WTS TUTORING

MATHEMATICS

GRADE 12

CAPS PAPERS & SOLUTIONS

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HERE YOU WILL FIND:

- EXAMINATION GUIDELINES
- INSTRUCTIONS AND INFORMATION
- CAPS PAST PAPERS AND SOLUTIONS {2014 TO 2017}
- WTS TUTORS CENTRES

PUBLISHERS

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EXAMINATION GUIDELINES

1. INTRODUCTION

The Curriculum and Assessment Policy Statement (CAPS) for Mathematics outlines the nature and purpose of the subject Mathematics. This guides the philosophy underlying the teaching and assessment of the subject in Grade 12.

The purpose of these Examination Guidelines is to:

- Provide clarity on the depth and scope of the content to be assessed in the Grade 12 National Senior Certificate (NSC) Examination in Mathematics.
- Assist teachers to adequately prepare learners for the examinations.

This document deals with the final Grade 12 external examinations. It does not deal in any depth with the School-Based Assessment (SBA).

These Examination Guidelines should be read in conjunction with:

- *The National Curriculum Statement (NCS) Curriculum and Assessment Policy Statement (CAPS):* Mathematics
- The National Protocol of Assessment: *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment (Grades R–12)*
- The national policy pertaining to the programme and promotion requirements of the National Curriculum Statement, Grades R–12

Included in this document is a list of Euclidean Geometry reasons, both in English and Afrikaans, which should be used as a guideline when teaching learners Euclidean Geometry.

The information sheet for Paper 1 and 2 is included in this document.

2. ASSESSMENT IN GRADE 12

All candidates will write two external papers as prescribed.

2.1 Format of question papers for Grade 12

Paper	Topics	Duration	Total	Date	Marking
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations and inequalities Differential Calculus Probability	3 hours	150	October/November	Externally
2	Euclidean Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150	October/November	Externally

Questions in both Papers 1 and 2 will assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of contexts.

An Information Sheet is included on p. 15.

2.2 Weighting of cognitive levels

Papers 1 and 2 will include questions across four cognitive levels. The distribution of cognitive levels in the papers is given below.

Cognitive level	Description of skills to be demonstrated	Weighting	Approximate number of marks in a 150-mark paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding • Learners are expected to solve problems by integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks

3. ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as learners should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $af(x)$ and $x=f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in Paper 2.

NUMBER PATTERNS, SEQUENCES AND SERIES

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FINANCE, GROWTH AND DECAY

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will not be examined in the examination.

ALGEBRA

1. Solving quadratic equations by completing the square will not be examined.
2. Solving quadratic equations using the substitution method (k-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

DIFFERENTIAL CALCULUS

1. The following notations for differentiation can be used: $f'(x)$, D_x, $\frac{dy}{dx}$ or y'.
2. In respect of cubic functions, candidates are expected to be able to:
   - Determine the equation of a cubic function from a given graph.

- Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
- Apply knowledge of transformations on a given function to obtain its image.

3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will not be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

PROBABILITY

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced are examinable.
3. Questions that require the learner to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
   - The opposite angles of a cyclic quadrilateral are supplementary;
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
   - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
   - Equiangular triangles are similar.
4. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circle
   - Equal chords subtend equal angles at the circumference
   - Equal chords subtend equal angles at the centre
   - In equal circles, equal chords subtend equal angles at the circumference
   - In equal circles, equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
   - Tangents drawn from a common point outside the circle are equal in length.

5. The theory of quadrilaterals will be integrated into questions in the examination.
6. Concurrency theory is excluded.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.
   In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR ; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers.

4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB ∥ CD, then the alternate angles are equal.	alt ∠s; AB ∥ CD
If AB ∥ CD, then the corresponding angles are equal.	corresp ∠s; AB ∥ CD
If AB ∥ CD, then the co-interior angles are supplementary.	co-int ∠s; AB ∥ CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS
The line segment joining the midpoints of two sides of a triangle is	Midpt Theorem

THEOREM STATEMENT	ACCEPTABLE REASON(S)
parallel to the third side and equal to half the length of the third side	
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt ∥ to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line ∥ one side of Δ **OR** prop theorem; name ∥ lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$ ⊙
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

6. GENERAL GUIDELINES FOR MARKING

- If a learner makes more than one attempt at answering a question and does not cancel any of them out, only the first attempt will be marked irrespective of which of the attempt(s) may be the correct answer.
- Consistent Accuracy marking regarding calculations will be followed in the following cases:
  - **Sub-question to sub-question:** When a certain variable is incorrectly calculated in one sub-question and needs to be substituted into another sub-question **full marks can be** awarded for the subsequent sub-questions provided the methods used are correct and the calculations are correct.
  - Assuming values/answers in order to solve a problem is unacceptable.

7. CONCLUSION

This Examination Guidelines document is meant to articulate the assessment aspirations espoused in the CAPS document. It is therefore not a substitute for the CAPS document which teachers should teach to.

Qualitative curriculum coverage as enunciated in the CAPS cannot be over-emphasised.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

MATHEMATICS P1

NOVEMBER 2014

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2-2x=14$ (correct to TWO decimal places) (4)

1.1.3 $2^{x+2}+2^x=20$ (3)

1.2 Solve the following equations simultaneously:

$x=2y+3$

$3x^2-5xy=24+16y$ (6)

1.3 Solve for x: $(x-1)(x-2)<6$ (4)

1.4 The roots of a quadratic equation are: $x=\dfrac{3\pm\sqrt{-k-4}}{2}$

For which values of k are the roots real? (2)

[21]

QUESTION 2

Given the arithmetic series: 2 + 9 + 16 + ... (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251^{st} term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)

[12]

QUESTION 3

3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \ldots$

3.1.1 Write down the value of p. (2)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: $16 ; 4 ; 1$

3.2.1 Calculate the value of the 12^{th} term. (Leave your answer in simplified exponential form.) (3)

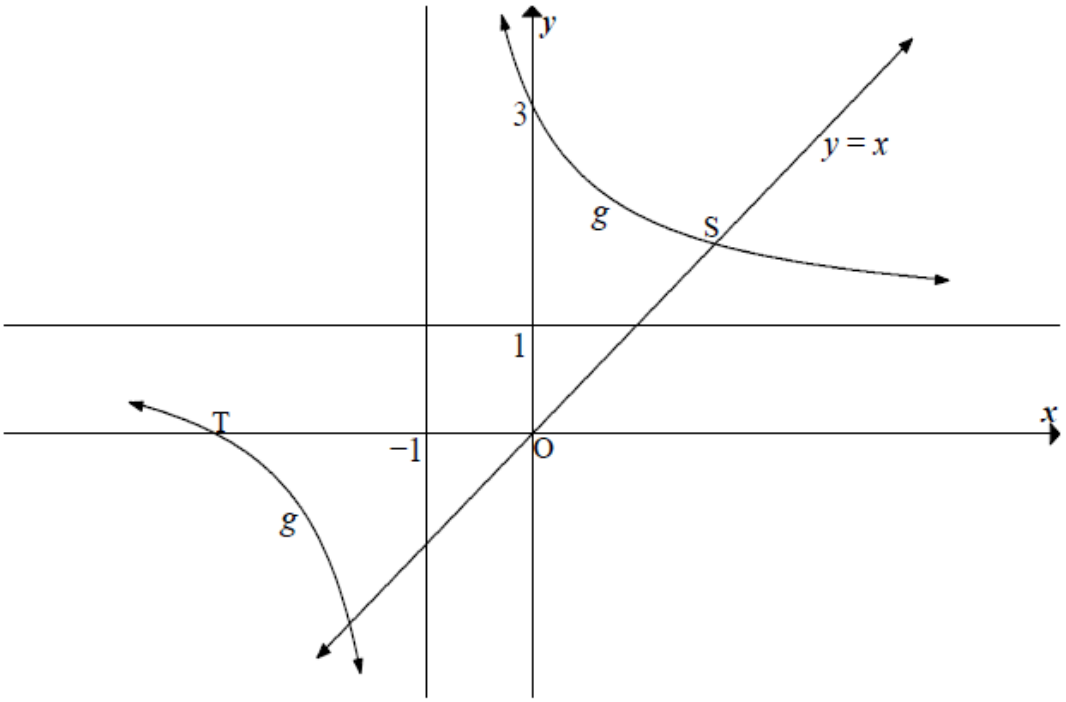
3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)$ ... up to 98 factors. (4)

[19]

QUESTION 4

The diagram below shows the hyperbola g defined by $g(x)=\frac{2}{x+p}+q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at $(0 ; 3)$. The line $y=x$ intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of p and q. (2)

4.2 Calculate the x-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x)=g(x+5)$ (1)

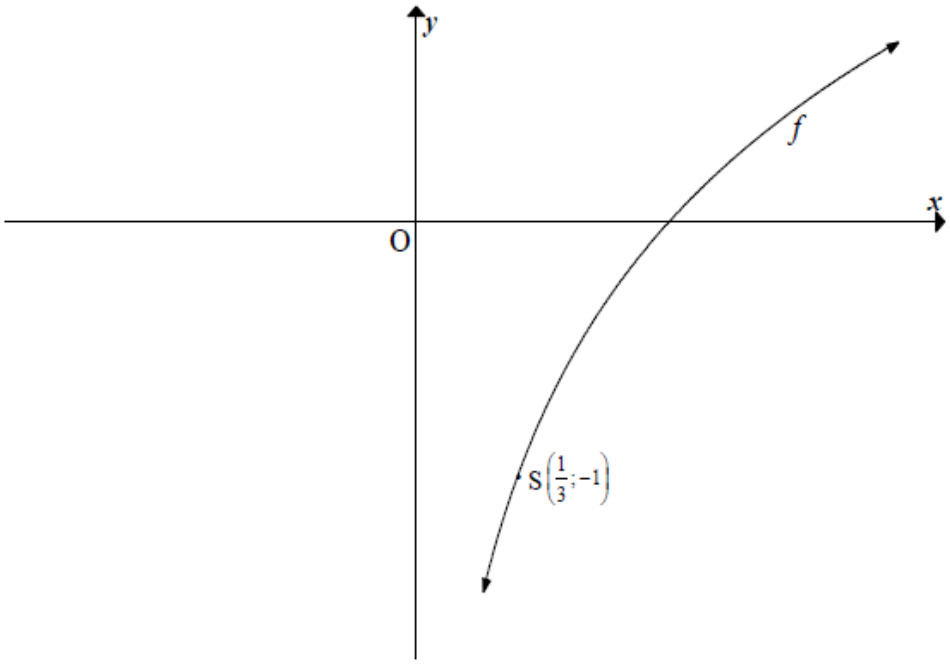
4.4 Calculate the length of OS. (5)

4.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1)

[11]

QUESTION 5

Given: $f(x)=\log_a x$ where $a>0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of f.



5.1 Prove that $a=3$. (2)

5.2 Write down the equation of h, the inverse of f, in the form $y=\ldots$ (2)

5.3 If $g(x)=-f(x)$, determine the equation of g. (1)

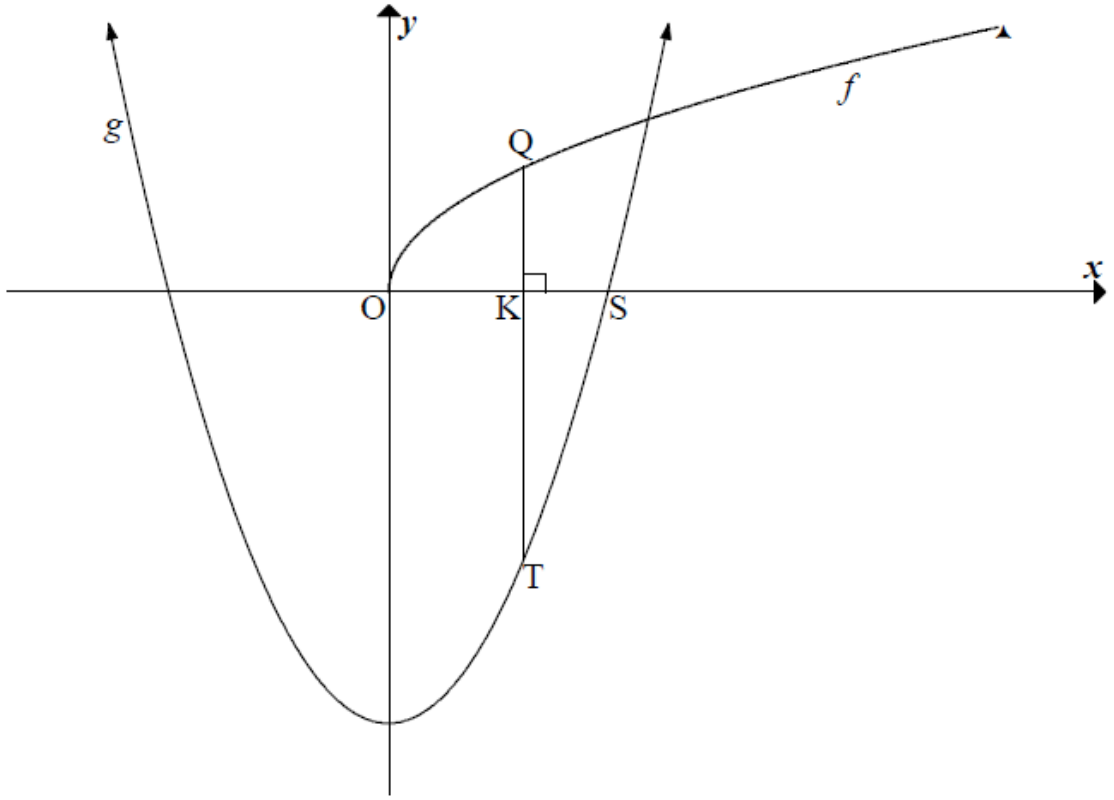
5.4 Write down the domain of g. (1)

5.5 Determine the values of x for which $f(x)\geq -3$. (3)

[9]

QUESTION 6

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



6.1 Determine the x-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3 6.3.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K. (3)

6.3.2 Calculate the maximum length of QT. (6)

[13]

QUESTION 7

7.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation. (3)

7.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

7.2.1 Calculate the value of Samuel's monthly instalment. (4)

7.2.2 Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

7.2.3 Who pays more interest, Samuel or Melissa? Justify your answer. (2)

[13]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = \left(x^6 - 1\right)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)

[14]

QUESTION 9

Given: $f(x) = (x+2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

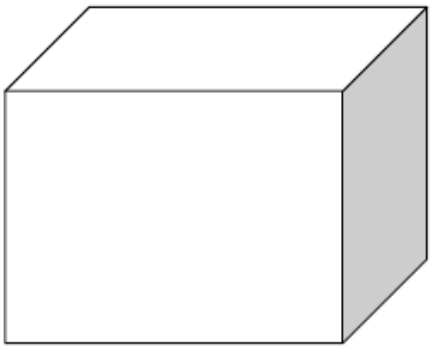
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

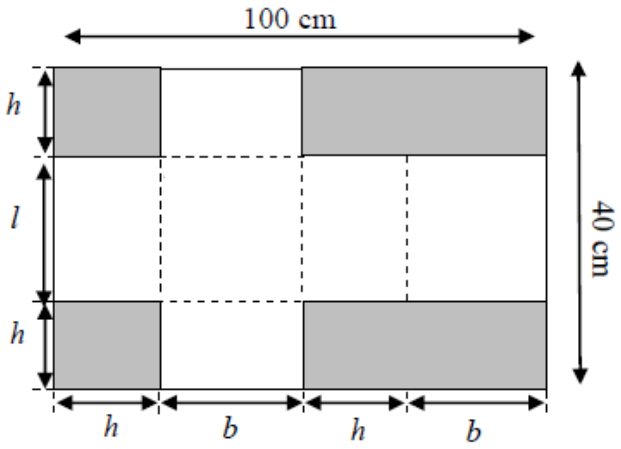
9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f'(x) < 0$? (3)

[13]

QUESTION 10





A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)

[9]

QUESTION 11

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

11.1 Determine the probability that a randomly selected staff member:

11.1.1 Is male (1)

11.1.2 Does not prefer visiting a game park (2)

11.2 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)
[7]

QUESTION 12

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 All the letters of the alphabet can be used (2)

12.1.2 The password must start with a 'D' and end with an 'L' (2)

12.2 Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

12.2.1 In how many different ways can ALL the cars be parked? (2)

12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)
[9]

TOTAL: 150

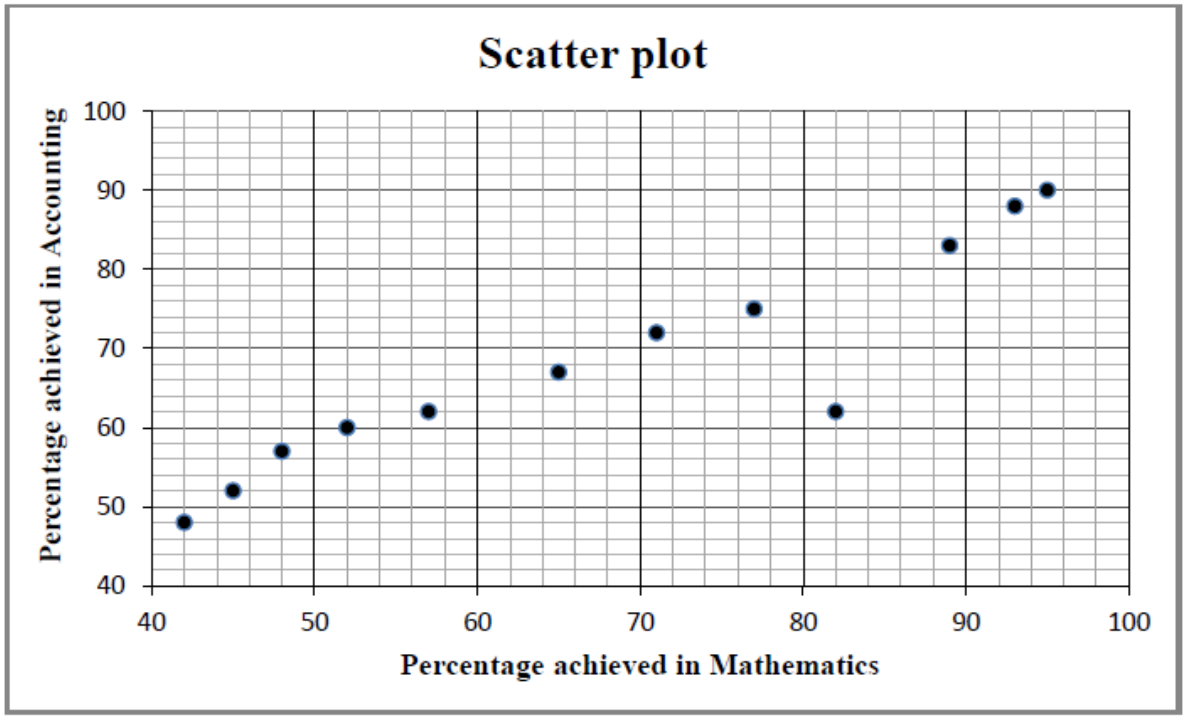
MATHEMATICS P2

NOVEMBER 2014

QUESTION 1

At a certain school, only 12 candidates take Mathematics and Accounting. The marks, as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62



1.1 Calculate the mean percentage of the Mathematics data. (2)

1.2 Calculate the standard deviation of the Mathematics data. (1)

1.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean. (3)

1.4 Calculate an equation for the least squares regression line (line of best fit) for the data. (3)

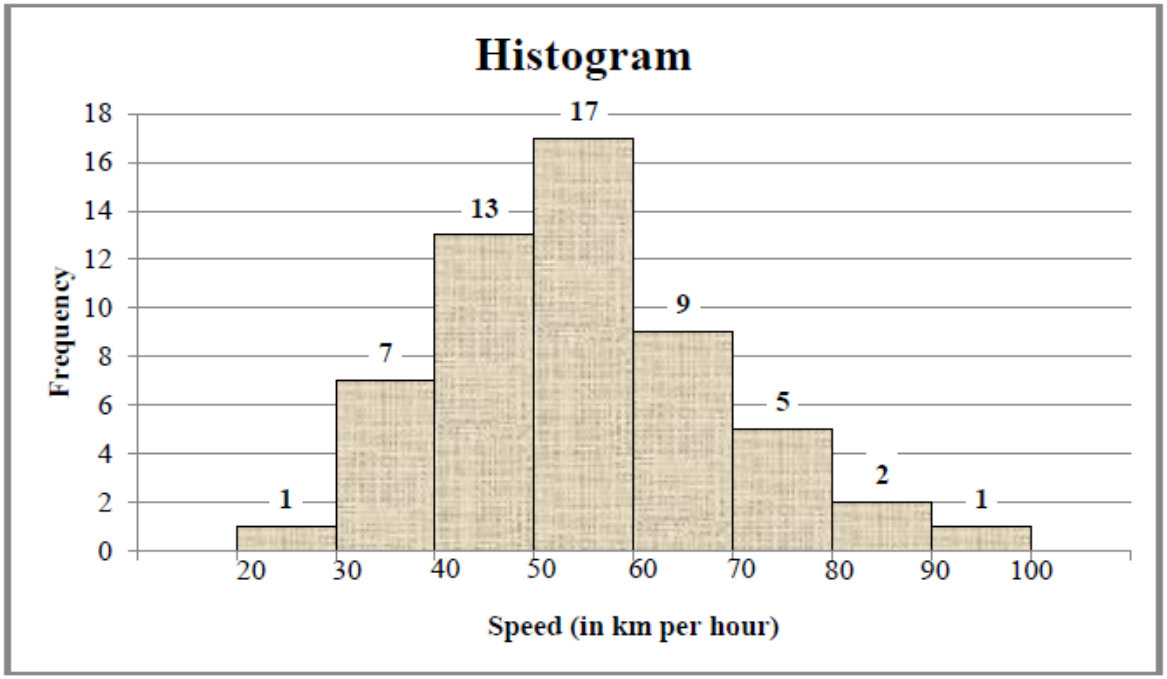
1.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in QUESTION 1.4. (Round off your answer to the NEAREST INTEGER.) (2)

1.6 Use the scatter plot and identify any outlier(s) in the data. (1)

[12]

QUESTION 2

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



2.1 Identify the modal class of the data. (1)

2.2 Use the histogram to:

2.2.1 Complete the cumulative frequency column in the table on DIAGRAM SHEET 1 (2)

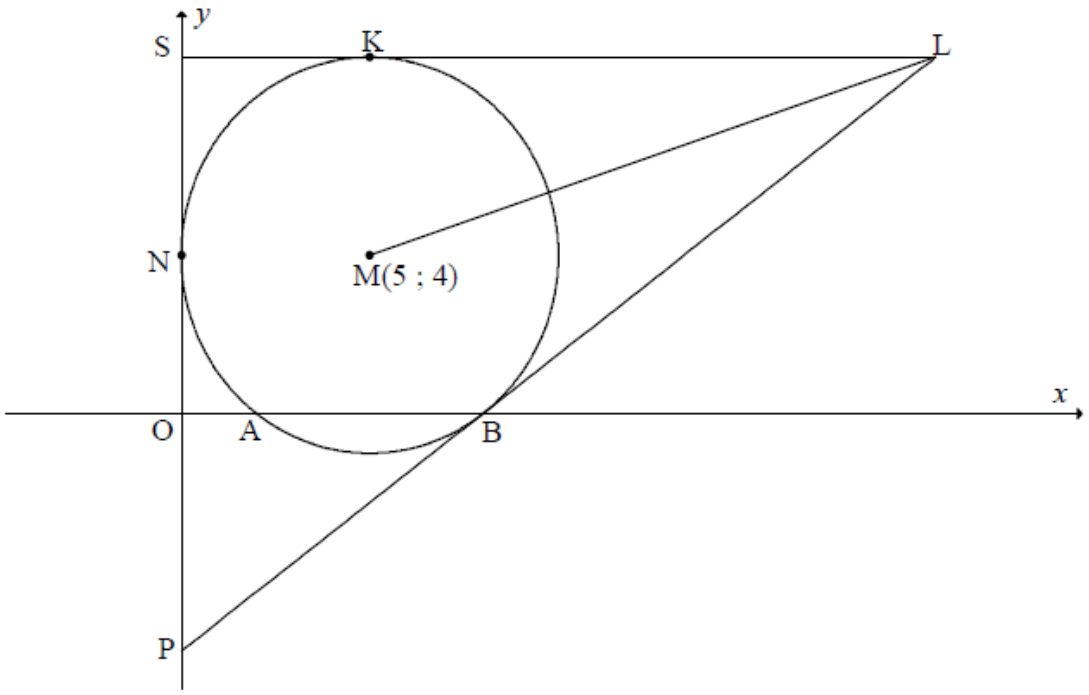
2.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid on DIAGRAM SHEET 1 (3)

2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2)

[8]

QUESTION 3

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y=mx+c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

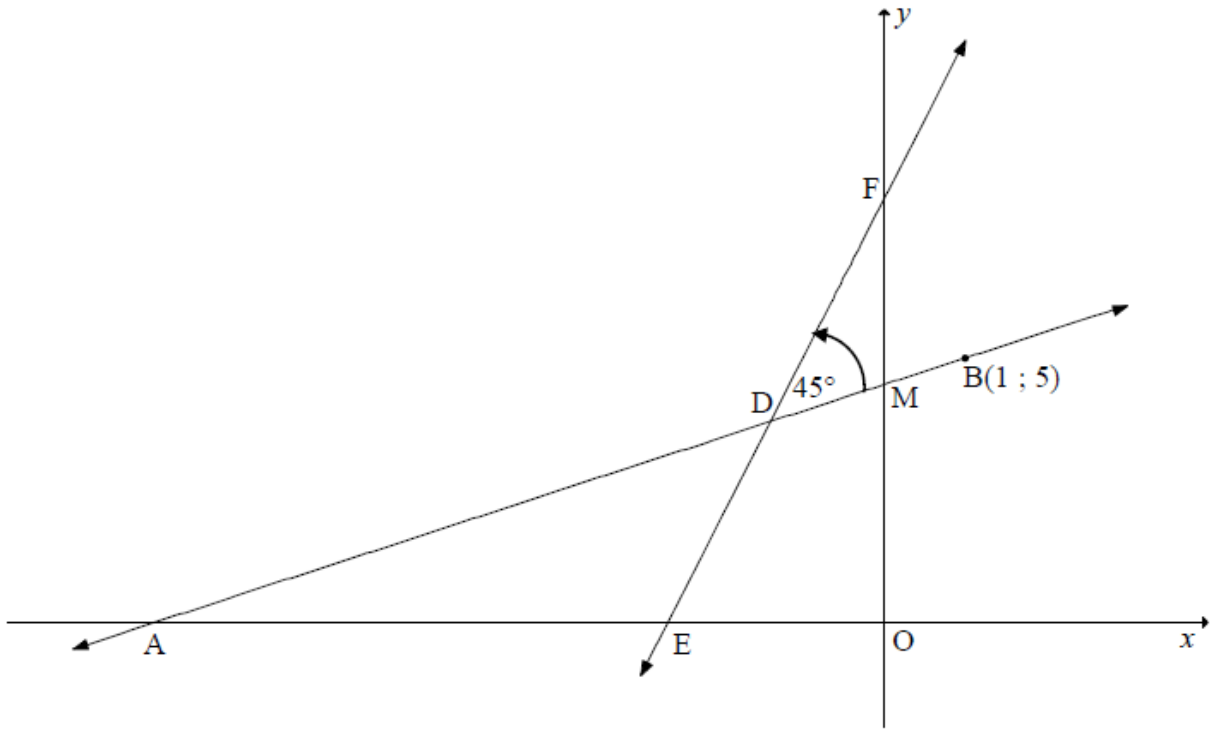
3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

QUESTION 4

In the diagram below, E and F respectively are the x- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has x- and y-intercepts A and M respectively.



4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

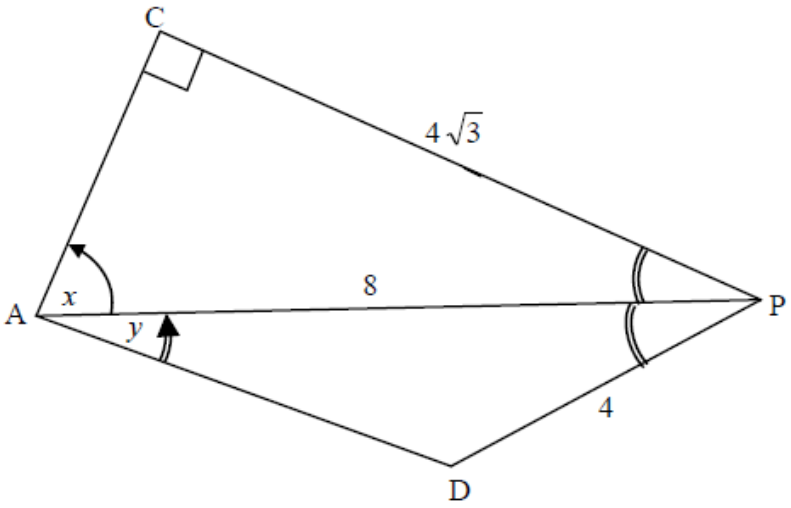
4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

4.5 Calculate the area of quadrilateral DMOE. (6)
[19]

QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



5.1 Show, by calculation, that $x = 60°$. (2)

5.2 Calculate the length of AD. (4)

5.3 Determine y. (3)

[9]

QUESTION 6

6.1 Prove the identity: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

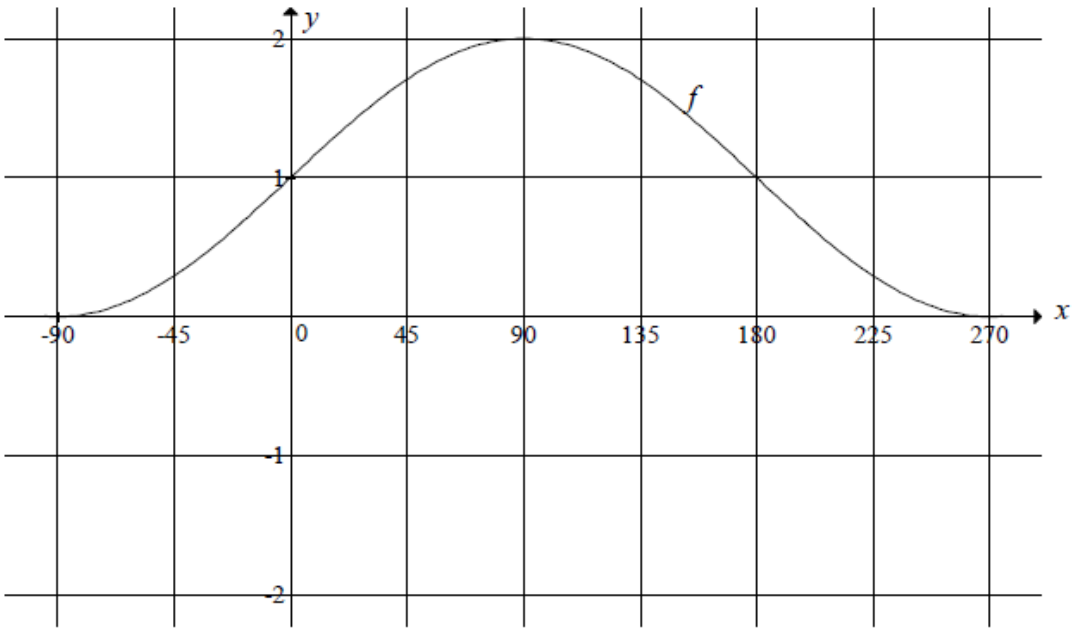
6.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$. (4)

[12]

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

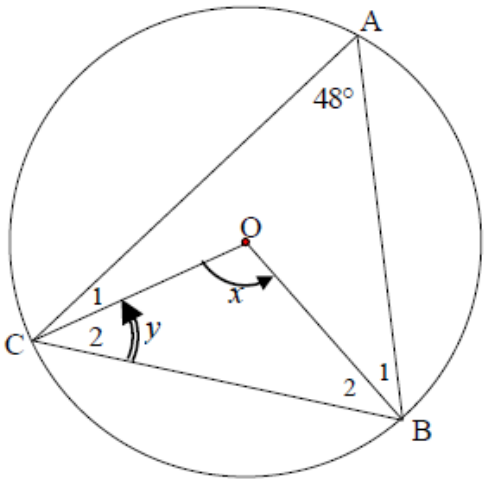
Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)

[19]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

8.1 In the diagram, O is the centre of the circle passing through A, B and C. $C\hat{A}B = 48°$, $C\hat{O}B = x$ and $\hat{C}_2 = y$.

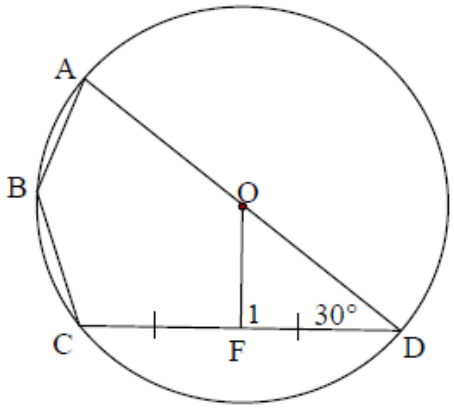


Determine, with reasons, the size of:

8.1.1 x (2)

8.1.2 y (2)

8.2 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. $O\hat{D}F = 30°$ and OF are joined.

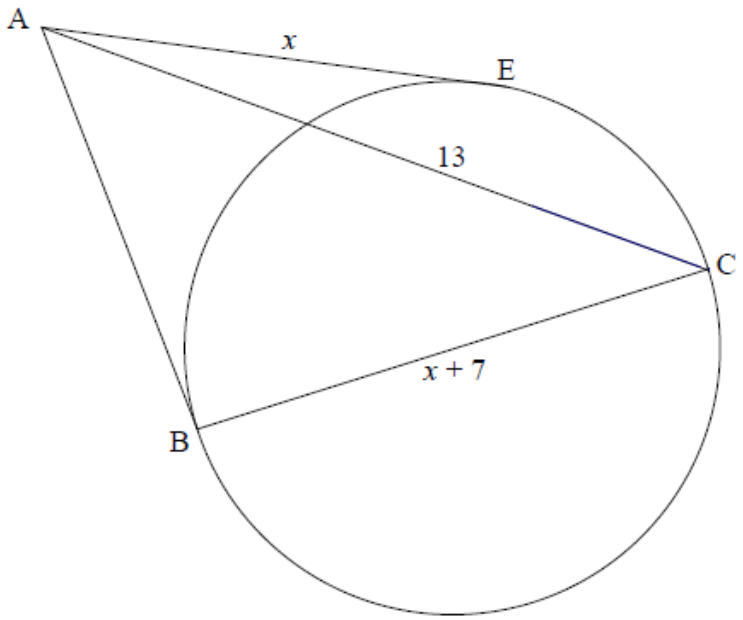


Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $A\hat{B}C$ (2)

8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. AC = 13, AE = x and BC = $x + 7$.



8.3.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 3.

	Statement	Reason
(a)	$A\hat{B}C = 90°$	
(b)	AB = x	

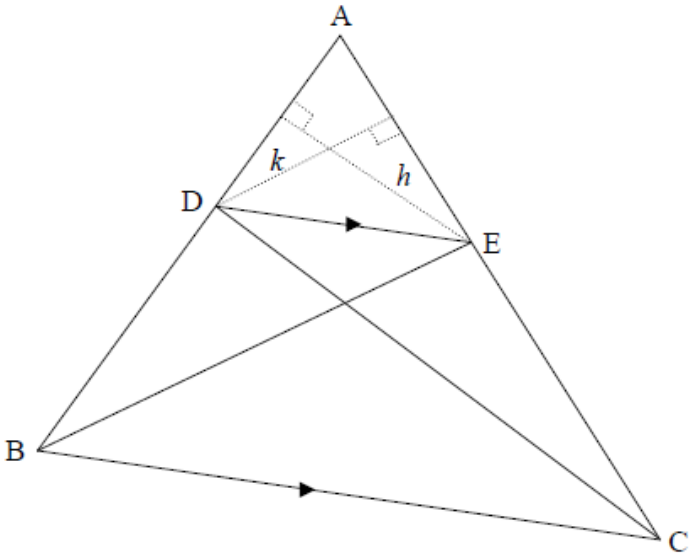
(2)

8.3.2 Calculate the length of AB. (4)

[14]

QUESTION 9

9.1 In the diagram, points D and E lie on sides AB and AC of ΔABC respectively such that DE || BC. DC and BE are joined.



9.1.1 Explain why the areas of ΔDEB and ΔDEC are equal. (1)

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \text{.....}$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \text{.....} = \frac{AE}{EC}$$

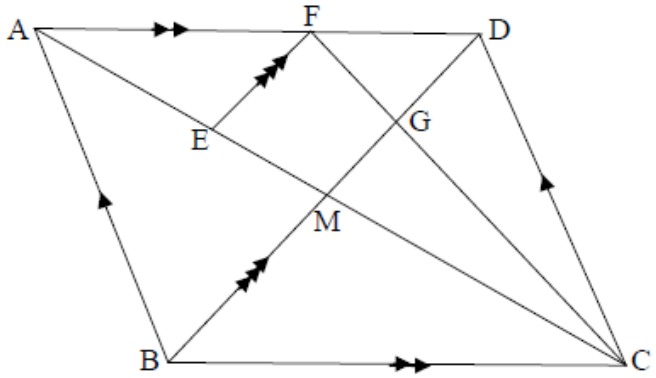
But area ΔDEB = ... (reason: .....)

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \text{.....}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF : FD = 4 : 3. E is a point on AM such that EF || BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

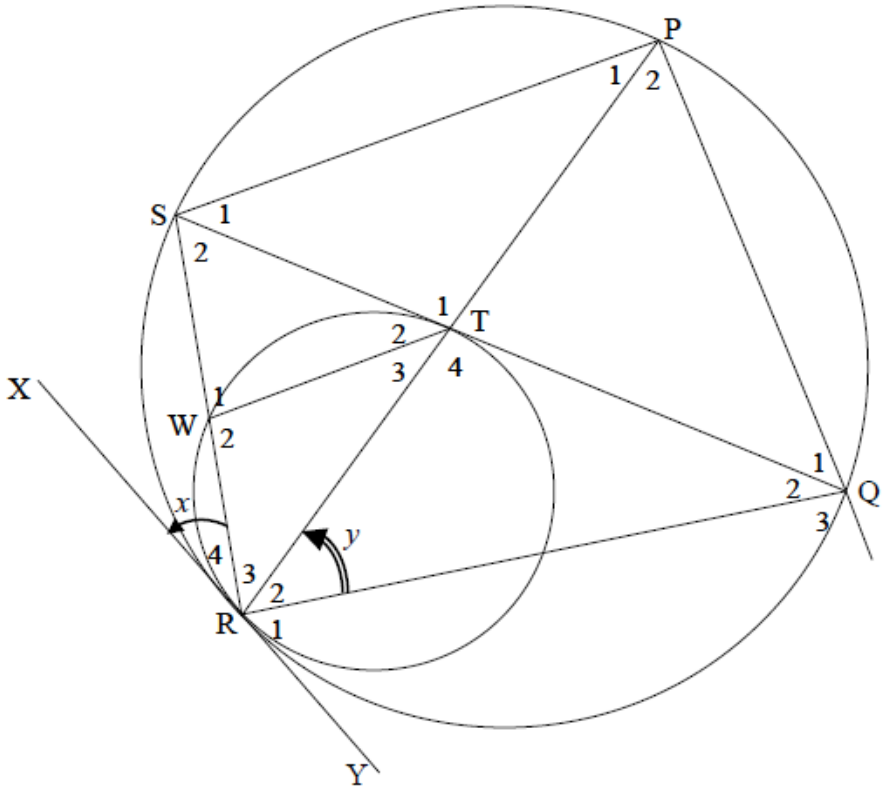
9.2.3 $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4)

[16]

QUESTION 10

The two circles in the diagram have a common tangent XRY at R. W is any point on the small circle. The straight line RWS meets the large circle at S. The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$



10.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 6.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT || SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

10.2 Prove that $RT = \frac{WR.RP}{RS}$ (2)

10.3 Identify, with reasons, another TWO angles equal to y. (4)

10.4 Prove that $\hat{Q}_3 = \hat{W}_2$. (3)

10.5 Prove that $\Delta RTS \,|||\, \Delta RQP$. (3)

10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)
[20]

TOTAL: 150

MATHEMATICS P1

FEBRUARY/MARCH 2015

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (2)

1.1.2 $2x^2 - 11x + 7 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5x^2 + 4 > 21x$ (5)

1.1.4 $2^{2x} - 6.2^x = 16$ (4)

1.2 Solve for x and y simultaneously:

$$y + 1 = 2x$$
$$x^2 - xy + y^2 = 7$$ (6)

1.3 The roots of a quadratic equation are given by $x = \frac{-5 \pm \sqrt{20 + 8k}}{6}$, where $k \in \{-3; -2; -1; 0; 1; 2; 3\}$.

1.3.1 Write down TWO values of k for which the roots will be rational. (2)

1.3.2 Write down ONE value of k for which the roots will be non-real. (1)

1.4 Calculate a and b if $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a(7^b)$ and a is not a multiple of 7. (4)

[27]

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

QUESTION 3

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \ldots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION 4

Given: $g(x) = \frac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

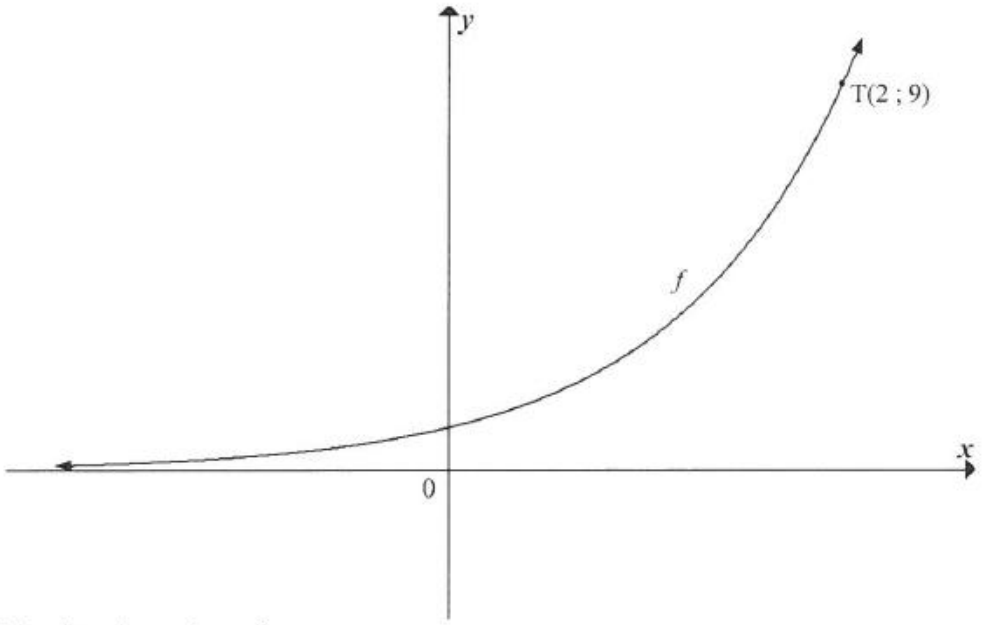
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\frac{6}{x+2} - 1 \geq -x - 3$. (2)

[13]

QUESTION 5

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

5.4 Is the inverse of f a function? Explain your answer. (2)

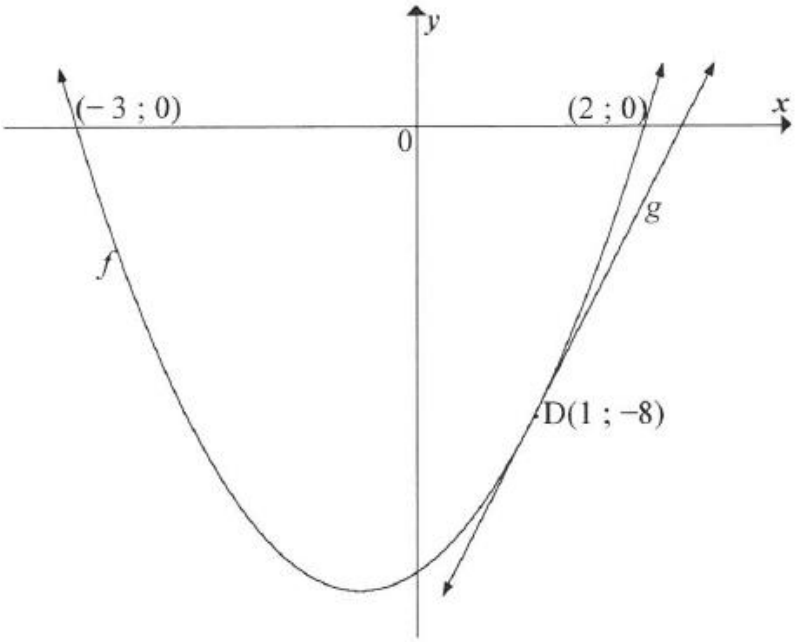
[7]

QUESTION 6

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D(1 ; – 8) is a common point on f and g.

- f intersects the x-axis at $(-3 ; 0)$ and $(2 ; 0)$.
- g is the tangent to f at D.



6.1 For which value(s) of x is $f(x) \leq 0$? (2)

6.2 Determine the values of a, b and c. (5)

6.3 Determine the coordinates of the turning point of f. (3)

6.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x - 7) + 2$. (2)

6.5 Calculate the gradient of g. (3)
[15]

QUESTION 7

7.1 Nomsa started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into an annuity fund. She continued doing this until she retired on 31 December 2013.

7.1.1 Determine the total amount of money that she paid into the fund. (2)

7.1.2 The interest rate on this fund was 8% p.a., compounded monthly. Calculate the value of the fund at the time that she retired. (5)

7.1.3 On 1 January 2014 Nomsa invested R2 million in an account paying interest at 10% p.a. compounded monthly. Nomsa withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If Nomsa wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount she could withdraw at the end of each month. (4)

7.2 For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.

At the end of 2012 the populations of these two towns were equal.

Determine the ratio of the population of town X (call it P_X) to the population of town Y (call it P_Y) at the beginning of 2010. (4)

[15]

QUESTION 8

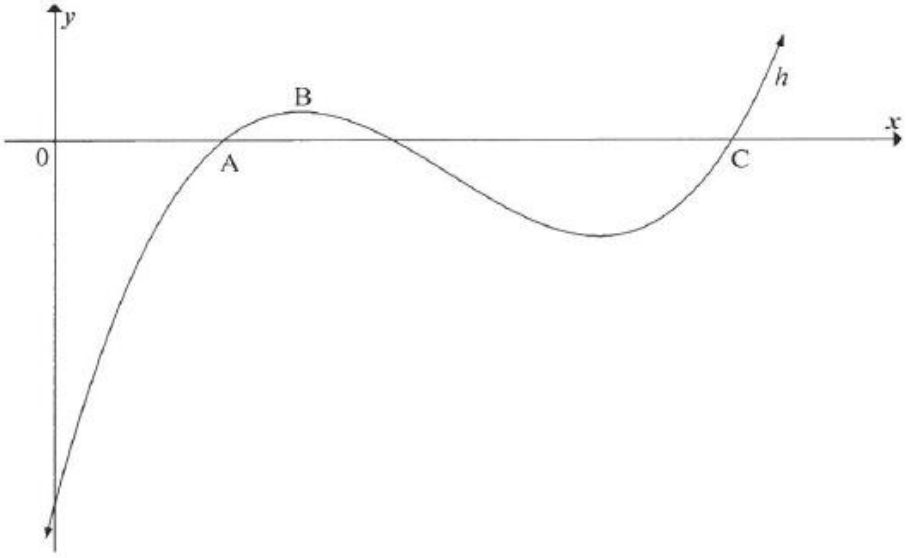
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

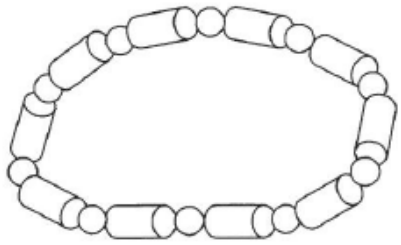
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)

[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

9.1 If the volume of a cylinder is $6\ cm^3$, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)

9.3 Determine the value of r so that the least amount of paint will be used. (4)

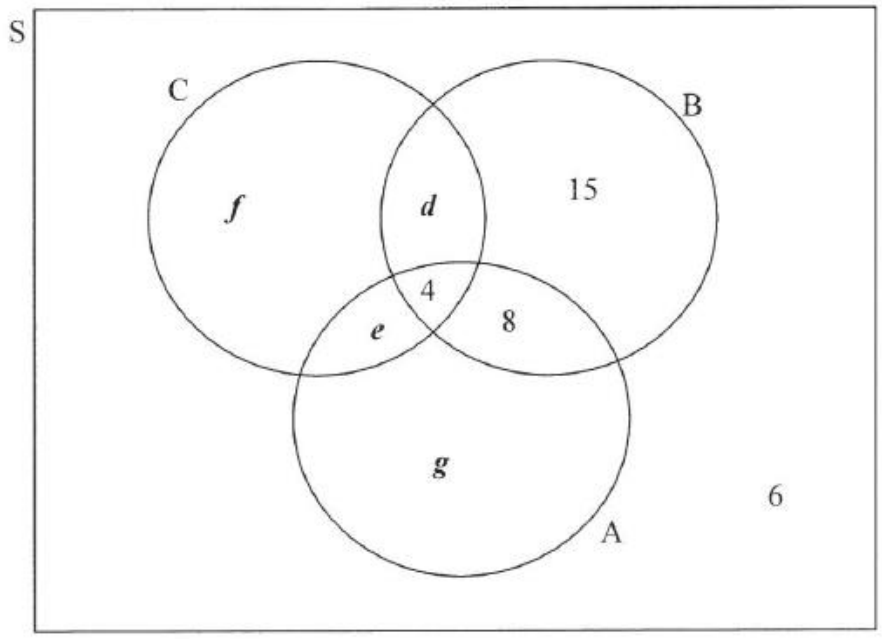
[9]

QUESTION 10

10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



10.1.1 Use the given information and the Venn diagram to determine the values of d, e, f and g. (4)

10.1.2 For a randomly selected country, calculate:

(a) P(A and B and C) (1)

(b) P(A or B or C) (1)

(c) P(only C) (1)

(d) P(that a country uses exactly two methods) (1)

10.2 Nametso may choose DVDs from three categories as listed in the table below:

Drama	Romance	Comedy
• *Last Hero*	• *One Heart*	• *Laughing Dragon*
• *Midnight*	• *You and Me*	• *Falling Down*
• *Stranger Calls*	• *Love Song*	• *Sitting on the Stairs*
• *Missing in Action*	• *Bird's First Nest*	
• *Only 40 Seconds Left*		

10.2.1 Nametso must choose ONE DVD from the Drama category. What is the probability that she will choose *Midnight*? (2)

10.2.2 How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy? (2)

10.2.3 Calculate the probability that she will have *Last Hero* and *Laughing Dragon* as part of her selection in QUESTION 10.2.2. (2)

[14]

TOTAL: 150

MATHEMATICS P2

FEBRUARY/MARCH 2015

QUESTION 1

The table below shows the distances (in kilometres) travelled daily by a sales representative for 21 working days in a certain month.

131	132	140	140	141	144	146
147	149	150	151	159	167	169
169	172	174	175	178	187	189

1.1 Calculate the mean distance travelled by the sales representative. (2)

1.2 Write down the five-number summary for this set of data. (4)

1.3 Use the scaled line on DIAGRAM SHEET 1 to draw a box-and-whisker diagram for this set of data. (2)

1.4 Comment on the skewness of the data. (1)

1.5 Calculate the standard deviation of the distance travelled. (2)

1.6 The sales representative discovered that his odometer was faulty. The actual reading on each of the 21 days was p km more than that which was indicated. Write down, in terms of p (if applicable), the:

1.6.1 Actual mean (1)

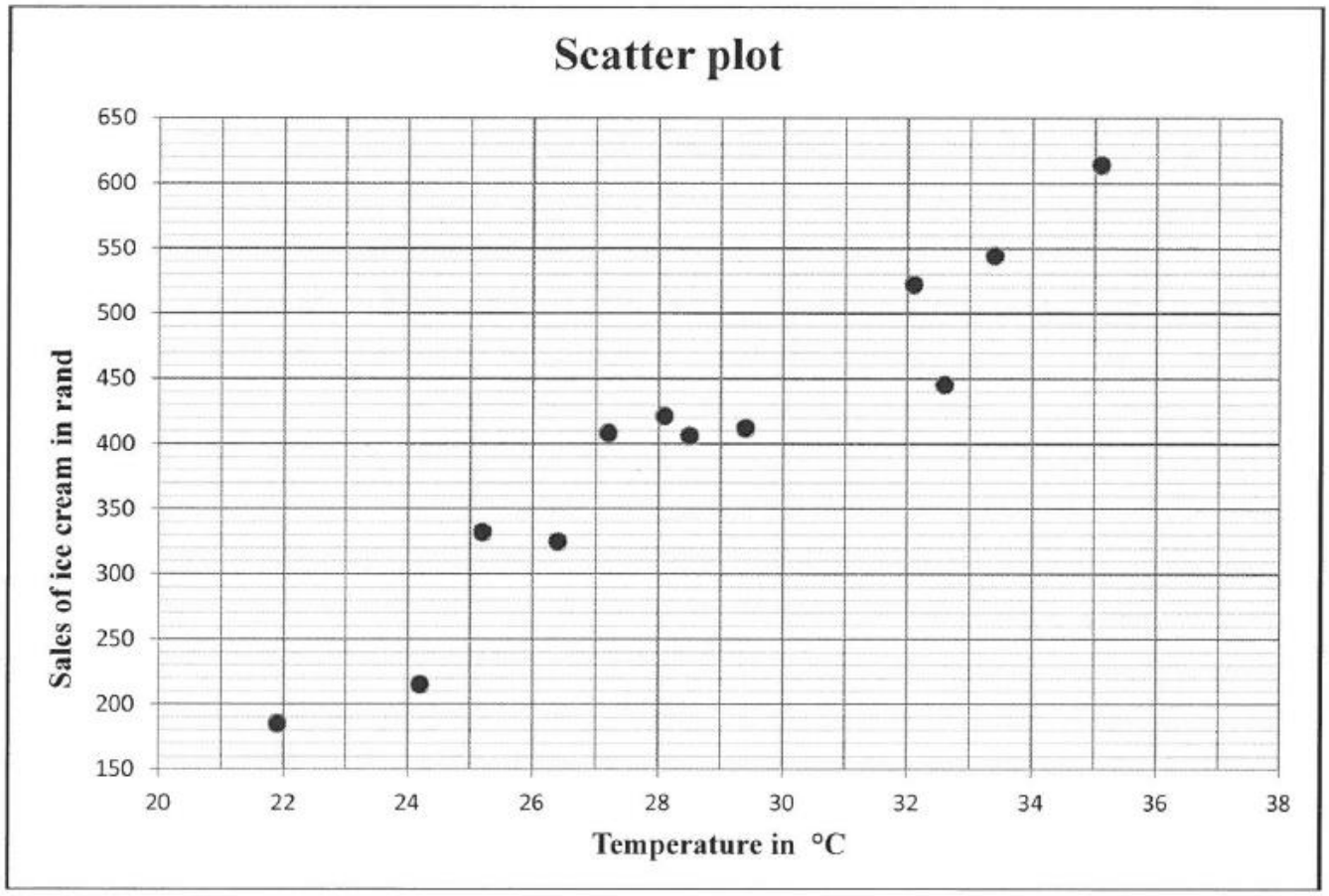
1.6.2 Actual standard deviation (1)

[13]

QUESTION 2

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature, in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

Temperature in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in rand	215	325	185	332	406	522	412	614	544	421	445	408



2.1 Describe the influence of temperature on the sales of ice cream in the scatter plot. (1)

2.2 Give a reason why this trend cannot continue indefinitely. (1)

2.3 Calculate an equation for the least squares regression line (line of best fit). (4)

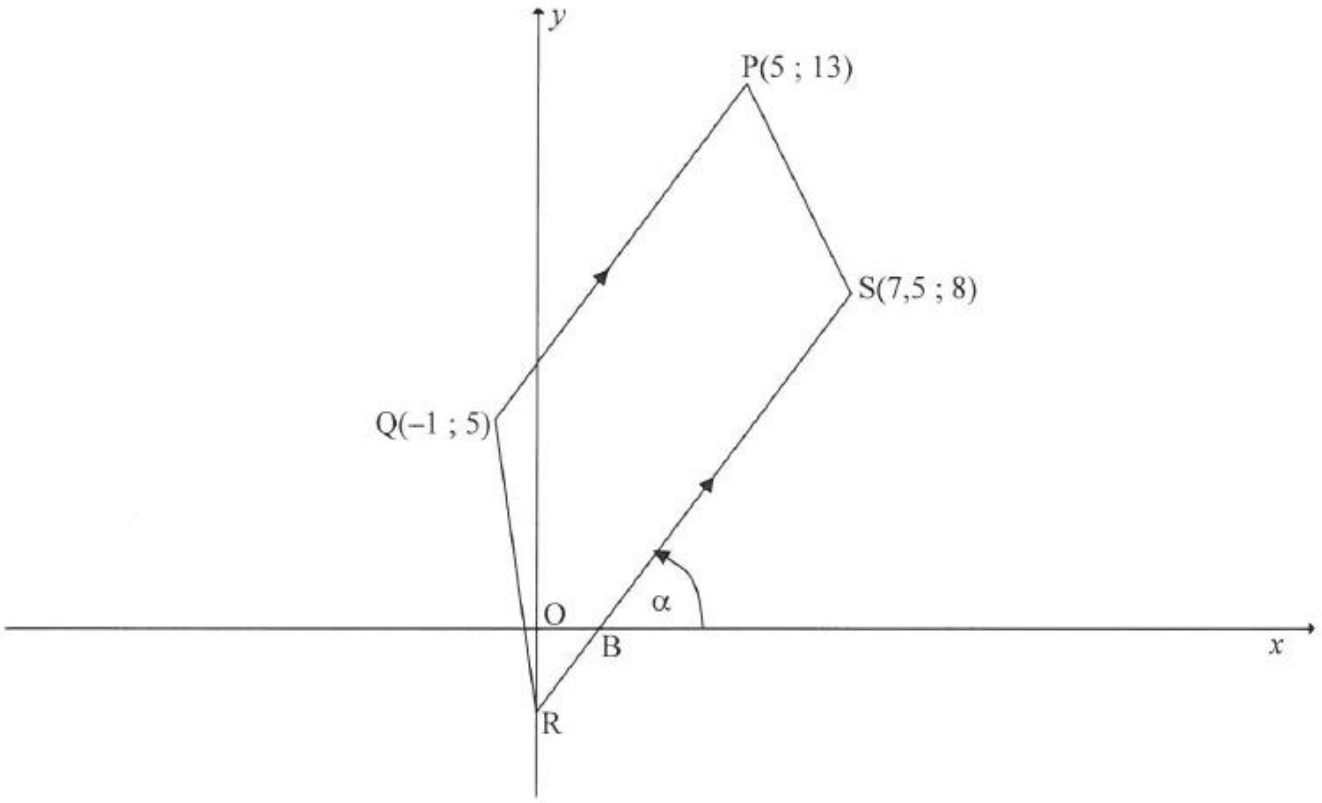
2.4 Calculate the correlation coefficient. (1)

2.5 Comment on the strength of the relationship between the variables. (1)

[8]

QUESTION 3

In the diagram below points P(5 ; 13), Q(–1 ; 5) and S(7,5 ; 8) are given. SR || PQ where R is the y-intercept of SR. The x-intercept of SR is B. QR is joined.



