learnXtra

MATHEMATICS

GRADE 12

exam school

2012

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INTRODUCTION

Have you heard about Mindset? Mindset Network, a South African non-profit organisation, was founded in 2002. We develop and distribute quality and contextually relevant educational resources for use in the schooling, health and vocational sectors. We distribute our materials through various technology platforms like TV broadcasts, the Internet (www.mindset.co.za/learn) and on DVDs. The materials are made available in video, print and in computer-based multimedia formats.

At Mindset we are committed to innovation. In the last three years, we have successfully run a series of broadcast events leading up to and in support of the Grade 12 NSC examinations

Now we are proud to announce our 2012 edition of Exam School. From 15th October till 20th November will bring you revision lessons in nine subjects - Mathematics, Physical Sciences, Life Sciences, Mathematical Literacy, English 1st Additional Language, Accounting, Geography, Economics and Business Studies.

In this exam revision programme we have selected Questions mainly from the Nov 2011 Papers and have tried to cover as many topics as we can. Each topic is about an hour long and if you work through the selected questions you will certainly have increased confidence to face your exams. In addition to the topics and questions in this booklet, we have schedule 1½ hour live shows a day or two before you write your exams. To get the most out of these shows, we need you to participate by emailing us questions, calling in or posting on twitter, peptxt or facebook.

Since you asked us for late night study sessions and that's what we've planned! You'll find repeats of our Live shows at 10:30pm every evening. Then from midnight to 6:00 am there are revision lessons too. So if you can't watch during the day, you can record or watch early in the morning! .

GETTING THE MOST FROM EXAM SCHOOL

You must read this booklet! You'll find the exam overviews and lots of study tips and hints here. in Start your final revision by working through the questions for a topic fully without looking up the solutions. If you get stuck and can't complete the answer don't panic. Make a note of any questions you have. Now you are ready to watch a Learn Xtra session. When watching the session, compare the approach you took to what the teacher does. Don't just copy the answers down but take note of the method used. Also make a habit of marking your work by checking the memo. Remember, there are usually more than one way to answer a question.If you still don't understand post your question on Facebook – you'll get help from all the other Mindsetters on the page. You can also send an email to helpdesk@learnxtra.co.za and we'll get back to you within 48 hours.

Make sure you keep this booklet. You can re-do the questions you did not get totally correct and mark your own work. Exam preparation requires motivation and discipline, so try to stay positive, even when the work appears to be difficult. Every little bit of studying, revision and exam practice will pay off. You may benefit from working with a friend or a small study group, as long as everyone is as committed as you are.

www.learnxtra.co.za

We are pleased to announce that we'll continue to run our special radio broadcasts on community radio stations in Limpopo, Eastern Cape and KZN. This programme is called MTN Learn. Find out more details at www.mtnlearning.co.za. You can also listen online or download radio broadcasts of previous shows. Tuning into radio will give you the chance to learn extra! Look out for additional notes in Newspaper supplements too.

Mindset believes that the 2012 Learn Xtra Spring School will help you achieve the results you want. All the best to the Class of 2012!

CONTACT US

We want to hear from you. So let us have your specific questions or just tell us what you think through any of the following:

LearnXtra

@learnxtra

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086 105 8262

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BROADCAST SCHEDULES

Exam School (Dstv and Toptv 319)

MATHEMATICS PAPER 2

DATE	TIMES	TOPICS
2 November	*22:30 – 12:00*	*Live: Trigonometry (Repeat)*
	12:00 – 01:00	*Data Handling*
	01:00 – 02:00	*Co-ordinate Geometry: Lines & Transformations*
	02:00 – 03:00	*Co-ordinate Geometry: Circles*
	03:00 – 04:00	*Trigonometry: Expressions, Equations & Graphs*
	04:00 – 05:00	*Trigonometry: Identities*
	05:00 – 06:00	*Trigonometry: 2D & 3D problems*
4 November	*9.00 - 10.00*	*Data Handling*
	10.00 - 11.00	*Co-ordinate Geometry: Lines & Transformations*
	11.00 - 12:00	*Co-ordinate Geometry: Circles*
	12:00 - 13:00	*Trigonometry: Expressions, Equations & Graphs*
	13:30 – 14:30	*Trigonometry: Identities*
	14.30 - 15.30	*Trigonometry: 2D & 3D problems*
	15.30 – 17:00	*Exam Revision Live*
	17.00 - 18.00	*Co-ordinate Geometry: Lines & Transformations*
	18.00 – 19:00	*Co-ordinate Geometry: Circles*
	22.30 – 24:00	*Exam Revision Live (Repeat)*
	24:00 – 06:00	*(Repeat of all Topics)*

MTN LEARN RADIO SCHEDULE

EXAM SCHOOL: MATHEMATICS

DATE	TIME	TOPICS
8-Oct	*17:00 - 18:00*	*Algebra*
9-Oct	*17:00 - 18:00*	*Analytical Geometry*
10-Oct	*17:00 - 18:00*	*Functions*
11-Oct	*17:00 - 18:00*	*Trigonometry*
13-Oct	*09:00 - 10:00*	*Calculus*
31-Oct	*17:00 - 18:00*	*Algebra*
	18:00 - 19:00	*Functions*
1-Nov	*17:00 - 18:00*	*Calculus*
	18:00 - 19:00	*Calculus Applications*
3-Nov	*09:00 - 10:00*	*Data Handling*
	10:00 - 11:00	*Analytical Geometry*
4-Nov	*17:00 - 19:00*	*Trigonometry*

MTN LEARN: PARTICIPATING COMMUNITY RADIO STATIONS

KwaZulu Natal:

Hindvani Radio	91.5 fm – Durban 102.3 fm – Rest or KZN
Maputaland Radio	170.6 fm

Limpopo Province:

Sekgosese Radio	100.3 fm
Greater Tzaneen Radio	104.8 fm
Mohodi FM	98.8 fm
Moletsi	98.6 fm
Univen	99.8 fm

Eastern Cape:

Vukani fm	90.6 – 99.9 fm
Fort Hare Community Radio	88.2 fm
Mdantsane fm	89.5 fm
Nkqubela fm	97.0 fm
Graaff Reinet	90.2 fm

PREPARING FOR EXAMINATIONS

1. Prepare well in advance for all your papers and subjects. You need to start your planning for success in the final examination now. You cannot guarantee success if you only study the night before an exam.
2. Write down the date of your prelim and final exam so that you can plan and structure a study time table for all your subjects.
3. Set up a study time-table according to your prelim and final Grade 12 exam time-table and stick to your study schedule. If you study a small section every day, you will feel you have achieved something and you will not be as nervous by the time you have to go and write your first paper.
4. Your study programme should be realistic. You need to spend no more than 2 hours per day on one topic. Do not try to fit too much into one session. When you cover small sections of work often, you will master them more quickly. The broadcast schedule may help you to make sure you have covered all the topics that are in the exam.
5. When studying don't just read through your notes or textbook. You need to be active by making summary checklists or mind maps. Highlight the important facts that you need to memorise. You may need to write out definitions and formulae a few times to make sure you can remember these. Check yourself as often as you can. You may find it useful to say the definitions out aloud.
6. Practise questions from previous examination papers. Follow these steps for using previous exam papers effectively:
    - Take careful note of all instructions - these do not usually change.
    - Try to answer the questions without looking at your notes or the solutions.
    - Time yourself. You need to make sure that you complete a question in time. To work out the time you have, multiply the marks for a question by total time and then divide by the total number of marks. In most exams you need to work at a rate of about 1 mark per minute.
    - Check your working against the memo. If you don't understand the answer given, contact the Learn Xtra Help desk (email: helpdesk@learnxtra.co.za).
    - If you did not get the question right, try it again after a few days.
7. Preparing for, and writing examinations is stressful. You need to try and stay healthy by making sure you maintain a healthy lifestyle. Here are some guidelines to follow:
    - Eat regular small meals including breakfast. Include fruit, fresh vegetables, salad and protein in your diet.
    - Drink lots of water while studying to prevent dehydration.
    - Plan to exercise regularly. Do not sit for more than two hours without stretching or talking a short walk.
    - Make sure you develop good sleeping habits. Do not try to work through the night before an exam. Plan to get at least 6 hours sleep every night.

EXAM TECHNIQUES

1. Make sure you have the correct equipment required for each subject. You need to have at least one spare pen and pencil. It is also a good idea to put new batteries in your calculator before you start your prelims or have a spare battery in your stationery bag.
2. Make sure you get to the exam venue early - don't be late.
3. While waiting to go into the exam venue, don't try to cram or do last minute revision. Try not to discuss the exam with your friends. This may just make you more nervous or confused.
4. Here are some tips as to what to do when you receive your question papers:
   Don't panic, because you have prepared well.
   - You are always given reading time before you start writing. Use this time to take note of the instructions and to plan how you will answer the questions. You can answer questions in any order.
   - Time management is crucial. You have to make sure that you answer all questions. Make notes on your question paper to plan the order for answering questions and the time you have allocated to each one.
   - It is a good idea always to underline the <u>key words</u> of a question to make sure you answer it correctly.
   - Make sure you look any diagrams and graph carefully when reading the question. Make sure you check the special answer sheet too.
   - When you start answering your paper, it is important to read every question twice to make sure you understand what to do. Many marks are lost because learners misunderstand questions and then answer incorrectly.
   - Look at the mark allocation to guide you in answering the question.
   - When you start writing make sure you number your answers exactly as they are in the questions.
   - Make sure you use the special answer sheet to answer selected questions.
   - Think carefully before you start writing. It is better to write an answer once and do it correctly than to waste time rewriting answers.
   - DO NOT use correction fluid (Tippex) because you may forget to write in the correct answer while you are waiting for the fluid to dry. Rather scratch out a wrong answer lightly with pencil or pen and rewrite the correct answer.
   - Check your work. There is usually enough time to finish exam papers and it helps to look over your answers. You might just pick up a calculation error.

EXAM OVERVIEW

MATHEMATICS PAPER 2 3HRS TOTAL MARKS: 150

Space, Shape and Measurement

Co-ordinate Geometry	±40 marks
Transformations	±25 marks
Trigonometry	±60 marks

Data Handling and Probability

Data Handling	±25 marks

DATA HANDLING & TRANSFORMATIONS

STUDY NOTES

Mean

The mean of a set of data is the average. To get the mean, you add the scores and divide by the number of scores.

Mode

This is the most frequently occurring score.

Quartiles

Quartiles are measures of dispersion around the median which is a good measure of central tendency. The median divides the data into two halves. The lower and upper quartiles further subdivide the data into quarters.

There are three quartiles:

The Lower Quartile (Q_1): This is the median of the lower half of the values.

The Median (M or Q_2): This is the value that divides the data into halves.

The Upper Quartile (Q_3): This is the median of the upper half of the values.

If there is an odd number of data values in the data set, then the specific quartile will be a value in the data set. If there is an even number of data values in the data set then the specific quartile will not be a value in the data set. A number which will serve as a quartile will need to be inserted into the data set (the average of the two middle numbers).

Range

The range is the difference between the largest and the smallest value in the data set. The bigger the range, the more spread out the data is.

The Inter-quartile Range (IQR)

The difference between the lower and upper quartile is called the inter-quartile range.

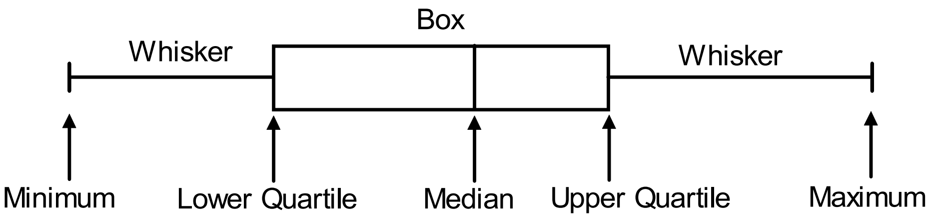
Five Number Summaries

The Five Number Summary uses the following measures of dispersion:

- Minimum: The smallest value in the data
- Lower Quartile: The median of the lower half of the values
- Median: The value that divided the data into halves
- Upper Quartile: The median of the upper half of the values
- Maximum: The largest value in the data

Box and Whisker Plots

A Box and Whisker Plot is a graphical representation of the Five Number Summary.



