STANMORE SECONDARY SCHOOL

ANALYTICAL GEOMETRY

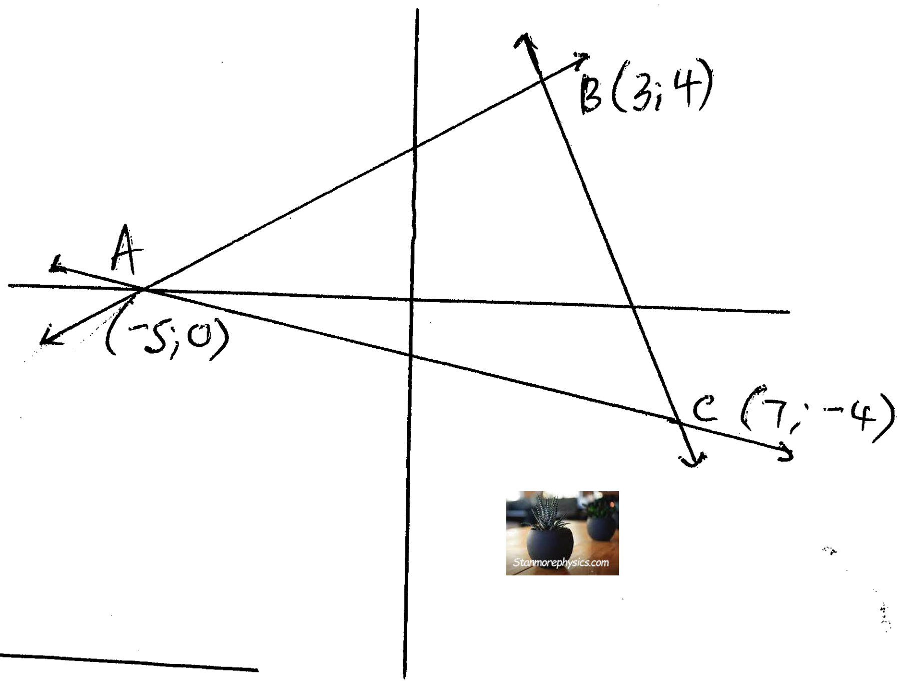
GRADE 10

COMPILED BY K.H.MOODLEY

Distance Formula

The distance formula is used to find the length of a line between two points.

Distance/length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$D_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-3)^2 + (0-4)^2}$

$= \sqrt{80} = 4\sqrt{5}$

$D_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3-7)^2 + (4-(-4))^2}$

$= \sqrt{80} = 4\sqrt{5}$

$D_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-(-5))^2 + (-4-0)^2}$

$= \sqrt{160} = 4\sqrt{10}$

$\therefore AB = BC$

$\therefore \Delta ABC$ is a Isosceles triangle

MIDPOINT

The midpoint of a line segment is the point halfway between two given points.
The midpoint is generally denoted as: $\mathbf{M}(x_M ; y_M)$

Midpoint Formula:

$$M(x_M ; y_M) = M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

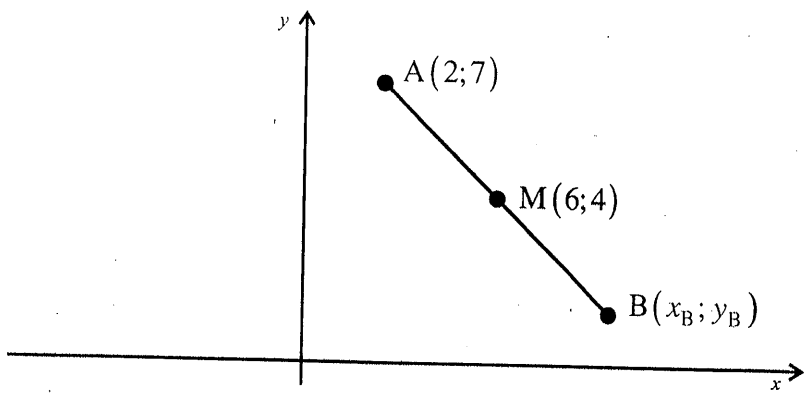
You can also work with each part of the formula separately to determine the x and y-coordinates of the midpoint:

$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

Example 1: Determine the midpoint of line AB if A is the point (5; 6) and B is the point (-3; -2).

$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$
$$= M\left(\frac{5 + (-3)}{2} ; \frac{6 + (-2)}{2}\right)$$
$$= M(1; 2)$$

Example 2: Determine the coordinates of B, if A is the point (2; 7) and the coordinates of M, the midpoint of AB, are (6; 4).



$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

$$\therefore 6 = \frac{2 + x_B}{2} \qquad 4 = \frac{7 + y_B}{2}$$

$$\therefore 12 = 2 + x_B \qquad 8 = 7 + y_B$$

$$\therefore x_B = 10 \qquad y_B = 1$$

$\therefore$ B is the point $(10; 1)$

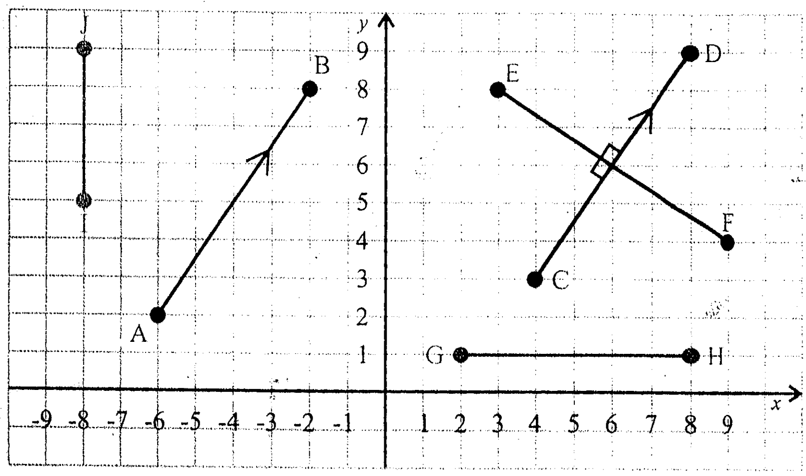
AVERAGE GRADIENT

Average gradient is the ratio of the change in y-values (the dependent variable) to the change in x-values (the independent variable).

Average Gradient

$$\text{gradient} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{\text{change in } y}{\text{change in } x}$$

Example: Determine the gradient of lines AB, CD, EF, GH and IJ.



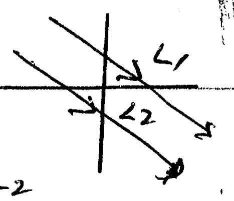
$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{-2-(-6)} = \frac{3}{2}$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{8-4} = \frac{3}{2}$$

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{3-9} = -\frac{2}{3}$$

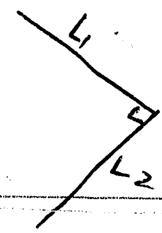
$$m_{GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{8-2} = 0$$

$$m_{IJ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{-8-(-8)} = \frac{4}{0} = \text{undefined} \rightarrow \text{you cannot divide by } 0$$



From the above example it can be seen that:

- **Parallel lines have equal gradients.** → $M_{L_1} = M_{L_2}$
- **The gradients of perpendicular lines are negative reciprocals of each other.**
  In other words $m_1 \times m_2 = -1$ or $m_1 = -\frac{1}{m_2}$
- **The gradient of a horizontal line is always equal to 0.**
- **The gradient of a vertical line is undefined.**



$M_{L_1} \times M_{L_2} = -1$

The gradient formula (m)

$M_{ab} = \frac{y_a - y_b}{x_a - x_b}$ or $M_{ab} = \frac{y_b - y_a}{x_b - x_a}$ NB! $x_a \neq x_b$

A) EXAMPLE1 (PARALLEL LINES)

$A(6;-2)$; $N(3;5)$; $P(1;9)$ and $Q(4;2)$. Show that $AN // PQ$.

