Basic Education

KwaZulu-Natal Department of Basic Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

COMMON TEST

JUNE 2016

NATIONAL SENIOR CERTIFICATE

GRADE 11

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages and 3 diagram sheets.

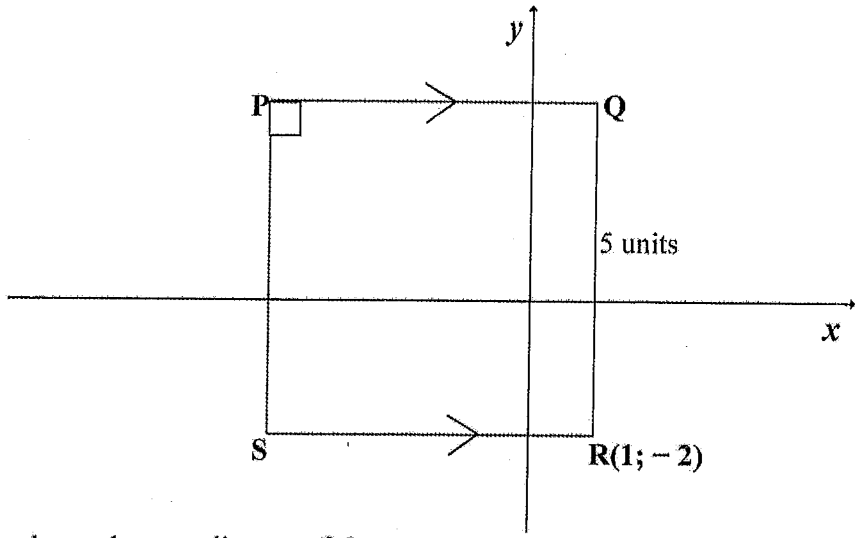
INSTRUCTIONS AND INFORMATION

Read the following instruction carefully before answering the questions.

1. The question paper consists of 6 questions.
2. Answer **ALL** the questions.
3. Clearly show all calculations and diagrams that you have used in determining your answers.
4. You may use an approved scientific calculator (non-programmable and non-graphical).
5. If necessary round off answers to **TWO** decimal places, unless otherwise stated.
6. Answers only will not be awarded full marks.
7. Diagrams not necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 In the diagram below PQRS is a square with sides of 5 units. The coordinates of R is $(1;-2)$. PQ is parallel to the x-axis.



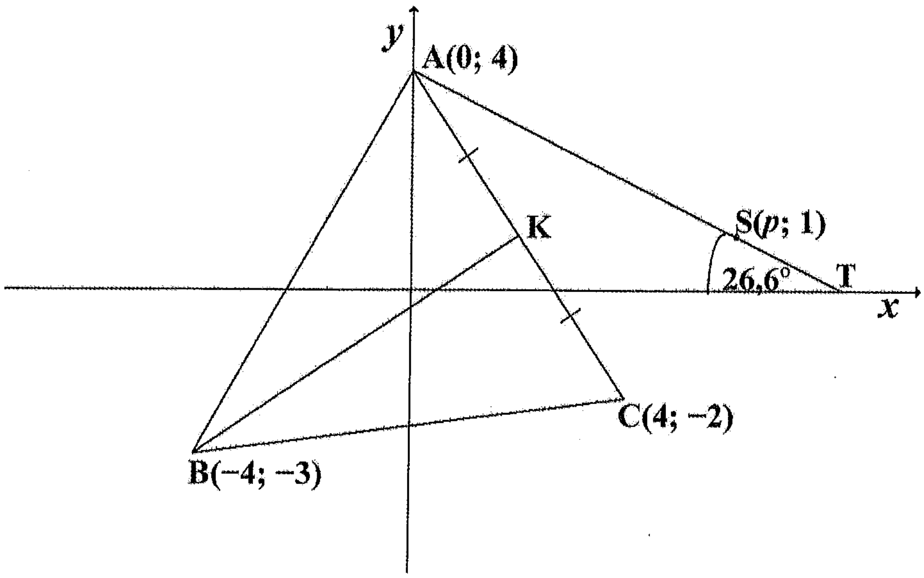
1.1.1 Write down the coordinates of Q. (1)

1.1.2 Write down the coordinates of S. (1)

1.1.3 Write down the equation of PQ. (1)

1.1.4 Write down the equation of QR. (1)

1.2 In the sketch below $A(0;4)$, $B(-4;-3)$ and $C(4;-2)$ are the vertices of ΔABC. K is the midpoint of AC. AT is drawn with T a point on the x-axis, such that the acute angle between AT and the x-axis is equal to $26,6°$. $S(p;\ 1)$ is a point on AT.



