Basic Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

JUNE 2015

NATIONAL SENIOR CERTIFICATE

GRADE 10

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages and a 2 page diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. TWO diagram sheets for answering QUESTION 6.1, QUESTION 6.2, QUESTION 7, and QUESTION 8 are attached at the end of this question paper. Write your name on these sheets in the spaces provided and insert the sheets inside the back cover of your ANSWER BOOK.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 If $k \in \{0\ ;\ 1\ ;\ 2\ ;\ 3\}$, determine the value of k such that $\sqrt{\frac{18}{k-1}}$

1.1.1 is an integer (1)

1.1.2 is irrational (1)

1.1.3 is undefined (1)

1.2 Determine the product of the following and simplify fully:

$(x+3y)(9y^2+x^2-3xy)$ (2)

1.3 Factorise the following completely:

x^3+x^2-x-1 (4)

1.4 Simplify the following expression fully:

$$\frac{2x^2-2y^2}{x^2-2xy+y^2} \div \frac{4x^2+4xy}{xy-y^2}$$ (5)

[14]

QUESTION 2

2.1 The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.

Write down the value of r, the radius of the cone, in terms of V, π and h. (2)

2.2 Solve simultaneously for x and y:

$3x = y-4$ and $2y = 5+3x$ (4)

2.3 Solve the following inequality: $-3 \leq 3+2x < 13$.

Hence, illustrate your answer on a number line if x is a real number. (4)

2.4 Solve for x: $2.3^{x+1} = 162$ (4)

2.5 The sides of a kite are x^2+3 and x^2+3x-3 units respectively.

2.5.1 Calculate the value of x for which the kite will be a rhombus. (3)

2.5.2 **Hence**, determine the length of the sides of the rhombus. (2)

[19]

QUESTION 3

3.1 Consider the following sequence: 9 ; 16 ; 23 ; 30 ; ...

3.1.1 Write down the next term in the sequence. (1)

3.1.2 Write down the general term (n^{th} term) of the sequence. (2)

3.1.3 Which term in the sequence has the value of 408? (2)

3.2 Write down the fourth term in terms of x in the following linear sequence:

$2x+1;\ 4x+3;\ 6x+5;\ ...$ (2)

3.3 The following pattern is given:

Row 1					1				
Row 2				1	3	5			
Row 3			1	3	5	7	9		
Row 4		1	3	5	7	9	11	13	
Row 5	1	3	5	7	9	11	13	15	17

3.3.1 Determine the last term in Row 6. (1)

3.3.2 Write down the rule to calculate the last number in the n^{th} row. (2)

3.3.3 Hence write down the last number in Row 70. (2)

3.3.4 Calculate the sum of the numbers in Row 15. (3)
[15]

QUESTION 4

The following functions are given: $f(x)=\left(\frac{1}{3}\right)^x$ where $x \in \mathrm{R}$ and $g(x)=-\frac{3}{x}$ where $x<0$

4.1 Sketch the graphs of f and g on the same set of axes.
Show all intercepts with the axes and asymptotes where applicable. (5)

