



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

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DATE: 13 MARCH 2017

**NATIONAL SENIOR CERTIFICATE: COMMON TEST MARCH 2017
GRADE 12**

**TO: THE CHIEF INVIGILATOR OF ALL SCHOOLS OFFERING
MATHEMATICS**

ERRATA

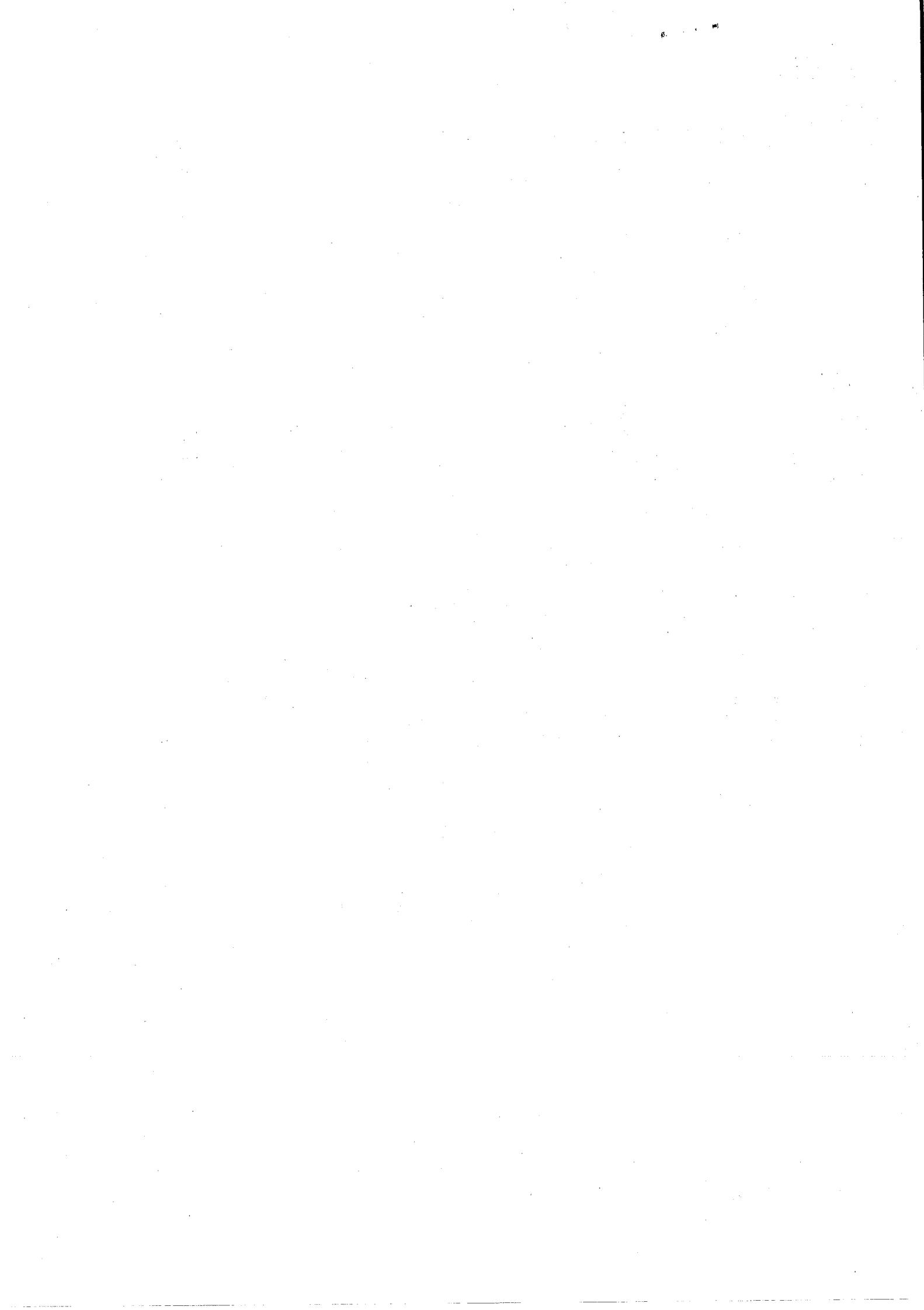
Please take note of the following change:

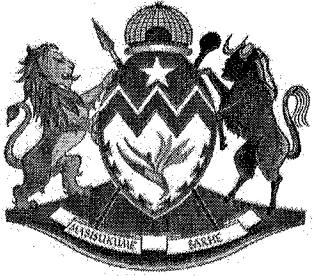
PAGE	NUMBER	ERROR	CORRECTION
5	QUESTION 4 (Statement)	... C ($\neg p$; q)	C (p; q)

Kindly ensure that candidates are informed of the Errata.

