

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

NOVEMBER 2016

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and a 22-page answer book.



INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. Write neatly and legibly.



QUESTION 1

The table below shows the number of cans of food collected by 9 classes during a charity drive.

5	8	15	20	25	27	31	36	75
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- 1.1 Calculate the range of the data. (1)
- 1.2 Calculate the standard deviation of the data. (2)
- 1.3 Determine the median of the data. (1)
- 1.4 Determine the interquartile range of the data. (3)
- 1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (3)
- 1.6 Describe the skewness of the data. (1)
- 1.7 Identify outliers, if any exist, for the above data. (1)
- [12]**

QUESTION 2

The table below shows the time (in minutes) that 200 learners spent on their cellphones during a school day.

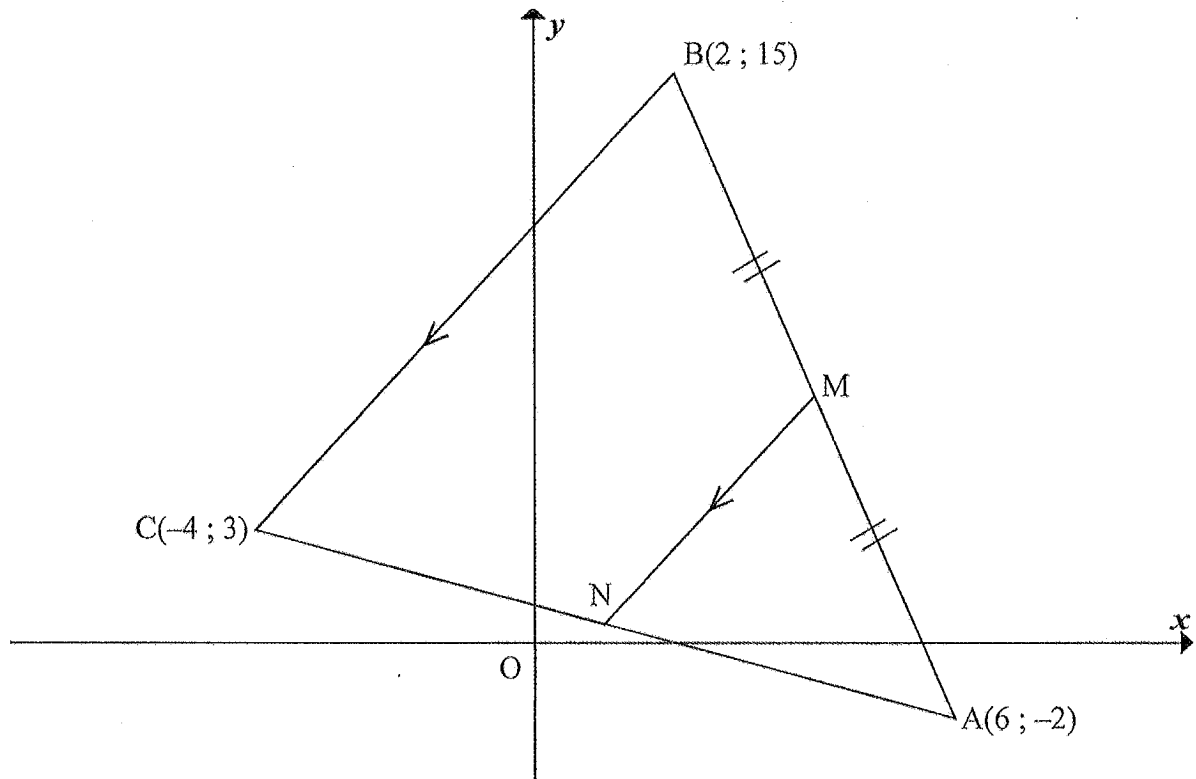
TIME SPENT (IN MINUTES)	FREQUENCY
$95 < x \leq 105$	15
$105 < x \leq 115$	27
$115 < x \leq 125$	43
$125 < x \leq 135$	52
$135 < x \leq 145$	28
$145 < x \leq 155$	21
$155 < x \leq 165$	10
$165 < x \leq 175$	4

- 2.1 Complete the cumulative frequency column in the table provided in the ANSWER BOOK. (2)
- 2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided. (3)
- 2.3 Use the cumulative frequency graph to determine the value of the lower quartile. (2)
- 2.4 Determine, from the cumulative frequency graph, the number of learners who used their cellphones for more than 140 minutes. (2)
- [9]**



QUESTION 3

In the diagram, $A(6; -2)$, $B(2; 15)$ and $C(-4; 3)$ are the vertices of $\triangle ABC$.
 M is the midpoint of AB . N is a point on CA such that $MN \parallel BC$.



- 3.1 Determine the coordinates of M , the midpoint of AB . (2)
- 3.2 Determine the gradient of line MN . (3)
- 3.3 Hence, or otherwise, determine the equation of line MN , in the form $y = mx + c$. (2)
- 3.4 Calculate, with reasons, the coordinates of point N . (4)
- 3.5 If $ABCD$ (in that order) is a parallelogram, determine the coordinates of point D . (4)
- [15]**

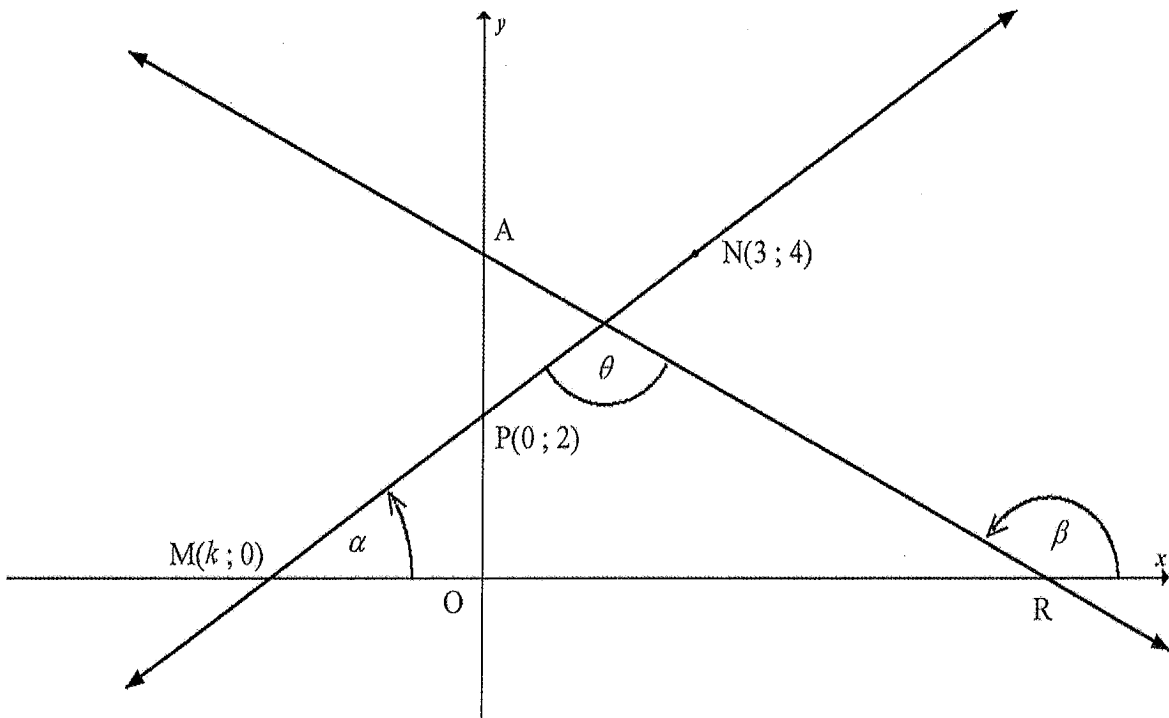


QUESTION 4

In the diagram, R and A are the x - and y -intercepts respectively of the straight line AR.