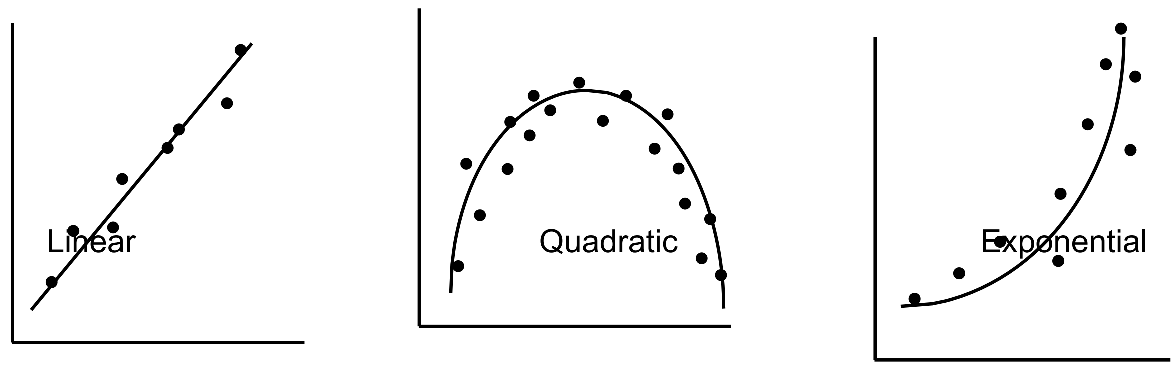
Standard Deviation using a Table

The standard deviation (SD) can be determined by using the following formula:

$$\mathrm{SD} = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

Scatter Plots and Lines and Curves of Best Fit

Plotting data on a scatter plot diagram will show trends in the data. Data could follow a linear, quadratic or exponential trend.



The Normal Distribution Curve

1. The mean, median and mode have the same value.

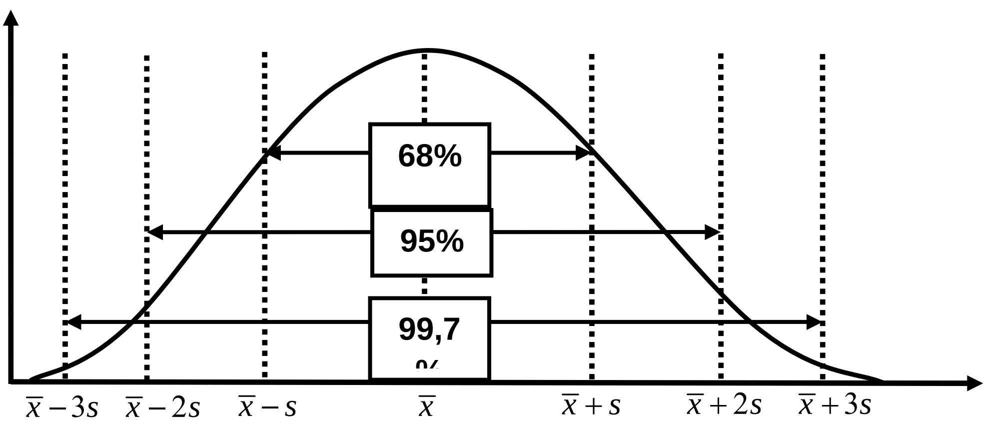
2. An equal number of scores lie on either side of the mean.

3. The majority of scores (99,7%) lie within three standard deviations from the mean, i.e. in the interval $(\overline{x} - 3s\,;\,\overline{x} + 3s)$ where $\overline{x}$ represents the mean and *s* represents the standard deviation.

4. About 95% of scores lie within two standard deviations from the mean, i.e. in the interval $(\overline{x} - 2s\,;\,\overline{x} + 2s)$ where $\overline{x}$ represents the mean and *s* represents the standard deviation.

5. About two-thirds of scores (68%) lie within one standard deviation from the mean, i.e. in the interval $(\overline{x} - s\,;\,\overline{x} + s)$ where $\overline{x}$ represents the mean and *s* represents the standard deviation.

6. The smaller the standard deviation, the thinner and taller the bell shape is. The bigger the standard deviation, the wider and flatter the bell shape is.



TRANSFORMATIONS

Translation Rules

If the point $(x\,;\,y)$ is translated to form the point $(x+a\,;\,y+b)$ where *a* is a horizontal move and *b* is a vertical move, then the following rules apply:

If $a>0$, the horizontal translation is to the right.
If $a<0$, the horizontal translation is to the left.
If $b>0$, the vertical translation is upward.
If $b<0$, the vertical translation is downward

Reflection Rules

Reflection about the *y*-axis:	$(x\,;\,y)\rightarrow(-x\,;\,y)$
Reflection about the *x*-axis:	$(x\,;\,y)\rightarrow(x\,;\,-y)$
Reflection about the line $y=x$:	$(x\,;\,y)\rightarrow(y\,;\,x)$

Rules of Rotation about the Origin

Rotation of **;2°** anti-clockwise:	$(x\,;\,y)\rightarrow(-y\,;\,x)$
Rotation of **;2°** clockwise:	$(x\,;\,y)\rightarrow(y\,;\,-x)$
Rotation of **3:2°** clockwise or anti-clockwise:	$(x\,;\,y)\rightarrow(-x\,;\,-y)$

Enlargement and Reduction Rules

$(x\,;\,y)\rightarrow(kx\,;\,ky)$

If $k>1$, the image is an <u>enlargement</u> of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image. The image will be k^2 times larger than the original figure.
If $0<k<1$, the image is a <u>reduction</u> of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image.
If $k<0$, the image is a <u>rotation</u> of $180°$ of the original figure followed by an <u>enlargement</u> of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image.
The following rules are extremely important:

$$\textbf{Ctgc 'qh'lo ci g} = k^4 \times \textbf{Ctgc 'qh'qtki kpcn}$$

$$\textbf{qt ""'}\ \frac{\textbf{Ctgc 'qh'lo ci g}}{\textbf{Ctgc 'qh'qtki kpcn}} = k^4$$

$$\textbf{qt ""'}\ \frac{\textbf{Ctgc 'qh'qtki kpcn}}{\textbf{Ctgc 'qh'lo ci g}} = \frac{3}{k^4}$$

Rules of Rotation of any Angle about the Origin

The coordinates of the image $A^{/}$ of any point A after rotation about the origin through any angle θ is given by:

$$A^{/}(x\cos\theta - y\sin\theta\,;\,y\cos\theta + x\sin\theta)$$

Anti-clockwise rotations are positive, while clockwise rotations are negative.

DATA HANDLING & TRANSFORMATIONS QUESTIONS

Question 1 *(Adapted from Nov 2011 Paper 2, Question 7.1 & 8)*

1.1 The following transformation is applied to all points:

- Firstly, a point is translated by 4 units to the right.
- Then is rotated through 180 about the origin.

Write down the general rule that represents the above transformation in the form (x;y) ...

1.2 If Q'(-2;-3) is the image of Q after rotation of 135^0 in an anticlockwise direction about the origin. Calculate the coordinates of Q. (Leave your answer in surd form)

Question 2 (*Adapted from Feb/March 2012 Paper 2, Question 4)*

In the grid below a, b, c, d, e, f, and g represent values in a data set written in an increasing order.

No value in the data set is repeated.

a	b	c	d	e	f	g

2.1 Determine the value of a, b, c, d, e, f and g if:

- the maximum value is 42
- the range is 35
- the median is 23
- the difference between the median and the upper quartile is 14
- the interquartile range is 22
- e=2c
- the mean is 25

2.2 Sketch a box-and-whisker diagram from the above information

Question 3 *(Adapted from Nov 2011 paper 2, Question 2)*

The scores for 8 golfers who played a single round of golf on the same golf course are shown below:

69	71	75	74	78	70	72	71

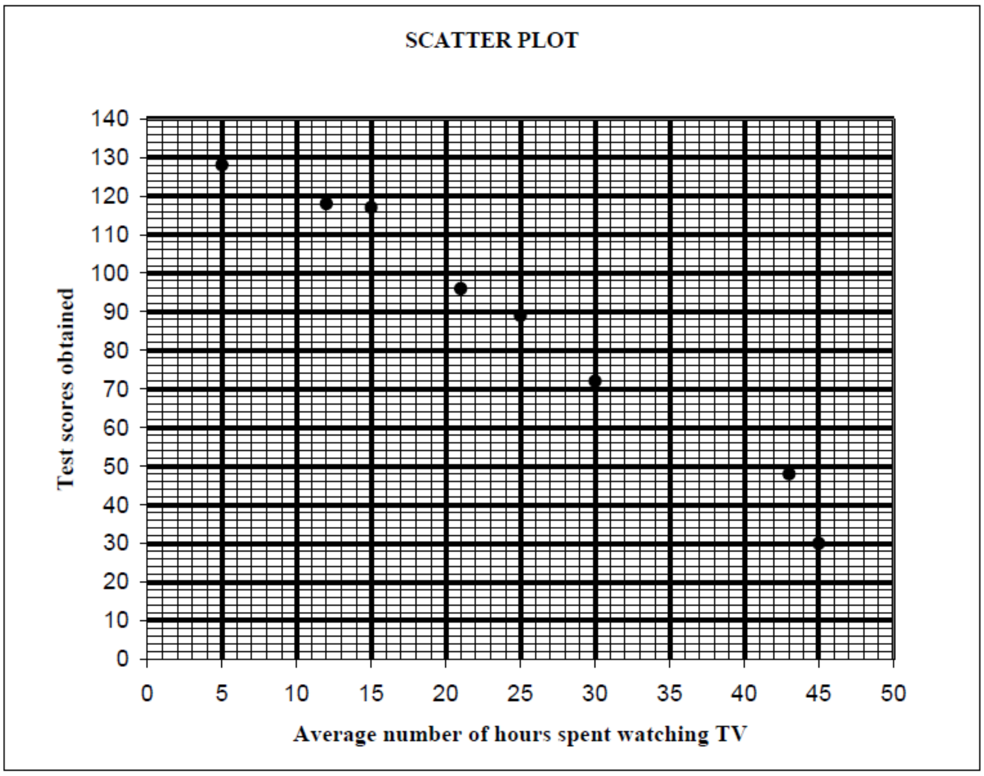
3.1 Calculate the mean score. (2)

3.2 Calculate the standard deviation for the data. (2)

3.3 How many golfers' scores lie outside one standard deviation of the mean? (2)

Question 4 *(Adapted from Nov 2011 paper 2, Question 3)*

A group of 8 learners was randomly selected from a class. The performance of these learners in a standardised test (which counted 150 marks) and the average number of hours they spend watching TV each week was recorded. The data is represented in the scatter plot below.



4.1 What is the lowest test score for this group of learners? (1)

4.2 Does the data display a linear, quadratic or exponential relationship?

Justify your choice. (2)

4.3 What conclusion can be reached about the learners' test scores and the average number of hours they spend watching TV? (1)

4.4 Another learner from the class watches 35 hours of TV per week. Using the given information, predict his/her performance in the test. (2)

Question 5 *(Adapted from Nov 2011 paper 2, Question 4)*

Thirty learners were asked to answer a question in Mathematics. The time taken, in minutes, to answer the question correctly, is shown in the frequency table below.

Time. T (in minutes)	Number of learners
$1 \leq t < 3$	3
$3 \leq t < 5$	6
$5 \leq t < 7$	7
$7 \leq t < 9$	8
$9 \leq t < 11$	5
$11 \leq t < 13$	1

5.1 Construct a cumulative frequency table for the data. (3)

5.2 Draw a cumulative frequency graph (ogive) of the above data (4)

5.3 If a learner answers the question correctly in less than 4 minutes, then he/she is classified as a 'gifted learner'. Estimate the percentage of 'gifted learners' in this group. (2)

CO-ORDINATE GEOMETRY: LINES

STUDY NOTES

If AB is the line segment joining the points $A(x_A ; y_A)$ and $B(x_B ; y_B)$, then the following formulas apply to line segment AB:

The Distance Formula

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

or $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

The Midpoint Formula

$M\left(\dfrac{x_A + x_B}{2} ; \dfrac{y_A + y_B}{2}\right)$ where M is the midpoint of AB.

The Gradient of a Line Segment Joining Two Points

Gradient of $AB = \dfrac{y_B - y_A}{x_B - x_A}$

Parallel Lines

Parallel lines have equal gradients. If AB||CD then $m_{AB} = m_{CD}$

Perpendicular Lines

The product of the gradients of two perpendicular lines is -1. If AB $\perp$ CD, then $m_{AB} \times m_{CD} = -1$

The Equation of the Line $y - y_A = m(x - x_A)$

Inclination of a Line

$\tan\theta = m_{AB}$

If $m_{AB} > 0$, then θ is acute

If $m_{AB} < 0$, then θ is obtuse

Collinear Points (A, B and C)

$m_{AB} = m_{BC}$

or $m_{AC} = m_{AB}$

or $m_{AC} = m_{BC}$

Rotation about the Origin

x '=xcosβ – ysinβ and y' = ycosβ+xsinβ. (Remember this formula is on the formula sheet)

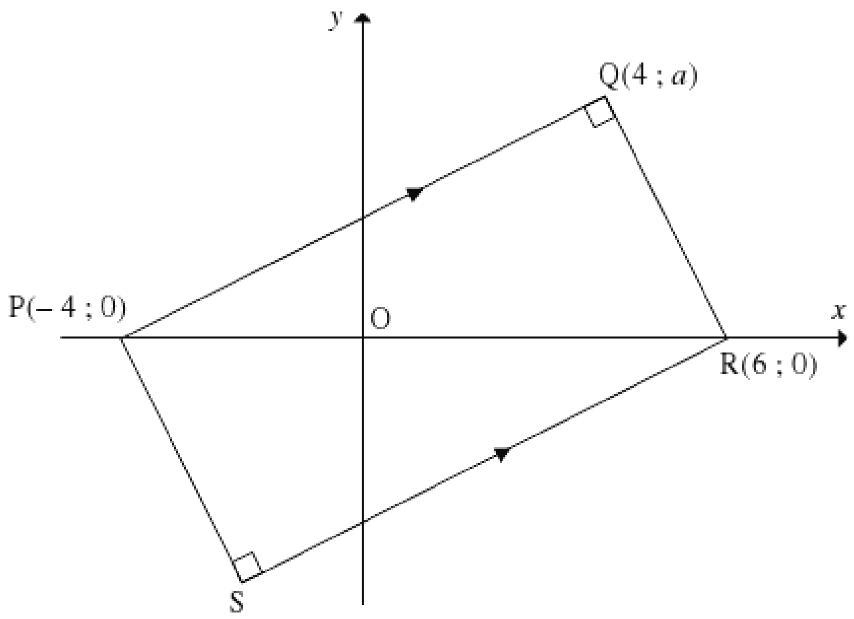
If the rotation is clockwise by θ: use an angle of β **=** 360 - θ in the formula for the image of (x, y)

A(x,y) → A'(-y, x) after rotation of 90° about origin
B(x,y) → B'(-x,-y) after rotation of 180° about origin

CO-ORDINATE GEOMETRY: LINES QUESTIONS

Question 1 (*Nov 2011 P2, Question5)*

In the diagram below, PQRS is a rectangle with vertices P(– 4 ; 0), Q(4 ; a), R(6 ; 0) and S.
Q lies in the first quadrant.