**MS N.V. MCAMBI
DEPUTY MANAGER
PROVINCIAL EXAMINATIONS SERVICES**

13/3/2017
DATE





Education

**KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA**

MATHEMATICS

COMMON TEST

MARCH 2017

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

Marks: 100

Time: 2 hours

N.B. This question paper consists of 8 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Determine the derivative of $f(x) = x^2 + 3x$ from first principles. (5)

1.2 Evaluate:

1.2.1 $\frac{dy}{dx}$ if $y = 3x^2 \cdot \sqrt[3]{8x^4}$ (4)

1.2.2 $f'(x)$ if $f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4}; x \neq 4$ (5)

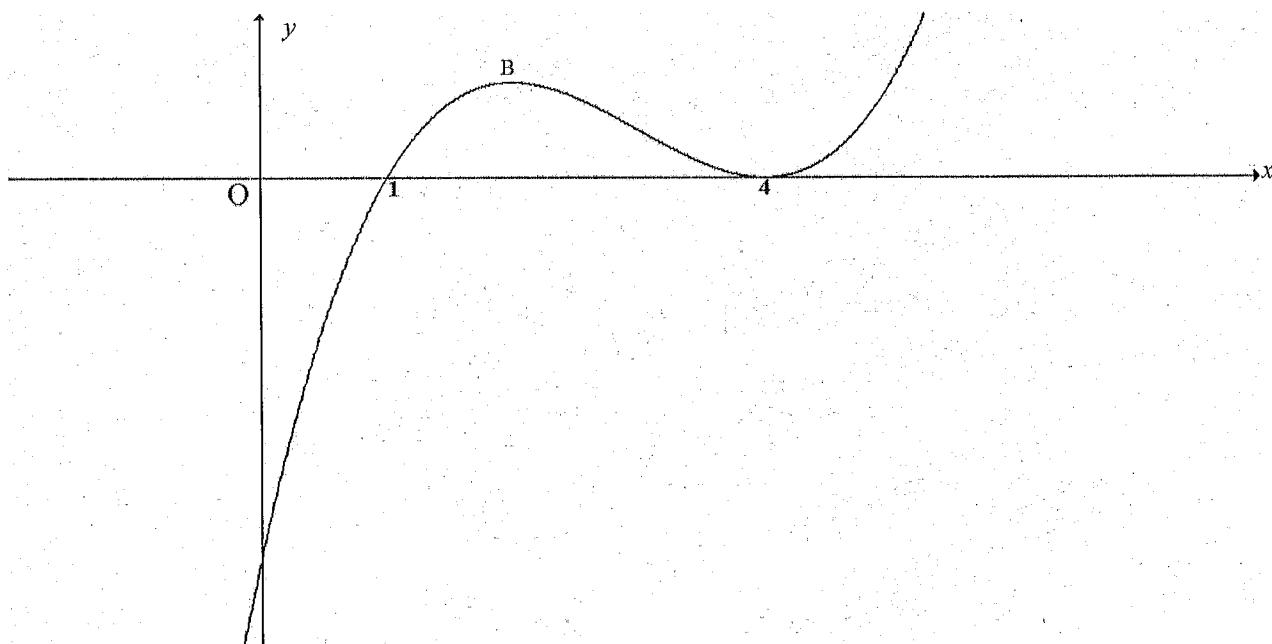
[14]

QUESTION 2

2.1 Determine co-ordinates of the points on the curve $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1 . (5)

2.2 The graph of a cubic function with equation $f(x) = x^3 + ax^2 + bx + c$ is drawn.

- $f(1) = f(4) = 0$
- f has a local maximum at B and a local minimum at $x = 4$.



2.2.1 Show that $a = -9$, $b = 24$ and $c = -16$. (4)

2.2.2 Calculate the coordinates of B. (5)

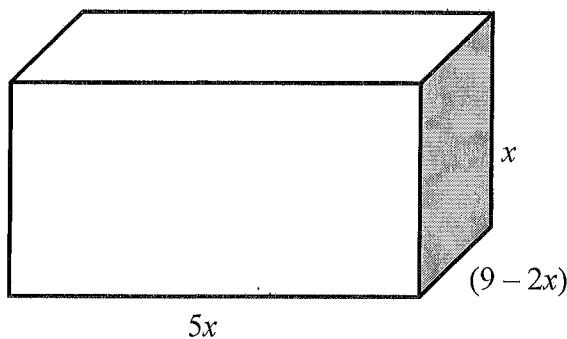
2.2.3 Write down the value(s) of k for which $f(x) = k$ has negative roots only. (2)

2.2.4 Determine the value(s) of x for which f is concave up. (3)

[19]

QUESTION 3

A rectangular box has a length of $5x$ units, breadth of $(9 - 2x)$ units and its height of x units.

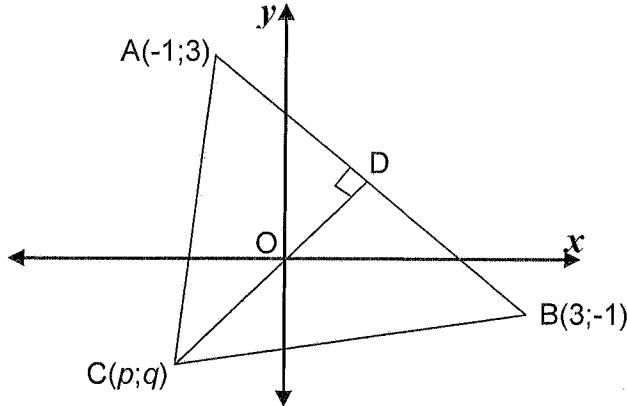


- 3.1 Show that the volume (V) of the box is given by $V = 45x^2 - 10x^3$. (2)
 3.2 Calculate the value of x for which the box will have maximum volume. (5)

[7]

QUESTION 4

In the figure below, A $(-1; 3)$, B $(3; -1)$ and C $(-p; q)$ are vertices of $\triangle ABC$.
 Area of $\triangle ABC = 12$ square units. CD is $\perp AB$.



