Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

COMMON TEST

JUNE 2015

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

TIME: 2.5 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(2x-1)=0$ (2)

1.1.2 $5^x(x+8)<0$ (3)

1.2 Solve for x and y simultaneously:

$x-2y=-3$ and $xy=20$ (6)

1.3 Calculate the sum of the digits of $2^{2009}\times 5^{2000}$. (5)

[16]

QUESTION 2

Given the quadratic sequence : $5\ ;12;21\ ;32;...$

2.1 Determine a formula for the n^{th} term of the sequence. (4)

2.2 Between which two consecutive terms in the sequence is the first difference equal to 245? (4)

2.3 Which term in the quadratic sequence has a value of 108237? (4)

[12]

QUESTION 3

3.1 Consider the following pattern:

$1+2+3 = 6$
$4+5+6 = 15$
$7+8+9 = 24$
$10+11+12 = 33$

Calculate the sum of the terms in the 2010^{th} row. (4)

3.2 Given: $16+3+8+3+4+3+2+....$

3.2.1 Determine the sum of the first 40 terms of the series, to the nearest integer. (4)

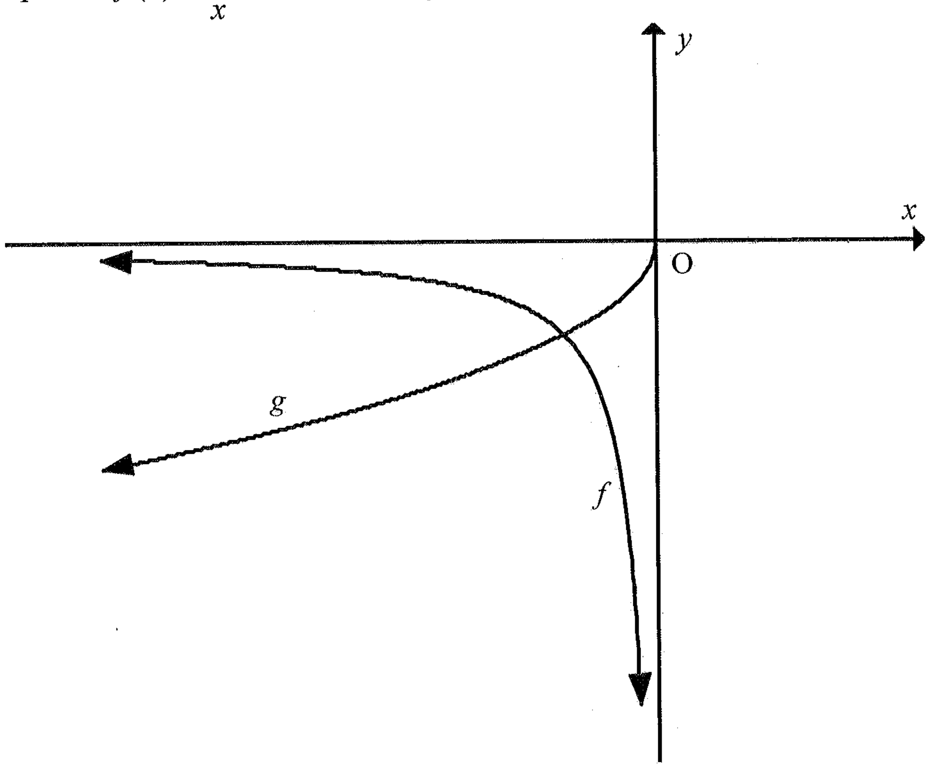
3.2.2 Write the series: $16+8+4+2+...$ in the form $\sum_{k=...}^{.....}T_k$,

where $T_k = ar^{k-1}$ and a and r are rational numbers. (3)

[11]

QUESTION 4

Given the graphs of $f(x)=\dfrac{1}{x}$ for x < 0 and $g(x)=-\sqrt{-x}$ for $x\leq 0$.



4.1 Prove that the graphs of f and g intersect at the point $(-1\ ;-1)$. (4)

4.2 Determine the equation of g^{-1} in the form $y=\ldots$. (3)

[7]

QUESTION 5

The graph of $f(x)=a^{x}$, where $a>0$ and $a\neq 1$, passes through the point $\left(3;\dfrac{27}{8}\right)$.

