calculate the value of x. There were a total of 160 people:

$\therefore 38 - x + x + 34 - x + 28 + 14 + 22 + 16 + 24 = 160$

$\therefore x = 16$



b) 24

c) 16

d) At least 2 subjects means 2 or more subjects (all intersections):
(Biology and Accounting) **or** (Biology and Science) **or** (Accounting and Science) **or** (Biology and Accounting and Science)

$\therefore P(\text{at least two subjects}) = \frac{16}{160} + \frac{28}{160} + \frac{22}{160} + \frac{14}{160}$

$= \frac{80}{160} = \frac{1}{2} = 0.5 = 50\%$

TREE DIAGRAM

Tree diagrams are constructed by showing all possible events. They can be used for dependent or independent events. When dealing with tree diagrams always multiply along branches and add probabilities moving down branches at the end.

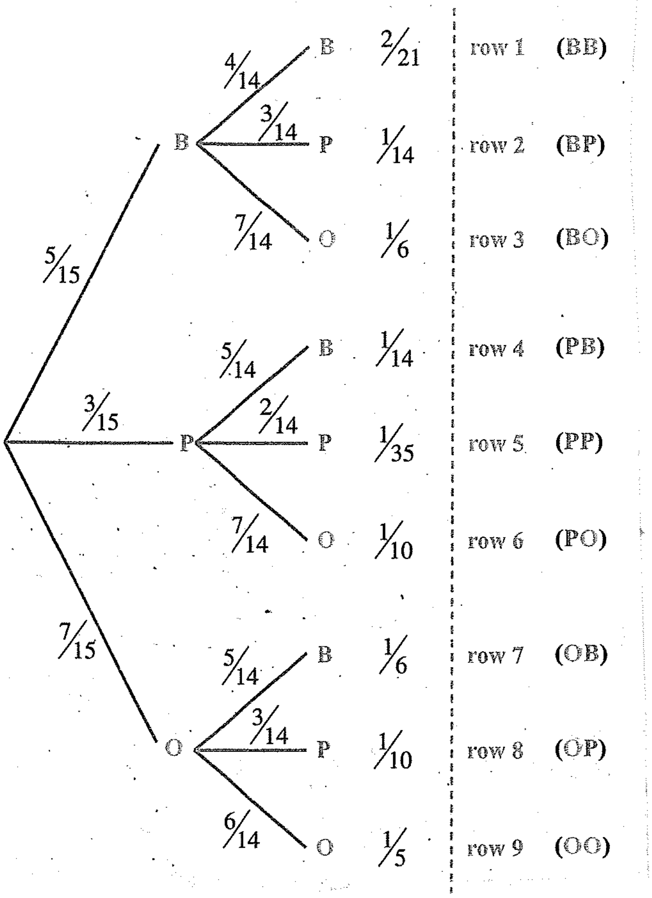
When dealing with tree diagrams multiply across branches and add when moving down. Write the probability of an event occurring at the top of the branches and the event at the end of the branch.

EXAMPLE OF TREE DIAGRAM

5 blue balls, 3 purple balls and 7 orange balls are placed in a bag. If 2 balls are then selected without replacement:

a) Draw a tree diagram to represent the above scenario.
b) Determine the probability of selecting a purple or blue ball.
c) Determine the probability of the 2nd ball being purple.
d) Determine the probability of selecting 2 orange balls.

(9)



b) $P(\text{B or P})=\frac{2}{21}+\frac{1}{14}+\frac{1}{6}+\frac{1}{14}+\frac{1}{35}+\frac{1}{10}+\frac{1}{6}+\frac{1}{10} \rightarrow$ all rows other than 9

$=\frac{4}{5}$

$=0,8$

$=80\%$

c) $P(\text{2nd ball being P})=\frac{1}{14}+\frac{1}{35}+\frac{1}{10} \rightarrow$ rows 2,5 and 8

$=\frac{1}{5}$

$=0,2$

$=20\%$

d) $P(\text{O and O})=\frac{1}{5} \rightarrow$ row 9

$=0,2$

$=20\%$

CONTINGENCY TABLE

Contingency table are statistical table that show the relationship between two or more variables. They can determine whether or not events are independent.

EXAMPLE 1

A group of 540 people with green or blue eyes were randomly selected in order to determine whether or not green or blue eyes are dependent on gender. The results are tabulated below:

	Male	Female	Totals
Green eyes	183	147	330
Blue Eyes	117	93	210
Totals	300	240	540

a) If a person is selected at random determine the probability that she is a female with green eyes.

b) If a person is selected at random and they have green eyes, determine the probability that she is female.

c) After analysing the results, a Grade 11 learner concludes that the probability of having green eyes is independent of gender. Is he correct? Substantiate your answer with relevant calculations. Give all answers correct to 2 decimal places.

a) $P(\text{female} \cap \text{green eyes}) = \frac{147}{540} \rightarrow$ 147 females with green eyes out of 540 people

$= \frac{49}{180}$

$= 27,22\%$

b) $P(\text{female} \mid \text{green eyes}) = \frac{147}{330} \rightarrow$ there are 330 people with green eyes

$= \frac{49}{110}$

$= 44,55\%$

c) For events to be independent: $P(\text{female} \cap \text{green eyes}) = P(\text{green eyes}) \times P(\text{female})$

$P(\text{green eyes}) \times P(\text{female}) = \frac{330}{540} \times \frac{240}{540} = \frac{22}{81} = 0,27$

$P(\text{female} \cap \text{green eyes}) = \frac{147}{540} = \frac{49}{180} = 0,27$

$\therefore P(\text{female} \cap \text{green eyes}) = P(\text{green eyes}) \times P(\text{female})$

$\therefore$ the events are independent and the learner is correct.