The equation of AR is $y = -\frac{1}{2}x + 4$. Another straight line cuts the y -axis at $P(0; 2)$ and passes through the points $M(k; 0)$ and $N(3; 4)$.

α and β are the angles of inclination of the lines MN and AR respectively.

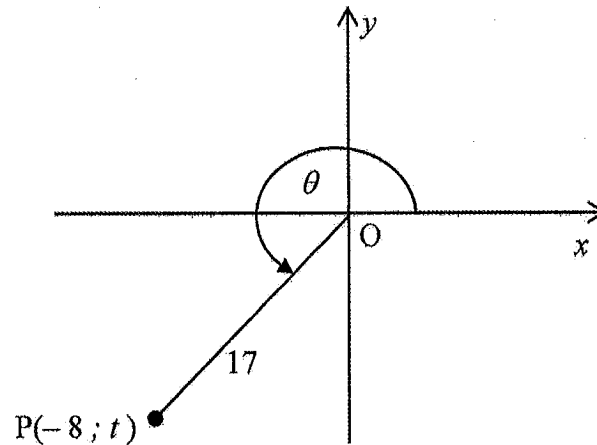


- 4.1 Given that M, P and N are collinear points, calculate the value of k . (3)
- 4.2 Determine the size of θ , the obtuse angle between the two lines. (4)
- 4.3 Calculate the length of MR. (3)
- 4.4 Calculate the area of $\triangle MNR$. (3)
- [13]**



QUESTION 5

- 5.1 In the diagram below, $P(-8 ; t)$ is a point in the Cartesian plane such that $OP = 17$ units and reflex $\widehat{XOP} = \theta$.



- 5.1.1 Calculate the value of t . (2)
- 5.1.2 Determine the value of each of the following WITHOUT using a calculator:
- (a) $\cos(-\theta)$ (2)
- (b) $1 - \sin \theta$ (2)
- 5.2 If $\sin 17^\circ = a$, WITHOUT using a calculator, express the following in terms of a :
- 5.2.1 $\tan 17^\circ$ (3)
- 5.2.2 $\sin 107^\circ$ (2)
- 5.2.3 $\cos^2 253^\circ + \sin^2 557^\circ$ (4)
- 5.3 Simplify fully, WITHOUT the use of a calculator:
- $$\frac{\cos(-225^\circ) \cdot \sin 135^\circ + \sin 330^\circ}{\tan 225^\circ}$$
- (6)

5.4 Prove the identity: $\frac{1}{(\cos x + 1)(\cos x - 1)} = \frac{-1}{\tan^2 x \cdot \cos^2 x}$ (4)

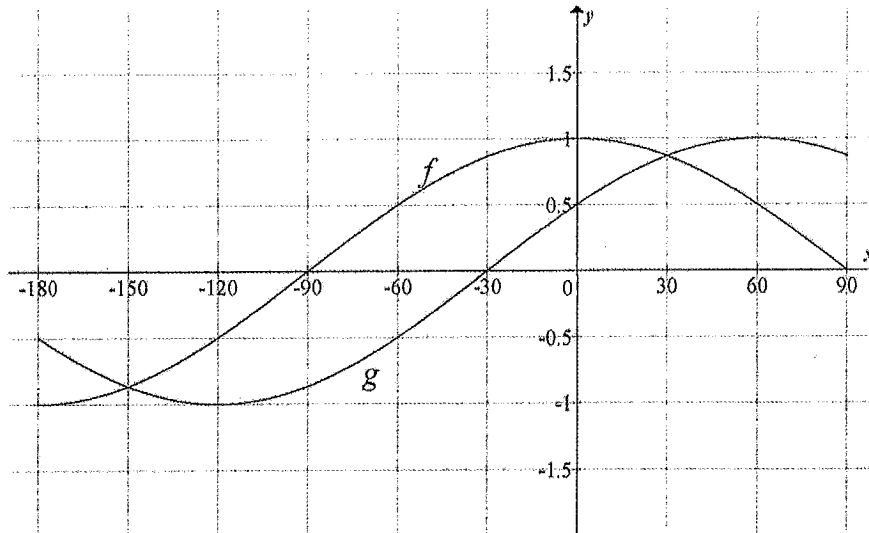
5.5 Determine the general solution for $2\sin x \cdot \cos x = \cos x$. (6)

[31]



QUESTION 6

In the diagram the graphs of $f(x) = \cos x$ and $g(x) = \sin(x + b)$ are drawn for the interval $-180^\circ \leq x \leq 90^\circ$.



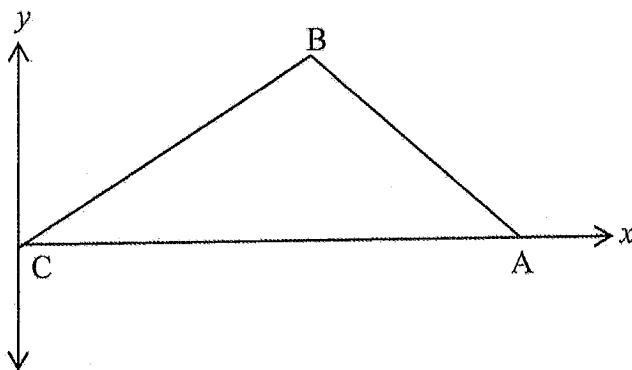
- 6.1 Write down the value of b . (1)
- 6.2 Write down the period of g . (1)
- 6.3 Write down the value(s) of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) - g(x) = 0$. (2)
- 6.4 For which values of x in the interval $-180^\circ \leq x \leq 90^\circ$ is $\sin(90^\circ - x) > g(x)$? (3)
- 6.5 The graph of h is obtained by shifting f 3 units upwards. Determine the range of h . (2)

[9]



QUESTION 7

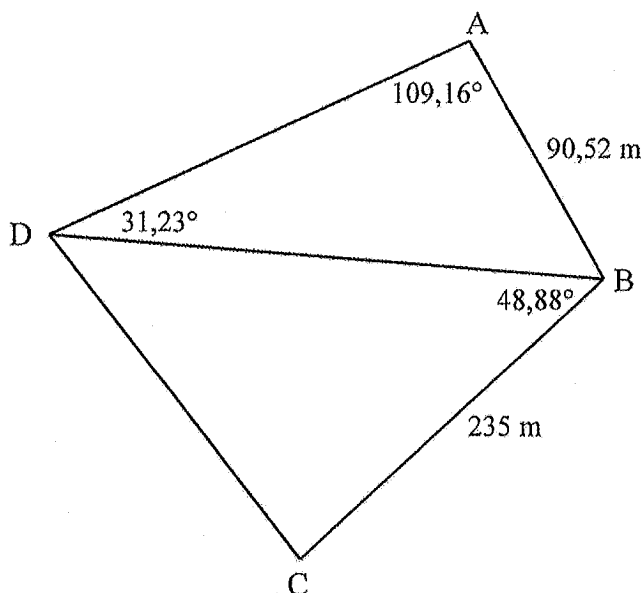
7.1 In the figure below, acute-angled $\triangle ABC$ is drawn having C at the origin.



7.1.1 Prove that $c^2 = a^2 + b^2 - 2ab \cos C$. (6)

7.1.2 Hence, deduce that $1 + \cos C = \frac{(a+b+c)(a+b-c)}{2ab}$. (4)

7.2 Quadrilateral ABCD is drawn with $BC = 235$ m and $AB = 90,52$ m. It is also given that $\hat{A}DB = 31,23^\circ$; $\hat{D}AB = 109,16^\circ$ and $\hat{C}BD = 48,88^\circ$.