- 4.1 Determine the coordinates of D, the midpoint of AB (2)
 4.2 Show that $p = q$ (5)
 4.3 If $p ; q < 0$, determine the co-ordinates of C. (5)

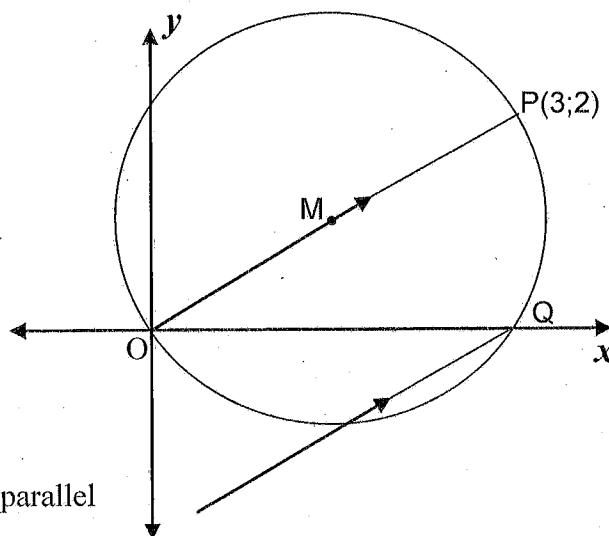
[12]

QUESTION 5

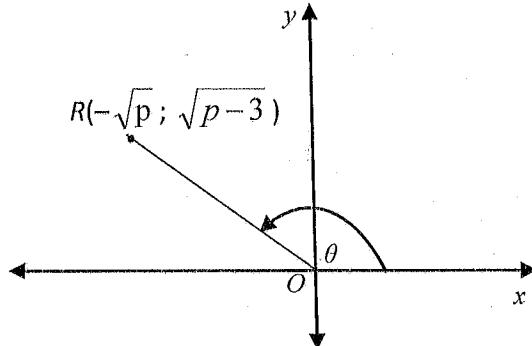
In the accompanying diagram alongside, $P(3; 2)$ lies on the circumference of a circle with centre M. The circle also passes through the origin. Q is the intercept of the circle with the x -axis.

Determine:

- 5.1 the co-ordinates of M. (2)
 - 5.2 the gradient of OP. (1)
 - 5.3 the equation of the circle M. (3)
 - 5.4 the co-ordinates of Q. (4)
 - 5.5 the equation of the line through Q, parallel to OP. (3)
- [13]

**QUESTION 6**

6. In the diagram alongside, $R(-\sqrt{p}; \sqrt{p-3})$ is a point in a Cartesian plane, $\hat{R}OX = \theta$



- 6.1 Express $\cos^2 \theta$ in terms of p . (5)
 - 6.2 Hence, for which real values of p is $\cos^2 \theta$ defined? (2)
 - 6.3 Determine $\frac{1}{\cos \alpha}$ in terms of p if α and θ are supplementary. (2)
- [9]

QUESTION 7

7.1 Simplify WITHOUT the use of a calculator

$$\frac{\cos 225^\circ \cdot \sin (-135^\circ) - \sin 330^\circ}{\tan 225^\circ} \quad (6)$$

7.2 Prove the identity:

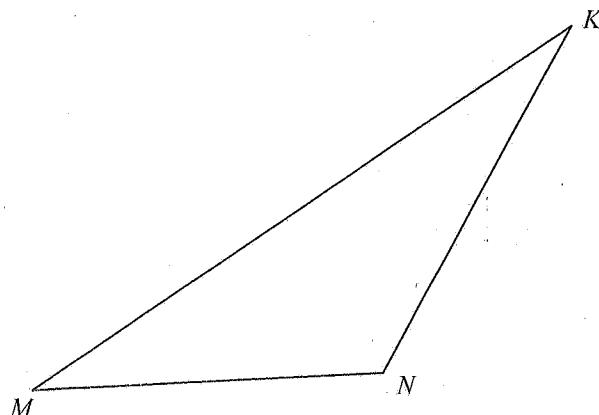
$$\frac{\tan x - \sin x}{\sin^3 x} = \frac{1}{\cos x (1 + \cos x)} \quad (5)$$

7.3 Determine the general solution of $\cos 2x + \cos x - 2 = 0$ (5)

[16]

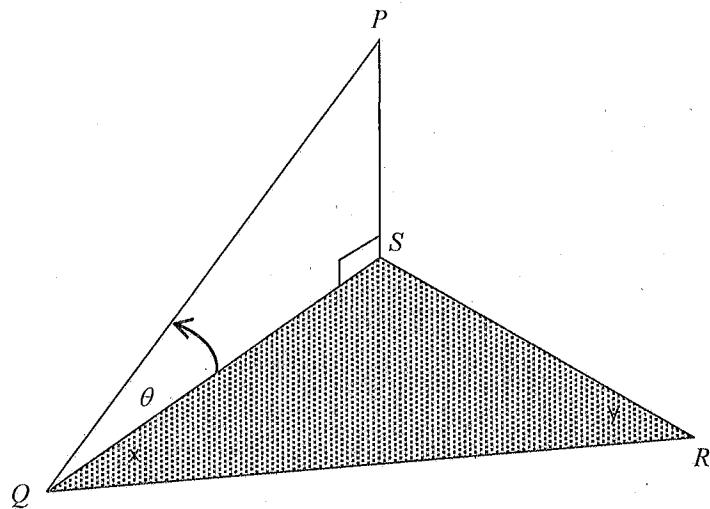
QUESTION 8

- 8.1 Given $\triangle KMN$, with \hat{N} obtuse, as shown in the diagram below:



$$\text{Prove that } \frac{\sin M}{m} = \frac{\sin N}{n} \quad (4)$$

- 8.2 In the diagram below, S, Q and R are points in the same plane. PS is a vertical telephone mast. The angle of elevation of P from Q is θ . $\hat{SQR} = x$, $\hat{SRQ} = y$, $QR = 10\text{m}$.