QUESTION 11 (NOVEMBER 2013)

11.1 The *Titanic* sank in 1912 without enough life boats for the passengers and crew. The contingency table below provides data on the passengers who were on board during the disaster. Use the information and determine, with reasons, whether the events M = {a passenger was male} and N = {a passenger did not survive} are dependent or independent.

***Titanic* survival data**

Gender

	Male	Female	Total
Yes	367	344	711
No	1 364	126	1 490
Total	1 731	470	2 201

(4)
[4]

QUESTION 12

12.1 It is given that A and B are independent events. P(A) = 0,4 and P(B) = 0,5.

Use a Venn diagram and calculate:

12.1.1 P(A or B) (4)

12.1.2 P(neither A or B) (1)

12.2 During a survey, 25 out of the 40 learners in a class indicated that they own a cellphone. Two learners are selected at random from the class, the first not being replaced before the second one is selected.

12.2.1 Draw a tree diagram that shows the possible outcomes of the situation. Write the probabilities on the relevant branches. (7)

12.2.2 What is the probability that of the two learners selected, one will own a cellphone and the other one not? (3)

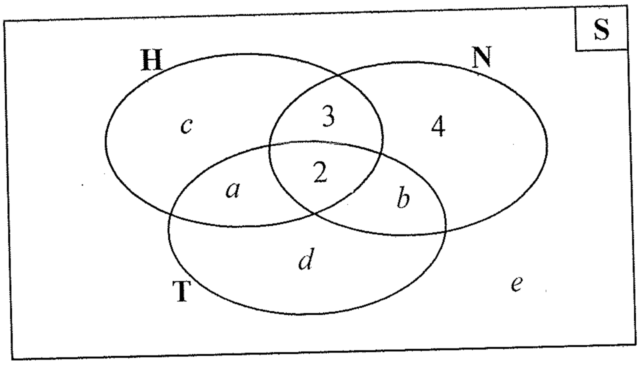
(7)

QUESTION 12

12.1 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

12.1.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e.



(5)

12.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1)

12.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1)

12.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1)

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6)

[14]

QUESTION 11

Given: $P(W) = 0{,}4$ $P(T) = 0{,}35$ $P(T \text{ and } W) = 0{,}14$

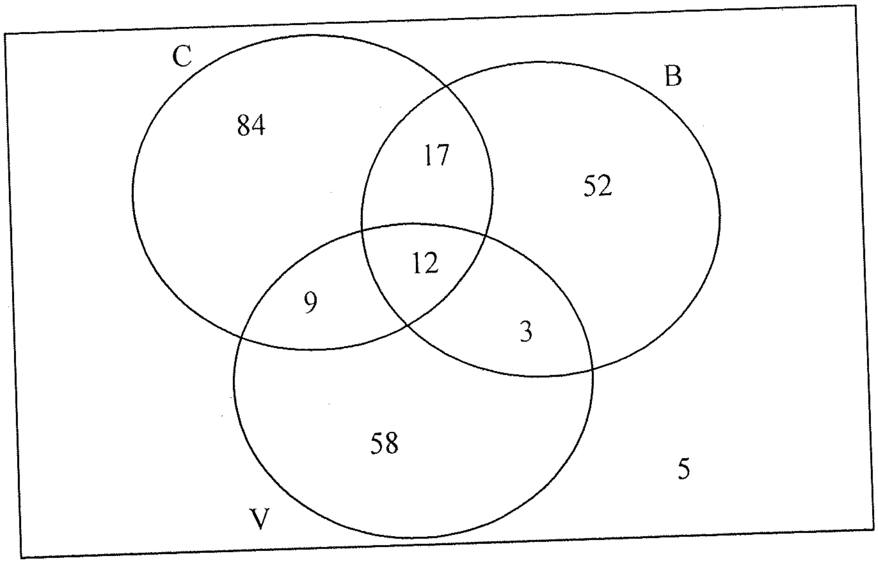
TOTAL:

11.1 Are the events W and T mutually exclusive. Give the reasons for your answer.

11.2 Are the events W and T independent. Give reasons

QUESTION 10 (DBE/NOVEMBER 2014)

A survey was carried out with 240 customers who bought food from a fastfood outlet on a particular day. The outlet sells cheese burgers (C), bacon burgers (B) and vegetarian burgers (V). The Venn diagram below shows the number of customers who bought different types of burgers on the day. (8)



10.1 How many customers did NOT buy burgers on the day? (1)

10.2 Are events B and C mutually exclusive? Give a reason for your answer. (2)

10.3 If a customer from this group is selected at random, determine the probability that he/she:

10.3.1 Bought only a vegetarian burger (1)

10.3.2 Bought a cheese burger and a bacon burger (1)

10.3.3 Did not buy a cheese burger (3)

10.3.4 Bought a bacon burger or a vegetarian burger (4)
[12]

QUESTION 11

Given: P(A) = 0,12
P(B) = 0,35
P(A or B) = 0,428

Determine whether events A and B are independent or not. Show ALL relevant calculations used in determining the answer. **[4]**

QUESTION 12

Paballo has a bag containing 80 marbles that are either green, yellow or red in colour. $\frac{3}{5}$ of the marbles are green and 10% of the marbles are yellow. Paballo picks TWO marbles out of the bag, one at a time and without replacing the first one.