Determine the length of:

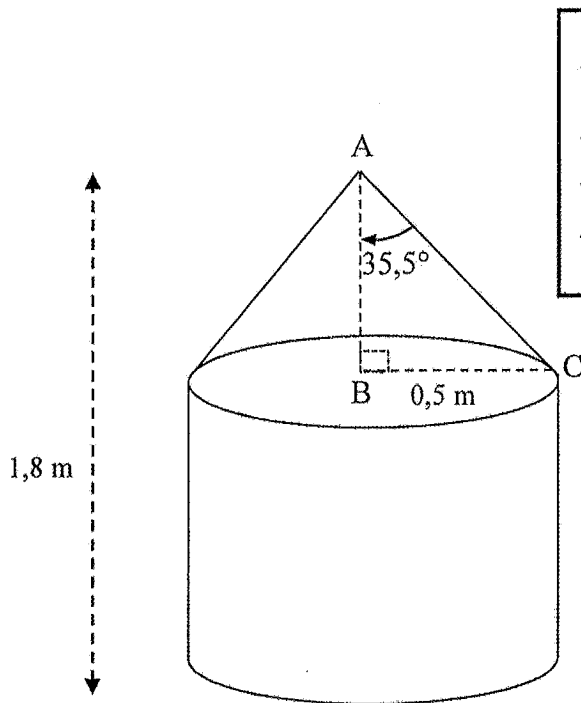
7.2.1 BD (3)

7.2.2 CD (3)
[16]



QUESTION 8

The diagram below shows a water tank which is made up of a cylinder and cone having equal radii. The height of the tank is 1,8 m and the radius is 0,5 m. The angle between the perpendicular height, AB, and the slant height, AC, of the conical section is $35,5^\circ$.



$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Total surface area of cone} = \pi r^2 + \pi r s$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Total surface area of cylinder} = 2\pi r^2 + 2\pi r h$$

- 8.1 Calculate the perpendicular height, AB, of the cone. (2)
- 8.2 When the tank is full, an electric pump switches on and pumps the water from the tank into an irrigation system at a rate of $0,52 \text{ m}^3/\text{h}$. The pump automatically switches off when the tank is $\frac{1}{4}$ full. Calculate how long, in hours, the pump feeds water into the irrigation system. (4)

[6]



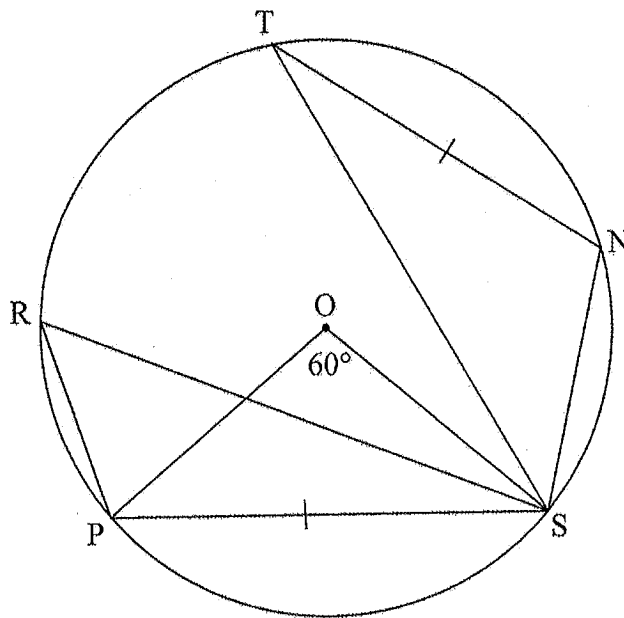
Give reasons for your statements and calculations in QUESTIONS 9, 10, 11 and 12.

QUESTION 9

9.1 Complete the statement so that it is TRUE:

The angle subtended by an arc at the centre of a circle is ... (2)

9.2 O is the centre of circle TNSPR. $\hat{POS} = 60^\circ$ and $PS = NT$.



Calculate the size of:

9.2.1 \hat{PRS} (2)

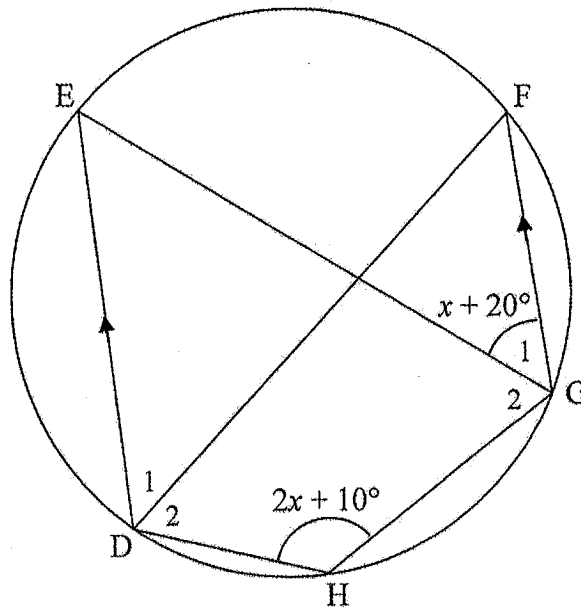
9.2.2 \hat{NST} (2)
[6]



QUESTION 10

D, E, F, G and H are points on the circumference of the circle.

$\hat{G}_1 = x + 20^\circ$ and $\hat{H} = 2x + 10^\circ$. $DE \parallel FG$.



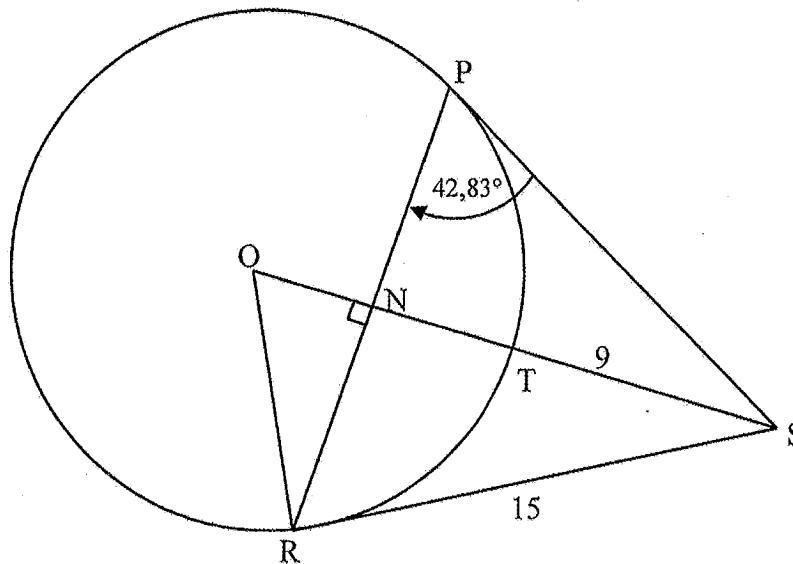
10.1 Determine the size of \hat{DEG} in terms of x . (2)

10.2 Calculate the size of \hat{DHG} . (4)
[6]



QUESTION 11

O is the centre of the circle PTR. N is a point on chord RP such that $ON \perp PR$.
RS and PS are tangents to the circle at R and P respectively.
RS = 15 units; TS = 9 units; $\hat{RPS} = 42,83^\circ$.