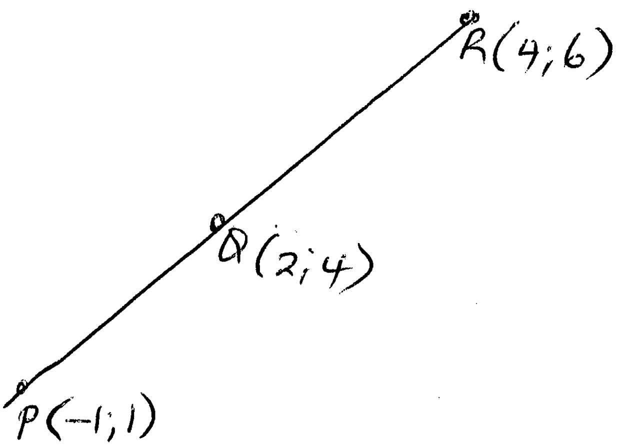
NB (If two straight lines are parallel, their gradients are equal)

- $M_{AN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_n - y_a}{x_n - x_a} = \frac{5-(-2)}{3-6} = \frac{7}{-3}$
- $M_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_q - y_p}{x_q - x_p} = \frac{2-9}{4-1} = \frac{-7}{3}$
- $M_{AN} = M_{PQ} = -\frac{7}{3}$ $\therefore$ $AN // PQ$

B) EXAMPLE 2! (COLLINEAR)

If $P(-1,1)$, $Q(2,4)$, and $R(4,6)$ are 3 points on a cartesian plane. Show that P, Q and R are collinear.

NB! (if points are collinear, they lie on the same straight line. If P, Q and R are collinear, then $M_{PQ} = M_{QR} = M_{PR}$)



- $M_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-(-1)} = 3/3 = 1$
- $M_{QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{4-2} = 2/2 = 1$
- $M_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-1}{4-(-1)} = 5/5 = 1$
- $M_{RQ} = M_{QR} = M_{PR} = 1$

$\therefore$ P.Q are R are collinear.

(c) PERPENDICULAR LINES

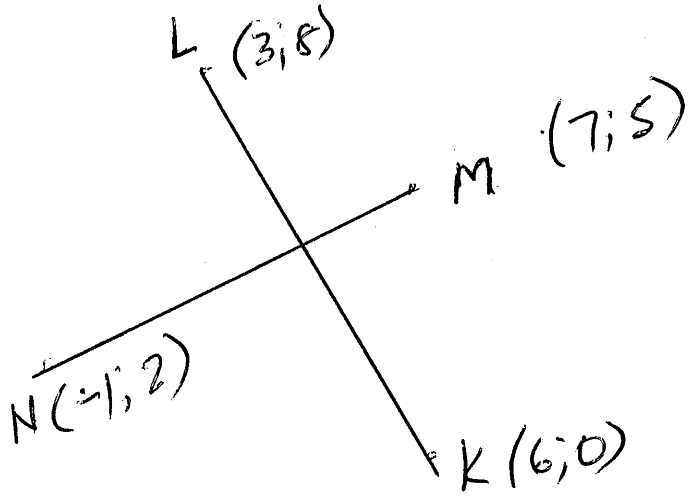
$m_{La} \times m_{Lb} = -1$

NB! EXAMPLES OF GRADIENTS OF PERPENDICULAR LINES

- If $m_{La} = 4/7$ then $m_{Lb} = -7/4$, since $(4/7) \times (-7/4) = -1$
- If $m_{La} = -5/2$ then $m_{Lb} = 2/5$ since $(-5/2) \times (2/5) = -1$
- If $m_{La} = 3$ then $m_{Lb} = -1/3$ since $(3) \times (-1/3) = -1$

Example 3:

If $K(6,0)$, $L(3,8)$, $M(7;5)$ and $N(-1,2)$, show that $KL \perp MN$.



- $m_{KL} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{3-6} = \frac{8}{-3}$
- $m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{7-(-1)} = \frac{3}{8}$

$\therefore m_{KL} \times m_{MN}$

$= (-8/3) \times (3/8) = -1$

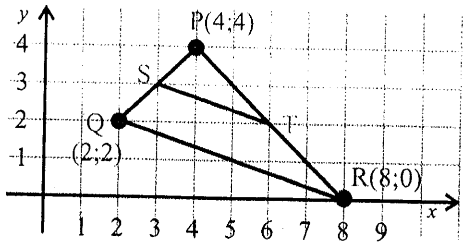
$\therefore KL \perp MN$

ΔPQR with vertices P(4;4), Q(2;2) and R(8;0) is sketched below.

a) Determine the coordinates of S and T, if they are the midpoints of PQ and PR respectively.

b) Show that the points P, T and R are collinear.
Collinear points are points that lie on the same straight line.

c) Prove that ST is half the length of QR.



a) Midpoint of PQ:

$$S\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)=S\left(\frac{4+2}{2};\frac{4+2}{2}\right)=S(3;3)$$

Midpoint of PR:

$$T\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)=T\left(\frac{4+8}{2};\frac{4+0}{2}\right)=T(6;2)$$

$\therefore$ S is the point $(3;3)$ and T is the point $(6;2)$

b) For P, T and R to be collinear the gradient of PT must be equal to the gradient of TR.

$$m_{PT}=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{4-6}=-\frac{2}{2}=-1 \qquad m_{TR}=\frac{y_2-y_1}{x_2-x_1}=\frac{2-0}{6-8}=-\frac{2}{2}=-1$$

$\therefore m_{PT}=m_{TR}$

$\therefore$ the points P, T and R are collinear

c)

$$QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(8-2)^2+(0-2)^2}$$
$$=\sqrt{40} \rightarrow \sqrt{40}=\sqrt{4\times 10}$$
$$=2\sqrt{10}\text{ units}$$

$$ST=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(6-3)^2+(2-3)^2}$$
$$=\sqrt{10}\text{ units}$$

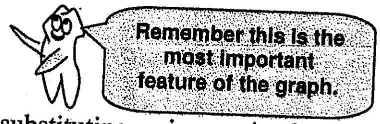
$$\therefore ST=\frac{1}{2}QR$$

TO DETERMINE A LINEAR / STRAIGHT LINE FUNCTION

When given information or even the graph, it is often important to be able to determine the equation (mathematical model) of the linear function. For the equation $y = mx + q$ the value of m (the gradient) and q (c) (vertical translation) must be determined. These values are then substituted back into the equation. Since there are two unknowns, two "bits" of information must be given.

To find m, the gradient:

Find m by using the relevant method



To find q:

The last value required in any equation is often found by substituting a given point for x and y into the equation found so far, and hence solve for q. Remember this is the vertical translation, so if it is given, use it.

EXAMPLE 1

Determine the equation of the straight line passing through (–1; –2) and (–5;6).