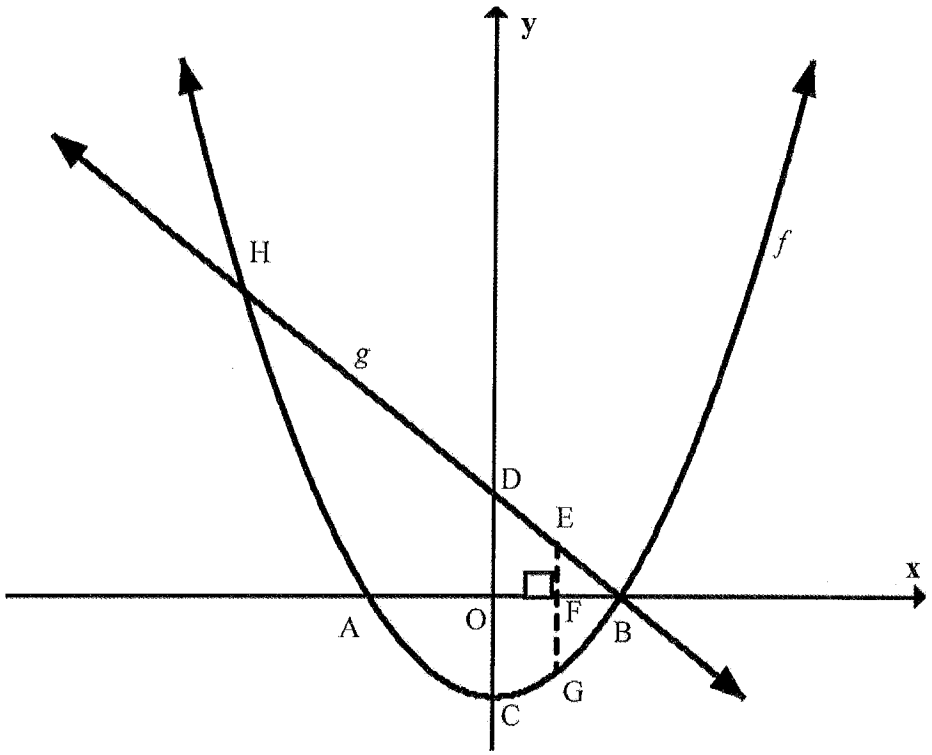
4.2 Write down the range of k if $k(x)=f(x)-2$. (2)

4.3 Write down the equations of the asymptotes of h if $h(x)=g(x)+3$. (2)

4.4 Write down the values of x for which $f(x) \leq g(x)$. (2)
[11]

QUESTION 5

Sketched below are the graphs of $f(x) = 2x^2 - 2$ and $g(x) = -2x + 2$.
The graph of f intersects the x–axis at A and B and the y–axis at C. The graph of g intersects the x–axis at B and the y–axis at D. f and g intersect at H and B.



Use the graphs and the information above to determine the following:

5.1 The coordinates of A and B. (4)

5.2 The coordinates of C. (1)

5.3 The coordinates of D. (1)

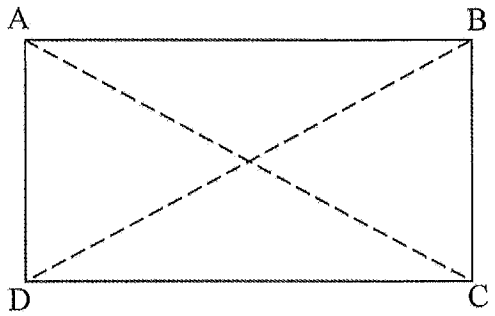
5.4 The length of EG if OF = $\frac{1}{2}$ unit and E lies on g and G lies on f. (5)

[11]

QUESTION 6

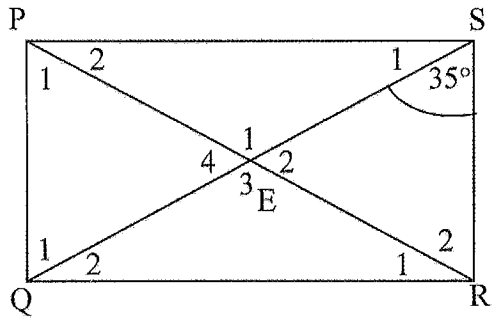
In this question, you must give a reason to justify each of your statements.

6.1 In the diagram below, ABCD is a rectangle. Use the diagram to prove the theorem which states that the diagonals of a rectangle are equal. THIS QUESTION MUST BE ANSWERED ON THE DIAGRAM SHEET.



(6)

6.2 In the diagram below, PQRS is a rectangle with E being the point of intersection of the diagonals. $Q\hat{S}R = 35°$. Calculate the size of $R\hat{P}S$ i.e. $\hat{P}_2$.