1.2.1 Determine the coordinates of point K. (2)

1.2.2 Calculate the length of AC, correct to 2 decimal places. (2)

1.2.3 Calculate the gradients of BK and AC and then show that $B\hat{K}C = 90°$. (5)

1.2.4 Determine the equation of line BK. (3)

1.2.5 Calculate the area of ΔABC, correct to 2 decimal places. (5)

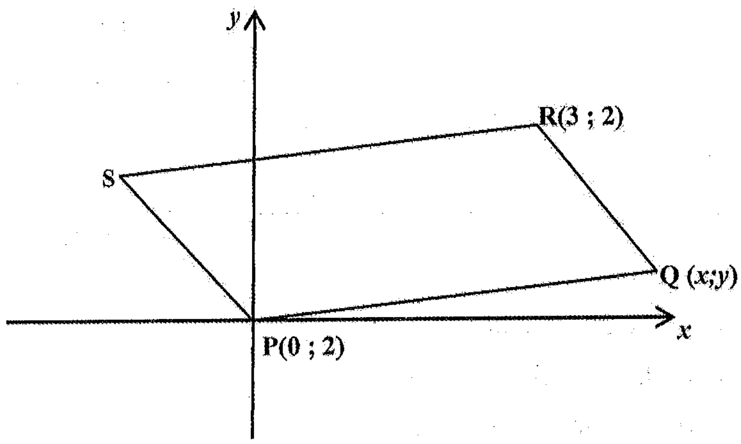
1.2.6 Calculate the value of p. (5)

[26]

QUESTION 2

2.1 PQRS is a parallelogram. The equation of PQ is $y = \frac{1}{4}x$.

The gradient of SP is equal to -1. R is the point $(3; 2)$.



2.1.1 Write down the gradient of RQ. (1)

2.1.2 Determine the equation of RQ. (2)

2.1.3 Calculate the coordinates of point Q. (4)

2.1.4 Calculate the size of $S\hat{P}Q$. (5)

2.2 Given A (6; 7), B (0; – 1) and C (4; p).

Calculate

2.2.1 The length of **AB**. (2)

2.2.2 The value of p if **AB = 2 BC**, $p < 0$ (5)

[19]

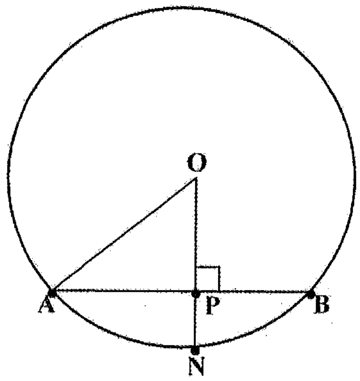
N.B: Give reasons for your statements and calculations in Questions 3 – 6.

QUESTION 3

3.1 COMPLETE: The line drawn from the centre of a circle to the midpoint of a chord

.... (1)

3.2



O is the centre of the circle. AB is a chord and $OP \perp AB$. OP is extended and intersects the circle at N. AB = 16 cm and PN = 2 cm.
Let OP = x.

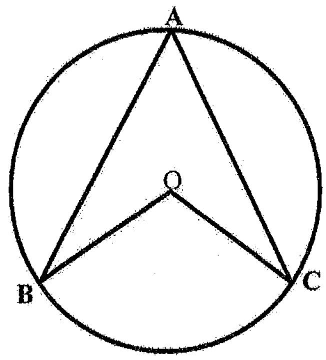
3.2.1 Calculate the length of **AP.** (2)

3.2.2 Calculate the length of the radius of the circle. (5)

[8]

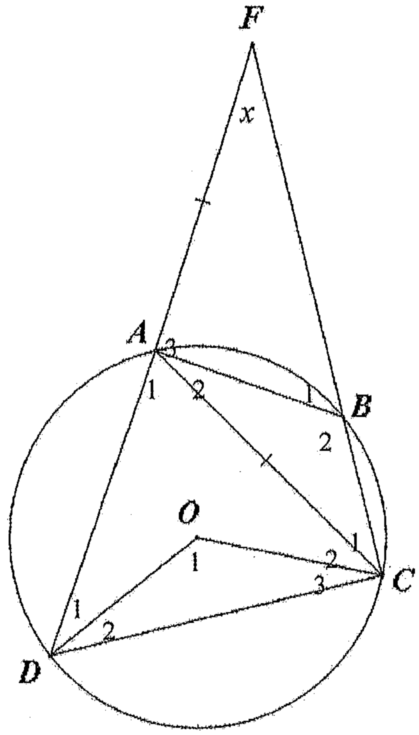
QUESTION 4

4.1 In the figure below O is the centre of the circle and A, B and C are three points on the circumference of the circle. Use the figure and prove the theorem that states that $B\hat{O}C = 2\hat{A}$.



(6)

4.2 In the figure O is the centre of the circle and ABCD is a cyclic quadrilateral. DA is produced to F such that FA = AC and CB is produced to meet DF at F.