- 8.2.1 Express PS in terms of QS and θ (2)

8.2.2 Show that $QS = \frac{10 \sin y}{\sin(x + y)}$ (4)

[10]

TOTAL: 100

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1 \quad S_\infty = \frac{a}{1-r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

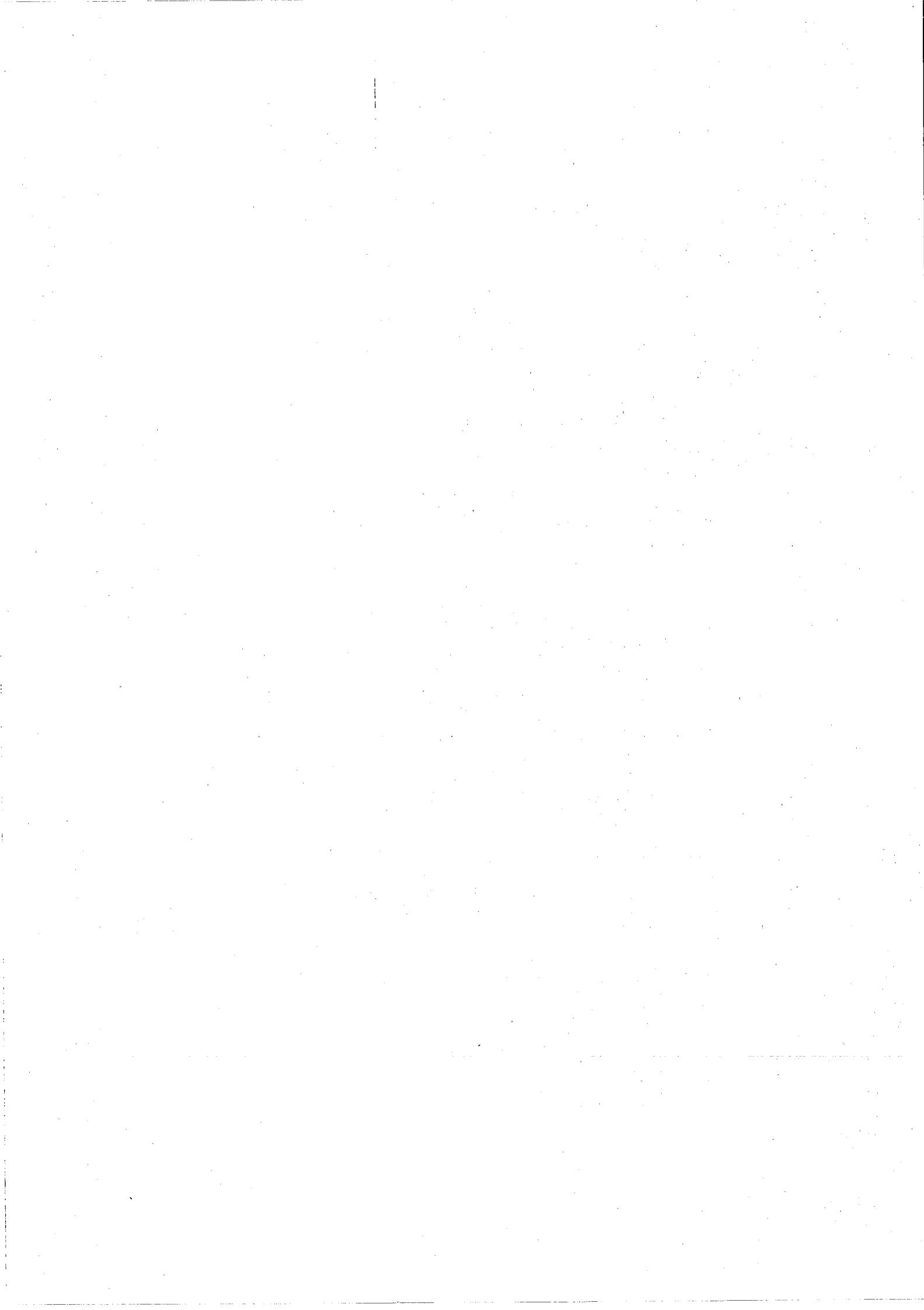
$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



QUESTION 1

Penalize once for notational error in Questions 1 to 3.