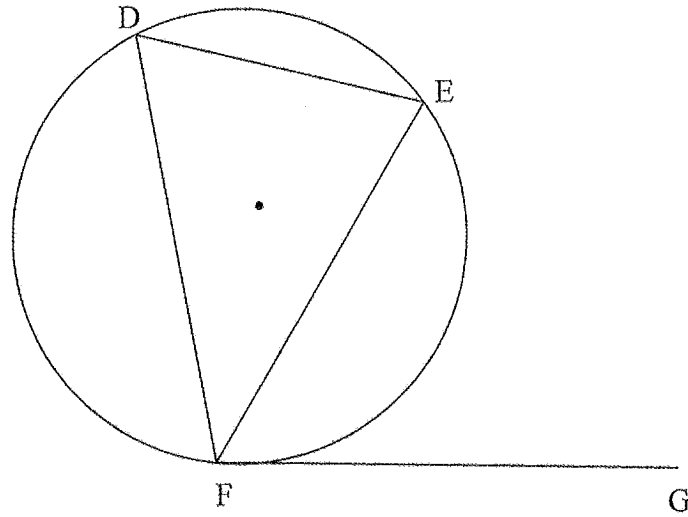


- 11.1 Calculate the size of \hat{NOR} . (5)
- 11.2 Calculate the length of the radius of the circle. (4)
- [9]



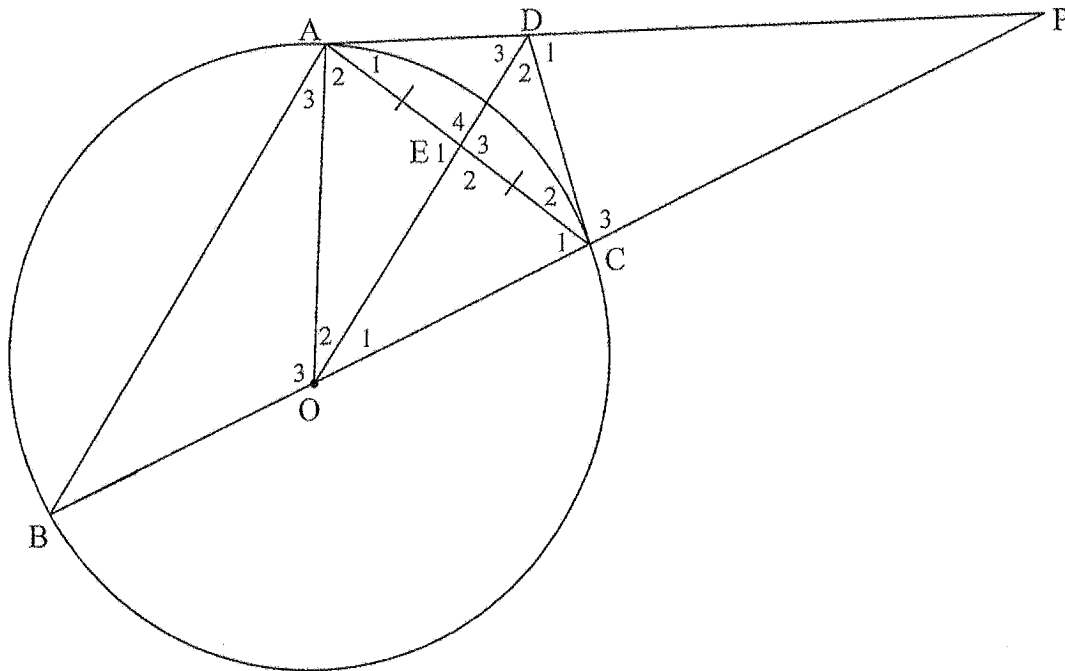
QUESTION 12

12.1 Use the diagram below to prove the theorem which states that $\hat{EFG} = \hat{EDF}$.



(5)

12.2 In the diagram below, BOC is a diameter of the circle. AP is a tangent to the circle at A and AE = EC.



Prove that:

12.2.1 $BA \parallel OD$ (4)

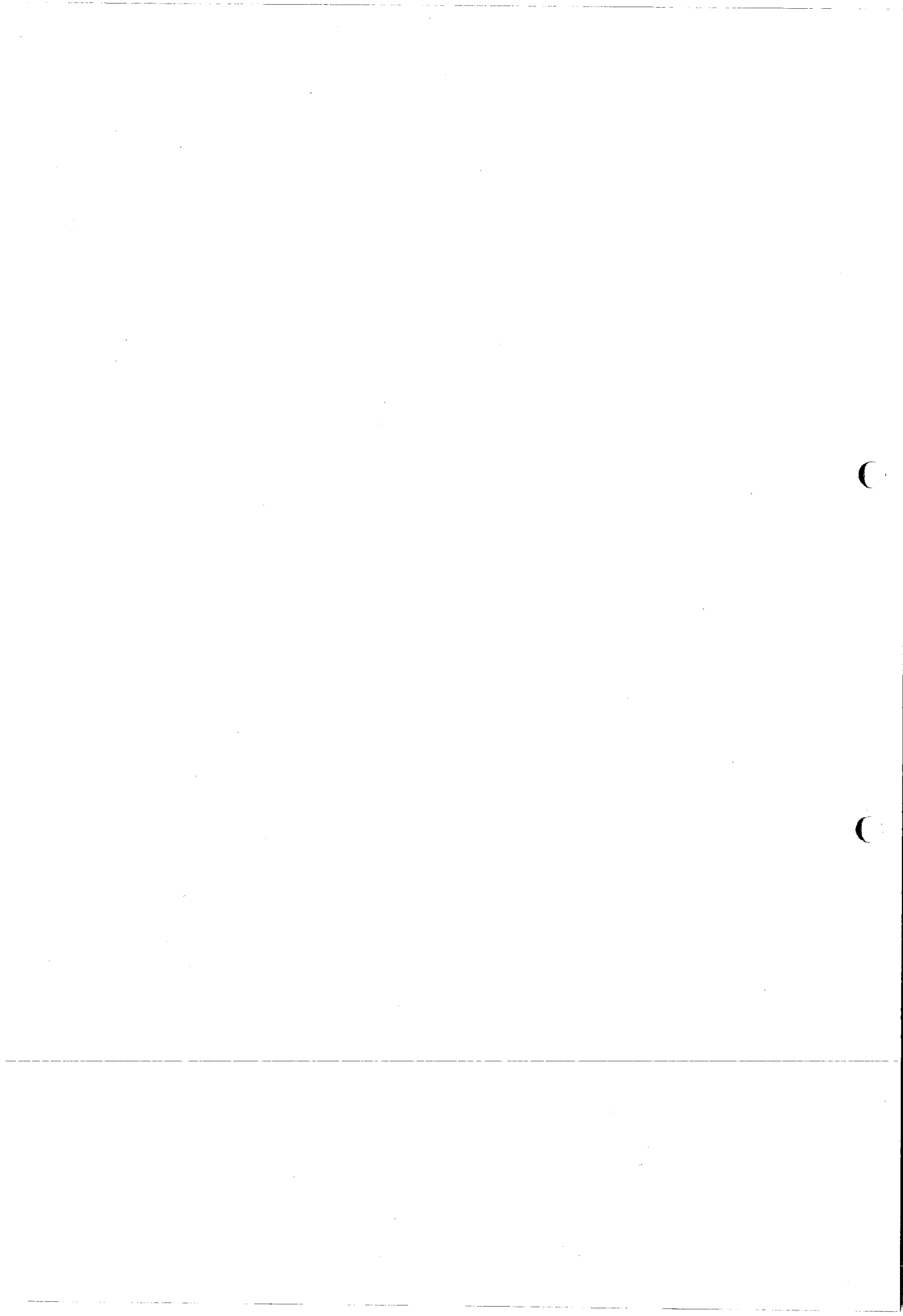
12.2.2 AOCD is a cyclic quadrilateral (5)

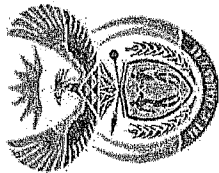
12.2.3 DC is a tangent to the circle at C (4)

[18]

TOTAL: 150

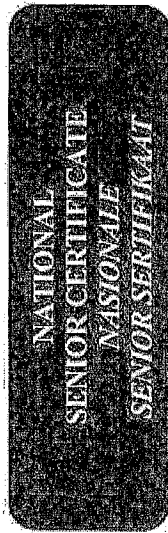






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REPUBLIC OF SOUTH AFRICA



GRADE/GRAD 11

MATHEMATICS P2/WISKUNDE V2
NOVEMBER 2016
MEMORANDUM

MARKS/PUNTE: 150

This memorandum consists of 21 pages.
Hierdie memorandum bestaan uit 21 bladsye.