4.2.1 If $\hat{F} = x$, write down, with reasons, in terms of x,

(a) the size of $\hat{A}_1$; (4)

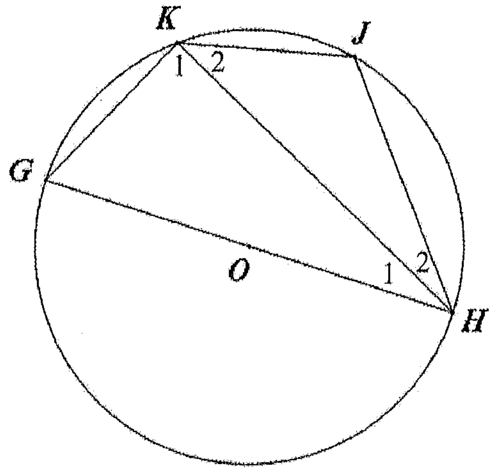
(b) the size of $\hat{O}_1$. (2)

4.2.2 If it is further given that FA = DA, find with reasons, the size of $\hat{A}_3$. (6)

[18]

QUESTION 5

5.1 In the diagram below O is the centre of the circle. G, H, J and K are points on the circumference of the circle.
GOH is a diameter.
Chords GH, JH, KH, KJ and GK have also been drawn.
$\hat{H}_1 = 26°$.

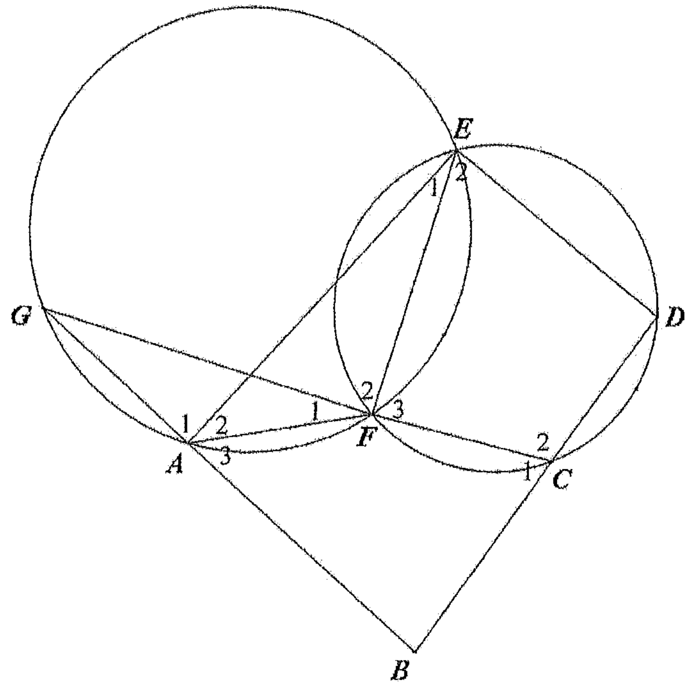


Calculate, with reasons, the size of

5.1.1 $\hat{K}_1$. (1)

5.1.2 $\hat{J}$. (3)

5.2 GAFE and FCDE are two circles. Chords GA, AF, EF, CF, CD and DE are drawn. GFC is a straight line. DC produced meets GA produced at B.
$\hat{E}_1 = 28°$ and $\hat{E}_2 = 64°$.



Calculate, with reasons, the size of

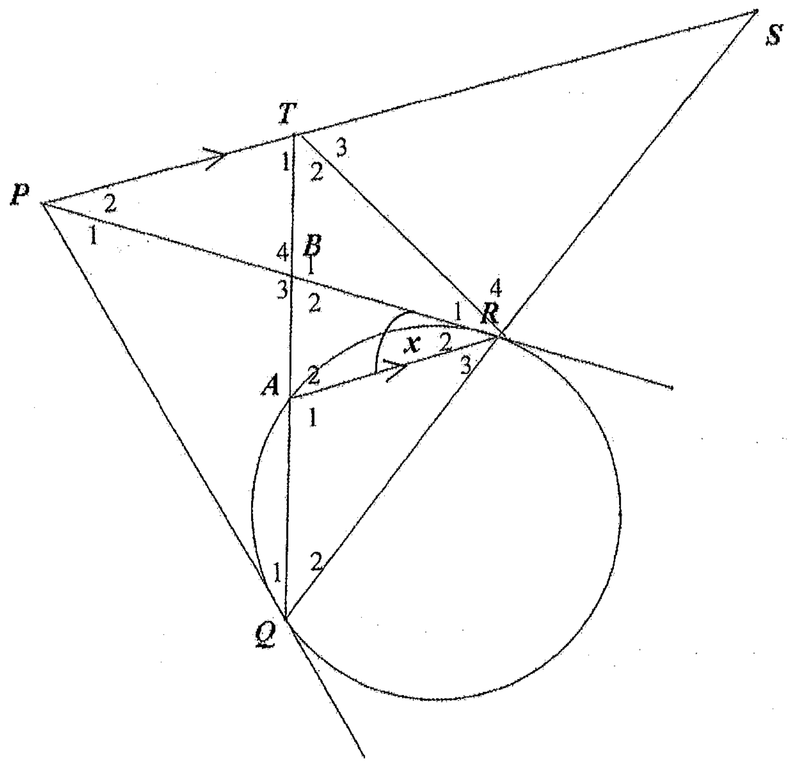
5.2.1 $\hat{G}$ (2)