5.1 Determine the value of a. (3)

5.2 Write down the equation of f^{-1} in the form $y=\ldots$. (2)

5.3 Determine the value(s) of x for which $f^{-1}(x)=-1$. (2)

5.4 If $h(x)=f(x-5)$, write down the domain of h. (1)

5.5 Determine the equation of t if $t(x)=f^{-1}(-x)$. (2)

[10]

QUESTION 6

The graph of $f(x) = a(x+p)^2 + q$ has its turning point at $(-3 ; -5)$ and passes through the point $(0 ; 4)$. $g(x) = 2x + c$ is a tangent to the graph of f.

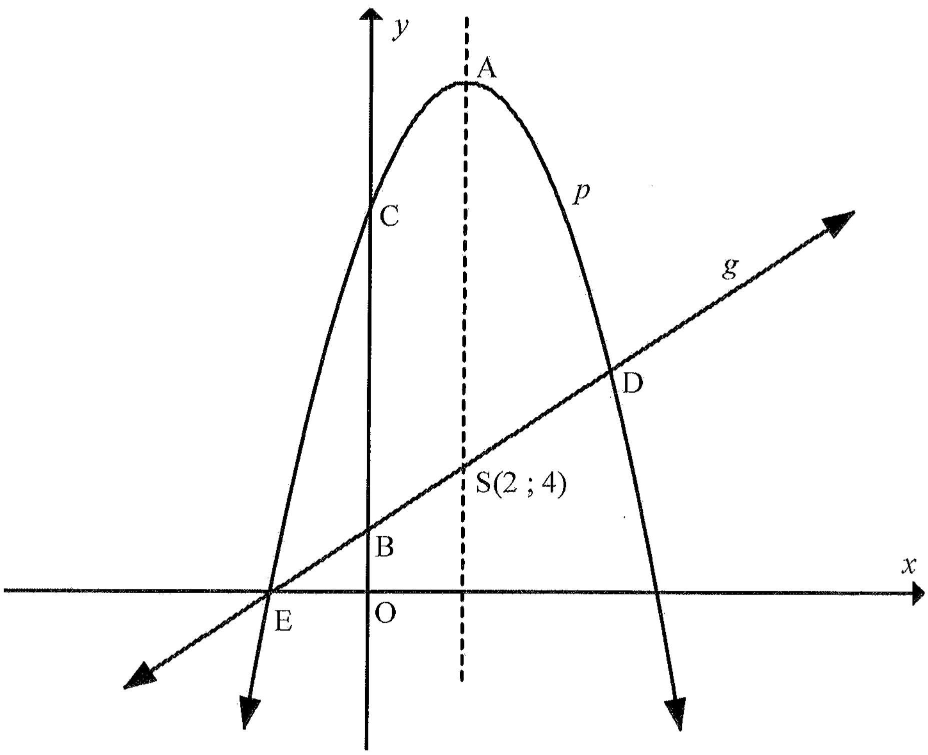
6.1 Determine the equation of f in the form $y = a(x+p)^2 + q$. (4)

6.2 Calculate the value of c. (5)

[9]

QUESTION 7

The sketch below represents the graphs of $p(x) = -x^2 + bx + c$ and $g(x) = x + k$.
The point S(2 ; 4) lies on the axis of symmetry of the graph p and on the line g.
A is the turning point of the graph of p. The graphs of p and g intersect at D and E.
The y–intercepts of p and g are C and B respectively.