1.1 Show that a = 4. (4)

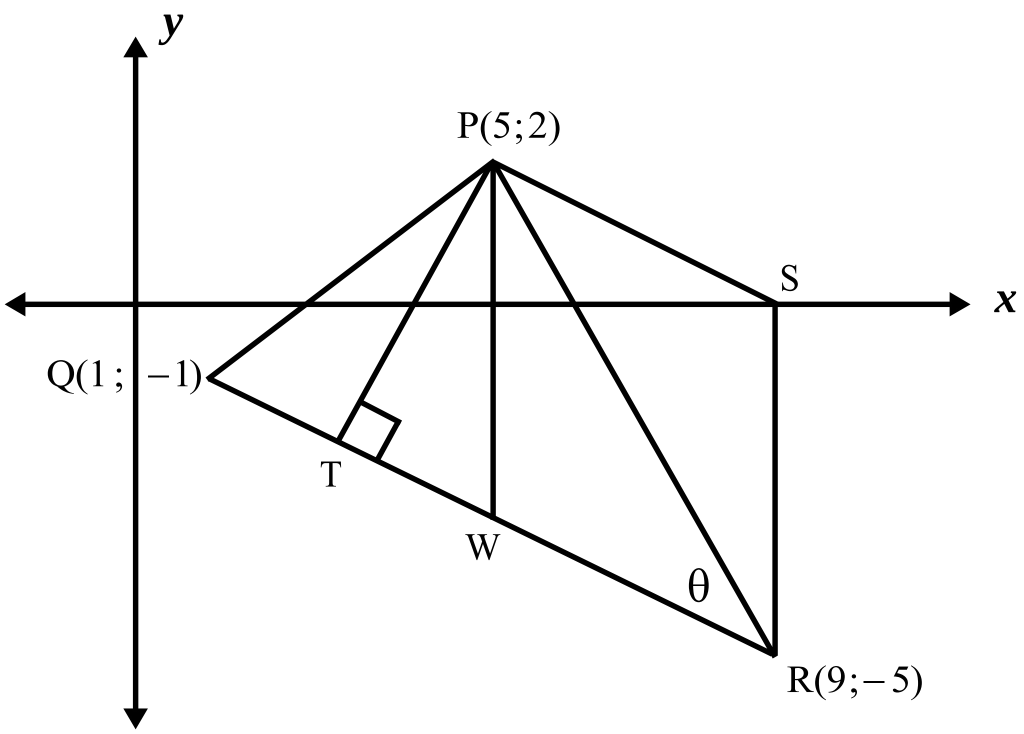
1.2 Determine the equation of the straight line passing through the points S and R in the form $y = mx + c$. (4)

1.3 Calculate the coordinates of S. (4)

1.4 Calculate the length of PR. (2)

1.5 Rectangle PQRS undergoes the transformation $(x ; y) \rightarrow (x + k ; y + l)$ where k and l are numbers. What is the minimum value of k + l so that the image of PQRS lies in the first quadrant (that is, $x \geq 0$ and $y \geq 0$)? (3)

Question 2
In the diagram, PQRS is a quadrilateral with vertices P(5; -2) Q(1; -1) S(9; 0) and R(9; -5).
PT is the perpendicular height of PQRS and W is the midpoint of QR. PRQ =θ



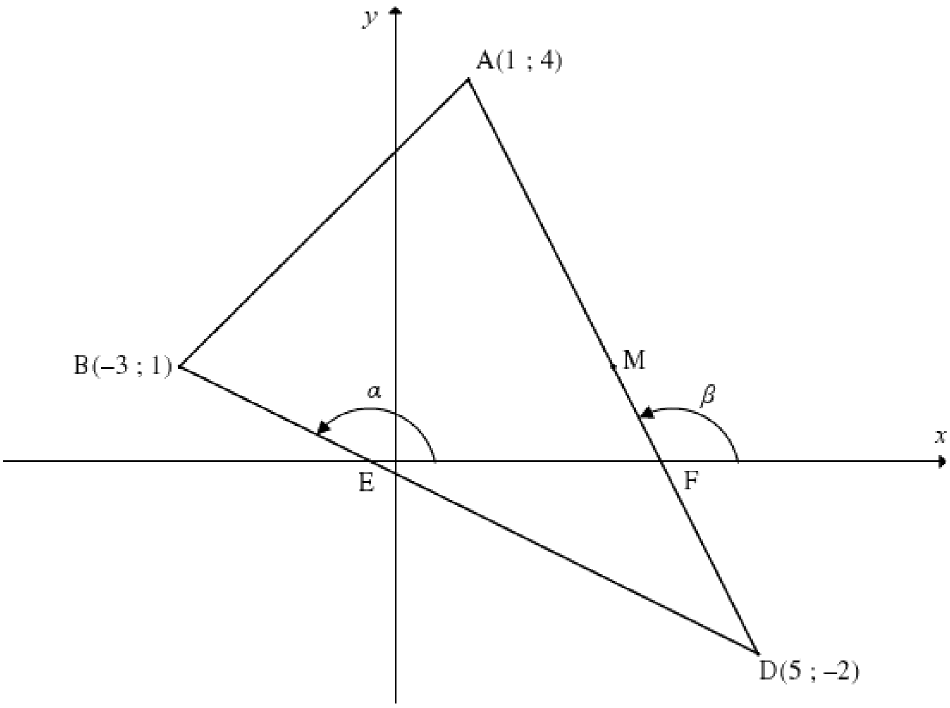
2.1 Prove that PQRS is a trapezium (4)

2.2 Given that point T has co-ordinates (3; -2). Show that $QT = \frac{1}{3}TR$. (5)

Question 3 *(Adapted from Feb 2012 P2, Question5)*

In the figure below, A(1 ; 4), B(–3 ; 1) and D(5 ; –2) are the coordinates of the vertices of ΔABD.

- BD and AD intersect the x-axis at E and F respectively.
- The angle of inclination of BD with the x-axis at E is α.
- The angle of inclination of AD with the x-axis at F is β.



3.1	Calculate the gradient of AD.	(2)
3.2	Determine the coordinates of M, the midpoint of AD.	(2)
3.3	C is a point such that line BC is parallel to AD. Determine the equation of line BC in the form ax + by + c = 0.	(3)
3.4	Calculate the size of β.	(2)
3.5	Calculate ALL the angles of ΔDEF.	(5)

CO-ORDINATE GEOMETRY: CIRCLES

STUDY NOTES

Circles and Tangents to Circles

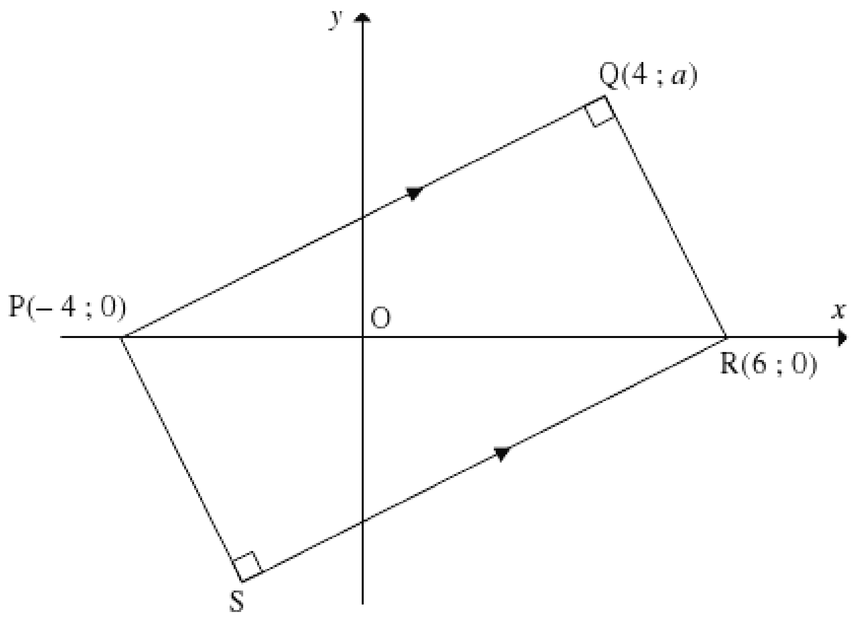
The equation of a circle centre the origin is given by: $x^2 + y^2 = r^2$

The equation of a circle centre $(a;b)$ is given by: $(x-a)^2 + (y-b)^2 = r^2$

The radius is perpendicular to the tangent: $m_{\text{radius}} \times m_{\text{tangent}} = -1$

CO-ORDINATE GEOMETRY: CIRCLES QUESTIONS

Question 1 *(Adapted from Nov 2011 P2, Question 5)*

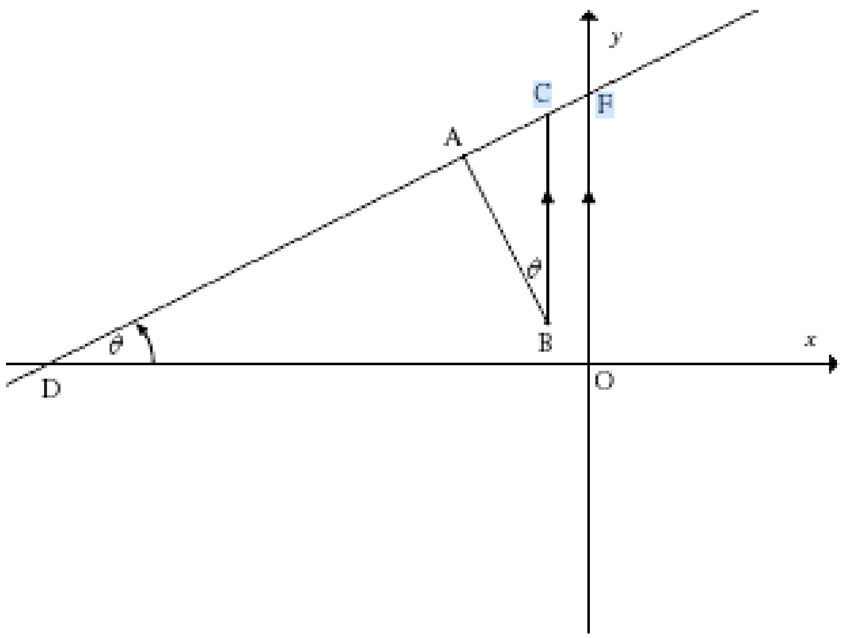


1.1 Determine the equation of the circle that has diameter PR. Give the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

1.2 Show that Q is a point on the circle in QUESTION 1.1. (2)

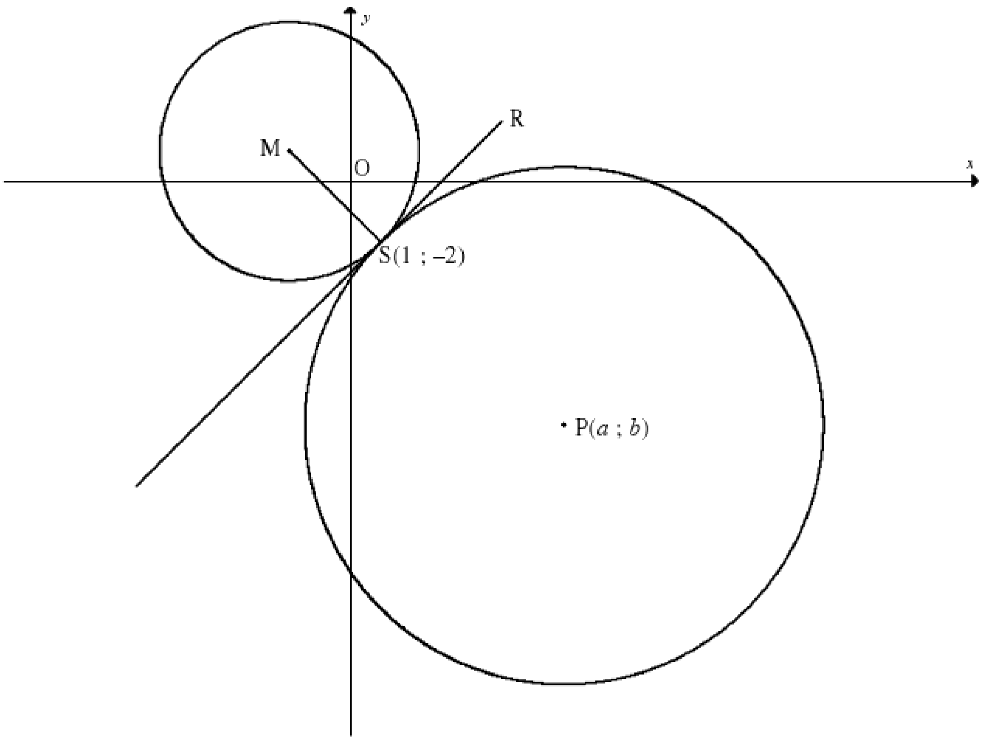
Question 2 *(Adapted from Nov 2011 Paper 2, Question 6)*
The circle with centre B(-1;1) and radius $\sqrt{20}$ is shown. BC is parallel to the y-axis and CB=5. The tangent to the circle at A passes through C.