5.2.2 $\hat{B}$ (4)

[10]

QUESTION 6

In the figure PR and PQ are two tangents drawn from point P to circle AQR. The straight line drawn through P parallel to AR meets QR produced at S, and QA produced at T. The tangent PR cuts QT at B.



Let $\hat{R}_2 = x$

6.1 Prove that PTRQ is a cyclic quadrilateral. (5)

6.2 If it is further given that QA = RA, prove that:

6.2.1 $\hat{S} = x$ (3)

6.2.2 PQ = RS (5)

6.2.3 PTS is a tangent to circle TAR. (6)
[19]

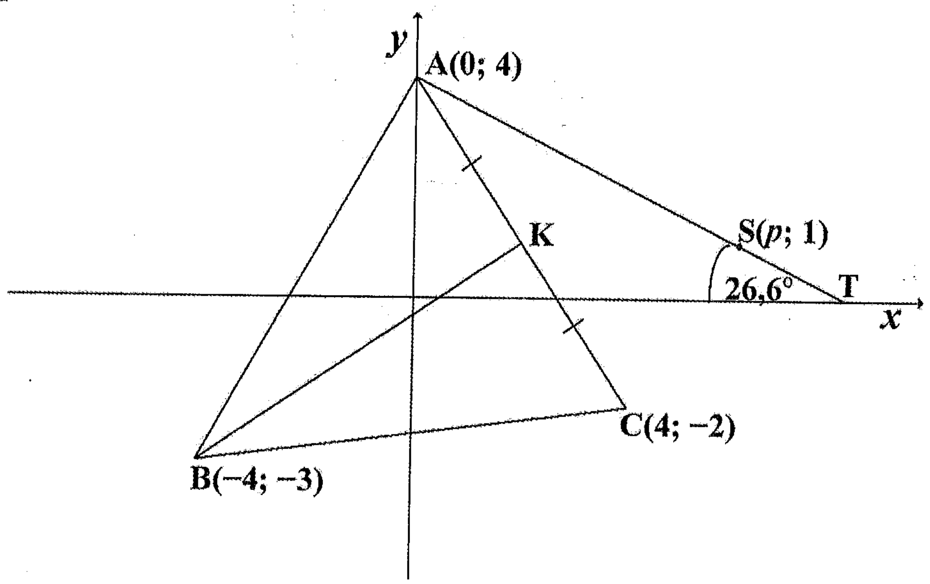
TOTAL MARKS: [100]

DIAGRAM SHEETS: HAND IN WITH YOUR ANSWER BOOK

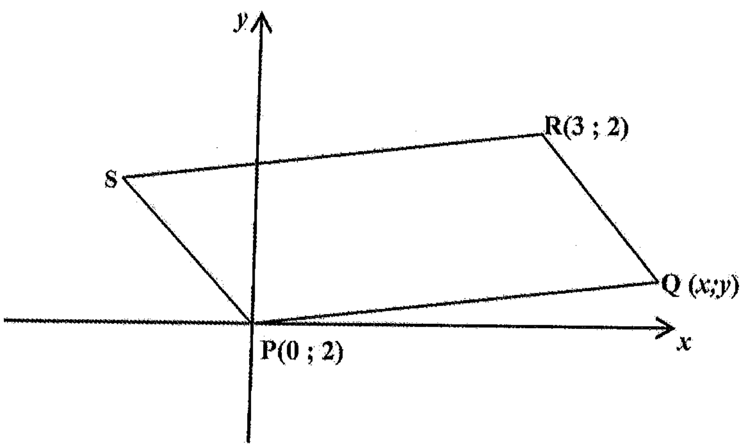
NAME: ______________________ **GRADE:** ______________