12.1 How many red marbles are in the bag? (2)

12.2 Draw a tree diagram to represent the above situation. (3)

12.3 What is the probability that Paballo will choose a GREEN and a YELLOW marble? (3)
[8]

(9)

QUESTION 9

9.1 Given: P(A) = 0,6
P(B) = 0,3
P(A or B) = 0,8 where A and B are two different events

Are the events A and B mutually exclusive? Justify your answer with appropriate calculations and/or a diagram. (4)

9.2 The table below shows data on the monthly income of employed people in two residential areas. Representative samples were used in the collection of the data.

MONTHLY INCOME (IN RANDS)	AREA 1	AREA 2	TOTAL
$x < 3\ 200$	500	460	960
$3\ 200 \leq x < 25\ 600$	1 182	340	1 522
$x \geq 25\ 600$	150	14	164
Total	1 832	814	2 646

9.2.1 What is the probability that a person chosen randomly from the entire sample will be:

(a) From Area 1 (2)

(b) From Area 2 and earn less than R3 200 per month (1)

(c) A person from Area 2 who earns more than or equal to R3 200 (2)

9.2.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides. (5)

9.2.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer. (3)
[17]

TOTAL: 150

(10)

QUESTION 9

(NOVEMBER 2016)

9.1 Given: $P(A) = 0{,}2$
$P(B) = 0{,}5$
$P(A \text{ or } B) = 0{,}6$ where A and B are two different events

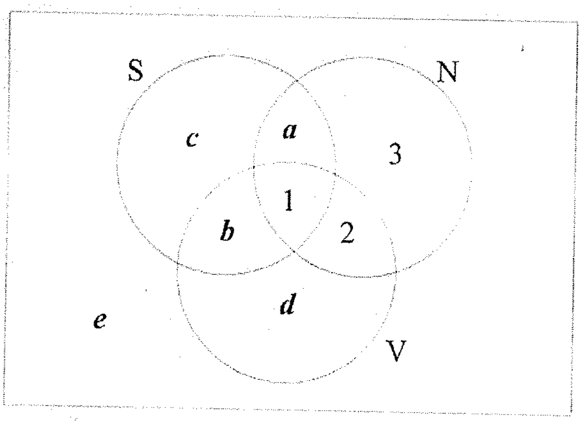
9.1.1 Calculate P(A and B). (2)

9.1.2 Are the events A and B independent ? Show your calculations. (3)

9.2 A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all 3 sports

The Venn diagram below shows the information above.



9.2.1 Determine the values of a, b, c, d and e. (5)

9.2.2 What is the probability that one of the learners chosen at random from this group plays netball or volleyball? (2)

9.3 The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the first answer is wrong, the probability of getting the next answer correct is only 0,3.

9.3.1 Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes. (3)

9.3.2 Calculate the probability of getting the second answer correct. (3)
[18]

QUESTION 8

8.1 A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.

8.1.1 Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the possible outcomes. (3)

8.1.2 Determine the probability of first taking a red marble and then taking a blue marble, in that order. (2)

8.2 A and B are two events. The probability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.

8.2.1 Are events A and B mutually exclusive?
Justify your answer with appropriate calculations. (3)

8.2.2 Are events A and B independent?
Justify your answer with appropriate calculations. (3)
[11]

QUESTION 9

A survey was done among 80 learners on their favourite sport.
The results are shown below.

- 52 learners like rugby (R)
- 42 learners like volleyball (V)
- 5 learners like chess (C) only
- 14 learners like rugby and volleyball but not chess
- 12 learners like rugby and chess but not volleyball
- 15 learners like volleyball and chess but not rugby
- x learners like all 3 types of sport
- 3 learners did not like any sport

9.1 Draw a Venn diagram to represent the information above. (5)

9.2 Show that $x = 8$. (2)

9.3 How many learners like only rugby? (1)

9.4 Calculate the probability that a learner, chosen randomly, likes at least TWO different types of sport. (3)
[11]

(12)

NOVEMBER 2018

QUESTION 8

A bag contains 6 red balls, 8 green balls and an unknown number of yellow balls. The probability of randomly choosing a green ball from the bag is 25%.

8.1 Show that there are 32 balls in the bag. (1)

8.2 A ball is drawn from the bag, the colour is recorded and it is not returned to the bag. Thereafter another ball is drawn from the bag, the colour is recorded and it is also not returned to the bag.

Draw a tree diagram to represent ALL the possible ways in which the two balls could have been drawn from the bag. Show the probabilities associated with EACH branch, as well as the outcomes. (4)

8.3 Calculate the probability that the two balls drawn from the bag will have the same colour. (4)

[9]

QUESTION 9

9.1 On a flight, passengers could choose between a vegetarian snack and a chicken snack. The snacks selected by the passengers were recorded. The results are shown in the table below.

SNACK	MALE	FEMALE	TOTAL
Vegetarian	12	20	32
Chicken	55	63	118
TOTAL	67	83	150

Was the choice of snack on this flight independent of gender? Motivate your answer with the necessary calculations. (5)

9.2 For any two events, A and B, it is given that P(A and B) = 0,12, P(A or B) = 0,83 and P(B) = 4 P(A).

9.2.1 Are events A and B mutually exclusive? Justify your answer. (2)

9.2.2 Calculate P(B). (4)

9.2.3 Calculate P(not A). (2)

[13]

GRADE 11 NOVEMBER 2019 EASTERN CAPE PAPER

QUESTION 8

8.1 A bag contains 3 red and 5 yellow tennis balls. A player picks a ball at random, observes the colour and does **not** replace it. She then picks a second ball.