$A\hat{B}C = A\hat{D}O = \theta$



2.1 Determine the coordinates of C. (2)
2.2 Calculate the length of CA. (3)
2.3 Write down the value of tan θ . (1)
2.4 Show that the gradient of AB is – 2. (2)
2.5 Determine the coordinates of A. (6)
2.6 Calculate the ratio of the area of Δ ABC to the area of Δ ODF. Simplify your answer. (5)

Question 3 *(Adapted from Feb 2012 Paper 2, Question 6)*
In the figure below, a circle with centre M is drawn. The equation of the circle is $(x + 2)^2 + (y - 1)^2 = r^2$. S(1 ; –2) is a point on the circle. SR is a tangent to the circle.

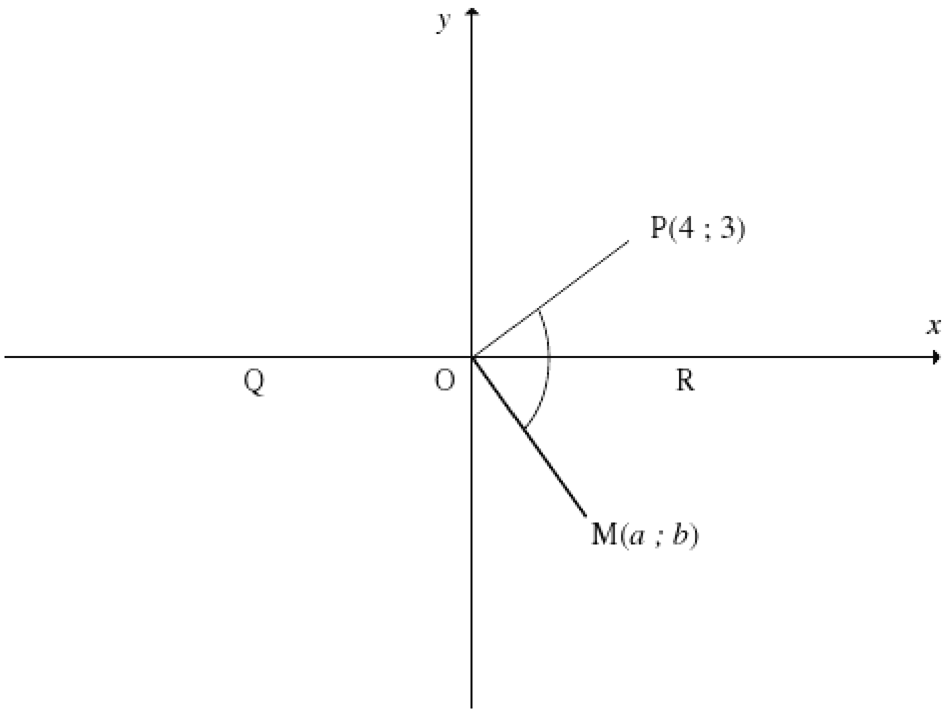


3.1 Write down the coordinates of M and the radius of the circle centre M. (4)

3.2 Determine the equation of the tangent RS in the form y = mx + c. (4)

3.3 The circles having centres P and M touch externally at point S. SR is a tangent to both these circles. If MS : MP = 1 : 3, determine the coordinates (a ; b) of point P. (8)

Question 4 (Nov 2011 Question 9.3)
P(4 ; 3) and M(a ; b) are points on a circle with the origin as centre.
Q and R are x-intercepts of the circle.

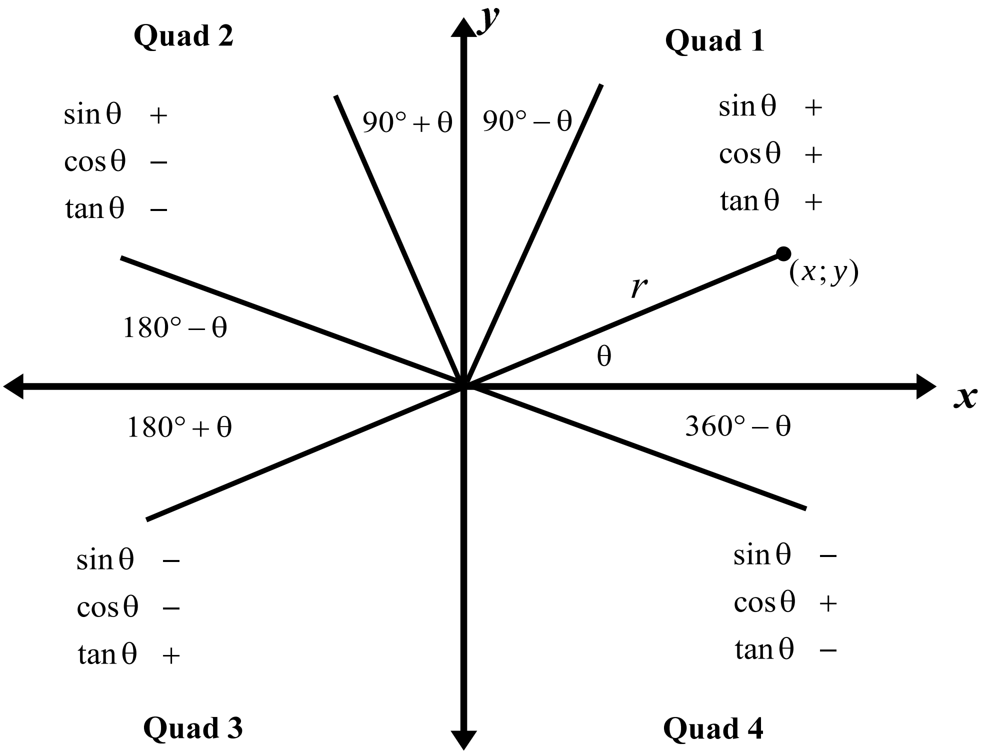


4.1 Write down the numerical value of sin ROP . (2)
4.2 Calculate the size of QOP. (2)
4.3 If obtuse POM = 115° , calculate the value of a, the x-coordinate of M, correct to TWO decimal places. (3)

TRIGONOMETRY: EXPRESSIONS, EQUATIONS AND GRAPHS

Summary of Trigonometry

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$



Reduction rules

$\sin(180° - \theta) = \sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$
$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$
$\sin(-\theta) = -\sin\theta$

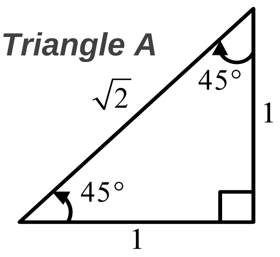
$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = \tan\theta$
$\sin(90° + \theta) = \cos\theta$
$\cos(90° + \theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

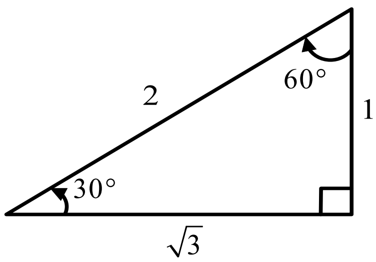
$\sin(360° - \theta) = -\sin\theta$
$\cos(360° - \theta) = \cos\theta$
$\tan(360° - \theta) = -\tan\theta$
$\tan(-\theta) = -\tan\theta$

Whenever the angle is greater than $360°$, keep subtracting $360°$ from the angle until you get an angle in the interval $[0°;360°]$.

Special angles

Triangle A





Triangle B

From Triangle A we have:

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{1}{1} = 1$$

From Triangle B we have:

$\sin 30° = \frac{1}{2}$ ***and*** $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ ***and*** $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ ***and*** $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Summary of Identities

You must know the following identities even though you can find them on the information sheet:

$$\cos^2\theta + \sin^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Compound angle identities

$$\begin{aligned}
\sin(A+B) &= \sin A\cos B + \cos A\sin B\\
\sin(A-B) &= \sin A\cos B - \cos A\sin B\\
\cos(A+B) &= \cos A\cos B - \sin A\sin B\\
\cos(A-B) &= \cos A\cos B + \sin A\sin B
\end{aligned}$$

Double angle identities

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

TRIG GRAPHS

The table below summarises the features of the three trig graphs you must know:

Parent Functions	Shape	Intercepts	Domain	Range	Period
y = sinx	Wave-like shape, starting at the origin	y-intercept = 0 x-intercept= 0^0 repeats every 180^0	x ε (-∞;∞)	y ε [-1; 1] max =1 when θ=90^0	360^0
y = cosx	Wave-like shape but when x = 0^0 y=1 (max)	y-intercept = 1 x-intercept =90^0 repeats every 180^0	x ε (-∞;∞)	y ε [-1; 1] max =1 when θ=0^0	360^0
y = tanx	Not wave-like shape. Curve that is repeated	y-intercept = 0 x-intercept= 0^0 repeats every 180^0	x ε (-∞;∞) except x =90^0 +k180^0 is undefined	y ε (-∞;∞)	180^0

Transformations of Trig functions

Trig functions can be transformed by changing the parameters that affect amplitude, frequency or rest position. We use the general form of the trig functions to describe these changes. The general form of the trig functions are:

f(x) = a sin (kx±p) + q f(x) = a cos (kx±p) + q f(x) = a tan (kx±p) + q

The effect of the parameters:

Amplitude (a) Changes the distance from the rest position. a = max value when q =0

Rest position (q): The y value of the horizontal line that is half way between the min & max value. When you add q, the graph shift up. When you subtract q, the graph shifts down.

Frequency (k): If k>1 then period decreases by k. If k <1 then period increases by 1/k

Shift (p): When you add p, the graph shift to the left. When you subtract q, the graph shifts to the right.

TRIGONOMETRY: EXPRESSIONS, EQUATIONS AND GRAPHS QUESTIONS

Question 1 (Adapted from November 2011 P2, Question 9)

Without using a calculator, determine the value of the following expressions:

1.1 $\dfrac{\cos 100^{\circ}}{\sin(-10^{\circ})} \times \tan^2 120^{\circ}$

1.2 $\dfrac{\tan(-60^{\circ})\cos(-156^{\circ})\cos 294^{\circ}}{\sin 492^{\circ}}$

Question 2 (Adapted from November 2011 P2, Question 9)

If $\tan\theta = \dfrac{3}{\sqrt{40}}$ and 0° < A < 90°, determine the values of the following with the aid of a sketch and without using a calculator. Leave your answers in surd form, if necessary.