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<p>QUESTION 1 Penalize once for notational error in Questions 1 to 3.</p> <p>1.1</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> $f(x) = x^2 + 3x$ $f(x+h) = (x+h)^2 + 3(x+h)$ $= x^2 + 2xh + h^2 + 3x + 3h$ $f(x+h) - f(x) = 2xh + h^2 + 3h$ $\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 3h}{h}$ $= \frac{h(2x + h + 3)}{h}$ $= 2x + h + 3$ $f'(x) = \lim_{h \rightarrow 0} (2x + h + 3)$ $= 2x + 3$ </td><td style="text-align: right; padding: 5px;"> 5 </td></tr> </table> <p>OR</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> $f(x) = x^2 + 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h}$ $= 2x + 3$ </td><td style="text-align: right; padding: 5px;"> 5 </td></tr> </table> <p>1.2.1</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> $\frac{dy}{dx} \text{ if } y = 3x^2 \cdot \sqrt[3]{8x^4}$ $y = 3x^2 \cdot 2x^{\frac{4}{3}}$ $y = 6x^{\frac{10}{3}}$ $\frac{dy}{dx} = 20x^{\frac{7}{3}} \text{ or } 20\sqrt[3]{x^7}$ </td><td style="text-align: right; padding: 5px;"> 4 \checkmark If exponent is an integer – maximum 2/4 Marks \checkmark A $2x^{\frac{4}{3}}$ \checkmark 6 $x^{\frac{10}{3}}$ CA \checkmark CA 20 \checkmark CA $x^{\frac{7}{3}}$ 4 </td></tr> </table>	$f(x) = x^2 + 3x$ $f(x+h) = (x+h)^2 + 3(x+h)$ $= x^2 + 2xh + h^2 + 3x + 3h$ $f(x+h) - f(x) = 2xh + h^2 + 3h$ $\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 3h}{h}$ $= \frac{h(2x + h + 3)}{h}$ $= 2x + h + 3$ $f'(x) = \lim_{h \rightarrow 0} (2x + h + 3)$ $= 2x + 3$	5	$f(x) = x^2 + 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h}$ $= 2x + 3$	5	$\frac{dy}{dx} \text{ if } y = 3x^2 \cdot \sqrt[3]{8x^4}$ $y = 3x^2 \cdot 2x^{\frac{4}{3}}$ $y = 6x^{\frac{10}{3}}$ $\frac{dy}{dx} = 20x^{\frac{7}{3}} \text{ or } 20\sqrt[3]{x^7}$	4 \checkmark If exponent is an integer – maximum 2/4 Marks \checkmark A $2x^{\frac{4}{3}}$ \checkmark 6 $x^{\frac{10}{3}}$ CA \checkmark CA 20 \checkmark CA $x^{\frac{7}{3}}$ 4	<p>Common Test March 2017</p> <p>1.1</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> GRADE 12 </td> <td style="text-align: right; padding: 5px;"> MARKS: 100 </td> </tr> </table> <p>1.2</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> TIME: 2 hours </td> <td style="text-align: right; padding: 5px;"> 5 </td> </tr> </table> <p>This memorandum consists of 12 pages</p>	GRADE 12	MARKS: 100	TIME: 2 hours	5
$f(x) = x^2 + 3x$ $f(x+h) = (x+h)^2 + 3(x+h)$ $= x^2 + 2xh + h^2 + 3x + 3h$ $f(x+h) - f(x) = 2xh + h^2 + 3h$ $\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 3h}{h}$ $= \frac{h(2x + h + 3)}{h}$ $= 2x + h + 3$ $f'(x) = \lim_{h \rightarrow 0} (2x + h + 3)$ $= 2x + 3$	5										
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$\frac{dy}{dx} \text{ if } y = 3x^2 \cdot \sqrt[3]{8x^4}$ $y = 3x^2 \cdot 2x^{\frac{4}{3}}$ $y = 6x^{\frac{10}{3}}$ $\frac{dy}{dx} = 20x^{\frac{7}{3}} \text{ or } 20\sqrt[3]{x^7}$	4 \checkmark If exponent is an integer – maximum 2/4 Marks \checkmark A $2x^{\frac{4}{3}}$ \checkmark 6 $x^{\frac{10}{3}}$ CA \checkmark CA 20 \checkmark CA $x^{\frac{7}{3}}$ 4										
GRADE 12	MARKS: 100										
TIME: 2 hours	5										

2.2.1	$y = (x-1)(x-4)^2$ $= (x-1)(x^2 - 8x + 16)$ $= x^3 - 9x^2 + 24x - 16$ OR $f'(4) = 0 \quad \text{and} \quad f'(4) = 0$ $(4)^3 + a(4)^2 + b(4) + c = 0$ $16a + 4b + c = -64 \rightarrow (1)$ $f'(x) = 3x^2 + 2ax + b$ $f'(4) = 3(4)^2 + 2a(4) + b = 0$ $8a + b = -48 \rightarrow (2)$ $f(1) = 1 + a + b + c = 0$ $a + b + c = -1 \rightarrow (3)$ $(1) - (3) : 1.5a + 3b = -63 \rightarrow (4)$ $(4) \div 3 : 5a + b = -21 \rightarrow (5)$ $(2) - (5) : 3a = -27$ $\therefore a = -9$ <i>substituting int o (5) : -45 + b = -21</i> $b = -21 + 45 = 24$ <i>Subst. a = -9 and b = 24 int o (3) :</i> $-9 + 24 + c = -1$ $c = -1 + 9 - 24 = -16$	4	$\checkmark A \sqrt{A(x-1)(x-4)^2}$ $\checkmark A \text{ squaring binomial}$ $\checkmark A \text{ Answer}$ OR $\checkmark A \text{ forming all three equations}$ $\checkmark A \text{ for derivative}$ $\checkmark A \text{ for equations (4) and (5)}$ $\checkmark A \text{ substitutions}$	
2.2.2	$y = x^3 - 9x^2 + 24x - 16$ $\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $x = 2$ $y = 4$ $B(2;4)$	5	$\checkmark A \text{ derivative}$ $\checkmark A \text{ derivative} = 0$ $\checkmark CA \text{ factors}$ $\checkmark CA \text{ }x\text{-value}$ $\checkmark CA \text{ }y\text{-value}$ <div style="border: 1px solid black; padding: 2px;">If the correct coordinate is not specified maximum 4/5 Marks</div>	5