3.1 Calculate the length of PQ. (3)

3.2 Calculate the gradient of PQ. (2)

3.3 Determine the equation of line RS in the form $ax + by + c = 0$. (4)

3.4 Determine the x-coordinate of B. (2)

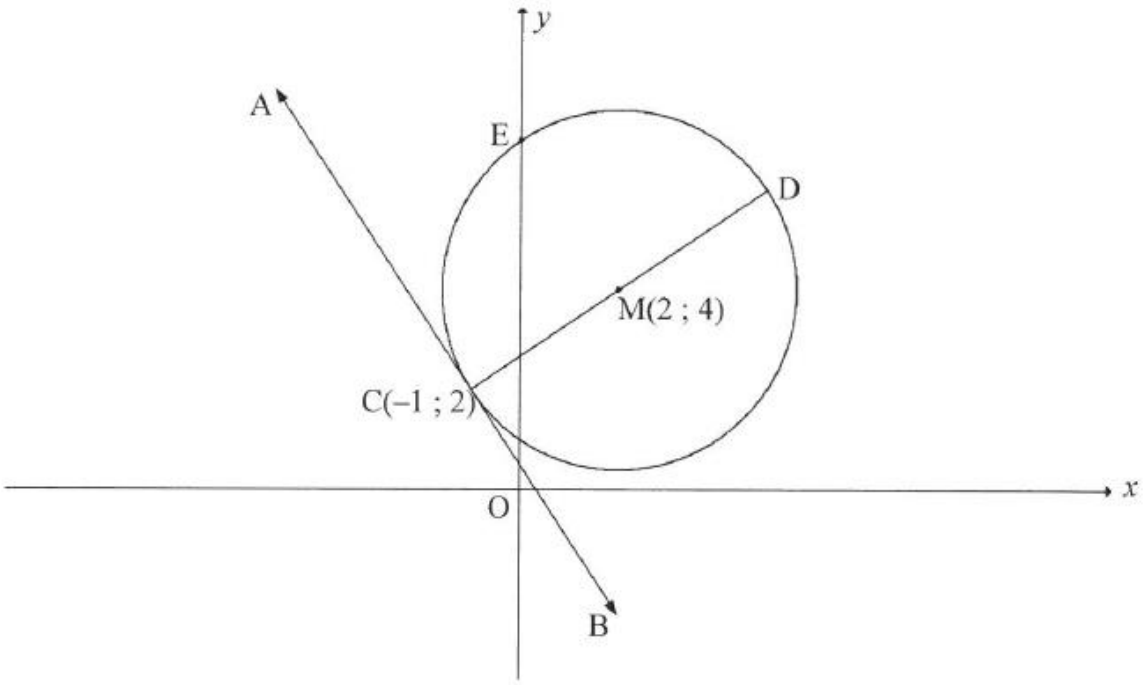
3.5 Calculate the size of $O\hat{R}B$. (3)

3.6 Prove that QBSP is a parallelogram. (4)

[18]

QUESTION 4

4.1 In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



4.1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Write down the coordinates of D. (2)

4.1.3 Determine the equation of AB in the form $y=mx+c$. (5)

4.1.4 Calculate the coordinates of E. (4)

4.1.5 Show that EM is parallel to AB. (2)

4.2 Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations. (6)

[22]

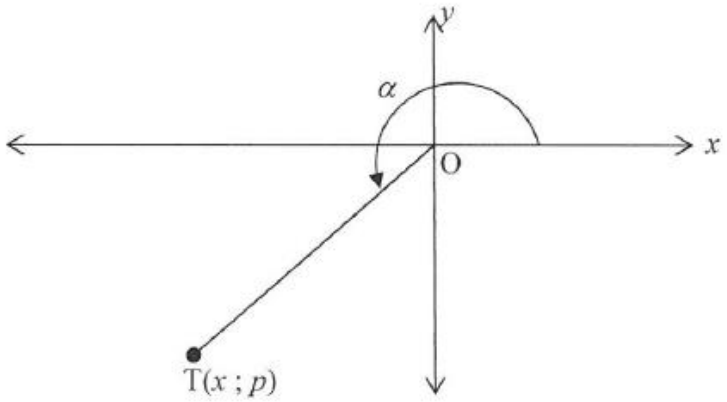
QUESTION 5

5.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$$ (6)

5.3 In the diagram below, $T(x\ ;\ p)$ is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180° + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$. (3)

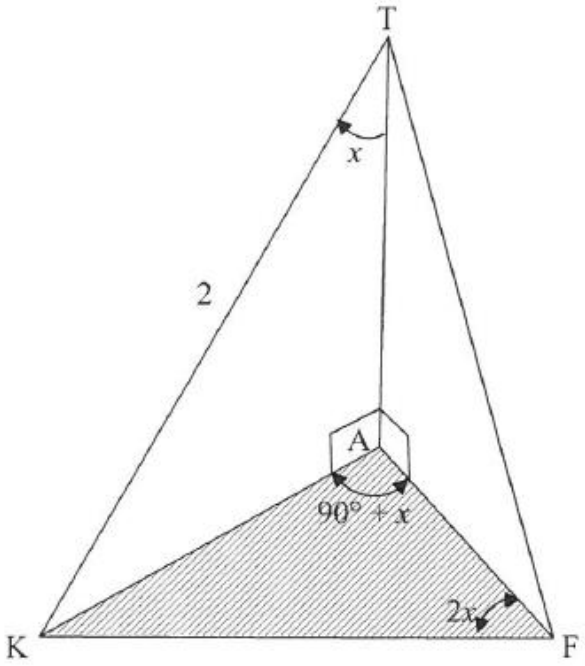
5.4 5.4.1 For which value(s) of x will $\frac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0° \le x \le 180°$? (3)

5.4.2 Prove the identity: $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

QUESTION 6

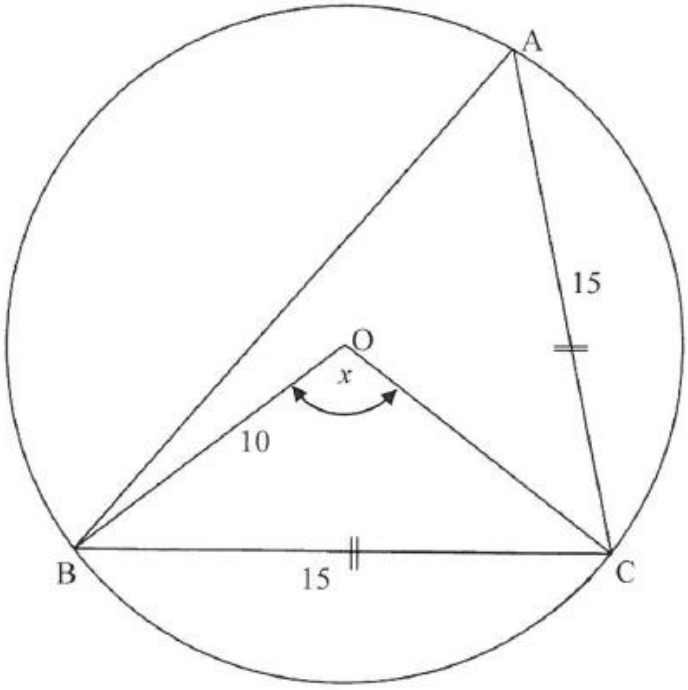
6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. TK = 2 units.



6.1.1 Express AK in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of KF. (5)

6.2 In the diagram below, a circle with centre O passes through A, B and C. BC = AC = 15 units. BO and OC are joined. OB = 10 units and $B\hat{O}C = x$.



Calculate:

6.2.1 The size of x (4)

6.2.2 The size of $A\hat{C}B$ (3)

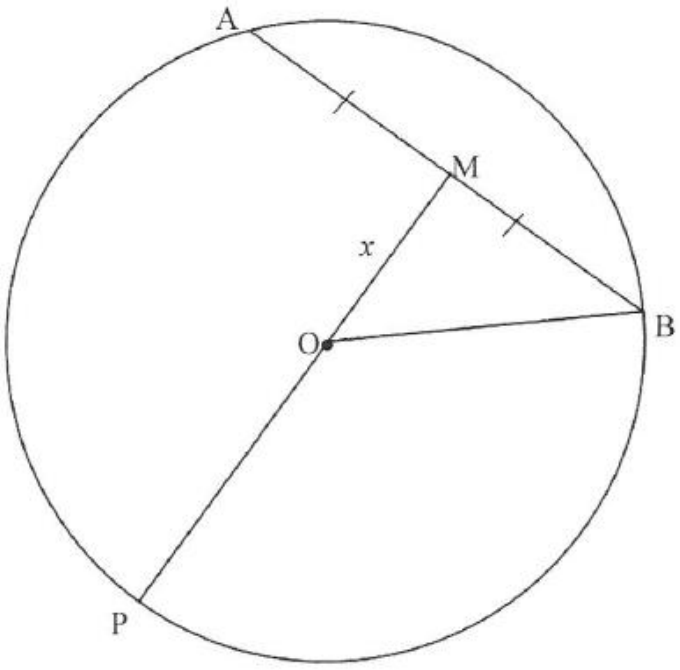
6.2.3 The area of ΔABC (2)

[16]

GIVE REASONS FOR YOUR ANSWERS IN QUESTIONS 7, 8, 9 AND 10.

QUESTION 7

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = x units, AB = 20 units and $\frac{PM}{OM} = \frac{5}{2}$.



7.1 Write down the length of MB. (1)

7.2 Give a reason why OM ⊥ AB. (1)

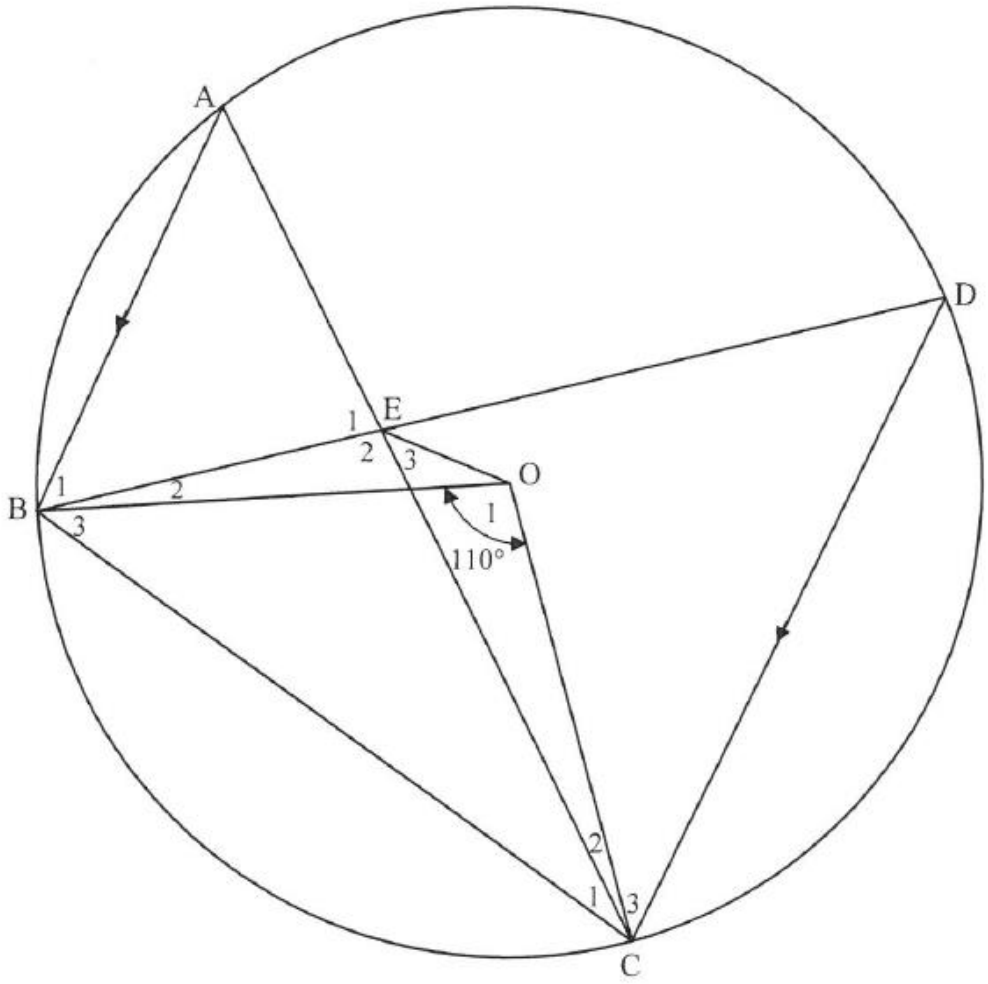
7.3 Show that OP = $\frac{3x}{2}$ units. (2)

7.4 Calculate the value of x. (3)

[7]

QUESTION 8

In the diagram below, the circle with centre O passes through A, B, C and D.
AB || DC and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

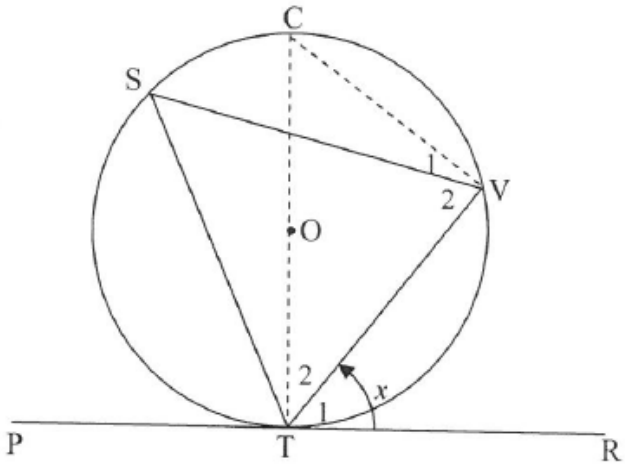
8.2 Prove that BEOC is a cyclic quadrilateral. (2)
[10]

QUESTION 9

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



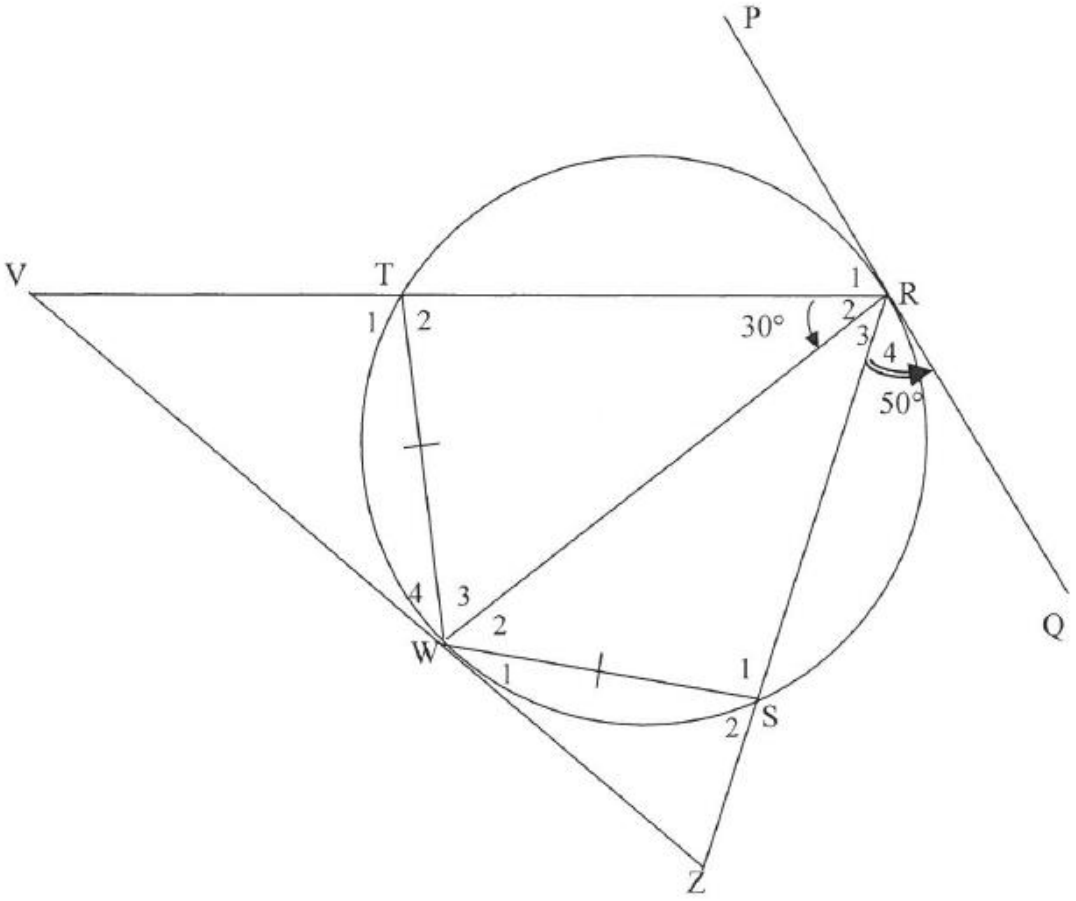
Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$.
Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.

Construction: Draw diameter TC and join CV.

Statement	Reason
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$.....	.....
$\hat{T}_2 = 90° - x$	.....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	.....
$\therefore V\hat{T}R = \hat{S}$	

(5)

9.3 In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



9.3.1 Give a reason why $\hat{R}_3 = 30°$. (1)

9.3.2 State, with reasons, TWO other angles equal to 30°. (3)

9.3.3 Determine, with reasons, the size of:

(a) $\hat{S}_2$ (3)

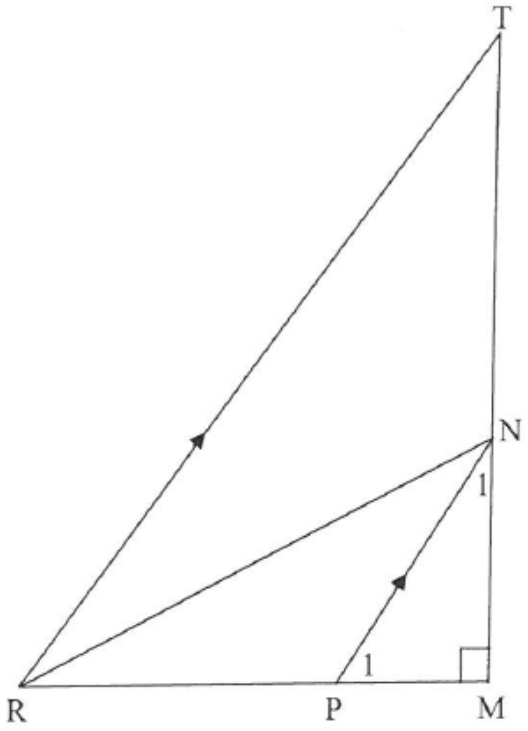
(b) $\hat{V}$ (4)

9.3.4 Prove that $WR^2 = RV \times RS$. (5)

[22]

QUESTION 10

In ΔTRM, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that $RT = 3PN$.



10.1 Give reasons for the statements below.
Use DIAGRAM SHEET 5.

	Statement	**Reason**
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	.....
	$\hat{M}$ is common	
10.1.2	$\therefore \Delta PNM \text{ ||| } \Delta RTM$	.....

(2)

10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$. (2)

10.3 Show that $RN^2 - PN^2 = 2RP^2$. (4)

[8]

TOTAL: 150

MATHEMATICS P1

NOVEMBER 2015

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ (correct to TWO decimal places) (4)

1.1.3 $2x^{\frac{-5}{3}} = 64$ (4)

1.1.4 $\sqrt{2-x} = x - 2$ (4)

1.1.5 $x^2 + 7x < 0$ (3)

1.2 Given: $(3x - y)^2 + (x - 5)^2 = 0$

Solve for x and y. (4)

1.3 For which value of k will the equation $x^2 + x = k$ have no real roots? (4)

[26]

QUESTION 2

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence. (2)

2.2 Determine the n^{th} term, T_n, in terms of n. (2)

2.3 Explain why the infinite series $10 + 5 + 2{,}5 + 1{,}25 + \ldots$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4)

[10]

QUESTION 3

Consider the series: $S_n = -3+5+13+21+...$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6-3$
$Q_3 = -6-3+5$
$Q_4 = -6-3+5+13$
$Q_5 = -6-3+5+13+21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the value of Q_{129}. (3)

[12]

QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of f. (1)

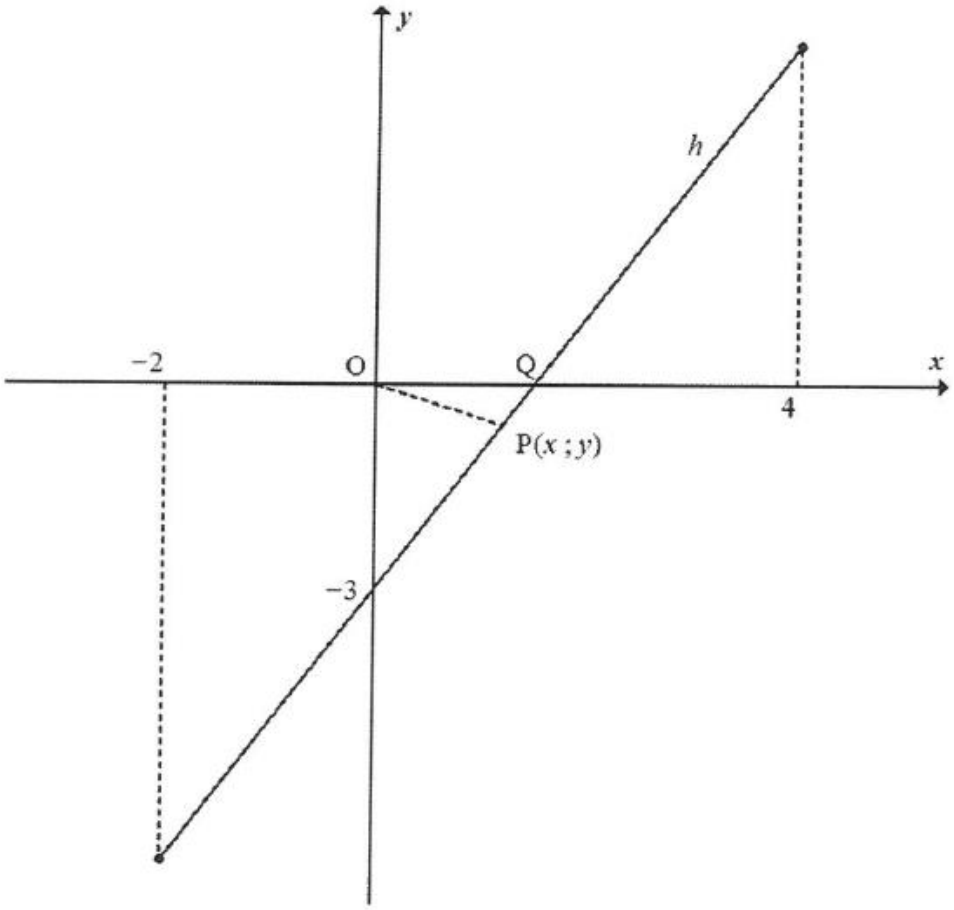
4.2 Sketch the graph of f. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g. (1)

[6]

QUESTION 5

Given: $h(x)=2x-3$ for $-2\le x\le 4$. The x-intercept of h is Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x)=h^{-1}(x)$? (3)

5.5 P(x ; y) is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x)=f'(x)$ where f is a function defined for $-2\le x\le 4$.

5.6.1 Explain why f has a local minimum. (2)

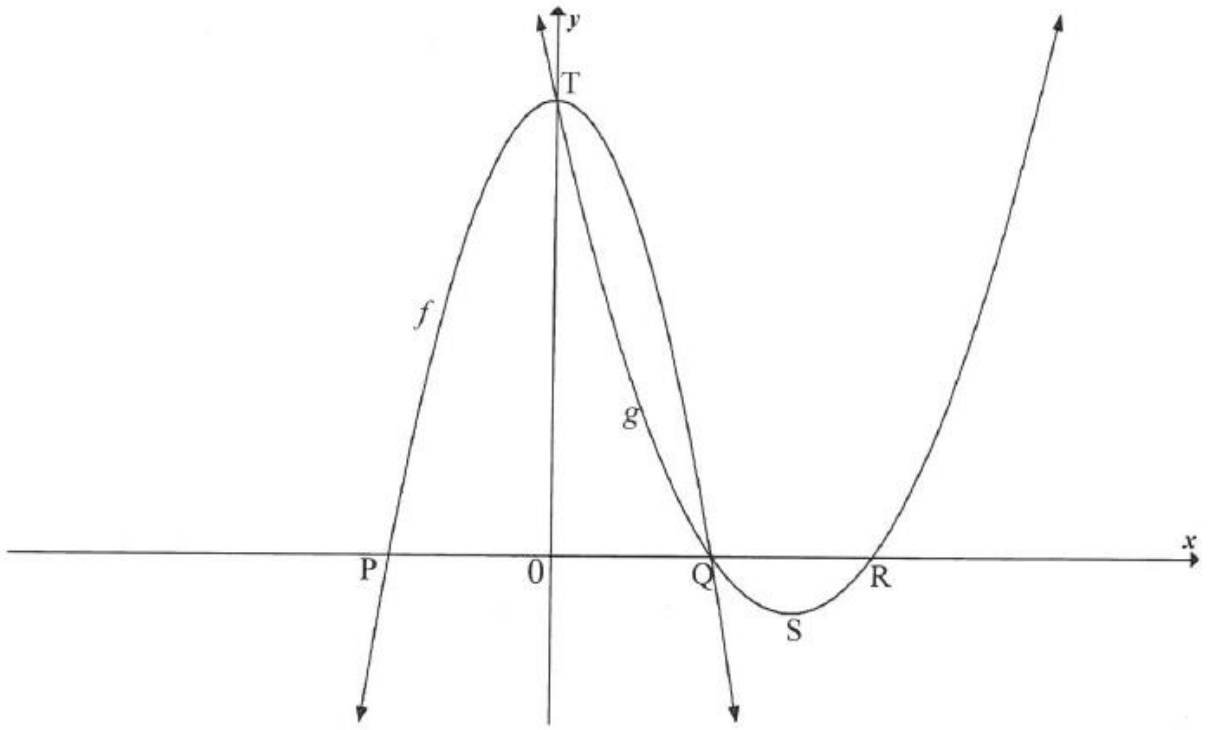
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

[19]

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

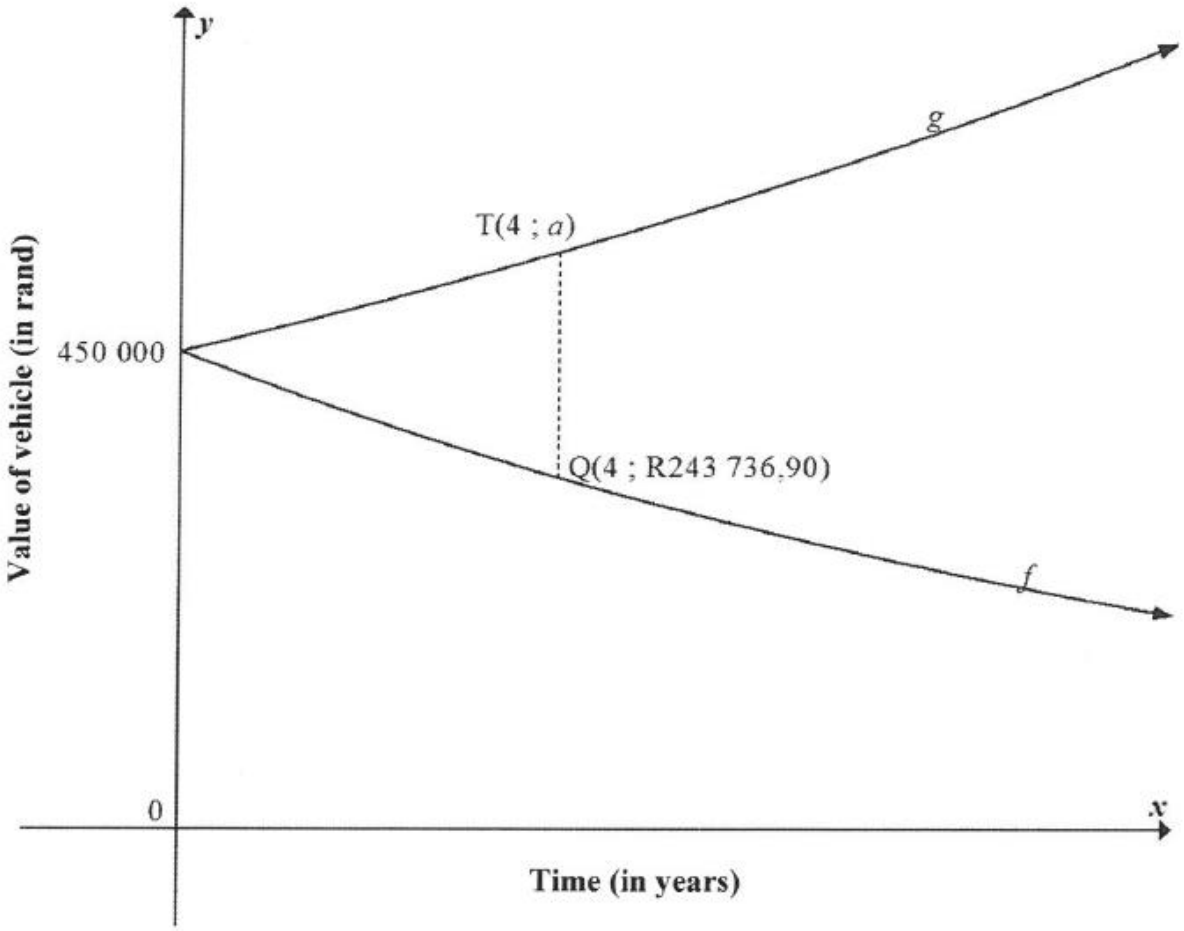
- The domain is $x \in R,\ x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[12]

QUESTION 7

The graph of f shows the book value of a vehicle x years after the time Joe bought it.
The graph of g shows the cost price of a similar new vehicle x years later.



7.1 How much did Joe pay for the vehicle? (1)

7.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)

7.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3)

7.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund. (5)
[13]

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

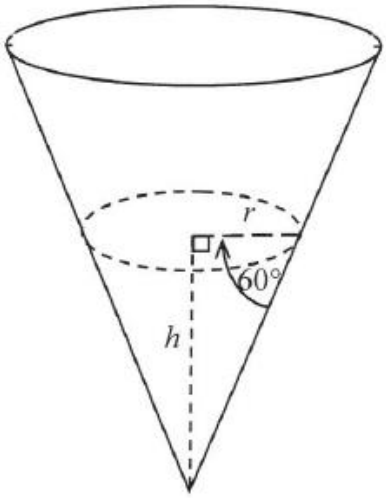
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

QUESTION 11

11.1 For two events, A and B, it is given that:

$P(A) = 0,2$
$P(B) = 0,63$
$P(A \text{ and } B) = 0,126$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)
[17]

TOTAL: 150

MATHEMATICS P2

NOVEMBER 2015

QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)

1.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)

1.3 Identify an outlier in the data set. (1)

1.4 Calculate the value of the correlation coefficient. (2)

1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)

[11]

QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

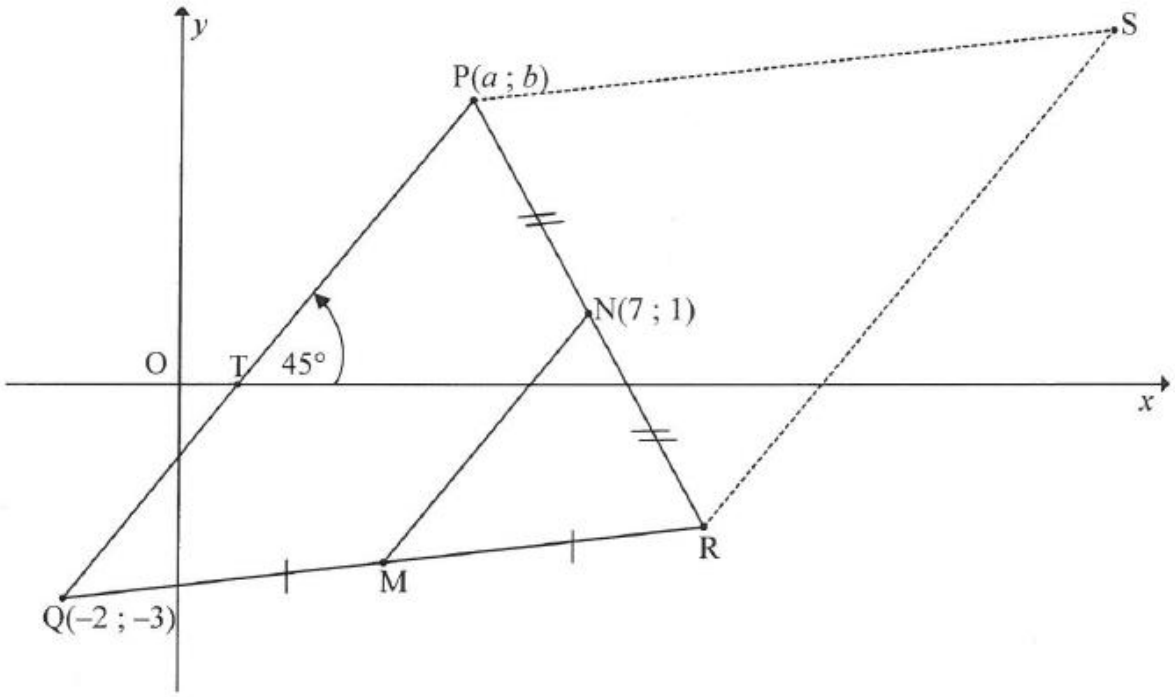
2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)

[9]

QUESTION 3

In the diagram below, the line joining Q(–2 ; –3) and P(a ; b), a and $b > 0$, makes an angle of 45° with the positive x-axis. QP = $7\sqrt{2}$ units. N(7 ; 1) is the midpoint of PR and M is the midpoint of QR.



Determine:

3.1 The gradient of PQ (2)

3.2 The equation of MN in the form $y = mx + c$ and give reasons (4)

3.3 The length of MN (2)

3.4 The length of RS (1)

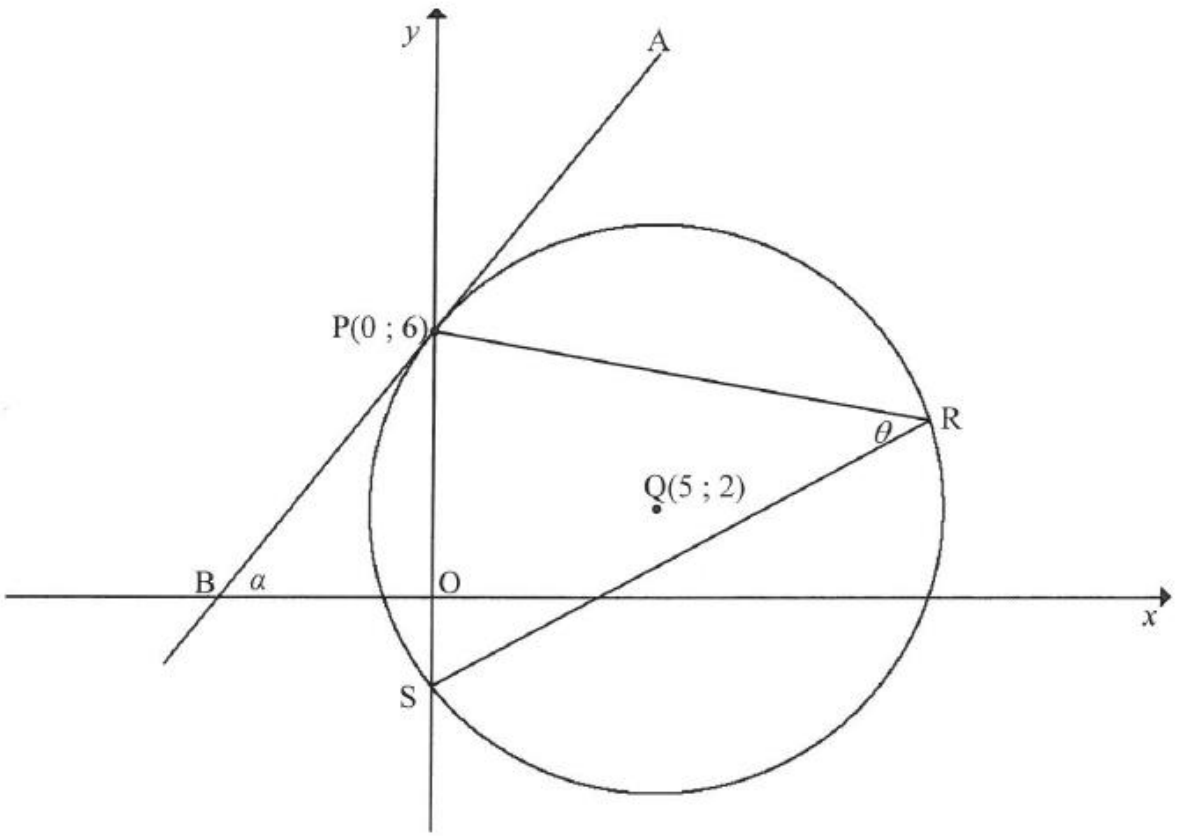
3.5 The coordinates of S such that PQRS, in this order, is a parallelogram (3)

3.6 The coordinates of P (6)

[18]

QUESTION 4

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4)

[20]

QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

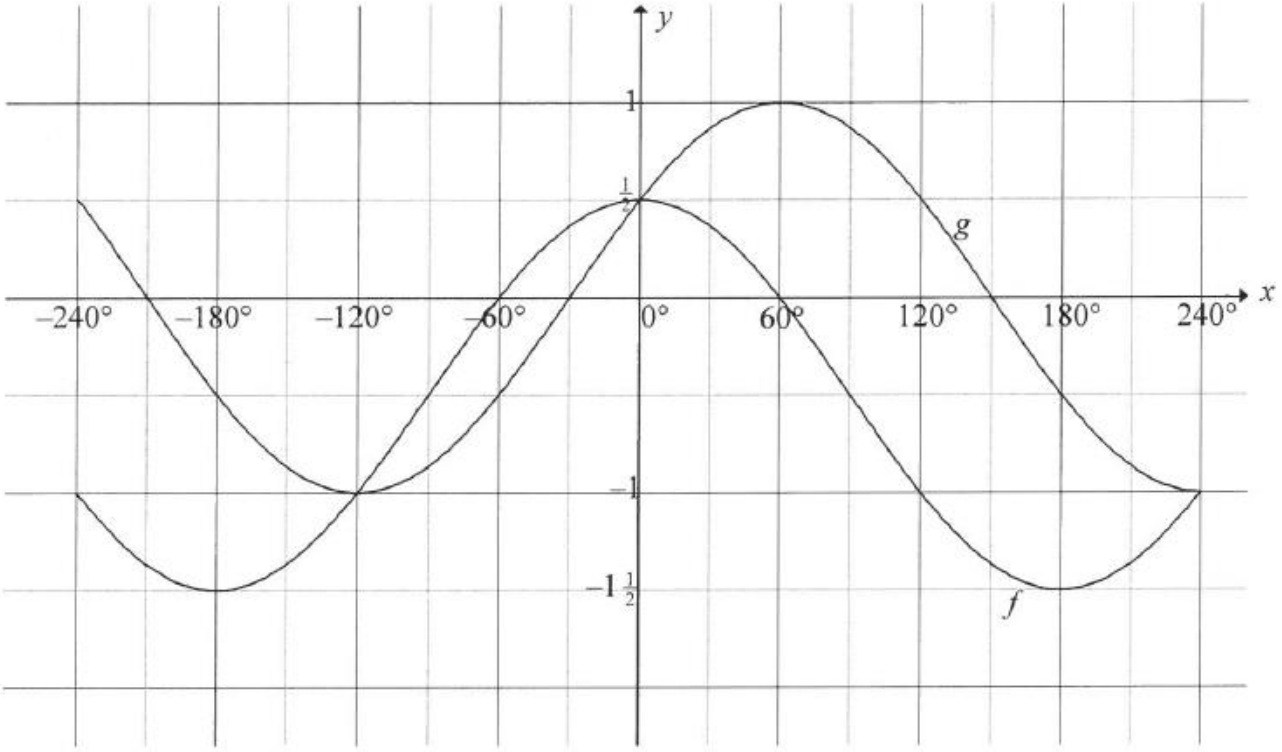
5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \leq x \leq 240°$. The graphs intersect at $\left(0° ; \frac{1}{2}\right)$, $(-120° ; -1)$ and $(240° ; -1)$.



6.1 Determine the values of p and q. (4)

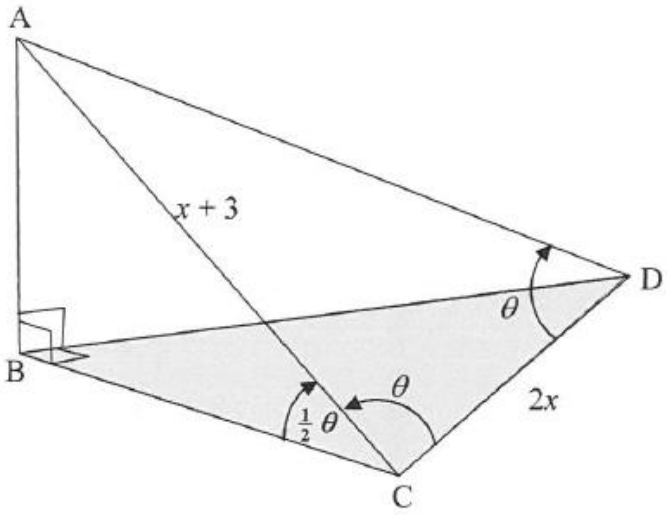
6.2 Determine the values of x in the interval $-240° \leq x \leq 240°$ for which $f(x) > g(x)$. (2)

6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2)

[8]

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.
The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.
Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

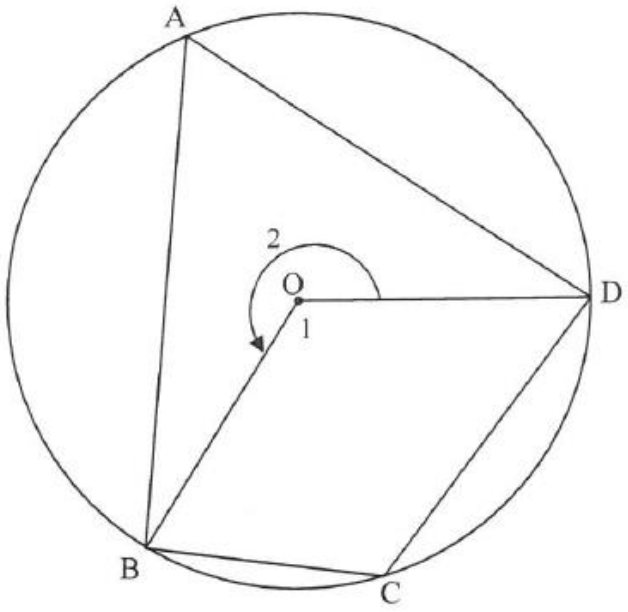
7.2 Prove that $\cos\theta = \frac{x}{x+3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)
[10]

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.

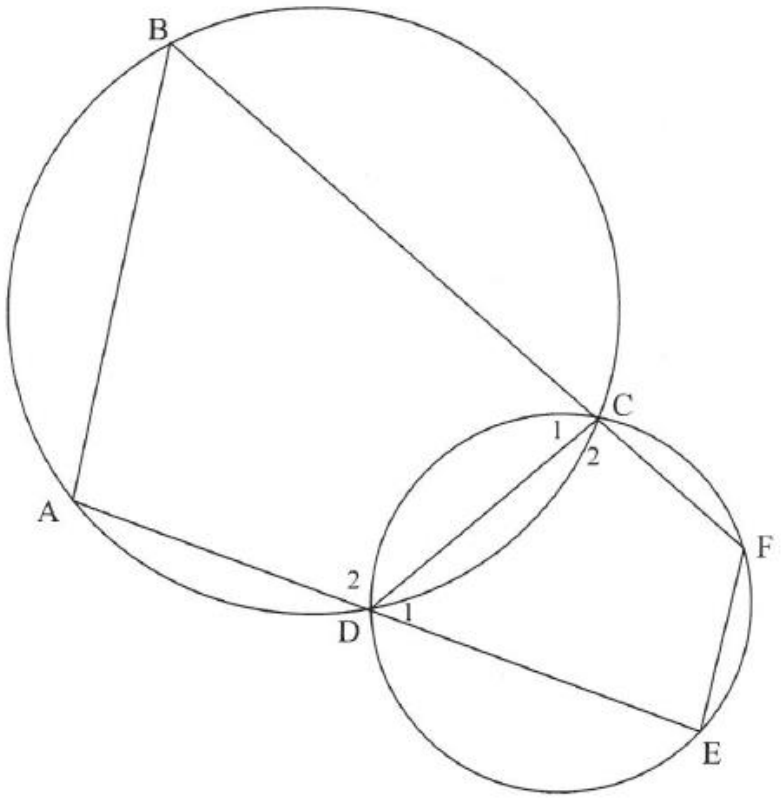


8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is … the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180°$. (3)

8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

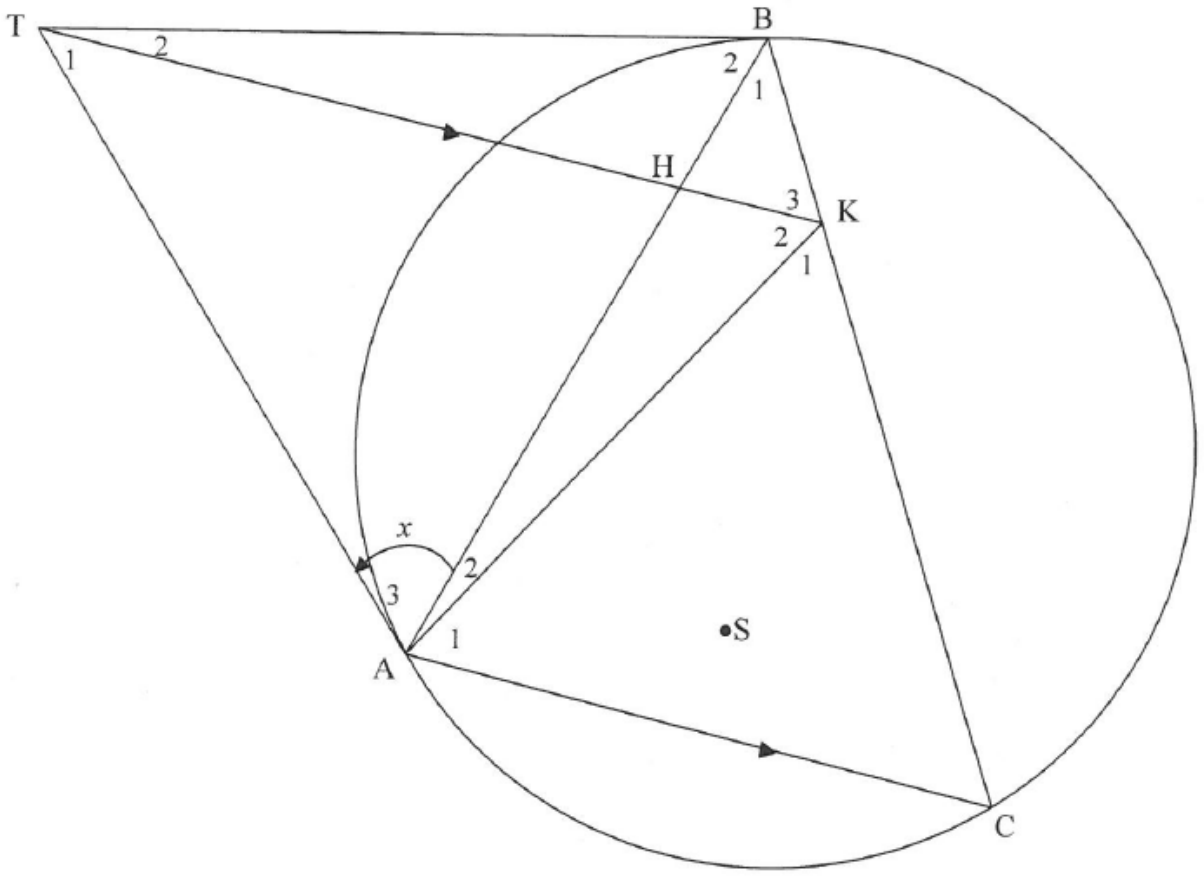


Prove that EF | | AB. (5)

[9]

QUESTION 9

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$. (4)

9.2 Prove that AKBT is a cyclic quadrilateral. (2)

9.3 Prove that TK bisects $A\hat{K}B$. (4)

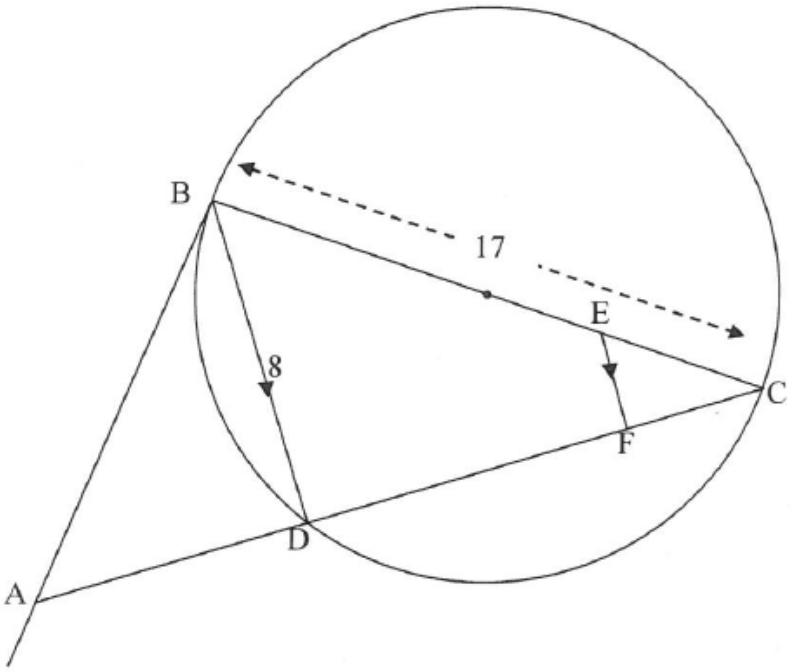
9.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic. (2)

[14]

QUESTION 10

In the diagram below, BC = 17 units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A.



10.1 Calculate, with reasons, the length of DC. (3)

10.2 E is a point on BC such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.

10.2.1 Calculate, with reasons, the length of CF. (3)

10.2.2 Prove that $\Delta BAC \,|||\, \Delta FEC$. (5)

10.2.3 Calculate the length of AC. (4)

10.2.4 Write down, giving reasons, the radius of the circle passing through points A, B and C. (2)

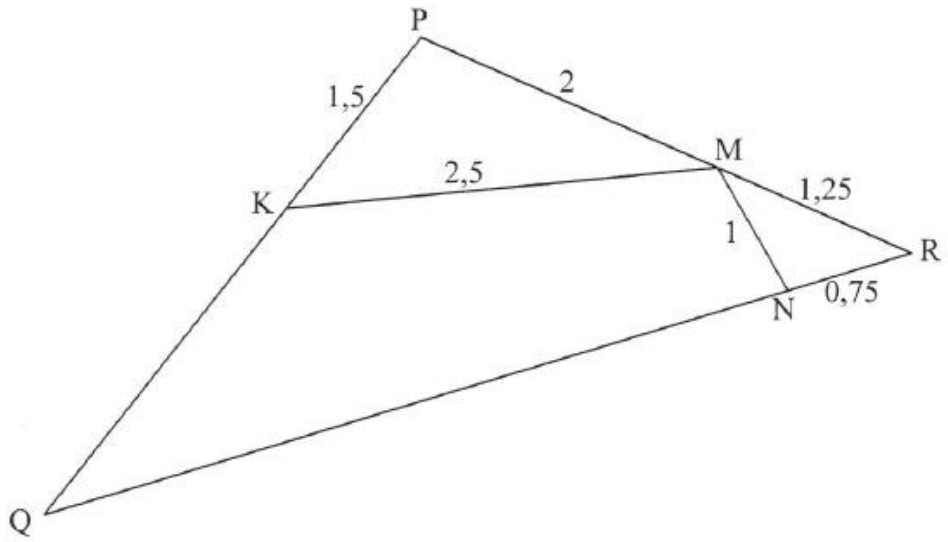
[17]

QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75.



11.2.1 Prove that ΔKPM ||| ΔRNM. (3)

11.2.2 Determine the length of NQ. (6)
[10]

TOTAL: 150

MATHEMATICS P1

FEBRUARY/MARCH 2016

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 12 = 0$ (3)

1.1.2 $x(x+3) - 1 = 0$ (Leave your answer in simplest surd form.) (3)

1.1.3 $x(4-x) < 0$ (3)

1.1.4 $x = \dfrac{a^2 + a - 2}{a - 1}$ if $a = 888\,888\,888\,888$ (2)

1.2 Solve the following equations simultaneously:

$y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$ (6)

1.3 Determine the range of the function $y = x + \dfrac{1}{x}$, $x \neq 0$ and x is real. (6)

[23]

QUESTION 2

2.1 Given the following quadratic sequence: $-2\ ;\ 0\ ;\ 3\ ;\ 7\ ;\ \ldots$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

[18]

QUESTION 3

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	**During the second year**	**During the third year**
100 mm	70 mm	49 mm

3.1 Chris noted that the sequence of height increases, namely 100 ; 70 ; 49 ..., was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)

3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$. (3)

3.3 What height will the tree eventually reach? (3)
[10]

QUESTION 4

Given: $f(x)=2^{-x}+1$

4.1 Determine the coordinates of the y-intercept of f. (1)

4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

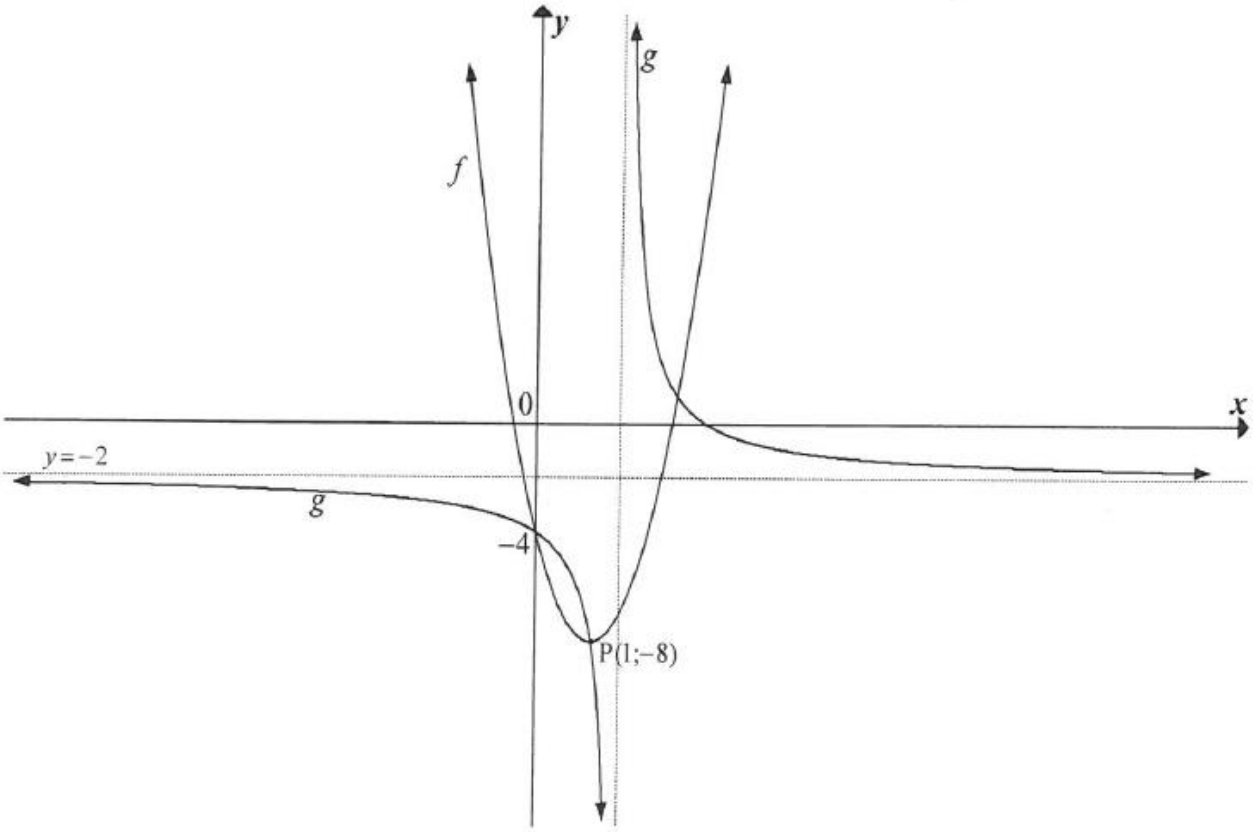
4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3)

4.4 If $h(x)=3f(x)$, write down an equation of the asymptote of h. (1)
[8]

QUESTION 5

The graphs of the functions $f(x)=a(x+p)^2+q$ and $g(x)=\frac{k}{x+r}+d$ are sketched below.

Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of f. The horizontal asymptote of g is $y=-2$.