TEAR-OFF SHEET

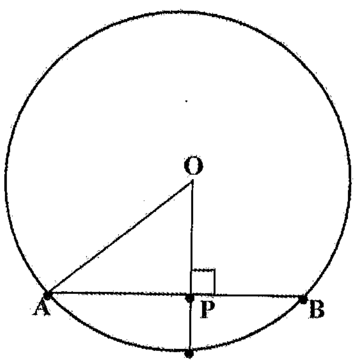
QUESTION 1.2



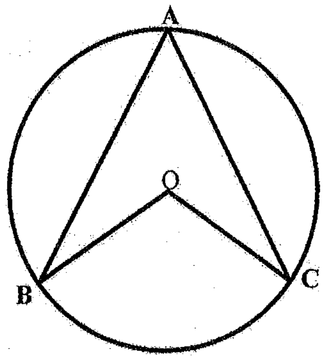
QUESTION 2.1



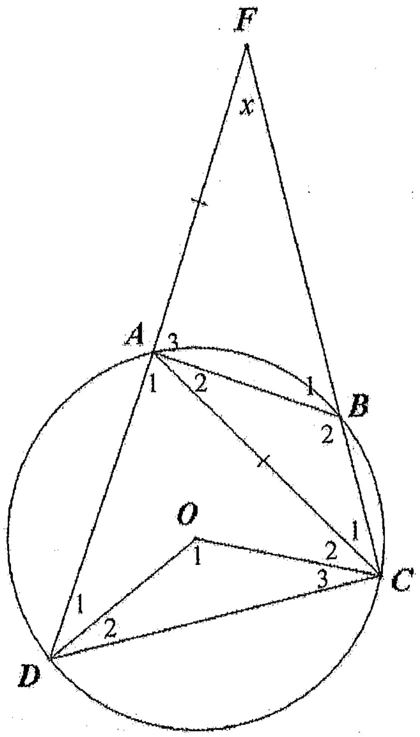
QUESTION 3.2



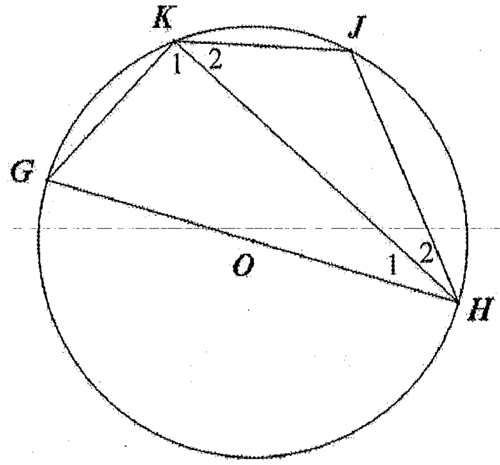
QUESTION 4.1



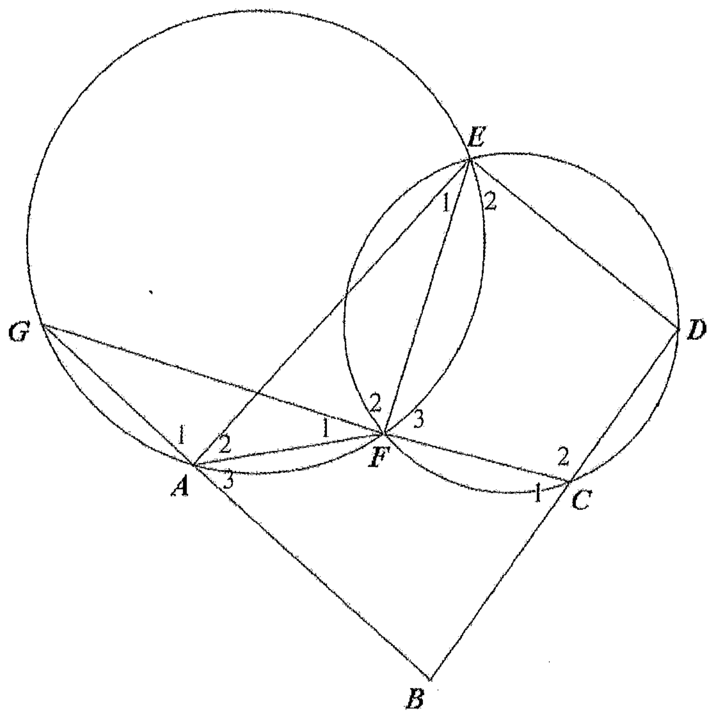
QUESTION 4.2



QUESTION 5.1

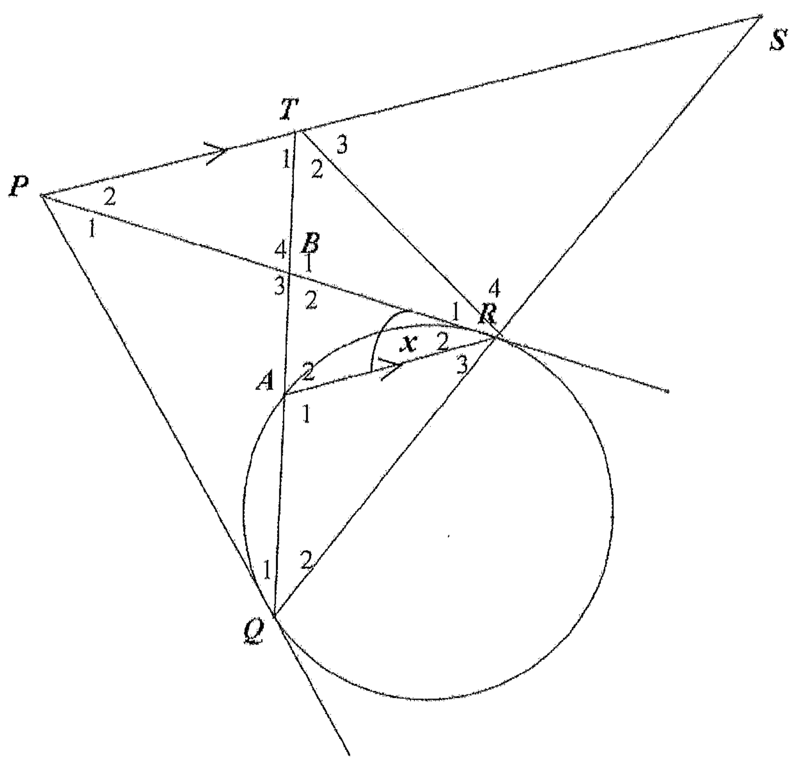


QUESTION 5.2



TEAR-OFF SHEET

QUESTION 6



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MATHEMATICS P2

JUNE 2016

MARKING MEMORANDUM

NATIONAL
SENIOR CERTIFICATE

GRADE 11

MARKS : 100

This memorandum consists of 8 pages.

Symbol	Explanation
CA	Consistent accuracy
A	Accuracy
S	Statement
R	Reason
S/R	Statement with reason

QUESTION1

1.1.1 Q(1;3)	1A for answer (1)
1.1.2 S(−4;−2)	1A for answer (1)
1.1.3 $y=3$	1A for answer (1)
1.1.4 $x=1$	1A for answer (1)
1.2.1 $\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $=\left(\frac{0+4}{2};\frac{4+(-2)}{2}\right)$ ✓ $=(2;1)$ ✓	1A for substitution 1CA for answer (2)
1.2.2 $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(4-0)^2+(-2-4)^2}$ ✓ $=\sqrt{52}$ $=7{,}21$ ✓	1A for substitution 1CA for answer (2)