DEPARTMENT OF BASIC
EDUCATION
PRIVATE BAG X908, PRETORIA 0001
2016 -11- 11
APPROVED MARKING GUIDELINE
PUBLIC EXAMINATION

W. White
15/11/2016

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedaen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraasheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

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QUESTION/VRAAG 1

5	8	15	20	25	27	31	36	75
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1.1	Range/Omvang = 75 – 5 = 70	✓ answer/antw (1)
1.2	Std dev/Std afwyking = 19,56	✓ rounding/afronding ✓ answer/antw (2)
1.3	Median/Meditaan = 25	✓ answer/antw (1)
1.4	$Q_1 = \frac{8+15}{2} = 11,5$ $Q_2 = \frac{31+36}{2} = 33,5$ IQR = $Q_3 - Q_1$ = 33,5 – 11,5 = 22	✓ $Q_1 = 11,5$ ✓ $Q_3 = 33,5$
1.5		✓ CA answer/antw (3) ✓ Q_1 and Q_3 ✓ Q_2 ✓ min and max min en maks
1.6	Skewed to the right/skeef na regs Positively skewed/positief skeef	✓ answer/antw (3)
1.7	Outlier/uitskieter = 75 OR/OF 33.5 + 1.5(22) = 66.5 Outlier/uitskieter = 75	✓ answer/antw (1) [12]

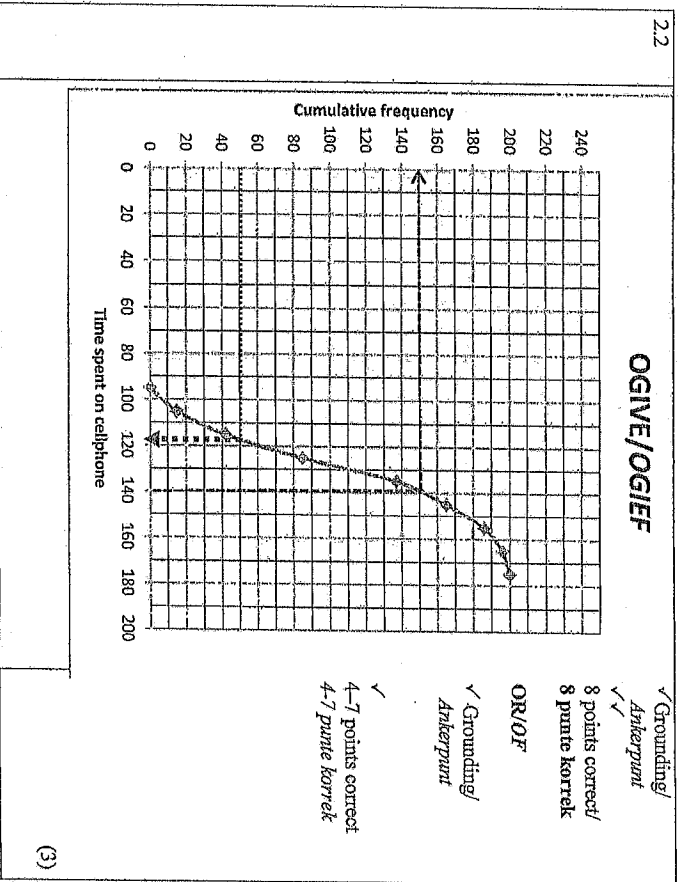
QUESTION/R44G 2

2.1

TIME SPENT/ TYD SPANDEER (IN MINUTES/ MINUTE)	FREQUENCY FREKWENSIE f	CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE c_f/n_f
$95 < x \leq 105$	15	15
$105 < x \leq 115$	27	42
$115 < x \leq 125$	43	85
$125 < x \leq 135$	52	137
$135 < x \leq 145$	28	165
$145 < x \leq 155$	21	186
$155 < x \leq 165$	10	196
$165 < x \leq 175$	4	200

✓ First 4 correct /
eersie 4 korrek
CF values
/KF waardes
✓ Last 4 correct CF
values/
laaste 4 korrekte KF
waardes

(2)



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2.3

$Q_1 = 118$	Accept any answer between (115 and 120)	✓ CA ✓ CA answer/antw	(2)
2.4	Number of learners / Getal leerders = 200 - 150 = 50	✓ CA 150 ✓ CA 50	(2)
	Accept 150 or any other reading between (145 and 155)		19

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2016 -11- 11
APPROVED MARKING GUIDELINE
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QUESTION/VRAAG 3

3.1	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{6+2}{2}, \frac{-2+15}{2} \right)$ $= \left(4; \frac{13}{2} \right)$	✓ x-ordinate x-koordinaat ✓ y-ordinate y-koordinaat (2)
3.2	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 3}{2 - (-4)} = 2$ $m_{MN} = m_{BC} = 2 \quad [BC \parallel MN]$ <p>OR/OF</p> $N \left(1; \frac{1}{2} \right)$ $m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 1}{2 - 2} = \frac{4 - 1}{4 - 1} = 2$	✓ subst into gradient form./subst in gradiëntform ✓ answer/antw ✓ gradients equal/ gradiënte gelyk $\sqrt{N \left(1; \frac{1}{2} \right)}$ ✓ subst into gradient form./subst in gradiëntform ✓ answer/antwoord
3.3	$y - y_1 = m(x - x_1)$ $y - \frac{13}{2} = 2(x - 4)$ $y = 2x - \frac{3}{2}$ <p>OR/OF</p> $y = mx + c$ $\frac{13}{2} = 2(4) + c$ $-\frac{3}{2} = c$ $y = 2x - \frac{3}{2}$	✓ subst $\left(4; \frac{13}{2} \right)$ and $m = 2$ into str line eq. / subst $\left(4; \frac{13}{2} \right)$ en $m = 2$ in reguitlyn verg. ✓ answer/antw (2)

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2016 -11- 11
APPROVED MARKING GUIDELINES

3.4	<p>N is a midpoint of AC / N is die middelpunt van AC [Line through midpoint of one side parallel to second side / omgekeerde van midpt stelling.]</p> $N \left(\frac{-4+6}{2}, \frac{3+(-2)}{2} \right)$ $= N \left(1; \frac{1}{2} \right)$ <p>OR/OF</p> $m_{AC} = \frac{3 - (-2)}{(-4) - 6} = -\frac{1}{2}$ <p>Equation of AC $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{2}(x - (-4))$ $y = -\frac{1}{2}x + 1$ $-\frac{1}{2}x + 1 = 2x - \frac{3}{2}$ $-x + 2 = 4x - 3$ $x = 1$ $y = 2(1) - \frac{3}{2} = \frac{1}{2}$ $N \left(1; \frac{1}{2} \right)$</p>	✓ S ✓ R ✓ x-value/waarde ✓ y-value/waarde ✓ gradient of AC ✓ equation of AC / vergelyking van AC ✓ equating/gelykstelling $\sqrt{N \left(1; \frac{1}{2} \right)}$ (4)
3.5	<p>N is the midpoint of BD and the midpoint of AC [diagonals of parm bisect] N is die midpt v BD en midpt v AC [hoekehoen van parm hakeer]</p> $\left(\frac{2+x}{2}, \frac{y+15}{2} \right) = \left(1; \frac{1}{2} \right)$ $\frac{2+x}{2} = 1 \quad \frac{y+15}{2} = \frac{1}{2}$ $x = 0 \quad y = -14$ <p>D (0 ; -14)</p>	✓ CA $\frac{2+x}{2} = 1$ ✓ CA $\frac{y+15}{2} = \frac{1}{2}$ ✓ CA $x = 0$ ✓ CA $y = -14$ Answer only: Full marks / Sleigs antwoord: Vol punte