5.1 Calculate the values of a, p and q. (4)

5.2 Calculate the values of k, r and d. (6)

5.3 Determine the value(s) of x in the interval $x\leq 1$ for which $g(x)\geq f(x)$. (2)

5.4 Determine the value(s) of k for which $f(x)=k$ has two, unequal positive roots. (2)

5.5 Write down an equation for the axis of symmetry of g that has a negative gradient. (3)

5.6 The point P is reflected in the line determined in QUESTION 5.5 to give the point Q. Write down the coordinates of Q. (2)

[19]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x - 2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

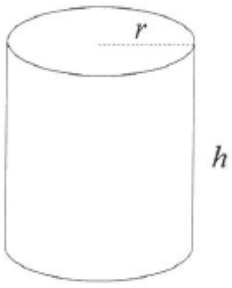
8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

QUESTION 9

A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.



9.1 Express h in terms of r. (2)

9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)

[8]

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = \ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)
[8]

QUESTION 7

7.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.

7.1.1 What is the quarterly interest rate for Diane's investment? (1)

7.1.2 Calculate the amount in Diane's savings account at the end of the 2 years. (3)

7.2 Motloi inherits R800 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.

7.2.1 How many withdrawals of R10 000 will Motloi be able to make from this fund? (5)

7.2.2 Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and to use it as a deposit towards a house.

(a) What is the value of Motloi's deposit, to the nearest rand? (4)

(b) Motloi's deposit is exactly 30% of the purchase price of the house. What is the purchase price of the house, to the nearest rand? (1)
[14]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x - 2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

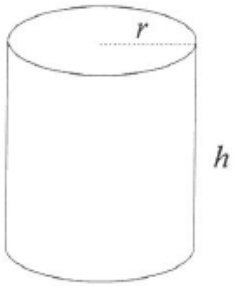
8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

QUESTION 9

A soft drink can has a volume of $340\ \text{cm}^3$, a height of h cm and a radius of r cm.



9.1 Express h in terms of r. (2)

9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)

[8]

QUESTION 10

10.1 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	***b***	***c***	80
Fruit juice	***d***	***e***	20
TOTAL	60	100	***a***

10.1.1 Write down the value of ***a***. (1)

10.1.2 What is the probability that a randomly selected passenger is male? (2)

10.1.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of ***b***. (4)

10.2 A Banana Airways aeroplane has 6 seats in each row.

10.2.1 How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)

10.2.2 Xoliswa, Anees and 4 other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must sit next to each other? (2)

10.2.3 Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? (4)

[15]

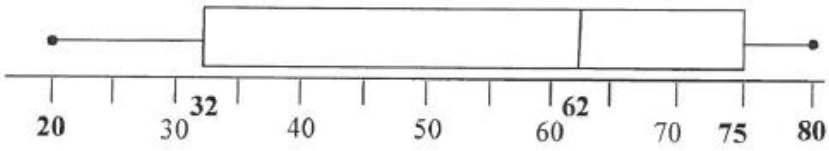
TOTAL: 150

MATHEMATICS P2

FEBRUARY/MARCH 2016

QUESTION 1

The box and whisker diagram below shows the marks (out of 80) obtained in a History test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (2)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (2)

1.4 In ascending order, the second mark is 28, the third mark 36 and the sixth mark 69. The seventh and eighth marks are the same. The average mark for this test is 54.

	28	36			69			

Fill in the marks of the remaining learners in ascending order. (6)

[11]

QUESTION 2

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

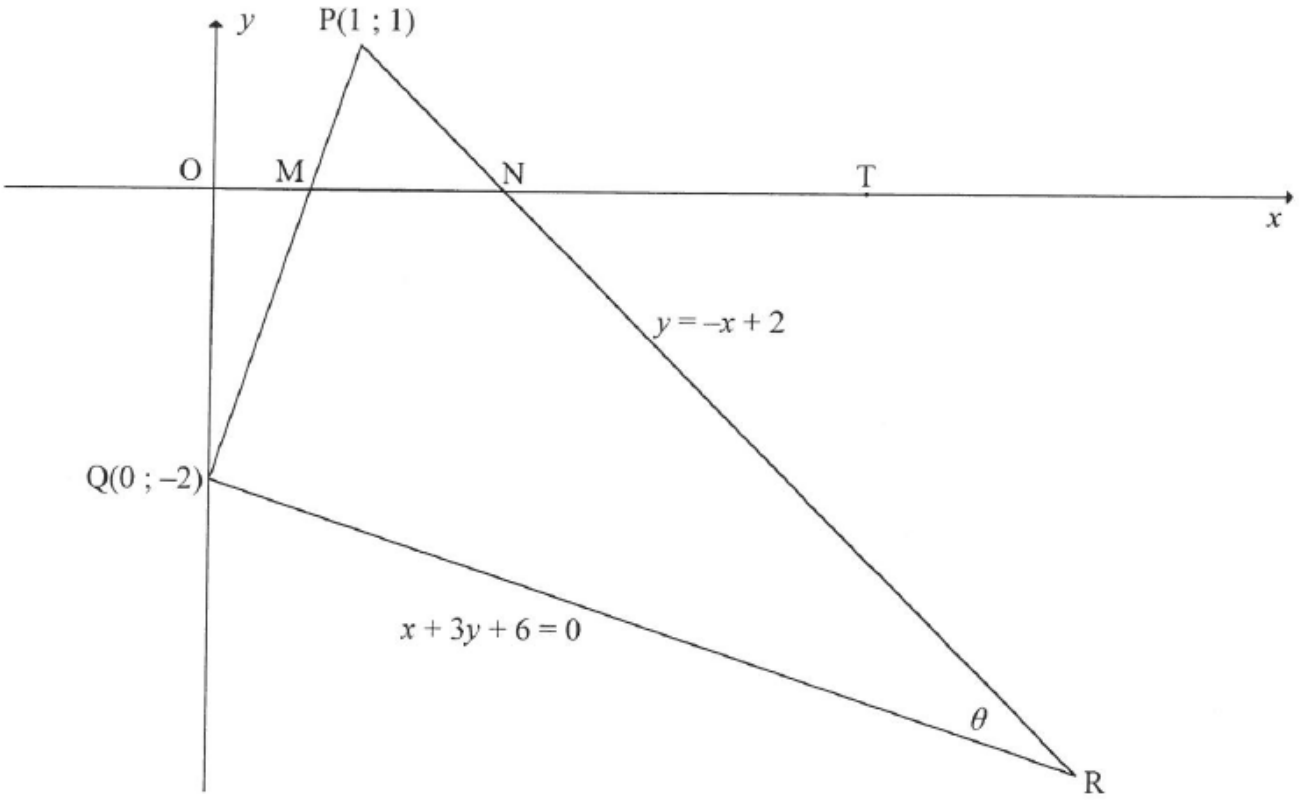
2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

2.3 Hence, estimate the number of days on which 65 or more messages were sent. (2)
[9]

QUESTION 3

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



3.1 Determine the gradient of QP. (2)

3.2 Prove that $P\hat{Q}R = 90°$. (2)

3.3 Determine the coordinates of R. (3)

3.4 Calculate the length of PR. Leave your answer in surd form. (2)

3.5 Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$. (6)

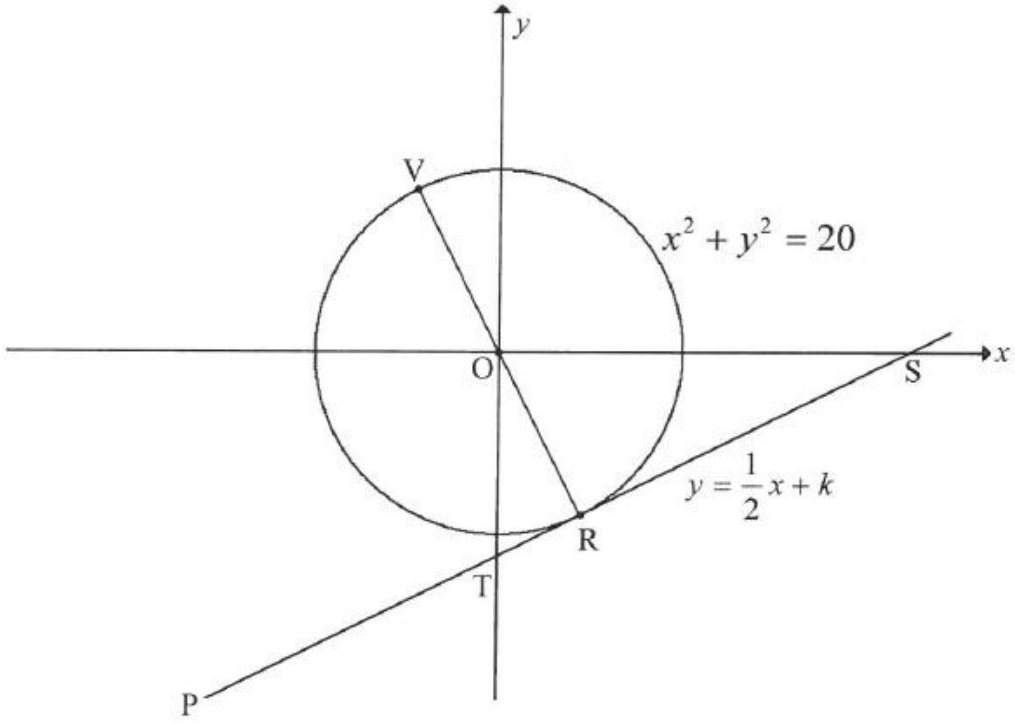
3.6 Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$. (3)

3.7 Calculate the size of θ. (5)

[23]

QUESTION 4

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1 Determine, giving reasons, the equation of OR in the form $y = mx + c$. (3)

4.2 Determine the coordinates of R. (4)

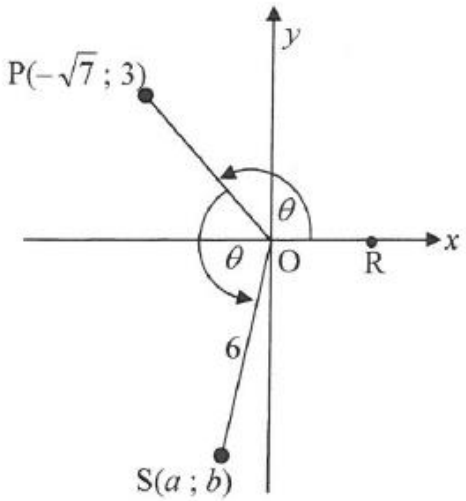
4.3 Determine the area of ΔOTS, given that R(2 ; –4). (6)

4.4 Calculate the length of VT. (4)

[17]

QUESTION 5

5.1 $P(-\sqrt{7}\,;3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and $OS = 6$.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x - 1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15° - 1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

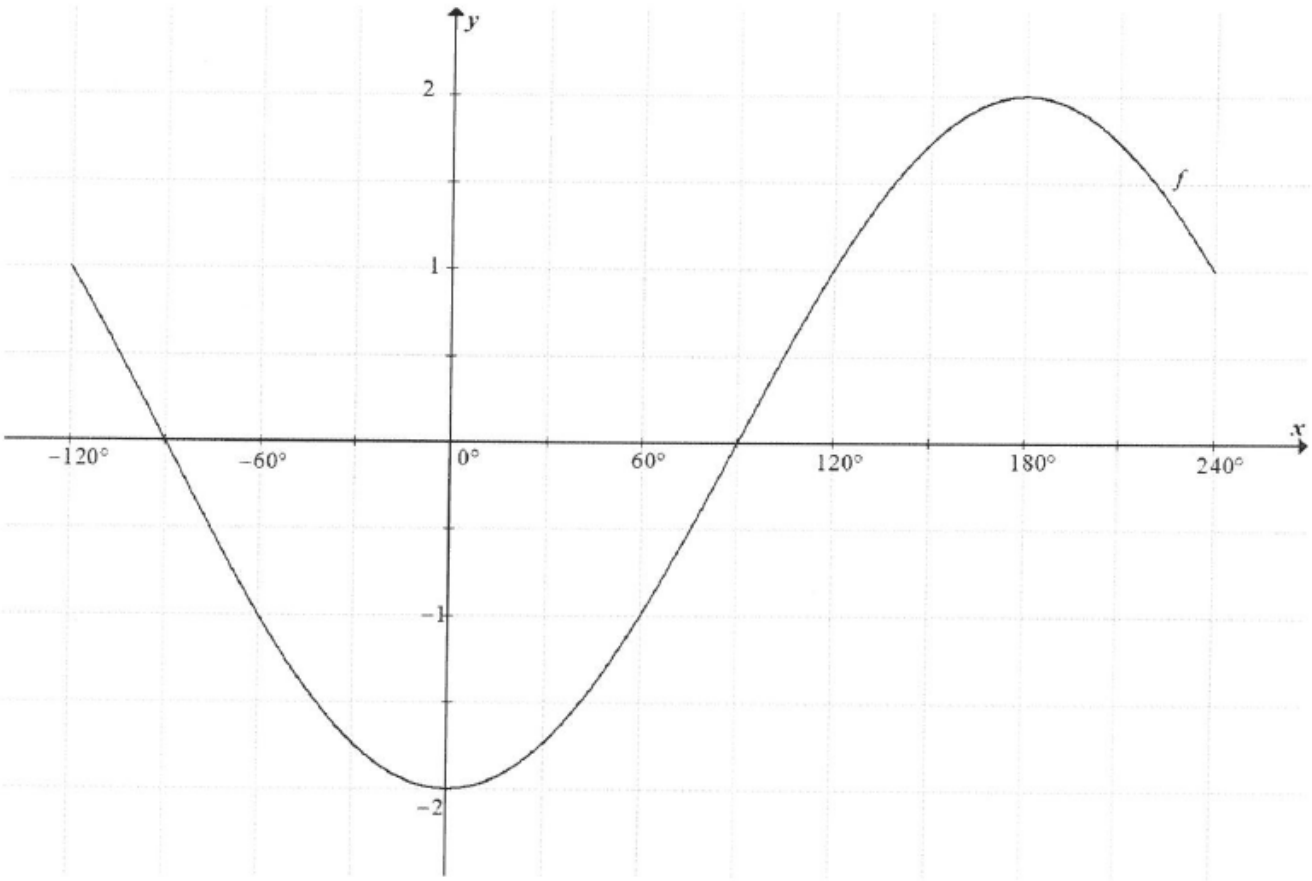
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



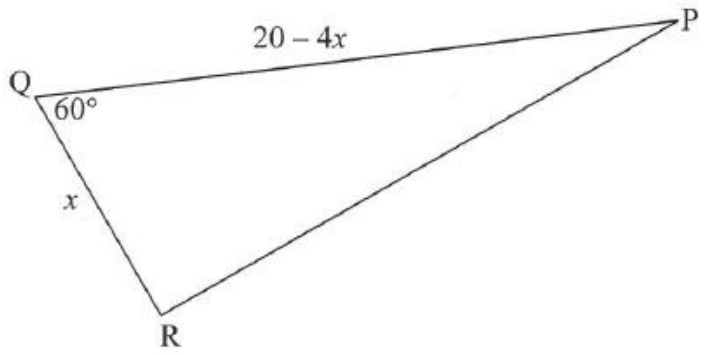
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

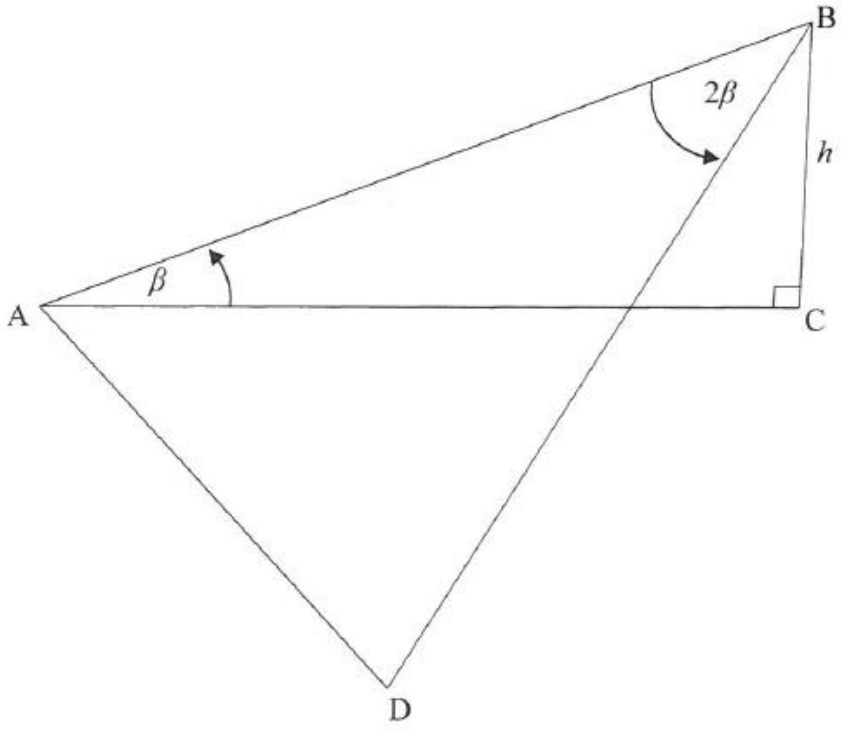


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and BA = BD.



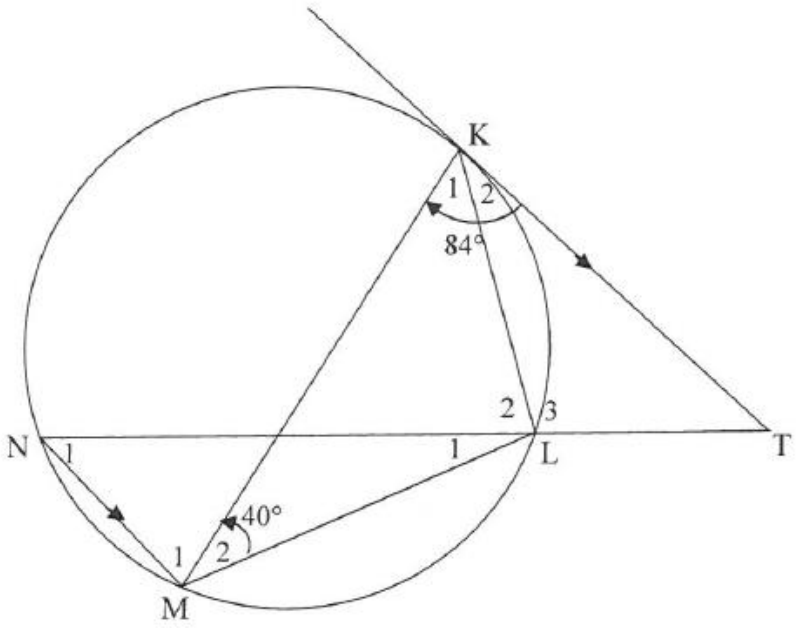
Determine the distance AD between the two anchors in terms of h. (7)

[15]

Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{K}_2$ (2)

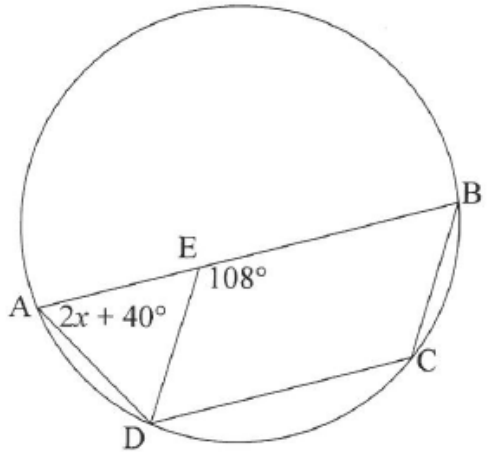
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

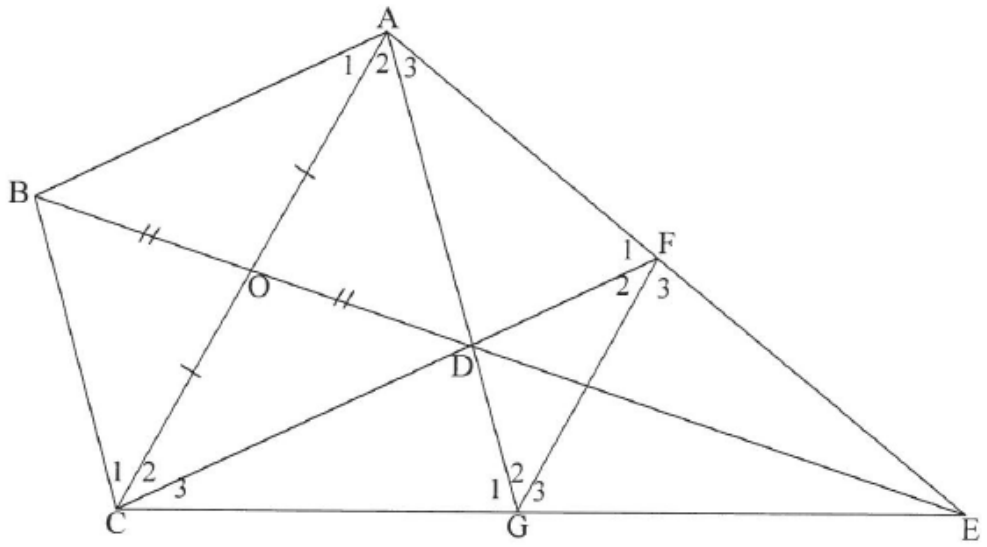
8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.



Calculate, giving reasons, the value of x. (5)
[15]

QUESTION 9

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that BO = OD. AD and CD produced meet EC and EA at G and F respectively.



9.1 Give a reason why ABCD is a parallelogram. (1)

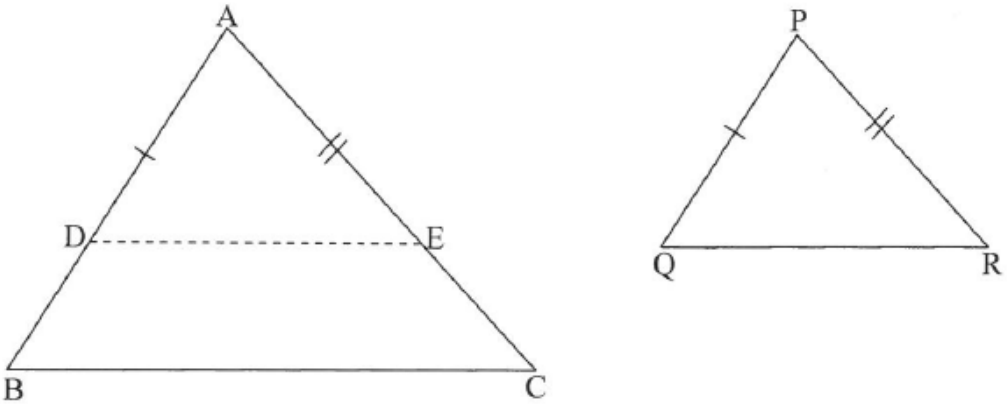
9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)

9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)

9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)
[13]

QUESTION 10

10.1 In the diagram below, ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



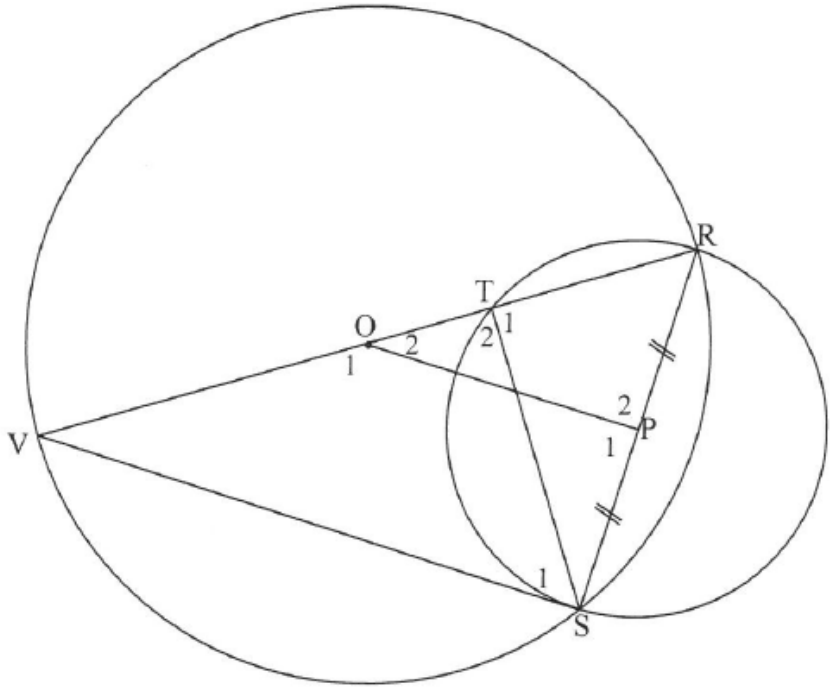
DE is drawn such that $AD = PQ$ and $AE = PR$.

10.1.1 Prove that $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Prove that $DE \parallel BC$. (3)

10.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1 Why is $OP \perp PS$? (1)

10.2.2 Prove that $\Delta ROP \,|||\, \Delta RVS$. (4)

10.2.3 Prove that $\Delta RVS \,|||\, \Delta RST$. (3)

10.2.4 Prove that $ST^2 = VT \cdot TR$. (6)

[21]

TOTAL: 150

MATHEMATICS P1

NOVEMBER 2016

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-7)=0$ (2)

1.1.2 $x^2-6x+2=0$ (correct to TWO decimal places) (3)

1.1.3 $\sqrt{x-1}+1=x$ (5)

1.1.4 $3^{x+3}-3^{x+2}=486$ (4)

1.2 Given: $f(x)=x^2+3x-4$

1.2.1 Solve for x if $f(x)=0$ (2)

1.2.2 Solve for x if $f(x)<0$ (2)

1.2.3 Determine the values of x for which $f'(x) \geq 0$ (2)

1.3 Solve for x and y: $x=2y$ and $x^2-5xy=-24$ (4)

[24]

QUESTION 2

Given the finite arithmetic sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87

2.1 Write down the fourth term (T_4) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87 ; ... ; –4 187
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)

[11]

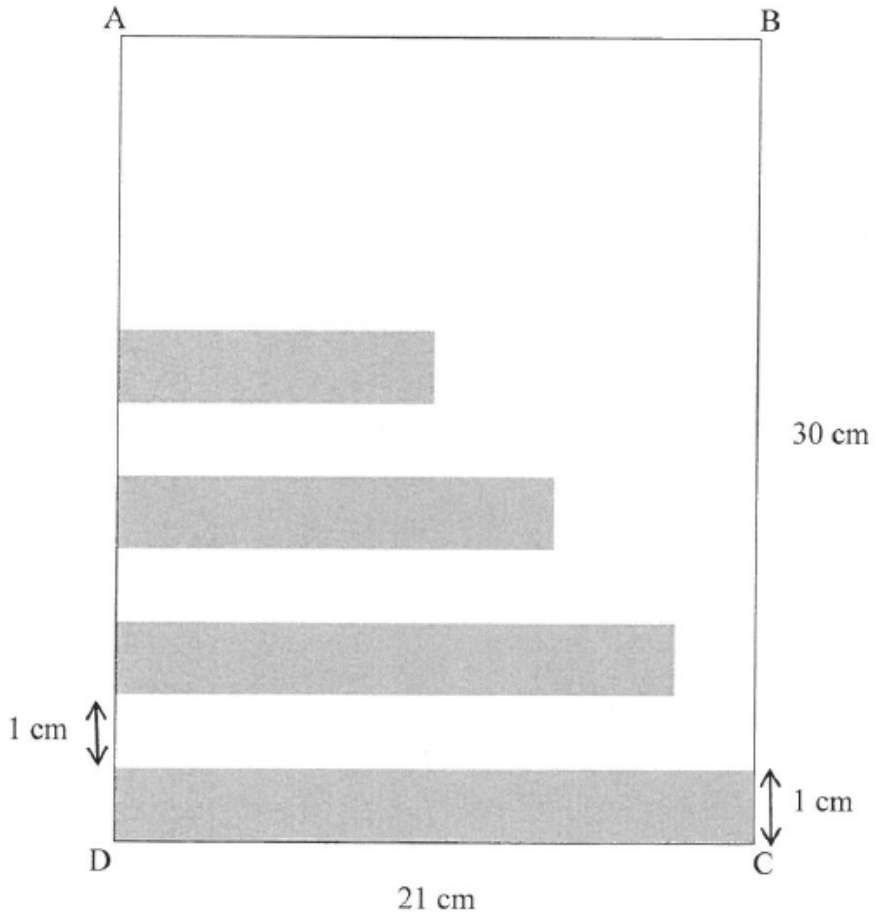
QUESTION 3

3.1 The first four terms of a quadratic number pattern are $-1 \; ; \; x \; ; \; 3 \; ; \; x+8$

3.1.1 Calculate the value(s) of x. (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.



3.2.1 Calculate the length of the 10^{th} rectangle. (3)

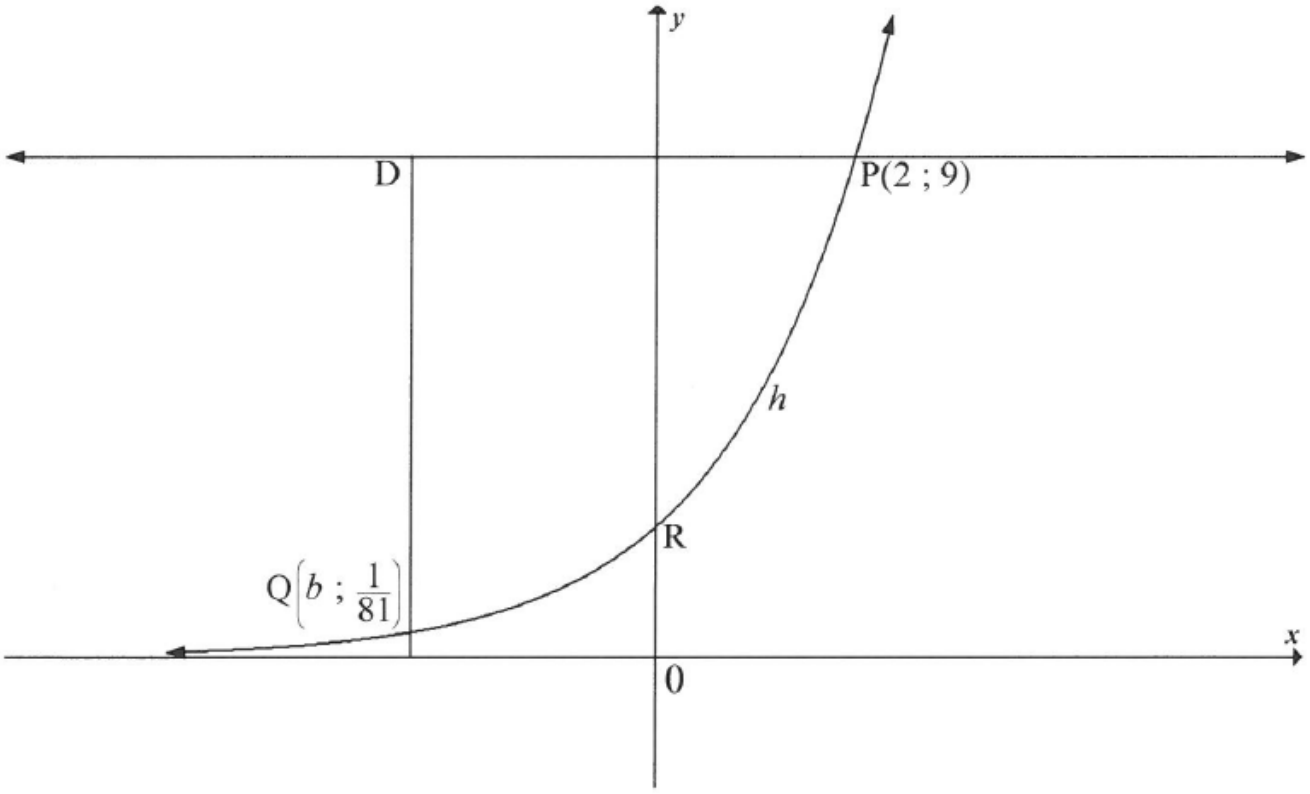
3.2.2 Calculate the percentage of the paper that is coloured grey. (4)

[15]

QUESTION 4

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the y-intercept of h.

The points P(2 ; 9) and $Q\left(b ; \frac{1}{81}\right)$ lie on h.



4.1 Write down the equation of the asymptote of h. (1)

4.2 Determine the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 D is a point such that DQ $\parallel$ y-axis and DP $\parallel$ x-axis. Calculate the length of DP. (4)

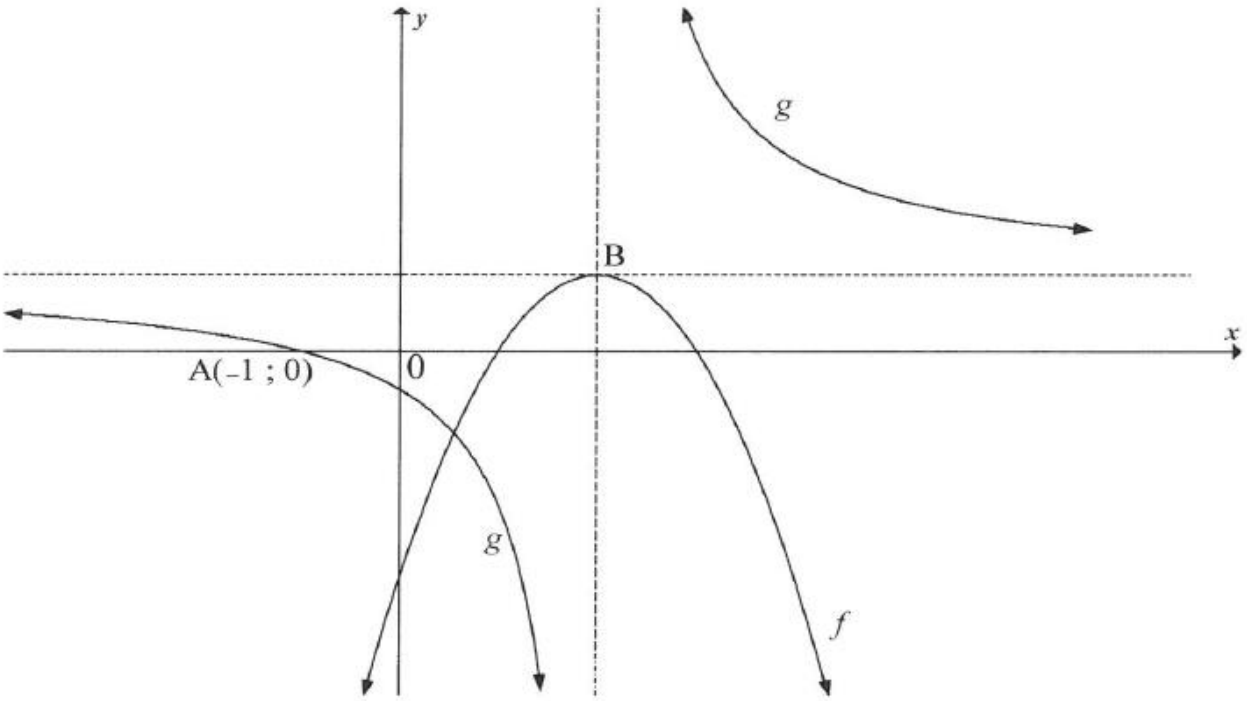
4.5 Determine the values of k for which the equation $h(x+2)+k=0$ will have a root that is less than -6. (3)

[11]

QUESTION 5

Sketched below is the parabola f with equation $f(x) = -x^2 + 4x - 3$ and a hyperbola g with equation $(x - p)(y + t) = 3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- A$(-1\,;\,0)$ is the x-intercept of g.



5.1 Show that the coordinates of B are $(2\,;\,1)$ (2)

5.2 Write down the range of f. (1)

5.3 For which value(s) of x will $g(x) \geq 0$? (2)

5.4 Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x + 4)$ (1)

5.5 Determine the values of p and t. (4)

5.6 Write down the values of x for which $f(x).g'(x) \geq 0$ (4)

[14]

QUESTION 6

Given: $f(x) = -x + 3$ and $g(x) = \log_2 x$

6.1 On the same set of axes, sketch the graphs of f and g, clearly showing ALL intercepts with the axes. (4)

6.2 Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$ (2)

6.3 Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3-x) = x$. (3)

6.4 Write down the solution to $\log_2(3-x) = x$. (1)

[10]

QUESTION 7

On 1 June 2016 a bank granted Thabiso a loan of R250 000 at an interest rate of 15% p.a. compounded monthly, to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2016 and ending 4 years later on 1 June 2020. However, Thabiso was unable to make the first two instalments and only commenced with the monthly instalments on 1 September 2016.

7.1 Calculate the amount Thabiso owed the bank on 1 August 2016, a month before he paid his first monthly instalment. (2)

7.2 Having paid the first monthly instalment on 1 September 2016, Thabiso will still pay his last monthly instalment on 1 June 2020. Calculate his monthly instalment. (4)

7.3 If Thabiso paid R9 000 as his monthly instalment starting on 1 September 2016, how many months sooner will he repay the loan? (5)

7.4 If Thabiso paid R9 000 as a monthly instalment starting on 1 September 2016, calculate the final instalment to repay the loan. (4)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives at the answer: $\lim\limits_{h\to 0}\dfrac{\sqrt{4+h}-2}{h}$

Write down the equation of g and the value of a. (2)

8.3 Determine $\dfrac{dy}{dx}$ if $y=\sqrt{x^3}-\dfrac{5}{x^3}$ (4)

8.4 $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x = 1$. Calculate the values of a and b. (5)

[16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x)=3x^2+8x-3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x)=ax^3+bx^2+cx+d$ and $f(0)=-18$, determine the equation of f. (5)

[13]

QUESTION 10

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t)=-t^3+3t^2+72t$, $0<t<10$.

10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)

[8]

QUESTION 11

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Male	80	*a*	
Female	48	12	
Totals	*b*	32	160

11.1 Calculate the values of a and b. (2)

11.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)

11.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:

11.3.1 Watched TV in the period during which the examinations were written? (2)

11.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2)

[8]

QUESTION 12

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? **[5]**

TOTAL: 150

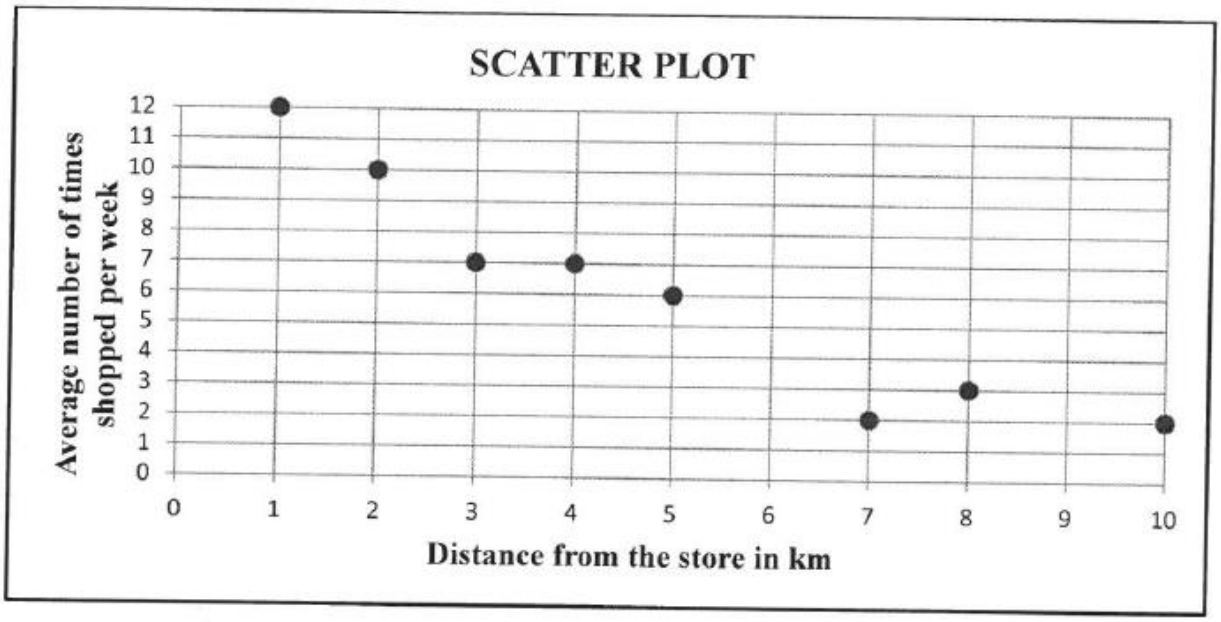
MATHEMATICS P2

NOVEMBER 2016

QUESTION 1

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



1.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

1.2 Calculate the correlation coefficient of the data. (1)

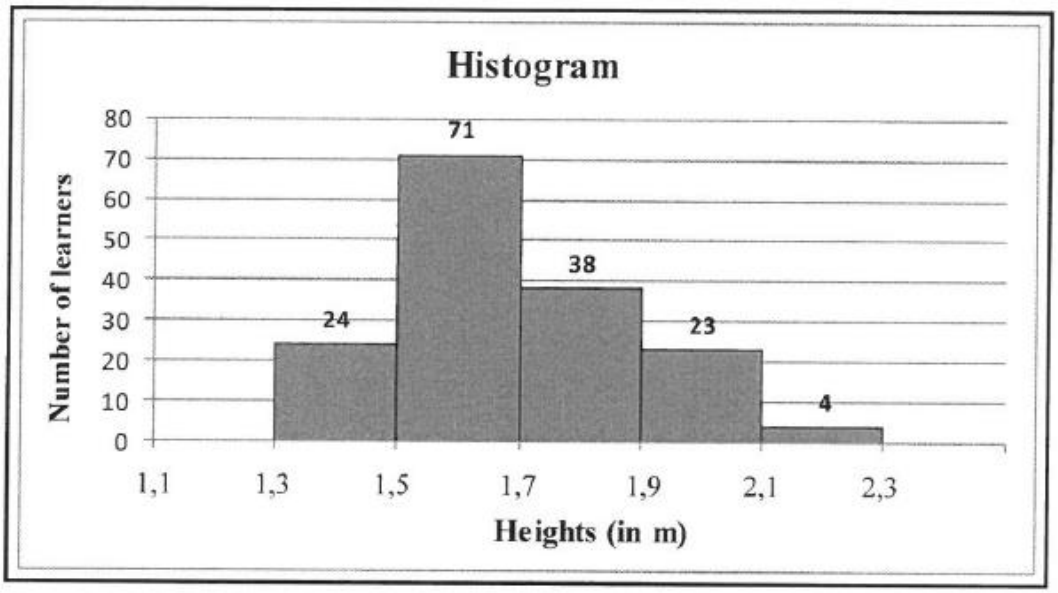
1.3 Calculate the equation of the least squares regression line of the data. (3)

1.4 Use your answer at QUESTION 1.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)

1.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)
[9]

QUESTION 2

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

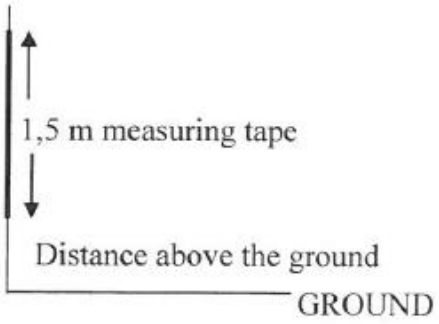
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

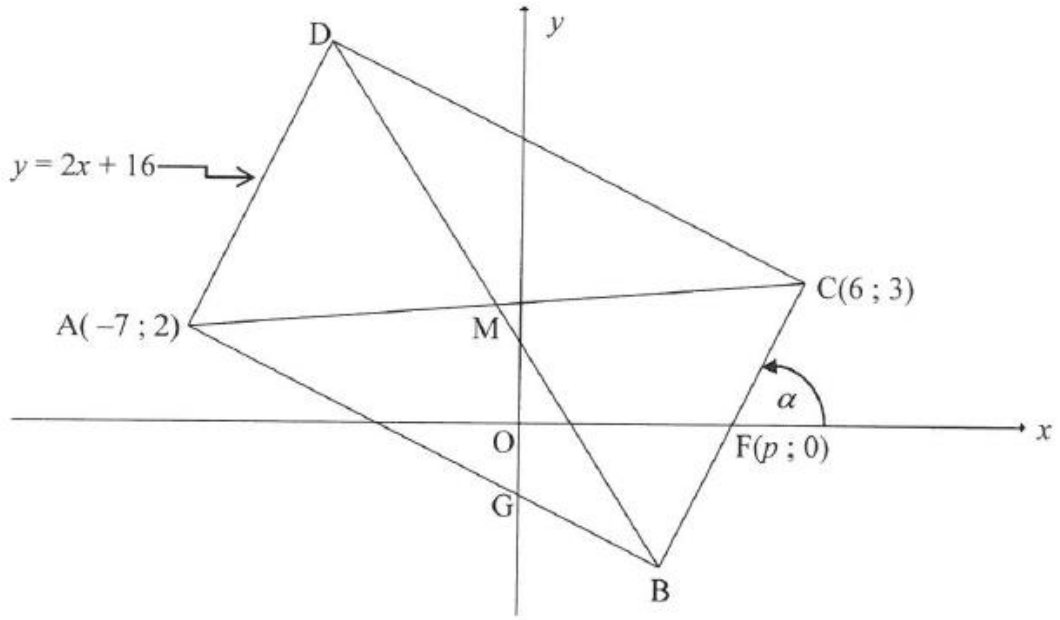
2.6.1 Mean of the data set (1)

2.6.2 Standard deviation of the data set (1)

[13]

QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

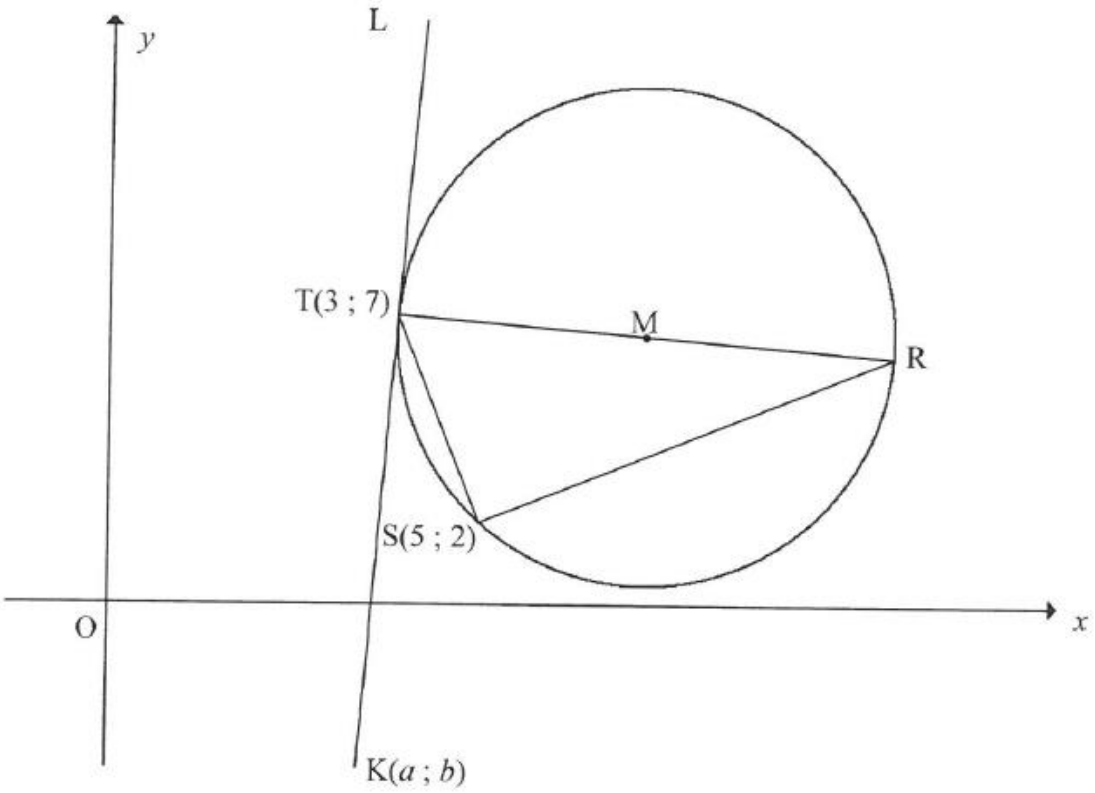
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2+(y-b)^2=r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)

[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

QUESTION 5

5.1 Given: $\sin 16° = p$

Determine the following in terms of p, **without using a calculator**.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

5.4.1 $\cos B$ (3)

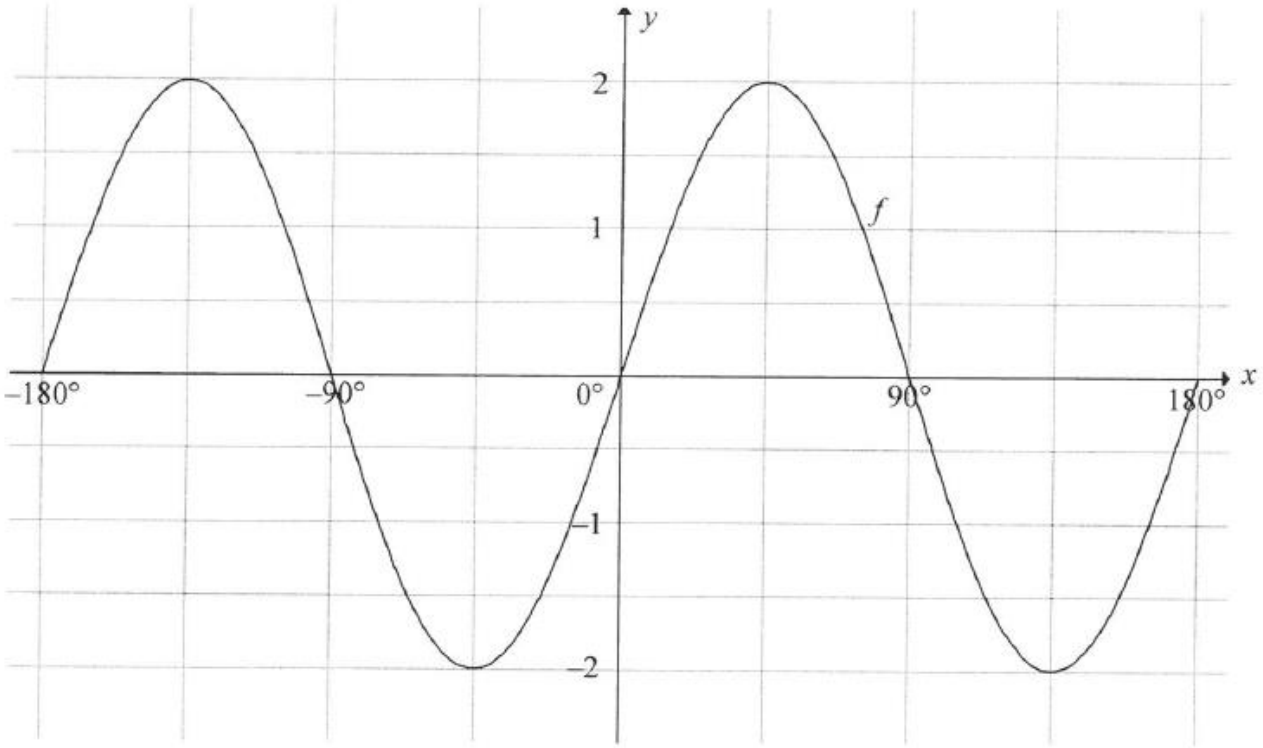
5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45°)$ (4)

[21]

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

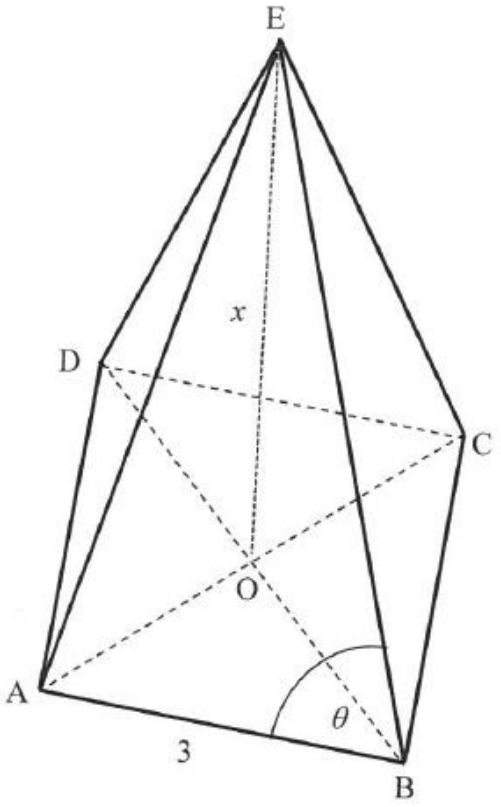
6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)

[12]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$



7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

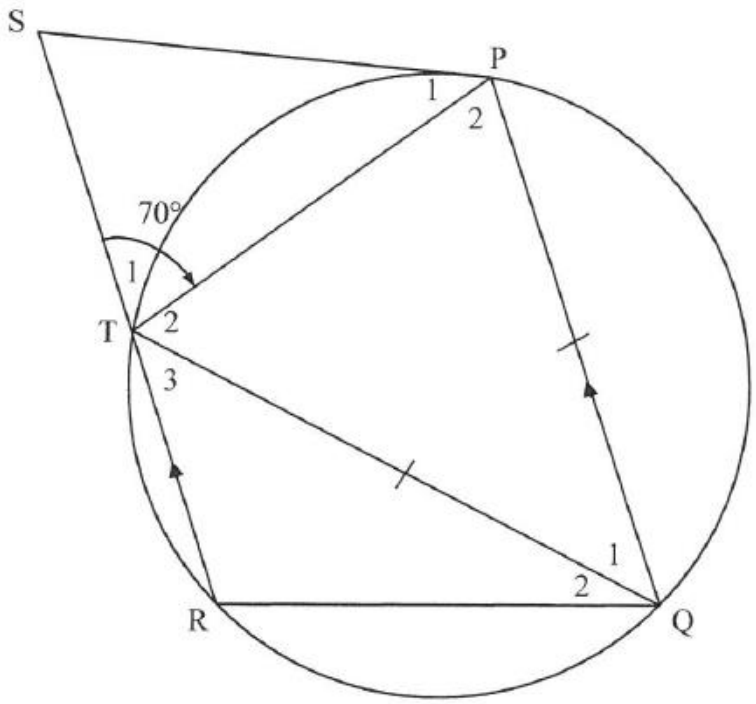
7.3 If the volume of the pyramid is 15 m^3, calculate the value of θ. (4)

[12]

Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below PQRT is a cyclic quadrilateral having RT || QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



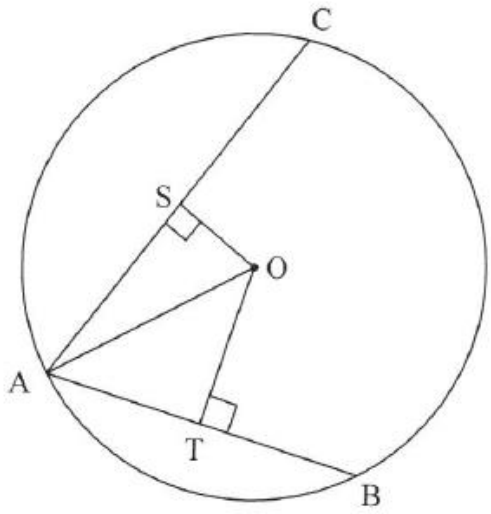
8.1.1 Give a reason why $\hat{P}_2 = 70°$. (1)

8.1.2 Calculate, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that $OS \perp AC$ and $OT \perp AB$. $AB = 40$ and $AC = 48$.