1.2.2	$f'(x) \text{ if } f(x) = \frac{x^3 - 5x^2 + 4x}{x-4}$ $f(x) = \frac{x(x^2 - 5x + 4)}{(x-4)}$ $= \frac{x(x-1)(x-4)}{(x-4)}$ $= x^2 - x$ $f'(x) = 2x - 1$	3	
1.2.2	$\checkmark A \text{ factorising}$ $\checkmark A \text{ factors}$ $\checkmark CA \text{ simplification of } f$ $\checkmark CA \text{ }x\text{-value}$ $\checkmark CA \text{ }y\text{-value}$ <div style="border: 1px solid black; padding: 2px;">If only one coordinate is given - maximum 4/5 Marks</div>	5	

QUESTION 4

<p>4.1</p> <table border="1" style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 5px;">$D \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$</td><td style="padding: 5px; text-align: right;">Answer only full marks</td></tr> </table> <p>$D \left(\frac{-1+3}{2}, \frac{3-1}{2} \right)$</p> <p>$D \left(1; 1 \right)$</p>	$D \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	Answer only full marks	<p>✓A Correct substitution</p> <p>✓A Answer</p>	<p>5</p>
$D \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	Answer only full marks			
<p>4.2</p> <p>$M_{AB} = \frac{-1-3}{3+1} = -1$</p> <p>$\therefore M_{CD} = 1$</p> <p>$k=0$</p> <p>equation of the line CD is $y=x$</p> <p>C lies on line CD, therefore co-ordinates of C must satisfy the above equation substituting $C(p, q)$ gives $q=p$.</p>	<p>✓A correct subst into formula</p> <p>✓A gradient value</p> <p>✓A $k=0$</p> <p>✓A $y=x$</p> <p>✓A conclusion</p>	<p>OR</p> <p>✓A correct subst into formula</p> <p>✓A gradient value</p> <p>✓✓AA $\frac{q-0}{p-0} = 1$</p> <p>✓A $\frac{q}{p} = 1$</p> <p>$p=q$</p>		

<p>QUESTION 3</p> <p>3.1 $f''(x) = 6x - 18 > 0$ $x > 3$</p> <table border="1" style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$x = \frac{2+4}{2} = 3$ or</td> <td style="padding: 5px;">$x = \frac{-b}{3a} = \frac{(-9)}{3(1)} = 3$</td> </tr> </table> <p>3.2 $V = l \times b \times h$ $= 5x(9-2x)(x)$ $= 45x^2 - 10x^3$</p> <p>Therefore the box will have a maximum at $x = 3$</p> <p>OR</p> <p>$V = 90x - 30x^2$ $90x - 30x^2 = 0$ $30x(3-x) = 0$ $x = 0 \text{ or } x = 3$</p>	$x = \frac{2+4}{2} = 3$ or	$x = \frac{-b}{3a} = \frac{(-9)}{3(1)} = 3$	<p>✓ CA, $6x - 18 > 0$</p> <p>✓ CA, $6x - 18 > 0$</p> <p>✓ CA answer (If x is between x-coordinates of both turning points)</p> <p>Answer only full marks</p> <p>[19]</p>
$x = \frac{2+4}{2} = 3$ or	$x = \frac{-b}{3a} = \frac{(-9)}{3(1)} = 3$		
<p>QUESTION 4</p> <p>4.1 $V = 90x - 30x^2$ $90x - 30x^2 = 0$ $30x(3-x) = 0$ $x = 0 \text{ or } x = 3$</p> <p>$f''(x) = 90 - 60x$ $f''(3) = 90 - 60(3) = -90 < 0$</p> <p>Therefore the box will have a maximum at $x = 3$</p>	<p>✓ CA choosing $x = 3$ (greater x value)</p> <p>OR</p> <p>✓ CA derivative</p> <p>✓ CA derivative equal to 0</p> <p>✓ CA factors</p> <p>✓ CA, x values</p> <p>✓ CA choosing $x = 3$ (greater x value)</p> <p>✓ CA, x values</p> <p>✓ CA derivative</p> <p>✓ CA derivative equal to 0</p> <p>✓ CA factors</p> <p>✓ CA, x values</p> <p>✓ CA choosing $x = 3$</p> <p>[7]</p>		
<p>QUESTION 5</p> <p>5.1 $V = 90x - 30x^2$ $90x - 30x^2 = 0$ $30x(3-x) = 0$ $x = 0 \text{ or } x = 3$</p> <p>$f''(x) = 90 - 60x$ $f''(3) = 90 - 60(3) = -90 < 0$</p> <p>Therefore the box will have a maximum at $x = 3$</p>	<p>✓ CA choosing $x = 3$</p> <p>5</p> <p>[7]</p>		