(4)
[10]

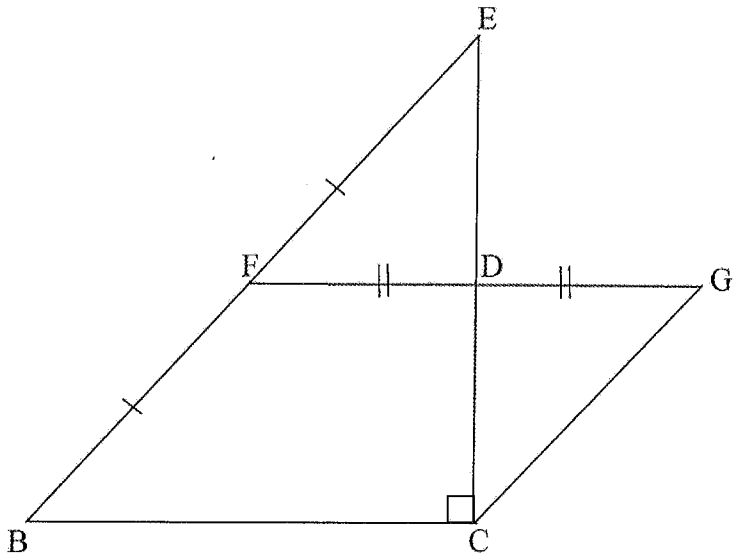
QUESTION 7

In this question, you must give a reason to justify each of your statements.

7.1 Complete the following statement:

The line drawn from the midpoint of one side of a triangle, parallel to the second side... (1)

7.2 In the figure below, BCE is a right-angled triangle. F is the midpoint of BE and D is a point on CE. FD is produced by its own length to G and forms parallelogram BCGF. FD = 30 mm and DE = 25 mm.



7.2.1 Why is ED = DC. (2)

7.2.2 Prove that EFCG is a rhombus. (2)

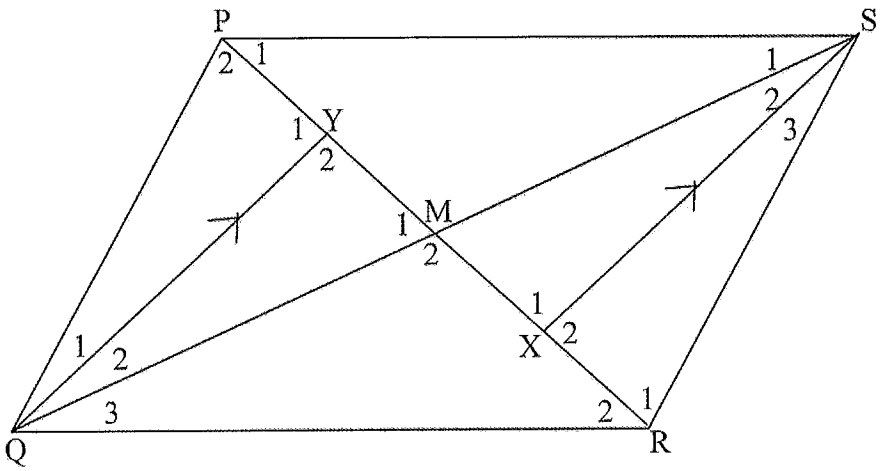
7.2.3 Calculate the area of parallelogram BCGF. (2)

7.2.4 Prove, by calculation, that:
the area of rhombus FCGE = the area of parallelogram BCGF. (2)
[9]

QUESTION 8

In this question, you must give a reason to justify each of your statements.

In the diagram below, PQRS is a parallelogram with diagonals PR and QS.
QY and XS are drawn such that QY // XS



Use the diagram to prove each of the following:

8.1 $\Delta QPY \equiv \Delta SRX$ (6)

8.2 QYSX is a parallelogram (3)

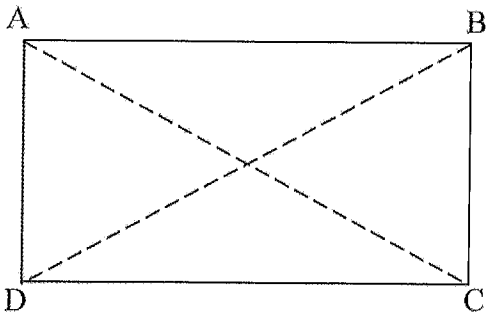
8.3 YM = MX (2)
[11]