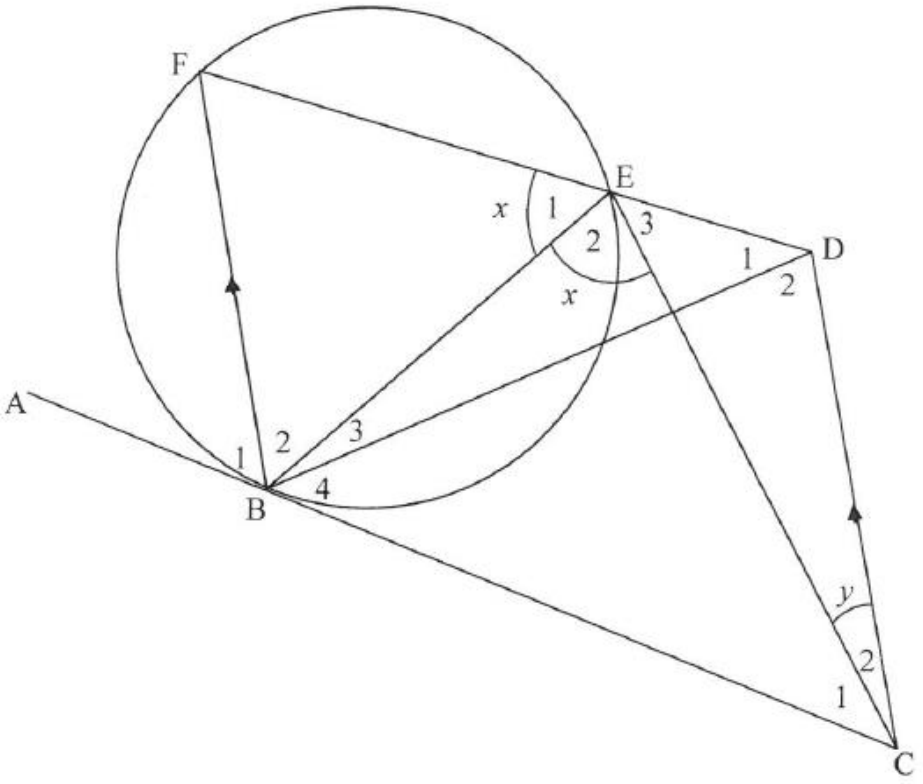
8.2.1 Calculate AT. (1)

8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

QUESTION 9

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1 Give a reason why EACH of the following is TRUE:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Prove that BCDE is a cyclic quadrilateral. (2)

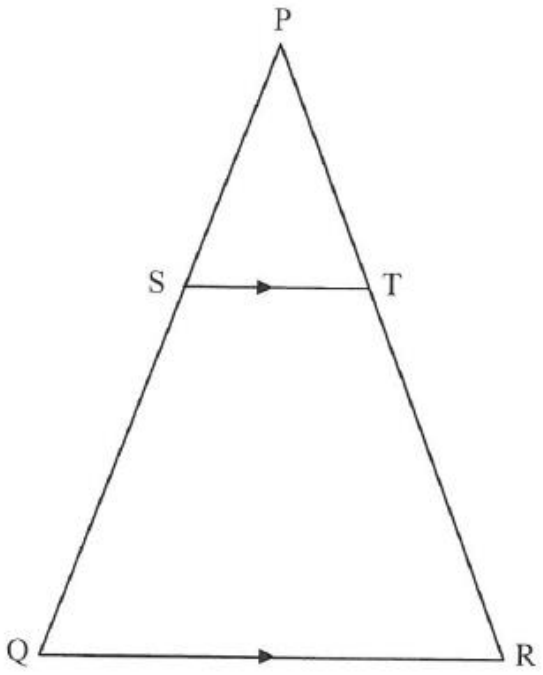
9.3 Which TWO other angles are each equal to x? (2)

9.4 Prove that $\hat{B}_2 = \hat{C}_1$. (3)

[9]

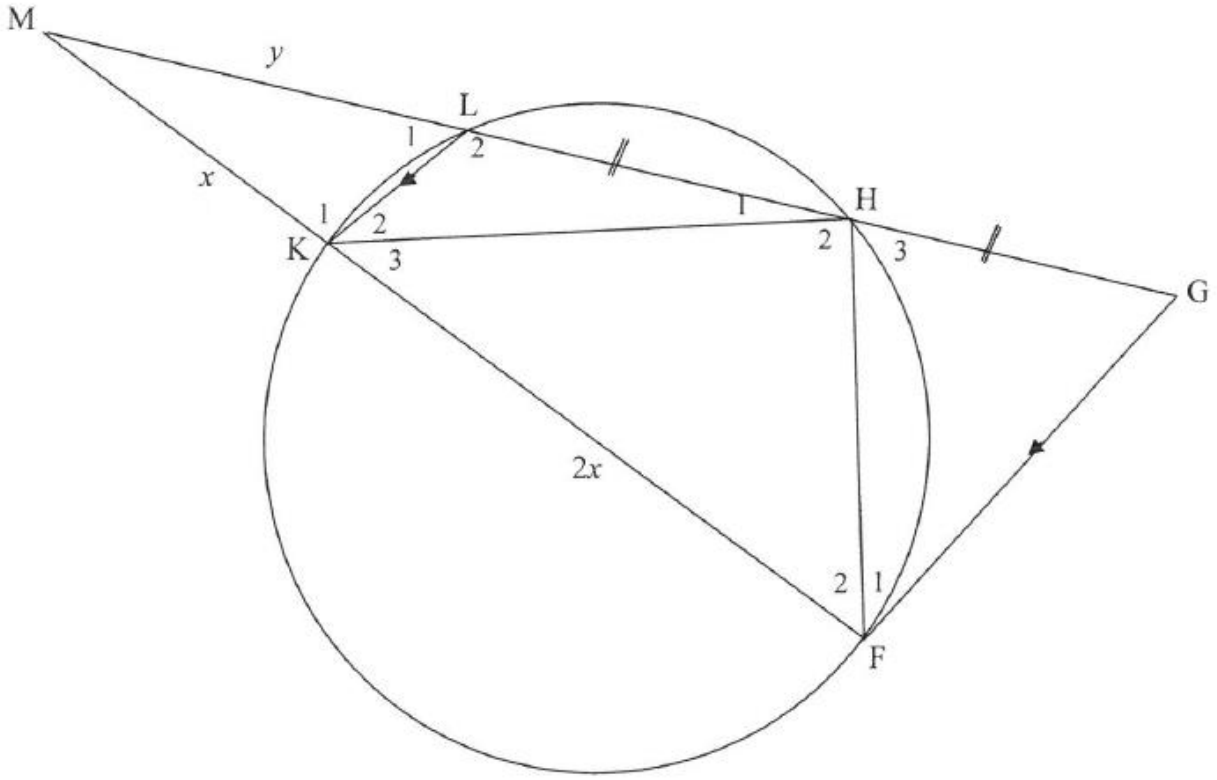
QUESTION 10

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that $ST \parallel QR$.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. MK = x, KF = $2x$, ML = y and LH = HG.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Show that $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

[20]

TOTAL: 150

MATHEMATICS P1

FEBRUARY/MARCH 2017

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+1)=0$ (2)

1.1.2 $\sqrt{x^3}=512$ (3)

1.1.3 $x(x-4)<0$ (2)

1.2 Given: $f(x)=x^2-5x+2$

1.2.1 Solve for x if $f(x)=0$ (3)

1.2.2 For which values of c will $f(x)=c$ have no real roots? (4)

1.3 Solve for x and y:

$$x=2y+2$$
$$x^2-2xy+3y^2=4$$ (6)

1.4 Calculate the maximum value of S if $S=\frac{6}{x^2+2}$. (2)

[22]

QUESTION 2

Given the geometric sequence: $-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ .....$

2.1 Calculate the possible values of b. (3)

2.2 If $b=\frac{1}{2}$, calculate the 19^{th} term (T_{19}) of the sequence. (3)

2.3 If $b=\frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)

[12]

QUESTION 3

3.1 6 ; 6 ; 9 ; 15 ; … are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term (T_5) of the pattern. (1)

3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

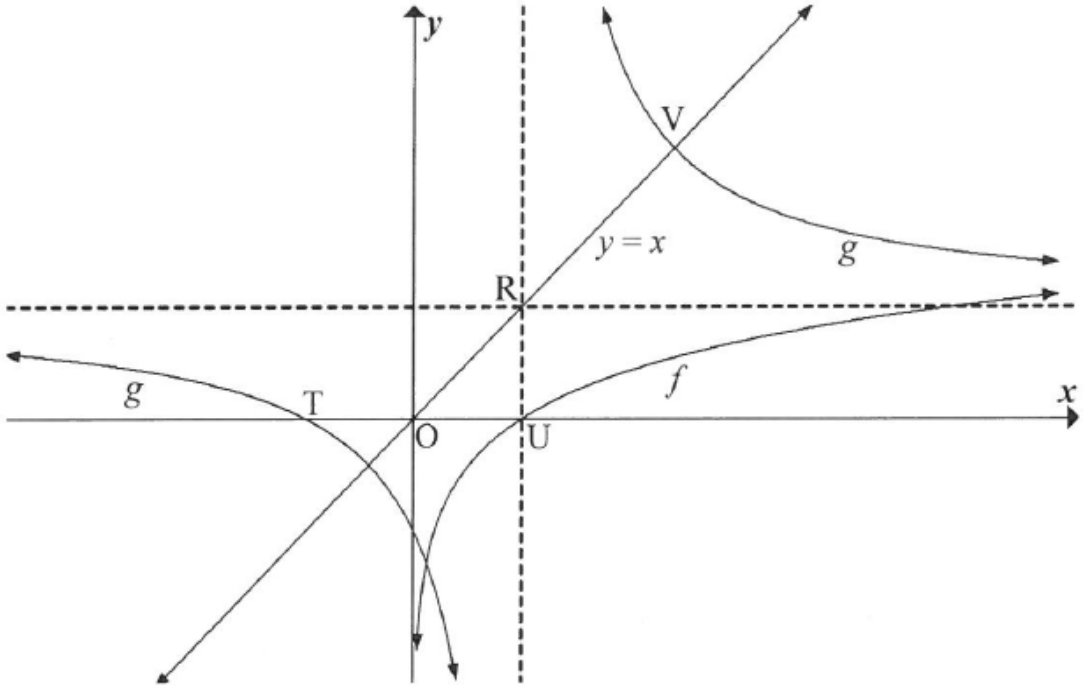
3.2 Determine the value(s) of x in the interval $x \in [0° ; 90°]$ for which the sequence -1 ; $2\sin 3x$; 5 ; ..... will be arithmetic. (4)

[13]

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



4.1 Write down the coordinates of U. (1)

4.2 Write down the equations of the asymptotes of g. (2)

4.3 Determine the coordinates of T. (2)

4.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = ...$ (2)

4.5 Write down the equation of the asymptote of $h(x-3)$. (1)

4.6 Calculate the coordinates of V. (4)

4.7 Determine the coordinates of T$'$ the point which is symmetrical to T about the point R. (2)

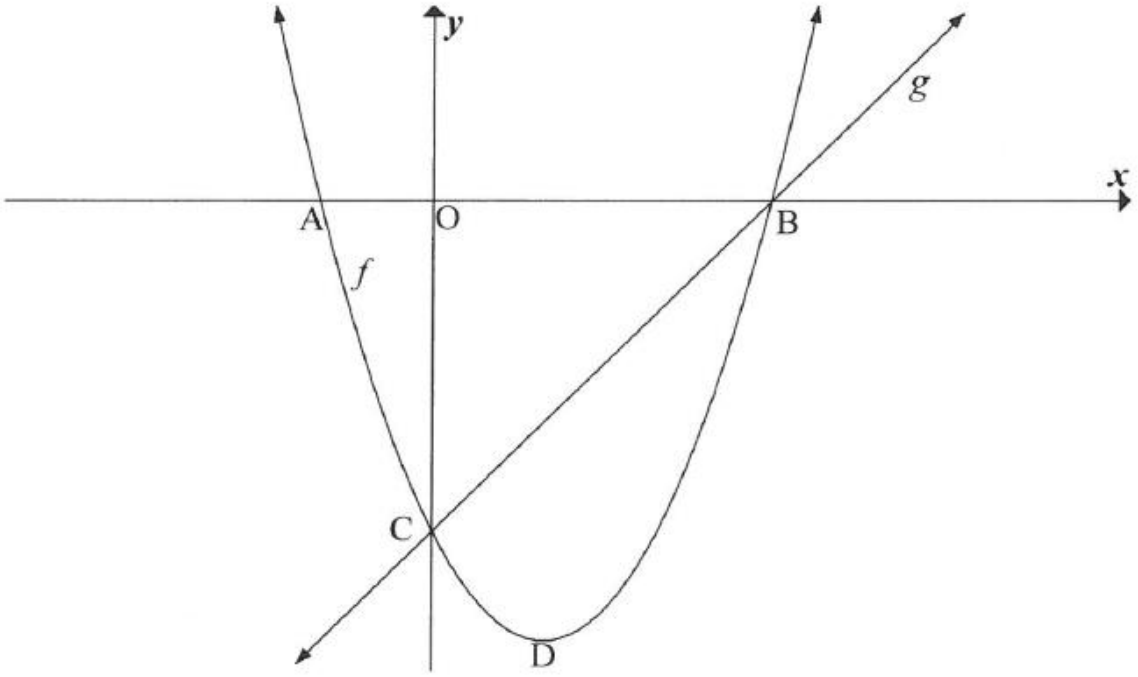
[14]

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1 Determine the coordinates of C. (1)

5.1.2 Calculate the length of AB. (4)

5.1.3 Determine the coordinates of D. (2)

5.1.4 Calculate the average gradient of f between C and D. (2)

5.1.5 Calculate the size of $O\hat{C}B$ (2)

5.1.6 Determine the values of k for which $f(x) = k$ will have two unequal positive real roots. (3)

5.1.7 For which values of x will $f'(x) \cdot f''(x) > 0$? (3)

5.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

QUESTION 6

6.1 On the 2nd day of January 2015 a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

6.1.1 Calculate the book value of the printer on the 2nd day of January 2017. (3)

6.1.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)

6.1.3 The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)

6.2 Lerato wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month after the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lerato can borrow from the bank. (5)

[16]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.
Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)

[14]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

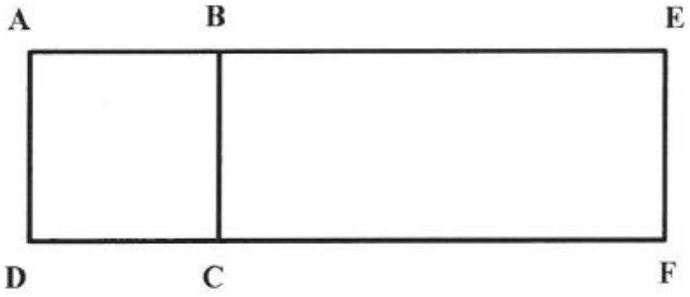
8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)
[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

QUESTION 10

10.1 The events S and T are independent.

- $P(S \text{ and } T) = \frac{1}{6}$
- $P(S) = \frac{1}{4}$

10.1.1 Calculate $P(T)$. (2)

10.1.2 Hence, calculate $P(S \text{ or } T)$. (2)

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

10.2.2 Repetition of digits IS allowed in the code (1)

10.3 A group of 3 South Africans, 2 Australians and 2 Englishmen are staying at the same hotel while on holiday. Each person has his/her own room and the rooms are next to each other in a straight corridor.

If the rooms are allocated at random, determine the probability that the 2 Australians will have adjacent rooms and the 2 Englishmen will also have adjacent rooms. (4)

[11]

QUESTION 11

The success rate of the Fana soccer team depends on a number of factors. The fitness of the players is one of the factors that influence the outcome of a match.

- The probability that all the players are fit for the next match is 70%
- If all the players are fit to play the next match, the probability of winning the next match is 85%
- If there are players that are not fit to play the next match, the probability of winning the match is 55%

Based on fitness alone, calculate the probability that the Fana soccer team will win the next match. **[5]**

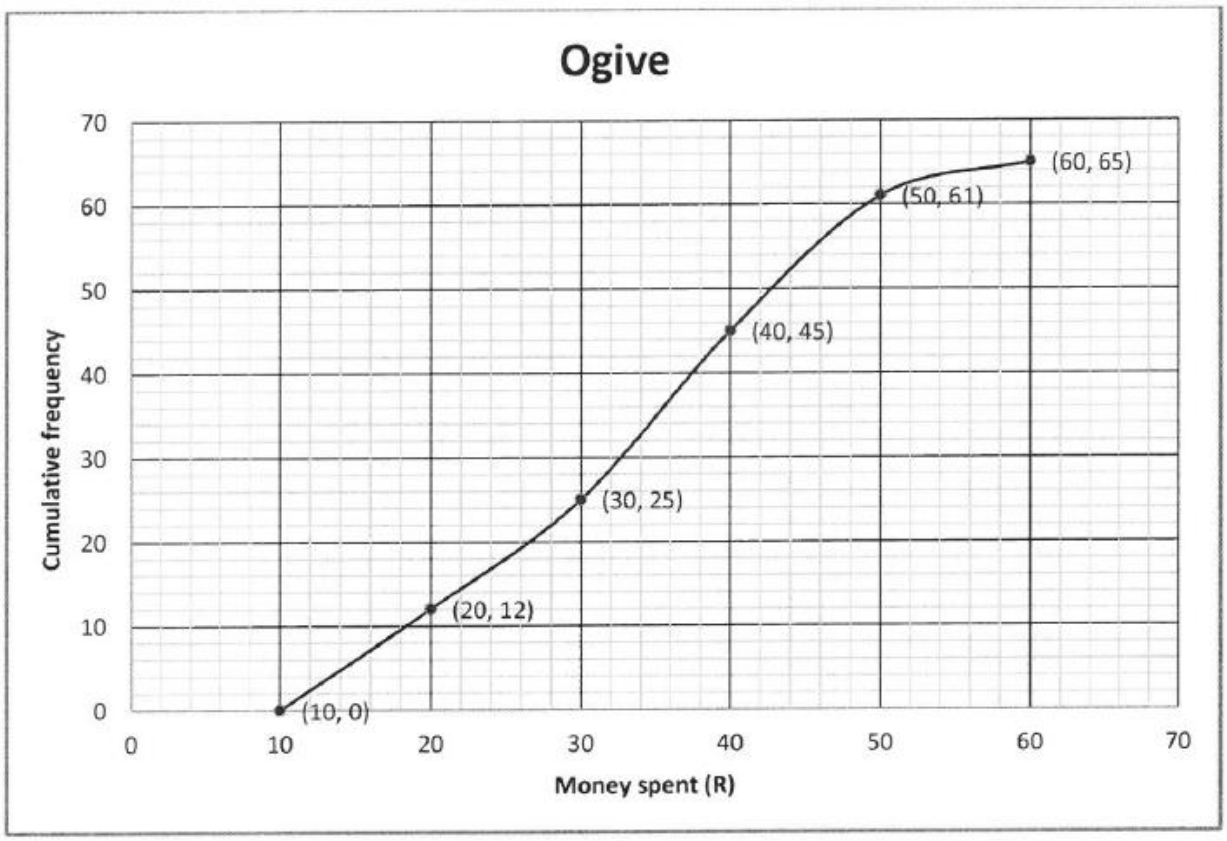
TOTAL: 150

MATHEMATICS P2

FEBRUARY/MARCH 2017

QUESTION 1

The amount of money, in rands, that learners spent while visiting a tuck shop at school on a specific day was recorded. The data is represented in the ogive below.



An incomplete frequency table is also given for the data.

Amount of money (in R)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency	a	13	20	b	4

1.1 How many learners visited the tuck shop on that day? (1)

1.2 Write down the modal class of this data. (1)

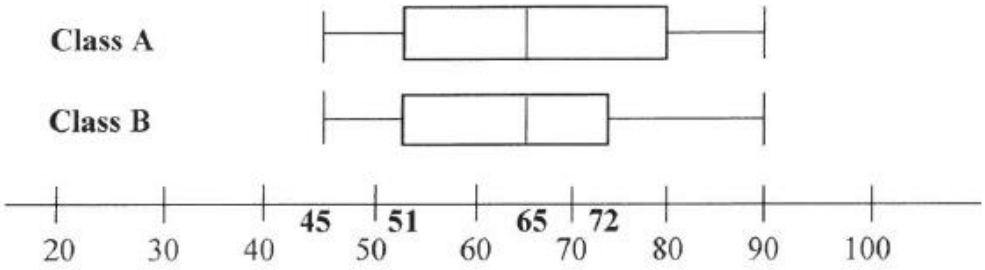
1.3 Determine the values of a and b in the frequency table. (2)

1.4 Use the ogive to estimate the number of learners who spent at least R45 on the day the data was recorded at the tuck shop. (2)

[6]

QUESTION 2

2.1 Mrs Smith has two classes, each having 30 learners. Their final marks (out of 100) for the year are represented in the box and whisker diagram below.



2.1.1 Determine the interquartile range of Class B. (2)

2.1.2 Explain the significance in the difference of the length of the boxes in the diagram. (2)

2.1.3 Mrs Smith studied the results and made the comment that there was no significant difference in the performance of the two classes. Give TWO reasons you think Mrs Smith will use to prove her statement. (2)

2.2 Eight couples entered a dance competition. Their performances were scored by two judges. The scores (out of 20) are given in the table below.

COUPLE	1	2	3	4	5	6	7	8
JUDGE 1	18	4	6	8	5	12	10	14
JUDGE 2	15	6	3	5	5	14	8	15

2.2.1 Determine the equation of the least squares regression line of the scores given by the two judges. (3)

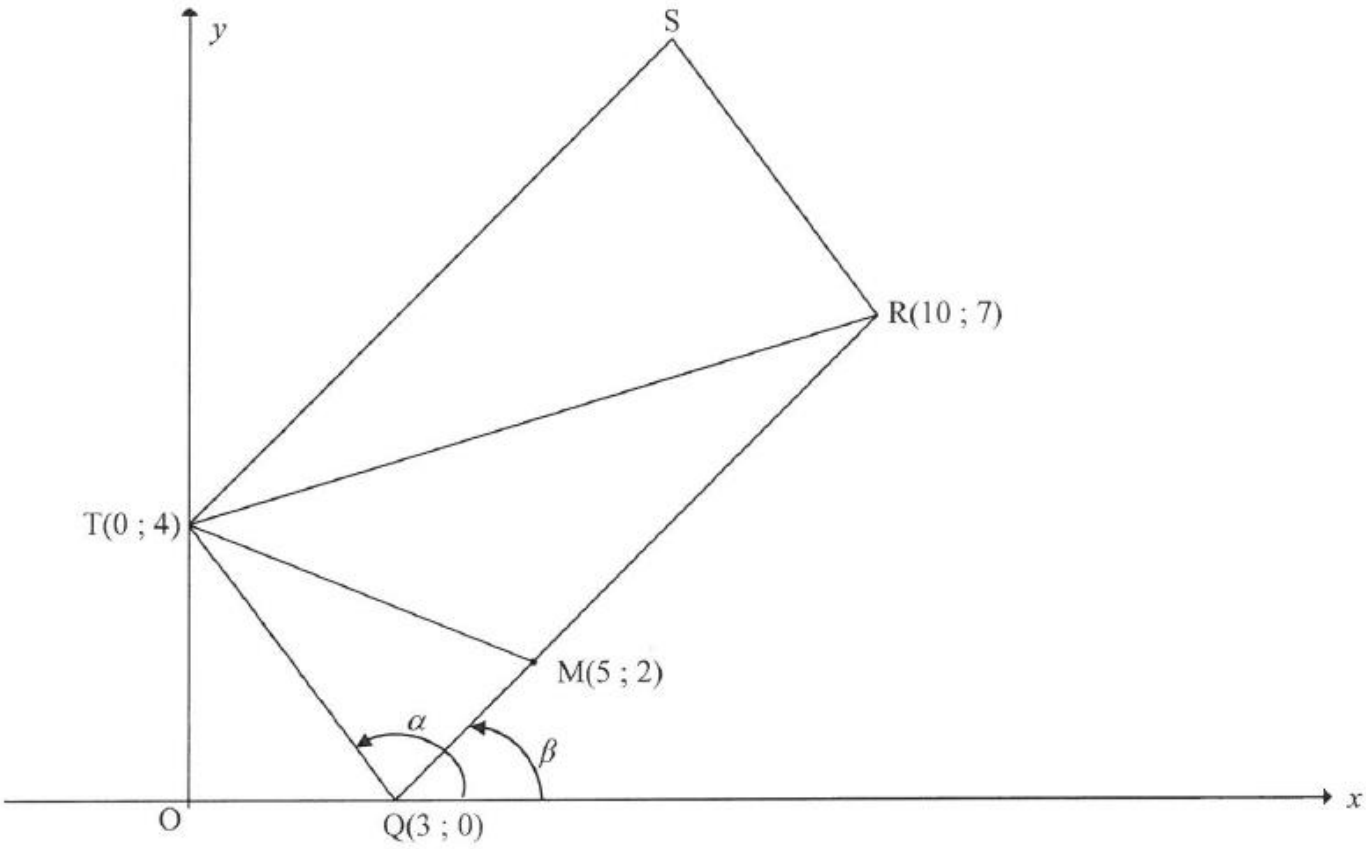
2.2.2 A ninth couple entered late for the competition and received a score of 15 from JUDGE 1. Estimate the score that might have been assigned by JUDGE 2 to the nearest integral value. (2)

2.2.3 Are the judges consistent in assigning scores to the performance of the couples? Prove your answer and support it with relevant statistics. (2)

[13]

QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F(k ; –8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (4)

3.5 Calculate the size of $T\hat{S}R$. (6)

3.6 Calculate, in the simplest form, the ratio of:

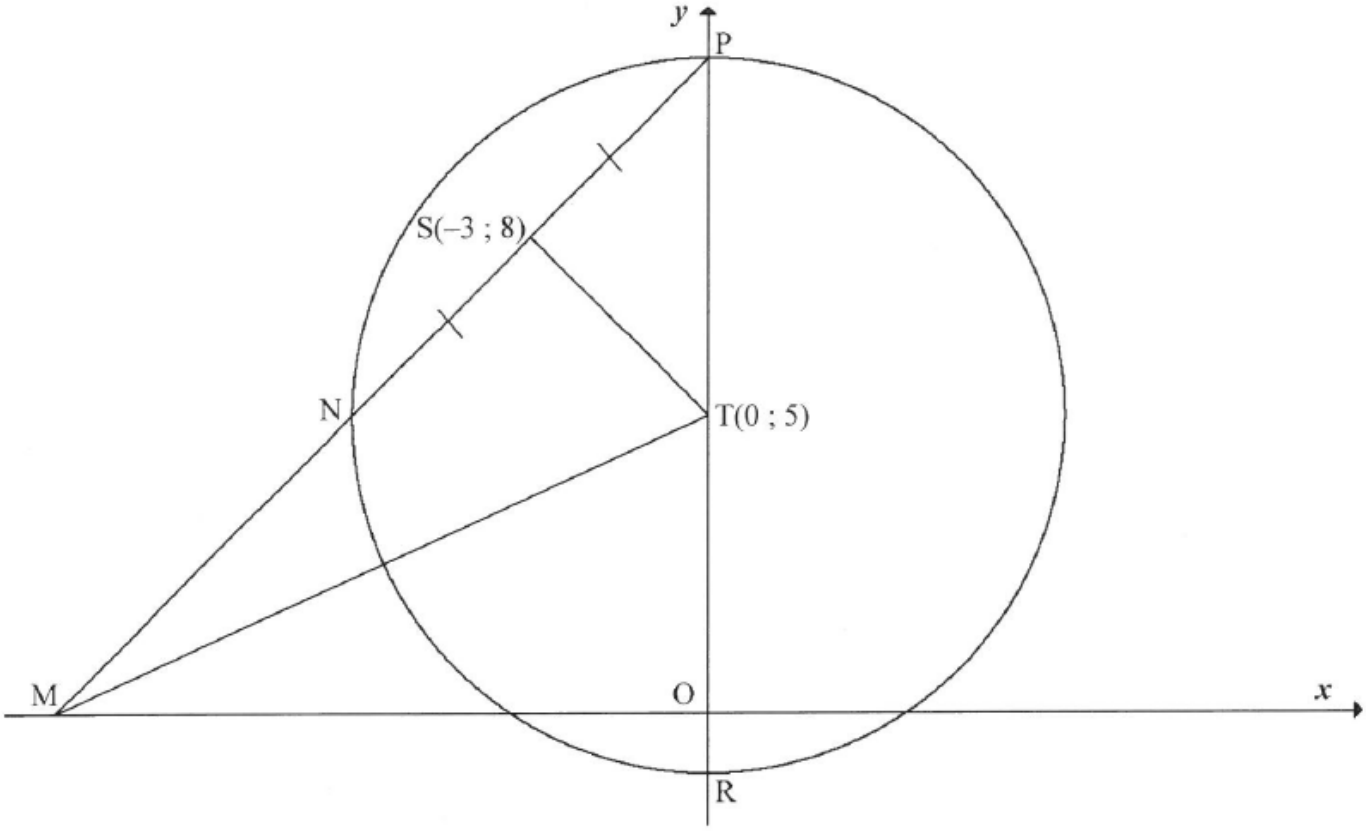
3.6.1 $\dfrac{MQ}{RQ}$ (3)

3.6.2 $\dfrac{\text{area of } \Delta TQM}{\text{area of parallelogram } RQTS}$ (3)

[23]

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

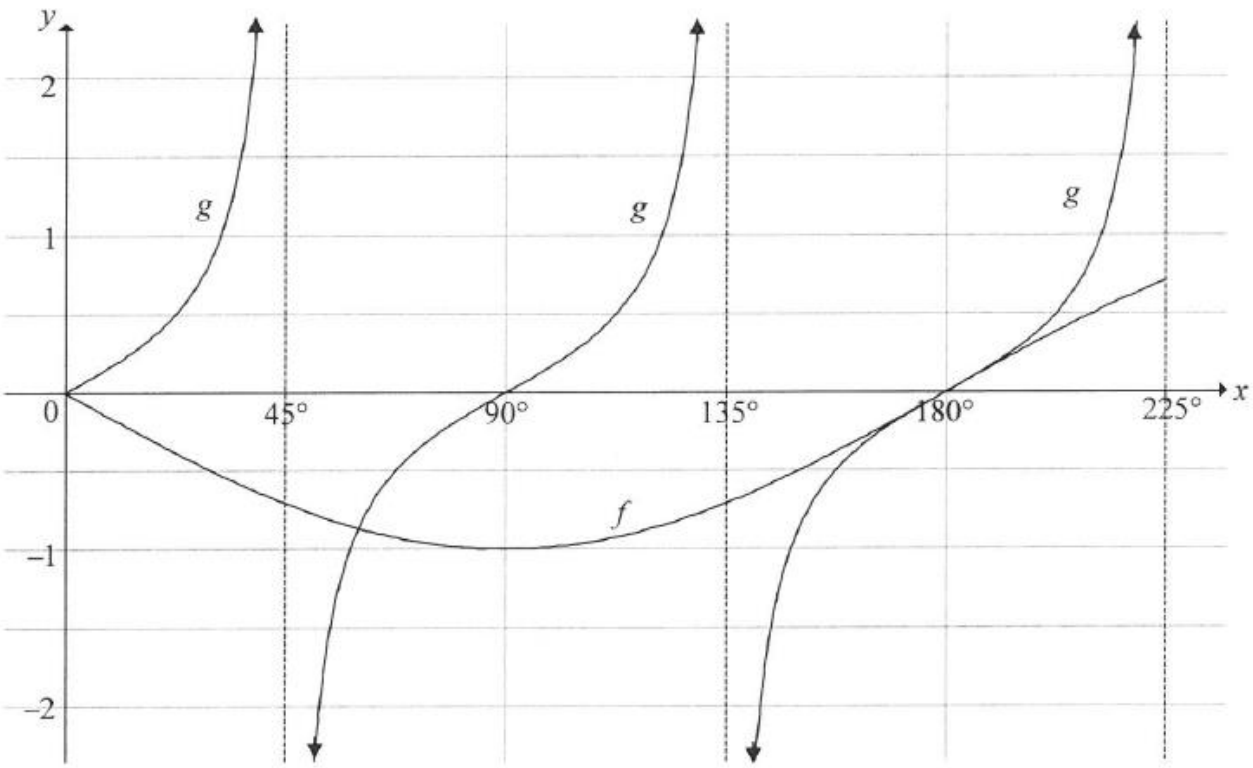
4.4 Determine the length of MT. (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)

[19]

QUESTION 5

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \le x \le 225°$.



5.1 Write down the values of a and b. (2)

5.2 Write down the period of $f(3x)$. (2)

5.3 Determine the values of x in the interval $90° \le x \le 225°$ for which $f(x).g(x) \le 0$. (3)

[7]

QUESTION 6

6.1 **Without using a calculator**, determine the following in terms of $\sin 36°$:

6.1.1 $\sin 324°$ (1)

6.1.2 $\cos 72°$ (2)

6.2 Prove the identity: $1-\dfrac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$ (6)

6.4 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Use the formula for $\cos(A-B)$ to derive a formula for $\sin(A-B)$. (4)

6.4.2 **Without using a calculator**, show that

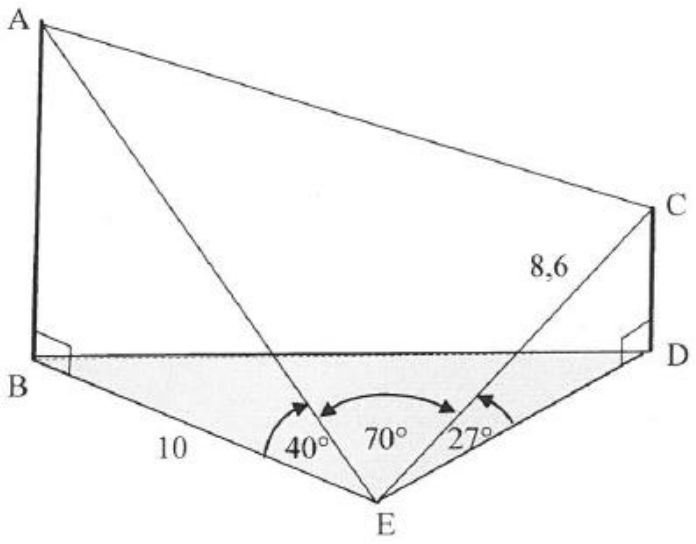
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

for all values of x. (6)

[23]

QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. CE = 8,6 m, BE = 10 m, $A\hat{E}B = 40°$, $A\hat{E}C = 70°$ and $C\hat{E}D = 27°$.



Calculate the:

7.1 Height of pole CD (2)

7.2 Length of cable AE (2)

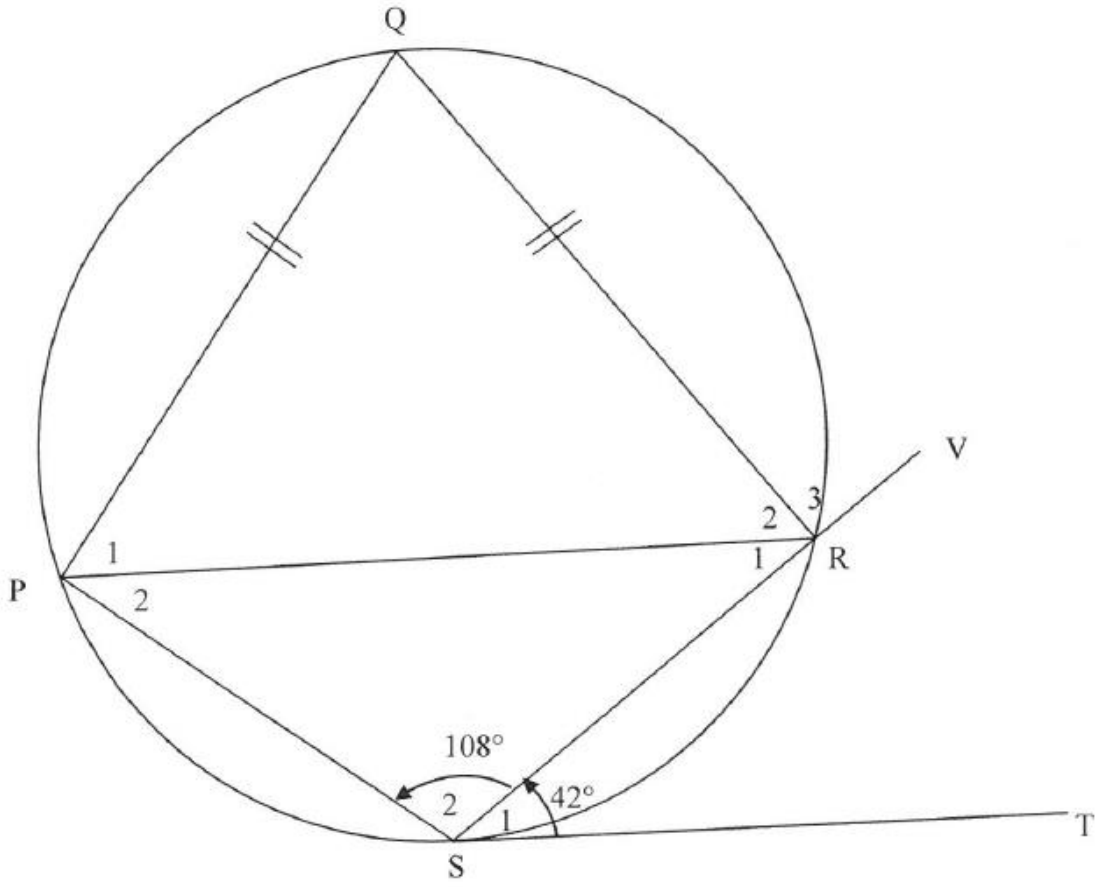
7.3 Length of cable AC (4)

[8]

Give reasons for ALL statements and calculations in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

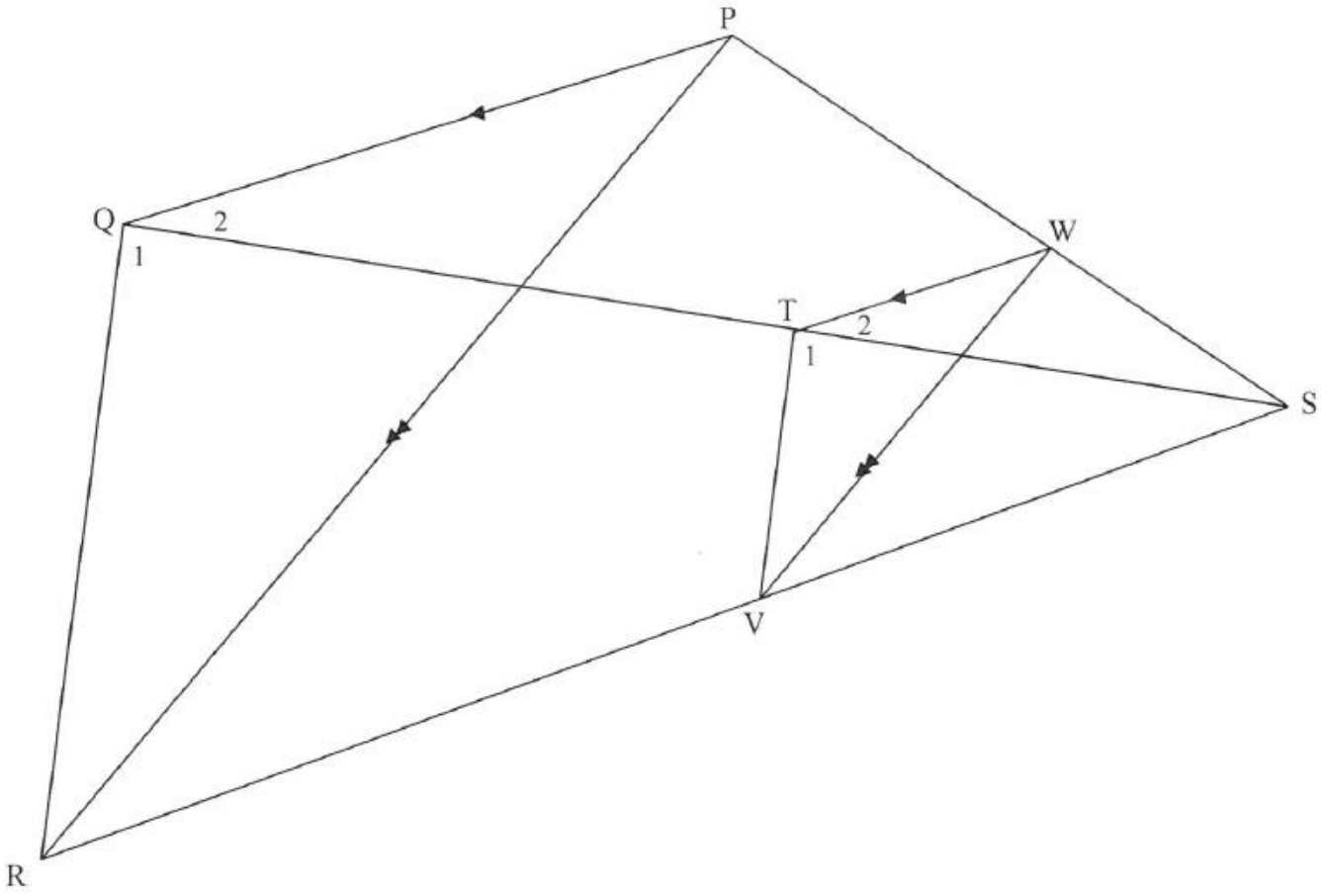
8.3 $\hat{P}_2$ (2)

8.4 $\hat{R}_3$ (2)

[8]

QUESTION 9

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn. PW : WS = 3 : 2.



9.1 Write down the value of the following ratios:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

9.2 Prove that $\hat{T}_1 = \hat{Q}_1$. (4)

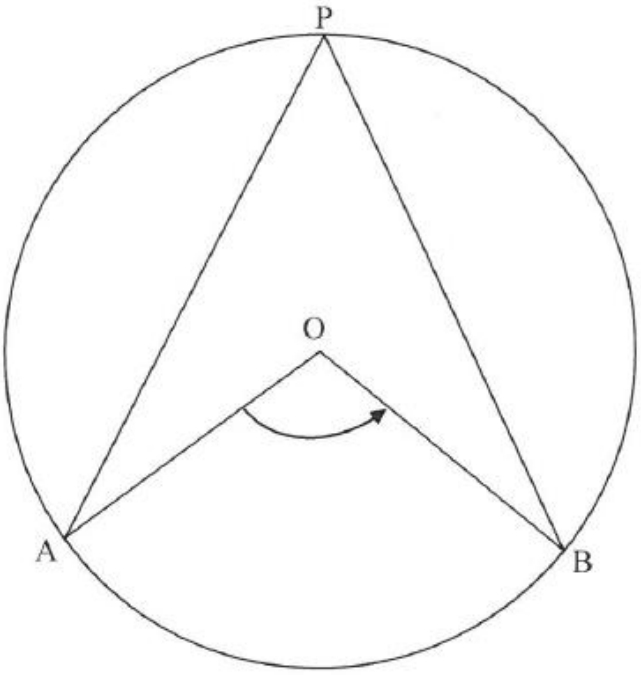
9.3 Complete the following statement: $\Delta VWS \;|||\; \Delta\ldots$ (1)

9.4 Determine WV : PR. (2)

[10]

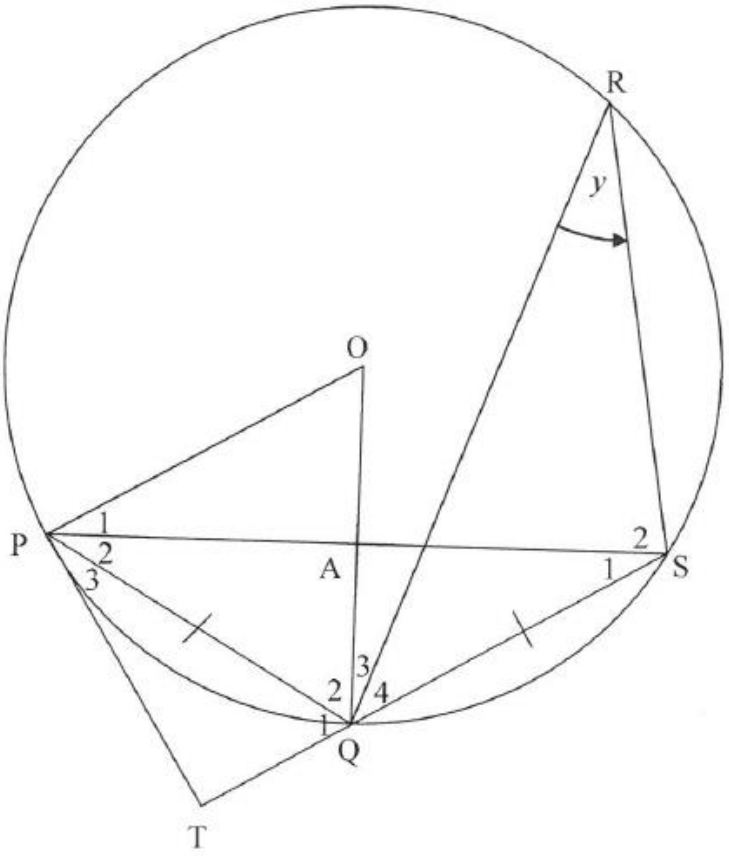
QUESTION 10

10.1 In the diagram, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$. (5)

10.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



10.2.1 Give a reason why $\hat{P}_2 = y$. (1)

10.2.2 Prove that PQ bisects $T\hat{P}S$. (4)

10.2.3 Determine $P\hat{O}Q$ in terms of y. (2)

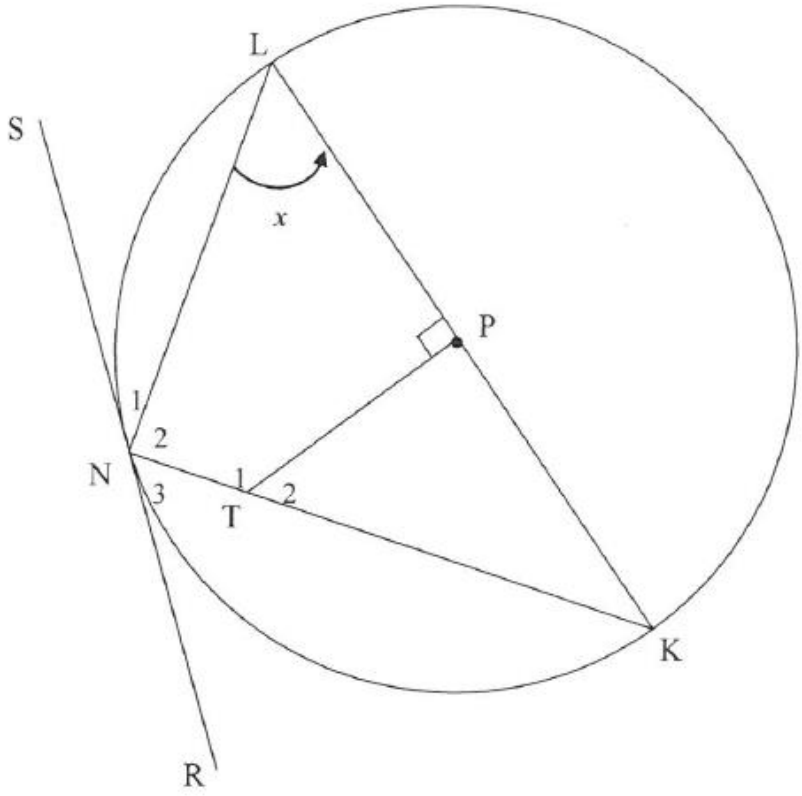
10.2.4 Prove that PT is a tangent to the circle that passes through points P, O and A. (2)

10.2.5 Prove that $O\hat{A}P = 90°$. (5)

[19]

QUESTION 11

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and TP ⊥ KL. $P\hat{L}N = x$.



11.1 Prove that TPLN is a cyclic quadrilateral. (3)

11.2 Determine, giving reasons, the size of $\hat{N}_1$ in terms of x. (3)

11.3 Prove that:

11.3.1 $\Delta KTP \,|||\, \Delta KLN$ (3)

11.3.2 $KT \cdot KN = 2KT^2 - 2TP^2$ (5)

[14]

TOTAL: 150

MATHEMATICS P1

NOVEMBER 2017

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:

$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

QUESTION 2

2.1 Given the following quadratic number pattern: 5 ; –4 ; –19 ; –40 ; …

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\,;\,\frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

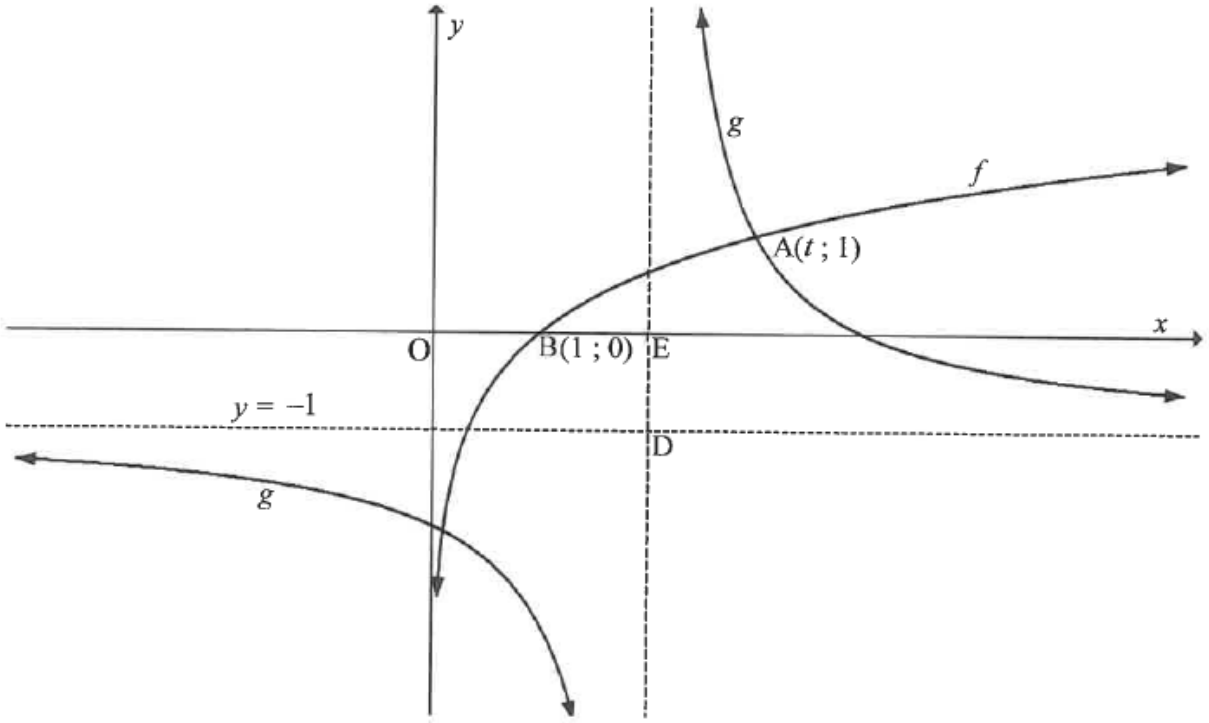
4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

[23]

QUESTION 5

The diagram below shows the graphs of $g(x)=\frac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$ (2)

5.5 For which values of x will $f^{-1}(x)<3$? (2)

5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

QUESTION 7

7.1 Given: $f(x)=2x^2-x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$

8.1 Show that f has a point of inflection at $x=2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y=-f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$. (2)

8.4.2 Claire claims that $f'(2)=1$.

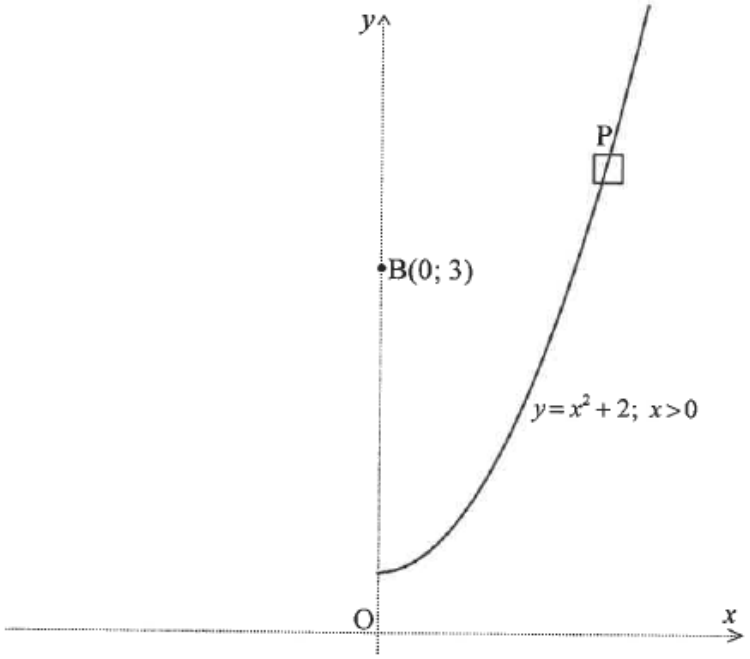
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y=x^2+2$, $x\geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
[6]

TOTAL: 150

MATHEMATICS P2

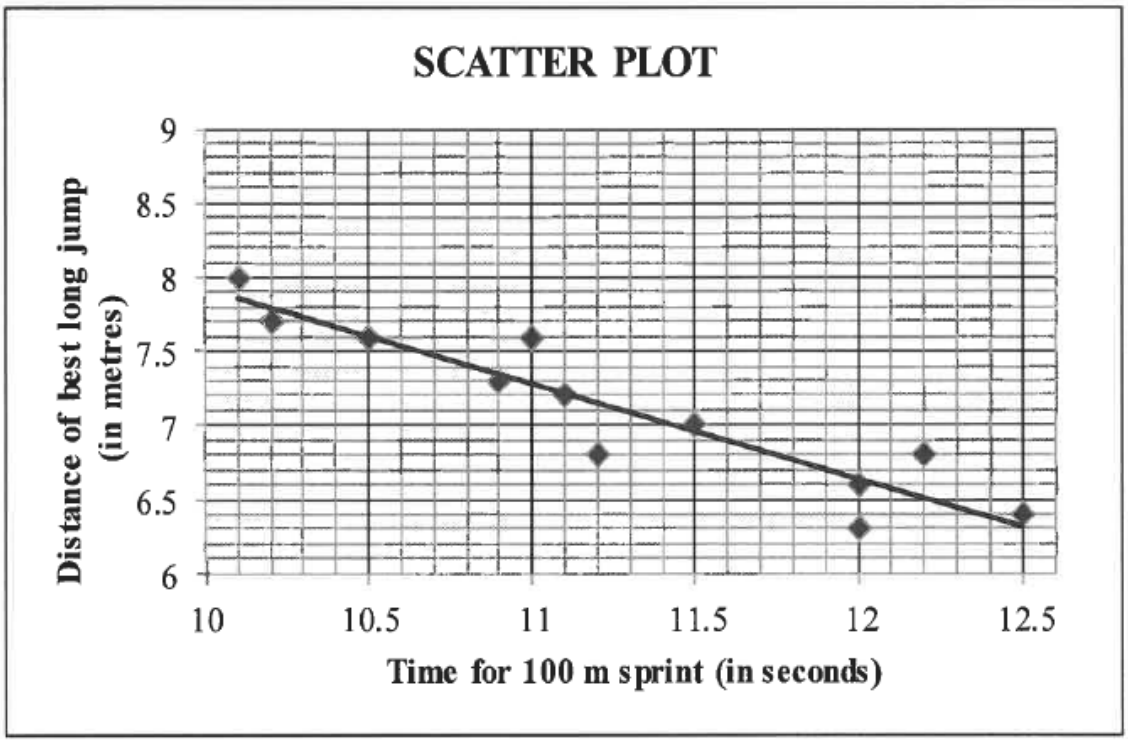
NOVEMBER 2017

QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)
[7]

QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

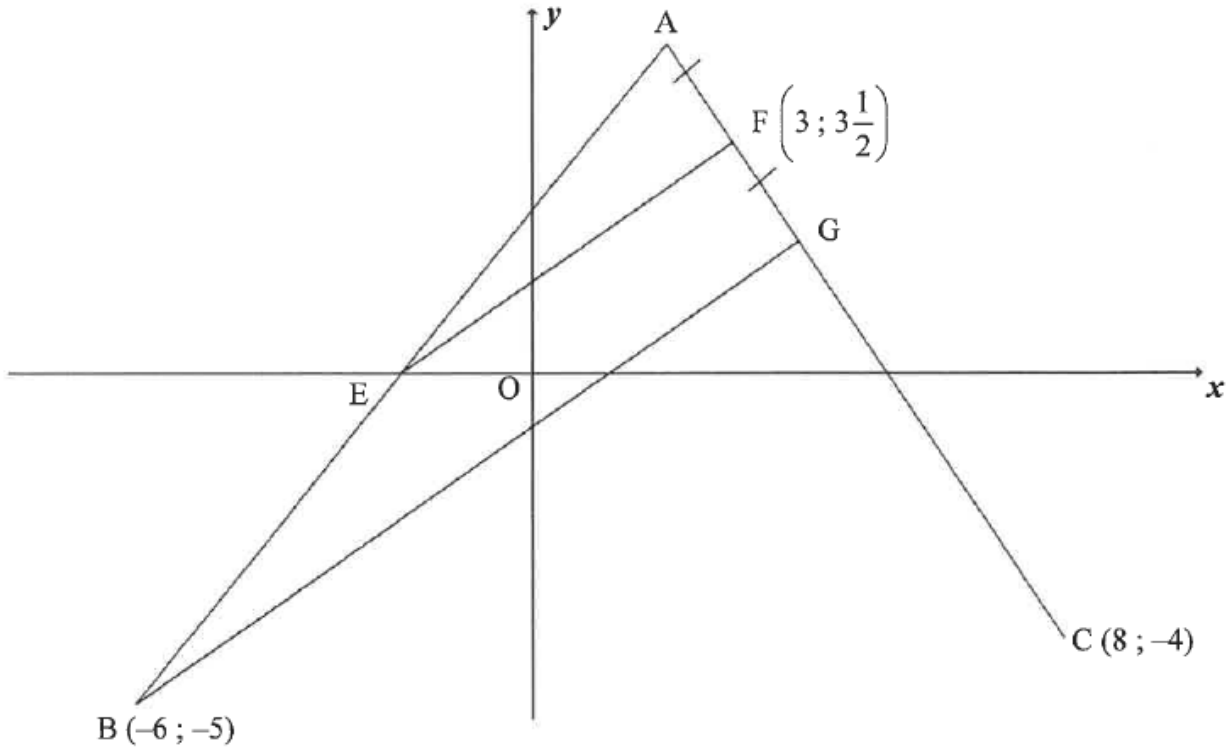
2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)

[13]

QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3; 3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

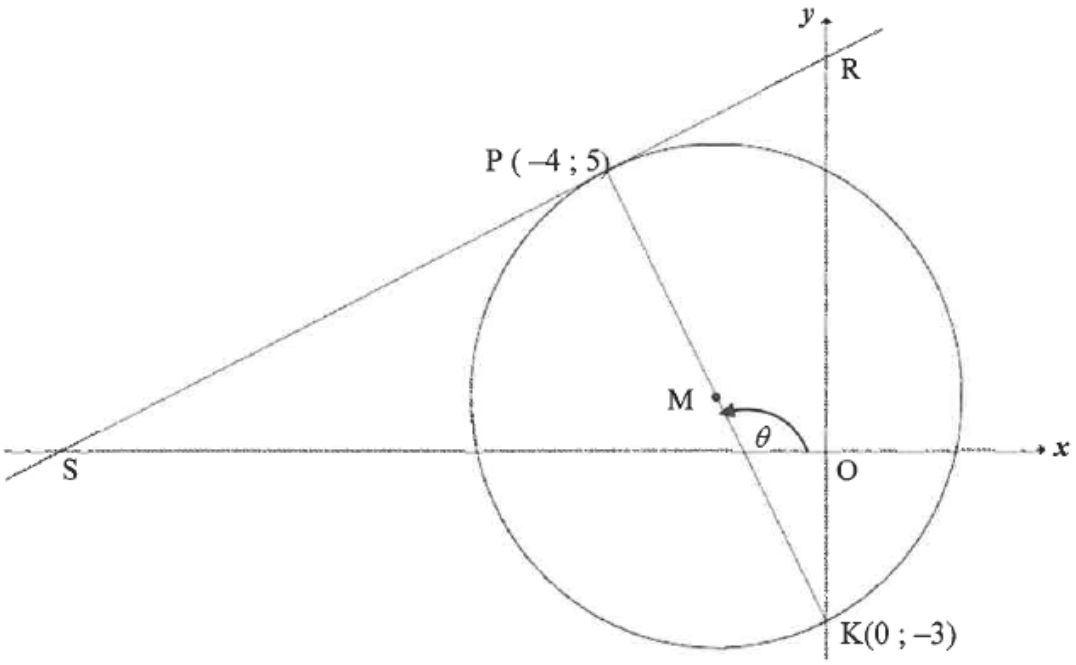
3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

3.3 Prove that EF || BG. (4)

3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

4.3 Calculate the area of ΔSMK. (5)

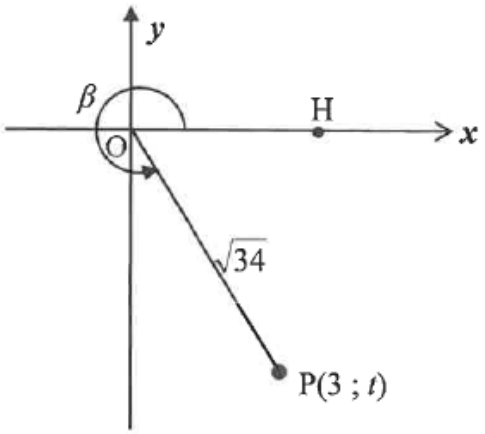
[23]

QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $\hat{HOP}=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

5.3 Prove:

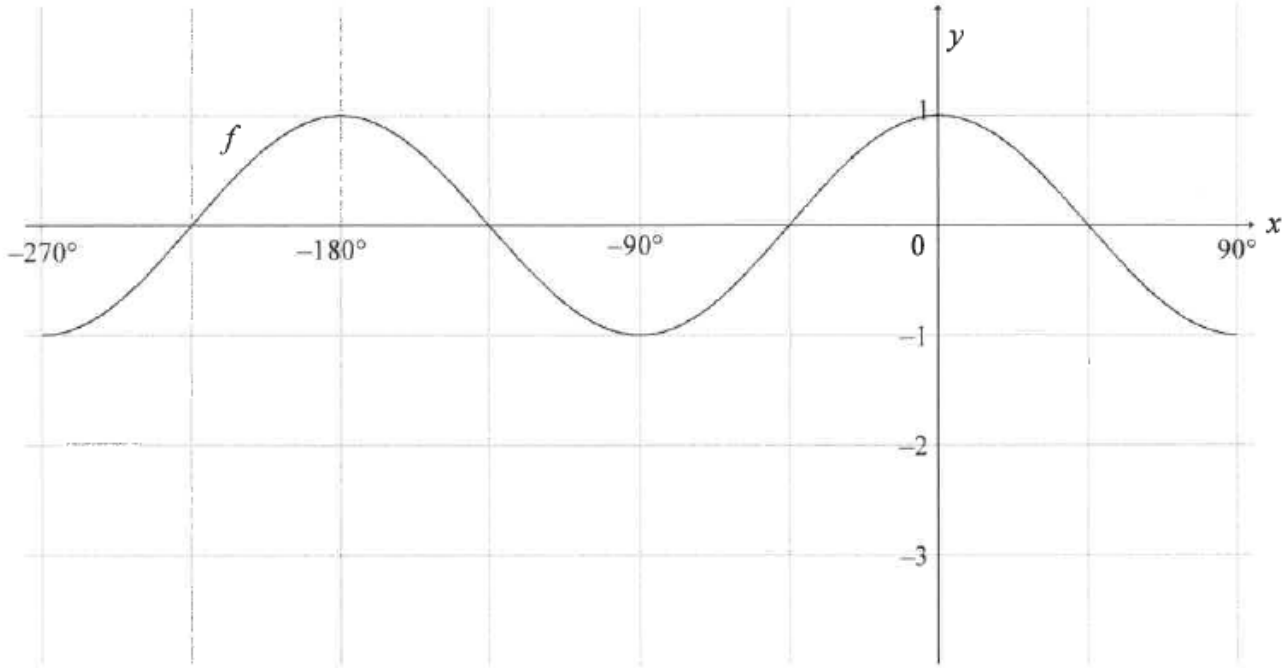
5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77°-\sin 43°=\sin 17°$ (4)

[19]

QUESTION 6

In the diagram, the graph of $f(x)=\cos 2x$ is drawn for the interval $x\in[-270°;90°]$.