QUESTION 5			
5.1	$M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $M \left(\frac{0+3}{2}, \frac{0+2}{2} \right)$ $M \left(\frac{3}{2}, 1 \right)$	<input type="checkbox"/> Answer only full marks <input checked="" type="checkbox"/> $\sqrt{A}x - \text{value}$ <input checked="" type="checkbox"/> $\sqrt{A}y - \text{value}$	2
5.2	$M_{OP} = \frac{2-0}{3-0} = \frac{2}{3}$	<input checked="" type="checkbox"/> \sqrt{A} answer	1
5.3	$\begin{aligned} OP^2 &= \left(\frac{3}{2} \right)^2 + (y-1)^2 = OP^2 \\ &= \left(-\frac{3}{2} \right)^2 + (1)^2 \\ &= \frac{13}{4} \\ \text{radius} &= \frac{\sqrt{13}}{2} \end{aligned}$ <p>∴ the equation of circle M is</p> $\left(x - \frac{3}{2} \right)^2 + (y-1)^2 = \frac{13}{4}$	<input checked="" type="checkbox"/> CA subst coordinates of centre into equation <input checked="" type="checkbox"/> CA subst of O(0,0) <input checked="" type="checkbox"/> CA radius value <input checked="" type="checkbox"/> CA equation of circle	4
5.4	$x - \text{int intercept}, y = 0$ $\left(x - \frac{3}{2} \right)^2 + (0-1)^2 = \frac{13}{4}$ $x^2 - 3x + \frac{9}{4} + 1 = \frac{13}{4}$ $x^2 - 3x - 1 = 0$ $x(x-3) = 0$ $x = 0 \text{ or } x = 3$ $\therefore Q(3,0)$ <p>Equation of the line through Q is</p>	<input checked="" type="checkbox"/> CA subst $y = 0$ into equation of circle <input checked="" type="checkbox"/> CA expansion <input checked="" type="checkbox"/> CA x values <input checked="" type="checkbox"/> CA coordinates of Q <input checked="" type="checkbox"/> CA gradient of $\frac{2}{3}$ <input checked="" type="checkbox"/> CA subst of coordinates of Q	4
5.5			

4.3	<p>using $\frac{1}{2}AB \cdot CD = 12$</p> $\frac{1}{2}\sqrt{(3+1)^2 + (-1-3)^2} \cdot CD = 12$ $\sqrt{4^2 + 4^2} \cdot CD = 24$ $CD = \frac{24}{\sqrt{32}} = 3\sqrt{2}$ <p>$(p-1)^2 + (q-1)^2 = (3\sqrt{2})^2$</p> $(p-1)^2 + (q-1)^2 = 18$ $2(p-1)^2 = 18$ $(p-1)^2 = 9$ $p-1 = \pm 3$ $p = -2 \text{ or } 4$ $q = -2 \text{ or } 4$ $\therefore p = -2; q = -2$ $\therefore C(-2, -2)$ <p>OR</p> $\Delta CDB \equiv \Delta CDA \quad SAS$ $CA = CB$ $C(p'; p)$ $CD = \sqrt{(p-1)^2 + (q-1)^2}$ $AB = \sqrt{(1-3)^2 + (1+1)^2} = \sqrt{8}$ $6 = \frac{1}{2}\sqrt{8\sqrt{(p+1)^2 + (p-3)^2}}$ $12 = \sqrt{8\sqrt{(p-1)^2 + (p-1)^2}}$ $144 = 8[(p-1)^2 + (p-1)^2]$ $144 = 8[2p^2 - 4p + 2]$ $144 = 16p^2 - 32p + 16$ $9 = p^2 - 2p + 1$ $p^2 - 2p - 8 = 0$ $(p-4)(p+2) = 0$ $p = 4 \text{ or } p = -2$ $n/a \quad q = -2$ $\therefore C(-2, -2)$	<p>✓ A Area formula of ΔABC and equating to 12</p> <p>✓ A distance of AB</p> <p>✓ CA length of CD</p> <p>✓ CA subst into distance formula for CD</p> <p>✓ CA coordinates of C (both values must be negative)</p> <p>OR</p> <p>✓ A length of CD</p> <p>✓ A distance of AB</p> <p>✓ CA Area formula of ΔCDA and equating to 6</p> <p>✓ CA p-values</p> <p>✓ CA coordinates of C</p>	5
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NSC**QUESTION 6**

6.1	$\begin{aligned} OR^2 &= (-\sqrt{p})^2 + (\sqrt{p-3})^2 \\ &= p+p-3 \\ &= 2p-3 \\ OR &= \sqrt{2p-3} \end{aligned}$ $\cos^2 \theta = \left(\frac{-\sqrt{p}}{\sqrt{2p-3}} \right)^2$ $= \frac{p}{2p-3}$ <p>OR</p> $\begin{aligned} OR^2 &= (-\sqrt{p})^2 + (\sqrt{p-3})^2 \\ &= p+p-3 \\ &= 2p-3 \\ OR &= \sqrt{2p-3} \end{aligned}$ $\cos^2 \theta = 1 - \sin^2 \theta$ $= 1 - \frac{p-3}{2p-3}$ $= \frac{2p-3-p+3}{2p-3}$ $= \frac{p}{2p-3}$	$\begin{aligned} \checkmark A \quad OR^2 &= (-\sqrt{p})^2 + (\sqrt{p-3})^2 \\ &\checkmark A \quad 2p-3 \\ &\checkmark CA \quad OR = \sqrt{2p-3} \\ &\checkmark CA \text{ substitution of } \cos \theta \\ &\checkmark CA \text{ answer} \end{aligned}$ <p>OR</p> $\begin{aligned} \checkmark A \quad OR^2 &= (-\sqrt{p})^2 + (\sqrt{p-3})^2 \\ &\checkmark A \quad 2p-3 \\ &\checkmark CA \quad OR = \sqrt{2p-3} \\ &\checkmark CA \text{ substitution of } \sin \theta \\ &\checkmark CA \text{ answer} \end{aligned}$	5
6.2	$\begin{aligned} 2p-3 > 0 \text{ and } p \geq 0 \\ p > \frac{3}{2} \end{aligned}$	$\begin{aligned} \checkmark \checkmark CA \quad p > \frac{3}{2} \end{aligned}$	2
6.3	$\begin{aligned} \frac{1}{\cos \alpha} &= \frac{1}{\cos(180^\circ - \theta)} = \frac{-1}{\cos \theta} \\ &= \frac{\sqrt{2p-3}}{\sqrt{p}} \end{aligned}$	$\begin{aligned} \checkmark A \quad \frac{-1}{\cos \theta} \\ \checkmark A \text{ answer} \end{aligned}$	2