8.1.1 Draw a tree diagram to represent the above information, showing all possible outcomes. (5)

8.1.2 Determine the probability that the player picks balls of a different colour. (4)

8.2 The probability that South Africa reaches the finals of the 2019 Rugby World Cup is 0,35 and the probability that New Zealand reaches the finals is 0,5. The probability that neither South Africa nor New Zealand reaches the final is 0,06.

8.2.1 Draw a Venn diagram to represent the information above. (3)

8.2.2 Determine the probability that both South Africa and New Zealand reach the finals. (2)

[14]

QUESTION 9

The partially-completed table below shows the number of distinctions obtained by male and female learners in a particular district, in three subjects: Mathematics, Accounting and Physical Sciences.

	Mathematics (M)	Accounting (A)	Physical Sciences (PS)	Total
Male	60	a	97	b
Female	65	81	114	260
Total	125	164	211	500

9.1 Determine the values of a and b. (2)

9.2 Use the information on the table to determine if the events
M = {a learner obtains a distinction in Maths} and
F = {a learner is female} , are independent events.
Support your answer with necessary calculations. (4)

9.3 If a learner is selected at random, calculate the probability that the learner is a female who achieved a distinction in Mathematics or Physical Sciences. (3)

[9]

(14)

QUESTION 9

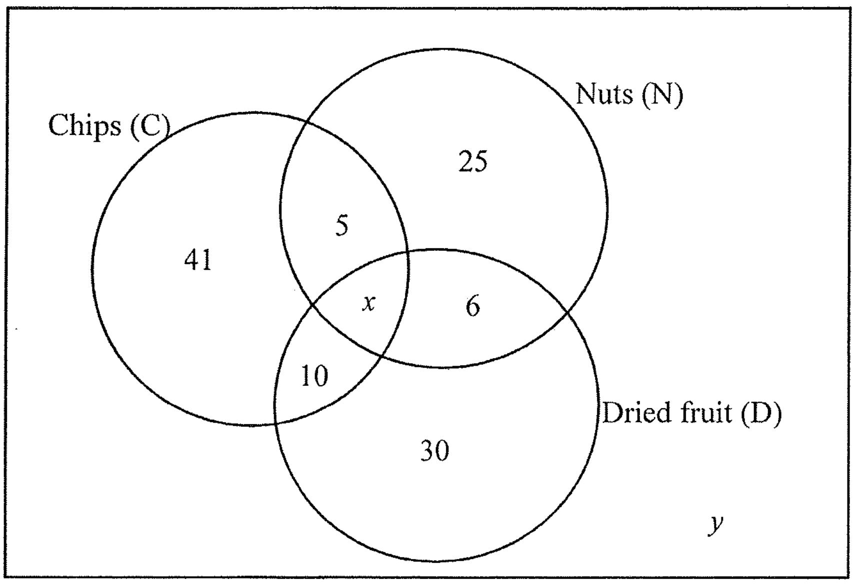
9.1 For any two events, A and B, it is given that $P(A)=0{,}48$ and $P(B)=0{,}26$.

Determine:

9.1.1 P(A and B) if A and B are independent events (2)

9.1.2 P(A or B) if A and B are mutually exclusive events (2)

9.2 A survey was conducted among 130 Grade 11 learners to establish which snack they prefer to eat while they watch television. The results were summarised in the Venn diagram below. However, some information is missing.



9.2.1 If 29 learners prefer at least two types of snacks, calculate the value of x and y. (4)

9.2.2 Determine the probability that a learner who does not eat nuts will either have another snack or no snack while watching television. (3)

9.3 A group of 200 tourists visited the same restaurant on two consecutive evenings. On both evenings, the tourists could either choose beef (B) or chicken (C) for their main meal. The manager observed that 35% of the tourists chose beef on the first evening and 70% of them chose chicken on the second evening.

9.3.1 Draw a tree diagram to represent the different choices of main meals made on the two evenings. Show on your diagram the probabilities associated with each branch as well as all the possible outcomes of the choices. (4)

9.3.2 Calculate the number of tourists who chose the same main meal on both evenings (3)

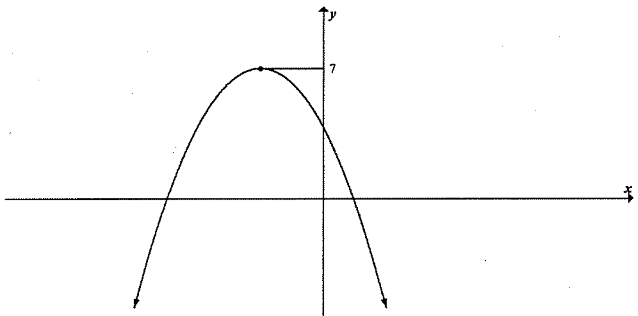
9.3.3 Show that more tourists opted not to change their choice of main meal during their two visits to the restaurant. (2)

[20]

TOTAL: 150

Memo Grade 11 – Probability

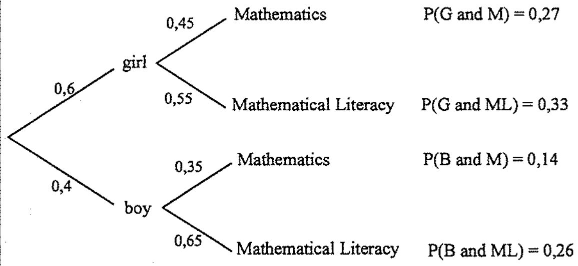
QUESTION 10

Range of f $(-\infty;7]$ $\Rightarrow$ y-part of turning point [Max value of $f(x)$] is 7 $a<0$ and shape ∩ $b<0$ $\Rightarrow$ b negative $\Rightarrow$ axis of symmetry on left of y-axis roots real, unequal & opposite signs $\Rightarrow$ x-ints on opposite sides of y-axis  	✓ shape ✓ turning point at $y=7$ ✓ axis of symmetry on left of y-axis ✓ roots are on opposite sides **[4]**