1.2.3 $m_{BK}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{1-(-3)}{2-(-4)}$ ✓ $=\frac{4}{6}$ $=\frac{2}{3}$ ✓ $m_{AC}=\frac{-2-4}{4-0}$ ✓ $=\frac{-6}{4}$ $=-\frac{3}{2}$ ✓ $m_{BK}\times m_{AC}=\frac{2}{3}\times-\frac{3}{2}=-1$ ✓ $\therefore BK\perp AC$ or $B\hat{K}C=90°$	1CA for substitution 1CA for value of m_{BK} 1CA for substitution 1CA for value of m_{AC} 1A for showing that product of two gradients equals – 1, and concluding (5)

<table>
<tr><td>1.2.4
$y = mx + c$
$= \frac{2}{3}x + c$ ✓
Substitute $(-4; -3)$:
$-3 = \frac{2}{3}(-4) + c$ ✓
$c = -3 + \frac{8}{3} = -\frac{1}{3}$
$y = \frac{2}{3}x - \frac{1}{3}$ ✓

OR

$y - y_1 = m(x - x_1)$
$= \frac{2}{3}(x - x_1)$ ✓
Substitute $(-4; -3)$:
$y - (-3) = \frac{2}{3}[x - (-4)]$ ✓
$y + 3 = \frac{2}{3}x + \frac{8}{3}$
$y = \frac{2}{3}x - \frac{1}{3}$ ✓</td><td>1CA for substitution of gradient of BK
1CA for substitution of coordinates of B (or K)
1CA for answer
(3)
OR
1CA for substitution of gradient of BK
1CA for substitution of coordinates of B (or K)
1CA for answer
(3)</td></tr>
<tr><td>1.2.5
Length of BK
$= \sqrt{[2-(-4)]^2 + [1-(-3)]^2}$ ✓
$= \sqrt{6^2 + 4^2}$
$= \sqrt{52}$
$= 7,21$ ✓
Area of ΔABC
$= \frac{1}{2} \times \text{base} \times \text{height}$ ✓
$= \frac{1}{2} \times AC \times BK$
$= \frac{1}{2} \times 7,21 \times 7,21$ ✓
$= 25,99$ ✓</td><td>1CA for substitution
1CA for answer to length of BK
1A for formula
1CA for substitution
1CA for answer (also accept: 26)
(5)</td></tr>
</table>