<p>OR/OF From B to A</p> <p>$(x; y) \rightarrow (x+4; y-17)$ $D(-4+4; 3-17)$ $D(0; -14)$</p>	<p>✓ $x+4$ ✓ $y-17$ ✓ subst ✓ $D(0; -14)$</p> <p>(4) (15)</p>
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 2016 -11-11
 APPROVED MARKING GUIDELINE
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QUESTION/VRAG 4

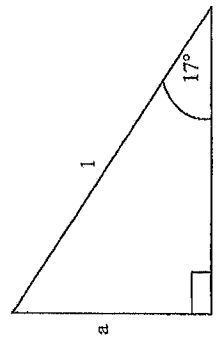
<p>4.1</p> <p>$m_{NP} = m_{PV}$ $2-0 = 4-2$ $0-k = 3-0$ $\frac{2}{-k} = \frac{2}{3}$ $k = -3$</p>	<p>✓ $m_{NP} = m_{PV}$ ✓ subst in gradient formula/vorm ✓ answer/antw</p> <p>(3)</p>
<p>4.2</p> <p>$\tan \alpha = m_{PN}$ $\tan \alpha = \frac{2}{3}$ $\alpha = 33,69^\circ$ $\tan \beta = m_{AB}$ $\tan \beta = -\frac{1}{2}$ $\beta = -26,57^\circ + 180^\circ$ $= 153,43^\circ$ OR/OF $\tan \beta = m_{AB}$ $\tan \beta = -\frac{1}{2}$ $KA = \tan^{-1}\left(\frac{1}{2}\right)$ $= 26,57^\circ$ $\beta = 180^\circ - 26,57^\circ$ $= 153,43^\circ$</p>	<p>✓ $\alpha = 33,69^\circ$ ✓ $\tan \beta = -\frac{1}{2}$ ✓ $153,43^\circ$</p>
<p>4.3</p> <p>$\theta = 153,43^\circ - 33,69^\circ$ $= 119,74^\circ$ $-\frac{1}{2}x+4=0$ $x=8$ $R(8;0)$ $MR = 8 - (-3)$ OR/OF $MR = \sqrt{(-3-8)^2 - 0^2}$ $= 11 \text{ units / eenhede}$</p>	<p>✓ $y=0$ ✓ $x=8$ ✓ CA answer/antw</p> <p>(4)</p>
<p>4.4</p> <p>Area of $\Delta MNR = \frac{1}{2}(MR) \cdot \perp \text{ height}$ $= \frac{1}{2}(11)(v - \text{value of } N)$ $= \frac{1}{2}(11)(4)$ $= 22 \text{ sq units/vt eenhede}$</p>	<p>✓ area formula/formule ✓ subst y-value of N ✓ subst y-waarde van N ✓ CA answer/antw</p> <p>(3)</p>

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OR/OF	$MN = \sqrt{(3 - (-3))^2 + (4 - 0)^2}$ $= \sqrt{36 + 16}$ $= \sqrt{52} \text{ units/eenh}$ <p>Area of Opp van $\Delta MNR = \frac{1}{2} \times \sqrt{52} \times 11 \times \sin 33,69^\circ$ $= 21,999$ $\approx 22 \text{ sq units/’k eenh}$</p>
	<p>✓ CA $\sqrt{52}$</p> <p>✓ subst in area form Subst in oppervlakt formule</p> <p>✓ CA answer/antw</p>
	(3) [13]

QUESTION/VRAAG 5

5.1.1	$x^2 + y^2 = r^2$ $(-8)^2 + (t)^2 = 17^2$ $t^2 = 225$ $t = -15$	<p>✓ subst in pyth</p> <p>✓ answer/antw</p>
5.1.2(a)	$\cos(-\theta)$ $= \cos \theta$ $= \frac{-8}{17}$	<p>✓ $\cos \theta$</p> <p>✓ answer/antw</p>
5.1.2(b)	$1 - \sin \theta = 1 - \frac{-15}{17}$ $= \frac{17}{17} + \frac{15}{17}$ $= \frac{32}{17}$	<p>✓ CA subst</p> <p>✓ CA answer/antw</p>
5.2.1	$\tan 17^\circ = \frac{a}{\sqrt{1-a^2}}$ 	<p>✓ Sketch/Pythagoras Skeits/Pythagoras</p> <p>✓ $\sqrt{1-a^2}$</p> <p>✓ CA answer/antw</p> <p>Answer only: Full marks/Slegs antwoord: 7 of punte</p>

5.2.2	$\sin 107^\circ$ $= \sin(90^\circ + 17^\circ)$ $= \cos 17^\circ$ $= \sqrt{1-a^2}$ <p>OR/OF</p> $\sin 107^\circ$ $= \sin(180^\circ - 73^\circ)$ $= \sin 73^\circ$ $= \sqrt{1-a^2}$	<p>✓ $\cos 17^\circ$</p> <p>✓ CA $\sqrt{1-a^2}$</p>
		(3)
		(2)

5.2.3	$\cos^2 253^\circ + \sin^2 557^\circ$ $= (-\cos 73^\circ)^2 + (-\sin 17^\circ)^2$ $= (-a)^2 + (-a)^2$ $= 2a^2$	<p>✓ $\cos^2 73^\circ$</p> <p>✓ $\sin^2 17^\circ$</p> <p>✓ subst of ratios/ subst van trig verhoudings</p> <p>✓ answer/antw</p>
5.3	$\cos(180^\circ + 45^\circ) \sin(180^\circ - 45^\circ) + \sin(360^\circ - 30^\circ)$ $= \frac{\tan(180^\circ + 45^\circ)}{\tan 45^\circ} \frac{(-\cos 45^\circ)(\sin 45^\circ) - \sin 30^\circ}{1}$ $= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1}$ $= -1$	<p>✓ $-\cos 45^\circ$</p> <p>✓ $\sin 45^\circ$</p> <p>✓ $-\sin 30^\circ$</p> <p>✓ $\tan 45^\circ$</p> <p>✓ Special angle ratios Spesiale hoekes</p> <p>✓ CA answer/antw</p>
		(4)
		(6)

5.4	<p>RHS</p> $= \frac{-1}{\tan^2 x \cos^2 x}$ $= \frac{\sin^2 x}{\cos^2 x} \times \cos^2 x$ $= \frac{-1}{\sin^2 x}$ $= \frac{-1}{1 - \cos^2 x}$ $= \frac{1}{\cos^2 x - 1}$ $= \frac{1}{(\cos x + 1)(\cos x - 1)}$ <p>LHS</p> $= \frac{1}{(\cos x + 1)(\cos x - 1)}$ $= \frac{1}{\cos^2 x - 1}$ $= \frac{-\sin^2 x}{1 - \sin^2 x}$ $= \frac{-1}{\sin^2 x}$ $= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x}$ $= \frac{-1}{\tan^2 x \cos^2 x}$	<p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ simplification/vereenv.</p> <p>✓ identity/identiteit</p> <p>✓ factors /faktore</p>
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5.5	<p>OR/OF</p> $RHS = \frac{-\cos^2 x}{\sin^2 x \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $LHS = \frac{1}{\cos^2 x - 1}$ $= \frac{1}{-\sin^2 x}$ <p>RHS = LHS</p>	<p>✓ $\frac{1}{\tan^2 x} = \frac{\cos^2 x}{\sin^2 x}$</p> <p>✓ $\frac{-1}{\sin^2 x}$</p> <p>✓ $\cos^2 x - 1 = -\sin^2 x$</p>
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5.5	<p>$2 \sin x \cos x - \cos x = 0$</p> <p>$\cos x(2 \sin x - 1) = 0$</p> <p>$\cos x = 0$ or $\sin x = \frac{1}{2}$</p> <p>$x = 90^\circ + 360^\circ, k, k \in \mathbb{Z}$ or $x = 30^\circ + 360^\circ, k, k \in \mathbb{Z}$</p> <p>$x = 270^\circ + 360^\circ, k, k \in \mathbb{Z}$ or $x = 150^\circ + 360^\circ, k, k \in \mathbb{Z}$</p> <p>or</p> <p>$x = 90^\circ + 180^\circ, k, k \in \mathbb{Z}$</p> <p>or</p> <p>$x = 190^\circ + 360^\circ, k, k \in \mathbb{Z}$</p>	<p>✓ factors /faktore</p> <p>✓ both equations/ beide verg.</p> <p>✓ BOTH general solutions for $\cos x = 0$ / <i>Alles algemene oplossings vir $\cos x = 0$</i></p> <p>✓ general solutions for $\sin x = \frac{1}{2}$ / <i>algemene oplossings vir $\sin x = \frac{1}{2}$</i></p> <p>✓ $k \in \mathbb{Z}$</p>
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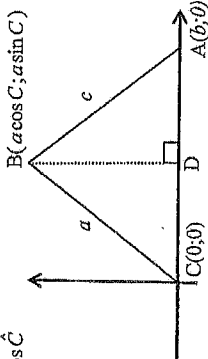
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QUESTION/VRAAG 6