QUESTION 11

11.1	No, W and T are **not** mutually exclusive Because P(W and T) $\neq 0$ **OR** No, W and T are **not** mutually exclusive Because P(W or T) $= 0{,}61 \neq 0{,}75 =$ P(W) + P(T)	✓ not mutually exclusive ✓ P(W and T) $\neq 0$ (2) ✓ not mutually exclusive ✓ P(W or T) $\neq$ P(W) + P(T) (2)
11.2	P(W and T) = 0,14 (given) and P(W)×P(T) = 0,4 × 0,35 = 0,14 $\Rightarrow$ P(W and T) = P(W)×P(T) Therefore yes, W and T are independent events	✓ P(W)×P(T) = 0,14 ✓ P(W and T) = P(W)×P(T) ✓ conclusion (yes) (3) **[5]**

QUESTION 12

12.1.1	$a=5$ $b=4$ $c=8$ $d=1$ $e=6$	✓ $a=5$ ✓ $b=4$ ✓ $c=8$ ✓ $d=1$ ✓ $e=6$ (5)
12.1.2	6	✓ answer (1)
12.1.3	$\frac{4}{33}$	✓ answer (1)
12.1.4	$\frac{4+3+2+a+b+c}{33}=\frac{26}{33}$	✓ answer (1)
12.2	 P(Mathematics) = P(G and M) + P(B and M) $= (0{,}6)(0{,}45) + (0{,}4)(0{,}35)$ $= 0{,}27 + 0{,}14$ $= 0{,}41$	✓ 0,4 ✓ 0,45 ✓ 0,35 ✓ P(G and M) = 0,27 ✓ P(B and M) = 0,14 ✓ answer (6) **[14]**

TOTAL: 150

$x_{tp} = \frac{3+(-1)}{2} = 1$

$\therefore$ tp $(1;-4)$

$y = a(x-1)^2 - 4$ ✓

sub $(0;-3)$

$-3 = a(0-1)^2 - 4$ ✓

$1 = a$

$\therefore y = (x-1)^2 - 4$ ✓

11. P(M) (male)   P(N) (not survive)

$= \frac{1731}{2201}$ ✓ $= \frac{1490}{2201}$ ✓

P(M and N) (male, not survive)

$= \frac{1364}{2201}$

$= \frac{44}{71}$ ✓   0,619...

$P(M) \times P(N)$

$= \frac{1731}{2201} \times \frac{1490}{2201}$

$= 0,532...$

$\therefore P(M \text{ and } N) \neq P(M) \times P(N)$

$\therefore$ M and N are dependent ✓ **4**

(not independent!)

12.1. $P(A) = 0,4$ ($\frac{2}{5}$) $P(B) = 0,5$ ($\frac{1}{2}$)

12.1.1. Independent

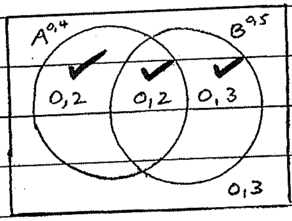
$\therefore$ P(A and B)

$= P(A) \times P(B)$

$= 0,4 \times 0,5$

$= 0,2$ ($\frac{1}{5}$)

S = 1



$\therefore$ P(A or B)