<table>
<tr><td>1.2.6
$m_{AT} = \tan 153,4°$
$= -\tan 26,6°$
$= -0,50$ ✓
Substitute (0; 4) and find equation of AT:
$y = mx + c$
$4 = -0,50(0) + c$ ✓
$c = 4$
$y = -0,50x + 4$ ✓
S lies on the line:
$1 = -0,50(p) + 4$ ✓
$p = 6$ ✓</td><td>1A for gradient of AT
1CA for substitution of (0; 4) and gradient
1CA for equation
1CA for substitution of (p; 1) into equation
1CA for answer
(5)
[26]</td></tr>
</table>

QUESTION 2

<table>
<tr><td>2.1.1 $m_{RQ} = m_{PQ} = -1$ ✓</td><td>1A for equating gradients
(1)</td></tr>
<tr><td>2.1.2
$y - y_1 = m(x - x_1)$
$y - 2 = -1(x - 3)$ ✓
$y = -x + 5$ ✓
OR
$y = mx + c$
$2 = (-1)(3) + c$ ✓
$c = 5$
$y = -x + 5$ ✓</td><td>1A for substitution
1CA for equation
OR
1CA for substitution
1CA for answer
(2)</td></tr>
<tr><td>2.1.3
$-x + 5 = \frac{1}{4}x$ ✓
$4x + 20 = x$ ✓
$-5x = -20$
$x = 4$ ✓
$y = 1$ ✓
Q(4; 1)</td><td>1CA for equating the equations
1 CA for simplification
1CA for value of x
1CA for value of y
(4)</td></tr>
</table>

2.1.4	$\tan X\hat{P}Q = \frac{1}{4}$ ✓ $X\hat{P}Q = 14{,}04°$ ✓ $\tan X\hat{P}S = -1$ $X\hat{P}S = 135°$ ✓ $S\hat{P}Q = 135° - 14{,}04°$ ✓ $S\hat{P}Q = 120{,}96°$ ✓	1A for substitution in correct equation 1A for correct angle 1A for correct angle 1CA for subtraction 1CA for answer (5)
2.2.1	$AB = \sqrt{(6-0)^2 + (7+1)^2}$ ✓ $= 10$ ✓	1A for substitution 1A for answer (2)
2.2.2	$AB = 2\,BC$ $AB^2 = (2\,BC)^2$ ✓ $100 = 4[(0-4)^2 + (-1-p)^2]$ ✓ ✓ $100 = 68 + 8p + 4p^2$ $4p^2 + 8p - 32 = 0$ ✓ $4(p^2 + 2p - 8) = 0$ $(p+4)(p-2) = 0$ $p = -4$ or $p = 2$ $p = -4$ ✓	1M for squaring 1A for substitution of AB^2 1A for substitution of $(2BC)^2$ 1CA for simplification 1CA for value of p
OR	$BC = \frac{1}{2}AB$ $= 5$ ✓ $5 = \sqrt{(0-4)^2 + (-1-p)^2}$ ✓ $25 = (0-4)^2 + (-1-p)^2$ ✓ $25 = 4^2 + 1^2 + 2p + p^2$ $p^2 + 2p - 8 = 0$ ✓ $(p+4)(p-2) = 0$ $p = 2$ or -4 $p = -4$ ✓	**OR** 1CA length of BC 1CA for substitution 1CA squaring 1CA simplification 1CA value of p (5) [19]

QUESTION 3

3.1	is perpendicular to the chord ✓	1S for correct conclusion (1)
3.2.1	$AP = 8$ ✓ (line from centre of circle to midpoint of chord) ✓	1S ; 1R (2)
3.2.2	$AO^2 = OP^2 + AP^2$ (Pythagoras) ✓ $(x+2)^2 = x^2 + 8^2$ ✓ ✓ $x^2 + 4x + 4 = x^2 + 64$ ✓ $4x = 60$ $x = 15$ radius $= 15 + 2 = 17$ ✓	1S/R for Pythagoras 2A for correct substitution 1CA for simplification 1CA for answer (5) [8]

QUESTION 4

4.1	Construction: Draw AO and extend to D ✓ Proof: Let $B\hat{A}O = x$ $OB = OA$ (radii) $\therefore \hat{B} = x$ (angles opposite equal sides) ✓ $\therefore \hat{O}_2 = 2x$ (exterior angle of Δ) ✓ Similarly let $C\hat{A}O = y$ $OA = OC$ (radii) $\therefore \hat{C} = y$ (angles opposite equal sides) $\therefore \hat{O}_1 = 2y$ ✓ (exterior angle of Δ) But $\hat{O}_1 + \hat{O}_2 = 2x + 2y$ ✓ $\therefore B\hat{O}C = 2(x+y)$ ✓ $= 2B\hat{A}C$ $= 2\hat{A}$	1M for correct construction 1S/R 1S/R 1 for $\hat{O}_1 = 2y$ 1 for $\hat{O}_1 + \hat{O}_2 = 2x + 2y$ 1 for $B\hat{O}C = 2(x+y)$ (6)