2.1 cos A (3)

2.2 sin (180 ° + A) (2)

Question 3

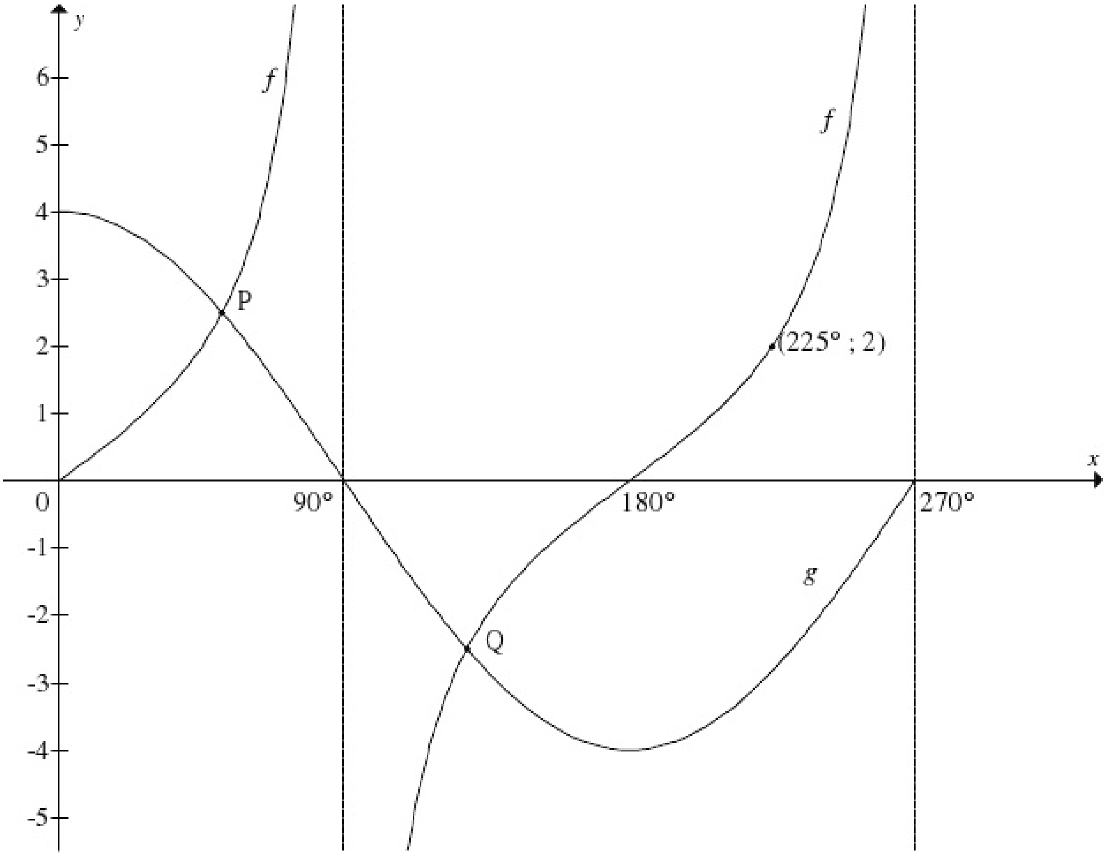
Determine the general solution of the equation $\cos 4\theta.\cos 40^{\circ} + \sin 4\theta.\sin 40^{\circ} = -1$ (4)

Question 4

Determine the general solution of the equation. $4\sin^2 x - 11\cos x = 1$ (6)

Question 5

The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0° \leq x \leq 270°$ are shown in the diagram below. The point (225°; 2) lies on *f*. The graphs intersect at points P and Q.



5.1 Determine the numerical values of a and b. (4)

5.2 Determine the minimum value of $g(x) + 2$. (2)

5.3 Determine the period of $f(\tfrac{1}{2}x)$ (2)

5.4 Show that, if the x-coordinate of P is θ, then the x-coordinate of Q is $(\theta - {}^\circ 180)$. (4)

TRIGONOMETRY: IDENTITIES

Summary of Identities

You must know the following identities even though you can find them on the information sheet:

$$\cos^2\theta + \sin^2\theta = 1 \qquad\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

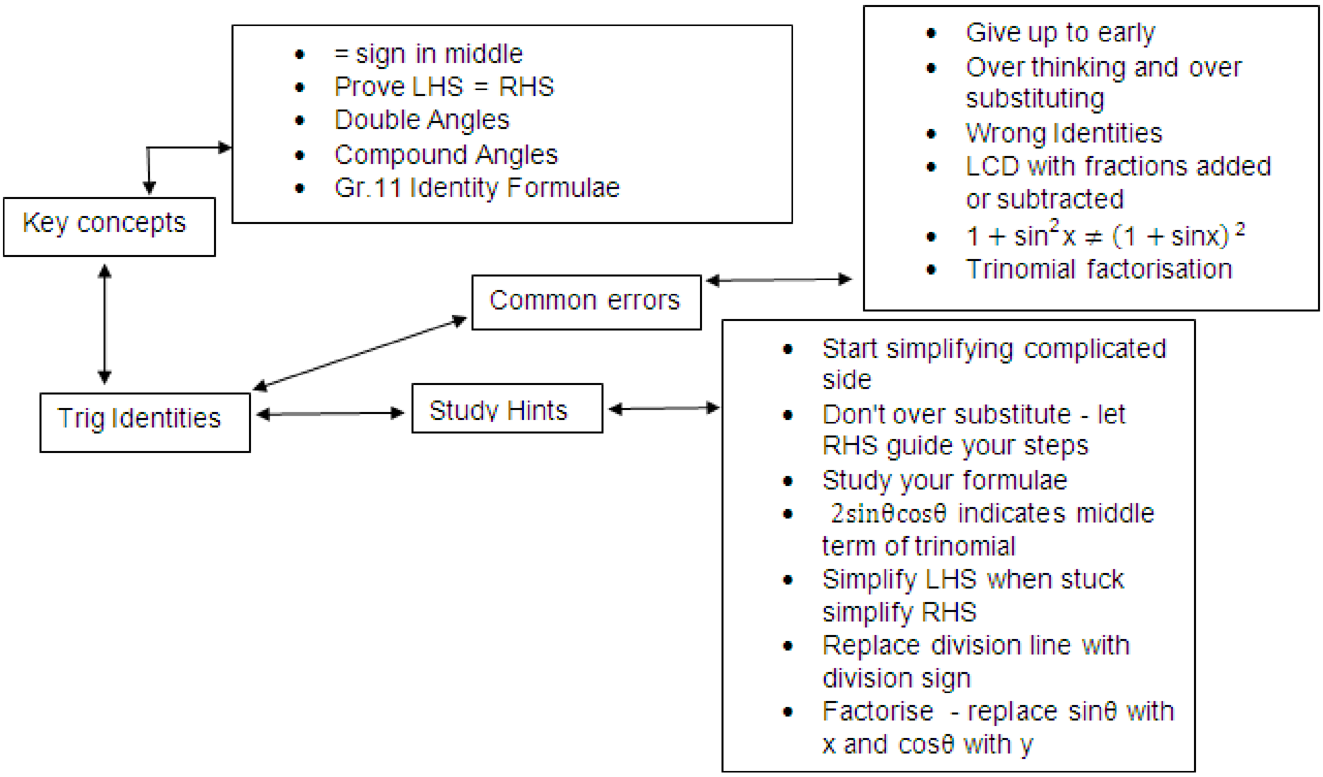
Compound angle identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

Double angle identities

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

Strategies for Solving Identities



TRIGONOMETRY: IDENTITIES QUESTIONS

Question 1

Prove the following using identities:

$$\frac{\sin(180°-\theta)-\sin(360°-2\theta)}{1+\cos(180°+\theta)+\cos(-2\theta)}=\tan(\theta-180°) \qquad (9)$$

Question 2

Prove that:

(a) $$\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x \qquad (6)$$

(b) $$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\tan\theta \qquad (6)$$

Question 3 (Adapted from Nov 2011 Paper 2, Question 12)

3.1 Prove that, for angles *A* and *B*,

$$\frac{\sin A}{\sin B}-\frac{\cos A}{\cos B}=\frac{2\sin(A-B)}{\sin 2B}$$

(4)

3.2 Hence, or otherwise, **without using a calculator,** show that

3.2.1

$$\frac{\sin 5B}{\sin B}-\frac{\cos 5B}{\cos B}=4\cos 2B$$

(3)

3.2.2

$$\frac{1}{\sin 18^{o}}=4\cos 36^{o}$$

(3)

3.2.3 sin 18° is a solution of the cubic equation $8x^3 - 4x + 1 = 0$

(4)

TRIGONOMETRY: 2D & 3D PROBLEMS

Summary

Solving Two-Dimensional Problems using the Sine, Cosine and Area Rules

- The **sine-rule** can be used when the following is known in the triangle:
  - more than 1 angle and a side
  - 2 sides and an angle (not included)

  $$\frac{\sin \mathrm{A}}{a} = \frac{\sin \mathrm{B}}{b} = \frac{\sin \mathrm{C}}{c}$$
- The cosine-rule can be used when the following is known of the triangle:
  - 3 sides
  - 2 sides and an included angle

  $$a^2 = b^2 + c^2 - 2bc\cos \mathrm{A}$$
- The **area** of any triangle can be found when at least two sides an included angle are known

  $$\text{Area of } \Delta\mathrm{ABC} = \frac{1}{2}ab\sin \mathrm{C}$$

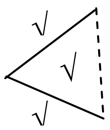
Study Tips

Sin Rule

In a solution of triangles question, use the sin rule to find a missing side or angle **only** if you have either two angles and one side, **or** two sides and an angle that is opposite one of the known sides. (Note: if the side opposite the given angle is the smaller of the 2 sides, there are 2 solutions)

Area Rule

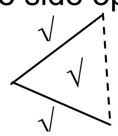
To use the area rule you need to know 3 things: 2 sides and an included angle.

area rule  ← missing side

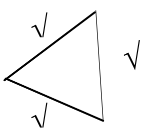
Cos Rule

In a solution of triangle questions use the cos rule

- To find the side opposite a given angle when we have 2 sides and an included angle

 ← missing side

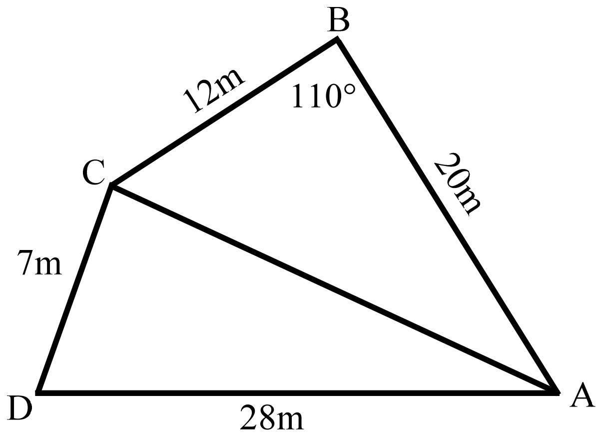
- To find an angle when we have 3 sides given

cos rule 

TRIGONOMETRY: 2D & 3D PROBLEMS QUESTIONS

QUESTION 1:

A piece of land has the form of a quadrilateral ABCD with $\mathrm{AB} = 20\mathrm{m}, \mathrm{BC} = 12\mathrm{m}, \mathrm{CD} = 7\mathrm{m}$ and $\mathrm{AD} = 28\mathrm{m}$. $\hat{\mathrm{B}} = 110°$. The owner decides to divide the land into two plots by erecting a fence from A to C.



(a) Calculate the length of the fence AC correct to one decimal place. (2)

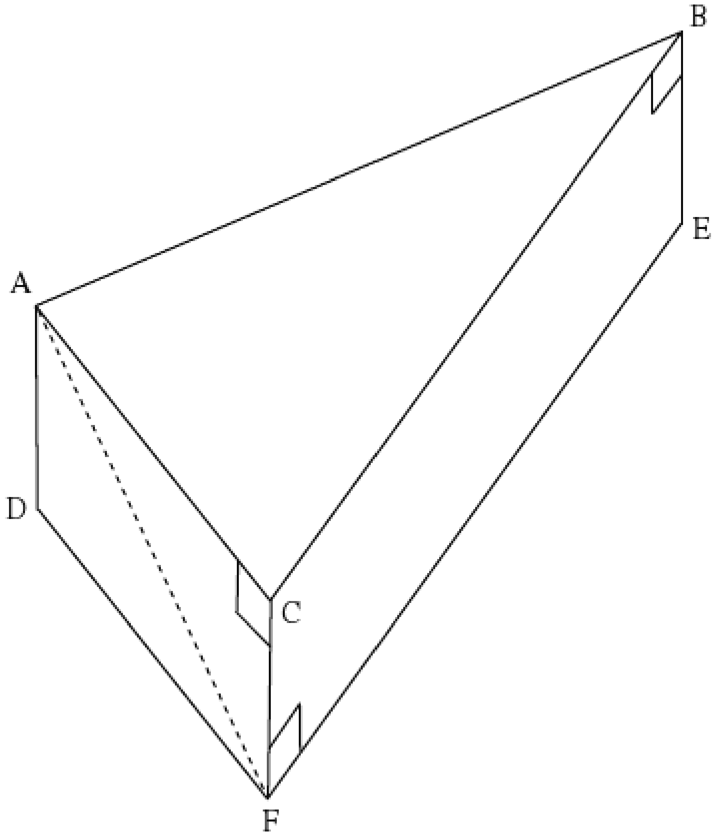
(b) Calculate the size of $\mathrm{B\hat{A}C}$ correct to the nearest degree. (2)

(c) Calculate the size of $\hat{\mathrm{D}}$, correct to the nearest degree. (3)

(d) Calculate the area of the entire piece of land ABCD, correct to one decimal place. (3)

[10]

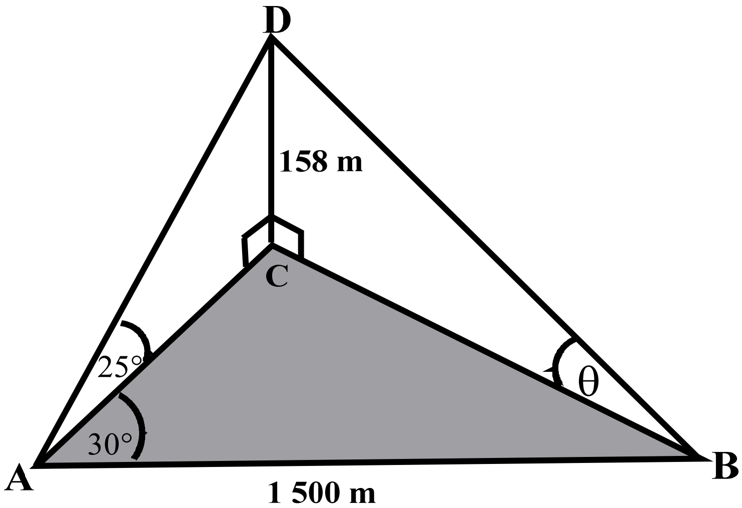
Question 2 (Nov 2011 P2 , Question 11)
The figure below represents a triangular right prism with BA = BC = 5 units, ABC = 50° and FAC = 25° .