7.1 Show that $k = 2$, $b = 4$ and $c = 12$. (6)

7.2 Determine the coordinates of A, the turning point of p. (3)

7.3 Calculate the length of BC. (1)

7.4 Determine the coordinates of D, the point of intersection of p and g. (5)

[15]

QUESTION 8

8.1 Determine the derivative of $f(x) = 2x^2 - 3x$ from first principles. (5)

8.2 Determine, using the rules for differentiation, the following:

8.2.1 $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{1}{2}x^4$ (3)

8.2.2 $D_x\left[\frac{x^2 - 5x + 4}{x^2}\right]$ (4)

8.3 Given: $g(x) = x^3 + 4x^2 + 8x$

8.3.1 Determine $g'(-2)$. (3)

8.3.2 Determine the equation of the tangent to g at $x = -2$ in the form $y = mx + c$. (4)

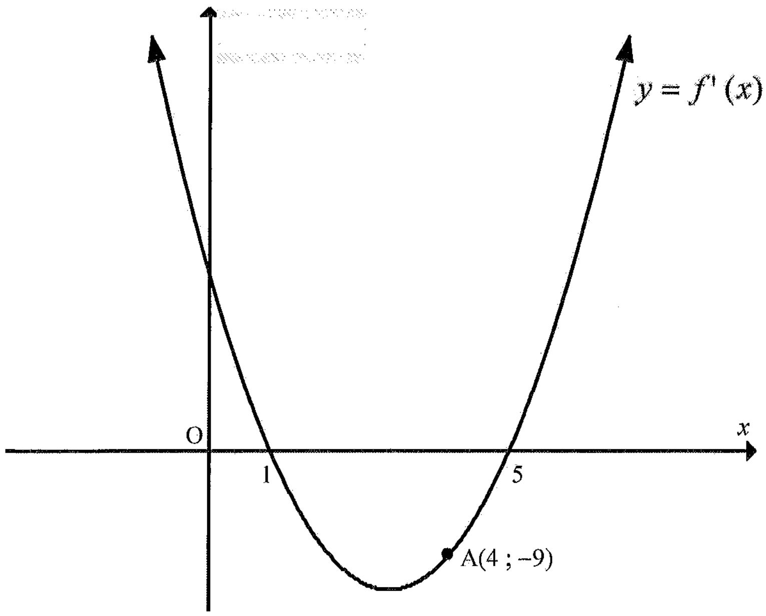
8.3.3 Calculate the coordinates of the point of inflection of g (2)

8.3.4 Show that g is increasing for all real value(s) of x. (3)

[24]

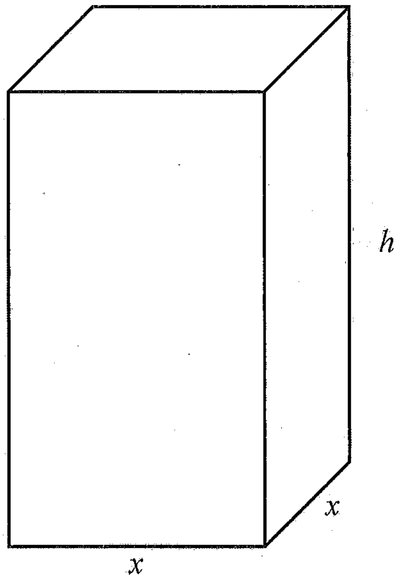
QUESTION 9

9.1 The diagram below shows the graph of $f'(x)$, the derivative of $f(x) = ax^3 + bx^2 + cx + d$. The graph of $f'(x)$ intersects the x–axis at 1 and 5. A(4 ; −9) is a point on the graph of $f'(x)$.



9.1.1	Write down the gradient of the tangent to f at $x = 4$.	(1)
9.1.2	Determine the x–coordinates of the turning points of f.	(2)
9.1.3	For which value(s) of x is f strictly decreasing?	(2)