6.1 Draw the graph of $g(x)=2\sin x-1$ for the interval $x\in[-270°;90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

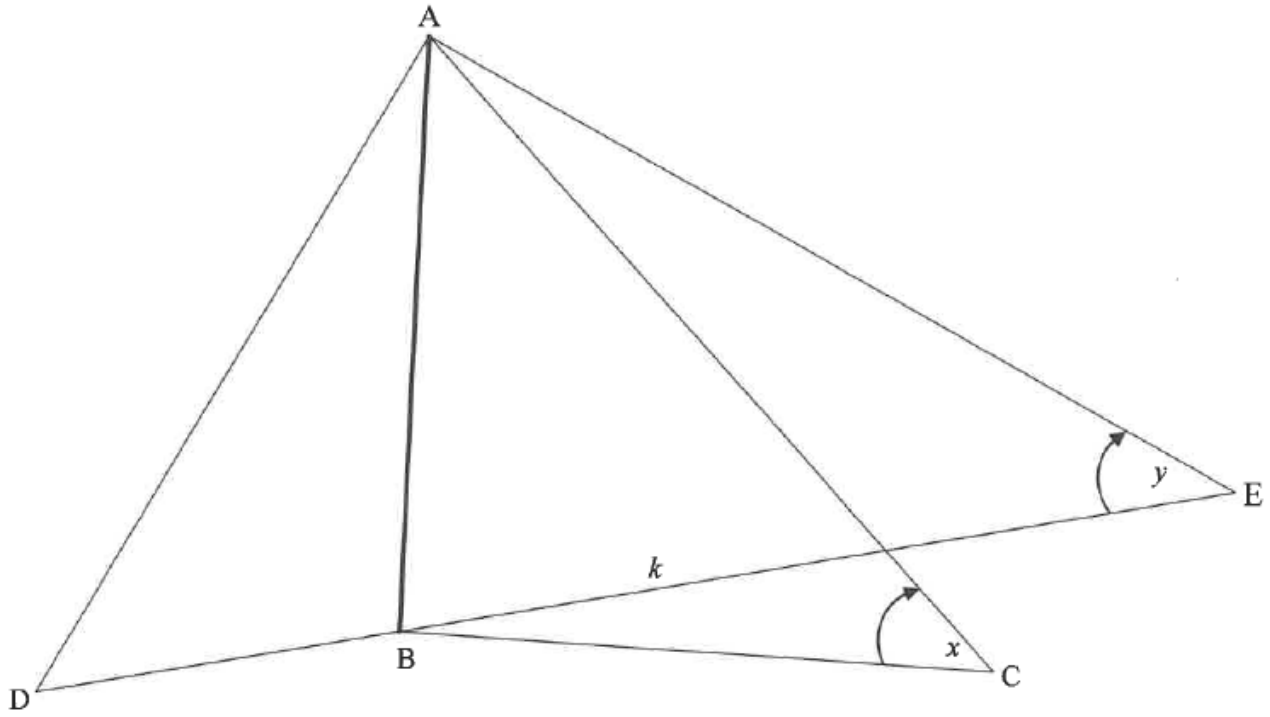
6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x=\dfrac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x\in[-270°;90°]$. (4)

[12]

QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

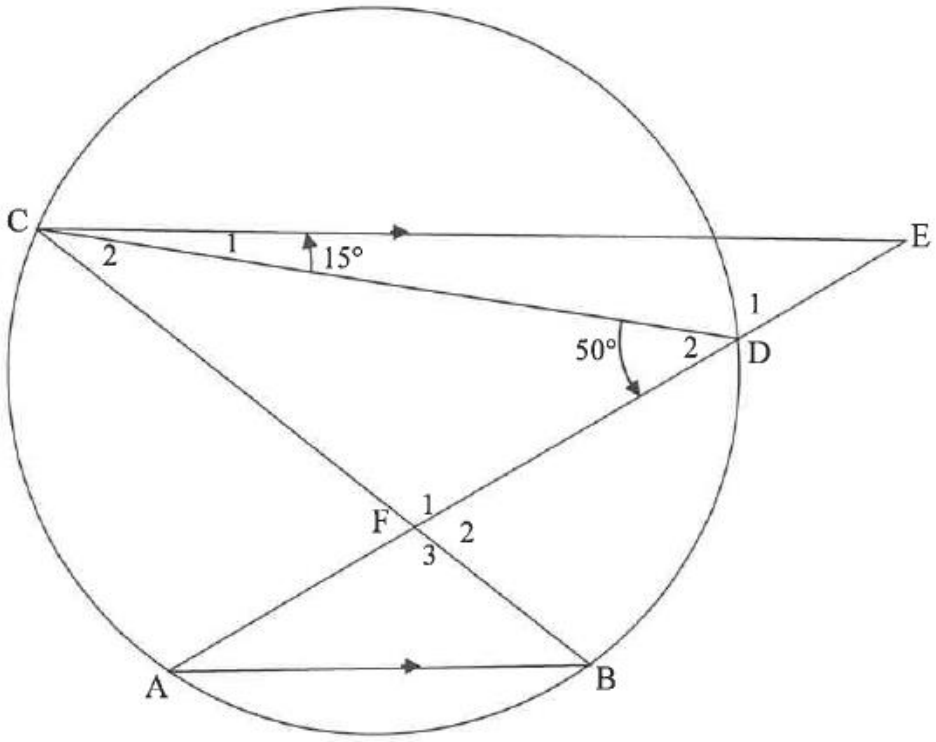
7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)

[10]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

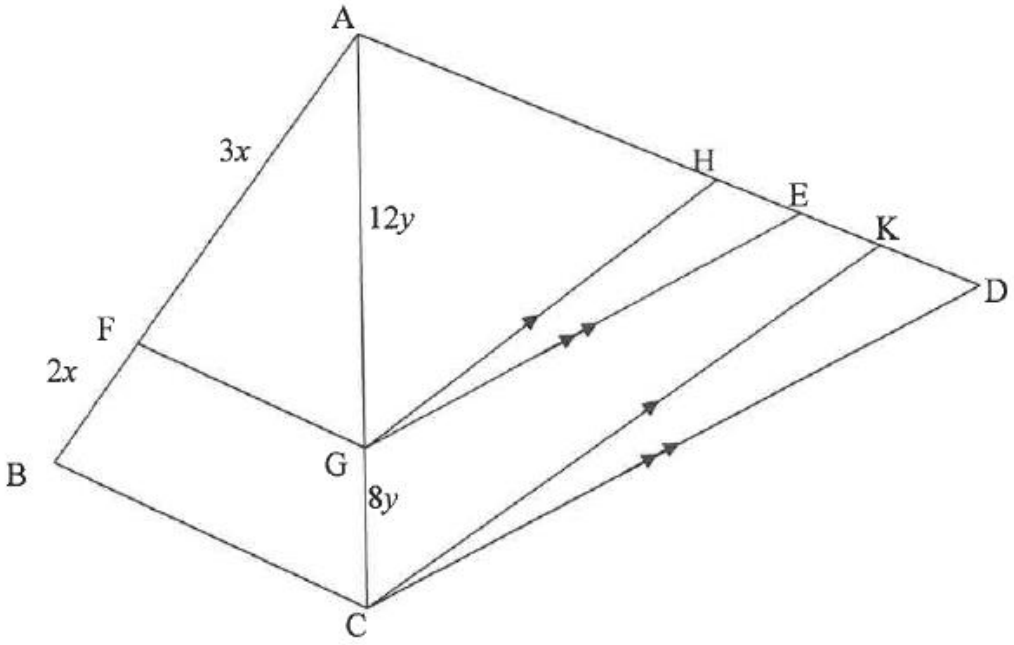
8.1.1 $\hat{A}$ (3)

8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)
[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

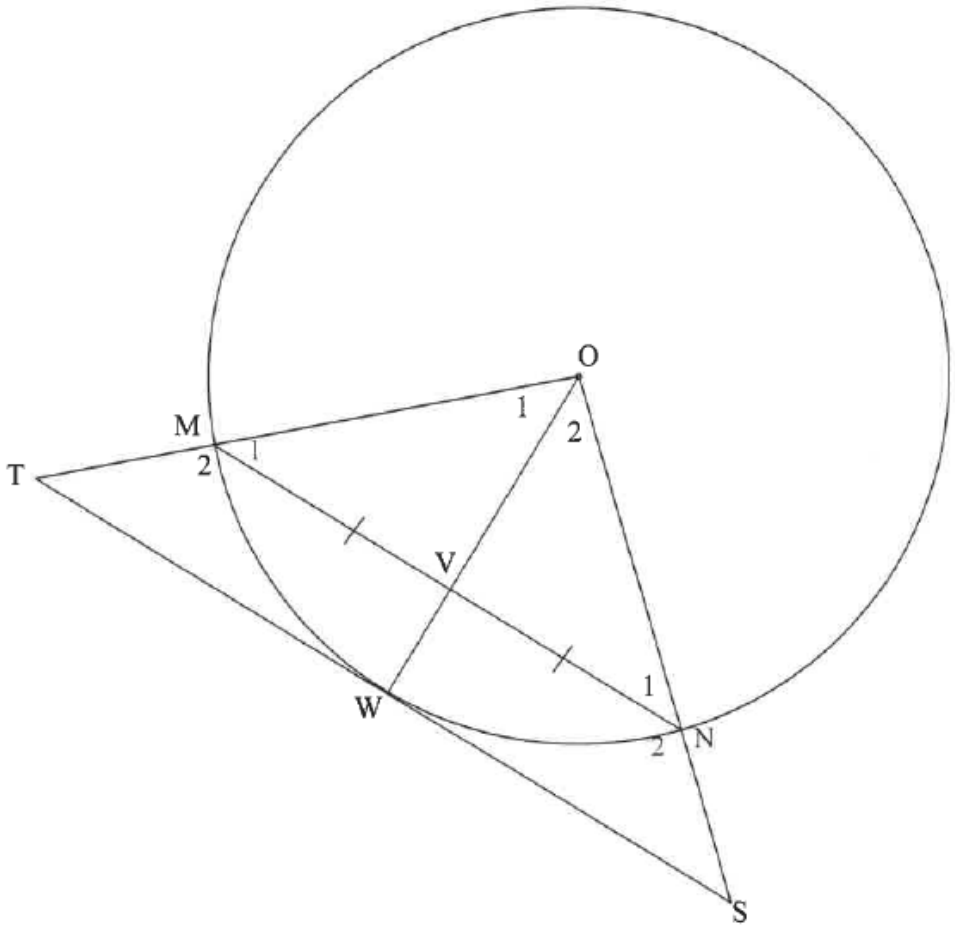
9.1.1 FG || BC (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)
[10]

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

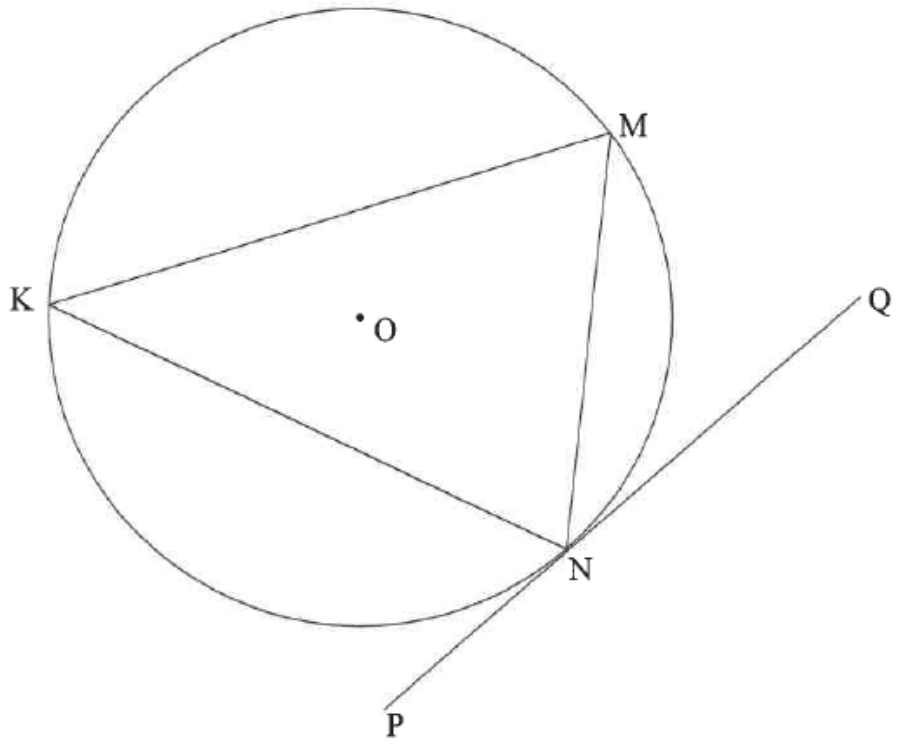
10.2.1 MN || TS (2)

10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)
[12]

QUESTION 11

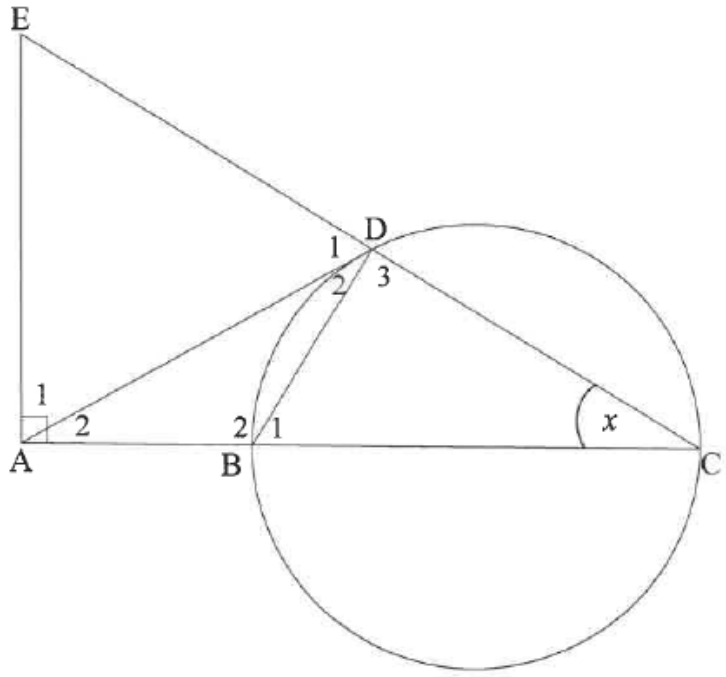
11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.

Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) ΔADB ||| ΔACD (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)

[20]

TOTAL: 150

MATHEMATICS P1

FEBRUARY/MARCH 2018

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6x - 16 = 0$ (3)

1.1.2 $2x^2 + 7x - 1 = 0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2 - 25 < 0$. (4)

1.3 Solve for x and y:

$-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6)

1.4 Evaluate: $\dfrac{3^{2018} + 3^{2016}}{3^{2017}}$ (2)

1.5 Given: $t(x) = \dfrac{\sqrt{3x-5}}{x-3}$

1.5.1 For which values of x will $\dfrac{\sqrt{3x-5}}{x-3}$ be real? (3)

1.5.2 Solve for x if $t(x) = 1$. (4)

[26]

QUESTION 2

2.1 Given the following geometric sequence: $30 \; ; \; 10 \; ; \; \dfrac{10}{3} \; ; \ldots$

2.1.1 Determine n if the n^{th} term of the sequence is equal to $\dfrac{10}{729}$. (4)

2.1.2 Calculate: $30 + 10 + \dfrac{10}{3} + \ldots$ (2)

2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d. (4)

[10]

QUESTION 3

The first three terms of an arithmetic sequence are -1 ; 2 and 5.

3.1 Determine the n^{th} term, T_n, of the sequence. (2)

3.2 Calculate T_{43}. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of n. (3)

3.4 A quadratic sequence, with general term T_n, has the following properties:

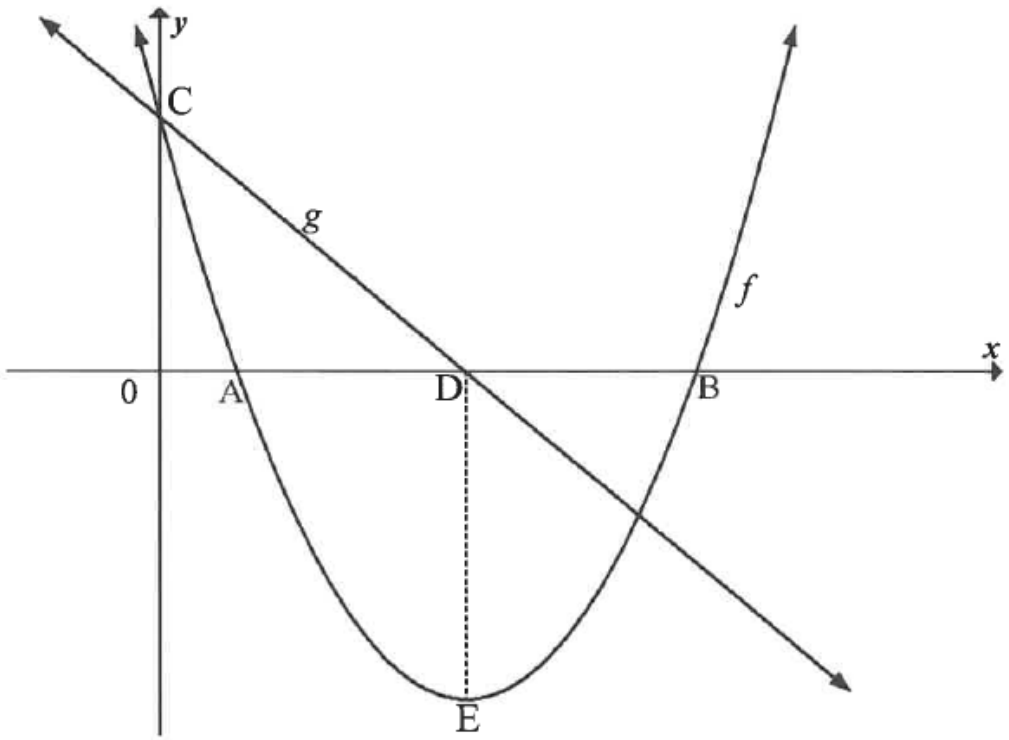
- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

Determine the first term of the sequence. (6)
[13]

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



4.1 Write down the coordinates of E. (2)

4.2 Calculate the coordinates of A. (3)

4.3 M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M. (3)

4.4 Determine the equation of g in the form $y = mx + c$. (3)

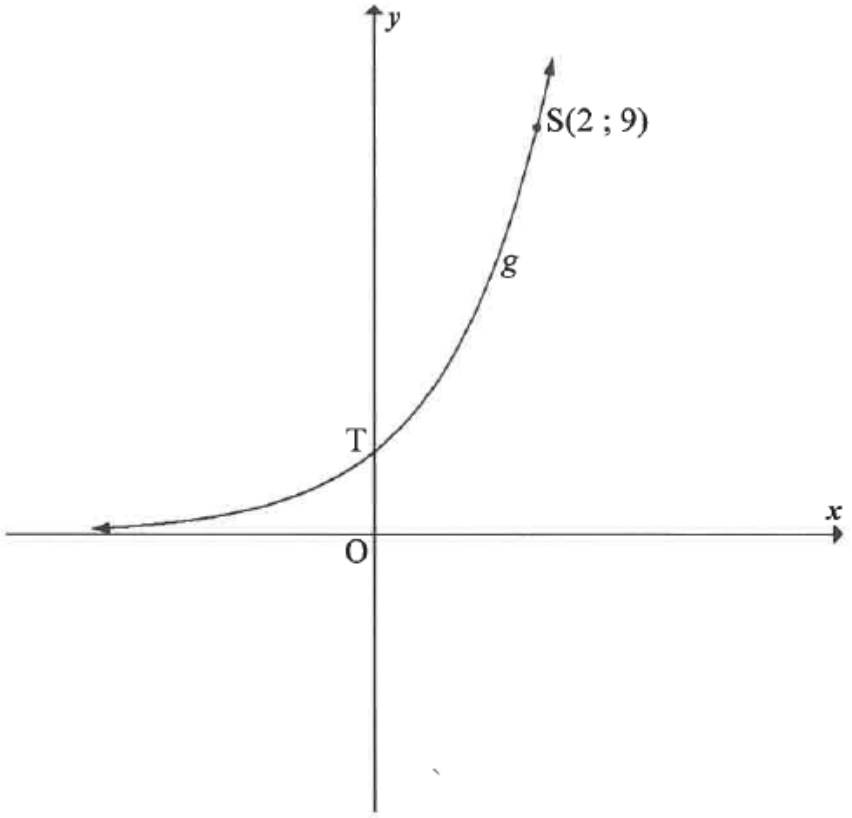
4.5 Write down the equation of g^{-1} in the form $y = \ldots$ (3)

4.6 For which values of x will $x(f(x)) \leq 0$? (4)

[18]

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1 Write down the coordinates of T. (2)

5.2 Calculate the value of a. (2)

5.3 The graph h is obtained by reflecting g in the y-axis. Write down the equation of h. (2)

5.4 Write down the values of x for which $0 < \log_3 x < 1$. (2)

[8]

QUESTION 6

The function f, defined by $f(x)=\frac{a}{x+p}+q$, has the following properties:

- The range of f is $y \in R,\ y \neq 1$.
- The graph f passes through the origin.
- $P\left(\sqrt{2}+2;\sqrt{2}+1\right)$ lies on the graph f.

6.1 Write down the value of q. (1)

6.2 Calculate the values of a and p. (5)

6.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)
[10]

QUESTION 7

7.1 On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018.

Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (3)

7.2 On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

7.2.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)

7.2.2 How many instalments of R3 200 must she pay? (5)

7.2.3 Calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan. (5)
[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

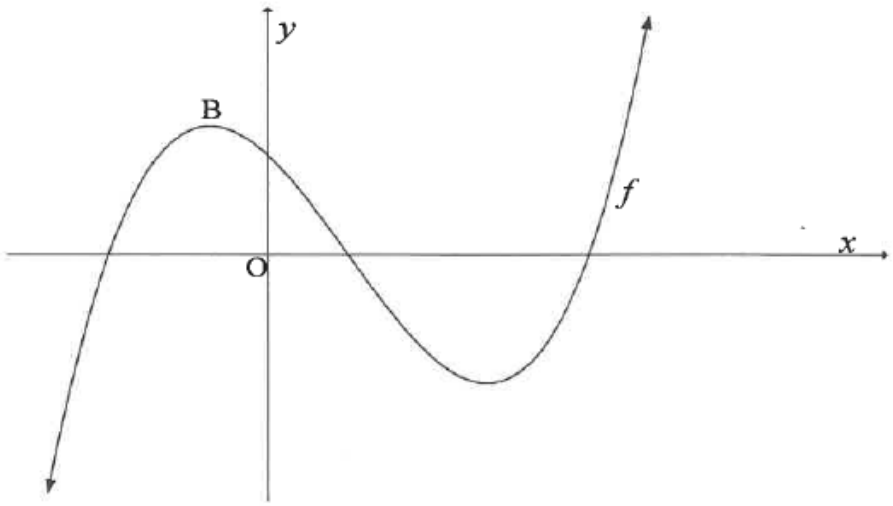
8.2.1 $D_x\left[\frac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 10

Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. **[6]**

QUESTION 11

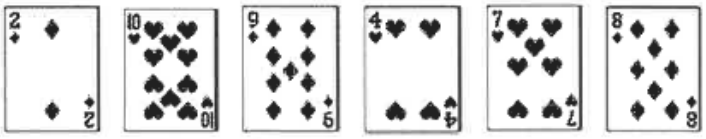
11.1 Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

11.1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)

11.1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3)

11.1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

11.2 The cards below are placed from left to right in a row.



11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)

[15]

TOTAL: 150

MATHEMATICS P2

FEBRUARY/MARCH 2018

QUESTION 1

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

DAYS	1	2	3	4	5	6	7	8	9	10
UNITS OF BLOOD	45	59	65	73	79	82	91	99	101	106

1.1 Calculate:

1.1.1 The mean of the units of blood donated per day over the period of 10 days (2)

1.1.2 The standard deviation of the data (2)

1.1.3 How many days is the number of units of blood donated at college X outside one standard deviation from the mean? (3)

1.2 The number of units of blood donated by the students of college X is represented in the box and whisker diagram below.

1.2.1 Describe the skewness of the data. (1)

1.2.2 Write down the values of **A** and **B**, the lower quartile and the upper quartile respectively, of the data set. (2)

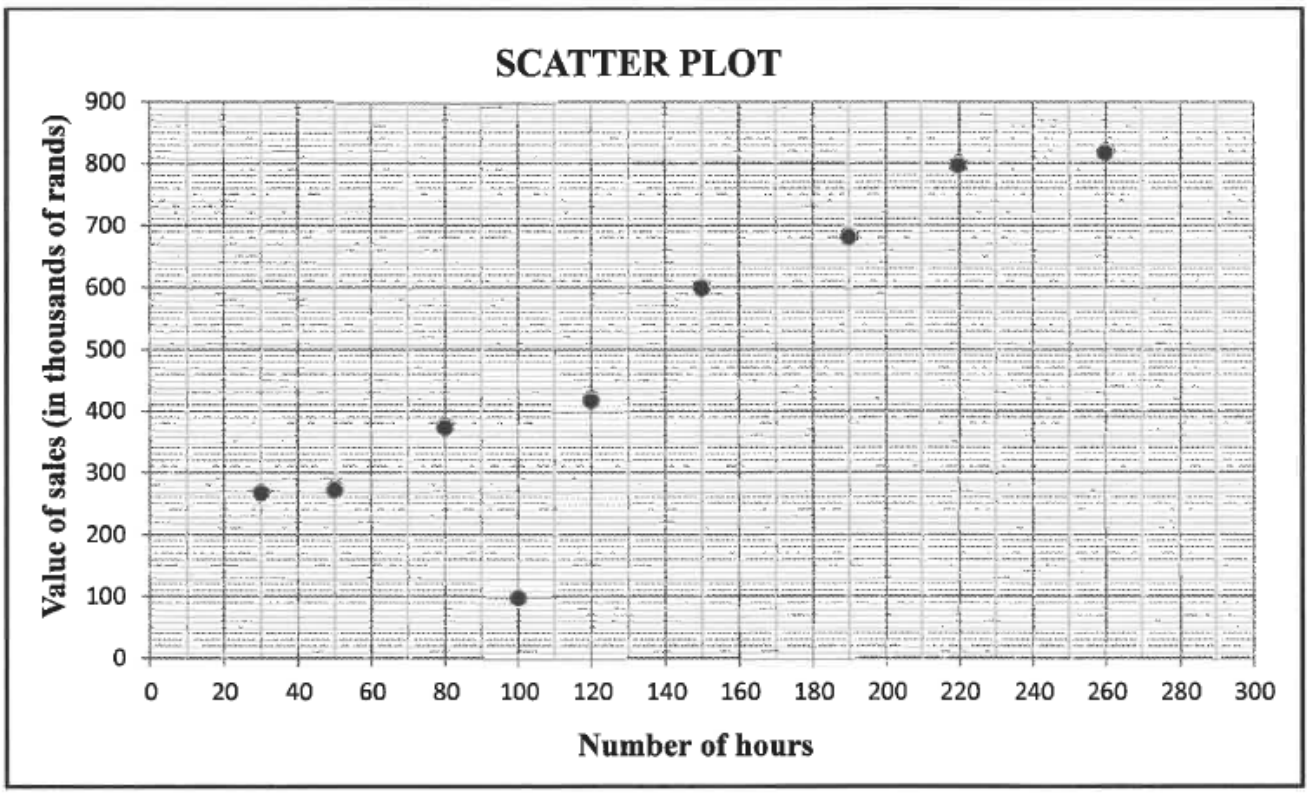
1.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1)

[11]

QUESTION 2

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

NUMBER OF HOURS	30	50	80	100	120	150	190	220	260
VALUE OF SALES (IN THOUSANDS OF RANDS)	270	275	376	100	420	602	684	800	820



2.1 Identify an outlier in the data above. (1)

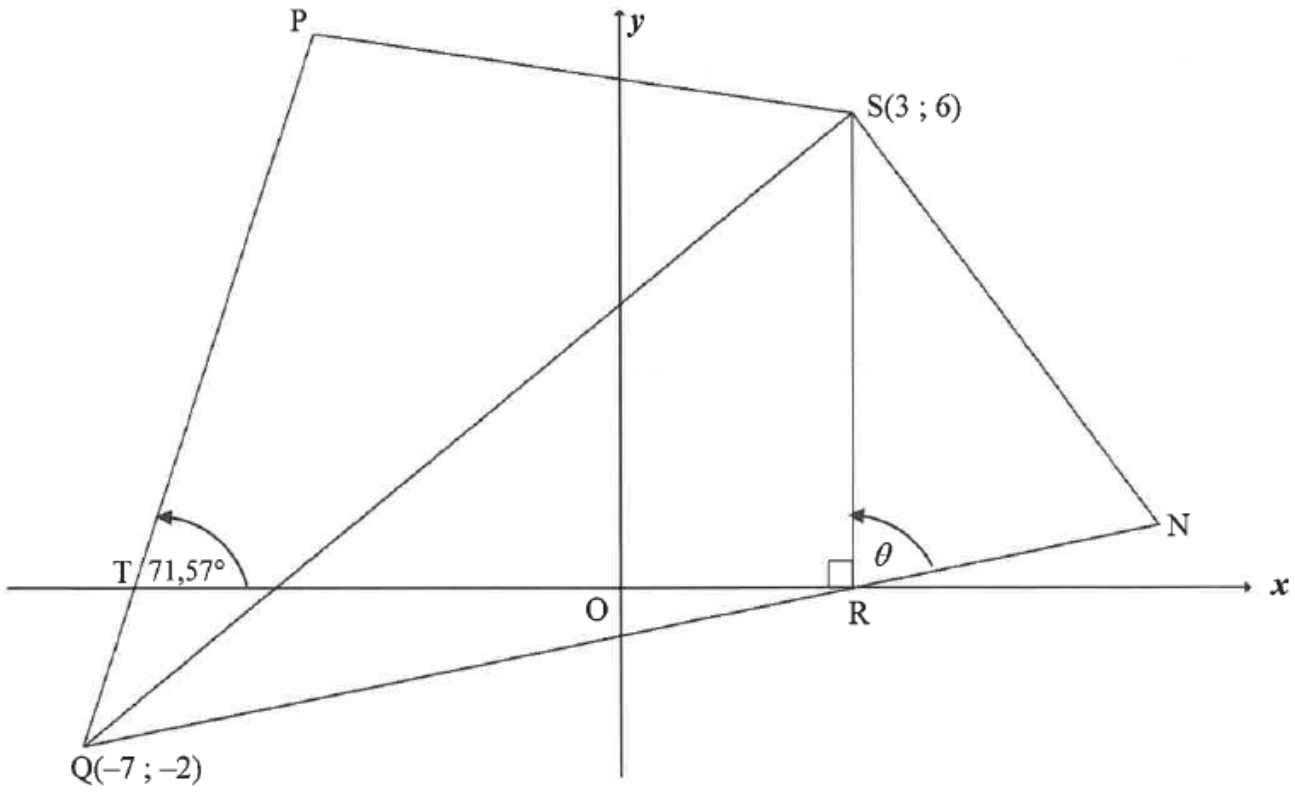
2.2 Calculate the equation of the least squares regression line of the data. (3)

2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year. (2)

2.4 What is the expected increase in sales for EACH additional hour spent with a client? (2)
[8]

QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.

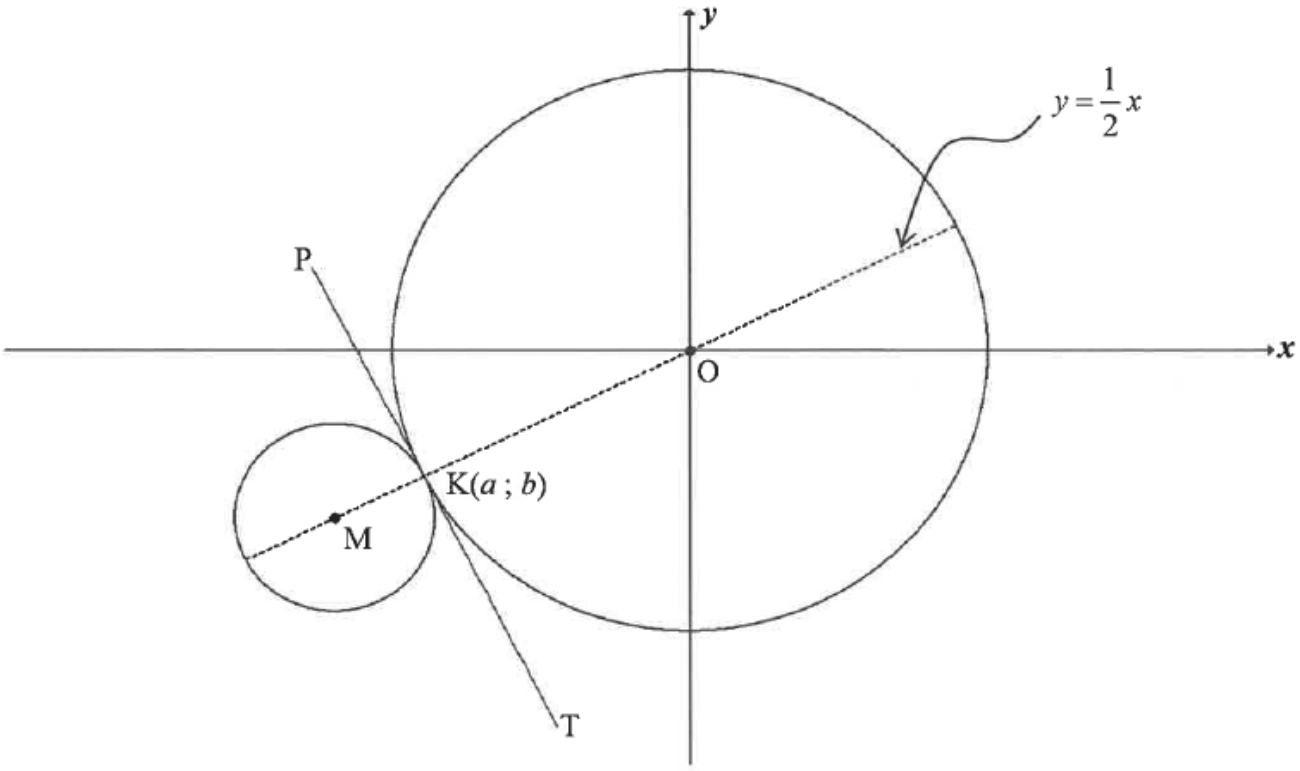


Determine:

3.1	The equation of SR	(1)
3.2	The gradient of QP to the nearest integer	(2)
3.3	The equation of QP in the form $y = mx + c$	(2)
3.4	The length of QR. Leave your answer **in surd form**.	(2)
3.5	$\tan(90° - \theta)$	(3)
3.6	The area of ΔRSN, **without using a calculator**	(6)
		[16]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

QUESTION 5

5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

5.1.2 $\sin^2 \theta$ (3)

5.2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio. (6)

5.3 Determine the value of $\sin 3x.\cos y + \cos 3x.\sin y$ if $3x + y = 270°$. (2)

5.4 Given: $2\cos x = 3\tan x$

5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)

5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)

5.4.3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

5.5 Consider: $g(x) = -4\cos(x + 30°)$

5.5.1 Write down the maximum value of $g(x)$. (1)

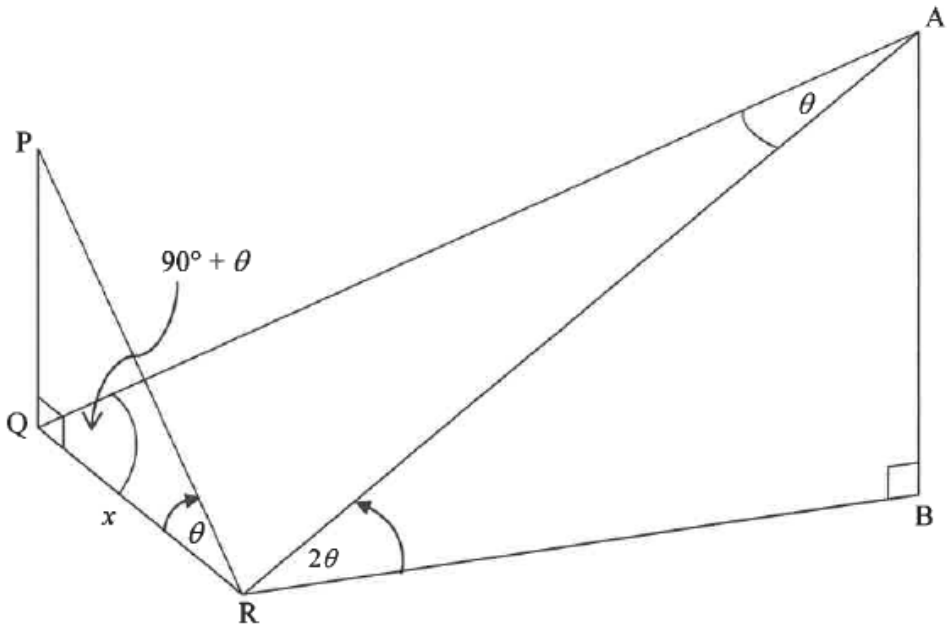
5.5.2 Determine the range of $g(x) + 1$. (2)

5.5.3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)

[33]

QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

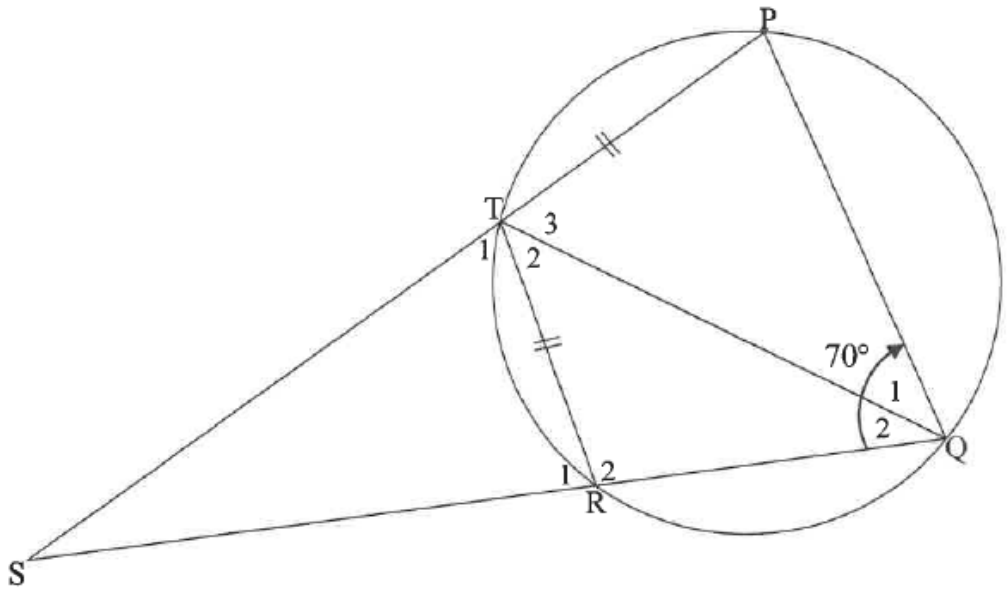
6.2 Show that $AB = 2x\cos^2\theta$ (4)

6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. $S\hat{Q}P = 70°$



7.1 Calculate, with reasons, the size of:

7.1.1 $\hat{T}_1$ (2)

7.1.2 $\hat{Q}_1$ (2)

7.2 If it is further given that PQ || TR:

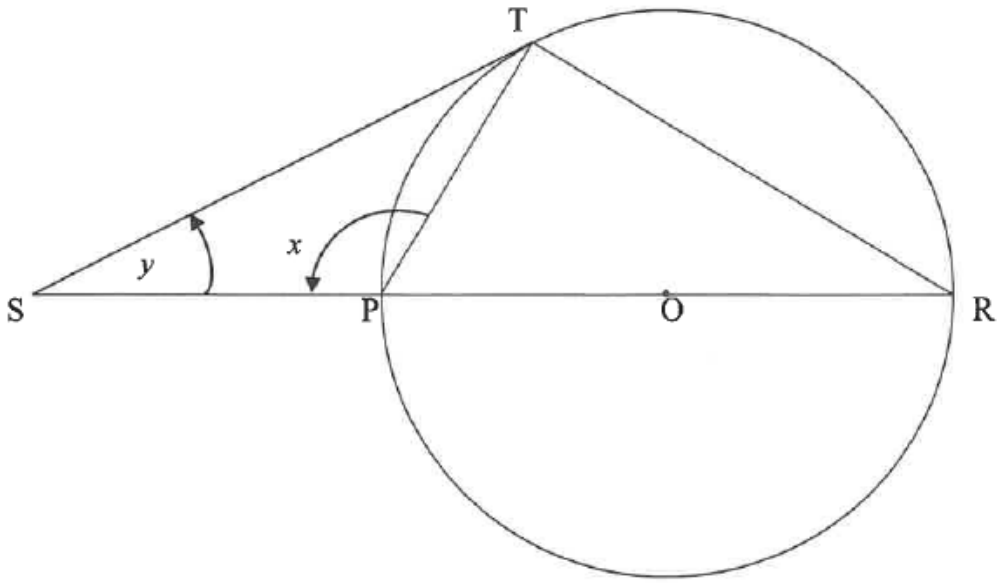
7.2.1 Calculate, with reasons, the size of $\hat{T}_2$ (2)

7.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$ (2)

[8]

QUESTION 8

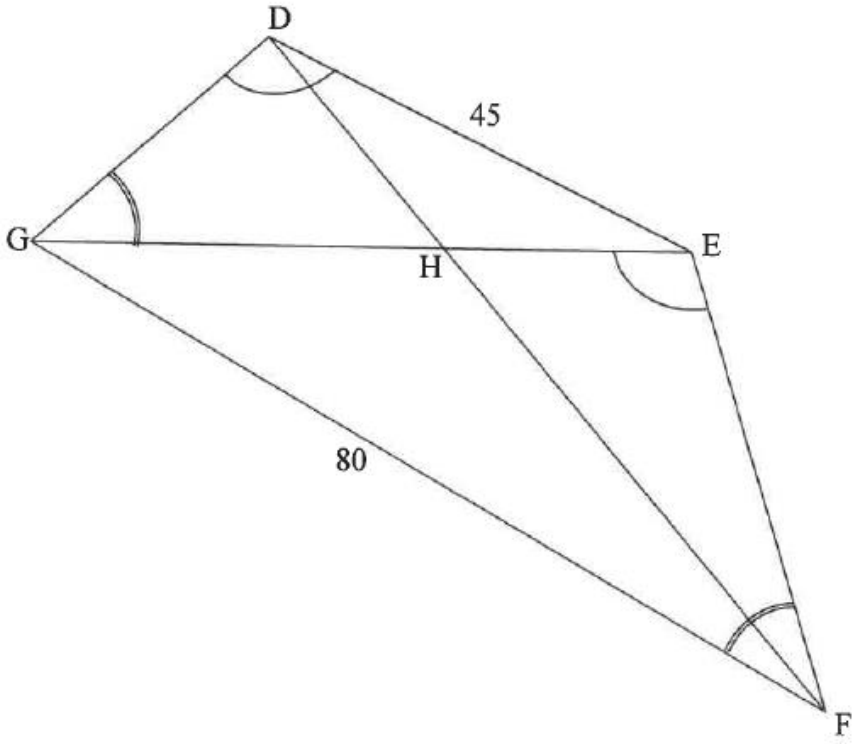
In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.



Determine, with reasons, y in terms of x. **[6]**

QUESTION 9

In the diagram, DEFG is a quadrilateral with DE = 45 and GF = 80. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



9.1 Give a reason why $\Delta DEG \,|||\, \Delta EGF$. (1)

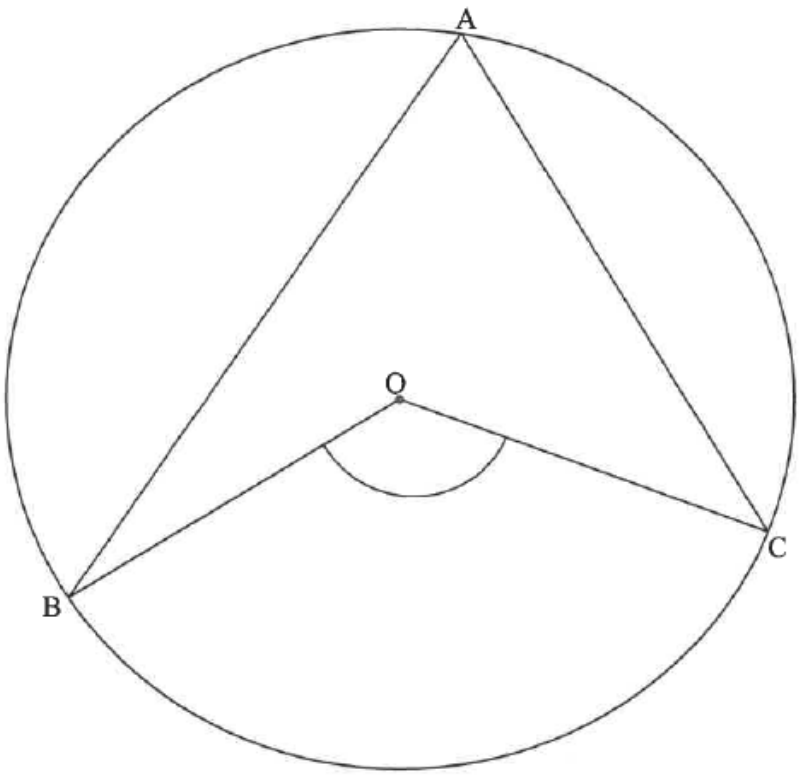
9.2 Calculate the length of GE. (3)

9.3 Prove that $\Delta DEH \,|||\, \Delta FGH$. (3)

9.4 Hence, calculate the length of GH. (3)
[10]

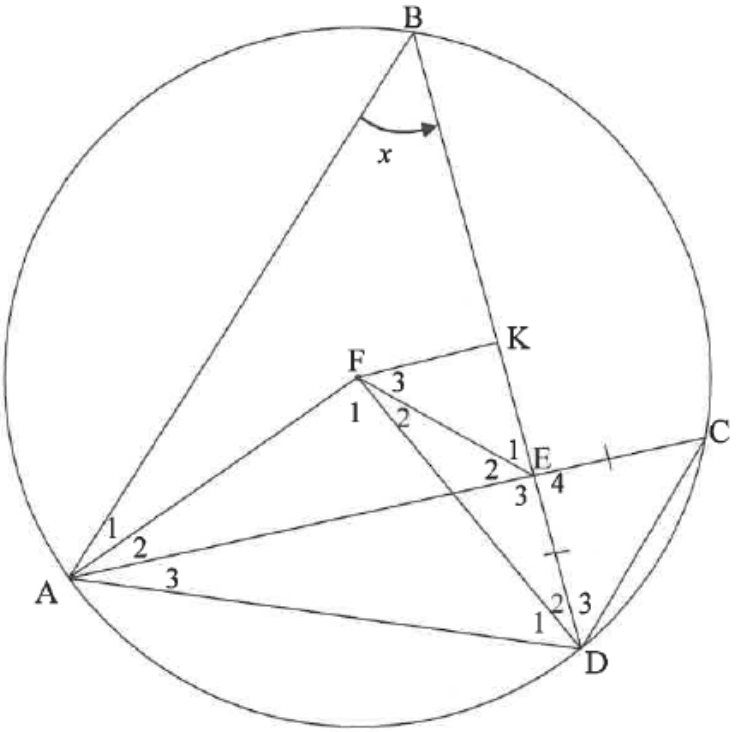
QUESTION 10

10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.



Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

10.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$ (2)

(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. (4)

10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

10.2.4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\frac{AE}{ED}$. (5)

[24]

TOTAL: 150

SOLUTIONS

MATHEMATICS P1/*WISKUNDE V1*

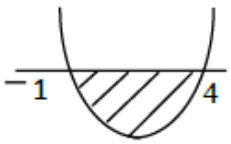
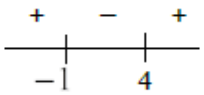
NOVEMBER 2014

MEMORANDUM

QUESTION/*VRAAG* 1

1.1.1	$(x-2)(4+x)=0$ $x=2 \quad \text{or} \quad x=-4$	✓$x=2$ ✓$x=-4$ (2)
1.1.2	$3x^2-2x-14=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\dfrac{2\pm\sqrt{(-2)^2-4(3)(-14)}}{2(3)}$ $=\dfrac{2\pm\sqrt{172}}{6}$ $x=2{,}52$ or/*of* $x=-1{,}85$ **OR/*OF*** $x^2-\dfrac{2}{3}x+\dfrac{1}{9}=\dfrac{14}{3}+\dfrac{1}{9}$ $\left(x-\dfrac{1}{3}\right)^2=\dfrac{43}{9}$ $x-\dfrac{1}{3}=\pm\dfrac{\sqrt{43}}{3}$ $\therefore x=\dfrac{1\pm\sqrt{43}}{3}$ $x=2{,}52 \quad \text{or/}of \quad x=-1{,}85$	✓ standard form/*standaardvorm* ✓ substitution into correct formula/ *substitusie in korrekte formule* ✓✓ answers/ *antwoorde* (4) ✓for adding $\frac{1}{9}$ on both sides/*tel* $\frac{1}{9}$ *by aan beide kante* ✓ $x=\dfrac{1\pm\sqrt{43}}{3}$ ✓✓answers (4)

1.1.3	$2^{x+2} + 2^x = 20$ $2^x(2^2 + 1) = 20$ $2^x = \frac{20}{5}$ $2^x = 2^2$ $\therefore x = 2$ **OR/*OF*** $2^x.2^2 + 2^x = 2^2.5$ $2^x(2^2 + 1) = 2^2.5$ $2^x.5 = 2^2.5$ $\therefore x = 2$ **OR/*OF*** $4.2^x + 2^x = 20$ $5.2^x = 20$ $2^x = 4 = 2^2$ $\therefore x = 2$	✓common factor/*gemeen. faktor* ✓ simplification/ *vereenvoudiging* ✓ answer/*antwoord* (3) ✓ common factor/*gemeen. faktor* ✓ simplification/ *vereenvoudiging* ✓ answer/*antwoord* (3) ✓ $5.2^x = 20$ ✓ $2^x = 4$ ✓answer/*antwoord* (3)
1.2	$x = 2y + 3$ .....(1) $3x^2 - 5xy = 24 + 16y$ .....(2) (1) in (2) : $3(2y + 3)^2 - 5(2y + 3)y = 24 + 16y$ $3(4y^2 + 12y + 9) - 10y^2 - 15y = 24 + 16y$ $12y^2 + 36y + 27 - 10y^2 - 15y - 24 - 16y = 0$ $2y^2 + 5y + 3 = 0$ $(2y + 3)(y + 1) = 0$ $y = -\frac{3}{2}$ or $y = -1$ $\therefore x = 2\left(-\frac{3}{2}\right) + 3$ *or* $x = 2(-1) + 3$ $x = 0$ *or* $x = 1$ $(0\,;-\frac{3}{2})$ $(1\,;-1)$ **OR/*OF***	✓ substitution/*substitusie* ✓ simplification/ *vereenvoudiging* ✓standard form/ *standaardvorm* ✓factorisation/*faktorisering* ✓*y*-values/*y-waardes* ✓*x*-values/*x-waardes* (6)

	$y=\frac{x-3}{2}$ $3x^2-5x\left(\frac{x-3}{2}\right)=24+16\left(\frac{x-3}{2}\right)$ $3x^2-\frac{5x^2-15x}{2}=24+\frac{16x-48}{2}$ $\times 2: 6x^2-5x^2+15x=48+16x-48$ $x^2-x=0$ $x(x-1)=0$ $x=0 \text{ or } x=1$ $y=-\frac{3}{2} \text{ or } y=-1$	✓substitution/*substitusie* ✓simplification/ *vereenvoudiging* ✓standard form / *standard vorm* ✓ factors/*faktore* ✓*x*- values/*x- waardes* ✓ *y*-values/*y*-waardes (6)
1.3	$(x-1)(x-2)<6$ $x^2-3x+2<6$ $x^2-3x-4<0$ $(x+1)(x-4)<0$  **OR/ *OF***  $-1<x<4$ or $x\in(-1;4)$	✓ standard form/ *standaardvorm* ✓factorisation/*faktorisering* ✓critical values in the context of inequality / *kritiese waardes in die konteks van die ongelykheid* ✓notation/*notasie* (4)
1.4	$-k-4\geq 0$ $k\leq -4$	✓ $-k-4\geq 0$ ✓ answer/*antwoord* (2) **[21]**

QUESTION/*VRAAG* 2

2.1	$T_4 = 23$	✓23 (1)
2.2	$T_{251} = a + (n-1)d$ $= 2 + (251-1)(7)$ $= 1752$	✓$a = 2$ and $d = 7$ ✓subst. into correct formula /*subt. in korrekte formule* ✓1752 (3)
2.3	$\sum_{n=1}^{251}(7n-5)$ **OR/*OF*** $\sum_{p=0}^{250}(7p+2)$	✓ general term/ *algemene term* ✓ complete answer /*volledige antwoord* (2) ✓ general term/ *algemene term* ✓ complete answer / *volledige antwoord* (2)
2.4	$S_n = \frac{n}{2}[a+l]$ $S_n = \frac{251}{2}[2+1752]$ $= 220127$ **OR/*OF*** $S_n = \frac{n}{2}[2a+(n-1)d]$ $= \frac{251}{2}[2(2)+(251-1)(7)]$ $= 220127$	✓substitution/*substitusie* ✓220127 (2) ✓substitution/*substitusie* ✓220127 (2)
2.5	The new series/*Die nuwe reeks* is 16 + 44 + 72 + …+1 752 $16 + 28(n-1) = 1752$ $1736 = 28(n-1)$ $62 = n-1$ $n = 63$ **OR/*OF*** 2 + 9 + <u>16</u> + 23 + 30 + 37 + <u>44</u> + 51 + ... + <u>1752</u> T_3 is divisible by /is deelbaar deur 4 Then $T_7, T_{11}, T_{15}, ..., T_{251}$ are divisible by 4, thus each 4th term is divisible by 4. Daarna is $T_7, T_{11}, T_{15}, ..., T_{251}$ deelbaar deur 4, d.w.s. elke 4de term is deelbaar deur 4. $\therefore$ number of terms divisible by 4 will be $= \frac{251-3}{4}+1 = 63$ $\therefore$ aantal terme deelbaar deur 4 sal wees $= \frac{251-3}{4}+1 = 63$ **OR/OF**	✓✓generating new series divisible by 4/ *vorming van nuwe reeks deelbaar deur 4* ✓ $T_n = 1752$ ✓ 63 (4) ✓ T_3 is divisible by 4/ *is deelbaar deur 4* ✓identifying terms divisible by 4/ *identifiseer terme deelbaar deur 4* ✓ reasoning/*redenering* ✓63 (4)

	Position of terms divisible by 4: 3 ; 7 ; 11 ; …; 247; 251 $T_n = 4n - 1 = 251$ $4n = 252$ $n = 63$	✓✓ generating sequence involving position of terms/*vorming van reeks i.t.v. posisie van terme* ✓ $T_n = 251$ ✓ 63 (4) **[12]**

QUESTION/*VRAAG* 3

3.1.1	$-1 \;;\; -7 \;;\; -11 \;;\; p \;;\ldots$ $\quad\searrow\quad\searrow\quad\quad\searrow$ $\quad -6 \quad -4 \quad p+11$ $\quad\quad\searrow\quad\searrow$ $\quad\quad 2 \quad 2$ $p+11-(-4)=2$ $p+15=2$ $p=-13$ **OR/*OF*** $-1 \;;\; -7 \;;\; -11 \;;\; p \;;\ldots$ $\quad\searrow\quad\searrow\quad\quad\searrow$ $\quad -6 \quad -4 \quad p+11$ $\quad\quad\searrow\quad\searrow$ $\quad\quad 2 \quad 2$ $p+11=-2$ $p=-13$	✓$p+15=2$ ✓$p=-13$ (2) ✓first differences/ *eerste verskille* ✓$p=-13$ (2)
3.1.2	$2a=2$ $a=1$ $3a+b=-6$ $3(1)+b=-6$ $b=-9$ $a+b+c=-1$ $1-9+c=-1$ $c=7$ $T_n=n^2-9n+7$ **OR/*OF*** $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)d_2}{2}$ $=-1+(n-1)(-6)+\frac{(n-1)(n-2)(2)}{2}$ $=-1-6n+6+\frac{2n^2-6n+4}{2}$ $=n^2-9n+7$	✓$a=1$ ✓$b=-9$ ✓$c=7$ ✓answer/*antwoord* (4) ✓formula/*formule* ✓substitution of first and second differences/*substitusie van eerste en tweede verskille* ✓simplification/*vereenvoudiging* ✓answer/*antwoord* (4)

OR/*OF*

$7;-1\ ;\ -7\ ;\ -11\ ;\ p\ ;\ \ldots$

$-8 \qquad -6 \qquad -4 \qquad p+11$

$2 \qquad 2 \qquad 2$

$T_0 = 7 = c$

$2a = 2 \quad \therefore a = 1$

$3a + b = -6 \quad \therefore b = -9$

$T_n = n^2 - 9n + 7$

✓ *c*-value/*c-waarde*
✓ *a*-value/*a-waarde*
✓ *b*-value/*b-waarde*

✓ answer/*antwoord*

(4)

OR/OF

$a = \frac{1}{2}(2) = 1$

$\therefore T_n = n^2 + bn + c$

$T_1 = -1 \ \therefore 1 + b + c = -1 \quad \ldots\ldots(1)$

$T_2 = -7 \ \therefore 4 + 2b + c = -7 \ \ldots\ldots(2)$

$(2) - (1): \quad 3 + b = -6$

$\therefore b = -9$

sub in $(1): c = 7$

$\therefore T_n = n^2 - 9n + 7$

✓ *a*-value/*a-waarde*

✓ *b*-value/*b-waarde*
✓ *c*-value/*c-waarde*

✓ answer/*antwoord*

(4)

3.1.3	The sequence of first differences is/*Die reeks van eerste verskille is*: $-6\ ;-4\ ;-2\ ;0\ ;\ldots$ $-6+(n-1)(2)=96$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$ **OR/OF** The sequence of first differences is/*Die reeks van eerste verskille is*: $-6\ ;-4\ ;-2\ ;0\ ;\ldots$ The formula for the sequence of first differences/*Die formule vir die reeks van eerste verskille* is $T_n=2n-8$ 1st difference/*1ste verskil*: $2n-8=96$ $2n=104$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$ **OR/*OF*** $T_n-T_{n-1}=96$ $(n^2-9n+7)-[(n-1)^2-9(n-1)+7]=96$ $n^2-9n+7-n^2+2n-1+9n-9-7=96$ $2n=106$ $n=53$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$-6+(n-1)(2)=96$ ✓52 ✓2 243 ✓2 339 (4) ✓$2n-8=96$ ✓52 ✓2 243 ✓2 339 (4) ✓$T_n-T_{n-1}=96$ ✓53 ✓2 243 ✓2 339 (4)
	OR/OF $T_{n+1}-T_n=96$ $[(n+1)^2-9(n+1)+7]-[n^2-9n+7]=96$ $n^2+2n+1-9n-9+7-n^2+9n-7=96$ $2n=104$ $n=52$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$T_{n+1}-T_n=96$ ✓52 ✓2 243 ✓2 339 (4)

3.2.1	$T_{12} = 16\left(\frac{1}{4}\right)^{12-1}$ $= \frac{1}{4^9}$ or 4^{-9} or $\frac{1}{2^{18}}$ or 2^{-18}	✓ $a = 16$ and $r = \frac{1}{4}$ ✓subst. into correct formula/ *subt in korrekte formule* ✓ answer/*antwoord* (3)
3.2.2	$S_{10} = \frac{16\left(1-\left(\frac{1}{4}\right)^{10}\right)}{1-\frac{1}{4}}$ $= 21{,}33$ **OR/OF** $S_{10} = \frac{16\left(\left(\frac{1}{4}\right)^{10}-1\right)}{\frac{1}{4}-1}$ $= 21{,}33$	✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2) ✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2)
3.3	$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{99}\right)$ $= \left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)\left(\frac{6}{5}\right)\ldots\left(\frac{100}{99}\right)$ $= \left(\frac{100}{2}\right)$ $= 50$ **OR/OF** $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{99}\right)$ $T_1 = \left(1+\frac{1}{2}\right) = \frac{3}{2}$ $T_2 = \frac{3}{2}\left(1+\frac{1}{3}\right) = \frac{3}{2}\times\frac{4}{3} = 2$ $T_3 = 2\left(1+\frac{1}{4}\right) = 2\times\frac{5}{4} = \frac{5}{2}$ $\frac{3}{2}, 2, \frac{5}{2}\ldots$ is an arithmetic sequence with $a = \frac{3}{2}$ and $d = \frac{1}{2}$ $\therefore T_{98} = \frac{3}{2} + (98-1)\frac{1}{2}$ $= \frac{100}{2} = 50$	✓ improper fractions/ *onegte breuke* ✓ $\left(1+\frac{1}{99}\right)$ or $\left(\frac{100}{99}\right)$ ✓✓ answer/*antwoord* (4) ✓$\left(1+\frac{1}{99}\right)$ ✓ giving the first three terms / *gee die eerste drie terme* ✓✓answer /*antwoord* (4) **[19]**

QUESTION/*VRAAG* 4

4.1	$p=1$ $q=1$	✓p value /*waarde* ✓q value /*waarde* (2)
4.2	$0=\frac{2}{x+1}+1$ $-x-1=2$ $x=-3$ **OR/*OF*** Reflect (0 ; 3) across $y=-x$ to get T(–3 ; 0) $x=-3$ *Reflekteer* (0 ; 3) *om* $y=-1$ *om* T(–3 ; 0) *te kry* $x=-3$	✓ $0=\frac{2}{x+1}+1$ ✓ $x=-3$ (2) ✓ reflect across/*reflekteer om* $y=-x$ ✓ $x=-3$ (2)
4.3	Shifting g five units to the left shifts (– 1 ; 0) five units to the left. $x=-6$	✓ answer/*antwoord* (1)
4.4	$\frac{2}{x+1}+1=x$ $2+x+1=x^2+x$ $x^2=3$ $\therefore x=\sqrt{3}$ since at S, $x>0$ $y=\sqrt{3}=1{,}73...$ $OS^2=x^2+y^2=3+3=6$ $\therefore OS=\sqrt{6}=2{,}45$ units/*eenhede* **OR/*OF***	✓ equating both graphs/*stel grafieke gelyk* ✓ $x^2=3$ ✓ $x=\sqrt{3}$ and $y=\sqrt{3}$ ✓ $OS^2=6$ ✓ answer/*antwoord* (5)
	Translate g one unit down and one unit to the right/*Transleer g een eenheid af en een eenheid na regs* The new equation/*Die nuwe vergelyking* : $p(x)=\frac{2}{x}$ Therefore the image of S is $S'(\sqrt{2};\sqrt{2})$/ *Daarom is die beeld van S nou* $S'(\sqrt{2};\sqrt{2})$ Now translate p back to g/*Transleer p terug na g*: $S(\sqrt{2}-1;\sqrt{2}+1)$ $OS^2=(\sqrt{2}-1)^2+(\sqrt{2}+1)^2=2-2\sqrt{2}+1+2+2\sqrt{2}+1$ $\therefore OS=\sqrt{6}=2{,}45$ units/*eenhede*	✓ $p(x)=\frac{2}{x}$ ✓✓coord. of/*koörd. van* S' ✓coord. of/*koörd. van* S ✓answer/*antwoord* (5)
4.5	$k<3$ will give roots with opposite signs/ $k<3$ *sal wortels met teenoorgestelde tekens gee*	✓$k<3$ (1) **[11]**

QUESTION 5

5.1	$y = \log_a x$ $-1 = \log_a \frac{1}{3}$ $a^{-1} = \frac{1}{3}$ $a = \left(\frac{1}{3}\right)^{-1}$ $\therefore a = 3$	✓subt. $\left(\frac{1}{3};-1\right)$ ✓ $a^{-1} = \frac{1}{3}$ or $a = \left(\frac{1}{3}\right)^{-1}$ (2)
5.2	$h: \quad x = \log_3 y$ $\therefore \quad y = 3^x$	✓swop x and y/*ruil x en y* ✓ answer/*antwoord* (2)
5.3	$g(x) = -\log_3 x$ **OR/*OF*** $g(x) = \log_3 \frac{1}{x}$ **OR/*OF*** $g(x) = \log_{\frac{1}{3}} x$ **OR/OF** $x = 3^{-y}$ **OR/OF** $x = \left(\frac{1}{3}\right)^y$	✓answer/*antwoord* (1) ✓answer/*antwoord* (1) ✓answer/*antwoord* (1) ✓answer/*antwoord* (1) ✓answer/*antwoord* (1)
5.4	$x > 0$ **OR/*OF*** $(0;\infty)$	✓ answer/*antwoord* (1) ✓ answer/*antwoord* (1)
5.5	$\log_3 x = -3$ $x = 3^{-3}$ $x = \frac{1}{27}$ $x \geq \frac{1}{27}$	✓ exponential form/ *eksponensiële vorm* ✓simplification/vereenvoudiging ✓answer/*antwoord* (3) **[9]**

QUESTION/*VRAAG 6*

6.1	$4x^2 - 6 = 0$ $x^2 = \frac{3}{2}$ $x = 1{,}22$ (x - coordinate of S is positive)	✓$y = 0$ ✓1,22 (2)
6.2	$(0 ; -6)$	✓ 0 ✓ -6 (2)
6.3.1	$QT = f(x) - g(x)$ $= 2\sqrt{x} - (4x^2 - 6)$ or $= 2\sqrt{x} - 4x^2 + 6$	✓✓ correct formula/ *korrekte formule* ✓substitution/*substitusie* (3)
6.3.2	$QT = 2x^{\frac{1}{2}} - 4x^2 + 6$ Derivative of $QT = x^{\frac{-1}{2}} - 8x = 0$ $\frac{1}{\sqrt{x}} = 8x$ $x^{\frac{3}{2}} = \frac{1}{8}$ or $\frac{1}{x} = 64x^2$ $x = \left(\frac{1}{8}\right)^{\frac{2}{3}}$ $x = \left(\frac{1}{2}\right)^2$ or $x^3 = \frac{1}{64}$ $x = \frac{1}{4} = 0{,}25$ Max/*Maks* $QT = 2\left(\frac{1}{4}\right)^{\frac{1}{2}} - 4\left(\frac{1}{4}\right)^2 + 6$ $= 6\frac{3}{4} = 6{,}75$units/*eenhede*	✓derivative/*afgeleide* ✓derivative equal to 0/ *afgeleide gelyk aan 0* ✓ $x^{\frac{3}{2}} = \frac{1}{8}$ ✓x-value/x-*waarde* ✓ substitution/*substitusie* ✓answer/*antwoord* (6) **[13]**

QUESTION/*VRAAG* 7

7.1	$A = P(1-i)^n$ $72\,500 = 145\,000\,(1-i)^5$ $i = 1-\sqrt[5]{\frac{72500}{145000}}$ $= 0{,}1294\ldots$ ∴ Rate of interest/*Rentekoers* is 12,94 % p.a./*p.j.* **OR/*OF*** $(1-i)^5 = \frac{1}{2}$ $\therefore i = 1-\left(\frac{1}{2}\right)^{\frac{1}{5}}$ $i = 0{,}1294$ ∴ Rate of interest/*Rentekoers* is 12,94 % p.a./*p.j.*	✓substitution/*substitusie* ✓writing in terms of i *herskryf in terme van i* ✓answer/*antwoord* (3) ✓substitution/*substitusie* ✓writing i.t.o i ✓answer (3)
7.2.1	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $500\,000 = \frac{x\left[1-\left(1+\frac{0{,}12}{12}\right)^{-240}\right]}{\frac{0{,}12}{12}}$ $x = \frac{500000\times\frac{0{,}12}{12}}{\left[1-\left(1+\frac{0{,}12}{12}\right)^{-240}\right]}$ $x = \text{R}5505{,}43$	✓ $i = \frac{0{,}12}{12}$ ✓ $n = 240$ ✓substitution into correct formula ✓answer/*antwoord* (4)

7.2.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $500000 = \frac{6000\left[1-\left(1+\frac{0,12}{12}\right)^{-n}\right]}{\frac{0,12}{12}}$ $\frac{500000}{6000} \times 0,01 = 1-(1,01)^{-n}$ $(1,01)^{-n} = 1-\frac{5}{6}$ $-n = \frac{\log\frac{1}{6}}{\log 1,01}$ $n = 180,07$ $\therefore$ Melissa settles the loan in 181 months	✓6000 ✓substitute into correct formula/*substitusie in korrekte formule* ✓ use of logs/*gebruik van logs* ✓answer/*antwoord* (4)
7.2.3	Samuel He is paying off his loan over a longer period thus more interest will be paid./*Hy betaal sy lening oor 'n langer tydperk af, dus sal hy meer rente betaal.* **OR/*OF*** Samuel He will pay/*Hy betaal* R5505,43 × 240 – R500 000 = R821 303,20 She will pay between/*Sy sal tussen* R580 000 and/*en* R586 000,00 *betaal.*	✓Samuel ✓reason/*rede* (2) ✓Samuel ✓reason/*rede* (2) **[13]**

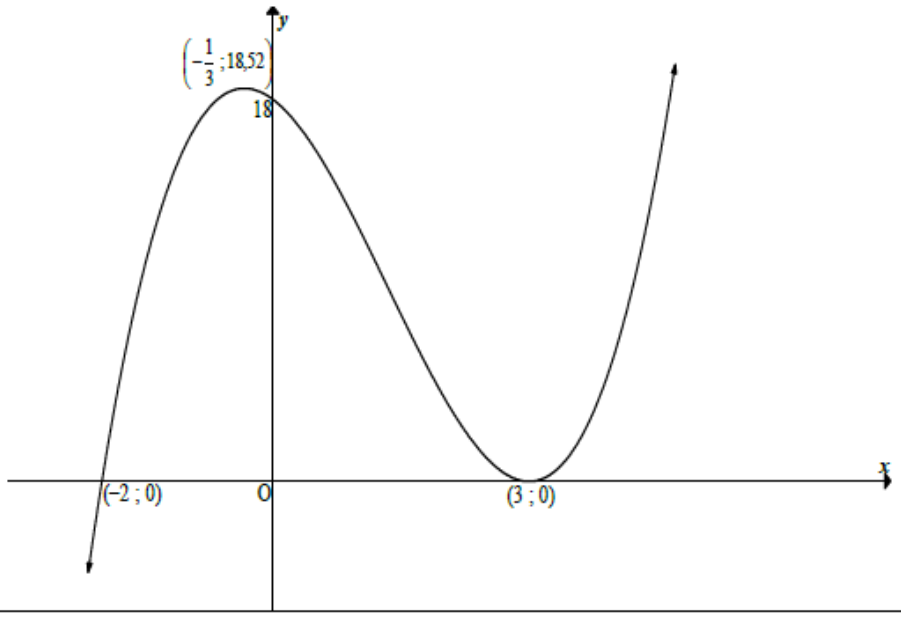
QUESTION/*VRAAG* 8

8.1	$f(x+h)=(x+h)^3=(x^2+2xh+h^2)(x+h)$ $=x^3+x^2h+2x^2h+2xh^2+h^2x+h^3$ $=x^3+3x^2h+3xh^2+h^3$ $f(x+h)-f(x)=x^3+3x^2h+3xh^2+h^3-x^3$ $=3x^2h+3xh^2+h^3$	✓simplifying/*vereenvouding*
	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	✓formula/*formule*
	$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$	✓subst. into formula/*subst. in formule*
	$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$	✓factorization/*faktorisering*
	$=\lim_{h\to 0}(3x^2+3xh+h^2)$ $=3x^2$	✓answer/*antwoord* (5)
	OR/OF	
	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	✓formula/*formule*
	$=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$	✓subst. into formula/*subst. in formule*
	$=\lim_{h\to 0}\frac{(x+h)(x+h)^2-x^3}{h}$	
	$=\lim_{h\to 0}\frac{(x+h)(x^2+2xh+h^2)-x^3}{h}$	
	$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$	✓simplifying/*vereenvoudiging*
	$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$	✓factorization/*faktorisering*
	$=\lim_{h\to 0}(3x^2+3xh+h^2)$ $=3x^2$	✓answer/*antwoord* (5)
	OR	

	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$ $=\lim_{h\to 0}\frac{(x+h-x)(x^2+2xh+h^2+x^2+xh+x^2)}{h}$ $=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$ $=\lim_{h\to 0}(3x^2+3xh+h^2)$ $=3x^2$	✓formula/*formule* ✓subst. into formula/*subst. in formule* ✓factorization/*faktorisering* ✓simplifying/*vereenvoudiging* ✓answer/*antwoord* (5)

8.2	$f'(x)=4x+2x^3$	✓$4x$ ✓$2x^3$ (2)
8.3	$y=x^{12}-2x^6+1$ $\frac{dy}{dx}=12x^{11}-12x^5$ $=12x^5(x^6-1)$ $=12x^5\sqrt{y}$	✓simplification/*vereenvoudiging* ✓derivative/*afgeleide* ✓factors/*faktore* (3)
8.4	$f(x)=2x^3-2x^2+4x-1$ $f'(x)=6x^2-4x+4$ $f''(x)=12x-4$ f is concave up when/*is konkaaf op as* $f''(x)>0$ $\therefore 12x-4>0$ $12x>4$ $x>\frac{1}{3}$	✓first derivative/*eerste afgeleide* ✓ second derivative/*tweede afgeleide* ✓ $f''(x)>0$ ✓ $x>\frac{1}{3}$ (4) **[14]**

QUESTION/*VRAAG* 9

9.1	$f'(x) = 3x^2 - 8x - 3 = 0$ $(3x+1)(x-3) = 0$ $x = -\frac{1}{3}$ or $x = 3$ $y = \frac{500}{27}$ $\left(\text{or } y = 18\frac{14}{27} \text{ or } 18{,}52\right)$ $y = 0$ Turning points are/*Draaipunte is* $\left(-\frac{1}{3}; \frac{500}{27}\right)$ and $(3;0)$	✓derivative/*afgeleide* ✓derivative/ *afgeleide* = 0 ✓factors/*faktore* ✓*x*-values/*waardes* ✓✓each *y*-values/*elke y-waarde* (6)
9.2		✓*x*-intercepts/*afsnitte* ✓*y*-intercept/*afsnit* ✓turning points/ *draaipunte* ✓shape/*vorm* (4)
9.3	$x < \frac{-1}{3}$ or $0 < x < 3$ **OR** $(-\infty; -\frac{1}{3}) \cup (0;3)$	✓ $x < \frac{-1}{3}$ ✓both critical points/ *beide kritieke-punte* ✓notation/*notasie* (3)

QUESTION/*VRAAG* 10

10.1	$l+2h=40$ $l=40-2h$	✓answer (1)
10.2	$2b+2h=100$ $b=50-h$ $V=lbh$ $V=h(40-2h)(50-h)$	✓ $2b+2h=100$ ✓ $b=50-h$ ✓volume formula (3)
10.3	$V=(50h-h^2)(40-2h)$ $V=2h^3-140h^2+2000h$ $V'=6h^2-280h+2000=0$ $h=\frac{280\pm\sqrt{(-280)^2-4(6)(2000)}}{2(6)}$ $h\neq 37{,}86$ or $h=8{,}80$ $\therefore$ for a box as large as possible, $h=8{,}80$ cm *vir die grootste moontlike boks = 8,80 cm*	✓simplifying/*vereenvoudig* ✓derivative / *afgeleide* ✓✓*h*-values in any form / *h-waardes in enige vorm* ✓answer/*antwoord* (5) **[9]**

QUESTION/*VRAAG* 11

11.1.1	P(male/*manlik*) = $\frac{83}{180}$ or 0,46 or 46,11%	✓answer/*antwoord* (1)
11.1.2	P(not game park/*nie wildreservaat*) = 1 – P(game park/*wildreservaat*) $=1-\frac{62}{180}$ $=\frac{59}{90}$ or 0,66 or 65,56% **OR/*OF*** P(not game park/*nie wildreservaat*) $=\frac{98}{180}+\frac{20}{180}$ $=\frac{118}{180}$ $=\frac{59}{90}$ or 0,66 or 65,56%	✓ $1-\frac{62}{180}$ ✓answer/*antwoord* (2) ✓ $\frac{98}{180}+\frac{20}{180}$ ✓answer/*antwoord* (2)

11.2

Events are independent if /*Gebeure is onafhanklike indien*
P(male) × P(home) = P(male and home)
P(*manlik*) × P(*huis*) = P(*manlik en huis*)

$P(\text{male/}manlik) = \frac{83}{180}$

and/*en* $P(\text{home/}huis) = \frac{20}{180}$ or 0,11 or 11,11%

P(male/*manlik*) × P(home/*huis*)

$= \frac{83}{180} \times \frac{20}{180}$

$= \frac{83}{1620}$

= 0,05123 or 5,12%

✓P(*m*) × P(*h*) and their values/*en hulle waardes*

✓answer of product

P(male and home/*manlik en huis*)

$= \frac{13}{180}$

= 0,07222… or 7,22%

✓P(*m* and/*en h*) value/*waarde*

Therefore P(male) × P(home) ≠ P(male and home)
Dus P(*manlik*) × P(*huis*) ≠ P(*manlik en huis*)
Thus the events are not independent./*Dus is die gebeure nie onafhanklik nie*

✓conclusion/*afleiding* (4)

OR/*OF*

	Home/*Huis*	Not Home/ *Nie huis*	
M	13	70	83
F	7	90	97
	20	160	180

P(female/*vroulik*) × P(not home/nie *huis*)

$= \frac{97}{180} \times \frac{160}{180}$

$= \frac{194}{405}$

= 0,479012345… or 47,90%

✓P(*f*) × P(not *h*) and their values/*en hulle waardes*

✓answer of product

P(female and not home/*vroulik en nie-huis*)

$= \frac{90}{180}$

= 0,5 or 50%

✓P(*f* and/*en* not *h*) value/*waarde*

Therefore P(female) × P(not home) ≠ P(female and not home)
Thus the events are not independent.
Dus P(*vroulik*) × P(nie-*huis*) ≠ P(*vroulik en nie-huis*)
Dus is die gebeure nie onafhanklik nie.

✓conclusion/*afleiding* (4)

[7]

QUESTION/*VRAAG* 12

12.1.1	$26\times25\times24\times23\times22$ $= 7\ 893\ 600$ **OR/*OF*** ${}^{26}P_5 = \frac{26!}{(26-5)!} = \frac{26!}{21!} = 7\ 893\ 600$	✓ $26\times25\times24\times23\times22$ ✓7 893 600 (2) ✓formula/*formule* ✓answer/*antwoord* (2)
12.1.2	$24\times23\times22$ $= 12\ 144$	✓ $24\times23\times22$ ✓12 144 (2)
12.2.1	$7\times6\times5\times4\times3\times2\times1$ $= 5\ 040$	✓product/*produk* ✓5 040 (2)
12.2.2	$(3\times2\times1)(5\times4\times3\times2\times1)$ $= 720$ **OR/*OF*** The five 'units' can be parked in $5\times4\times3\times2\times1$ ways./*Die vyf 'eenhede' kan op $5\times4\times3\times2\times1$ maniere geparkeer word.* The three silver cars can be parked in $3\times2\times1$ ways./*Die drie silwer motors kan op $3\times2\times1$ maniere parkeer word.* So there are $(3\times2\times1)(5\times4\times3\times2\times1) = 720$ ways to park the cars./*Dus is daar $(3\times2\times1)(5\times4\times3\times2\times1) = 720$ maniere om die motors te parkeer.* **OR/*OF*** Suppose for the moment the 3 silver cars are at one end./*Veronderstel die drie silwer motors is op die punt.* The 3 cars can be arranged in $3\times2\times1 = 6$ ways./*Die 3 motors kan op $3\times2\times1 = 6$ maniere gerangskik word.* For each of them the remaining four cars can be arranged in $4\times3\times2\times1 = 24$ ways./*Die 4 oorblywende motors kan op $4\times3\times2\times1 = 24$ maniere rangskik word.* So $6 \times 24 = 144$ ways if all 3 cars at one end./*Dus is daar $6 \times 24 = 144$ maniere as die 3 motors op die punt is.*   Together, the silver cars can only occupy 5 different positions amongst the 7 positions. */Saam kan die silwer motors slegs 5 verskillende posisies hê tussen die 7 moontlike posisies.* ∴ Total ways/*Totale getal maniere* $= 5 \times 144 = 720$	✓$3\times2\times1$ ✓$5\times4\times3\times2\times1$ ✓720 (3) ✓$5\times4\times3\times2\times1$ ✓$3\times2\times1$ ✓720 (3) ✓$6 \times 24 = 144$ ✓5×144 ✓720 (3) **[9]**

TOTAL/*TOTAAL*: 150

MATHEMATICS P2/*WISKUNDE V2*

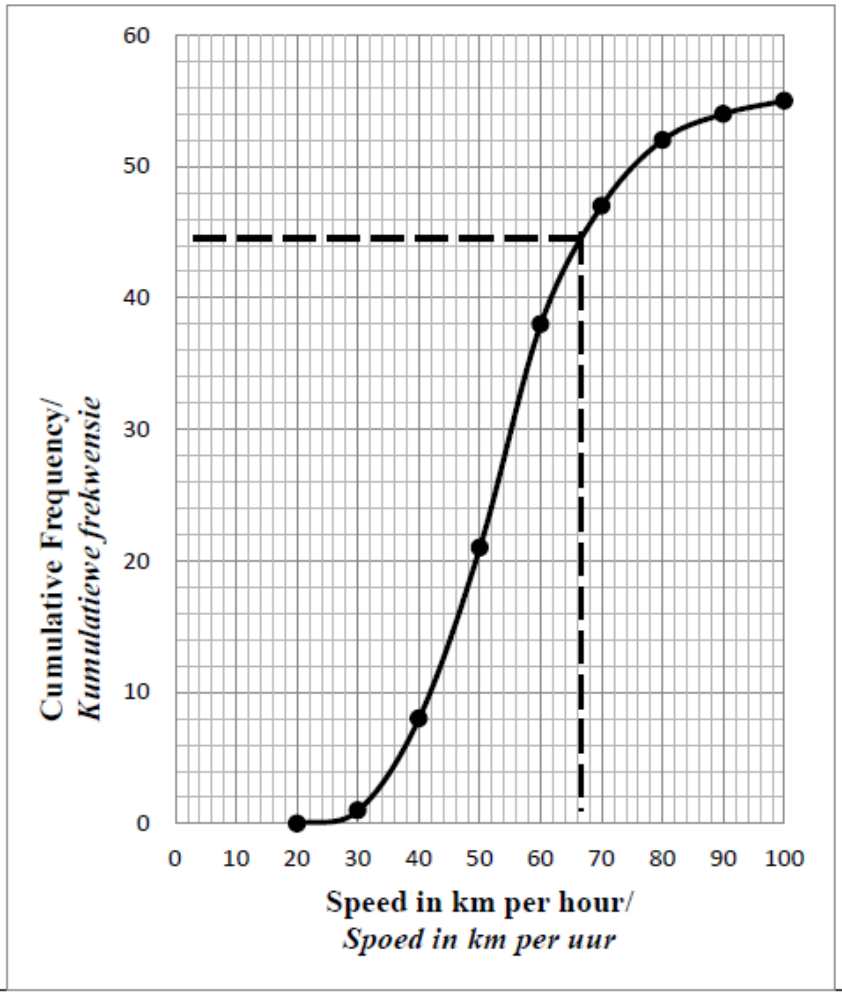
NOVEMBER 2014

MEMORANDUM

QUESTION/*VRAAG* 1

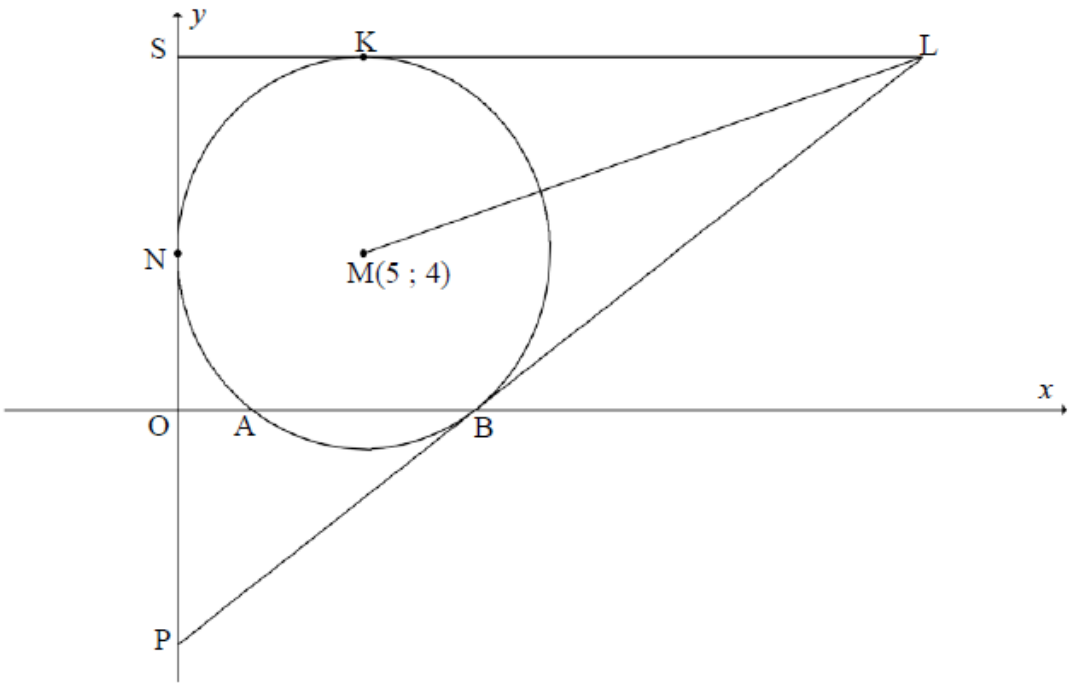
1.1	$\bar{x} = \frac{816}{12} = 68$	✓ $\frac{816}{12}$ ✓ 68 (2)
1.2	$\sigma = 18{,}42$	✓ answer/*antw* (1)
1.3	$(68 - 18{,}42 \; ; \; 68 + 18{,}42) = (49{,}58 \; ; \; 86{,}42)$ $\therefore$ 6 candidates had a mark within one standard deviation of the mean/*6 kandidate het 'n punt binne een standaardafwyking vanaf die gemiddelde.*	✓✓ interval ✓ answer/*antw* (3)
1.4	$a = 22{,}828... = 22{,}83$ $b = 0{,}66429... = 0{,}66$ $\therefore \hat{y} = 0{,}66x + 22{,}83$ **OR/*OF*** $\hat{y} = 22{,}83 + 0{,}66x$	✓ value of *a/ waarde van a* ✓ value of *b/ waarde van b* ✓ equation/*vgl* (3)
1.5	$\hat{y} = 0{,}66x + 22{,}83$ $y = 0{,}66(60) + 22{,}83$ $62{,}43...\% \approx 62\%$ **OR/*OF*** $62{,}69\% \approx 63\%$	✓ subs of 60 into equation ✓ answer/*antw* (2) ✓✓ answer/*antw* (2)
1.6	(82 ; 62)	✓ answer/*antw* (1) **[12]**

QUESTION/*VRAAG* 2

2.1	$50 < x \leq 60$ **OR/*OF*** $50 \leq x < 60$ **OR/*OF*** between 50 and 60/*tussen 50 en 60*	✓ answer/*antw* (1)
2.2.1	See table below	✓ 8 ✓ 55 (2)
2.2.2		✓ grounding at (20 ; 0)/ *anker by* (20 ; 0) ✓ plotting at upper limits/ *plot by boonste limiete* ✓ smooth shape of curve/*gladde kurwe* (3)
2.3	55 – 44 (accept/*aanvaar* 43 – 45) ≈ 11 motorists/*motoriste* (accept/*aanvaar* 10 – 12 motorists/*motoriste*)	✓ 44 ✓ 11 (2) **[8]**

Class *Klas*	**Frequency** *Frekwensie*	**Cumulative frequency** *Kumulatiewe frekwensie*
$20 < x \leq 30$	1	1
$30 < x \leq 40$	7	8
$40 < x \leq 50$	13	21
$50 < x \leq 60$	17	38
$60 < x \leq 70$	9	47
$70 < x \leq 80$	5	52
$80 < x \leq 90$	2	54
$90 < x \leq 100$	1	55

QUESTION/*VRAAG* 3

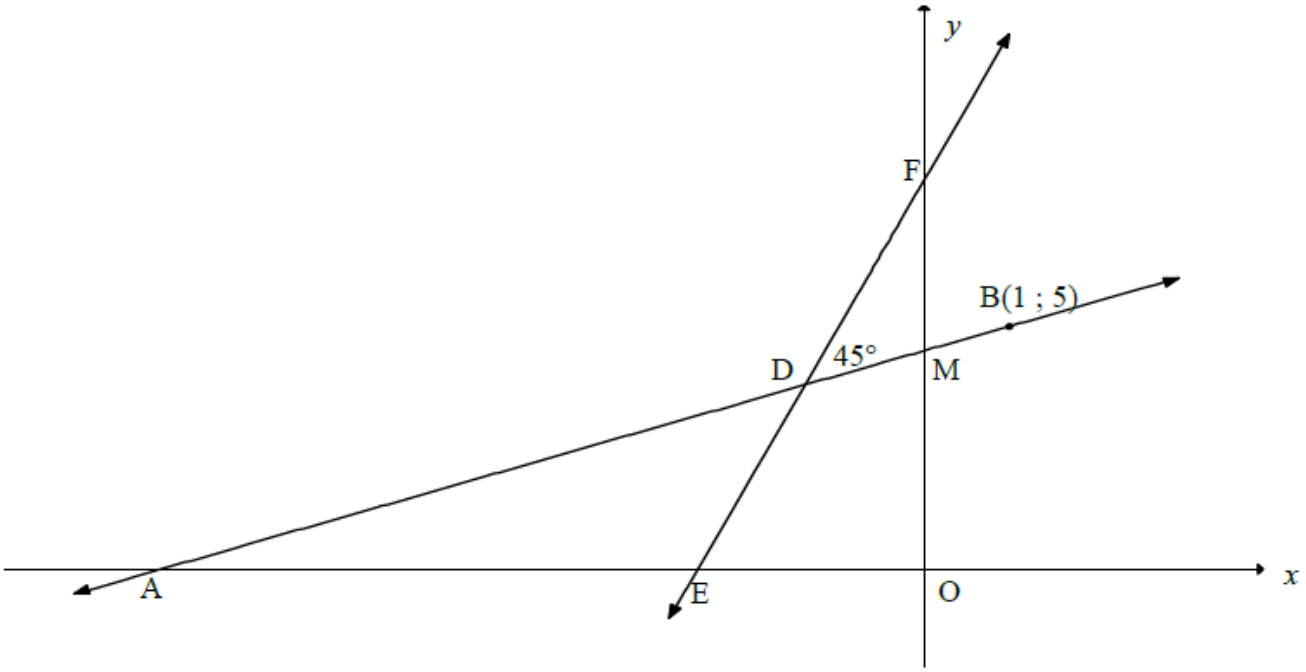


3.1	$r = MN = 5$	✓ answer/*antw* (1)
3.2	$(x-5)^2 + (y-4)^2 = 25$	✓equation/*vgl* (1)
3.3	A(x ; 0) $(x-5)^2 + (0-4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x-8)(x-2) = 0$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2 + (0-4)^2 = 25$ $(x-5)^2 + 16 = 25$ $(x-5)^2 = 9$ $(x-5) = \pm 3$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y = 0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
3.4.1	$m_{MB} = \frac{4-0}{5-8}$ $= -\frac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{MB} = -\frac{4}{3}$ (2)

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent $\perp$ radius/ *rkl* $\perp$ *radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
3.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20 - 5)^2 + (9 - 4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9 + 1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
3.8	**MK $\perp$ KL OR/OF M$\hat{K}$L** $= 90°$ (radius $\perp$ tangent/*radius* $\perp$ *rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5 + 20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4 + 9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore \quad (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK ⊥ KL OR/OF $\hat{MKL} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $(x-12,5)^2 + (y-6,5)^2 = r^2$ subst (5 ; 4): $(5-12,5)^2 + (4-6,5)^2 = r^2$ $62,5 = r^2$ $\therefore \quad (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$ **OR/*OF*** By symmetry about LM/*deur simmetrie om LM*: **MK ⊥ KL OR/OF $\hat{MKL} = 90°$** (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* ML is a diameter /*ML is 'n middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore \quad (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$	✓ S ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ value of/*waarde van* r^2 ✓ answer in correct form/*antw in korrekte vorm* (5) ✓ S ✓ value of/*waarde van r* ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ answer in correct form/*antw in korrekte vorm* (5) **[21]**

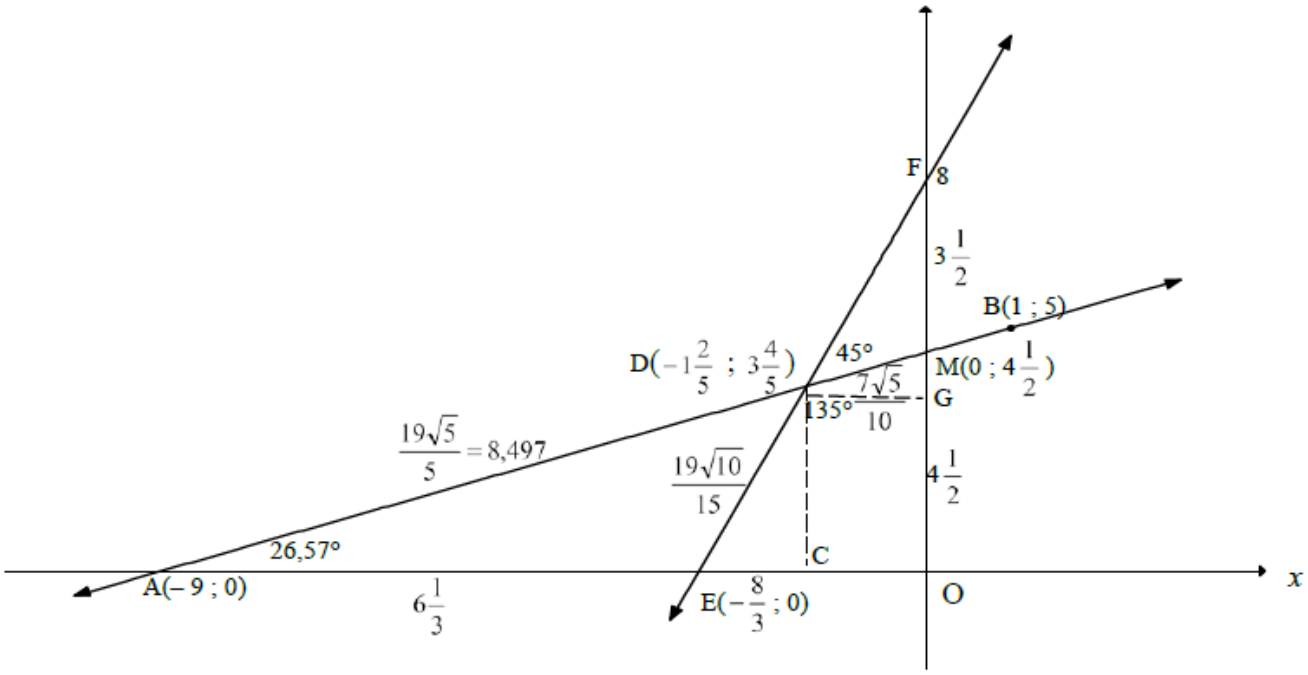
QUESTION/VRAAG 4



4.1	$y = 0$: $3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore \text{E}\left(-2\frac{2}{3};0\right)$ **OR/*OF*** $\text{E}\left(-\frac{8}{3};0\right)$	✓ *y*-value/*waarde* ✓ *x*-value/*waarde* (2)
4.2	$\tan \text{D}\hat{\text{E}}\text{O} = m_{\text{DE}} = 3$ $\therefore \text{D}\hat{\text{E}}\text{O} = 71{,}565... = 71{,}57°$ $\text{D}\hat{\text{A}}\text{E} = 71{,}565...° - 45°$ $= 26{,}57°$	✓ $\tan \text{D}\hat{\text{E}}\text{O} = 3$ ✓ 71,565…° ✓ 26,57° (3)
4.3	$m_{\text{AB}} = \tan 26{,}57°$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ **OR/*OF*** $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{\text{AB}} = \tan 26{,}57°$ ✓ $m_{\text{AB}} = \frac{1}{2}$ ✓ subst of *m* and (1 ; 5)into formula/ *subst m en (1 ; 5) in formule* ✓ equation/*vgl* (4)

4.4	Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously: $x - 2(3x+8) + 9 = 0$ $x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $-1\frac{2}{5} - 2y + 9 = 0$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/OF** $x = 2y - 9$ $y = 3(2y - 9) + 8$ $y = 6y - 27 + 8$ $\therefore y = 3\frac{4}{5}$ $x = 2(3\frac{4}{5}) - 9$ **OR/OF** $3\frac{4}{5} = 3x + 8$ $x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF*** $3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$ $6x + 16 = x + 9$ $5x = -7$ $\therefore \quad x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF***	✓ subst/*vervang* ✓ *x*-value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4) ✓ subst/*vervang* ✓ *y* value/*waarde* ✓ subst/*vervang* ✓ *x*-value/*waarde* (4) ✓ equating/*gelyk stel* ✓ *x* value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4)

	$x - 2y = -9$ ……(1) $-6x + 2y = 16$ ……(2) (1) + (2): $-5x = 7$ $\therefore x = -1\frac{2}{5}$ $\therefore -1\frac{2}{5} - 2y = -9$ **OR/*OF*** $y = 3(-1\frac{2}{5}) + 8$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF*** $y = 3x + 8$ ……(1) $6y = 3x + 27$ ……(2) (1) – (2): $-5y = -19$ $\therefore y = 3\frac{4}{5}$ $3\frac{4}{5} = 3x + 8$ $x = 2(3\frac{4}{5}) - 9$ **OR/*OF*** $x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ adding/*optelling* ✓ *x*-value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4) ✓ subtracting/*aftrekking* ✓ *y*-value/*waarde* ✓ subst/*vervang* ✓ *x*-value/*waarde* (4)



4.5	area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore$ A(–9 ; 0) area ΔAMO $= \frac{1}{2}$. AO. OM $= \frac{1}{2}(9)(4\frac{1}{2})$ = 20,25 area ΔADE $= \frac{1}{2}$. AE . y_D $= \frac{1}{2}$. (AO – EO). y_D $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ = 12,03 **OR/OF** area ΔADE $= \frac{1}{2}$AD.AE.sin DÂE $= \frac{1}{2}(\frac{19\sqrt{5}}{5}).6\frac{1}{3}$.sin 26,57° = 12,03 $\therefore$ area DMOE = 8,22 square units/*vk eenh* **OR/*OF***	✓ correct method/ *korrekte metode* ✓ $x_A = -9$ ✓ $\frac{1}{2}(9)(4\frac{1}{2})$ ✓ $AE = 9 - 2\frac{2}{3} = 6\frac{1}{3}$ ✓ $y_D = 3\frac{4}{5}$ **OR/OF** ✓ $AD = \frac{19\sqrt{5}}{5}$ ✓ $AE = 6\frac{1}{3}$ ✓ answer/*antw* (6)

area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC

$= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$

= 8,22 square units/*vk eenh*

✓ correct method/ *korrekte metode*
✓ $3\frac{4}{5}$
✓ $1\frac{2}{5}$ ✓ 0,7
✓ $\frac{19}{15}$
✓ answer

(6)

OR/*OF*

area DMOE = area ΔEDO + area ΔODM

$= \frac{1}{2}(\text{EO} \times y_D) + \frac{1}{2}(\text{OM} \times -x_D)$

$= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$

$= \frac{1}{2}\left(\frac{304+189}{30}\right)$

$= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh*

✓ correct method/ *korrekte metode*
✓ $y_D = \frac{19}{5}$ or $3\frac{4}{5}$
✓ $\text{EO} = \frac{8}{3}$
✓ $-x_D = \frac{7}{5}$
✓ $\text{OM} = \frac{9}{2}$ or $4\frac{1}{2}$
✓ answer/*antw*

(6)

OR/*OF*

area DMOE = area ΔEOF − area ΔDMF

$= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$

$= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$

$= \frac{1}{2}\left(\frac{640-147}{30}\right)$

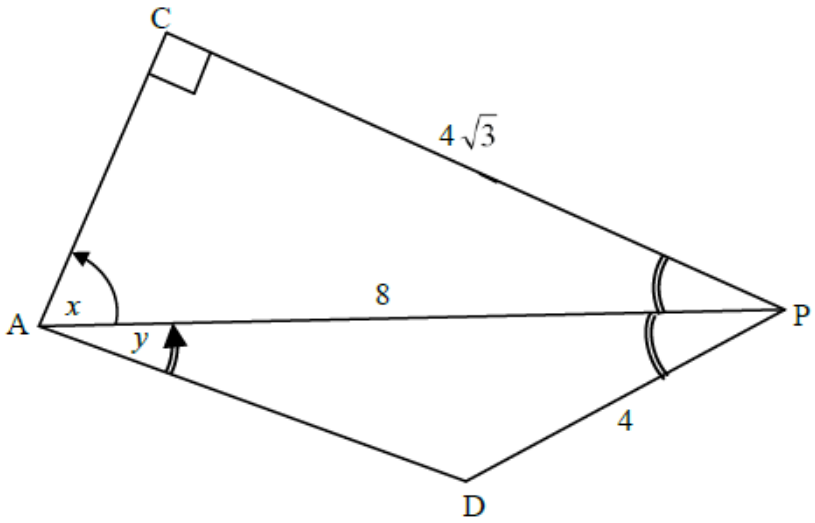
$= \frac{493}{60}$ or $8\frac{13}{60}$ or 8,22 square units/*vk eenh*

✓ correct method/ *korrekte metode*
✓ $y_F = 8$
✓ $\text{EO} = \frac{8}{3}$
✓ $-x_D = \frac{7}{5}$
✓ $\text{FM} = 3\frac{1}{2}$
✓ answer/*antw*

(6)

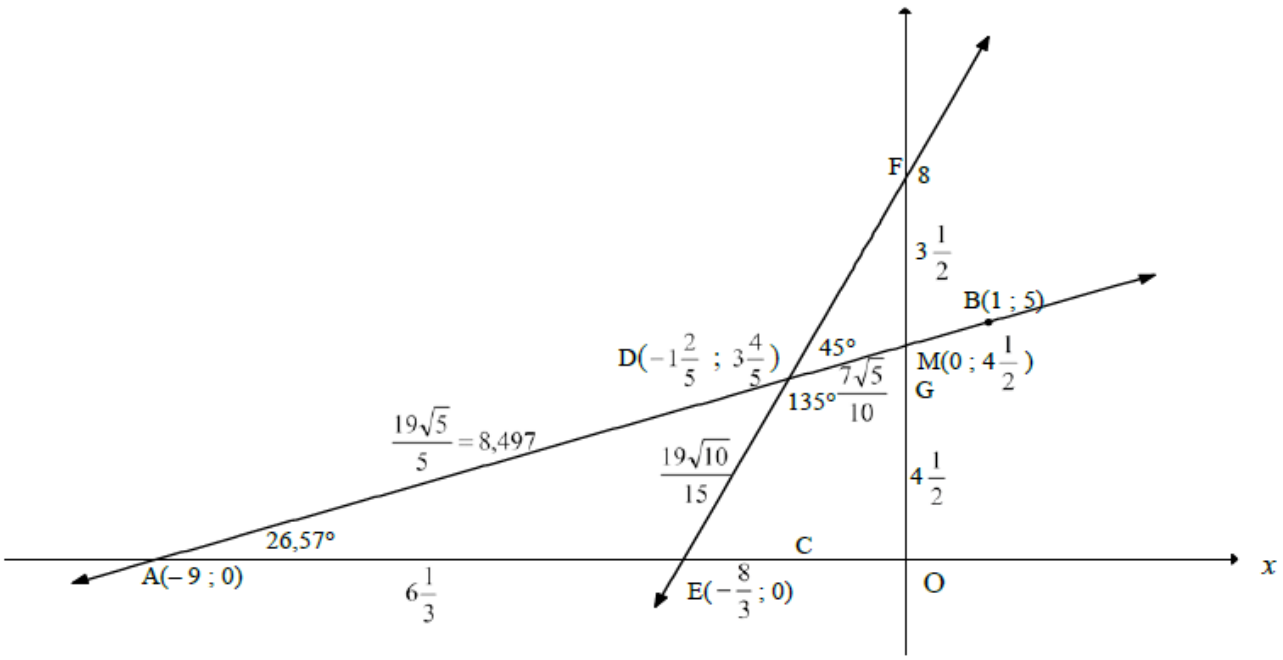
OR/*OF*

QUESTION/VRAAG 5

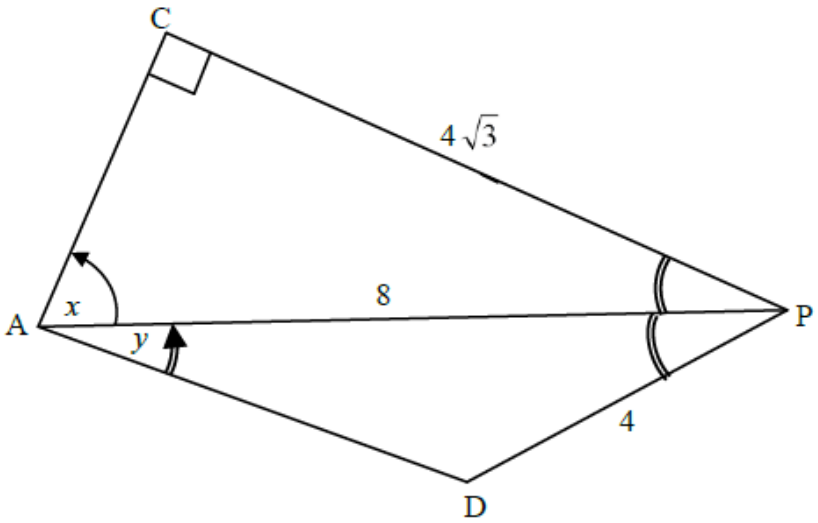


5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$	✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2) ✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2)
5.2	$C\hat{P}A = D\hat{P}A = 30°$ (AP bisects $D\hat{P}C$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24,57...$ $AD = 4,96$	✓ $D\hat{P}A = 30°$ ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ 24,57... ✓ 4,96 (4)

	$\text{area } \Delta EOM = \frac{1}{2}(EO \times OM)$ $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $= 6 \text{ sq units/}vk\ eenh$	✓ area ΔEOM
	$ED = \sqrt{\left(-\frac{7}{5}+\frac{8}{3}\right)^2+\left(\frac{19}{5}\right)^2}$ and $DM = \sqrt{\left(\frac{7}{5}\right)^2+\left(\frac{9}{2}-\frac{19}{5}\right)^2}$ $= \frac{19\sqrt{10}}{15}$ or 4,005... $= \frac{7\sqrt{5}}{10}$ or 1,565..	✓ $ED = \frac{19\sqrt{10}}{15}$ ✓ $DM = \frac{7\sqrt{5}}{10}$
	$\text{area } \Delta EDM = \frac{1}{2}(ED \times DM \times \sin E\hat{D}M)$ $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$ $= \frac{133}{60}$ or 2,216...	✓ area ΔEDM ✓ correct method/ *korrekte metode*
	$\therefore$ area DMOE = area ΔEOM + area ΔEDM $= 6 + 2{,}216...$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/$eenh^2$	✓ answer/*antw* (6) **[19]**



QUESTION/VRAAG 5



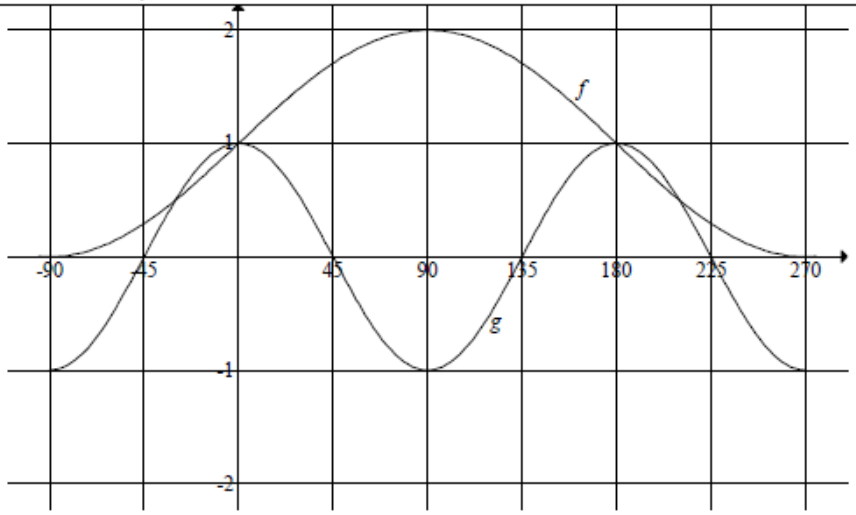
5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$	✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2) ✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2)
5.2	$C\hat{P}A = D\hat{P}A = 30°$ (AP bisects $D\hat{P}C$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24,57...$ $AD = 4,96$	✓ $D\hat{P}A = 30°$ ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ 24,57... ✓ 4,96 (4)

5.3	$\frac{\sin D\hat{A}P}{DP} = \frac{\sin A\hat{P}D}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30°}{4,96}$ $\sin y = \frac{4\sin 30°}{4,96}$ $= 0,403...$ $y = 23,78°$ **OR/*OF*** $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos D\hat{A}P$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96).\cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148...$ $y = 23,82°$	✓ correct subst into sine rule/ *korrekte subst in sin-reël* ✓ sin y subject ✓ 23,78° (3) ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ cos y subject ✓ 23,82° (3) **[9]**

QUESTION/VRAAG 6

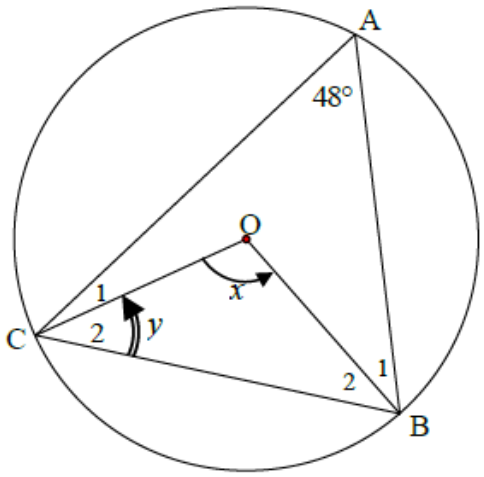
6.1	$\cos^2(180°+x)+\tan(x-180°)\sin(720°-x)\cos x$ $=(-\cos x)^2+[-(-\tan x)](-\sin x)(\cos x)$ $=\cos^2 x+\left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x)$ $=\cos^2 x-\sin^2 x$ $=\cos 2x$	✓ $(-\cos x)^2$ or $\cos^2 x$ ✓ $\tan x$ or $-(-\tan x)$ ✓ $-\sin x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\cos^2 x-\sin^2 x$ (5)
6.2	$\sin(\alpha-\beta)$ $=\cos[90°-(\alpha-\beta)]$ $=\cos[(90°-\alpha)+\beta]$ $=\cos(90°-\alpha)\cos\beta-\sin(90°-\alpha)\sin\beta$ $=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ **OR/*OF*** $\sin(\alpha-\beta)$ $=\cos[90°-(\alpha-\beta)]$ $=\cos[(90°+\beta)+(-\alpha)]$ $=\cos(90°+\beta)\cos(-\alpha)-\sin(90°+\beta)\sin(-\alpha)$ $=(-\sin\beta)\cos\alpha-\cos\beta(-\sin\alpha)$ $=\sin\alpha\cos\beta-\cos\alpha\sin\beta$	✓rewrite as/*herskryf* $\cos[(90°-\alpha)+\beta]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3) ✓ rewrite as/*herskryf* $\cos[(90°+\beta)+(-\alpha)]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3)
6.3	x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=-(\cos^2 76°-\sin^2 76°)$ $=-\cos 2(76°)$ $=-\cos 152°$ $=-(-\cos 28°)$ $=\cos 28°$ $=\cos(90°-62°)$ $=\sin 62°$ **OR/*OF*** $=-\cos(90°+62°)$ $=-(-\sin 62°)$ $=\sin 62°$ **OR/*OF*** x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=\sin 76°\sin 76°-\cos 76°\cos 76°$ $=\sin 76°\cos 14°-\cos 76°\sin 14°$ $=\sin(76°-14°)$ $=\sin 62°$ **OR/*OF*** x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=\cos^2 14°-\sin^2 14°$ $=\cos 2(14°)$ $=\cos 28°$ $=\sin 62°$	✓$-(\cos^2 76°-\sin^2 76°)$ ✓ recognition of cos double angle ✓ $-\cos 152°$ ✓ $\cos 28°$ (4) ✓ $\cos 14°$ ✓ $\sin 14°$ ✓ recognition of sine compound angle ✓ $\sin(76°-14°)$ (4) ✓ $\cos^2 14°$ ✓ $\sin^2 14°$ ✓ recognition of cos double angle ✓ $\cos 28°$ (4) **[12]**