$= 0,2 + 0,2 + 0,3$

$= 0,7$ ✓ **4**

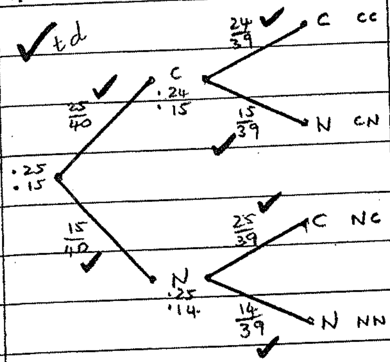
12.2. $P((A \cup B)')$

$= 0,3$ ✓ **1**

NOVEMBER 2013   M2

12.2.1. C = cell   N = no cell

✓ td



7

12.2.2. P(CN or NC)

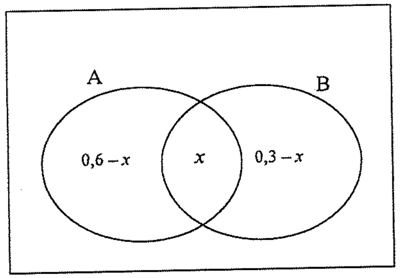
$= P(CN) + P(NC)$

$= \frac{25}{40} \times \frac{15}{39} + \frac{15}{40} \times \frac{25}{39}$

$= \frac{25}{104}$ ✓ $+ \frac{25}{104}$ ✓

$= \frac{25}{52}$ ✓   0,48 **3**

QUESTION/*VRAAG* 9

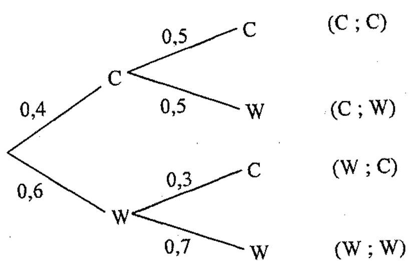
9.1	Given/*Gegee*: P(A) = 0,6 P(B) = 0,3 P(A or B) = 0,8 P(A and/*en* B) = 0,6 + 0,3 – 0,8 = 0,1 $\neq$ 0 Therefore A and B are not mutually exclusive. *Dus is A en B nie onderling uitsluitend nie* **OR/*OF***  $0{,}6 - x + x + 0{,}3 - x = 0{,}8$ $0{,}9 - x = 0{,}8$ $x = 0{,}1$ There is an intersection between A and B/*Daar is snyding tussen A en B* Therefore A and B are not mutually exclusive/*Dus is A en B nie onderling uitsluitend nie.*	✓✓P(A and/*en* B) = 0,1 ✓justification/*regv* ✓Not mutually exclusive/*Nie onderling uitsluitend* (4) ✓Venn diagram ✓$x = 0{,}1$ ✓justification/*regv* ✓Not mutually exclusive/*Nie onderling uitsluitend* (4)
9.2.1a	$\frac{1832}{2646} = 69{,}24\%$	✓1832 ✓2646 (2)
9.2.1b	$\frac{460}{2646} = \frac{230}{1323} = 17{,}38\%$	✓answer/*antwoord* (1)
9.2.1c	$\frac{340+14}{2646} = \frac{59}{441} = 13{,}38\%$	✓✓answer/*antwoord* (2)

9.2.2	Let the event of a randomly selected person living in Area 1 be A. Let the event of a randomly selected person earning less than R3 200 be B. *Laat die gebeurtenis van 'n persoon wat willekeurig gekies is wat in Gebied 1 woon, A wees.* *Laat die gebeurtenis van 'n persoon wat willekeurig gekies is wat minder as R3 200 verdien, B wees.* $P(A \text{ and } B) = \frac{500}{2646} = 18{,}90\%$ $P(A) \times P(B) = \frac{1832}{2646} \times \frac{960}{2646} = 25{,}12\%$ Clearly/*Duidelik*, $P(A \text{ and } B) \neq P(A) \times P(B)$ Hence A and B are not independent/*Vervolgens is A en B nie onafhanklik nie.* **OR/*OF*** Let the event of a randomly selected person living in Area 2 be C. Let the event of a randomly selected person earning less than R3200 be D. *Laat die gebeurtenis van 'n persoon wat willekeurig gekies is wat in Gebied 2 woon, C wees.* *Laat die gebeurtenis van 'n persoon wat willekeurig gekies is wat minder as R3 200 verdien, D wees.* $P(C \text{ and } D) = \frac{460}{2646} = 17{,}38\%$ $P(C) \times P(D) = \frac{814}{2646} \times \frac{960}{2646} = 11{,}16\%$ Clearly, $P(C \text{ and } D) \neq P(C) \times P(D)$ Hence C and D are not independent/*Vervolgens is C en D nie onafhanklik nie.*	✓P(A and B) ✓P(A) ✓P(B) ✓P(A) × P(B) ✓conclusion with justification/*gevolgtrekking met motivering* (5) ✓P(C and D) ✓P(C) ✓P(D) ✓P(C) × P(D) ✓conclusion with justification/*gevolgtrekking met motivering* (5)
9.2.3	P(Area 1 person earns less than R3200) $= \frac{500}{1832} = 27{,}29\%$ P(Area 2 person earns less than R3200) $= \frac{460}{814} = 56{,}51\%$ A person from Area 2 is more likely to earn less than R3200 P(*Gebied* 1 *persoon verdien minder as* R3200) $= \frac{500}{1832} = 27{,}29\%$ P(*Gebied* 2 *persoon verdien minder as* R3200) $= \frac{460}{814} = 56{,}51\%$ *Dis meer waarskynlik dat 'n persoon uit Gebied 2 minder as R3 200 sal verdien.*	✓27,29% ✓56,51% ✓conclusion/*vgl* (3) **[17]**