- Determine m:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{6-(-2)}{-5-(-1)}$

$m = \frac{6+2}{-5+1}$

$m = -2$

- Equation so far: $y = -2x + q$
- Substitute either point for x and y:

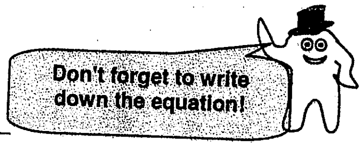
Using(-1;-2) $x = -1; y = -2$

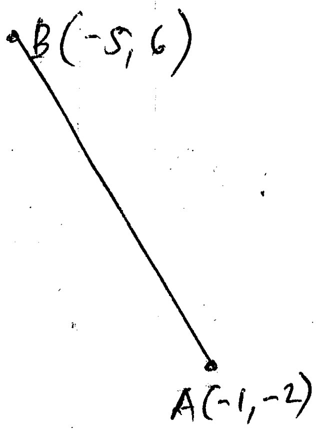
$-2 = -2(-1) + q$

$-2 = 2 + q$

$-4 = q$

$\therefore y = -2x - 4$





EXAMPLE 2

Find the equation of the straight line passing through (4; –1) perpendicular to $4 - y = 2x$.

- Determine m:

First find the gradient of the given line - $4 - y = 2x$.

$y = -2x + 4 \quad \therefore m = -2$

For perpendicular lines $\begin{cases} m_1 \times m_2 = -1 \\ \therefore -2 \times m_2 = -1 \\ m = \frac{-1}{-2} = \frac{1}{2} \end{cases}$

- Equation so far: $y = \frac{1}{2}x + q$
- Substitute point for x and y:

Using(4;-1) $x = 4; y = -1$

$-1 = \frac{1}{2}(4) + q$

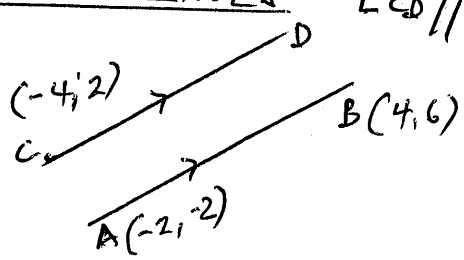
$-1 = 2 + q$

$-3 = q$

$\therefore y = \frac{1}{2}x - 3$

DETERMINE EQUATION OF PARALLEL LINES

$L_{CD} // L_{AB}$



Equation of AB: $y = mx + c$

$M_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-2)}{4-(-2)} = 8/6 = 4/3$

$\therefore M_{CD} = 4/3 \quad (L_{CD} // L_{AB})$

$\therefore y = mx + c$

$y = 4/3\, x + c$

$(-4; 2): \; 2 = 4/3(-4) + c$

$2 = -16/3 + c$

$2 + 16/3 = c$

$22/3 = c$

$\therefore L_{CD}: y = 4/3\, x + 22/3$

Co-ordinate Geometry

Distance formula:

$A(x_1; y_1)$; $B(x_2; y_2)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. Find the distance between each pair of points:

1.1. $A(2;3)$; $B(5;7)$

1.2. $A(0;1)$ and $C(6;9)$

1.3. $C(-4;8)$ and $D(2;0)$

1.4. $E(-3;0)$ and $F(0;4)$

1.5. $G(5;-3)$; $H(-2;-7)$

1.6. $M(2;-3)$; $N(-2;-3)$

1.7. $P(-1;7)$; $Q(-2;-3)$

2. Determine whether the Δ whose vertices are given in each case is scalene, equilateral or isosceles.

2.1. $A(1;2)$; $B(6;2)$; $C(6;1)$

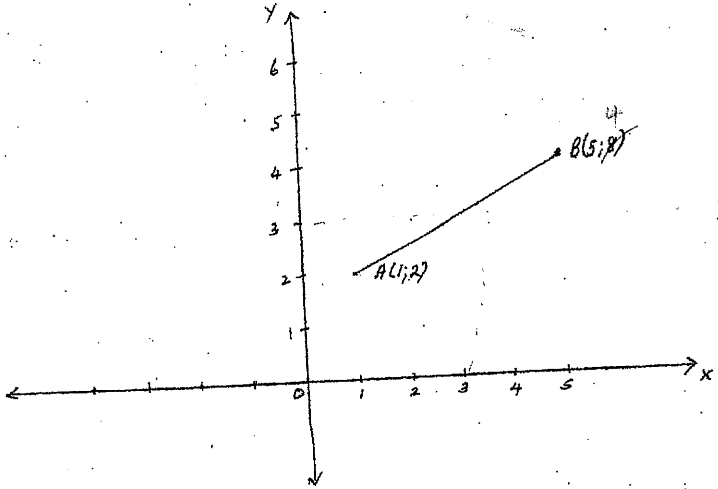
2.2. $P(-4;1)$; $Q(3;0)$; $R(1;-3)$

2.3. $U(-5;-2)$; $V(-1;-1)$ and $W(-13;1)$

3. Find the perimeter of a Δ with vertices: $P(-1;2)$; $Q(1;6)$ and $R(4;0)$. What type of Δ is PQR?

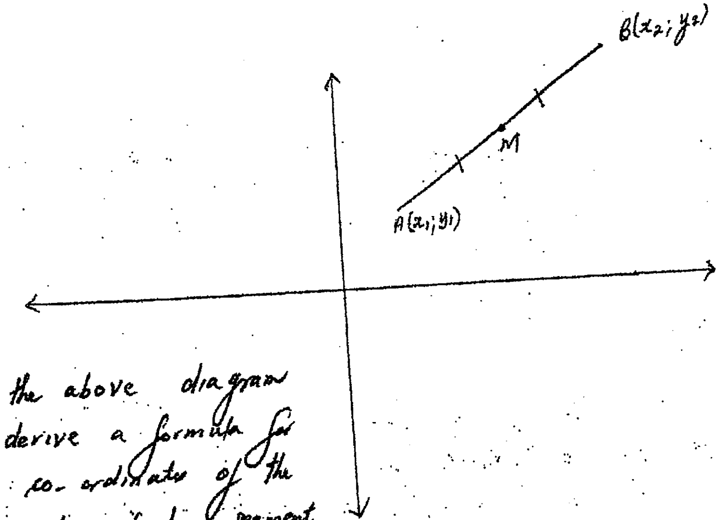
The midpoint of a line segment.

1.



Use the above diagram to find the co-ordinates of the midpoint of line segment AB.

2.



Use the above diagram to derive a formula for the co-ordinates of the midpoints of line segment AB.

3. Find the mid-point of the line segment joining the points.

3.1. A(1;4) and B(3;6)

3.2. A(3;7) and B(5;9)

3.3. A(−3;−1) and B(−5;−5)

3.4. A(−3;−4) and B(5;6)

4. Det x and y if:

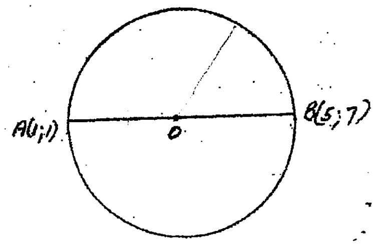
4.1. M(−4;1) is the midpoint of the line segment joining A(−2;4) and B(x;y).

4.2. M(0;y) is the midpoint of the line segment joining A(−1;−3) and B(x;7)

5. O is the centre of the circle.

5.1 Find the co-ordinates of O.

5.2. Find the radius of the circle.



Gradient of a straight line.