2.1	Determine the area of ΔABC.	(2)
2.2	Calculate the length of AC.	(3)
2.3	Hence, determine the height FC of the prism.	(3)

QUESTION 3

In the diagram below, AB is a straight line 1 500 m long. DC is a vertical tower 158 metres high with C, A and B points in the same horizontal plane. The angles of elevation of D from A and B are $25°$ and θ. $C\hat{A}B = 30°$.



(a)	Determine the length of AC.	(3)
(b)	Find the value of θ.	(5)
(c)	Calculate the area of ΔABC.	(2)
(d)	Calculate the size of $A\hat{D}B$	(6)
		[16]

Question 4

ΔABC is an isosceles triangle with $AB = BCm$, $AB = c$, $AC = b$ and $BC = a$.

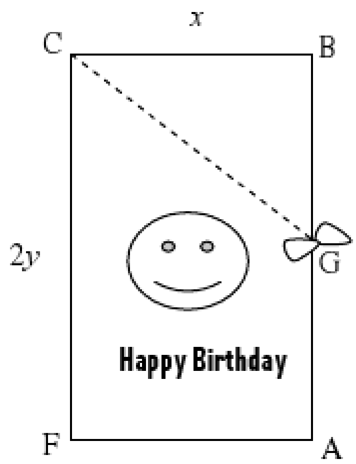
Prove that $\cos B = 1 - \dfrac{b^2}{2a^2}$

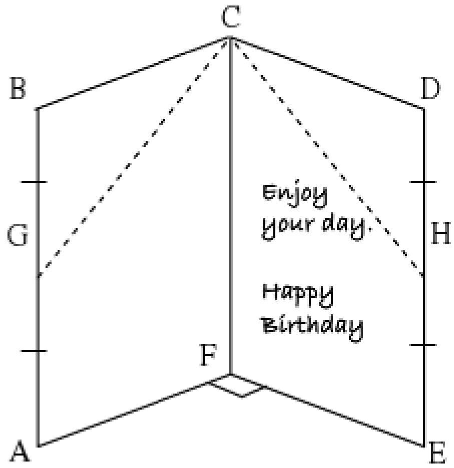
[4]

Question 5

A rectangular birthday card is tied with a ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle AFE between the front cover and the back cover of the card is 90° . The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch.

Let the shorter side of the card, BC = x, and the longer side, CF = 2y.





Prove that $\cos G\hat{C}H = \dfrac{y^2}{x^2+y^2}$.

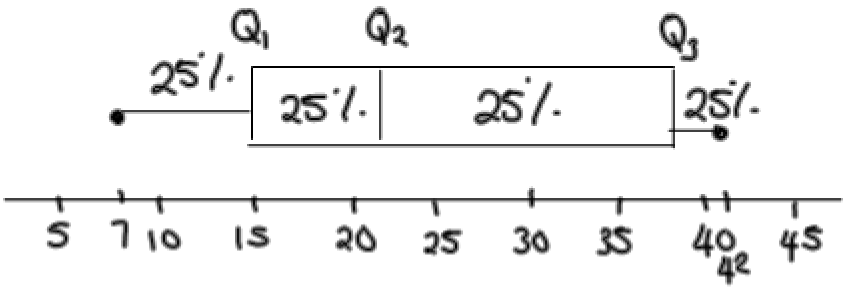
SOLUTIONS TO DATA HANDLING & TRANSFORMATIONS

1.1	$(x;y) \rightarrow (x+4;y) \rightarrow (-x-4;-y)$ OR $(x;y) \rightarrow (-x-4;-y)$	✓ $x+4$ ✓ y ✓ $-x-4$ ✓ $-y$ (4)
1.2	$x_Q = x\cos\theta + y\sin\theta$ $x_Q = -2\cos 135° + (-3)\sin 135°$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$ or $\frac{-\sqrt{2}}{2}$ or $-0{,}71$ $y_Q = y\cos\theta - x\sin\theta$ $y_Q = -3\cos 135° - (-2)\sin 135°$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3{,}54$ $Q\left(\frac{-1}{\sqrt{2}};\frac{5}{\sqrt{2}}\right)$	✓ subst -2 and -3 into correct formula for x_Q ✓ using 135° ✓ x coordinate (in any format) ✓ subst -2 and -3 into correct formula for y_Q ✓ for y coordinate (in any format) (5)

2.1

$a=7$	$b=15$	$c=17$	$d=23$	$e=34$	$f=37$	$g=42$

2.2



.: POSITIVELY SKEWED

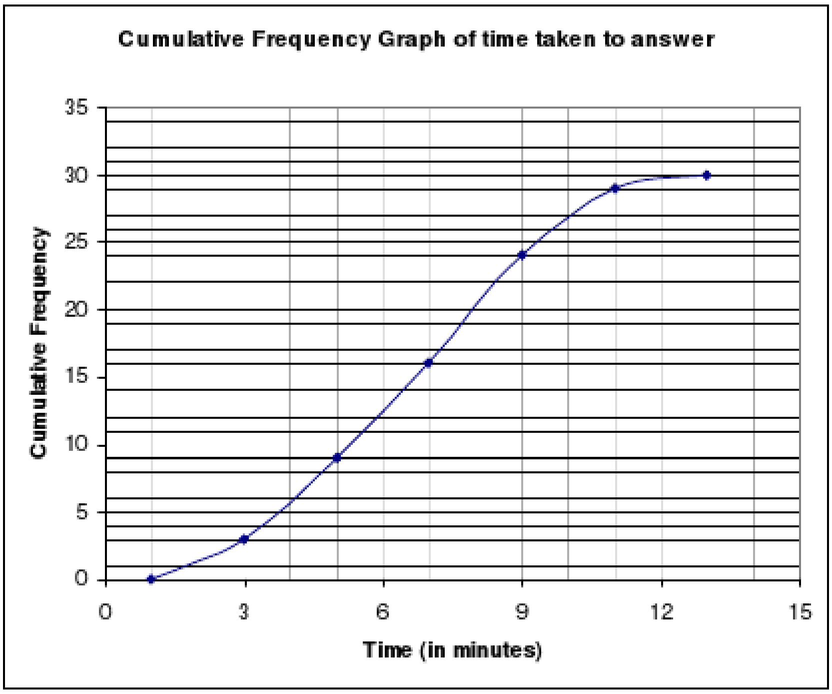
MEAN> MEDIAN.

3.1	Mean $= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{580}{8} = 72,5$ **Note**: If rounded off to 73: 1 mark	✓ 580 ✓ answer (2)
3.2	Standard deviation $(\sigma) = 2,78$ (2,783882181...) **Note**: If rounded off to 2,8: 1 mark	✓✓ answer (2)
3.3	$\therefore$ 2 golfers' scores lie outside 1 standard deviation of the mean. The interval for 1 standard deviation of the mean is $(72,5 - 2,78 ; 72,5 + 2,78) = (69,72 ; 75,28)$	✓ interval ✓ number (2) **[6]**

4.1	30	✓ 30 (1)
4.2	Linear, the points seem to form a straight line.	✓ linear ✓ reason (2)
4.3	The greater the number of hours spent watching TV, the lower the test scores **OR** The less time a person spends watching TV, the higher the test score. **OR** Negative correlation between the variables **OR** Indirect relationship between the variables	✓ deduction (1)
4.4	60 marks. (Accept 50 -70 marks)	✓✓ deduction (2) **[6]**

5.1

Time, T (in minutes)	Number of learners	Cumulative frequency
$1 \le t < 3$	3	3
$3 \le t < 5$	6	9
$5 \le t < 7$	7	16
$7 \le t < 9$	8	24
$9 \le t < 11$	5	**29**
$11 \le t < 13$	1	30

5.2		✓upper limit ✓ cumulative frequency (at least 4 of 6 y-values correctly plotted) ✓ grounding at (1 ; 0) ✓ shape (not joined by a ruler; smooth curve) (4)
5.3	Estimated number of learners that took less than 4 minutes: approximately 5 learners (Accept 6) Approximate percentage = 16,67% (Accept 20%)	✓ 5 learners ✓ 16,67% (2)

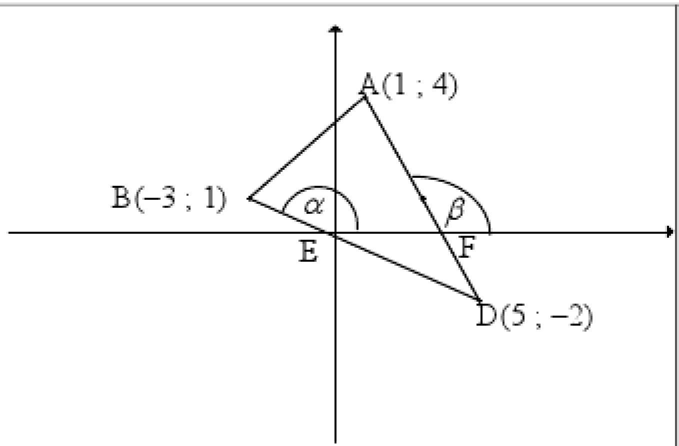
SOLUTIONS TO CO-ORDINATE GEOMETRY: LINES

1.1	$m_{PQ} \times m_{QR} = -1$ $\left(\frac{a-0}{4+4}\right)\left(\frac{a-0}{4-6}\right) = -1$ $\left(\frac{a}{8}\right)\left(\frac{a}{-2}\right) = -1$ $\frac{a^2}{-16} = -1$ $a^2 = 16$ $a = \pm 4$ $a = 4;$ since $a > 0$	✓ $\frac{a-0}{4+4}$ or $\frac{a}{8}$ ✓ $\frac{a-0}{4-6}$ or $\frac{a}{-2}$ ✓ using gradient of perpendicular lines ✓ $a^2 = 16$ (4)
1.2	Equation of line SR: $m_{PQ} = \frac{4-0}{4-(-4)} = \frac{1}{2}$ $m_{SR} = m_{PQ} = \frac{1}{2}$ PQ || SR $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{2}(x-6)$ $y = \frac{1}{2}x - 3$	✓ $m_{PQ} = \frac{1}{2}$ ✓ $m_{SR} = \frac{1}{2}$ ✓ substitution of m and (6 ; 0) ✓ standard form (4)
1.3	Eq. of RS: $y = \frac{1}{2}x - 3$ Eq. of SP: $y - 0 = -2(x+4)$ $\therefore \frac{1}{2}x - 3 = -2(x+4)$ $\therefore x = -2$ $y = -4$	✓ m = − 2 ✓ eq. of SP ✓ value of x ✓ value of y (4)

1.4	$PR = 6-(-4)$ $= 10$ $PR^2 = (6+4)^2 + (0-0)^2$ $PR = 10$	✓ $6-(-4)$ ✓ 10 (2) ✓ substitution in correct formula ✓ 10 (2)

1.5	P needs to shift at least 4 units to the right and S needs to shift at least 4 units up for the image of PQRS in first quadrant. $\therefore$ minimum value of k is 4 and minimum value of l is 4 $\therefore$ minimum value of $k+l$ is 8	✓ $k=4$ ✓ $l=4$ ✓ $k+l=8$ (3)

2.1	Gradient of PS $=\frac{2-0}{5-9} = \frac{2}{-4} = -\frac{1}{2}$ Gradient of QR $= \frac{-1-(-5)}{1-9} = \frac{4}{-8} = -\frac{1}{2}$ $\therefore$ PS || QR PQRS is a trapezium	
2.2	$QT^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore QT^2 = 4+1$ $\therefore QT^2 = 5$ $\therefore QT = \sqrt{5}$ $TR^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore TR^2 = 36+9$ $\therefore TR^2 = 45$ $\therefore TR = \sqrt{45} = \sqrt{9\times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}TR = \sqrt{5} = QT$ $\therefore QT = \frac{1}{3}TR$	✓ correct substitution to get QT ✓ answer for QT ✓ correct substitution to get TR ✓ answer for TR ✓ establishing that $QT = \frac{1}{3}TR$ (5)