TOTAL MARKS: 100

NAME: _____

DIAGRAM SHEET 1

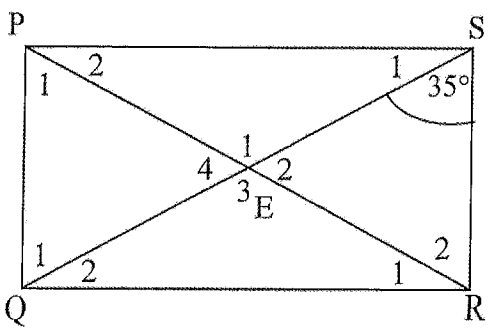
QUESTION 6.1



PLEASE TEAR ON DOTTED LINES

	Solution	Marks
6.1		(6)

QUESTION 6.2

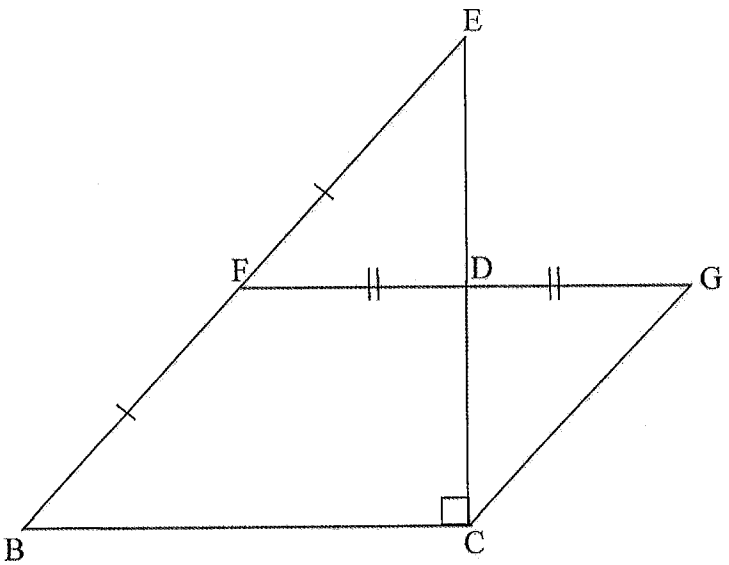


PLEASE TEAR ON DOTTED LINES

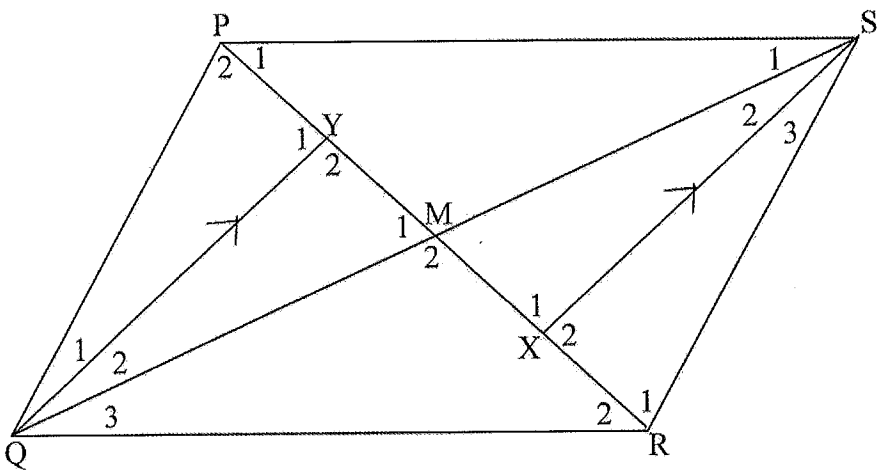
NAME: _____

DIAGRAM SHEET 2

QUESTION 7



QUESTION 8



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MEMORANDUM

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GRADE 10

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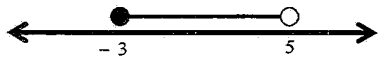
This memorandum consists of 9 pages.