1. $A(x_1; y_1)$ and $B(x_2; y_2)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

2. Parallel lines have equal gradients.

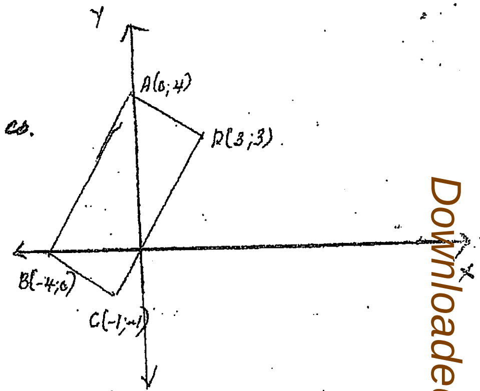
3. Perpendicular lines: $L_1 \perp L_2$
$\Rightarrow m_1 \times m_2 = -1$

4. Determine which of the following sets of points are collinear.

4.1. $A(-1;0)$; $B(1;2)$ and $C(3;4)$
4.2. $P(-1;0)$; $Q(2;-1)$ and $R(5;5)$

5. What is the gradient of the line passing through the origin and $(-3;2)$?

6.



6.1. Use the distance formula to find the length of AB and CD.

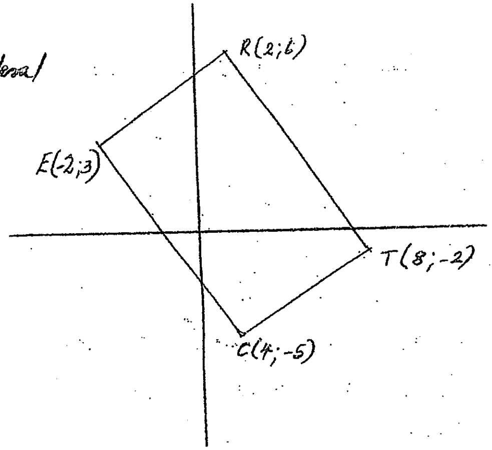
6.2 Prove that AB // CD.

6.3. Prove BC // AD.

6.4. What type of figure is ABCD?

6.5. Determine whether the diagonals of the above figure bisect each other; in other words, determine whether the midpoints of the diagonal coincide.

7.
Show that the quadrilateral is a rectangle.

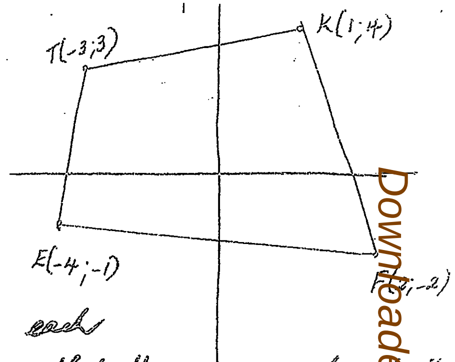


8. A rhombus is a quad in which all 4 sides are of equal length. A quad has co-ordinates A(3;6); B(-1;4) C(3;-2) and D(7;2) 8.1. Draw a diagram to illustrate the quad. 8.2. Show that the quad is a rhombus. 8.3 Check whether it is also a square. Show your working out.

9.
A kite is a quad that has 2 pairs of adjacent sides equal.

9.1. Show that the quad KTEF is a kite.

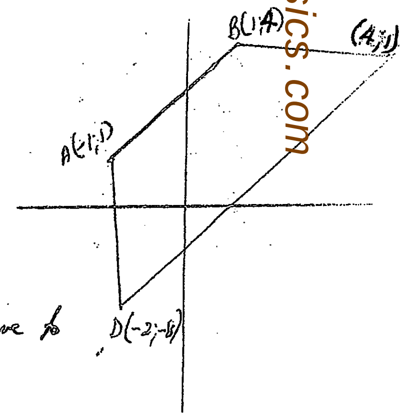
9.2. Show that the diagonals of the kite do not bisect each other. In other words show that the midpoints of the diagonals do not coincide.



10.1 Det. the gradients of: AB; BC; CD and AD.

10.2. Which lines are parallel to each other? Give a reason.

10.3. What name would you give to ABCD?



PAST YEAR PAPERS

QUESTIONS

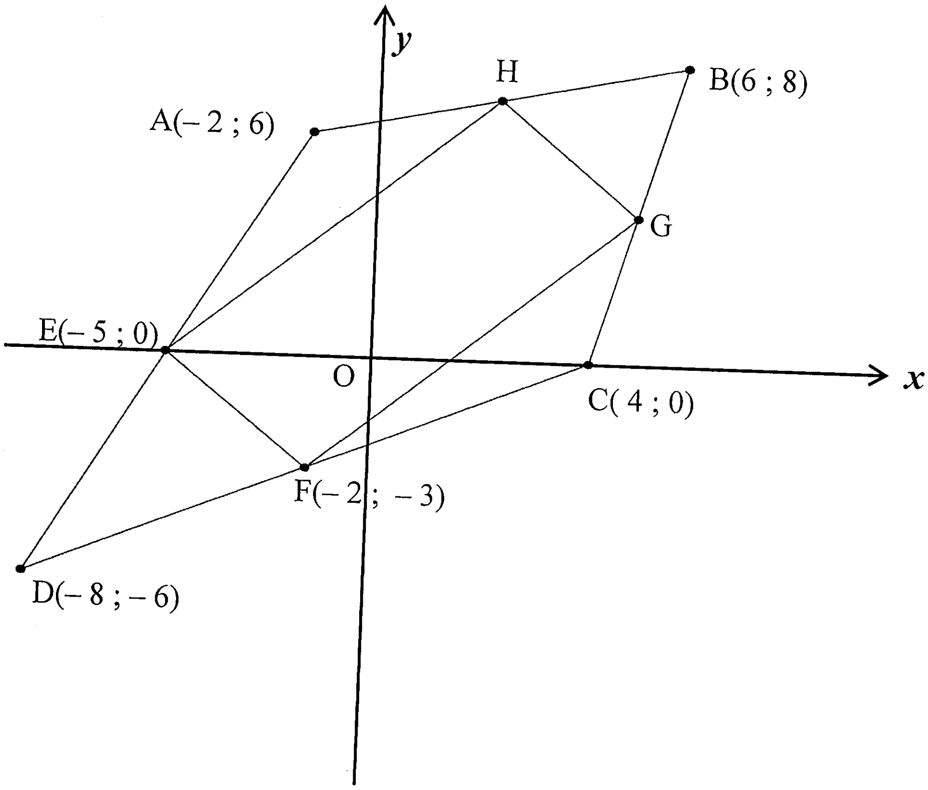
&

ANSWERS

NOVEMBER 2019

QUESTION 2

In the diagram below, H and G are the midpoints of AB and BC respectively. The coordinates of A(– 2 ; 6) , B(6 ; 8) , C(4 ; 0) , D(– 8 ; – 6) , E (– 5 ; 0) and F(– 2 ; – 3) are given. The diagram is not necessarily drawn to scale.