9
NSC**QUESTION 6**

	$\begin{aligned} y &= \frac{2}{3}x + c \\ \text{subst } (3; 0) \\ 0 &= \frac{2}{3}(3) + c \\ 0 &= 2 + c \\ \therefore c &= -2 \\ \therefore y &= \frac{2}{3}x - 2 \end{aligned}$	$\checkmark CA \text{ equation of the line}$	3
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QUESTION 7

7.2	LHS	$\frac{\tan x - \sin x}{\sin^3 x}$	$\sqrt{A} \frac{1 - \cos x}{\cos x (\sin^2 x)}$	$\sqrt{A} \sin^2 x = 1 - \cos^2 x$	5
		$= \frac{\sin x - \sin x}{\sin^3 x}$			
7.2	RHS	$\frac{\cos x + \cos x - 2}{\cos x (1 + \cos x)}$			
		$= \frac{1 - \cos x}{1 + \cos x}$			
7.3		$\cos 2x + \cos x - 2 = 0$	$\sqrt{A} \text{ writing } \cos 2x \text{ as } 2\cos^2 x - 1$	$\sqrt{A} \text{ std. form equation}$	5
		$2\cos^2 x - 1 + \cos x - 2 = 0$			
7.3		$2\cos^2 x + \cos x - 3 = 0$	$\sqrt{A} \text{ factors}$	$\sqrt{A} \cos x = 1$	[16]
		$(2\cos x + 3)(\cos x - 1) = 0$			
7.3		N/A	$\sqrt{A} \text{ answer}$	5	[16]
		$\therefore \cos x = 1$			
7.3		$x = 0^\circ + n \cdot 360^\circ, n \in \mathbb{Z}$	$\sqrt{A} \text{ answer}$	5	[16]

7.1	$\frac{\cos 225^\circ \cdot \sin (-135^\circ) - \sin 330^\circ}{\tan 225^\circ}$	$\sqrt{A} - \cos 45^\circ$	$\sqrt{A} - \sin 45^\circ$	6
	$= \frac{\cos (180^\circ + 45^\circ) \cdot -\sin (180^\circ - 45^\circ) - \sin (360^\circ - 30^\circ)}{\tan (180^\circ + 45^\circ)}$			
7.1	$= \frac{-\cos 45^\circ \cdot (-\sin 45^\circ) - (-\sin 30^\circ)}{\tan 45^\circ}$	$\sqrt{A} \tan 45^\circ$	$\sqrt{A} \tan 45^\circ$	6
	$= \frac{-\frac{1}{\sqrt{2}} \cdot \left(\frac{-1}{\sqrt{2}}\right) + \frac{1}{2}}{1}$			
7.1	$= \frac{\frac{1}{2} + \frac{1}{2}}{1}$	$\sqrt{CA} \text{ substitution}$	$\sqrt{CA} \text{ answer}$	6
	$= 1$			

QUESTION 8

<p>8.1 By area formula</p> $\text{Area of } \triangle KMN = \frac{1}{2} k m \sin N = \frac{1}{2} k m \sin M = \frac{1}{2} k m \sin N$ <p>dividing by $\frac{1}{2} k m$ yields</p> $\frac{\sin M}{m} = \frac{\sin N}{n}$	<p>$\checkmark \checkmark \text{AA}$</p> <p>$\text{Area of } \triangle KMN = \frac{1}{2} k m \sin M = \frac{1}{2} k m \sin N$</p> <p>$\checkmark \checkmark \text{AA dividing by } \frac{1}{2} k m \text{ yields}$</p> <p>4</p>
	<p><i>Draw $KL \perp MN$ extended</i> (180°-KNM)</p> <p>$\Delta KLM : KL = n \sin M$ and</p> <p>$\Delta KLN : KL = m \sin(180^\circ - KNM)$</p> $= m \sin KNM$ <p>$\therefore n \sin M = m \sin KNM$</p> $\therefore \frac{\sin M}{m} = \frac{\sin N}{n}$
<p>8.2.1 $PS = QS \tan \theta$</p>	<p>$\checkmark \checkmark \text{AA Answer}$</p> <p>2</p>
<p>8.2.1 $\frac{QS}{\sin R} = \frac{QR}{\sin S}$</p> <p>but $QSR = 180^\circ - (x+y)$</p> $\therefore \frac{QS}{\sin y} = \frac{10}{\sin(180^\circ - (x+y))}$ $\frac{QS}{\sin y} = \frac{10}{\sin(x+y)}$ <p>$\therefore QS = \frac{10 \sin y}{\sin(x+y)}$</p>	<p>$\checkmark \text{A Application of sine rule}$</p> <p>$\checkmark \text{A calculation of angle QSR}$</p> <p>$\checkmark \text{A substitution into sine rule}$</p> <p>$\checkmark \text{A reduction formulae}$</p> <p>4</p>

Total Marks: 100