QUESTION 1

1.1.1	$k=3$	✓ answer (1)
1.1.2	$k=2$	✓ answer (1)
1.1.3	$k=1$	✓ answer (1)
1.2	$(x+3y)(9y^2+x^2-3xy)$ $=9xy^2+x^3-3x^2y+27y^3+3x^2y-9xy^2$ $=x^3+27y^3$	✓ multiplying out ✓ answer (2) N.B Answer only: full marks
1.3	x^3+x^2-x-1 $=x^2(x+1)-1(x+1)$ $=(x+1)(x^2-1)$ $=(x+1)(x+1)(x-1)$	✓ $x^2(x+1)$ ✓ $-1(x+1)$ ✓ $(x+1)(x^2-1)$ ✓ answer (4)
1.4	$\frac{2x^2-2y^2}{x^2+y^2-2xy}\div\frac{4x^2+4xy}{xy-y^2}$ $=\frac{2x^2-2y^2}{x^2-2xy+y^2}\times\frac{xy-y^2}{4x^2+4xy}$ $=\frac{2(x^2-y^2)}{(x-y)(x-y)}\times\frac{y(x-y)}{4x(x+y)}$ $=\frac{2(x+y)(x-y)}{(x-y)(x-y)}\times\frac{y(x-y)}{4x(x+y)}$ $=\frac{y}{2x}$	✓ inverting fraction and multiplying ✓ $y(x-y)$ or $4x(x+y)$ ✓ $(x-y)(x-y)$ ✓ $2(x+y)(x-y)$ ✓answer (5)
		[14]

QUESTION 2

2.1	$V=\frac{1}{3}\pi r^2h$ $3V=\pi r^2h$ $r^2=\frac{3V}{\pi h}$ $r=\sqrt{\frac{3V}{\pi h}}$	✓ $3V=\pi r^2h$ ✓ $r=\sqrt{\frac{3V}{\pi h}}$ **NOTE:** Answer only : 2/2 marks (2)
2.2	$3x=y-4$ $\Rightarrow y=3x+4$ $2y=5+3x$ $2(3x+4)=5+3x$ $6x+8=5+3x$ $6x-3x=5-8$ $3x=-3$ $x=-1$ $y=3(-1)+4$ $=1$	✓ $y=3x+4$ ✓ substitution ✓ $x=-1$ ✓ $y=1$ (4)
2.3	$-3\le 3+2x<13$ $-3-3\le 3+2x-3<13-3$ $-6\le 2x<10$ $\frac{-6}{2}\le\frac{2x}{2}<\frac{10}{2}$ $-3\le x<5$  	✓subtracting 3 throughout ✓ dividing throughout by 2 ✓ answer ✓ number line with correct values, shading and points joined (4)
2.4	$2.3^{x+1}=162$ $3^{x+1}=\frac{162}{2}$ $3^{x+1}=81$ $3^{x+1}=3^4$ $x+1=4$ $x=3$	✓ dividing throughout by 2 ✓ $3^{x+1}=3^4$ ✓ $x+1=4$ ✓ $x=3$ (4)

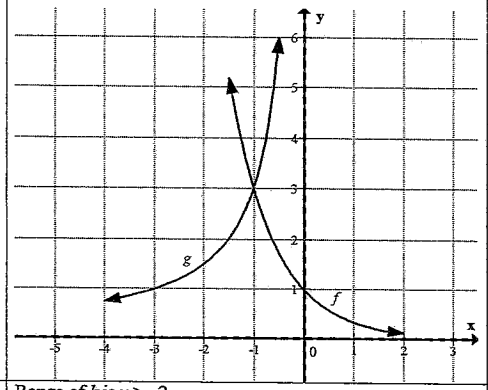
2.5.1	In order for a kite to be a rhombus, the sides must be equal in length. $x^2+3x-3=x^2+3$ $3x=6$ $x=2$	✓ equating the lengths ✓simplifying ✓answer (3)
2.5.2	The length of the sides are: $(2)^2+3$ $=7$ units **OR** The length of the sides are: $(2)^2+3(2)-3$ $=7$ units	✓ substitution ✓ answer ✓ substitution ✓ answer **NOTE:** Answer only : 2/2 marks (2)
		[19]