2.1 Show by calculation that AB = BC.

2.2 If it is further given that AD = DC, what type of quadrilateral is ABCD? Motivate your answer.

2.3 Determine the coordinates of G and H.

2.4 If line BD is drawn and it is also given that EH|| BD, prove that ΔAEH|||ΔCDB.

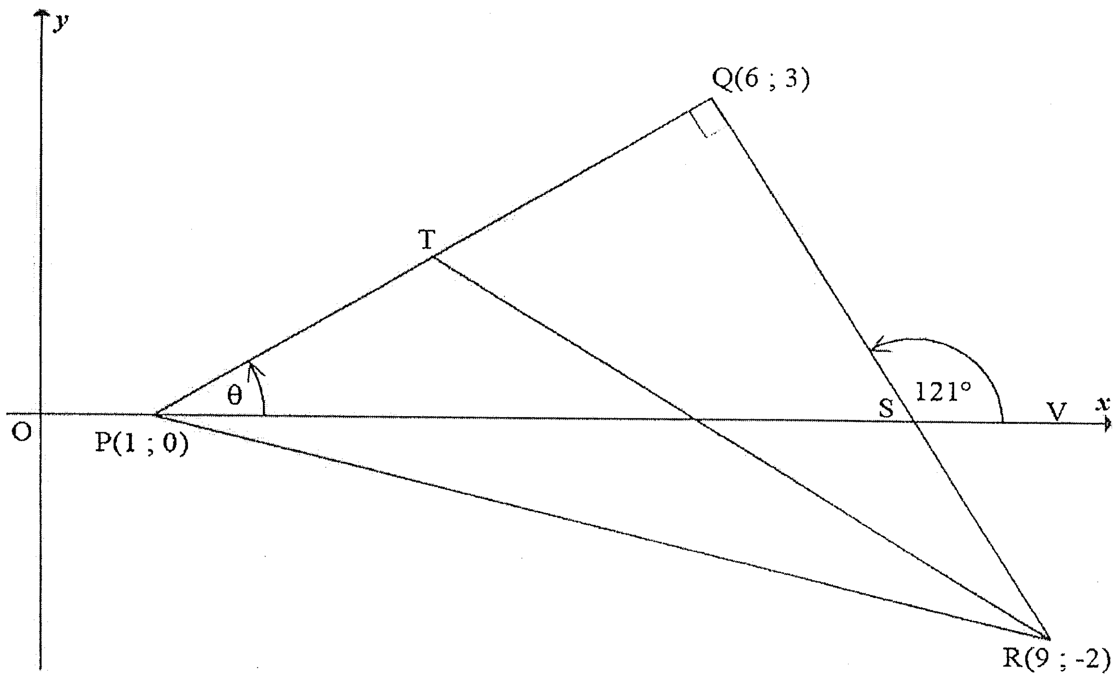
NOVEMBER 2019

QUESTION 2

2.1	A(– 2 ; 6) , B(6 ; 8) and C(4 ; 0) $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6-(-2))^2 + (8-6)^2}$ $= 2\sqrt{17}$ $d_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4-6)^2 + (0-8)^2}$ $= 2\sqrt{17}$ $\therefore$ AB = BC.
2.2	ABCD is a kite adjacent sides are equal
2.3	A(– 2 ; 6) , B(6 ; 8) and C(4 ; 0) Midpoint of BC $= \left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$ $= (\frac{-2+6}{2} ; \frac{8+6}{2})$ = G(2 ; 7) Midpoint of AB $= \left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$ $= (\frac{4+6}{2} ; \frac{0+8}{2})$ = H(5 ; 4)
2.4	$B\hat{A}D = B\hat{C}D$ (opposite ∠'s of a kite are =) $A\hat{E}H = E\hat{D}B$ (corresponding ∠'s , EG || DB) but $E\hat{D}B = B\hat{D}C$ (diagonals of a kite) $\therefore A\hat{E}G = B\hat{D}C$ $\therefore \Delta AEG \| \| \| \Delta CDB$. (A A A)

QUESTION 2

In the diagram below, P(1 ; 0), Q(6 ; 3) and R (9; –2) are the vertices of a triangle such that $PQ = QR$ and $PQ \perp QR$. T is a point on PQ such that T is the midpoint of PQ. S is the point of intersection of RQ and the x-axis. V is a point on the x-axis such that $Q\hat{S}V = 121°$. $Q\hat{P}S = \theta$



2.1 Determine the:

2.1.1 Length of PQ. Leave your answer in surd form. (2)

2.1.2 Gradient of PQ (2)

2.1.3 Coordinates of T (2)

2.2 Calculate the:

2.2.1 Area of ΔQTR (3)

2.2.2 Size of θ, with reasons (2)

2.2.3 Coordinates of S (3)

2.3 Determine, with reasons, the gradient of the line through T and the midpoint of PR. (3)

[17]

QUESTION/*VRAAG* 2

2.1.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(1-6)^2 + (0-3)^2}$ $= \sqrt{25+9}$ $= \sqrt{34}$ Answer only: 2/2 marks	✓subst./*verv.* ✓answer/*antwoord* (2)
2.1.2	$m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{3-0}{6-1}$ $= \dfrac{3}{5}$ Answer only: 2/2 marks	✓subst./*verv.* ✓answer/*antwoord* (2)
2.1.3	$x_T = \dfrac{x_1 + x_2}{2}$ $\quad y_T = \dfrac{y_1 + y_2}{2}$ $= \dfrac{1+6}{2}$ $\quad = \dfrac{0+3}{2}$ $= \dfrac{7}{2}$ $\quad = \dfrac{3}{2}$ $T\left(\dfrac{7}{2} ; \dfrac{3}{2}\right)$	✓*x*-value/*x-waarde* ✓*y*-value/*y-waarde* (2)
2.2.1	$QR = QP = \sqrt{34}$ $QT = \dfrac{1}{2}PQ$ $QT = \dfrac{1}{2}\sqrt{34}$ **OR/*OF*** $QT = \sqrt{\left(\dfrac{7}{2}-6\right)^2 + \left(\dfrac{3}{2}-3\right)^2}$ $QT = \dfrac{\sqrt{34}}{2}$ Area of $\Delta QTR = \dfrac{1}{2}(QR)(QT)$ $= \dfrac{1}{2}(\sqrt{34})\left(\dfrac{1}{2}.\sqrt{34}\right)$ $= \dfrac{17}{2} = 8{,}5$ sq units/eenhede **OR/*OF***	✓ $QR = \sqrt{34}$ ✓ $QT = \dfrac{1}{2}\sqrt{34}$ ✓answer/*antwoord* (3)