3.1	$m_{AD} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{-2-4}{5-1}$ $= -\dfrac{6}{4} \quad = -\dfrac{3}{2}$	✓ for substitution ✓ for answer (2)
3.2	$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(5-1)^2+(-2-4)^2}$ $= \sqrt{16+36}$ $= \sqrt{52}$	✓ for substitution ✓ $\sqrt{52}$ (2)
3.3	$M = \left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $M = \left(\dfrac{1+5}{2};\dfrac{4-2}{2}\right)$ $M = (3\,;\,1)$	✓ x-value ✓ y-value (2)
3.4	$m_{AD} = -\dfrac{3}{2}$ $\tan\beta = -\dfrac{3}{2}$ $\beta = 180° - 56{,}31°$ $\beta = 123{,}69$  	✓ $\tan\beta = m_{AD}$ ✓ 123,69° (2)
3.5	$m_{BD} = \dfrac{-2-1}{5-(-3)} = \dfrac{-3}{8}$ $\tan\alpha = -\dfrac{3}{8}$ $\alpha = 180° - 20{,}56°$ $\alpha = 159{,}44°$ $F\hat{E}D = 180° - 159{,}44° = 20{,}56°$ $E\hat{F}D = 123{,}69°$ $F\hat{D}E = 180° - (20{,}56° + 123{,}69°) = 35{,}75°$	✓ $m_{BD} = \dfrac{-3}{8}$ ✓159,44° ✓20,56° ✓123,69° ✓35,75° (5)

SOLUTIONS TO CO-ORDINATE GEOMETRY: CIRCLES

1.1	midpoint PR $= \left(\frac{6+(-4)}{2} ; \frac{0+0}{2}\right) = (1; 0)$ radius of circle $= \frac{1}{2}\text{PR} = 5$ units $\therefore (x-1)^2 + (y-0)^2 = 5^2$ $(x-1)^2 + y^2 = 25$ Answer only: FULL MARKS	✓ midpoint ✓ radius ✓ eq. of circle in correct form (3)
1.2	$(x-1)^2 + y^2 = 25$ substitute Q(4 ; 4): LHS $= (4-1)^2 + 4^2$ $= 25$ $=$ RHS $\therefore$ Q is a point on the circle **Note**: If substitute point into equation resulting in $25 = 25$: 1 mark No conclusion: 1 mark	✓ substitute Q(4;4) ✓ LHS = RHS (2)

2.1	$x_C = x_B = -1$ $y_C = y_B + 5 = 6$ $\therefore$ C$(-1\,;\,6)$	✓ value of x ✓ value of y (2)
2.2	BA $\perp$ CA (tangent $\perp$ radius) $\therefore\ CA^2 = BC^2 - AB^2$ (Pythagoras) $= (5)^2 - (\sqrt{20})^2 = 5$ $\therefore\ CA = \sqrt{5}$ or 2,24 units	✓ BA$\perp$CA or $B\hat{A}C = 90°$ ✓ substitution into Pythagoras ✓ answer (3)
2.3	$\tan\theta = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$	✓ tan ratio (in any form) (1)
2.4	$m_{DC} \times m_{AB} = -1$ $m_{DC} = \tan\theta = \frac{1}{2}$ $m_{DC} = \frac{1}{2}$ $m_{AB} = -2$	✓ $m_{DC} \times m_{AB} = -1$ ✓ $m_{DC} = \tan\theta = \frac{1}{2}$ (2)
2.5	Eq. of DC: $y - 6 = \frac{1}{2}(x+1)$ $y = \frac{1}{2}x + \frac{13}{2}$ Eq. of AB: $y - 1 = -2(x+1)$ $y = -2x - 1$ $-2x - 1 = \frac{1}{2}x + \frac{13}{2}$ $-\frac{5}{2}x = \frac{15}{2}$ $x = -3$ $y = -2(-3) - 1$ $y = 5$ $\therefore$ A $(-3\,;\,5)$ Answer only: $(-3\,;\,5)$: 1 mark	✓ DC: subst m and $(-1\,;\,6)$ ✓ eq. of DC ✓ eq. of AB ✓ equating equations ✓ value of x ✓ value of y (6)
2.6	Area Δ ABC $= \frac{1}{2}(\sqrt{5})(\sqrt{20}) = 5$ Eqn. of DC is $y = \frac{1}{2}x + \frac{13}{2}$ Therefore OF $= \frac{13}{2}$ and OD $= 13$. Area Δ ODF $= \frac{1}{2}\left(\frac{13}{2}\right)(13) = \frac{169}{4}$ Area ΔABC: Area ΔODF $= 5 : \frac{169}{4} = 20 : 169$	✓ $\frac{1}{2}(\sqrt{5})(\sqrt{20})$ ✓ OF $= \frac{13}{2}$ ✓OD $= 13$ ✓ $\frac{1}{2}\left(\frac{13}{2}\right)(13)$ ✓ answer (5)

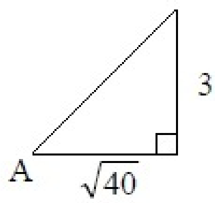
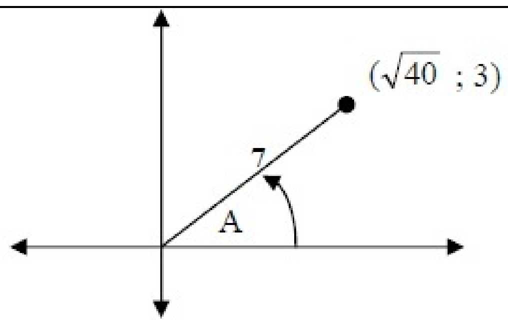
3.1	Coordinates of centre M $(-2\ ;\ 1)$ $(1+2)^2+(-2-1)^2=18=r^2$ Radius $=\sqrt{18}$ or $3\sqrt{2}$	✓✓ coordinates of centre ✓ calculation ✓ value (4)
3.2	$m_{MS}=\frac{-3}{3}=-1$ $m_{MS}\, x\, m_{RS}=-1$ OR tangent $\perp$ radius $m_{RS}=1$ $y-y_1=m\,(x-x_1)$ $y+2=1(x-1)$ $y=x-3$	✓ gradient MS ✓ gradient RS ✓ subst $(1\ ;\ -2)$ ✓ equation (4)
3.3	$\frac{MS}{MP}=\frac{1}{3}$ $\therefore MP=3MS$ $MP^2=9MS^2$ $(a+2)^2+(b-1)^2=9(3^2+3^2)=162$ (1) $MS\perp SR$ and $PS\perp SR$ $\therefore m_{PS}=m_{MS}$ $\frac{b+2}{a-1}=\frac{3}{-3}=-1$ $b+2=-a+1$ $b=-a-1$ (2) Subst (2) into(1)	✓ MP = 3MS ✓ equation ✓ equal gradients ✓ gradient ✓ b = -a - 1
	$(a+2)^2+(-a-1-1)^2=162$ $(a+2)^2+(a+2)^2=162$ $2(a+2)^2=162$ $(a+2)^2=81$ $a+2=9$ or -9 $a=7$ or -11 $b=-a-1=-8$ $P(7\ ;-8)$	✓substitution ✓ $a=7$ ✓ b = -8 (8)

SOLUTIONS TO TRIGONOMETRY: EXPRESSIONS, EQUATIONS AND GRAPHS

Question 1

1.1		$\frac{\cos 100° \times \tan^2 120°}{\sin(-10°)}$ $= \frac{(-\cos 80°)(-\tan 60°)^2}{(-\sin 10°)}$ $= \frac{(-\cos 80°)\times\left((-\sqrt{3})^2\right)}{(-\cos 80°)}$ $= 3$ **Note**: Answer only: 0 marks **Note**: If $\frac{+\cos 80°}{+\sin 10°}$ (assume two negatives cancelled), no penalty	✓ $-\cos 80°$ ✓ $-\tan 60°$ or $\tan^2 60°$ ✓ $-\sin 10°$ ✓ $-\sqrt{3}$ ✓ $\sin 10° = \cos 80°$ ✓ 3 (6)
1.2		$\frac{\tan(-60°)\cos(-156°)\cos 294°}{\sin 492°}$ $= \frac{(-\tan 60°)(\cos 156°)(-\cos 66°)}{(\sin 132°)}$ $= \frac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{(\sin 48°)}$ $= \frac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{2\sin 24° \cos 24°}$ $= -\frac{\sqrt{3}}{2}$	✓ $(-\tan 60°)(\cos 156°)$ ✓ $-\cos 66°$ ✓ $\sin 48°$ ✓ $-\sqrt{3}$ ✓ $-\sin 24°$ ✓ $2\sin 24° \cos 24°$ ✓ $\frac{\sqrt{3}}{2}$ (7)

Question 2

2.1	 or  $r^2 = 40 + 9$ $r = 7$ $\cos A = \frac{\sqrt{40}}{7}$	✓ sketch ✓ $r = 7$ ✓ $\frac{\sqrt{40}}{7}$ (3)

2.2	$\sin(180° + A)$ $= -\sin A$ $= -\frac{3}{7}$ **OR** $\sin(180°+A) = \sin 180°.\cos A + \cos 180°.\sin A$ $= 0.\cos A - 1.\sin A$ $= -\sin A$ $= -\frac{3}{7}$	✓ $-\sin A$ ✓ $-\frac{3}{7}$ (2) ✓ $-\sin A$ ✓ $-\frac{3}{7}$ (2)

Question 3

3	$\cos 4\theta.\cos 40° + \sin 4\theta.\sin 40° = -1$ $\therefore \cos(4\theta - 40°) = -1$ $\therefore (4\theta - 40°) = \pm\cos^{-1}(-1)$ $\therefore (4\theta - 40°) = \pm 180° + k.360°$ where $k \in Z$ $\therefore 4\theta = \pm 180° + 40° + k.360°$ $\therefore 4\theta = 220° + k.360°$ or $4\theta = -140° + k.360°$ $\therefore \theta = 55° + k.90°$ or $\theta = -35° + k.360°$	✓ $\cos(4\theta - 40°) = -1$ ✓ $(4\theta - 40°) = \pm 180° + k.360°$ ✓ $\theta = 55° + k.90°$ ✓ $\theta = -35° + k.360°$ (4)

Question 4

4	$4\sin^2 x - 11\cos x = 1$ $\therefore 4(1-\cos^2 x) - 11\cos x - 1 = 0$ $\therefore 4 - 4\cos^2 x - 11\cos x - 1 = 0$ $\therefore -4\cos^2 x - 11\cos x + 3 = 0$ $\therefore 4\cos^2 x + 11\cos x - 3 = 0$ $\therefore (4\cos x - 1)(\cos x + 3) = 0$ $\therefore \cos x = \frac{1}{4}$ or $\cos x = -3$ Since $-1 \le \cos x \le 1$, $\cos x = -3$ has no solution $\cos x = \frac{1}{4}$ $\therefore x = \pm\cos^{-1}\left(\frac{1}{4}\right) + k.360°$ $\therefore x = \pm 75{,}52° + k.360°$ $(k \in Z)$	✓ $\sin^2 x = 1 - \cos^2 x$ ✓ $4\cos^2 x + 11\cos x - 3 = 0$ ✓ $(4\cos x - 1)(\cos x + 3) = 0$ ✓ $\cos x = \frac{1}{4}$ or $\cos x = -3$ ✓ $\cos x = -3$ has no solution ✓ $x = \pm 75{,}52° + k.360°$ (6)

Question 5

5.1	$f(225°) = 2$ $\therefore a \tan 225° = 2 \quad \therefore a = 2$ $g(0) = 4$ $\therefore b \cos 0° = 4 \quad \therefore b = 4$ Answer only: Full marks	✓ substitution ✓ $a = 2$ ✓ substitution ✓ $b = 4$ (4)
5.2	Minimum value of $g(x) + 2 = -4 + 2 = -2$ Answer only: Full marks	✓ -4 ✓ -2 (2)
5.3	$\text{Period} = \dfrac{180°}{\frac{1}{2}} = 360°$ Answer only: Full marks	✓ $\dfrac{180°}{\frac{1}{2}}$ ✓ $360°$ (2)
5.4	At P $f(\theta) = g(\theta)$ $2\tan\theta = 4\cos\theta$ for $180° - \theta$: $2\tan(180° - \theta) = -2\tan\theta$ and $4\cos(180° - \theta) = -4\cos\theta$ $2\tan\theta = 4\cos\theta$ at P $\therefore -2\tan\theta = -4\cos\theta$ $\therefore 2\tan(180° - \theta) = 4\cos(180° - \theta)$ at Q	✓ $2\tan\theta = 4\cos\theta$ ✓ $2\tan(180° - \theta) = -2\tan\theta$ ✓ $4\cos(180° - \theta) = -4\cos\theta$ ✓ $2\tan(180° - \theta) = 4\cos(180° - \theta)$ (4)