9.2

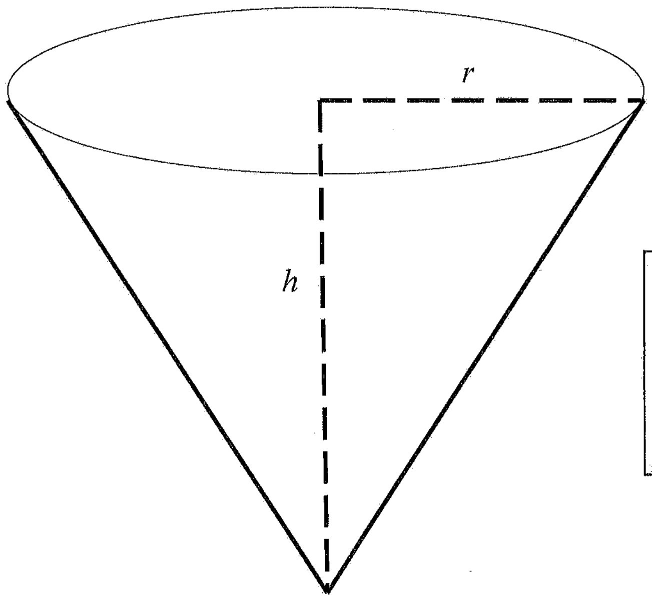


A solid square right prism is made of 8 m^3 melted metal. The length of the sides of the base are x metres and the height is h metres. The block will be coated with one layer of paint.

9.2.1 Show that the surface area of the block is given by $A = 2x^2 + \frac{32}{x}$ (4)

9.2.2 Calculate the dimensions of the block that will ensure that a minimum quantity of paint will be used. (6)

9.3 A water tank in the shape of a right circular cone has a height of h cm. The top rim of the tank is a circle with radius of r cm. The ratio of the height to the radius is 5:2. Water is being pumped into the tank at a constant rate. Determine the rate of change of the volume of water flowing into the tank when the depth is 5 cm. (6)



Surface Area of Cone $= \pi r^2 + \pi r s$

Volume of Cone $= \frac{1}{3}\pi r^2 h$

[21]

TOTAL 125

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ P(A or B) = P(A) + P(B) - P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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COMMON TEST

JUNE 2015

MEMORANDUM

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 125

This memorandum consists of 12 pages.

NOTE:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.
- If a candidate has made two consecutive errors, stop marking.
- If a candidate has made an error, and continued correctly , then subsequently made another error, then continued correctly, continue marking until two consecutive errors are made.

QUESTION 1

1.1.1	$x=0$ or $x=\frac{1}{2}$	✓✓ values of x (2)
1.1.2	$5^x>0$ for all $x\in R$ $5^x(x+8)<0$ iff $x+8<0$ $\therefore x<-8$	✓$5^x>0$ for all $x\in R$ ✓$x+8<0$ ✓answer (3)
1.2	$x-2y=-3\rightarrow(1)$ $xy=20 \quad \rightarrow(2)$ $x=2y-3\rightarrow(3)$ subst. (3) into (1) $y(2y-3)=20$ $2y^2-3y-20=0$ $(2y+5)(y-4)=0$ $y=\frac{-5}{2}$ or $y=4$ $x=-8$ or $x=5$	✓making x the subject ✓substitution ✓std form ✓factors ✓ y values ✓ x values (6)
1.3	$2^{2009}\times5^{2000}$ $=2^9.2^{2000}\times5^{2000}$ $=512.(2.5)^{2000}$ $=512\times10^{2000}$ $=51200000000......0$ (2000 zeros) Sum of digits $=5+1+2+0+0+0+.......+0$ $=8$	✓$2^9.2^{2000}$ ✓512 ✓$(2.5)^{2000}$ ✓512000000 ......0 ✓answer (5)

[16]

QUESTION 2

2.1	1st difference : 7 ; 9 ; 11 ; ... 2nd difference : 2 ; 2 ; 2; ..... $2a=2$ $a=1$ $3a+b=7$ $b=4$ $c=T_0=0$ $T_n=n^2+4n$ OR $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)}{2}d_2$ $=5+(n-1)(7)+\frac{(n-1)(n-2)(2)}{2}$ $=5+7n-7+n^2-3n+2$ $=n^2+4n$	✓a value ✓b value ✓c value ✓T_n (4) ✓substituting T_1 ✓substituting d_1 and d_2 ✓simplifying ✓ T_n (4)
2.2	1st difference n^{th} term : $T_n=2n+5$ $2n+5=245$ $2n=240$ $n=120$ $T_{120}=120^2+4(120)=14880$ $T_{121}=121^2+4(121)=15125$ $T_{121}-T_{120}=245$	✓$T_n=2n+5$ ✓$2n+5=245$ ✓n value ✓T_{120} & T_{121} (4)
2.3	$n^2+4n=108237$ $n^2+4n-108237=0$ $(n-327)(n+331)=0$ $\therefore n=327$ or $n=-331(n/a)$	✓ $n^2+4n=108237$ ✓standard form ✓factors ✓ $n=327$ (n natural) (4)
		[12]