	$QR = QP = \sqrt{34}$ Area of $\Delta QTR = \dfrac{1}{2}$Area of ΔQPR $= \dfrac{1}{2}\left(\dfrac{1}{2}.QR.QP\right)$ $= \dfrac{1}{2} \times \dfrac{1}{2}.(\sqrt{34})(\sqrt{34})$ $= \dfrac{17}{2}$ sq units/eenhede	✓ $QR = \sqrt{34}$ ✓ $\dfrac{1}{2}\sqrt{34}$ ✓answer/*antwoord* (3)
2.2.2	$\theta = 121^\circ - 90^\circ$ (ext ∠ Δ/*buitehoek van Δ*) $= 31^\circ$ **OR/*OF*** $Q\hat{S}P = 59^\circ$ (∠ str line/*hoek op reguitlyn*) $\theta = 31^\circ$ (∠ sum Δ/*binnehoek van Δ*)	✓ reason ✓answer/*antwoord* (2) ✓ ∠ sum Δ/*binnehoek van Δ* ✓ answer/*antwoord* (2)
2.2.3	$\cos\theta = \dfrac{PQ}{PS}$ $\cos 31^\circ = \dfrac{\sqrt{34}}{PS}$ $PS = \dfrac{\sqrt{34}}{\cos 31^\circ}$ $PS = 6{,}80$ **OR/*OF*** $\sin Q\hat{S}P = \dfrac{PQ}{PS}$ $\sin 59^\circ = \dfrac{\sqrt{34}}{PS}$ $PS = \dfrac{\sqrt{34}}{\sin 59^\circ}$ $PS = 6{,}80$ $S(6{,}8 + 1 ; 0)$ $S(7{,}8 ; 0)$ **OR/*OF*** $m_{QR} = -\dfrac{5}{3}$ $\dfrac{3-0}{6-x} = -\dfrac{5}{3}$ $9 = -30 + 5x$ $x = 7{,}8$ **OR/*OF*** $m_{QR} = -\dfrac{5}{3}$ Equation of QR $y - 3 = -\dfrac{5}{3}(x-6)$ $y = -\dfrac{5}{3}x + 13$ $0 = -\dfrac{5}{3}x + 13$ $x = 7{,}8$ $S(7{,}8 ; 0)$	✓ $\cos\theta = \dfrac{PQ}{PS}$ or/*of* $\sin Q\hat{S}P = \dfrac{PQ}{PS}$ ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* (3) ✓ $m_{QR} = m_{QS}$ ✓ $y = 0$ ✓ *x*-value/*x-waarde* (3) ✓ equation of QR/*verhouding van QR* ✓ $y = 0$ ✓ *x*-value/*x-waarde*

		(3)
2.3	$m_{QR} = \frac{3-(-2)}{6-(9)}$ $= -\frac{5}{3}$ $m_{T\text{-midpoint}} = m_{QR}$ (Midpoint Theorem) $m_{T\ \text{midpoint}} = -\frac{5}{3}$	✓ m_{QR} ✓ $m_{T\text{-midpoint}} = m_{QR}$ ✓Midpoint theorem/ *Middelpunt-stelling* (3)
	OR/*OF* Midpoint PR$\left(\frac{9+1}{2};\frac{-2+0}{2}\right)$ Midpoint PR$(5;-1)$ $m_{T\ and/en\ PR} = \frac{\frac{3}{2}-(-1)}{\frac{7}{2}-(5)}$ $= -\frac{5}{3}$	✓midpoint of PR ✓ subst ✓answer (3)
		[17]

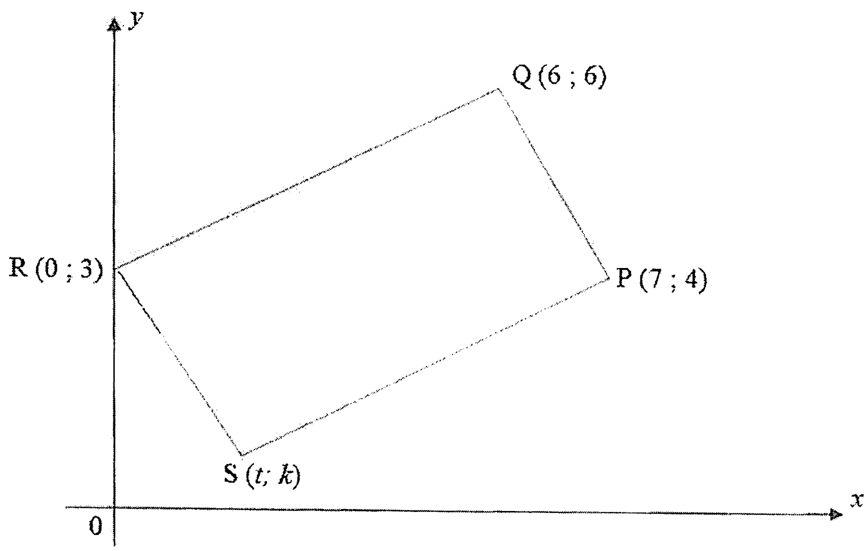
QUESTION/*VRAAG* 3

3.1.1	$\tan(90^\circ - R) = \frac{PR}{QP}$ OR/*OF* $\frac{q}{r}$	✓answer/*antwoord* (1)
3.1.2	sec Q	✓ answer/*antwoord* (1)
	OR/*OF* cosec R	✓ answer/*antwoord* (1)
	OR/*OF* cosec (90° – Q)	✓ answer/*antwoord* (1)
	OR/*OF* sec (90° – R)	✓ answer/*antwoord* (1)
3.2.1	$OS = \sqrt{(-3)^2+(-4)^2}$ (Pythagoras) $= 5$ Answer only: 2/2 marks	✓subst./*verv.* ✓answer/*antwoord* (2)

15

QUESTION 3

In the diagram below, P(7 ; 4), Q(6 ; 6), R(0 ; 3) and S(t ; k) are the vertices of quadrilateral PQRS.



3.1 Calculate the length of PQ. Leave your answer in surd form. (2)

3.2 If $T\left(\frac{7}{2};\frac{7}{2}\right)$ is the midpoint of QS, determine the coordinates of S. (3)

3.3 If the coordinates of S are (1 ; 1), show that PR = QS. (2)

3.4 Show that $QR \perp RS$. (4)

3.5 Hence, what type of special quadrilateral is PQRS? Motivate your answer. (2)

3.6 Calculate the size of $R\hat{S}Q$. (3)

[16]

QUESTION/*VRAAG* 2

2.1	30 days/*dae*	✓answ./*antw.* (1)
2.2	$28 \le T < 32$	✓answ./*antw.* (1)
2.3	The mean/*Gemiddeld* $(\overline{X}) = \frac{44+104+270+170+266+126}{30}$ $= \frac{980}{30}$ $= 32{,}666$ $= 32{,}67°C$.	✓ addition/*optel* ✓30 ✓answ./*antw.* (3)
2.4	$9+5+7+3 = 24$ days/*dae* % of number of days/*getal dae* $= \frac{24}{30} \times 100$ $= 80\%$	✓addition/*optel* ✓answ./*antw.* (2)
		[7]