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ *kritieke waardes* ✓ notation/*notasie* (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/*st vorm* (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ $x = 0° + k.360°$ or $x = 210° + k.360°$ or $x = 180° + k.360°$ $x = 330° + k.360°, k \in Z$ **OR/*OF*** $x = k.180°, k \in Z$	✓ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ ✓ 0° ; 180° **OR/*OF*** $x = k.180°$ ✓ 210° ; 330° ✓ $k.360°, k \in Z$ (4)
7.4		✓ *y*-intercept/*afsnit* ✓ *x*-intercepts/*afsnitte* ✓ min/max points/ *min/maks punte* (3)
7.5	$f(x) = g(x)$ at/*by*: $x = -30° ; 0° ; 180° ; 210°$ $\therefore f(x + 30°) = g(x + 30°)$ at/*by*: $x = -60° ; -30° ; 150° ; 180°$	✓ $-30° ; 0° ; 180° ; 210°$ ✓✓ $-60° ; -30° ; 150° ; 180°$ (3)
7.6	Series will converge if/*Reeks sal konvergeer as:* $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore 30° < x < 60°$ or $x \in (30° ; 60°)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓✓ $30° < x < 60°$ (5) **[19]**

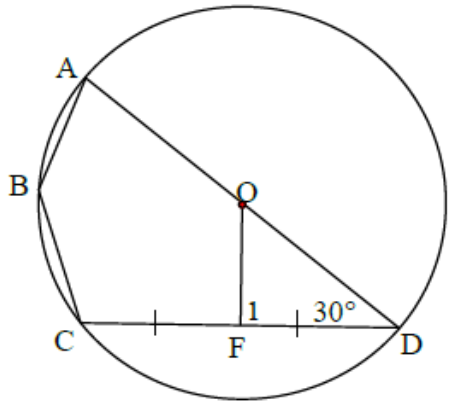
QUESTION/VRAAG 8

8.1



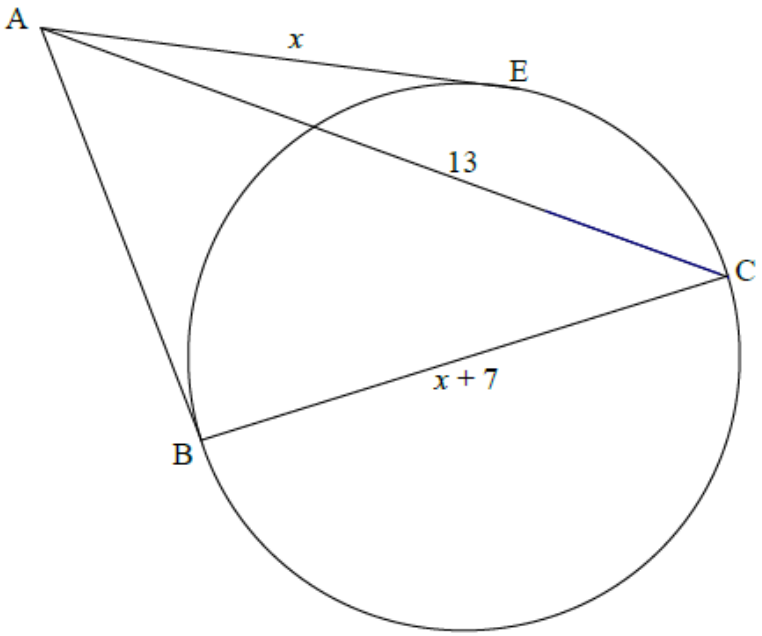
8.1.1	$x = 96°$	($\angle$ at centre = 2$\angle$ at circumference/ *$\angle$ by midpt = 2$\angle$ by omtrek*)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180° - 96° = 84°$ $y = \hat{B}_2 = 42°$	(sum of $\angle$s in Δ/ *som v$\angle$e in Δ*) ($\angle$s opp = sides/*$\angle$e teenoor = sye*)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90°$	(line from centre to midpt chord/ *lyn vanaf midpt na midpt kd)*	✓ S ✓ R (2)
8.2.2	$A\hat{B}C = 150°$	(opposite $\angle$s of cyclic quad/ *tos $\angle$e v koordevh*)	✓ S ✓ R (2)

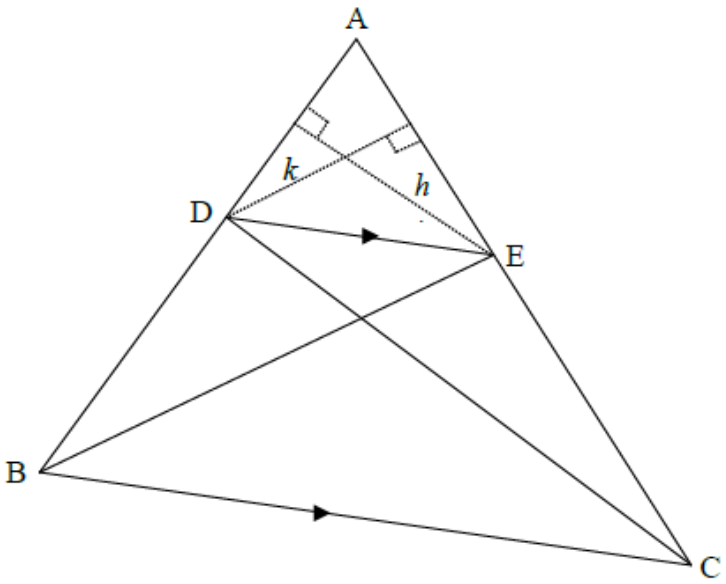
8.3



8.3.1 (a)	tangent ⊥ radius/diameter / *raaklyn ⊥ radius/middellyn*	✓ R (1)
8.3.1 (b)	tangents from common pt **OR** tangents from same pt / *raaklyne v gemeensk pt **OF** raaklyne vanaf dies pt*	✓ R (1)
8.3.2	$AB^2 + BC^2 = AC^2$ $x^2 + (x+7)^2 = 13^2$ (Theorem of/*Stelling van*Pythagoras) $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x-5)(x+12) = 0$ $x = 5 \quad (x \neq -12)$	✓ $AB^2 + BC^2 = AC^2$ ✓ $x^2 + (x+7)^2 = 13^2$ ✓ standard form ✓ answer (4) **[14]**

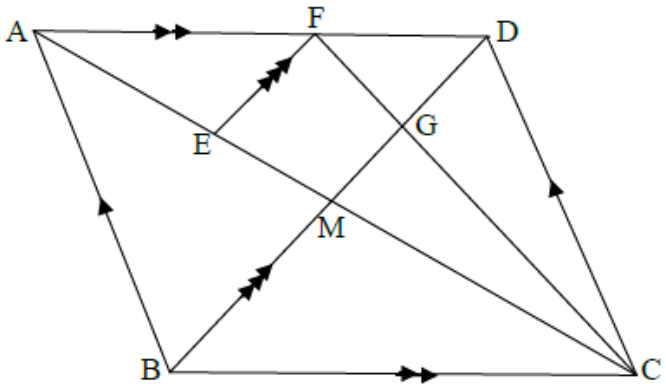
QUESTION/VRAAG 9

9.1



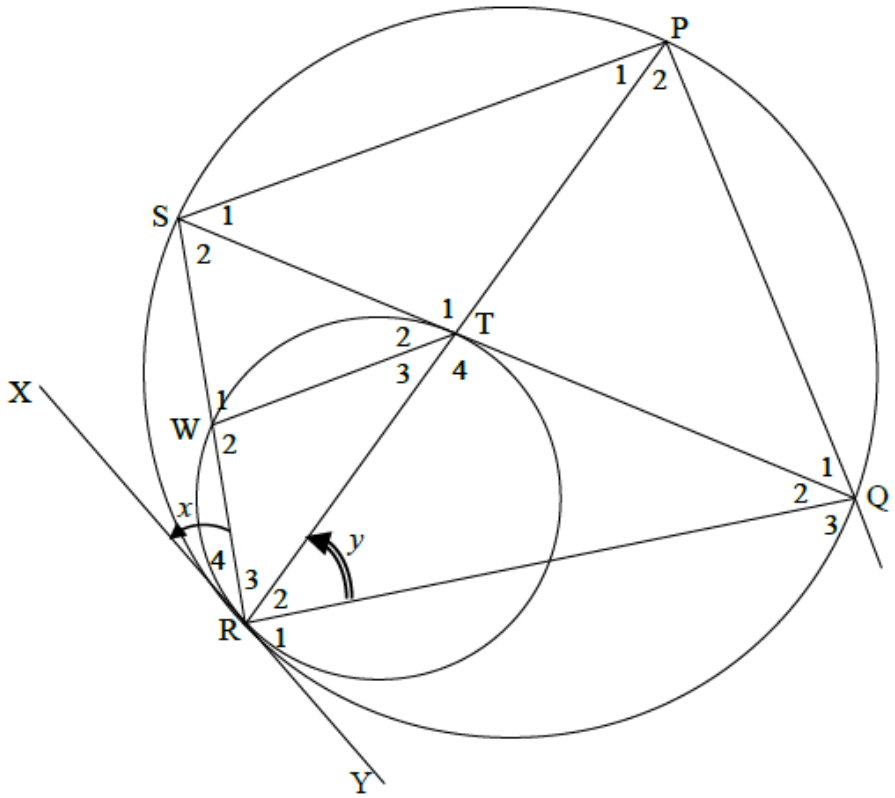
9.1.1	Same base (DE) and same height (between parallel lines) *Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)*	✓ same base/*dies basis* between || lines/ *tussen || lyne* (1)
9.1.2	$\frac{AD}{DB}$ $\frac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \times k}$ But/*Maar* area ΔDEB = area ΔDEC (Same base and same height/*dieselfde basis en dieselfde hoogte*) $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

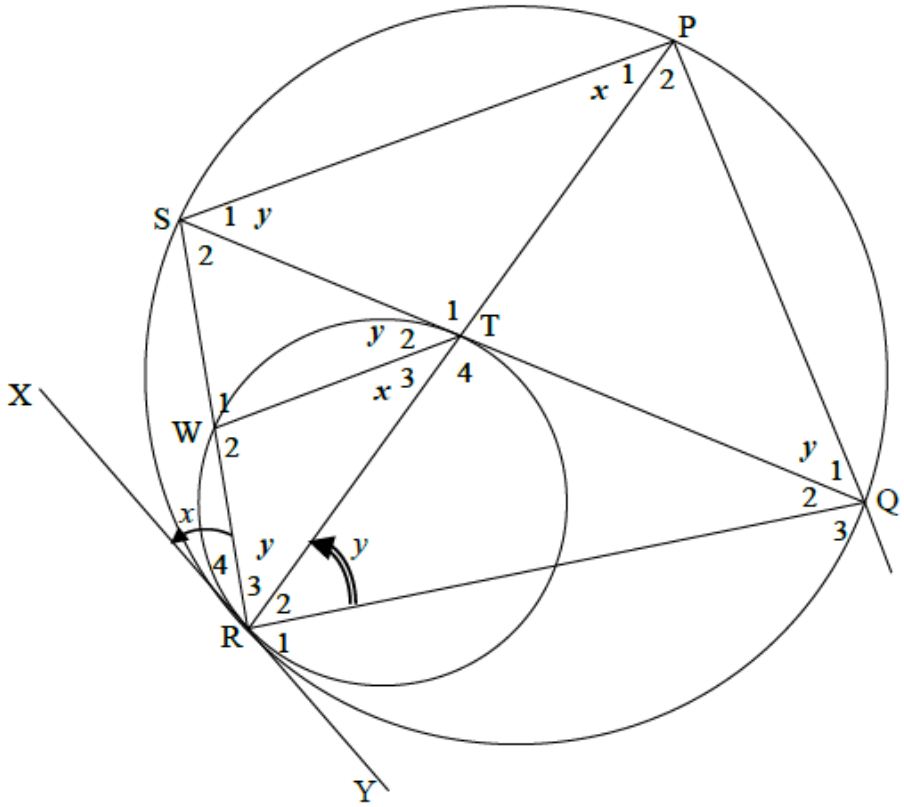


9.2.1	$\frac{EM}{AM}=\frac{FD}{AD}$	(Line parallel one side of Δ **OR** prop th; EF || BD) *(Lyn ewewydig aan sy v Δ* ***OF** eweredigst; EF ||BD)*	✓ S ✓R ✓ answer/*antw* (3)
9.2.2	CM = AM $\frac{CM}{ME}=\frac{AM}{ME}=\frac{7}{3}$	(diags of parm bisect/*hoekl parm halv*) (from 9.2.1/*vanaf 9.2.1*)	✓ S ✓R ✓ answer/*antw* (3)
9.2.3	*h* of ΔFDC = *h* of ΔBDC $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{\frac{1}{2}FD.h}{\frac{1}{2}BC.h}$ $=\frac{FD}{AD}$ $=\frac{3}{7}$ **OR/*OF*** $\frac{\text{area }\Delta FDC}{\text{area }\Delta ADC}=\frac{FD}{AD}=\frac{3}{7}$ But Area ΔADC = Area ΔBDC $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{3}{7}$	(AD || BC) (opp sides of parm =) *(tos sye v parm =)* (same heights) *(dieselfde hoogtes)* (diags of parm bisect area) *(hoekl v parm halv opp)*	✓ AD || BC ✓ subst into area form/ *subst in opp formule* ✓ S ✓ answer/*antw* (4) ✓ S ✓ R ✓ S ✓ answer/*antw* (4) **[16]**

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.2	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.3	Corresponding angles equal/*Ooreenkomstige ∠e gelyk*	✓ R (1)
10.1.4	∠s subtended by chord PQ **OR** ∠s in same segment *∠e onderspan deur dieselfde koord* ***OF*** *∠e in dieselfde segment*	✓ R (1)
10.1.5	alternate ∠s/*verwisselende ∠e* ; WT || SP	✓ R (1)
10.2	$\frac{RW}{RS}=\frac{RT}{RP}$ (Line parallel one side of Δ **OR** prop th; WT || SP) $\therefore RT=\frac{WR.RP}{RS}$ (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredighst: WT || SP)* **OR/*OF*** ΔRTW ||| ΔRPS (∠; ∠; ∠) $\therefore \frac{RW}{RS}=\frac{RT}{RP}$ (ΔRTW ||| ΔRPS) $\therefore RT=\frac{RW.RP}{RS}$	✓ S ✓ R (2) ✓ S ✓ S (2)
10.3	$y=\hat{T}_2=\hat{R}_3$ (tan chord theorem/*Rkl-koordst*) $y=\hat{R}_3=\hat{Q}_1$ (∠s in same segment/*∠e in dieselfde segment*)	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = P\hat{S}R$	(ext ∠ of cyc quad/*buite∠ v kdvh*)	✓ S ✓ R
	$P\hat{S}R = \hat{W}_2$	(corresp∠s/*ooreenk ∠e* ; WT || SP)	✓ S
	$\therefore \hat{Q}_3 = \hat{W}_2$		(3)
	OR/OF		
	$\hat{Q}_2 = x$	(∠s in same segment/*∠e in dies segment*)	✓ R
	$\hat{Q}_3 = 180° - (x + y)$	(∠s on straight line/*∠e op reguitlyn*)	✓ S
	$\hat{W}_2 = 180° - (x + y)$	(∠s of ΔWRT/*∠e v ΔWRT*)	✓ S
	$\therefore \hat{Q}_3 = \hat{W}_2$		(3)
10.5	In ΔRTS and ΔRQP:		
	$\hat{R}_3 = \hat{R}_2 = y$	(proven above/*hierbo bewys*)	✓ S
	$\hat{S}_2 = \hat{P}_2$	(∠s in same segment/*∠e in dies segment*)	✓ S/R
	$R\hat{T}S = R\hat{Q}P$	(3^{rd} angle of Δ)	✓ S **OR/OF** (∠; ∠; ∠)
	$\therefore$ ΔRTS ||| ΔRQP	(∠; ∠; ∠)	(3)

10.6	$\frac{RT}{RQ}=\frac{RS}{RP}$	$(\Delta RTS \mid\mid\mid \Delta RQP)$	✓ S
	$\frac{RS}{RP}\times\frac{RS}{RP}=\frac{RT}{RQ}\times\frac{RS}{RP}$		✓ $\times\frac{RS}{RP}$ on both sides
	$\left(\frac{RS}{RP}\right)^2=\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$		
	$=\left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right)$	(proven in 10.2/*bewys in 10.2*)	✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ (3)
	$=\frac{RW}{RQ}$		
	OR/OF		
	$\frac{RT}{RQ}=\frac{RS}{RP}$	$(\Delta RTS \mid\mid\mid \Delta RQP)$	✓ S
	But $RT=\frac{WR.RP}{RS}$	(proven in 10.2/*bewys in 10.2*)	✓ $RT=\frac{WR.RP}{RS}$
	$\therefore\frac{RT}{RQ}=\frac{WR.RP}{RQ.RS}=\frac{RS}{RP}$		
	$WR.RP^2=RQ.RS^2$		✓multiplication/ *vermenigvuldig*
	$\therefore\frac{WR}{RQ}=\frac{RS^2}{RP^2}$		(3)
	OR/OF		
	$\frac{RT}{RS}=\frac{RQ}{RP}$	$(\Delta RTS \mid\mid\mid \Delta RQP)$	✓ S
	$RQ=\frac{RT.RP}{RS}$		
	and $WR=\frac{RT.RS}{RP}$	(proven in 10.2/*bewys in 10.2*)	✓ $WR=\frac{RT.RS}{RP}$
	$\frac{WR}{RQ}=\frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$		
	$=\frac{RT.RS}{RP}\times\frac{RS}{RT.RP}$		✓ simplification/ *vereenvoudiging*
	$=\frac{RS^2}{RP^2}$		(3) **[20]**
			TOTAL/TOTAAL: 150

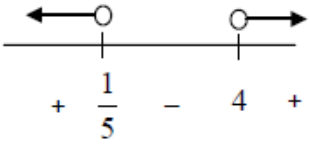
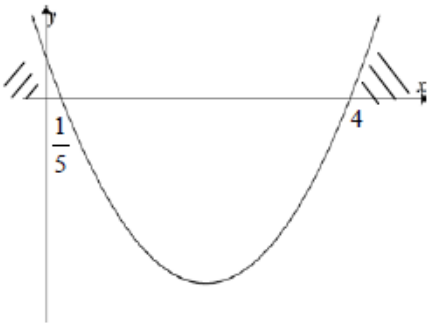
MATHEMATICS P1/*WISKUNDE V1*

FEBRUARY/MARCH/FEBRUARIE/MAART 2015

MEMORANDUM

QUESTION/*VRAAG* 1

1.1.1	$(x+4)(x-5)=0$ $\therefore x=-4$ or $x=5$	✓ factors/*faktore* ✓ answers/*antwoorde* (2)
1.1.2	$2x^2-11x+7=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\dfrac{-(-11)\pm\sqrt{(-11)^2-4(2)(7)}}{2(2)}$ $=$ 4,77 or 0,73 **OR/*OF*** $2x^2-11x+7=0$ $x^2-\dfrac{11}{2}x+\dfrac{7}{2}=0$ $x^2-\dfrac{11}{2}x+\left(\dfrac{1}{2}.\dfrac{11}{2}\right)^2+\dfrac{7}{2}-\left(\dfrac{1}{2}.\dfrac{11}{2}\right)^2=0$ $\left(x-\dfrac{11}{4}\right)^2+\dfrac{7}{2}-\dfrac{121}{16}=0$ $\left(x-\dfrac{11}{4}\right)^2=\dfrac{121-56}{16}$ $x-\dfrac{11}{4}=\pm\sqrt{\dfrac{65}{16}}$ $\therefore x=\dfrac{11}{4}+\dfrac{\sqrt{65}}{4}$ or $x=\dfrac{11}{4}-\dfrac{\sqrt{65}}{4}$ $x=4,77$ $x=0,73$	✓ substitution into correct formula/*substitusie* in korrekte formule ✓4,77 ✓0,73 (3) ✓ correct completion of the square/*korrekte voltooiing van die vierkant* ✓4,77 ✓0,73 (3)

1.1.3	$5x^2 - 21x + 4 > 0$ $(5x-1)(x-4) > 0$ $x < \frac{1}{5}$ or/*of* $x > 4$  **or**  	✓ standard form/ *standaardvorm* ✓ factors/*faktore* ✓ $x < \frac{1}{5}$ ✓ $x > 4$ ✓ of (5)
1.1.4	$2^{2x} - 6.2^x = 16$ $2^{2x} - 6.2^x - 16 = 0$ $(2^x - 8)(2^x + 2) = 0$ $2^x = 2^3$ or/*of* $2^x = -2$ $x = 3$ or/*of* No Solution or $2^x \neq -2$	✓ factors/*faktore* ✓no solution to/ *geen oplossing* $2^x = -2$ ✓ $2^x = 2^3$ ✓ answer/*antw.* (4)

1.2	$y = 2x - 1$ $x^2 - x(2x-1) + (2x-1)^2 = 7$ $x^2 - 2x^2 + x + 4x^2 - 4x + 1 = 7$ $3x^2 - 3x - 6 = 0$ $x^2 - x - 2 = 0$ $(x-2)(x+1) = 0$ $x = 2$ or/*of* $x = -1$ $y = 3$ or/*of* $y = -3$ **OR/*OF*** $x = \frac{y}{2} + \frac{1}{2}$ $\left(\frac{y}{2} + \frac{1}{2}\right)^2 - \left(\frac{y}{2} + \frac{1}{2}\right)y + y^2 = 7$ $\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - \frac{y^2}{2} - \frac{y}{2} + y^2 = 7$ $\times 4: \; y^2 + 2y + 1 - 2y^2 - 2y + 4y^2 - 28 = 0$ $3y^2 - 27 = 0$ $y^2 - 9 = 0$ $(y-3)(y+3) = 0$ $\therefore y = 3$ or $y = -3$ $\therefore x = \frac{3}{2} + \frac{1}{2}$ $x = \frac{-3}{2} + \frac{1}{2}$ $x = 2$ $x = -1$	✓*y* the subject/ *die onderwerp* ✓substitution/*substitusie* ✓simplification/*vereenv.* ✓ factors/*faktore* ✓*x*-values/*waardes* ✓*y*-values/*waardes* (6) ✓ *x* the subject/ *die onderwerp* ✓substitution/*substitusie* ✓simplification/*vereenv.* ✓ factors/*faktore* ✓*y*-values/*waardes* ✓*x*-values/*waardes* (6)
1.3.1	$k = -2$ or/*of* $k = 2$	✓✓answer/*antw.* (2)
1.3.2	$k = -3$	✓ -3 (1)

1.4	$\sqrt{\frac{7^{2014}-7^{2012}}{12}}$ $=\sqrt{\frac{7^{2012}\left(7^2-1\right)}{12}}$ $=\sqrt{\frac{7^{2012}.48}{12}}$ $=\sqrt{7^{2012}.4}$ $=2.7^{1006}$ $a = 2;\ b = 1006$	✓ $\frac{7^{2012}\left(7^2-1\right)}{12}$ ✓ $\sqrt{7^{2012}.4}$ ✓ 2.7^{1006} ✓ **OR/*OF*** ✓$a = 2$ ✓$b = 1006$ (4) **[27]**

QUESTION/*VRAAG* 2

2.1	$S_n = a + (a+d) + (a+2d) + \ldots + a + (n-1)d$ $S_n = a + (n-1)d + a + (n-2)d + a + (n-3)d + \ldots + a$ $2S_n = n(2a + (n-1)d)$ $S_n = \frac{n}{2}[2a + (n-1)d]$	✓ first series/*eerste reeks* ✓ series reversed/*reeks omgekeer* ✓ sum/*som* ✓ division/*deling* (4)
2.2	$\sum_{k=1}^{50}(100-3k) = 97 + 94 + 91 + \ldots$ $T_1 = a = 97$ $d = -3$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{50}{2}[2(97) + 49(-3)]$ $= 1175$ **OR/*OF*** $T_1 = a = 97$ $l = 100 - 3(50) = -50$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[a + l]$ $= \frac{50}{2}[97 - 50]$ $= 1175$	✓$a = 97$ ✓$d = -3$ ✓$n = 50$ ✓ answer/*antwoord* (4) ✓$a = 97$ ✓$l = -50$ ✓$n = 50$ ✓ answer/*antwoord* (4)

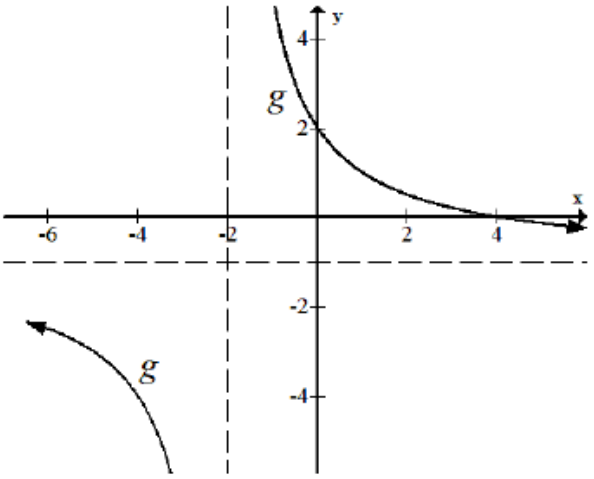
2.3.1 (a)	$T_5 - T_4 = 25$	✓ answer/*antwoord* (1)
2.3.1 (b)	$T_{70} - T_{69} = 7 + (69-1)(6)$ $= 415$	✓ $n = 69$ ✓ $7 + (69-1)(6)$ ✓ answer/*antw.* (3)
2.3.2	$T_{89} - T_{69} = (T_{70} - T_{69}) + (T_{71} - T_{70}) + ... + (T_{89} - T_{88})$ $= 415 + 421 + ...$to 20 terms $= \frac{20}{2}[2(415) + 19(6)]$ $= 9440$ $T_{69} = T_{89} -$ (sum of the differences from/*som van die verskille van* T_{69} to T_{89}) $T_{69} = 23594 - 9440$ $= 14154$ **OR/*OF*** 7 13 19 25 6 6 6 $\therefore 2a = 6$ $a = 3$ $3a + b = 7$ $b = -2$ $T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $\therefore T_{69} = 3(69)^2 - 2(69) + 9$ $\therefore T_{69} = 14154$	✓ expansion/*uitbreiding* ✓ $n = 20$ ✓ method/*metode* ✓ $a = 415$ ✓ answer/*antwoord* (5) ✓ a and/*en* b ✓ T_{89} (subst $n = 89$) ✓ T_n ✓ substitution/*substitusie* ✓ answer/*antwoord* (5)

	OR/*OF* 7 13 19 25 6 6 6 $\therefore 2a = 6$ $a = 3$ $7 - 6 = 1$ $T_1 - T_0 = 1$ $a + b + c - c = 1$ $3 + b = 1$ $b = -2$ $T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $\therefore T_{69} = 3(69)^2 - 2(69) + 9$ $\therefore T_{69} = 14154$	✓ a and/*en* b ✓ T_{89} (subst $n = 89$) ✓ T_n ✓substitution/*substitusie* ✓ answer/*antwoord* (5) **[17]**
	OR/*OF* $T_{n+1} - T_n = 7 + 6(n-1)$ $\therefore T_{89} - T_1 = \sum_{n=1}^{88}(T_{n+1} - T_n)$ $= \frac{n}{2}[2a + (n-1)d]$ $= \frac{88}{2}[14 + 87 \times 6]$ $= 23584$ $\therefore T_1 = 23594 - 23584 = 10$ $\therefore T_{69} - 10 = \sum_{n=1}^{68}(T_{n+1} - T_n)$ $= 34(15 + 67 \times 6) = 14144$ $\therefore T_{69} = 14154$	✓formula/*formule* ✓value of/*waarde van* S_{88} ✓first term value/ *eerste term waarde* ✓substitution/*substitusie* ✓answer/*antwoord* (5) **[17]**

QUESTION 3

3.1	$r=\frac{40,5}{45}=0,9$ $T_{12}=45(0,9)^{12-1}$ $=14,12147682...$ $=14,12$	✓$r=0,9$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (3)
3.2	$r=0,9$ $-1<0,9<1$	✓answer/*antwoord* (1)
3.3	$S_\infty=\frac{45}{1-0,9}$ $S_\infty=450$	✓ substitution/*substitusie* ✓ 450 (2)
3.4	$S_\infty-S_n<1$ $S_\infty-S_n=450-\frac{45\left(1-(0,9)^n\right)}{1-0,9}$ $S_\infty-S_n=450-450\left(1-(0,9)^n\right)$ $450(0,9)^n<1$ $(0,9)^n<\frac{1}{450}$ $\log(0,9)^n<\log\frac{1}{450}$ $n.\log(0,9)<\log\frac{1}{450}$ $n>\frac{\log\frac{1}{450}}{\log(0,9)}$ $n>57,98...$ Smallest value/*Kleinste waarde:* $n=58$	✓ $450-\frac{45\left(1-(0,9)^n\right)}{1-0,9}$ ✓ $(0,9)^n=\frac{1}{450}$ ✓ introducing/*gebruik* logs ✓making n the subject/*maak n die onderwerp* ✓ $n=58$ (5) **[11]**

QUESTION/*VRAAG* 4

4.1	$x=-2$ $y=-1$	✓ $x=-2$ ✓ $y=-1$ (2)
4.2.1	$g(0)=\frac{6}{0+2}-1$ $=2$ y-intercept/*afsnit* $(0\ ;\ 2)$	✓ answer/*antwoord* (1)
4.2.2	$0=\frac{6}{x+2}-1$ $1=\frac{6}{x+2}$ $x+2=6$ $x=4$ x-intercept/*afsnit* $(4\ ;\ 0)$	✓ equating to/*stel gelyk aan* 0 ✓ answer/*antwoord* (2)
4.3		✓ asymptotes/*asimptote* ✓ intercepts/*afsnitte* ✓ shape/*vorm* (3)
4.4	$y+1=-(x+2)$ $y=-x-3$ **OR/*OF*** Using general formula/*Gebruik algemene formule*: $y=-(x+p)+q$ $y=-(x+2)-1$ $y=-x-3$	✓$m=-1$ ✓substitution of $(-2\ ;-1)$ ✓answer (3) ✓formula/*formule* ✓substitution of p and q values/*substitusie van p- en q-waardes* ✓answer/*antwoord* (3)
4.5	$x>-2$	✓✓ answer (2) **[13]**

QUESTION/*VRAAG* 5

5.1	$9 = a^2$ $a = 3$ **OR/*OF*** $f^{-1}(x) = \log_a x$ $2 = \log_a 9$ $a^2 = 9 = 3^2$ $\therefore a = 3$	✓ $9 = a^2$ ✓ $a = 3$ (2) ✓ $9 = a^2$ ✓ $a = 3$ (2)
5.2	$g(x) = 3^{-x}$ **OR/*OF*** $g(x) = \left(\frac{1}{3}\right)^x$	✓ answer/*antwoord* (1) ✓ answer/*antwoord* (1)
5.3	$x \geq 9$ **OR/*OF*** $f^{-1}(x) = \log_3 x$ $\log_3 x = 2$ $x = 3^2 = 9$ $\therefore x \geq 9$ **OR/*OF*** $\log_3 x \geq 2$ $x \geq 3^2$ $\therefore x \geq 9$	✓✓ answer/*antwoord* (2) ✓✓ answer/*antwoord* (2) ✓✓ answer/*antwoord* (2)
5.4	Yes/*Ja*. For every y-value there is only one x such that/*Vir elke y-waarde is daar slegs een x sodanig dat* $y = f(x)$. **OR/*OF*** Yes/*Ja*. f is a one-to-one relation/*is 'n een-tot-een-relasie*.	✓ Yes/*Ja* ✓ Reason/*Rede* (2) ✓ Yes/*Ja* ✓ Reason/*Rede* (2) **[7]**

QUESTION/*VRAAG 6*

6.1	$-3 \le x \le 2$	✓ critical values/ *kritiese waardes* ✓ notation/*notasie* (2)
6.2	$f: \quad y = a(x-x_1)(x-x_2)$ $y = a(x+3)(x-2)$ $-8 = a(1+3)(1-2)$ $-8 = -4a$ $2 = a$ $y = 2(x+3)(x-2)$ $y = 2x^2 + 2x - 12$ $b = 2$ and/*en* $c = -12$ **OR/*OF*** $y = a\left(x+\frac{1}{2}\right)^2 + q$ $0 = a\left(2+\frac{1}{2}\right)^2 + q \quad \rightarrow \quad 0 = \frac{25}{4}a + q \quad ....(1)$ $-8 = a\left(1+\frac{1}{2}\right)^2 + q \quad \rightarrow \quad -8 = \frac{9}{4}a + q \quad .....(2)$ $(1)-(2): 8 = 4a$ $a = 2$ $q = 0 - \frac{25}{4}(2) = -\frac{25}{2} = -12{,}5$ $y = 2\left(x+\frac{1}{2}\right)^2 - 12\frac{1}{2}$ $y = 2\left(x^2 + x + \frac{1}{4}\right) - 12\frac{1}{2}$ $y = 2x^2 + 2x + \frac{1}{2} - 12\frac{1}{2}$ $y = 2x^2 + 2x - 12$ $\therefore b = 2$ and $c = -12$ **OR/*OF***	✓ $y = a(x+3)(x-2)$ ✓ substitute/*vervang* $(1\ ;-8)$ ✓ $a = 2$ ✓ $b = 2$ and/*en* ✓ $c = -12$ (5) ✓ equation/*vergelyking* 1 ✓ equation/*vergelyking* 2 ✓ $a = 2$ ✓ substitution/*substitusie* ✓ $b = 2$ and/*en* ✓ $c = -12$ (5)

	$f'(x) = 2ax + b$ $f'\left(-\frac{1}{2}\right) = 2a\left(-\frac{1}{2}\right) + b = 0$ $\therefore a = b$ $(-3;0): 9a - 3b + c = 0$ $\therefore 6a + c = 0 \ldots\ldots\ldots\ldots (1)$ $(1;-8): a + b + c = -8$ $\therefore 2a + c = -8 \ldots\ldots\ldots\ldots (2)$ $(1) - (2): \quad 4a = 8$ $\therefore a = 2$ $\Rightarrow b = 2$ $\therefore c = -12$	✓equation/*vergelyking* 1 ✓equation/*vergelyking* 2 ✓ $a = 2$ ✓ $b = 2$ ✓ $c = -12$ (5)
6.3	$x = -\frac{b}{2a}$ $x = -\frac{2}{2(2)} = -\frac{1}{2}$ $y = \frac{1}{2} - 1 - 12$ $y = -12\frac{1}{2}$ $\text{TP}\left(-\frac{1}{2}\ ;\ -12\frac{1}{2}\right)$ **OR/*OF***	✓ $x = -\frac{1}{2}$ ✓substitution/*substitusie* ✓*y*-value/*waarde* (3)

	$y = 2[x^2 + x - 6]$ $y = 2\left[x^2 + x + \left(\frac{1}{2}.1\right)^2 - 6 - \left(\frac{1}{2}.1\right)^2\right]$ $= 2\left[\left(x + \frac{1}{2}\right)^2 - 6{,}25\right]$ $= 2\left(x + \frac{1}{2}\right)^2 - 12{,}5$ $\text{TP}\left(-\frac{1}{2}; -12{,}5\right)$	✓ method/*metode* ✓ x-value/*waarde* ✓ y-value/*waarde* (3)
	OR/*OF* $x = \frac{-3+2}{2} = -\frac{1}{2}$ $y = 2\left(-\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right) - 12$ $y = -12\frac{1}{2}$ $\text{TP}\left(-\frac{1}{2}; -12{,}5\right)$	✓ method/*metode* ✓ x-value/*waarde* ✓ y-value/*waarde* (3)
	OR/*OF* $f(x) = y = 2x^2 + 2x - 12$ $f'(x) = 4x + 2$ $4x + 2 = 0$ $4x = -2$ $x = -\frac{1}{2}$ $\therefore y = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 12 = -\frac{25}{2}$ $\text{TP}\left(-\frac{1}{2}; -\frac{25}{2}\right)$	✓ method/*metode* ✓ x-value/*waarde* ✓ y-value/*waarde* (3)
6.4	$x = \frac{13}{2}$	✓✓ answer/i (2)
6.5	$f'(x) = 4x + 2$ $m = f'(1) = 4(1) + 2$ $m = 6$	✓ $y' = 4x + 2$ ✓ subst. $x = 1$ ✓ answer/*antwoord* (3) **[15]**

QUESTION/*VRAAG* 7

7.1.1	$R400\times(44\times12)$ $=R211200$	✓ $R400\times(44\times12)$ ✓ $R211200$ (2)
7.1.2	$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $=\frac{400\left[\left(1+\frac{0,08}{12}\right)^{528}-1\right]}{\frac{0,08}{12}}$ $=R1\ 943\ 524,42$	✓ $x=400$ ✓ $n=528$ ✓ $i=\frac{0,08}{12}$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (5)
7.1.3	$P=\frac{x[1-(1+i)^{-n}]}{i}$ $2000000=\frac{x\left[1-\left(1+\frac{0,1}{12}\right)^{-300}\right]}{\frac{0,1}{12}}$ $x=R18\ 174,01$ **OR/*OF*** $2000000\left(1+\frac{0,1}{12}\right)^{300}=\frac{x\left(\left(1+\frac{0,1}{12}\right)^{300}-1\right)}{\frac{0,1}{12}}$ $x=R18174,01$	✓ P = 2000000 ✓ $n=300$ and/*en* $i=\frac{0,1}{12}$ ✓ substituting into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (4) ✓ P = 2000000 ✓ $n=300$ and/*en* $i=\frac{0,1}{12}$ ✓ equating/stel gelyk ✓ answer/*antwoord* (4)
7.2	Let P_X and P_Y be the populations of the two towns at the beginning of 2010./*Laat* P_X *en* P_Y *die bevolkings wees van die twee dorpe aan die begin van 2010.* $A_X=A_Y$ $P_X(1-0,08)^3=P_Y(1+0,12)^3$ $\frac{P_X}{P_Y}=\frac{(1+0,12)^3}{(1-0,08)^3}$ $=\frac{1,404...}{0,778...}$ $=1,8:1$	✓ equating/*stel gelyk* ✓ $A_X=P_X(1-0,08)^3$ ✓ $A_Y=P_Y(1+0,12)^3$ ✓ answer/*antwoord* (4) **[15]**

QUESTION/*VRAAG* 8

8.1	$f(x+h)=2(x+h)^2+4$ $=2x^2+4xh+2h^2+4$ $f(x+h)-f(x)=2x^2+4xh+2h^2+4-2x^2-4$ $=4xh+2h^2$ $f'(x)=\lim_{h\to 0}\frac{4xh+2h^2}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h)}{h}$ $=\lim_{h\to 0}(4x+2h)$ $=4x$	✓ $2x^2+4xh+2h^2+4$ ✓ $4xh+2h^2$ ✓ $\lim_{h\to 0}\frac{h(4x+2h)}{h}$ ✓ $4x$ (4)
8.2.1	$f(x)=-3x^2+5\sqrt{x}$ $f(x)=-3x^2+5x^{\frac{1}{2}}$ $f'(x)=-6x+\frac{5}{2}x^{-\frac{1}{2}}$	✓ $5x^{\frac{1}{2}}$ ✓ $-6x$ ✓ $\frac{5}{2}x^{-\frac{1}{2}}$ (3)
8.2.2	$p(x)=\left(\frac{1}{x^3}+4x\right)^2$ $=\frac{1}{x^6}+\frac{8}{x^2}+16x^2$ $=x^{-6}+8x^{-2}+16x^2$ $p'(x)=-6x^{-7}-16x^{-3}+32x$ **OR/*OF*** $p(x)=(x^{-3}+4x)^2$ by making use of the chain rule : $p'(x)=2(x^{-3}+4x)(-3x^{-4}+4)$ $p'(x)=-6x^{-7}-16x^{-3}+32x$	✓ $\frac{1}{x^6}+\frac{8}{x^2}+16x^2$ ✓ $x^{-6}+8x^{-2}+16x^2$ ✓✓ answer/*antwoord* (4) ✓✓ $2(x^{-3}+4x)$ ✓✓ $(-3x^{-4}+4)$ (4)
8.3.1	$h'(x)=3x^2-14x+14$	✓ finding/*kry* $h'(x)$ (1)
8.3.2	At/*By* B: $h'(x)=0$ $3x^2-14x+14=0$ $x=\frac{14\pm\sqrt{(-14)^2-4(3)(14)}}{2(3)}$ $=1{,}45$ or $3{,}22$ n/a	✓ derivative equal to/ *afgeleide gelyk aan* 0 ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ x-value of/*x-waarde van* 1,45 (3)

8.3.3	$x^3 - 7x^2 + 14x - 8 = (x-1)(x^2 - 6x + 8)$ $= (x-1)(x-2)(x-4)$ $C(4;0)$ **OR/*OF*** $x_c > 3{,}22$ $h(4) = (4)^3 - 7(4)^2 + 14(4) - 8 = 0$ $\therefore x_c = 4$	✓ $(x-1)$ ✓ $x^2 - 6x + 8$ ✓ $(x-2)(x-4)$ ✓ coordinates of/*koördinate van* C (4) ✓ $x_c > 3{,}22$ ✓ substitution of/ *substitusie van* 4 ✓ $h(4) = 0$ ✓ x_c (4)
8.3.4	$h'(x) = 3x^2 - 14x + 14$ $h''(x) = 6x - 14$ $6x - 14 < 0$ $6x < 14$ $\therefore x < \frac{7}{3}$ $\therefore k = \frac{7}{3}$	✓ $h''(x) = 6x - 14$ ✓ $6x - 14 < 0$ ✓ $k = \frac{7}{3}$ (3) **[22]**

QUESTION/*VRAAG 9*

9.1	$\pi r^2 h = 6$ $h = \frac{6}{\pi r^2}$	✓ $h = \frac{6}{\pi r^2}$ (1)
9.2	$S = 10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ $= 10\left[2\pi rh + 6\pi r^2\right]$ $= 20\pi rh + 60\pi r^2$ $= 20\pi r\left(\frac{6}{\pi r^2}\right) + 60\pi r^2$ $= 60\pi r^2 + \frac{120}{r}$ **OR/*OF*** Area of/*van* 10 spheres/*sfere* $= 10 \times 4 \times \pi \times r^2 = 40\pi r^2$ Area of/*van* 10 cylinders/*silinders* $= 10(2\pi r^2 + 2\pi r h)$ $= 10(2\pi r^2 + 2\pi r \frac{6}{\pi r^2})$ $= 20\pi r^2 + \frac{120}{r}$ Total area/*Totale area* $= 40\pi r^2 + 20\pi r^2 + \frac{120}{r}$ $= 60\pi r^2 + \frac{120}{r}$	✓✓ $10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ ✓ $20\pi rh + 60\pi r^2$ ✓ substitution/*substitusie* (4) ✓area of 10 spheres/ *area van 10 sfere* ✓ area of 10 cylinders/ *area van 10 silinders* ✓ substitution/*substitusie* ✓simplification/*vereen-voudiging* (4)
9.3	$S' = 120\pi r - 120r^{-2} = 0$ $120\pi r - \frac{120}{r^2} = 0$ $120\pi r^3 - 120 = 0$ $r^3 = \frac{120}{120\pi}$ $\therefore r = \frac{1}{\pi^{\frac{1}{3}}} = 0{,}68$ cm	✓ $120\pi r - 120r^{-2}$ ✓ $= 0$ ✓ $r^3 = \frac{120}{120\pi}$ ✓answer/*antwoord* (4) **[9]**

QUESTION/*VRAAG* 10

10.1.1	$d = 5$ $e = 4$ $f = 7$ $g = 5$	✓$d = 5$ ✓$e = 4$ ✓$f = 7$ ✓$g = 5$ (4)
10.1.2a	P(A and/*en* B and/*en* C) $= \frac{4}{54} = \frac{2}{27}$	✓$\frac{4}{54} = \frac{2}{27}$ (1)
10.1.2b	P(A or/*of* B or/*of* C) $= \frac{48}{54} = \frac{8}{9}$	✓$\frac{48}{54} = \frac{8}{9}$ (1)
10.1.2c	P(only/*slegs* C) $= \frac{7}{54}$	✓$\frac{7}{54}$ (1)
10.1.2d	P(that a country uses exactly two methods/*dat 'n land presies twee metodes gebruik*) $= \frac{5+4+8}{54} = \frac{17}{54}$	✓$\frac{17}{54}$ (1)
10.2.1	P(selects *Midnight* as drama/*kies Midnight as drama*) $= \frac{1}{5}$	✓✓ answer/*antwoord* (2)
10.2.2	Number of different selections of drama, romance and comedy/*Aantal verskillende keuses van drama, liefdesverhale en komedie* = 5 × 4 × 3= 60	✓ product/*produk* ✓answer/*antwoord* (2)
10.2.3	P(select *Last Hero* and *Laughing Dragon*/*kies Last Hero en Laughing Dragon*) $= \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$ **OR/*OF*** P(select *Last Hero* and *Laughing Dragon*/*kies Last Hero en Laughing Dragon*) $= \frac{1 \times 4 \times 1}{60} = \frac{1}{15}$	✓ product/*produk* ✓answer/*antwoord* (2) ✓ product/*produk* ✓ answer/*antwoord* (2) **[14]**
	TOTAL/TOTAAL:	**150**

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2015

MEMORANDUM

QUESTION/*VRAAG* 2

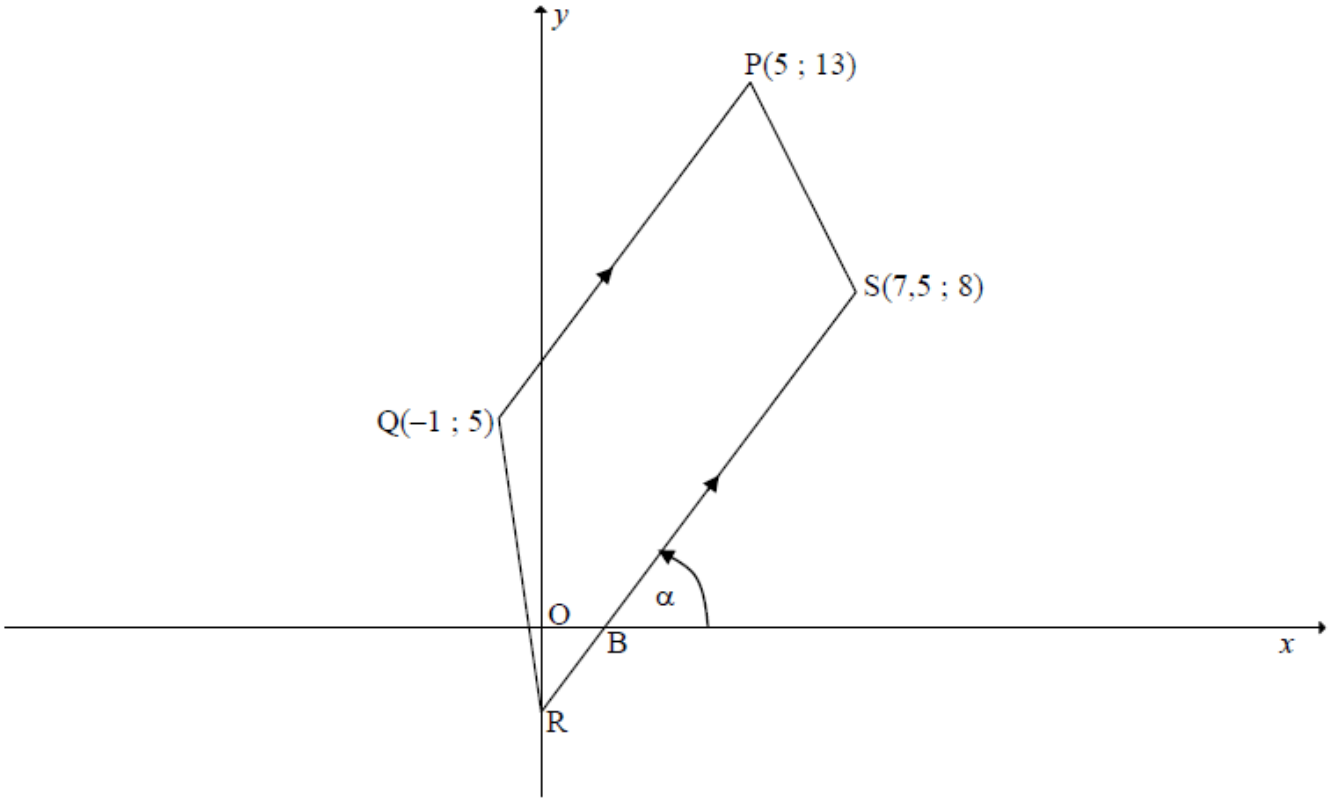
2.1	As the temperature increases, the sales of ice-creams increase/*Soos die temperatuur styg, neem die verkope toe.* **OR/*OF*** As the temperature decreases, the sales of ice-creams decrease/*Soos die temperatuur daal, neem die verkope af.*	✓ reason/*rede* (1) ✓ reason/*rede* (1)
2.2	The liveable temperature cannot keep on increasing/*Die leefbare temperatuur kan nie aanhou styg nie.*	✓ reason/*rede* (1)
2.3	$a = -460{,}35$ $b = 30{,}09$ $\hat{y} = 30{,}09x - 460{,}35$ **OR/*OF*** $\hat{y} = -460{,}35 + 30{,}09x$ Answer only: Full marks *slegs antw: volpunte*	✓✓ –460,35 ✓ 30,09 ✓ equation/*vgl* (4)
2.4	$r = 0{,}96$	✓ 0,96 (1)
2.5	There is a very strong positive relationship (correlation)/*Daar is 'n baie sterk positiewe verband (korrelasie).*	✓ very strong/*baie sterk* (1) **[8]**

		antwoora (1)
1.5	$\sigma = 17{,}27$	✓✓answer/ *antwoord* (2)
1.6.1	$\overline{x} = 157{,}62 + p$	✓ answer (1)
1.6.2	$\sigma = 17{,}27$	✓ answer/ *antwoord* (1) **[13]**

QUESTION/*VRAAG* 2

2.1	As the temperature increases, the sales of ice-creams increase/*Soos die temperatuur styg, neem die verkope toe.* **OR/*OF*** As the temperature decreases, the sales of ice-creams decrease/*Soos die temperatuur daal, neem die verkope af.*	✓ reason/*rede* (1) ✓ reason/*rede* (1)
2.2	The liveable temperature cannot keep on increasing/*Die leefbare temperatuur kan nie aanhou styg nie.*	✓ reason/*rede* (1)
2.3	$a = -460{,}35$ $b = 30{,}09$ $\hat{y} = 30{,}09x - 460{,}35$ **OR/*OF*** $\hat{y} = -460{,}35 + 30{,}09x$ Answer only: Full marks *slegs antw: volpunte*	✓✓ –460,35 ✓ 30,09 ✓ equation/*vgl* (4)
2.4	$r = 0{,}96$	✓ 0,96 (1)
2.5	There is a <u>very strong</u> positive relationship (correlation)/*Daar is 'n <u>baie sterk</u> positiewe verband (korrelasie).*	✓ very strong/*baie sterk* (1) **[8]**

QUESTION/VRAAG 3

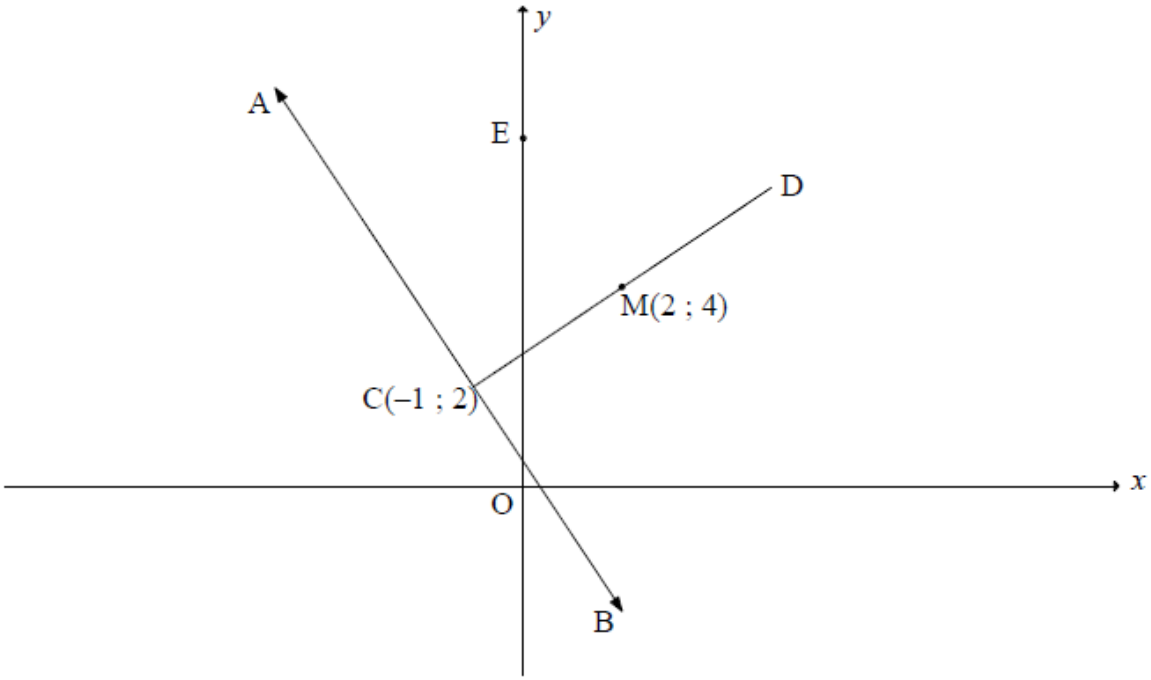


3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)
3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/ *korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, | | lines/=*gradiënte, | | lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and *m* into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of *c* /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the *x*-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan\ \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = \text{O}\hat{\text{B}}\text{R}$ (vert opp ∠s/*regoorst ∠e*) $\text{OR}\hat{\text{B}} = 180° - (90° + 53{,}13°)$ (∠s of Δ/*∠e van Δ*) $= 36{,}87°$	✓ $\tan\ \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$\text{BS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ ∥ BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ∥/*1 pr tos sye =en* ∥) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar*) **OR/*OF***	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right)$ ✓ $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right)$ ✓ $\left(\frac{13}{4}; \frac{13}{2}\right)$ ✓ reason/*rede* (4)

	$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore$ QB || PS PQ || BS PQBS = parallelogram (both pairs opp sides ||/*beide pr tos sye* ||)	✓ m_{QB} ✓ m_{PS} ✓ QB || PS ✓ reason/*rede* (4)
	OR/*OF*	
	$BS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{\left(\frac{15}{2}-\frac{3}{2}\right)^2+(8-0)^2}$ $\therefore$ PQ = BS $= 10$	✓ correct subst into form/*korrekte subst in formule* ✓ PQ = 10
	$QB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-1-1,5)^2+(5-0)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{5\sqrt{5}}{2}$ or 5,59 $PS = \sqrt{(5-7,5)^2+(13-8)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{\sqrt{125}}{2}$ or 5,59 QB = PS PQBS = parallelogram (both pairs opp sides =/ *beide pr tos sye* =)	✓ QB = PS ✓ reason/*rede* (4) **[18]**

QUESTION/VRAAG 4



4.1.1	Radius = $\sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR/*OF*** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore\ (x-2)^2+(y-4)^2=13$	✓ $\sqrt{(2+1)^2+(4-2)^2}$ or/*of* $\sqrt{13}$ ✓ $(x-2)^2+(y-4)^2$ ✓ 13 (3) ✓ $(x-2)^2+(y-4)^2$ ✓ $(-1-2)^2+(2-4)^2$ ✓ 13 (3)
4.1.2	At/*by* D: $\frac{-1+x_D}{2}=2$ and/*en* $\frac{2+y_D}{2}=4$ $-1+x_D=4$ and/*en* $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR/*OF*** By inspection/*deur inspeksie*: D(5 ; 6)	✓ x – value/*waarde* ✓ y - value/*waarde* (2) ✓ x - value/*waarde* ✓ y - value/*waarde* (2)

4.1.3	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ (Tangent $\perp$ radius/*raaklyn* $\perp$ *radius*) $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ **OR/OF** $y = mx + c$ $y - 2 = -\frac{3}{2}(x+1)$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$ $y = -\frac{3}{2}x + \frac{1}{2}$	✓ $m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ ✓ $m_{AB} \times m_{MC} = -1$ ✓ $m_{AB} = -\frac{3}{2}$ ✓subst m and $(-1 ; 2)$ into eq /*subst m en (–1 ; 2) in vgl* ✓ eq in standard form/ *vgl in st vorm* (5)
4.1.4	At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR/*OF*** At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)	✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4) ✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4)
4.1.5	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore$ EM $\|\|$ AB $(m_{EM} = m_{AB})$	✓ $m_{EM} = -\frac{3}{2}$ ✓ reason/*rede* (2)

4.2	The centres of the circles are / *Die middelpunte van die sirkels is* P(–2 ; 4) and / *en* Q(5 ; –1) $QP^2 = (-2-5)^2 + (4-(-1))^2$ $QP = \sqrt{74} \approx 8{,}60$ units $r_M + r_p = 5 + 3$ $= 8$ $\therefore\ r_M + r_p < QP$ $\therefore$ The two circles do not intersect/*Die twee sirkels sny nie*	✓ both centres/*albei Midpte* ✓ QP ✓ correct subst into form/*korrekte subst in formule* ✓ distance between 2 centres/*afstand tussen 2 midpte* ✓✓ $r_M + r_p < QP$ (6) **[22]**

QUESTION/VRAAG 5

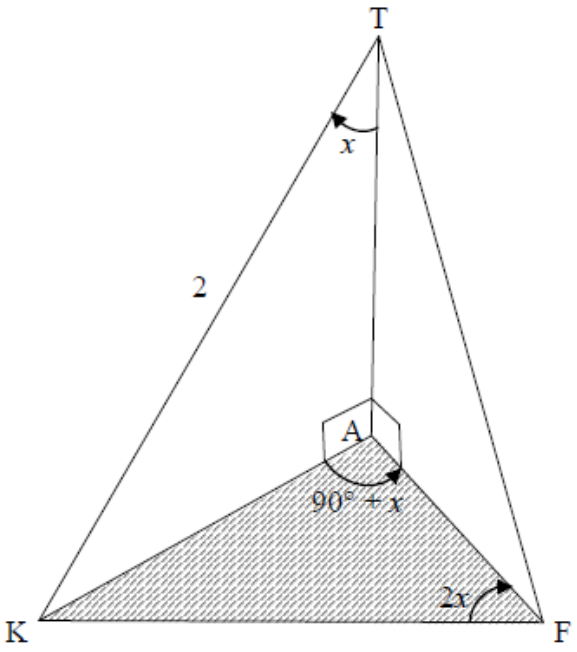
5.1	$x^2 + y^2$ $=(3\sin\theta)^2 + (3\cos\theta)^2$ $=9\sin^2\theta + 9\cos^2\theta$ $=9(\sin^2\theta + \cos^2\theta)$ $=9(1)$ $=9$	✓ simpl/*vereenv* ✓ CF/*GF* = 9 ✓ answer/*antw* (3)
5.2	$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $\sin(180° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	✓ $\sin(540° - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180° - x) = -\cos x$ ✓ $\sin(90° + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin\alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ **OR/*OF*** (P lies in 3rd quadrant) $x^2 + y^2 = r^2$ $x^2 + p^2 = \left(\sqrt{1 + p^2}\right)^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ (P lies in 3rd quadrant)	✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin\alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3) ✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)
5.3.2	$\cos(180° + \alpha)$ $= -\cos\alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	✓ $-\cos\alpha$ ✓ answer/*antw* (2)