QUESTION 3

3.1.1	The next number is 37.	✓ 37 (1)
3.1.2	$Tn=7n+2$	✓ $7n$ ✓ $+2$ (2)
3.1.3	$7n+2=408$ $7n=406$ $n=58$	✓ $T_n=408$ ✓ $n=58$ (2)
3.2	The next term is $8x+7$.	✓ $8x$ ✓ $+7$ (2)
3.3.1	The last term in Row 6 is 21.	✓ 21 (1)
3.3.2	The rule for the last number in row n is $4n-3$.	✓ $4n$ ✓ -3 (2)
3.3.3	The last number in the 70^{th} row is $4(70)-3$ $=277$	✓ substitute 70 into rule found in 3.3.2 ✓ 277 (2) N.B. Answer only: full marks
3.3.4	Sum of numbers in Row $15=29^2$ $=841$	✓ 29 ✓squaring ✓ answer (3)
		[15]

QUESTION 4

$f(x)=\left(\frac{1}{3}\right)^x$ and $g(x)=-\frac{3}{x}$ for $x<0$.

4.1		✓ y – intercept of f ✓ another point and shape of f ✓ g passing through $(-1\,;3)$ ✓ another point and shape of g ✓ asymptotes (5)
4.2	Range of k is $y>-2$ **OR** $y\in(-2\,;\infty)$	✓✓ answer (2)
4.3	Equations of the asymptotes of h are $x=0$ and $y=3$.	✓ $x=0$ ✓ $y=3$ (2)
4.4	$f(x)\le g(x)$ for $-1\le x<0$	✓ critical values ✓ notation (2)
		[11]

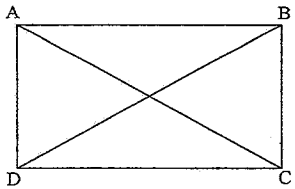
QUESTION 5

The sketches of $f(x)=2x^2-2$ and $g(x)=-2x+2$ are given.

5.1	The x-intercepts of f are: $2x^2-2=0$ $2(x^2-1)=0$ $2(x+1)(x-1)=0$ $x=-1$ or $x=1$ A$(-1;0)$ and B$(1\,;0)$	✓ $y=0$ ✓ factors ✓ $x=-1$ or $x=1$ coordinates of A and B (4)
5.2	C$(0\,;-2)$	✓ answer (1)
5.3	D$(0\,;2)$	✓ answer (1)
5.4	At E: $y=-2\left(\frac{1}{2}\right)+2$ $=1$ At G: $y=2\left(\frac{1}{2}\right)^2-2$ $=-1{,}5$ EG $=1-(-1{,}5)$ $=2{,}5$	✓ substitute $\frac{1}{2}$ into g ✓ 1 ✓ substitute $\frac{1}{2}$ into f ✓ $-1{,}5$ ✓ $2{,}5$ (5)
		[11]

QUESTION 6

6.1

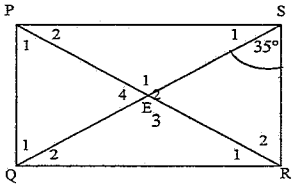


RTP: AC = BD
Construction: Join diagonals AC and BD.
Proof: In ΔADC and ΔBCD

(1) AD = BC	(opp sides of rectangle)	✓ construction ✓ correct triangles ✓ AD = BC with reason
(2) $A\hat{D}C = B\hat{C}D$	(angles of rectangle = 90°)	✓ $A\hat{D}C = B\hat{C}D$ with reason
(3) CD is common		✓ CD is common
ΔADC ≡ ΔBCD	(S; A; S)	✓(S; A; S)
∴ AC = BD		