SOLUTIONS TO TRIGONOMETRY: IDENTITIES

Question 1

1	LHS $= \dfrac{\sin(180° - \theta) - \sin(360° - 2\theta)}{1 - \cos(180° + \theta) + \cos(-2\theta)}$ $= \dfrac{\sin\theta - (-\sin 2\theta)}{1 - (-\cos\theta) + \cos 2\theta}$ $= \dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ $= \dfrac{\sin\theta + 2\sin\theta\cos\theta}{1 + \cos\theta + 2\cos^2\theta - 1}$ $= \dfrac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta$ RHS $= \tan(\theta - 180°)$ $= \tan\left[-(180° - \theta)\right]$ $= -\tan(180° - \theta)$ $= -(-\tan\theta)$ $= \tan\theta$ $\therefore$ LHS = RHS	✓ $\sin\theta + \sin 2\theta$ ✓ $1 + \cos\theta + \cos 2\theta$ ✓ $2\sin\theta\cos\theta$ ✓ $2\cos^2\theta - 1$ ✓ factorising ✓ $\tan\theta$ ✓ $-\tan(180° - \theta)$ ✓ $\tan\theta$ ✓ LHS = RHS (9)

Question 2

2(a)	$\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \dfrac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x\cos x - \cos x}$ $= \dfrac{1 - 1 + 2\sin^2 x - \sin x}{2\sin x\cos x - \cos x}$ $= \dfrac{2\sin^2 x - \sin x}{2\sin x\cos x - \cos x}$ $= \dfrac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$ $= \dfrac{\sin x}{\cos x} = \tan x$	✓ $1 - 2\sin^2 x$ ✓ $2\sin x\cos x$ ✓ $\sin x(2\sin x - 1)$ ✓ $\cos x(2\sin x - 1)$ ✓ $\dfrac{\sin x}{\cos x}$ ✓ $\tan x$ (6)

2(b)	$\dfrac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$ $=\dfrac{\sin\theta+2\sin\theta.\cos\theta}{1+\cos\theta+\left(2\cos^2\theta-1\right)}$ $=\dfrac{\sin\theta\left(1+2\cos\theta\right)}{\cos\theta+2\cos^2\theta}$ $=\dfrac{\sin\theta\left(1+2\cos\theta\right)}{\cos\theta\left(1+2\cos\theta\right)}$ $=\dfrac{\sin\theta}{\cos\theta}$ $=\tan\theta$	✓ $2\sin\theta\cos\theta$ ✓ $2\cos^2\theta-1$ ✓ $\sin\theta\left(1+2\cos\theta\right)$ ✓ $\cos\theta\left(1+2\cos\theta\right)$ ✓ $\dfrac{\sin\theta}{\cos\theta}$ ✓ $\tan\theta$ (6)

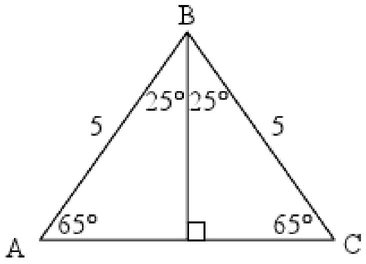
Question 3

3.1	LHS: $\dfrac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ $=\dfrac{\sin(A-B)}{\sin B\cos B}$ $=\dfrac{2\sin(A-B)}{2\sin B\cos B}$ $=\dfrac{2\sin(A-B)}{\sin 2B}$ $=RHS$	✓ writing as single fraction ✓ comp. angle expansion ✓mult. by 2 ✓ comp. angle expansion (4)
3.2.1	A = 5B $\dfrac{\sin 5B}{\sin B}-\dfrac{\cos 5B}{\cos B}=\dfrac{2\sin(5B-B)}{\sin 2B}$ $=\dfrac{2\sin 4B}{\sin 2B}$ $=\dfrac{4\sin 2B\cos 2B}{\sin 2B}$ $=4\cos 2B$	✓ recognising A = 5B ✓ substituting A = 5B ✓ sin 4B = 2sin 2B cos 2B (3)

3.2.2	$B = 18°$ $\frac{\sin 90°}{\sin 18°} - \frac{\cos 90°}{\cos 18°} = 4\cos 2(18)°$ $\therefore \frac{1}{\sin 18°} - 0 = 4\cos 36°$ $\therefore \frac{1}{\sin 18°} = 4\cos 36°$	✓ recognising $B = 18°$ ✓ substituting $B = 18°$ ✓ simplify (3)
3.2.3	Let $\sin 18° = a$ $\frac{1}{\sin 18°} = 4\cos 36°$ $\frac{1}{\sin 18°} = 4(1 - 2\sin^2 18°)$ $\therefore \frac{1}{a} = 4(1 - 2a^2)$ $\therefore 1 = 4a - 8a^3$ $\therefore 8a^3 - 4a + 1 = 0$ Hence $\sin 18°$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$	✓ $\sin 18° = a$ ✓ $\cos 36° = 1 - 2\sin^2 18°$ ✓ substitution of a ✓ simplification (4)

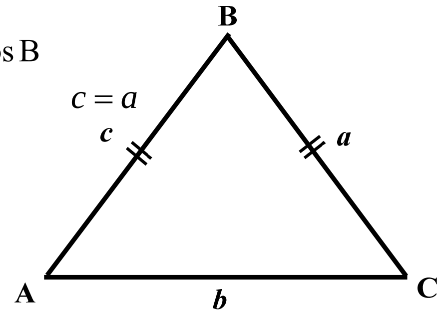
SOLUTIONS TO TRIGONOMETRY: 2D & 3D PROBLEMS

1(a)	$AC^2 = (12m)^2 + (20m)^2 - 2(12m)(20m)\cos 110°$ $\therefore AC^2 = 708{,}1696688$ $\therefore AC = 26{,}6m$	✓ substitution into cosine rule ✓ answer (2)
1(b)	$\dfrac{\sin B\hat{A}C}{12m} = \dfrac{\sin 110°}{26{,}6m}$ $\therefore \sin B\hat{A}C = \dfrac{12 \times \sin 110°}{26{,}6m}$ $\therefore \sin B\hat{A}C = 0{,}4239214831$ $\therefore B\hat{A}C = 25°$ OR $(12m)^2 = (20m)^2 + (26{,}6m)^2 - 2(20m)(26{,}6m)\cos B\hat{A}C$ $\therefore 1064\cos B\hat{A}C = 963{,}56m^2$ $\therefore \cos B\hat{A}C = 0{,}9056015038$ $\therefore B\hat{A}C = 25°$	✓ substitution into sine or cosine rule ✓ answer (2)
1(c)	$(26{,}6m)^2 = (7m)^2 + (28m)^2 - 2(7m)(28m)\cos\hat{D}$ $\therefore 392\cos\hat{D} = 125{,}44$ $\therefore \cos\hat{D} = 0{,}32$ $\therefore \hat{D} = 71°$	✓ substitution into cosine rule ✓ $\cos\hat{D} = 0{,}32$ ✓ answer (3)
1(d)	Area ABCD $= \dfrac{1}{2}(12m)(20m)\sin 110° + \dfrac{1}{2}(7m)(28m)\sin 71°$ $= 205{,}4m^2$	✓ $\dfrac{1}{2}(12m)(20m)\sin 110°$ ✓ $\dfrac{1}{2}(7m)(28m)\sin 71°$ ✓ answer (3) **[10]**

2.1	Area $\Delta ABC = \frac{1}{2}.AB.BC.\sin 50°$ $= \frac{1}{2}(5)(5)\sin 50°$ $= 9{,}58 \ units^2$	✓ substitution into correct formula ✓ answer (2)
	OR Area of ΔABC $= \frac{1}{2}(2)(5)\sin 25°)(5\cos 25°)$ $= 9{,}58 \ units^2$ 	✓ base and height in terms of 5 and 25° ✓ answer (2)
	OR Area of ΔABC $= [\frac{1}{2}(5\cos 65°)(5\sin 65°)](2)$ $= 9{,}58 \ units^2$	✓ base and height in terms of 5 and 65° ✓ answer (2)

2.2	$AC^2 = 5^2 + 5^2 - 2(5)(5)\cos 50°$ $AC^2 = 17{,}86061952$ $AC = 4{,}23\ units$ **OR** $\hat{A} = \hat{C} = 65°$ (angles opposite equal sides) $\frac{\sin 65°}{5} = \frac{\sin 50°}{AC}$ $AC = \frac{5\sin 50°}{\sin 65°}$ $= 4{,}23\ units$	✓ use of cosine rule ✓ substitution ✓ answer (3) ✓ use of sine rule ✓ substitution ✓ answer (3)
2.3	$\tan 25° = \frac{CF}{AC}$ $\therefore CF = 4{,}23 \times \tan 25°$ $\therefore CF = 1{,}97\ units$ **OR** $\frac{FC}{\sin 25°} = \frac{4{,}23}{\sin 65°}$ $FC = \frac{4{,}23\sin 25°}{\sin 65°}$ $= 1{,}97\ units$	✓ ratio ✓ CF as subject ✓ answer (3) ✓ sine rule ✓ FC as subject ✓ answer (3) **[8]**

3(a)	In Δ ADC: $\hat{D} = 65°$ ($\angle$s of Δ) $\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$ $\therefore AC.\sin 25° = 158.\sin 65°$ $\therefore AC = \frac{158.\sin 65°}{\sin 25°}$ $\therefore AC = 338,83\text{ m}$	✓ $\hat{D} = 65°$ ✓ $\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$ ✓ $AC = 338,83\text{ m}$ (3)
3(b)	In Δ ACB: $BC^2 = 338,83^2 + 1500^2 - 2(338,83)(1500)\cos 30°$ $\therefore BC^2 = 1\ 484\ 499,606$ $\therefore BC = 1218,4\text{ m}$ In Δ DCB: $\tan\theta = \frac{DC}{BC}$ $\therefore \tan\theta = \frac{158}{1218,4}$ $\therefore \theta = 7,39°$	✓ cosine rule to get BC ✓ $BC = 1218,4\text{ m}$ ✓ $\tan\theta = \frac{DC}{BC}$ ✓ $\tan\theta = \frac{158}{1218,4}$ ✓ $\theta = 7,39°$ (5)
3(c)	$\text{Area } \Delta ABC = \frac{1}{2}(338,83)(1500)\sin 30°$ $\therefore \text{Area } \Delta ABC = 127061,25m^2$	✓ area rule ✓ answer (2)
3(d)	$AD^2 = (338,83)^2 + (158)^2$ $\therefore AD^2 = 139769,7689$ $\therefore AD = 373,86m$ $BD^2 = (1218,4)^2 + (158)^2$ $\therefore BD^2 = 1509462,56$ $\therefore BD = 1228,60m$ $(1500)^2 = (373,86)^2 + (1228,60)^2 - 2(373,86)(1228,60)\cos A\hat{D}B$ $\therefore 2(373,86)(1228,60)\cos A\hat{D}B = (373,86)^2 + (1228,60)^2 - (1500)^2$ $\therefore 918648,792\cos A\hat{D}B = -600770,7404$ $\therefore \cos A\hat{D}B = -0,6539721661$ $\therefore A\hat{D}B = 130,84°$	✓ Pythagoras ✓ $AD = 373,86m$ ✓ $BD = 1228,60m$ ✓ cosine rule ✓ substitution ✓ answer (6)

4.	$b^2 = a^2 + c^2 - 2ac\cos B$ $\therefore b^2 = a^2 + a^2 - 2a \times a \times \cos B$ $\therefore b^2 = 2a^2 - 2a^2\cos B$ $\therefore b^2 = 2a^2(1 - \cos B)$ $\therefore \dfrac{b^2}{2a^2} = 1 - \cos B$ $\therefore \cos B = 1 - \dfrac{b^2}{2a^2}$  	✓ equal sides ✓ cos rule ✓ substitution ✓ simplification **[4]**

5.	In Δ CBG and ΔCDH: $CG^2 = x^2 + y^2$ Pythagoras $CH^2 = x^2 + y^2$ Pythagoras In ΔFAE $AE^2 = x^2 + x^2$ $= 2x^2$ $= GH^2$ In Δ CGH $GH^2 = CG^2 + CH^2 - 2\,CG.CH.\cos GCH$ $\cos G\hat{C}H = \dfrac{CG^2 + CH^2 - GH^2}{2CG.CH}$ $\cos G\hat{C}H = \dfrac{x^2 + y^2 + x^2 + y^2 - 2x^2}{2\sqrt{x^2+y^2}.\sqrt{x^2+y^2}}$ $\cos G\hat{C}H = \dfrac{2y^2}{2(x^2+y^2)}$ $\cos G\hat{C}H = \dfrac{y^2}{x^2+y^2}$	✓ CG^2 ✓ CH^2 ✓ AE^2 ✓ $AE^2 = GH^2$ ✓ use of cos rule ✓ manipulation of formula ✓ substitution ✓ $\cos G\hat{C}H = \dfrac{2y^2}{2(x^2+y^2)}$ (8) **[8]**