QUESTION 3

3.1	First term in each row has the sequence: 1 ; 4 ; 7 ; 10 ; ... $n^{th}term=3n-2$ $T_{2010}=3(2010)-2$ $=6028$ Therefore the sum of the terms in the 2010th row $6028+6029+6030=18087$	✓ $n^{th}term=3n-2$ ✓substituting 2010 into linear pattern ✓6028 ✓answer (4)
3.2.1	Two sequences are in the given sequence $16+8+4+2+....$ and $3+3+3+.....$ $S_{40}=\frac{16\left(1-\frac{1}{2}^{20}\right)}{\frac{1}{2}}+(3\times 20)$ $=32+60$ $=92$	✓correct substitution into sum formula for geometric ✓32 ✓60 ✓92 (4)
3.2.2	$\sum_{k=1}^{\infty}16\left(\frac{1}{2}\right)^{k-1}$	✓$k=1$ ✓upper limit is ∞ ✓correct values of a and r in T_n (3)
		[11]

QUESTION 4

4.1	$\frac{1}{x} = -\sqrt{-x}$ $\frac{1}{x^2} = -x$ $x^3 = -1$ $x = -1$ $y = \frac{1}{-1} = 1$ Therefore f and g intersect at $(-1\,;-1)$.	✓equating ✓squaring both sides correctly ✓$x^3 = -1$ ✓substituting $x = -1$ (4)
4.2	g : $y = -\sqrt{-x}$ Interchanging x and y: $x = -\sqrt{-y}$ $x^2 = -y$ $y = -x^2\,; x \leq 0$	✓ $x = -\sqrt{-y}$ ✓$x^2 = -y$ ✓ $y = -x^2;\ x \leq 0$ (3)
		[7]

QUESTION 5

5.1	$y = a^x$ $\frac{27}{8} = a^3$ $\left(\frac{3}{2}\right)^3 = a^3$ $a = \frac{3}{2}$	✓substituting $\left(3;\frac{27}{8}\right)$ ✓$\left(\frac{3}{2}\right)^3$ ✓ a value (3)
5.2	$y = \left(\frac{3}{2}\right)^x$ Interchanging x and y: $x = \left(\frac{3}{2}\right)^y$ $f^{-1}(x): y = \log_{\frac{3}{2}} x$	✓ $x = \left(\frac{3}{2}\right)^y$ ✓answer answer only full marks (2)

5.3	$\log_{\frac{3}{2}} x = -1$ $x = \left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$	✓setting equation ✓$x = \frac{2}{3}$ (2)
5.4	$x \in R$	✓answer (1)
5.5	$t(x) = \log_{\frac{3}{2}}(-x)\ ;\ \ x < 0$	✓$t(x) = \log_{\frac{3}{2}}(-x)$ ✓$x < 0$ (2)
		[10]

QUESTION 6

6.1	$y = a(x+p)^2 + q$ $y = a(x+3)^2 - 5$ $4 = a(0+3)^2 - 5$ $9 = 9a$ $a = 1$ $y = (x+3)^2 - 5$	✓substituting the turning point ✓ substituting the point (0 ; 4) ✓$a = 1$ ✓answer (4)
6.2	$f: y = x^2 + 6x + 4$ $f'(x) = 2x + 6$ $2x + 6 = 2$ $2x = -4$ $x = -2$ $y = -4$ $-4 = 2(-2) + c$ $c = 0$	✓derivative ✓equating derivative to 2 ✓ x value ✓ y value ✓ c value (5)
		[9]