QUESTION/*VRAAG* 3

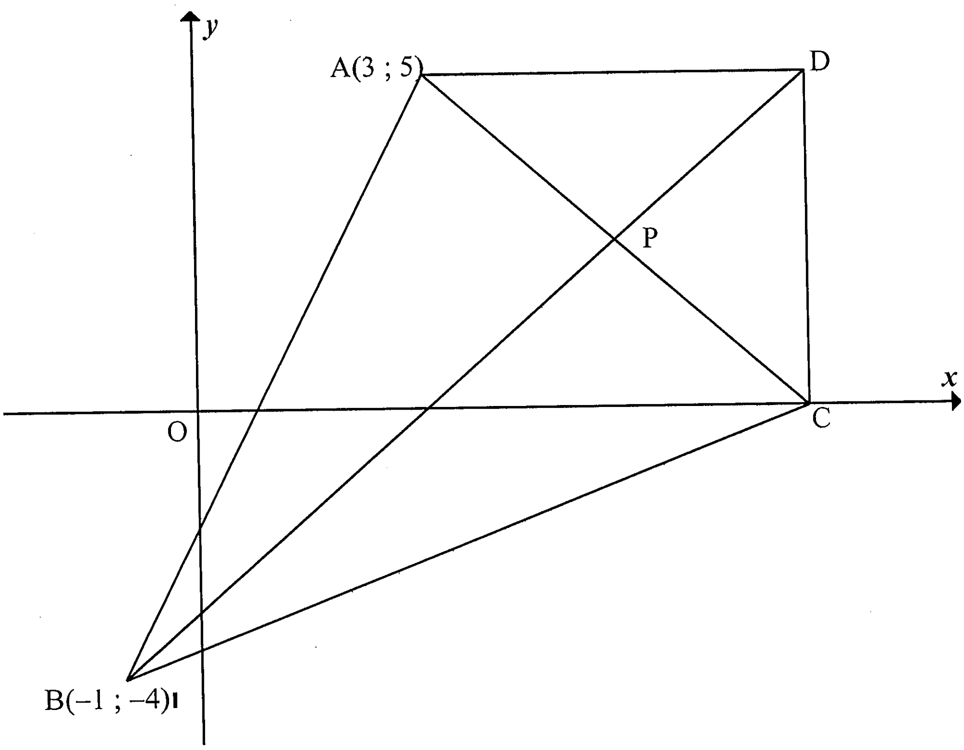
3.1	$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(7-6)^2+(4-6)^2}$ $= \sqrt{(1)^2+(-2)^2}$ $= \sqrt{5}$	✓subst./*verv.* ✓answ./*antw.* (2)
3.2	$M_{QS} = T(x;y)$ $\left(\frac{6+x}{2};\frac{6+y}{2}\right) = \left(\frac{7}{2};\frac{7}{2}\right)$ $\frac{6+x}{2} = \frac{7}{2}$ $\quad \frac{6+y}{2} = \frac{7}{2}$ $x = 1$ $\quad y = 1$ $S(1;1)$	✓ $\frac{6+x}{2} = \frac{7}{2}$ ✓ $\frac{6+y}{2} = \frac{7}{2}$ ✓answ./*antw.* (3)

3.3	$PR = \sqrt{(x_p-x_R)^2+(y_p-y_R)^2}$ $= \sqrt{(7-0)^2+(4-3)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$ $= 7{,}07$ **OR/*OF*** $QS = \sqrt{(x_S-x_Q)^2+(y_S-y_Q)^2}$ $= \sqrt{(1-6)^2+(1-6)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$ $= 7{,}07$ $\therefore PR = QS$	✓answ./*antw.* ✓answ./*antw.* (2)
3.4	$m_{QR} = \frac{6-3}{6-0} = \frac{1}{2}$ $m_{RS} = \frac{3-1}{0-1} = -2$ $m_{QR} \times m_{RS}$ $= \frac{1}{2} \times -2$ $= -1$ $m_{QR} \times m_{RS} = -1$ $\therefore QR \perp RS$	✓ $m_{QR} = \frac{1}{2}$ ✓ $m_{RS} = -2$ ✓ $\frac{1}{2} \times -2$ ✓ $m_{QR} \times m_{RS} = -1$ (4)
3.5	Rectangle./*Reghoek.* The diagonals are equal and one of the interior angles is equal to 90°. *Die hoeklyne is gelyk en een van die binnehoeke is gelyk aan 90°.*	✓Rectangle/*Reghoek* ✓reason/*rede* (2)
3.6	$\cos R\hat{S}Q = \frac{\sqrt{5}}{5\sqrt{2}}$ $R\hat{S}Q = 71{,}57°$	✓✓ $\cos R\hat{S}Q = \frac{\sqrt{5}}{5\sqrt{2}}$ ✓ answ./*antw.* (3)
		[16]

QUESTION 3

3.1 Show that a triangle ABC, with vertices A(1 ; 1); B(3 ; 6) and C(6 ; 3), is an isosceles triangle. (4)

3.2 In the diagram below, ADCB is a kite with A(3 ; 5) and B(–1 ; –4). AD = DC and AB = BC. D is a point such that AD is parallel to the *x*-axis and AD = 5 units. CD is perpendicular to the *x*-axis. The diagonals intersect at P.



3.2.1 Show that the coordinates of C are (8 ; 0). (2)

3.2.2 Write down the coordinates of point P. (2)

3.2.3 Calculate the gradient of line BD. (2)

3.2.4 Calculate the length of line AC. (2)

3.2.5 Calculate the area of the kite ADCB. (3)
[15]

QUESTION 2/*VRAAG 2*

2.1	Modal class(*Module klas*) $100 \leq x < 110$	✓ answer/*antwoord* (1)
2.2	$110 \leq x < 120$	✓✓answer/*antwoord* (2)
2.3	Estimate Mean IQ of students/*Geskatte gemiddelde IK* $= \frac{3480}{30}$ $= 116$	✓ 3480 ✓ 30 ✓ answer/*antwoord* (3) **[6]**