6.1	$b = 30^\circ$	✓ answer/antw	(1)
6.2	360°	✓ answer/antw	(1)
6.3	$f(x) = g(x)$ $x = -150^\circ$ $x = 30^\circ$	✓ $x = -150^\circ$ ✓ $x = 30^\circ$	(2)
6.4	$\sin(90^\circ - x) > g(x)$ $\cos x > g(x)$ $f(x) > g(x)$ $x \in (-150^\circ; 30^\circ)$ or $-150^\circ < x < 30^\circ$	✓ $\cos x$ ✓ end points/eindpunte ✓ notation/notasie	(3)
6.5	Range: $y \in [2; 4]$ or / of $2 \leq y \leq 4$	✓ end points/eindpunte ✓ notation/notasie	(2) (9)

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QUESTION/VRAAG 7

7.1	$AB^2 = AD^2 + BD^2$ [Pythagoras] $c^2 = (b - a \cos \hat{C})^2 + (a \sin \hat{C})^2$ $= b^2 - 2ab \cos \hat{C} + a^2 \cos^2 \hat{C} + a^2 \sin^2 \hat{C}$ $= b^2 - 2ab \cos \hat{C} + a^2 (\cos^2 \hat{C} + \sin^2 \hat{C})$ $= a^2 + b^2 - 2ab \cos \hat{C}$ 	✓ $B(a \cos C; a \sin C)$ ✓ $A(b; 0)$ ✓ distance formula/ afstand formule. ✓ expansion/ontwikkel common factor/ gemene faktor ✓ square identity/ vierkants identiteit (6)
7.1.2	$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$ $\therefore \cos \hat{C} = \frac{a^2 + b^2 - c^2}{2ab}$ $1 + \cos \hat{C} = 1 + \frac{a^2 + b^2 - c^2}{2ab}$ $= \frac{2ab}{2ab} + \frac{a^2 + b^2 - c^2}{2ab}$ $= \frac{a^2 + 2ab + b^2 - c^2}{2ab}$ $= \frac{(a+b)^2 - c^2}{2ab}$ $= \frac{(a+b+c)(a+b-c)}{2ab}$	$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$ $\therefore \cos \hat{C} = \frac{a^2 + b^2 - c^2}{2ab}$ ✓ making $\cos C$ the subject of formula/maak $\cos C$ die onderwerp van die formule ✓ Adding 1 on both sides/ Tel 1 by albei kante ✓ simplifying/ vereenvoudig ✓ factorising/ faktorisering

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<p>OR/OF</p> $c^2 = a^2 + b^2 - 2ab \cos C$ $\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $RHS = \frac{(a+b)^2 - c^2}{2ab}$ $= \frac{a^2 + 2ab + b^2 - c^2}{2ab}$ $= \frac{a^2 + b^2 - c^2}{2ab} + \frac{2ab}{2ab}$ $= \cos C + 1$ <p>= LHS</p> <p>OR/OF</p> $c^2 = a^2 + b^2 - 2ab \cos C$ $2ab \cos C = a^2 + b^2 - c^2$ $2ab + 2ab \cos C = a^2 + 2ab + b^2 - c^2$ $2ab(1 + \cos C) = (a+b)^2 - c^2$ $1 + \cos C = \frac{(a+b)^2 - c^2}{2ab}$	<p>$c^2 = a^2 + b^2 - 2ab \cos C$</p> $\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$ <p>✓ Making cos C the subject of the formula/ Onderwerp van die formule</p> <p>✓ writing as a difference of 2 squares/ Skryf as die verskil tussen twee vierkante</p> <p>✓ expansion/ontwikkel</p> <p>✓ splitting up the fraction / Deel die breuk in twee</p> <p>✓ making $2ab \cos C$ the subject of the formula/ Maak $2ab \cos C$ die onderwerp van die formule</p> <p>✓ adding $2ab$ on both sides of equation/ tel $2ab$ aan beide kante van die vergelyking</p> <p>✓ common factor/ gemene faktor</p> <p>✓ factorise the trinomial/faktorisier die drieterm</p>
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PLAASLIK OORTOEGEKEN

<p>7.2.1</p> <p>In $\triangle ABD$</p> $\frac{BD}{\sin 109,16^\circ} = \frac{90,52}{\sin 31,23^\circ}$ $BD = \frac{90,52 \times \sin 109,16^\circ}{\sin 31,23^\circ}$ $= 164,92 \text{ m}$	<p>✓ sine rule/sinusreël</p> <p>✓ subst</p> <p>✓ answer/antw.</p>	<p>(3)</p>
<p>7.2.2</p> $CD^2 = 164,92^2 + 235^2 - 2 \times 164,92 \times 235 \times \cos 48,88^\circ$ $CD^2 = 31448,4874$ $CD = 177,34 \text{ m}$	<p>✓ cos rule/cosinusreël</p> <p>✓ subst</p> <p>✓ CA answer/antw.</p>	<p>(3)</p>

QUESTION/VRAAG 8

<p>8.1</p> $\tan 35,5^\circ = \frac{0,5}{AB}$ $AB = \frac{0,5}{\tan 35,5^\circ}$ $= 0,7 \text{ m}$	<p>✓ subst</p> <p>✓ answer/antw</p>	<p>(2)</p>
<p>8.2</p> <p>Volume of cone</p> $= \frac{1}{3} \times \pi (0,5)^2 \times 0,7$ $= 0,18 \text{ m}^3$ <p>Volume of a cylinder = $\pi (0,5)^2 \times 1,1$</p> $= 0,86 \text{ m}^3$ <p>$\frac{3}{4}$ of volume = $\frac{3}{4} \times (0,18 + 0,86)$</p> $= \frac{3}{4} \times (1,04) \text{ m}^3$ $= 0,78 \text{ m}^3$ <p>Time taken by pump = $\frac{0,78 \text{ m}^3}{0,52 \text{ m}^3/\text{h}}$</p> $= 1,5 \text{ hours}$	<p>✓ CA V of cone/keël</p> <p>✓ CA V of cylinder/silinder</p> <p>✓ CA answer/antw</p>	<p>(4)</p>

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QUESTION/VRAAG 9

9.1	Equal to twice the angle subtended by the arc at the circumference	✓ ✓ <i>answ/antw</i>	(2)
9.2.1	$\hat{R} = 30$ [\angle at centre = $2 \times \angle$ at circumference]	✓ S ✓ R	(2)
9.2.2	$\hat{NST} = 30^\circ$ [equal chords subtends equal angles]	✓ S ✓ R	(2)
			[6]