4.2.1(a)	$\hat{C}_6 = \hat{F} = x$ ✓ (FA = CA; angles opposite equal sides) ✓ $\hat{A}_1 = 2x$ ✓ (exterior angle of Δ) ✓	1S ; 1R 1S ; 1R (4)
4.2.1(b)	$D\hat{O}C = 4x$ ✓ (∠ at centre = 2 x ∠ at circumference) ✓	1S ; 1R (2)
4.2.2	DA = CA (both = FA) $\therefore A\hat{D}C = A\hat{C}D$ ✓ (angles opposite equal sides) $= \frac{180° - \hat{A}_1}{2}$ ✓ (sum of ∠s of ΔADC) ✓ $= \frac{180° - 2x}{2}$ $= 90° - x$ $B\hat{C}D = 90° - x + x$ ✓ $= 90°$ $\hat{A}_3 = 90°$ ✓ (exterior ∠ of cyclic quad) ✓	1S 1S; 1R 1S for calculating $B\hat{C}D$ 1S ; 1R (6) **[18]**

QUESTION 5

5.1.1	$\hat{K}_1 = 90°$ (angle in a semicircle) ✓	1S/R (1)
5.1.2	$\hat{G} = 180° - (\hat{K}_1 + \hat{H}_1)$ (sum of angles of Δ) $= 180° - (90° + 26°)$ ✓ $= 74°$ $\hat{J} = 180° - \hat{G}$ (opposite ∠'s of cyclic quad) ✓ $= 180° - 74°$ $= 106°$ ✓	1S 1S/R 1 answer (3)
5.2.1	$\hat{G} = \hat{E}_1 = 28°$ ✓ (angles in the same segment) ✓	1S ; 1R (2)
5.2.2	$\hat{C}_1 = \hat{E}_2 = 64°$ ✓ (exterior ∠ of cyclic quad) ✓ $\hat{B} = 180° - (\hat{G} + \hat{C}_1)$ (sum of ∠s of Δ) ✓ $= 180° - (28° + 64°)$ $= 88°$ ✓	1S ; 1R 1S/R 1 answer (4) **[10]**

QUESTION 6

6.1	$\hat{Q}_2 = \hat{R}_2 = x$ ✓ (tan-chord-theorem) ✓ $\hat{P}_2 = \hat{R}_2 = x$ ✓ (alt. ∠s; AR ∥ PT) ✓ $\therefore \hat{P}_2 = \hat{Q}_2$ (both = x) PTRQ is a cyclic quadrilateral (converse: angles in the same segment) ✓	1S ; 1R 1S ; 1R 1R (5)
6.2.1	$\hat{Q}_2 = \hat{R}_3 = x$ (angles opposite equal sides) ✓ $P\hat{R}S = 180° - (\hat{R}_2 + \hat{R}_3)$ (QRS is a straight line) $= 180° - 2x$ ✓ $\hat{S} = 180° - (180° - 2x + x)$ (sum of ∠s of ΔPRS) $= x$ ✓	1S/R 1S 1S (3)
6.2.1	$\therefore \hat{P}_2 = \hat{S}$ ✓ (both = x) ∴ PR = RS (sides opposite = ∠s) ✓ But PR = PQ ✓ (2 tangents from same point) ✓ ∴PQ = RS ✓	1S 1S/R 1S; 1R 1S (5)
6.2.3	$\hat{Q}_1 = \hat{R}_3 = x$ (tan chord theorem) ✓ $\therefore P\hat{Q}R = 2x$ ✓ ($Q_2 = x$; already proved) $P\hat{Q}R = \hat{T}_3$ (exterior ∠ of cyclic quad) ✓ $\therefore \hat{T}_3 = 2x$ $\hat{A}_2 = \hat{Q}_2 + \hat{R}_3$ (exterior angle of Δ) ✓ $= x + x$ $= 2x$ $\hat{T}_3 = \hat{A}_2$ (both = $2x$) ✓ PTS is a tangent to circle TAR (converse: tan chord theorem) ✓ **OR** $\hat{Q}_1 = \hat{R}_3 = x$ (tan chord theorem) ✓ $\hat{Q}_1 = \hat{R}_1 = x$ (angles in the same segment) ✓ $T\hat{R}A = \hat{R}_1 + \hat{R}_2 = 2x$ ✓ But $\hat{T}_1 = \hat{R}_2 + \hat{R}_3 = 2x$ (angles in the same segment) ✓ $\therefore \hat{T}_1 = T\hat{R}A$ (both = $2x$) ✓ ∴PTS is a tangent to circle TAR (converse: tan chord theorem) ✓	1S/R 1S 1S/R 1S/R 1S 1R **OR** 1S/R 1 S/R 1S 1S/R 1S 1R (6) **[19]** **TOTAL: 100**