QUESTION 7

7.1	$y = x + k$ $4 = 2 + k$ $k = 2$ $\therefore y = x + 2$ $x = \frac{-b}{2a} = 2$ $\frac{-b}{2(-1)} = 2$ $\therefore b = 4$ $y = -x^2 + 4x + c$ $E(-2;0)$: $0 = -(-2)^2 + 4(-2) + c$ $c = 12$	✓substituting point S into g ✓ axis of symmetry ✓substituting (-1)in axis of symmetry equation ✓ equating to 2 ✓ $y = -x^2 + 4x + c$ ✓substituting coordinates of E in p (6)
7.2	Maximum Value: $y = -(2)^2 + 4(2) + 12$ $= 16$ $A(2;16)$	✓substituting 2 into p ✓16 ✓coordinates (3)
7.3	BC = 10 units	✓ answer (1)
7.4	$-x^2 + 4x + 12 = x + 2$ $x^2 - 3x - 10 = 0$ $(x+2)(x-5) = 0$ $x = -2$ or $x = 5$ $y = 0$ or $y = 7$ $D(5;7)$	✓equating both graphs ✓factors ✓x values ✓ y values ✓coordinates of D (5)
		[15]

QUESTION 8

8.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0}\frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$ $= \lim_{h\to 0}\frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$ $= \lim_{h\to 0}\frac{4xh + 2h^2 - 3h}{h}$ $= \lim_{h\to 0}\frac{h(4x + 2h - 3)}{h}$ $= 4x - 3$	✓formula ✓correct substitution into formula ✓simplification ✓removing h as common factor ✓answer (5)
8.2.1	$y = \sqrt{x^3} - \frac{1}{2}x^4$ $y = x^{\frac{3}{2}} - \frac{1}{2}x^4$ $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2x^3$ OR $\frac{dy}{dx} = \frac{3}{2}\sqrt{x} - 2x^3$	✓ $x^{\frac{3}{2}}$ ✓✓each term (3)
8.2.2	$D_x\left[\frac{x^2 - 5x + 4}{x^2}\right]$ $D_x\left[1 - 5x^{-1} + 4x^{-2}\right]$ $= 5x^{-2} - 8x^{-3}$ OR $= \frac{5}{x^2} - \frac{8}{x^3}$	✓1 & $+4x^{-2}$ ✓ $-5x^{-1}$ (simplifying) ✓✓each answer (4)
8.3.1	$g'(x) = 3x^2 + 8x + 8$ $g'(-2) = 3(-2)^2 + 8(-2) + 8$ $= 4$	✓derivative ✓correct substitution of -2 into derivative ✓answer (3)
8.3.2	$g(-2) = (-2)^3 + 4(-2)^2 + 8(-2)$ $y = -8$ $y - y_1 = m(x - x_1)$ $y + 8 = 4(x + 2)$ $y = 4x$	✓correct substitution into original equation ✓y value ✓correct substitution into equation of a line ✓answer (4)

8.3.3	$g''(x)=6x+8=0$ $x=\frac{-4}{3}$ $y=\left(\frac{-4}{3}\right)^3+4\left(\frac{-4}{3}\right)^2+8\left(\frac{-4}{3}\right)$ $=\frac{-160}{27}$ Point of Inflection : $\left(\frac{-4}{3};\frac{-160}{27}\right)$	✓ x value ✓ y value (2)
8.3.4	$g'(x)=3x^2+8x+8$ $=3\left[x^2+\frac{8}{3}x+\frac{16}{9}-\frac{16}{9}+\frac{8}{3}\right]$ $=3\left[\left(x+\frac{4}{3}\right)^2+\frac{8}{9}\right]$ $=3\left(x+\frac{4}{3}\right)^2+\frac{8}{3}$ $3\left(x+\frac{4}{3}\right)^2\geq 0$ for all $x\in R$ $3\left(x+\frac{4}{3}\right)^2+\frac{8}{3}\geq\frac{8}{3}$ $\Rightarrow g'(x)>0$ for all $x\in R$ $\therefore g$ is increasing for all real value(s) of x	✓$3\left(x+\frac{4}{3}\right)^2+\frac{8}{3}$ ✓$3\left(x+\frac{4}{3}\right)^2\geq 0$ for all $x\in R$ ✓ $g'(x)>0$ for all $x\in R$ (3)
		[24]