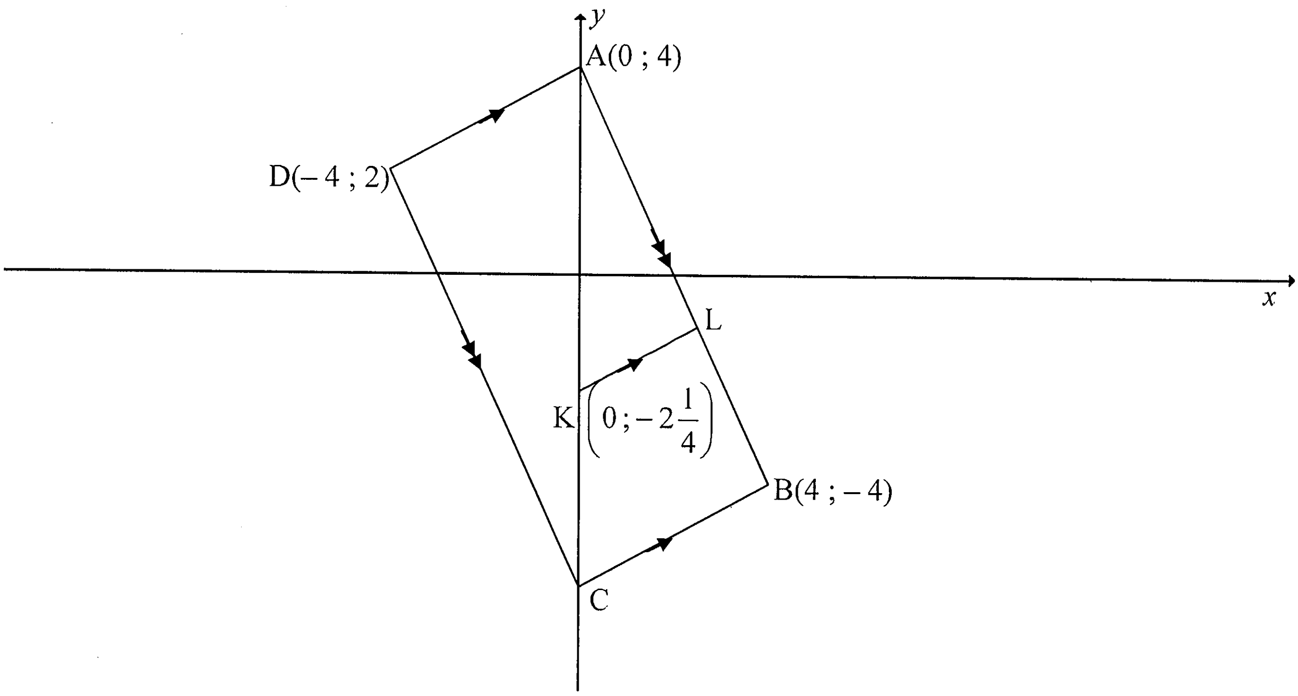
*
QUESTION 3/*VRAAG 3*

3.1	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(3-1)^2 + (6-1)^2}$ $= \sqrt{29}$ $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6-1)^2 + (3-1)^2}$ $= \sqrt{29}$ AB = AC ∴ ΔABC is isosceles/*gelykbenig*	✓subst. in corr. formula/*vervang in korrekte formule* ✓ distance/*afstand* AB ✓subst. in corr. formula/*vervang in korrekte formule* ✓AB = AC (4)
3.2.1	AD is parallel to the *x*-axis/*AD parallel aan x-as* ∴ A and D have the same *y*-coordinates/*A en D het dieselfde y-koördinate* but AD = 5 units/*eenhede* ∴ D(8 ; 5) CD is perpendicular to the *x*-axis/*CD is loodreg met x-as* ∴ C and D have the same *x*-coordinate/*C en D het dieselfde x-koördinate* But C lies on the *x*-axis./*C lê op x-as* ∴ C(8 ;0)	✓coordinates D/ *koördinate D* ✓ coordinates C/ *koördinate C* (2)

3.2.2	P is midpoint of AC the diagonals of the kite/ *P is middelpunt van AC, die hoeklyne van die ruit* $\therefore P\left(\frac{3+8}{2} ; \frac{5+0}{2}\right)$ $P\left(\frac{11}{2} ; \frac{5}{2}\right)$	✓ *x*-value/*waarde* ✓ *y*-value/*waarde* (2)
3.2.3.	B(–1 ; –4) D(8 ; 5) $m_{bd} = \frac{5+4}{8+1}$ $= 1$	✓ substitution/*vervang* ✓answer/*antwoord* (2)
3.2.4	A(3 ; 5) C(8 ; 0) $AC = \sqrt{(0-5)^2 + (8-3)^2}$ $= \sqrt{50}$	✓ substitution *vervang* ✓ answer/*antwoord* (2)
3.2.5	B(–1 ; –4) D(8 ; 5) $BD = \sqrt{(5+4)^2 + (8+1)^2}$ $= \sqrt{162}$ $Area = \frac{1}{2}(BD.AC)$ $= \frac{1}{2}(\sqrt{162} . \sqrt{50})$ $= 45$	✓ length/*lengte* BD ✓ substitution/ *vervang* ✓ answer/*antwoord* (3) **[15]**

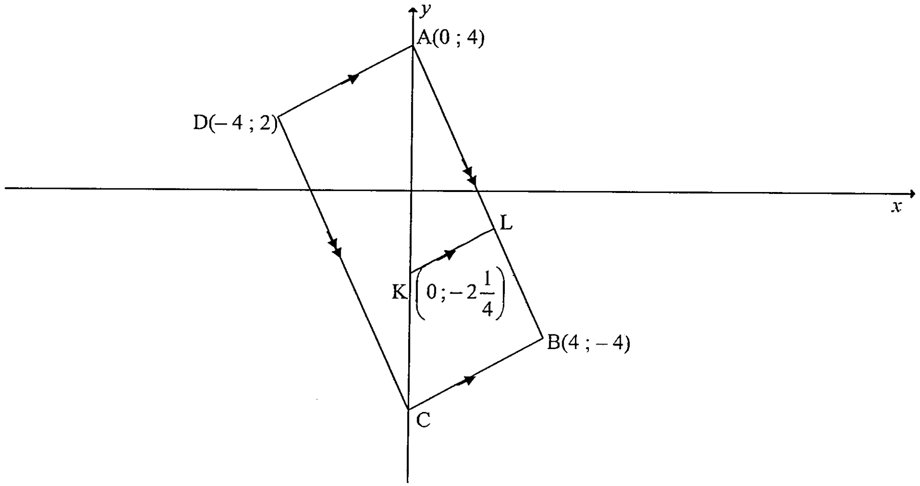
QUESTION 3

In the diagram, C is a point on the y-axis such that A(0 ; 4), B(4 ; – 4), C and D(– 4 ; 2) are vertices of parallelogram ABCD. K is the point $\left(0;-2\frac{1}{4}\right)$ and L is a point on AB such that KL | | CB.



3.1	Calculate the length of diagonal DB.	(3)
3.2	Calculate the coordinates of M, the midpoint of DB.	(3)
3.3	Calculate the gradient of AD.	(3)
3.4	Prove that AD $\perp$ AB.	(3)
3.5	Give a reason why parallelogram ABCD is a rectangle.	(1)
3.6	Determine the equation of KL in the form $y = mx + c$.	(2)
3.7	Write down, with reasons, the coordinates of C.	(3)
		[18]

QUESTION/*VRAAG* 3



3.1	$DB=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ $=\sqrt{(-4-4)^2+(2-(-4))^2}$ $=\sqrt{64+36}$ $=\sqrt{100}$ $=10$	✓ correct formula/ *korrekte formule* ✓ subst ✓ answer/*antw* (3)
3.2	$M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $M\left(\frac{-4+4}{2};\frac{2-4}{2}\right)$ $\therefore M(0;-1)$	✓ correct formula/ *korrekte formule* ✓ *x*-value/*waarde* ✓ *y*-value/*waarde* (3)
3.3	$m_{AD}=\frac{y_1-y_2}{x_1-x_2}$ $=\frac{4-2}{0-(-4)}$ $=\frac{2}{4}=\frac{1}{2}$	✓ correct formula/ *korrekte formule* ✓ subst into/*in* gradient form/ gradiëntvorm ✓ answer/*antw* (3)

3.4	$m_{AB}=\frac{y_1-y_2}{x_1-x_2}$ $=\frac{4-(-4)}{0-4}$ $=\frac{8}{-4}=-2$ $\therefore m_{AD}\times m_{AB}=\frac{1}{2}\times-2=-1$ $\therefore AD\perp AB$	✓ subst ✓ gradient of AB/ *gradiënt van AB* ✓ $m_{AD}\times m_{AB}=-1$ (3)
3.5	parallelogram with one internal angle = 90° *parallelogram met een binnehoek = 90°*	✓ R (1)
3.6	$m_{KL}=m_{AD}=\frac{1}{2}$ [KL | | AD] $\therefore y=\frac{1}{2}x-2\frac{1}{4}$	✓ gradient of KL/ *gradiënt van KL* ✓ equation/*vgl* (2)
3.7	AC = DB = 10 units [diag of rectangle =/*hkle v regh* =] $4-y_C=10$ $y_C=-6$ $\therefore C(0;-6)$ **OR/*OF*** $m_{BC}=m_{AD}=\frac{1}{2}$ [sides of rectangle | |/*sye v regh* | |] $\frac{-4-y_C}{4-0}=\frac{1}{2}$ $-8-2y_C=4$ $y_C=-6$ $\therefore C(0;-6)$	✓ R ✓ equation/*vgl* ✓ answer/*antw* (3) ✓ R ✓ equation/*vgl* ✓ answer/*antw* (3)
		[18]