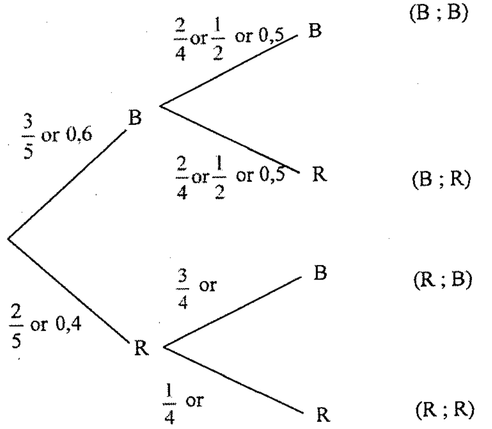
TOTAL/*TOTAAL*: 150

M4

QUESTION/VRAAG 9

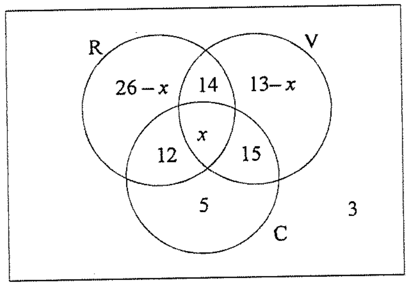
9.1	Given/*Gegee*: $P(A) = 0{,}2$ $P(B) = 0{,}5$ $P(A \text{ or } B) = 0{,}6$	
9.1.1	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0{,}6 = 0{,}2 + 0{,}5 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0{,}1$	✓$0{,}6 = 0{,}2 + 0{,}5 - P(A \text{ and } B)$ ✓ $P(A \text{ and } B) = 0{,}1$ (2)
9.1.2	$P(A \text{ and } B) = 0{,}1$ $P(A) \times P(B) = 0{,}2 \times 0{,}5$ $= 0{,}1$ $\therefore P(A \text{ and } B) = P(A) \times P(B)$ $\therefore$ A and B are independent	✓ $P(A) \times P(B) = 0{,}1$ ✓ $P(A \text{ and } B) = P(A) \times P(B)$ ✓ conclusion (3)
9.2.1	$a = 15$ $b = 1$ $c = 38$ $d = 3$ $e = 37$	✓$a = 15$ ✓$b = 1$ ✓$c = 38$ ✓$d = 3$ ✓$e = 37$ (5)
9.2.2	$P(\text{one learner plays netball or volleyball}) = \frac{25}{100} = \frac{1}{4}$	✓25 ✓answer/*antwoord* (2)
9.3.1		✓ branch at first level ✓ branches at second level ✓ probabilities and outcomes (3)

QUESTION/*VRAAG* 8

8.1.1		✓branches/*takke* ✓probabilities/*waarskynlikhede* ✓outcomes/*uitkomste* (3)
8.1.2	$P(R,B)=\frac{2}{5}\times\frac{3}{4}$ $=\frac{3}{10}$	✓ $\frac{2}{5}\times\frac{3}{4}$ ✓answer/*antwoord* (2)
8.2.1	P(A) = 0,4 P(B) = 0,3 P(A or/ B) = 0,58 P(A or B) = P(A) + P(B) – P(A and B) 0,58 = 0,4 + 0,3 – P(A and B) P(A and B) = 0,12 ≠ 0 Events A and B are not mutually exclusive/*Gebeurtenis A en B is nie onderlinguitsluitend nie*	✓0,58 = 0,4 + 0,3 – P(A and B) ✓ P(A and B) = 0,12 ≠ 0 ✓Not mutually exclusive/ *nie onderling uitsluitend nie* (3)
8.2.2	P(A and B) = 0,12 P(A) × P(B) = 0,4 × 0,3 = 0,12 ∴ P(A and B) = P(A) × P(B) A and B are independent events/*is onafhanklik*	✓ P(A) × P(B) = 0,4 × 0,3 ✓ P(A and B) = P(A) × P(B) ✓A and B are independent/*is onafhanklik* (3)
		[11]

M5

QUESTION/*VRAAG* 9

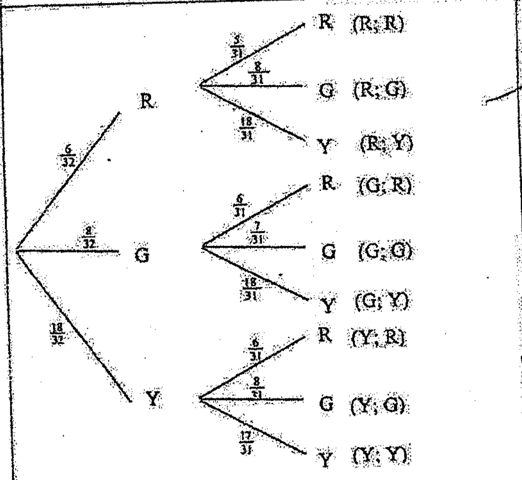
9.1		✓ 14 or/of 12 or/of 15 ✓ 26 – x ✓ 13 – x ✓5 ✓ 3 (5)
9.2	$26-x+14+x+12+5+15+13-x+3=80$ $88-80=x$ $x=8$	✓ $26-x+14+x+12+5+15+13-x+3$ ✓equating to/*gelyk aan* 80 (2)
9.3	Number who chose Rugby only/*aantal wat net rugby kies* = 26 – 8 = 18	✓answer/*antw.* (1)
9.4	P(At least 2 types of sports /*ten minste 2 sportsoorte*) $=\frac{12+14+15+8}{80}$ $=\frac{49}{80}$ **OR/*OF*** P(at least 2 types of sport/*ten minste 2 sportsoorte*) $=1-\frac{18+5+5+3}{80}$ $=1-\frac{31}{80}$ $=\frac{49}{80}$	✓numerator/*Noemer* ✓denominator/*Teller* ✓answer/*antw.* (3) ✓ $\frac{18+5+5+3}{80}$ ✓method/*metode* ✓answer/*antw.* (3)
		[11]

TOTAL/*TOTAAL*: 150

M6

QUESTION/VRAAG 8

25% ≈ 8 balls ∴ 32 balls
75% ≈ 8 × 3 = 24 balls

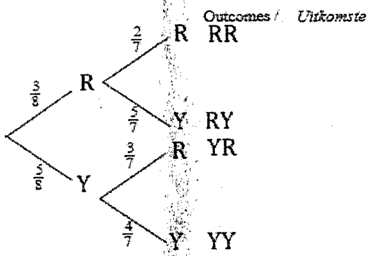
8.1	Given/Gegee: P(G) = 0,25 Let x be the total number of balls $P(G)=\frac{8}{x}=\frac{1}{4}$ $x=32$ $n(S)=32$ **OR/OF** Let x be the number of yellow balls ∴ $x+14$ be the total number of balls $P(G)=\frac{8}{x+14}=\frac{1}{4}$ $x+14=32$ $n(S)=32$	✓ $\frac{8}{x}=\frac{1}{4}$ A (1) ✓ $\frac{8}{x+14}=\frac{1}{4}$ A (1)
8.2		Refer to addendum 2 marks only ✓18 (number of yellow balls/ *aantal geel balle*) A ✓branches/*takke* A ✓probabilities/*waarskynlikhede* A ✓outcomes/*uitkomste* A (4)
8.3	P(G, G) + P(R, R) + P(Y, Y) $=\left(\frac{8}{32}\times\frac{7}{31}\right)+\left(\frac{6}{32}\times\frac{5}{31}\right)+\left(\frac{18}{32}\times\frac{17}{31}\right)$ $=\frac{49}{124}$ → answer must be between 0 & 1 0,395 / 39,5%	✓$\left(\frac{8}{32}\times\frac{7}{31}\right)$ CA ✓$\left(\frac{6}{32}\times\frac{5}{31}\right)$ CA ✓$\left(\frac{18}{32}\times\frac{17}{31}\right)$ CA ✓answer/*antw* CA (4) [9]