QUESTION/VRAAG 10

10.1	$\hat{DEG} = x + 20^\circ$ [alt \angle 's, ED FG] OR/OF $\hat{DEG} = 170^\circ - 2x$ [opp angles of cyclic quad]	✓ S ✓ R	(2)
10.2	$x + 20^\circ + 2x + 10^\circ = 180^\circ$ [opp \angle of cyclic quad] $3x = 150^\circ$ $x = 50^\circ$ $\hat{DHG} = 2(50^\circ) + 10^\circ = 110^\circ$ OR/OF $x + 20^\circ = 170^\circ - 2x$ [alt 's, ED FG] $3x = 150^\circ$ $x = 50$ $\hat{DHG} = 2(50^\circ) + 10^\circ = 110^\circ$	✓ S ✓ R ✓ <i>answ/antw</i> ✓ 110° ✓ S ✓ R ✓ <i>answ/antw</i> ✓ 110°	(4) [6]

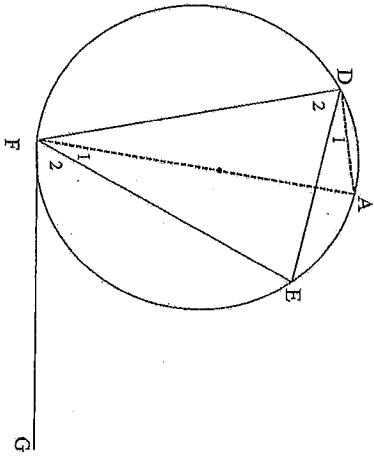
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QUESTION/VRAAG 11

11.1	SP = SR [tangents from the same point] $\hat{PRS} = 42,83^\circ$ [\angle 's between equal sides] $\hat{ORS} = 90^\circ$ [tan \perp rad] $\hat{ORN} = 90^\circ - 42,83^\circ = 47,17^\circ$ $\hat{NOR} = 90^\circ - 47,17^\circ$ [sum \angle 's of Δ] $= 42,83^\circ$	✓ S ✓ S ✓ S/R ✓ ORN ✓ <i>answ/antw</i>	(5)
11.2	Let OR = x $OS = x + 9$ $\hat{ORS} = 90^\circ$ [tan \perp rad] $x^2 + 15^2 = (x + 9)^2$ [Pythagoras] $x^2 + 225 = x^2 + 18x + 81$ $18x = 144$ $x = 8$ units radius = 8 units OR/OF In ΔNRS $\cos 42,83^\circ = \frac{NR}{15}$ $NR = 15 \cos 42,83^\circ = 11$ In ΔORN $\sin 42,83^\circ = \frac{11}{OR}$ $OR = \frac{11}{\sin 42,83^\circ} = 16,18$	✓ S ✓ S ✓ Using Pythagoras / Gebruik Pythagoras ✓ <i>answ/antw</i> ✓ $\cos 42,83^\circ = \frac{NR}{15}$ ✓ 11 ✓ $\sin 42,83^\circ = \frac{11}{OR}$ ✓ 16,18	(4)
			[9]

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QUESTION/VRAG 12



12.1	<p>Construction: Draw diameter AOF. Join A to D.</p> <p>$\hat{F}_1 + \hat{F}_2 = 90^\circ$ [tan \perp diameter] $\hat{D}_1 = \hat{F}_1$ [\angle's in the same segment] $\hat{D}_1 + \hat{D}_2 = 90^\circ$ [\angle in a semi circle] $\therefore \hat{F}_2 = \hat{D}_2$</p> <p>$EF \hat{G} = FD \hat{E}$ OR / OF</p> <p>Construction: Draw diameter AOF. Join A to E.</p> <p>$\hat{F}_1 + \hat{F}_2 = 90^\circ$ [tan \perp diameter] $\hat{A} \hat{E} \hat{F} = 90^\circ$ [\angle's in the semi circle] $\hat{C} + \hat{F}_1 = 90^\circ$ [sum of \angle's in Δ] $\therefore \hat{F}_1 + \hat{F}_2 = \hat{C} + \hat{F}_1$ $\therefore \hat{F}_2 = \hat{C}$</p> <p>but $\hat{C} = \hat{D}$ [\angle's in the same segment] $\therefore \hat{F}_2 = \hat{D}_2$</p> <p>$EF \hat{G} = FD \hat{E}$</p>	<p>✓ Constr/Kons ✓ S ✓ R ✓ S ✓ R</p>
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12.2.1	<p>Construction: Draw radii OF and OE OR / OF</p> <p>Let $\hat{E}OF = 2x$ $\therefore \hat{D} = x$ [\angle at centre = $2 \times \angle$ at circumference] $\hat{O} \hat{F} \hat{E} = 90^\circ - x$ [sum of int \angle's of Δ] $\therefore EF \hat{G} = x$ [rad \perp tan] $\therefore EF \hat{G} = FD \hat{E}$</p> <p>$\hat{B} \hat{A} \hat{C} = 90^\circ$ [\angle in a semi circle] $\hat{E}_2 = 90^\circ$ [line drawn from centre to midpoint of chord] $\therefore \hat{B} \hat{A} \hat{C} = \hat{E}_2$</p> <p>$BA \parallel OD$ [corresp. \angle's are equal] OR / OF</p> <p>$\hat{B} \hat{A} \hat{C} = 90^\circ$ [\angle in a semi circle] $\hat{E}_1 = 90^\circ$ [line from centre to midpoint of chord] $\hat{B} \hat{A} \hat{C} = \hat{E}_1$</p> <p>$\Rightarrow BA \parallel OD$ [Alt \angle's are equal]</p>	<p>✓ S / R ✓ S ✓ R ✓ R ✓ S / R ✓ S / R ✓ R</p>
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(4)

12.2.2	<p>$\hat{A}_1 = x$ $\hat{B} = x$ [tan-chord theorem] $\hat{Q}_1 = x$ [corresp \angle's equal, $AB \parallel OD$] $\hat{A}_1 = \hat{Q}_1$ $\therefore AOC \hat{D}$ is a cyclic quadrilateral [conv. \angle's in the same segment] OR / OF</p> <p>Let $\hat{O}_1 = a$ $\hat{C}_1 = 90^\circ - a$ [int. \angle's of Δ] $\therefore \hat{A}_2 = 90^\circ - a$ [\angle's opp = sides] $\therefore \hat{A}_1 = a$ [tan \perp rad] $\therefore \hat{Q}_1 = \hat{A}_1$</p> <p>$\therefore AOC \hat{D}$ is a cyclic quadrilateral. [Converse \angle's in the same segment]</p>	<p>✓ S ✓ R ✓ S ✓ R ✓ R ✓ S ✓ S ✓ S ✓ R</p>
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<p>12.2.3</p>	<p>$\angle AOC = 2x$ [\angle at centre = $2 \times \angle$ at circumference.] $\hat{O}_1 = x$ $\therefore \hat{O}_2 = x$ $\hat{C}_2 = \hat{O}_2 = x$ [\angle's in the same segment] $\therefore \hat{C}_2 = \hat{B} = x$ $\therefore DC$ is a tangent to circle [conv. tan – chord] OR/OF $\hat{O}CD = 90^\circ$ [opp \angle's of cyclic quadrilateral] $\therefore CD$ is a tangent. [Converse tan \perp rad] OR/OF $\hat{B} = \hat{A}_3$ [\angle's opp = sides] $\hat{A}_3 = \hat{O}_2$ [Alt \angle's]; $\hat{O}_2 = \hat{C}_2$ [Angles in the same segment] $\therefore \hat{C}_2 = \hat{B}$ $\therefore DC$ is a tangent. [Converse tan – chord]</p>	<p>✓ S ✓ R ✓ S / R ✓ R ✓ S ✓ R ✓ S ✓ R ✓ S / R ✓ S / R ✓ S / R ✓ R (4)</p>
	<p>[18]</p>	

TOTAL/TOTAAL: 150

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