(6)

6.2



$\hat{S}_1 = 90° - 35°$ $= 55°$	(angles in a rectangle = 90°)	✓ $\hat{S}_1 = 55°$ with reason
QS = PR	(diagonals of rectangle are equal)	✓ QS = PR with reason
∴ SE = PE	(diagonals bisect each other)	✓ SE = PE with reason
$\hat{P}_2 = \hat{S}_1$ $= 55°$	(angles opp equal sides)	✓ $\hat{P}_2 = 55°$ with reason

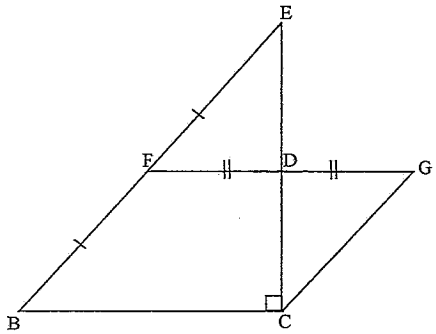
(4)

[10]

QUESTION 7

7.1	... bisects the third side	✓ answer (1)

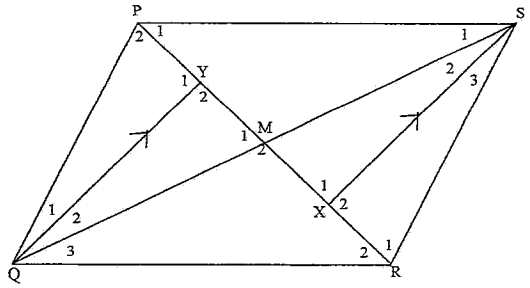
7.2



7.2.1	In ΔBEC, BF = FE (given) BC ∥ FD (opp sides of parm) ∴ DC = DE (line drawn from the midpoint of one side, parallel to the 2nd side, bisects the 3rd side)	✓statement with reason ✓statement with reason (2)
7.2.2	In EFCG, ED = DC and FD = DG $F\hat{D}C = 90°$ (co-interior angles FD ∥ BC) ∴ EFCG is a rhombus (diagonals of a quad bisect each other at right angles)	✓statement with reason ✓reason (2)
7.2.3	BC = 2 FD (opposite sides of parm) = 60 mm ∴ Area of parallelogram BCGF = BC × CD = 60 × 25 = 1 500 mm²	✓ BC = 60 mm ✓ answer (2)
7.2.4	Area of rhombus FCGE $= \frac{1}{2}FG \times EC$ $= \frac{1}{2}(60 \times 50)$ $= 1500\,mm^2$ ∴ Area of rhombus FCGE = Area of parallelogram BCGF	✓ $\frac{1}{2}(60 \times 50)$ ✓ answer (2)

[9]

QUESTION 8



8.1	$\hat{Y}_2 = \hat{X}_1$	alt angles QY ∥ SX	✓ $\hat{Y}_2 = \hat{X}_1$ with reason
	$180° - \hat{Y}_2 = 180° - \hat{X}_1$		✓ $\hat{Y}_1 = \hat{X}_2$
	$\hat{Y}_1 = \hat{X}_2$		
	In ΔQPY and ΔSRX		
	(1) QP = SR	(opp sides of parm)	✓ QP = SR ✓ reason
	(2) $\hat{P}_2 = \hat{R}_1$	(alt angles PQ ∥ RS)	✓ $\hat{P}_2 = \hat{R}_1$ with reason
	(3) $\hat{Y}_1 = \hat{X}_2$	(proven above)	
	ΔQPY ≡ ΔSRX	(A; A; S)	✓(A; A; S) (6)
8.2	QY = SX	(deduction from 8.1)	✓ QY = SX
	But QY ∥ SX	(given)	✓ QY ∥ SX
	∴ QYSX is a parallelogram	(one pair of opp sides in a quad are equal and parallel)	✓ reason (3)
8.3	YM = MX	(diagonals of parm QYSX bisect each other)	✓✓reason (2)
			[11]

TOTAL MARKS: 100