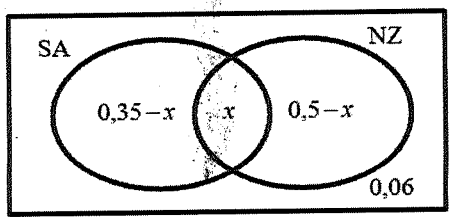
QUESTION/VRAAG 9

9.1	method 1 P(V) × P(M) $\frac{32}{150}\times\frac{67}{150}=0,095$ → answers very close P(V and/en M) $=\frac{12}{150}=0,08$ P(V and/en M) ≠ P(V) × P(M) The events are not independent/*Die gebeurtenisse is nie onafhanklik* **OR/OF** method 2 P(V) × P(F) $\frac{32}{150}\times\frac{83}{150}=0,118$ P(V and F) $=\frac{20}{150}=0,133$ P(V and F) ≠ P(V) × P(F) The events are not independent/*Die gebeurtenisse is nie onafhanklik*	✓ $\frac{32}{150}$ A ✓ $\frac{67}{150}$ A ✓P(V) × P(M) = 0,095 A ✓P(V and/*en* M) = 0,08 A ✓conclusion/*gevolgtr.* CA (5) ✓ $\frac{32}{150}$ ✓ $\frac{83}{150}$ ✓P(V) × P(F) = 0,118 ✓P(V and F) = 0,133 ✓conclusion/*gevolgtr.* (5)
9.2.1	P(A and/en B) = 0,12 ≠ 0 Events are not mutually exclusive/ *Gebeurtenisse nie onderling uitsluitend nie*	✓P(A and B) ≠ 0 A ✓conclusion/*gevolgtr.* A (2)
9.2.2	P(A or B) = P(A) + P(B) − P(A and B) 0,83 = P(A) + 4P(A) − 0,12 0,95 = 5P(A) P(A) = 0,19 P(B) = 4(0,19) = 0,76 0,83 = x + 4x − 0,12 (P(A), P(B)) 0,95 = 5x 0,19 = x	✓formula/ *formule* A ✓substitution/*verv.* A ✓P(A) CA ✓P(B) CA (4)
9.2.3	P(not A) = 1 − P(A) = 1 − 0,19 = 0,81	✓P(not A) = 1 − P(A) CA ✓answer/*antw.* CA (2) [13]

TOTAL/TOTAAL: 150

QUESTION 8/*VRAAG 8*

<table>
<tr><td>8.1.1</td><td>Let Red ballsbe R andYellowbe Y
Laat Rooi balle R en Geel balle Y wees

</td><td>✓firstbranch/ eerste tak
✓probabilities of firstbranch waarskynlikhede van eerste tak
✓secondbranches / tweede takke
✓probabilities of secondbranches waarskynlikhede van tweede takke
✓outcomes / uitkomste
(5)</td></tr>
<tr><td>8.1.2</td><td>P(different colours/ verskillende kleure)
$=P(RY)$ or / of $P(YR)$
$=P(RY)+P(YR)$
$=\left(\frac{3}{8}\times\frac{5}{7}\right)+\left(\frac{5}{8}\times\frac{3}{7}\right)$
$=\frac{15}{28}$ or / of $0{,}54$
OR / OF
P(different colors) $=1-P(\text{same colour})$
P(verskillende kleure) $=1-P(\text{dieselfde kleur})$
$=1-(P(RR)+P(YY))$
$=1-\left(\left(\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{8}\times\frac{4}{7}\right)\right)$
$=1-\frac{13}{28}$
$=\frac{15}{28}$ or / of $0{,}54$</td><td>✓ $P(RY)+P(YR)$
✓ $\frac{3}{8}\times\frac{5}{7}$ ✓ $\frac{5}{8}\times\frac{3}{7}$
✓answer / antwoord
(4)

✓ $1-(P(RR)+P(YY))$
✓ $\frac{3}{8}\times\frac{2}{7}$ ✓ $\frac{5}{8}\times\frac{4}{7}$
✓answer / antwoord
(4)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>8.2.1</td><td>Let x be the probability that both SA & NZ reach finals
Laat x die waarskynlikheid wees dat beide SA & NZ die finaal haal

</td><td>✓ $0{,}06$
✓ $0{,}35-x$ & $0{,}5-x$
✓ x
(3)</td></tr>
<tr><td>8.2.2</td><td>$0{,}35-x+x+0{,}5-x+0{,}06=1$
$\therefore x=0{,}09$</td><td>✓equation/vergelyking
✓answer/antwoord
(2)</td></tr>
<tr><td></td><td></td><td>[14]</td></tr>
</table>

M7