QUESTION 9

9.1.1	Gradient of tangent $=-9$	✓answer (1)
9.1.2	$x=1$ or $x=5$	✓✓ answers (2)
9.1.3	$1<x<5$	✓ end points ✓inequalities (2)
9.2.1	$A = x^2+x^2+4xh$ $V=x^2h=8$ $\therefore h=\frac{8}{x^2}$ $\Rightarrow A=2x^2+4x\left(\frac{8}{x^2}\right)$ $\therefore A=2x^2+\frac{32}{x}$	✓surface area in terms of x ✓$x^2h=8$ ✓$h=\frac{8}{x^2}$ ✓substitution of $h=\frac{8}{x^2}$ (4)
9.2.2	$A=2x^2+\frac{32}{x}$ $A=2x^2+32x^{-1}$ $A'=4x-32x^{-2}=0$ $4x=\frac{32}{x^2}$ $x^3=8$ $x=2$ $h=\frac{8}{2^2}$ $=2$ sides of the base are $2m$ and the height is $2m$	✓✓derivative = 0 ✓ $4x=\frac{32}{x^2}$ ✓$x^3=8$ ✓$x=2$ ✓height = 2 (6)
9.3	$V=\frac{1}{3}\pi r^2h$ $=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2h$ $=\frac{4}{75}\pi h^3$ $V'=\frac{12}{75}\pi h^2$ $V'(5)=\frac{12}{75}\pi(5)^2$ $=12{,}57\ cm^3/cm$	✓formula ✓substitution of $r=\left(\frac{2}{5}h\right)$ ✓$\frac{4}{75}\pi h^3$ ✓derivative ✓correct substitution of 5 into derivative ✓answer (6)
		[21]

Alternatives:

2.2 Alternative

$T_n - T_{n-1} = 245$ $(n^2+4n) - [(n-1)^2 + 4(n-1)] = 245$ $n^2+4n-n^2+2n-1-4n+4 = 245$ $2n = 242$ $n = 121$ $\therefore T_{121} - T_{120} = 245$	✓ $T_n - T_{n-1} = 245$ ✓substitution ✓simplification ✓values of n (4)
OR	
$T_{n+1} - T_n = 245$ $[(n+1)^2 + 4(n+1)] - (n^2+4n) = 245$ $n^2+2n+1+4n+4-n^2-4n = 245$ $2n = 240$ $n = 120$ $T_{121} - T_{120} = 245$	✓ $T_{n-1} - T_n = 245$ ✓substitution ✓simplification ✓values of n (4)

6.2 Alternative

$x^2+6x+4 = 2x+c$ $x^2+4x+4-c = 0$ $b^2-4ac = 0$ $(4)^2 - 4(1)(4-c) = 0$ $16-16+4c = 0$ $4c = 0$ $c = 0$	✓equating both graphs ✓standard form ✓correct substitution into discriminant ✓simplifying ✓c value (5)

1.2

$y = \frac{20}{x} \rightarrow (3)$ $x - 2\left(\frac{20}{x}\right) = -3$ $x^2+3x-40 = 0$ $(x-5)(x+8) = 0$ $x = 5$ *or* $x = -8$ $y = 4$ *or* $y = -\frac{5}{2}$	✓making y the subject ✓substitution ✓standard form ✓factors ✓x values ✓y values (6)
OR $x = \frac{20}{y} \rightarrow (3)$ $\frac{20}{y} - 2y = -3$ $2y^2-3y-20 = 0$ $(y-4)(2y+5) = 0$ $y = 4$ *or* $y = -\frac{5}{2}$ $x = 5$ *or* $x = -8$	✓making x the subject ✓substitution ✓standard form ✓factors ✓y values ✓x values (6)

2.1

$2a = 2$ $a = 1$ $3a+b = 7$ $b = 4$ $a+b+c = 5$ $c = 0$ $T_n = n^2+4n$	✓a value ✓b value ✓c values ✓$T_n = n^2+4n$ (4)

2.3

$n^2+4n = 108237$ $n^2+4n-108237 = 0$ $n = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-108237)}}{2(1)}$ $= \frac{-4 \pm \sqrt{432964}}{2} = \frac{-4 \pm 658}{2}$ $\therefore n = 327$ or $n = -331 n/a$	✓ $n^2+4n = 108237$ ✓standard form ✓substitution into quadratic formula ✓327 (4)