5.3.3	$\cos 2\alpha$ $= \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - \left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= \frac{1}{1+p^2} - \frac{p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓✓ squaring each term/*kwadreer elke term* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 1 - 2\sin^2\alpha$ $= 1 - 2\left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= 1 - 2\left(\frac{p^2}{1+p^2}\right)$ $= 1 - \frac{2p^2}{1+p^2}$ $= \frac{1+p^2-2p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓ squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 2\cos^2\alpha - 1$ $= 2\left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - 1$ $= 2\left(\frac{1}{1+p^2}\right) - 1$ $= \frac{2}{1+p^2} - 1$ $= \frac{2-1-p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓ squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)

5.4.1	The identity is undefined for/*die identiteit is ongedefinieerd as*: $2\sin^2 x = 0$ $\therefore$ $\underline{\sin x = 0:}$ $x = 0° ; 180°$ or/*of* $\underline{\tan x = \infty:}$ $x = 90°$ $\therefore$ $x = 0° ; 90° ; 180°$	✓ $x = 0°$ ✓ $x = 90°$ ✓ $x = 180°$ (3)
5.4.2	LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x}$ $= \left(\dfrac{2\sin x - 2\sin x\cos^2 x}{\cos x}\right) \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(\sin^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK **OR/*OF*** LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x} \times \dfrac{\cos x}{\cos x}$ $= \dfrac{2\sin x - 2\sin x\cos^2 x}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x.\sin^2 x}{2\sin^2 x\cos x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK	✓ $\dfrac{\sin x}{\cos x}$ ✓ 2sinx.cosx ✓ simplify numerator/*vereenv teller* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to/*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) ✓ $\dfrac{\sin x}{\cos x}$ ✓ 2sinx.cosx ✓ simpl/*vereenv* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to /*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) **[26]**

QUESTION/VRAAG 6

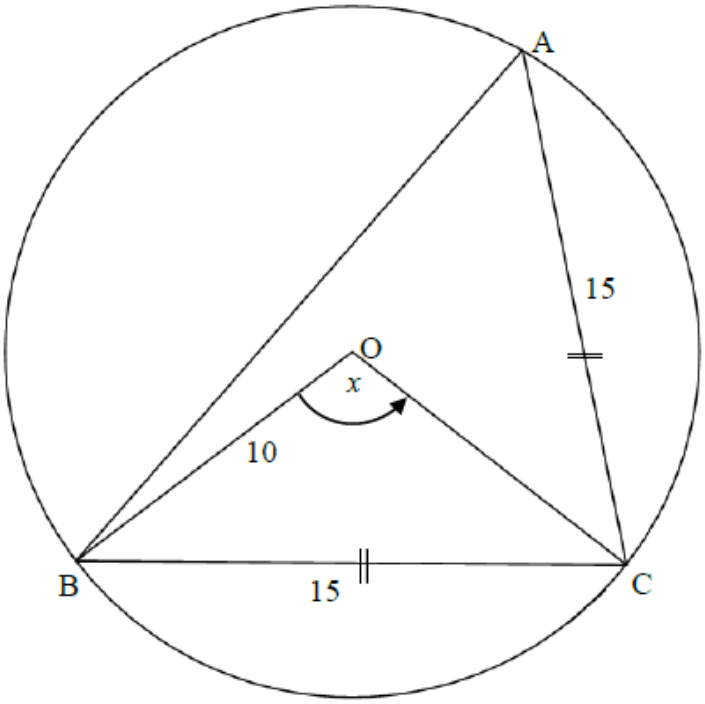
6.1



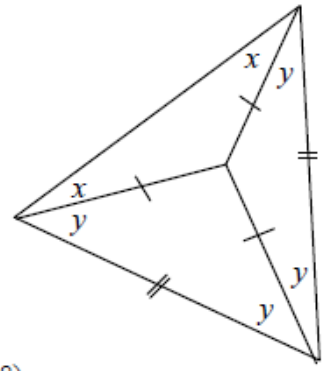
6.1.1	In ΔTAK: $\frac{AK}{KT} = \sin K\hat{T}A$ $AK = KT.\sin x$ $= 2\sin x$ **OR/*OF*** $\frac{\sin K\hat{T}A}{AK} = \frac{\sin K\hat{A}T}{KT}$ $\frac{\sin 90°}{2} = \frac{\sin x}{AK}$ $AK = 2\sin x$	✓ correct trig ratio/ *korrekte trigverh.* ✓ answer/*antw* (2) ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ answer/*antw* (2)

6.1.2	In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{2\sin x.\cos x}{2\sin x.\cos x}$ $= 1$ **OR/*OF*** In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{AT.\tan x.\cos x}{2\sin x.\cos x}$ $= \frac{2\cos x.\frac{\sin x}{\cos x}.\cos x}{2\sin x.\cos x}$ $= 1$ $\cos x = \frac{AT}{2}$ $\therefore AT = 2\cos x$	✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin(90° + x) = \cos x$ ✓ $2\sin x.\cos x$ ✓ 1 (5) ✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin(90° + x) = \cos x$ ✓ $2\sin x.\cos x$ ✓ 1 (5)

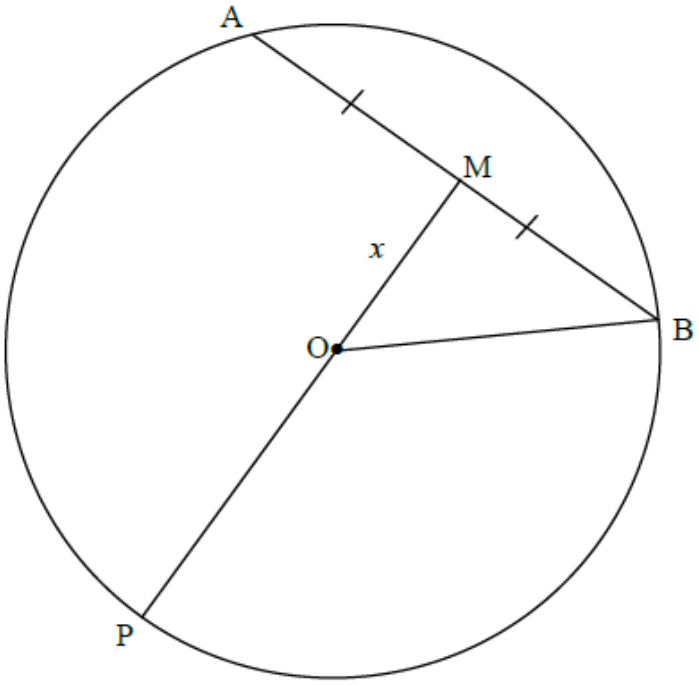
6.2



6.2.1	In ΔBOC: $BC^2 = BO^2 + CO^2 - 2.BO.\ CO\ .\cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10).\ \cos x$ $200 \cos x = -25$ $\cos x = -0{,}125$ $x = 180° - 82{,}82°$ $= 97{,}18°$ **OR/*OF*** Draw a line OD⊥ BC: BD = DC (line from centre ⊥ on chord) ΔOBD ≡ ΔOCD (90°; h; s) $\sin \frac{x}{2} = \frac{7{,}5}{10}$ $\frac{x}{2} = 48{,}59°$ $\therefore \quad x = 97{,}18°$	✓ using cosine rule/ *gebruik cos-reël* ✓correct subst/ *korrekte subst* ✓ $\cos x = -0{,}125$ ✓ 97,18° (4) ✓ S/R ✓correct ratio/ *korrekte verh* ✓ value of/*waarde van* $\frac{x}{2}$ ✓ 97,18° (4)

6.2.2	$B\hat{A}C = 48{,}59°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠omt*) $A\hat{B}C = B\hat{A}C = 48{,}59°$ (∠'s opp equal sides/∠*e teenoor = sye*) $\therefore A\hat{C}B = 82{,}82°$ (sum of ∠s of Δ/*som van ∠e van Δ*) **OR/*OF*** $A\hat{C}B = \frac{1}{2}A\hat{O}B$ (∠at centre = 2× ∠at circle) (∠*by midpt=2×∠omt*) $= \frac{1}{2}[360° - 2(97{,}18°)]$ $= 82{,}82°$ **OR/*OF*** $O\hat{C}B = \frac{1}{2}(180° - 97{,}18°)$ (∠'s opp equal sides; sum of ∠s of Δ) (∠*e teenoor = sye; som van ∠e van Δ*) $= 41{,}41°$  $A\hat{C}B = 2(41{,}41°)$ $= 82{,}82°$	✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3)
6.2.3	Area/*Oppervlakte* ΔABC $= \frac{1}{2}(BC)(AC)\sin A\hat{C}B$ $= \frac{1}{2}(15)(15)(\sin 82{,}82°)$ $= 111{,}62\ cm^2$	✓correct subst into area rule/*korrekte subst in opp-reël* ✓ 111,62 cm^2 (2) **[16]**

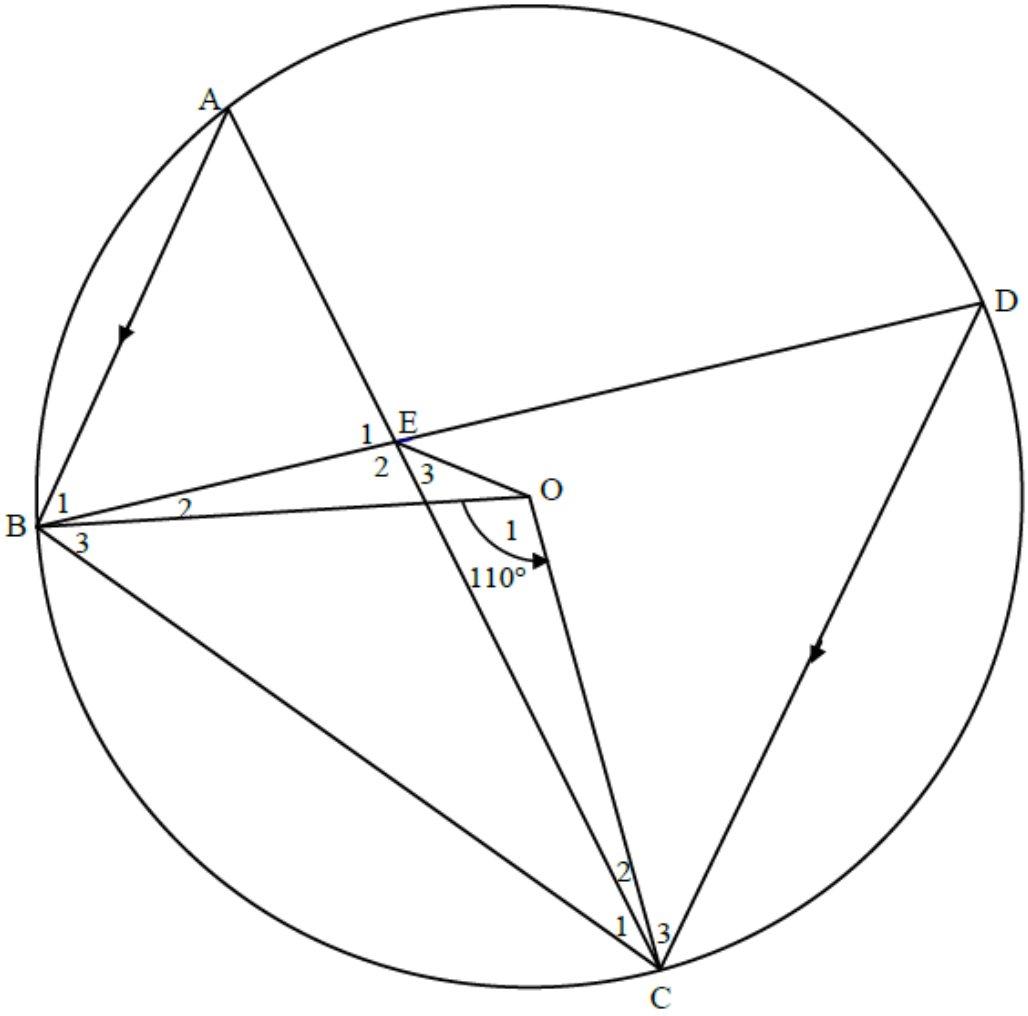
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/*antw* (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/*lyn vanaf midpt na midpt van koord is loodreg op koord* **OR/*OF*** line from centre bisects chord/*lyn vanaf midpt halveer koord*	✓ answer/*antw* (1) ✓ answer/*antw* (1)
7.3	$\frac{MP}{OM}=\frac{5}{2}$ $\frac{x+OP}{x}=\frac{5}{2}$ $2x+2OP=5x$ $OP=\frac{3x}{2}$ **OR/*OF*** $\frac{OP}{OM}=\frac{3}{2}$ $OP=\frac{3x}{2}$	✓ $\frac{x+OP}{x}=\frac{5}{2}$ ✓ $OP=\frac{3x}{2}$ (2) ✓ $\frac{OP}{OM}=\frac{3}{2}$ ✓ $OP=\frac{3x}{2}$ (2)

7.4	$OM^2 + MB^2 = OB^2$ $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8{,}94$ or $4\sqrt{5}$ or $\sqrt{80}$	✓ subst into/*subst* Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/*antw* (3) [7]

QUESTION/VRAAG 8

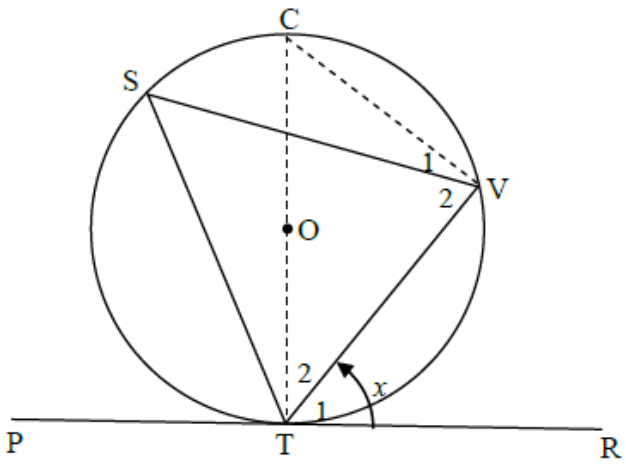


8.1.1	$\hat{D} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/*∠ by midpt=2×∠by omt*)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/*∠ by midpt=2×∠by omt*) **OR/*OF*** $\hat{A} = \hat{D} = 55°$ (∠s in same segment/*∠e in dieselfde segment*)	✓S ✓R (2) ✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55°$ (alternate ∠s/*verwiss∠e*; AB || DC) $\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext∠ of∆ = sum of opp∠^{s}/*buite∠ v∆=som v tos∠e*) $= 55° + 55°$ $\hat{E}_2 = 110°$	✓S ✓R ✓R ✓ answer/*antw* (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110°$ (proven in/*bewys in* 8.1.3) BEOC is a cyclic quadrilateral (equal ∠s subtended by line/ *gelyke ∠e onderspan deur lyn*)	✓S ✓R (2) **[10]**

QUESTION/VRAAG 9

9.1	the interior opposite angle/*die teenoorstaande binnehoek.*	✓ answer/*antw* (1)

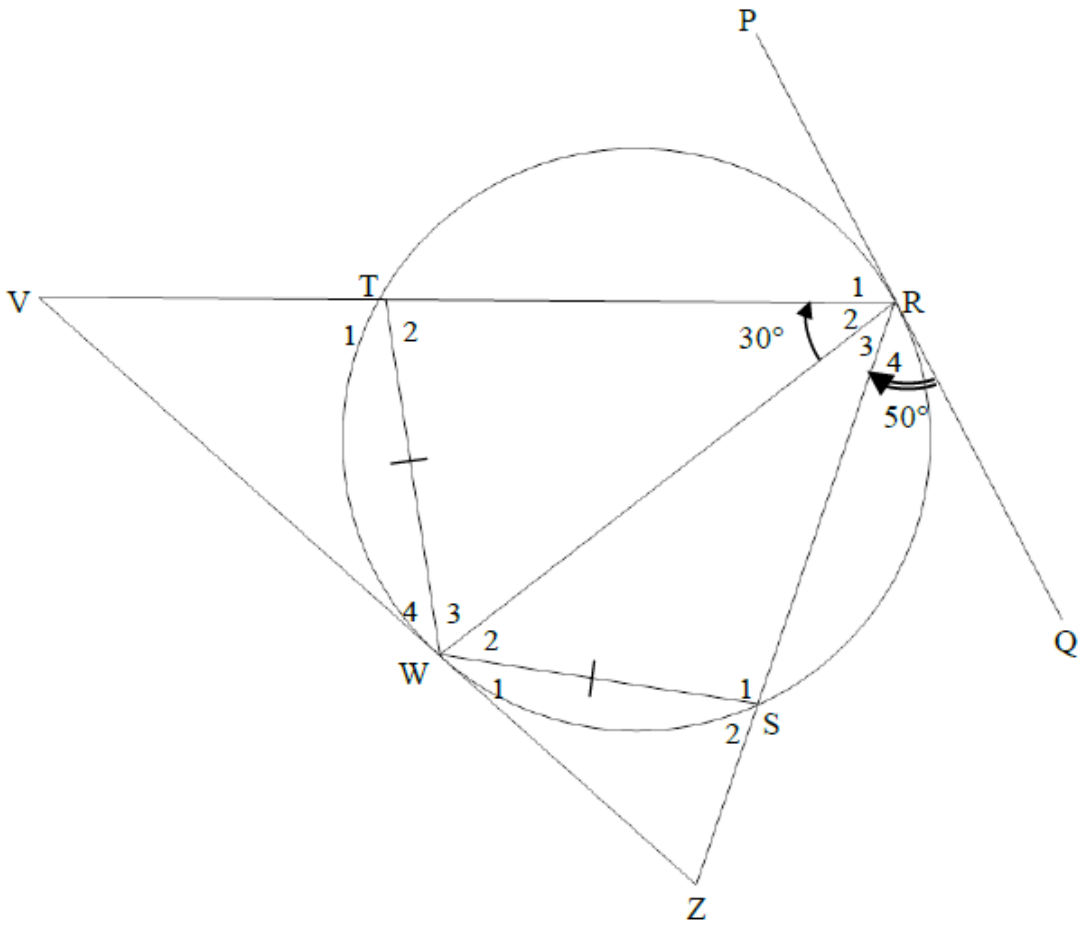
9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = 90°$	**∠ in semi-circle/∠ *in halfsirkel***	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius/*raaklyn ⊥ middellyn/radius***	✓ R
$\therefore \hat{C} =$ ***x***	Sum of the angles of triangle/*Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment/*∠e in dieselfde segment***	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

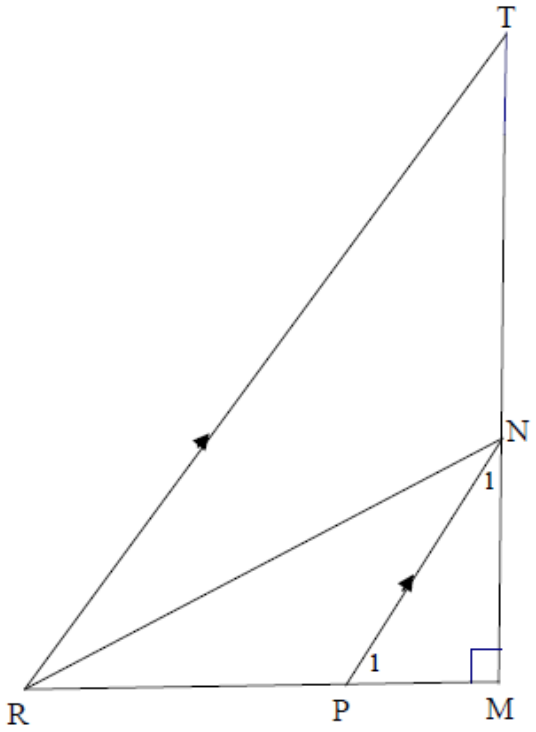
9.3



9.3.1	Equal chords subtend equal ∠s/*Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
9.3.2	$\hat{W}_4 = 30°$ (tan chord theorem/*rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer/*antw* ✓ reason/*rede* ✓ answer/*antw* (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext∠ of Δ/*buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend =∠s /= *kde onderspan=∠e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext ∠ of cyclic quad/*buite ∠ van koordevh*) $V + \hat{W}_4 = \hat{T}_2$ (ext∠ of Δ/*buite ∠ van Δ*) $\therefore\ \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
9.3.4	In ΔRVW and/*en* ΔRWS:	✓ using the correct Δs/ *gebruik korrekte Δe*
	$\hat{R}_2 = \hat{R}_3 = 30°$ (proven/*bewys* in 9.3.1)	✓ S
	$\hat{V} = \hat{W}_2 = 50°$ (proven/*bewys* in 9.3.3)	✓ S
	$V\hat{W}R = \hat{S}_1$ (3rd ∠ in Δ) $\therefore \Delta RVW \mid\mid\mid \Delta RWS$ (∠∠∠)	✓ R (3rd ∠ in Δ) or (∠∠∠)
	$\therefore \frac{WR}{RV} = \frac{RS}{WR}$ (ΔRVW ||| ΔRWS) $\therefore WR^2 = RV.RS$	✓ S (5) **[22]**

QUESTION/VRAAG 10



10.1.1	corresponding ∠s/*ooreenkomstige ∠e*; PN || RT	✓ answer/*antw* (1)
10.1.2	∠; ∠; ∠ **OR/*OF*** ∠; ∠	✓ answer/*antw* (1)
10.2	$\frac{PM}{RM}=\frac{PN}{RT}$ $(\Delta PNM \|\|\| \Delta RTM)$ $=\frac{PN}{3PN}$ $=\frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM}=\frac{1}{3}$ $\therefore \frac{RP}{RM}=\frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $=\left(\frac{3}{2}RP\right)^2-\left(\frac{1}{2}RP\right)^2$ $=\frac{9}{4}RP^2-\frac{1}{4}RP^2$ $= 2RP^2$ **OR/*OF***	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM=\frac{3}{2}RP$ ✓ $PM=\frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (3PM)^2 - PM^2$ $= 8PM^2$ $= 2(2PM)^2$ $= 2RP^2$ **OR/*OF*** $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (RP + PM)^2 - PM^2$ $= RP^2 + 2RP.PM + PM^2 - PM^2$ $= RP^2 + 2RP.\frac{1}{2}RP$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ $(3PM)^2 - PM^2$ ✓ RP = 2PM (4) ✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ expansion/ *uitbreiding* ✓ $PM = \frac{1}{2}RP$ (4) **[8]**

TOTAL/*TOTAAL*: 150

MATHEMATICS P1/*WISKUNDE V1*

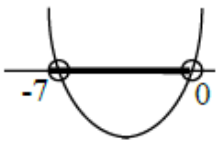
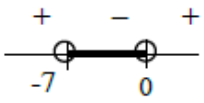
NOVEMBER 2015

MEMORANDUM

QUESTION/*VRAAG* 1

1.1.1	$x^2 - 9x + 20 = 0$ $(x-4)(x-5) = 0$ $x = 4$ or $x = 5$	✓factors ✓ $x = 4$ ✓ $x = 5$ (3)
1.1.2	$3x^2 + 5x - 4 = 0$ $x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-4)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2{,}26$ or $x = 0{,}59$ **OR/*OF*** $x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{4}{3} + \frac{25}{36}$ $\left(x + \frac{5}{6}\right)^2 = \frac{73}{36}$ $x + \frac{5}{6} = \pm\frac{\sqrt{73}}{6}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2{,}26$ or $x = 0{,}59$	✓standard form ✓substitution into correct formula ✓✓answers (4) ✓for adding $\frac{25}{36}$ on both sides ✓ $x = \frac{-5 \pm \sqrt{73}}{6}$ ✓✓answers (4)
1.1.3	$2x^{\frac{-5}{3}} = 64$ $x^{\frac{-5}{3}} = 32$ $x = \left(2^5\right)^{\frac{-3}{5}}$ $x = 2^{-3}$ or $\frac{1}{8}$ or $0{,}125$ **OR/*OF***	✓dividing both sides by 2 ✓ $32 = 2^5$ or $64 = 2^6$ ✓raising RHS to $\frac{-3}{5}$ ✓answer (4)

	$2x^{\frac{-5}{3}} = 64$ $x^{\frac{-5}{3}} = 32$ $x = (32)^{\frac{-3}{5}}$ $x = \sqrt[5]{32^{-3}}$ $x = 2^{-3}$ or $\frac{1}{8}$ or 0,125 **OR/*OF*** $\left(2x^{\frac{-5}{3}}\right)^{\frac{-3}{5}} = 64^{\frac{-3}{5}}$ $0,659x = 0,0825$ $x = 0,125$ **OR/*OF*** $x^{\frac{-5}{3}} = 32$ $\frac{-5}{3}\log x = \log 32$ $\log x = \frac{3}{-5}\log 32$ $\log x = -0,903$ $x = 10^{-0,903}$ $= 0,125$ or $\frac{1}{8}$	✓dividing both sides by 2 ✓raising RHS to $\frac{-3}{5}$ ✓$\sqrt[5]{32^{-3}}$ ✓answer (4) ✓raising both sides to $\frac{-3}{5}$ ✓0,659 and 0,0825 ✓dividing both sides by 0,659 ✓answer (4) ✓dividing both sides by 2 ✓logs on both sides ✓$\log x = -0,903$ ✓answer (4)
1.1.4	$\sqrt{2-x} = x-2$ $2-x = (x-2)^2$ $2-x = x^2-4x+4$ $x^2-3x+2=0$ $(x-1)(x-2)=0$ $x=1$ or $x=2$ if $x=1$, $\sqrt{2-x}=1$ and $x-2=-1$ $x=2$ only **OR/*OF***	✓squaring both sides ✓factors ✓$x=1$ or $x=2$ ✓$x=2$ only (4)

	$\sqrt{2-x}=x-2$ $2-x=(x-2)^2$ $2-x=(2-x)^2$ $2-x=1$ or $2-x=0$ $x=1$ or $x=2$ if $x=1$, $\sqrt{2-x}=1$ and $x-2=-1$ $\therefore x=2$ only **OR/*OF*** $\sqrt{2-x}=x-2$ $2-x\geq 0$ and $x-2\geq 0$ $x\leq 2$ and $x\geq 2$ $\therefore x=2$ only	✓squaring both sides ✓$2-x=1$ or $2-x=0$ ✓$x=1$ or $x=2$ ✓$x=2$ only (4) ✓$2-x\geq 0$ ✓ $x-2\geq 0$ ✓$x\leq 2$ and $x\geq 2$ ✓$x=2$ (4)
1.1.5	$x^2+7x<0$ $x(x+7)<0$  **OR/*OF***  $-7<x<0$ **OR/*OF*** $x\in(-7;0)$	✓factors ✓✓inequality or interval (3)

1.2	The square of any number is always positive or zero So for the sum of two squares to be zero, both squares must be zero, i.e. *Die kwadraat van enige getal is altyd positief of nul. Vir die som van twee kwadrate om nul te wees, moet beide die kwadrate nul wees, d.i.* $(3x-y)^2=0$ and/*en* $(x-5)^2=0$ $3x-y=0$ and/*en* $x-5=0$ $x=5$ $3(5)-y=0$ $y=15$	 ✓$3x-y=0$ ✓$x-5=0$ ✓$x=5$ ✓$y=15$ (4)

1.3

$$x^2 + x = k$$
$$x^2 + x - k = 0$$
$$\Delta < 0$$
$$b^2 - 4ac < 0$$
$$1^2 - 4(1)(-k) < 0$$
$$1 + 4k < 0$$
$$k < \frac{-1}{4}$$

✓standard form
✓ $\Delta < 0$
✓ $1^2 - 4(1)(-k)$
✓ $k < \frac{-1}{4}$

OR/*OF*

(4)

$$x^2 + x = k$$
$$x^2 + x + \frac{1}{4} = k + \frac{1}{4}$$
$$\left(x + \frac{1}{2}\right)^2 = k + \frac{1}{4}$$

for nonreal roots $k + \frac{1}{4} < 0$

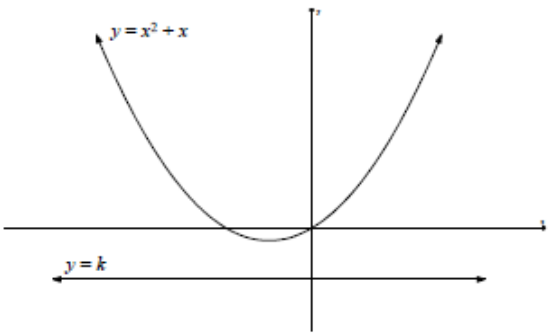
$$k < \frac{-1}{4}$$

✓adds $\frac{1}{4}$ to both sides
✓ $\left(x + \frac{1}{2}\right)^2 = k + \frac{1}{4}$
✓ $k + \frac{1}{4} < 0$
✓ $k < \frac{-1}{4}$

OR/*OF*

(4)

Consider the functions $y = x^2 + x$ and $y = k$
Beskou die funksies $y = x^2 + x$ *en* $y = k$



✓sketch or explanation

Turning point of/*Draaipunt van* $y = x^2 + x$ is $\left(\frac{-1}{2}; \frac{-1}{4}\right)$

✓ $x = -\frac{1}{2}$
✓ $y = -\frac{1}{4}$

$x^2 + x = k$ does not have real roots when the line $y = k$ does not intersect $y = x^2 + x$.
$x^2 + x = k$ *het geen reële wortels as die lyn* $y = k$ *nie met* $y = x^2 + x$ *sny nie.*

Therefore $k < \frac{-1}{4}$

✓ $k < \frac{-1}{4}$

(4)
[26]

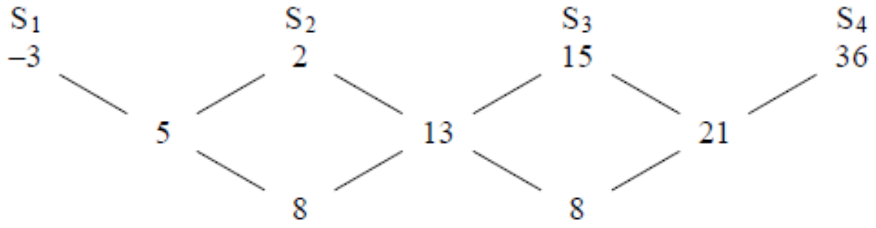
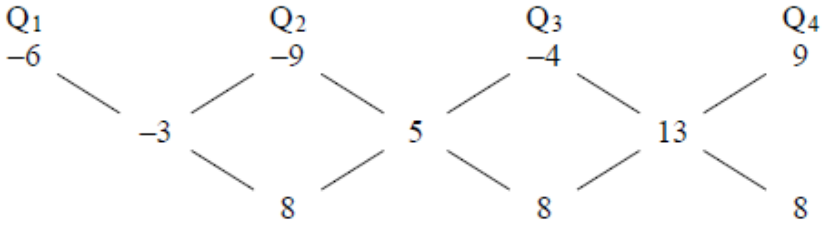
QUESTION/*VRAAG* 2

2.1	$r=\frac{T_2}{T_1}$ $=\frac{5}{10}$ $=\frac{1}{2}$ $T_5=1{,}25\left(\frac{1}{2}\right)$ **OR/*OF*** $T_5=10\left(\frac{1}{2}\right)^4$ $=\frac{5}{8}$ or 0,625 $=\frac{5}{8}$ or 0,625	✓ $r=\frac{1}{2}$ ✓answer (2)
2.2	$T_n=10\left(\frac{1}{2}\right)^{n-1}$	✓substitutes $a=10$ into GP formula ✓substitutes $r=\frac{1}{2}$ into GP formula (2)
2.3	For convergence/*Om te konvergeer* $-1<r<1$ Since/*Aangesien* $r=\frac{1}{2}$ and/*en* $-1<\frac{1}{2}<1$ the sequence converges/*die ry konvergeer*	✓ $-1<r<1$ ✓ show that $r=\frac{1}{2}$ is $-1<r<1$ (2)
2.4	$S_\infty - S_n=\frac{a}{1-r}-\frac{a\left(1-r^n\right)}{1-r}$ $=\frac{10}{1-\frac{1}{2}}-\frac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $=20-20\left(1-\frac{1}{2}^n\right)$ $=20-20+20\left(\frac{1}{2}\right)^n$ $=20\left(\frac{1}{2}\right)^n$ **OR/*OF***	✓ $\frac{10}{1-\frac{1}{2}}$ ✓ $\frac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ ✓ $20\left(1-\frac{1}{2}^n\right)$ ✓answer (4) ✓constructing the series

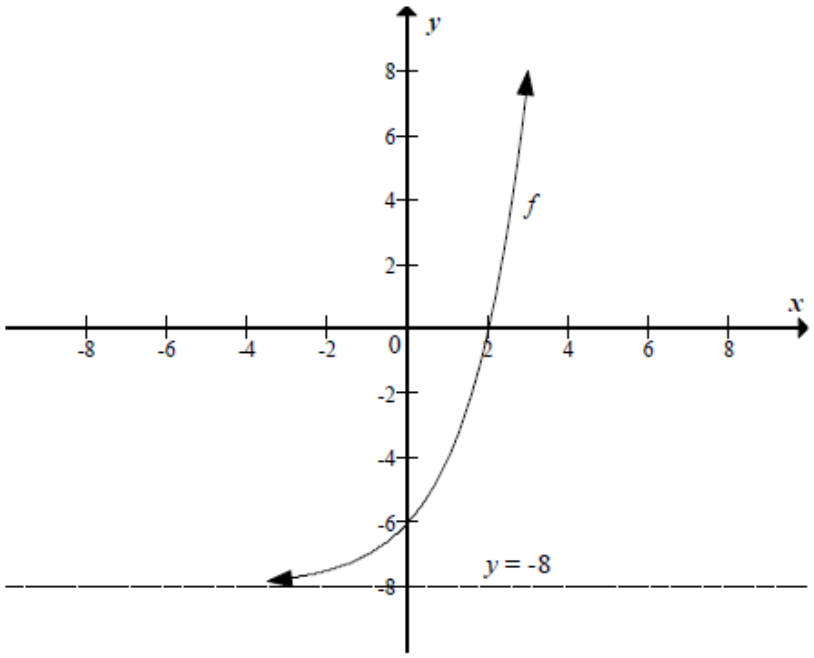
	$S_\infty - S_n = T_{n+1} + T_{n+2} + T_{n+3} + \ldots$ $= 10\left(\frac{1}{2}\right)^n\left[1+\frac{1}{2}+\frac{1}{4}+\ldots\right]$ $= 10\left(\frac{1}{2}\right)^n\left[\frac{1}{1-\frac{1}{2}}\right]$ $= 20\left(\frac{1}{2}\right)^n$ **OR/*OF*** $S_\infty - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{a-a+ar^n}{1-r}$ $= \frac{ar^n}{1-r}$ $= \frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ $= 20\left(\frac{1}{2}\right)^n$	✓ $10\left(\frac{1}{2}\right)^n\left[1+\frac{1}{2}+\frac{1}{4}+\ldots\right]$ ✓ $\frac{1}{1-\frac{1}{2}}$ ✓answer (4) ✓ $\frac{a-a+ar^n}{1-r}$ ✓ $\frac{ar^n}{1-r}$ ✓ $\frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ ✓answer (4) **[10]**

QUESTION/*VRAAG* 3

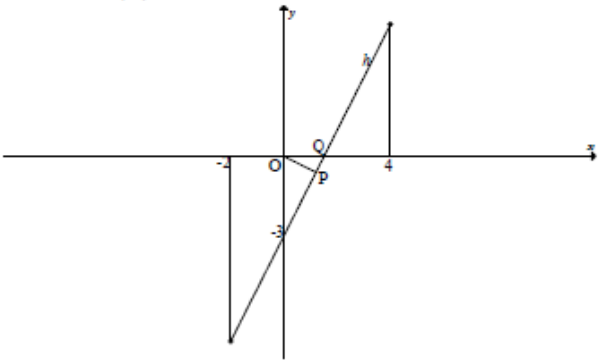
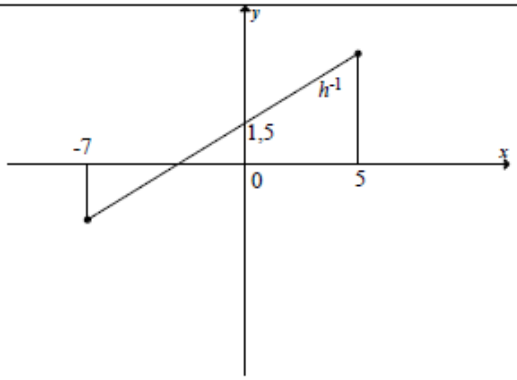
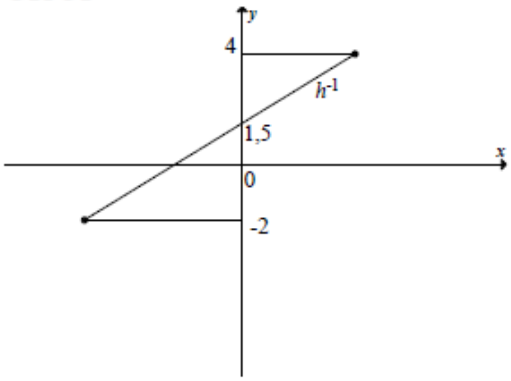
3.1	$d = 8$ $T_k = a + (k-1)d$ $= -3 + (k-1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓answer (2)
3.2	$\sum_{k=1}^{n}(8k-11)$ **OR/*OF*** $\sum_{k=0}^{n-1}(8(k+1)-11) = \sum_{k=0}^{n-1}(8k-3)$	✓for general term ✓lower and upper values in sigma notation (2)
3.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= n(4n - 7)$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[a + l]$ $= \frac{n}{2}[-3 + 8n - 11]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$	✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3)

	OR/*OF*  $S_n = an^2 + bn + c$ $a = \frac{8}{2}$ $a = 4$ $S_1 = 4 + b + c = -3$ $b + c = -7$ .....(1) $S_2 = 16 + 2b + c = 2$ $2b + c = -14$.....(2) $b = -7$.....(2) − (1) $c = 0$ Hence $S_n = 4n^2 - 7n$	$S_2 = -3 + 5 = 2$ $S_3 = 2 + 13 = 15$ $S_4 = 15 + 21$ ✓calculates S_1, S_2, S_3 and S_4. ✓ $a = 4$ ✓solves simultaneously for b and c. (3)
3.4.1	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	✓✓ answer (2)
3.4.2	$Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ **OR/*OF***  $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6$ $b + c = -10$ .....(1) $Q_2 = 16 + 2b + c = -9$ $2b + c = -25$.....(2) $b = -15$.....(2) − (1) $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\ 634$	✓✓ $-6 + 4(128)^2 - 7(128)$ ✓answer (3) ✓$a = 4$ ✓ $Q_n = 4n^2 - 15n + 5$ ✓answer (3) **[12]**

QUESTION/*VRAAG* 4

Given: $f(x)=2^{x+1}-8$		
4.1	$y=-8$	✓ $y=-8$ (1)
4.2		✓ x-intercept ✓ y-intercept ✓ shape ✓ asymptote (only if the graph does not cut the asymptote) (4)
4.3	$g(x)=2^{-x+1}-8$ **OR/*OF*** $g(x)=\left(\frac{1}{2}\right)^{x-1}-8$	✓ answer (1) ✓ answer (1) **[6]**

QUESTION/*VRAAG* 5

Given $h(x) = 2x - 3$ for $-2 \le x \le 4$. 		
5.1	For x-intercepts, $y = 0$ $2x - 3 = 0$ $x = 1{,}5$ $Q(1{,}5\ ;\ 0)$	✓ $x = 1{,}5$ ✓ $y = 0$ (2)
5.2	h: $x = -2$: $y = 2(-2) - 3 = -7$ $x = 4$: $y = 2(4) - 3 = 5$ Domain of h^{-1}: $-7 \le x \le 5$ **OR/*OF*** $[-7;\ 5]$	✓ $h(-2) = -7$ ✓ $h(4) = 5$ ✓ $-7 \le x \le 5$ (3)
5.3	 **OR/*OF*** 	✓ y-intercept on a straight line ✓ line segment ✓ accurate endpoints (x or y or both) (3)

5.4	$h(x)=2x-3$	
	For the inverse of h,	
	$x=2y-3$	
	$y=\frac{x+3}{2}$	✓ $y=\frac{x+3}{2}$
	$h^{-1}(x)=\frac{x+3}{2}$	
	$h(x)=h^{-1}(x)$	
	$2x-3=\frac{x+3}{2}$	✓ $2x-3=\frac{x+3}{2}$
	$4x-6=x+3$	
	$x=3$	✓ $x=3$
		(3)
	OR/*OF*	
	$h(x)=2x-3$	
	h and h^{-1} intersect when $y=x$	
	$h(x)=x$	✓ $h(x)=x$
	$2x-3=x$	✓ $2x-3=x$
	$x=3$	✓ $x=3$
		(3)
	OR/*OF*	
	$h(x)=2x-3$	
	For the inverse of h,	✓ $y=\frac{x+3}{2}$
	$x=2y-3$	
	$y=\frac{x+3}{2}$	
	$h^{-1}(x)=x$	✓ $\frac{x+3}{2}=x$
	$\frac{x+3}{2}=x$	
	$x+3=2x$	✓ $x=3$
	$x=3$	
		(3)

5.5	$OP^2 = (x-0)^2 + (y-0)^2$ $= x^2 + (2x-3)^2$ $= x^2 + 4x^2 - 12x + 9$ $= 5x^2 - 12x + 9$ For OP to be at its minimum, OP^2 has to be a minimum *Vir OP om minimum te wees, moet OP^2 'n minimum wees* $\frac{d(OP^2)}{dx} = 0$ **OR** / *OF* $x = -\frac{b}{2a}$ $10x - 12 = 0$ $= -\frac{-12}{2(5)}$ $\therefore x = \frac{6}{5}$ Minimum length of OP $= \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9} = \sqrt{\frac{9}{5}}$ or $\frac{3}{\sqrt{5}}$ or 1,34 units **OR/*OF*** For minimum distance OP $\perp$ the line $m_h = 2$ (given) $m_{OP} = \frac{-1}{2}$ $\therefore$ OP has equation $y = \frac{-1}{2}x$ $\frac{-1}{2}x = 2x - 3$ $-x = 4x - 6$ $5x = 6$ $x_P = 1,2$ $y_P = -\frac{1}{2}(1,2) = -0,6$ $OP = \sqrt{(1,2-0)^2 + (-0,6-0)^2}$ $= 1,34$ or $\sqrt{1,8}$ units	✓ $OP^2 = x^2 + y^2$ ✓ substitute $y = 2x - 3$ ✓ $5x^2 - 12x + 9$ ✓ x-value ✓ answer (5) ✓ $m_{OP} = \frac{-1}{2}$ ✓ equation of OP ✓ $\frac{-1}{2}x = 2x - 3$ ✓ x-value ✓ answer (5)

OR/*OF*

For minimum distance OP ⊥ the line

$O(0;0)\quad P(x;\,2x-3)\quad Q\left(\frac{3}{2};0\right)$

✓ $OP^2 = x^2 + y^2$

✓ substitute $y = 2x - 3$

$$OP^2 + PQ^2 = OQ^2 \quad \text{(pythag)}$$

$$(x-0)^2 + (2x-3-0)^2 + \left(x-\frac{3}{2}\right)^2 + (2x-3-0)^2 = \left(\frac{3}{2}\right)^2$$

✓ $10x^2 - 27x + 18$

$$x^2 + 4x^2 - 12x + 9 + x^2 - 3x + \frac{9}{4} + 4x^2 - 12x + 9 = \frac{9}{4}$$

$$10x^2 - 27x + 18 = 0$$

$$(5x-6)(2x-3) = 0$$

✓ x-value

$$x = \frac{6}{5} \text{ or } \frac{3}{2}$$

Hence, $x = \frac{6}{5}$ at P

$$OP^2 = x^2 + (2x-3)^2$$

$$= \left(\frac{6}{5}\right)^2 + \left(2\left(\frac{6}{5}\right) - 3\right)^2$$

✓ answer (5)

$$= \frac{36}{25} + \frac{9}{25}$$

$$= \frac{9}{5}$$

$$OP = 1{,}34$$

OR/*OF*

For minimum distance OP ⊥ the line

✓ $\tan\hat{Q} = 2$

✓ $\hat{Q} = 63{,}43^\circ$

✓ $\sin 63{,}43^\circ$

✓ $\frac{OP}{1{,}5}$

✓ answer (5)

$$\tan\hat{Q} = 2$$

$$\hat{Q} = 63{,}43^\circ$$

$$\sin 63{,}43^\circ = \frac{OP}{1{,}5}$$

$$OP = 1{,}34$$

OR/*OF*

$$OP = \sqrt{(x-0)^2 + (y-0)^2}$$ ✓ $OP = \sqrt{(x-0)^2 + (y-0)^2}$

$$= \sqrt{(x-0)^2 + (2x-3-0)^2}$$ ✓ substitute $y = 2x - 3$

$$= \sqrt{x^2 + 4x^2 - 12x + 9}$$

$$= \sqrt{5x^2 - 12x + 9}$$ ✓ $5x^2 - 12x + 9$

By using the chain rule (which is not in the CAPS):

$$\frac{dOP}{dx} = \frac{1}{2}\left(5x^2 - 12x + 9\right)^{\frac{-1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}\left(5x^2 - 12x + 9\right)^{\frac{-1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}(10x - 12)$$

$$0 = 5x - 6$$

$$x = \frac{6}{5}$$ ✓ x-value

$$OP = \sqrt{5\left(\frac{5}{6}\right)^2 - 12\left(\frac{6}{5}\right) + 9}$$

$$= 1{,}34$$ ✓ answer

(5)

OR/*OF*

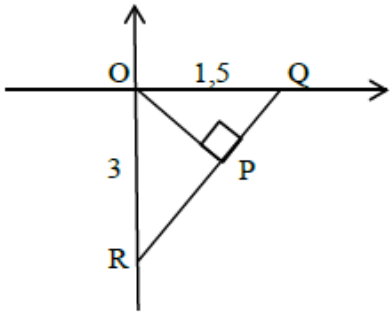
For minimum distance OP ⊥ the line

Let the y-intercept be R

OR = 3 units

OQ = $\frac{3}{2}$ units

RQ = $\frac{3}{2}\sqrt{5}$ (Pythagoras) ✓ RQ = $\frac{3}{2}\sqrt{5}$



Area OQR = $\frac{1}{2}$ × base × ⊥height

$$\frac{1}{2}.OR.OQ = \frac{1}{2}.\left(\frac{3}{2}\sqrt{5}\right).OP$$ ✓ $\frac{1}{2}\left(\frac{3}{2}\sqrt{5}\right).OP$

$$\frac{1}{2}.3.\left(\frac{3}{2}\right) = \frac{1}{2}.\left(\frac{3}{2}\sqrt{5}\right).OP$$ ✓ $\frac{1}{2}.3.\left(\frac{3}{2}\right)$

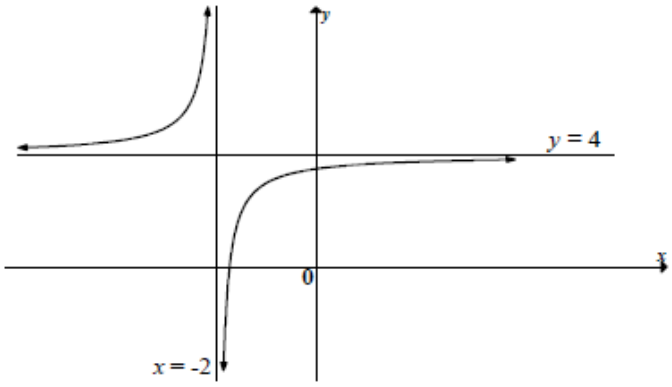
✓ equating

$$OP = \frac{3}{\sqrt{5}} = 1{,}34$$ ✓ answer

(5)

5.6.1	$f'(x)=2x-3$ Turning point at $x=\frac{3}{2}$ $f''(x)=2>0$ or $f''\left(\frac{3}{2}\right)>0$ f has a local minimum at $x=\frac{3}{2}$ *f het 'n lokale minimum by* $x=\frac{3}{2}$ **OR/*OF*** $h(x)=f'(x)<0$ for $x\in(-2\ ;\ 1{,}5)\Rightarrow f$ is decreasing on the left of Q / *f is dalend links van Q.* $h(x)=f'(x)>0$ for $x\in(1{,}5\ ;\ 4)\Rightarrow f$ is increasing on the right of Q / *f is stygend regs van Q.* $\therefore f(x)$ has a local minimum when $x=\frac{3}{2}$ / *$\therefore f(x)$ het 'n lokael minimum by* $x=\frac{3}{2}$ **OR/*OF*** $f(x)=x^2-3x+c$ f has a minimum value since $a>0$ *f het 'n minimum waarde omdat $a>0$*	✓ Turning point at $x=\frac{3}{2}$ ✓ $f''(x)=2>0$ (2) ✓decreasing left of Q ✓increasing right of Q (2) ✓ $f(x)=x^2-3x+c$ ✓ explanation (2)
5.6.2	$m=f'(4)=h(4)=5$	✓answer (1) **[19]**

QUESTION/*VRAAG 6*

6.1.1	$T(0;18)$	✓ (0;18) (1)
6.1.2	$-2x^2+18=0$ $(x-3)(x+3)=0$ Q(3 ; 0) **OR/*OF*** $-2x^2+18=0$ $x^2=9$ Q(3 ; 0)	✓$y=0$ ✓factors ✓$x=3$ (3) ✓ $y=0$ ✓ $x^2=9$ ✓ $x=3$ (3)
6.1.3	x-coordinate of S is 4,5/*x-koördinaat van S is 4,5* By symmetry about the line $x=4{,}5$/*Deur simmetrie om die lyn $x=4{,}5$:* R = (4,5 + 4,5–3 ; 0) = (6 ; 0)	✓ $x=6$ ✓ $y=0$ (2)
6.1.4	For all $x \in \mathbf{R}$ **OR/*OF*** $(-\infty;\infty)$	✓✓answer (2)
6.2	If C $(x;y)$is the centre of the hyperbola/*As C $(x;y)$die middelpunt is van die hiperbool* $y=x+6$ and $x=-2$ $\therefore y=-2+6=4$  	✓✓ asymptote $y=4$ ✓asymptote $x=-2$ ✓shape (increasing hyperbolic function) (4) **[12]**

QUESTION/*VRAAG* 7

7.1	R450 000	✓answer (1)
7.2	$A = P(1-i)^n$ $f(x) = 450000(1-i)^x$ $243\ 736{,}90 = 450000(1-i)^4$ $i = 1 - \sqrt[4]{\frac{243\ 736{,}90}{450000}}$ $i = 0{,}1421$ The rate of depreciation is 14,21 % p.a. *Die waardeverminderingskoers is 14,21% p.j.*	✓substitution of 450 000 into correct formula ✓substitution of (4; 243 736,90) into correct formula ✓making i the subject ✓answer (4)
7.3	At T : $A = P(1+i)^n$ $g(x) = 450000(1+i)^x$ $a = 450000(1+0{,}081)^4$ $= R614490{,}66$	✓$i = 0{,}081$ & $n = 4$ ✓correct substitution into formula ✓answer (3)
7.4	Future Value = R614 490, 66 – R243 736 ,90 = R370 753,76 Let x be the value of monthly payment $F_v = \frac{x\left[(1+i)^n - 1\right]}{i}$ $370753{,}76 = \frac{x\left[\left(1+\frac{0.062}{12}\right)^{36} - 1\right]}{\frac{0.062}{12}}$ $x = R9397{,}11$	✓R370 753,76 ✓$i = \frac{0{,}062}{12}$ ✓$n = 36$ ✓substitution into correct formula ✓answer (5) **[13]**

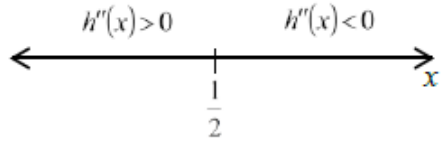
QUESTION/*VRAAG* 8

8.1	$f(x+h)=(x+h)^2-3(x+h)$ $=x^2+2xh+h^2-3x-3h$ $f(x+h)-f(x)=x^2+2xh+h^2-3x-3h-(x^2-3x)$ $=2xh+h^2-3h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$	✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓formula ✓factorisation ✓answer (5) ✓formula ✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓factorisation ✓answer (5)
8.2.1	$y=\left(x^2-\frac{1}{x^2}\right)^2$ $y=x^4-2+\frac{1}{x^4}$ $=x^4-2+x^{-4}$ $\frac{dy}{dx}=4x^3-4x^{-5}$ **OR/*OF***	✓ $x^4-2+\frac{1}{x^4}$ ✓ $4x^3$ ✓ $-4x^{-5}$ (3)

	By using the chain rule (which is not part of CAPS): $y = \left(x^2 - x^{-2}\right)^2$ $\frac{dy}{dx} = 2\left(x^2 - x^{-2}\right)\left(2x + 2x^{-3}\right)$ $= 2\left(2x^3 + 2x^{-1} - 2x^{-1} - 2x^{-5}\right)$ $= 2\left(2x^3 - 2x^{-5}\right)$ $= 4x^3 - 4x^{-5}$	✓✓✓ $2(x^2 - x^{-2})(2x + 2x^{-3})$ (3)
8.2.2	$D_x\left[\frac{(x-1)(x^2+x+1)}{x-1}\right]$ $= D_x\left[x^2 + x + 1\right]$ $= 2x + 1$ **OR/*OF*** By using the quotient rule (with is not part of CAPS): $D_x\left[\frac{x^3-1}{x-1}\right]$ $= \frac{3x^2(x-1) - (x^3-1)}{(x-1)^2}$	✓factorisation ✓ $x^2 + x + 1$ ✓ $2x + 1$ (3) ✓✓✓ $\frac{3x^2(x-1) - (x^3-1)}{(x-1)^2}$ (3) **[11]**

QUESTION/*VRAAG* 9

9.1	Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$ .....line 1 $h'(x) = -3x^2 + 2ax + b$ At Q : $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$ .....line 2 line 2 – line 1 : $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1 : $b = 6$	✓substitute Q into h ✓finding derivative ✓ $h'(2)$ ✓equating derivative to 0 ✓solving simultaneously for a and b (5)
9.2	$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3,5$ Average gradient/*Gemiddelde gradiënt* $= \frac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/ *Gemiddelde gradiënt* $= \frac{10 - (-3,5)}{2 - (-1)}$ $= 4,5$	✓ $f(-1) = -3,5$ ✓formula ✓substitution ✓answer (4)

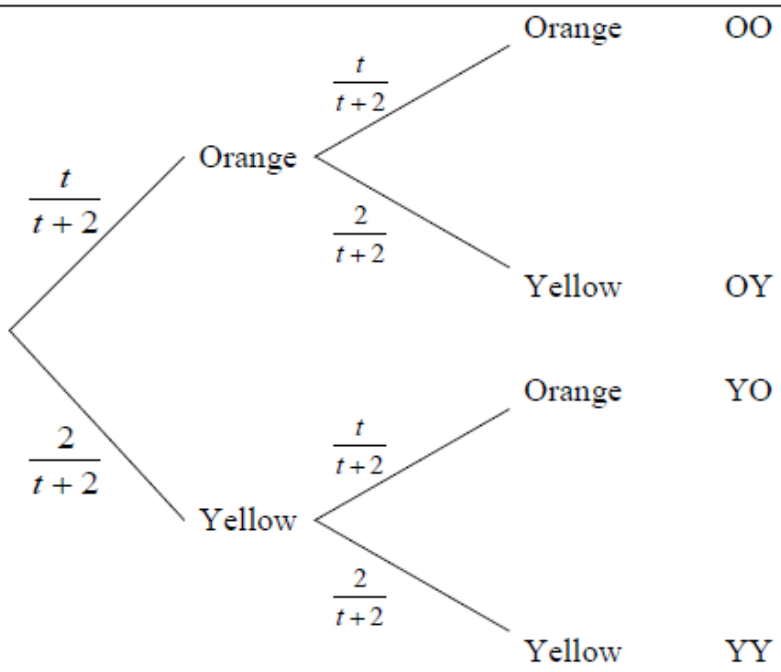
9.3	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $= -3(2x - 1)$  For $x < \frac{1}{2}$, h is concave up and for $x > \frac{1}{2}$, h is concave down *Vir* $x < \frac{1}{2}$,*is* h *konkaaf na bo en vir* $x > \frac{1}{2}$,*is* h *konkaaf na onder* $\therefore$ concavity changes at $x = \frac{1}{2}$/ $\therefore$ *konkwiteit verander by* $x = \frac{1}{2}$	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h''(x) = -6x + 3$ ✓explanation using $h''(x)$ (3)
9.4	The graph of h has a point of inflection at $x = \frac{1}{2}$ / *Die grafiek van h het 'n buigpunt by* $x = \frac{1}{2}$. **OR/*OF*** The graph of h changes from concave up to concave down at $x = \frac{1}{2}$ / *Die grafiek van h verander by* $x = \frac{1}{2}$ *van konkaaf op na konkaaf af*	✓answer (1) ✓answer (1)
9.5	Gradient of g is -12/*Gradiënt van g is –12* Gradient of tangent is/*Gradiënt van die raaklyn is*: $h'(x) = -3x^2 + 3x + 6$ $h'(x) = -12$ $-3x^2 + 3x + 6 = -12$ $3x^2 - 3x + 18 = 0$ $x^2 - x + 6 = 0$ $(x - 3)(x + 2) = 0$ $x = -2$ only	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h'(x) = -12$ ✓factors ✓selection of x-value (4) **[17]**

QUESTION/*VRAAG* 10

10.1	$\frac{h}{r} = \tan 60^0$ $r = \frac{h}{\tan 60^0}$ $\therefore r = \frac{h}{\sqrt{3}}$	✓ $\frac{h}{r} = \tan 60^0$ ✓ answer (2)
10.2	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$ $= \frac{1}{9}\pi h^3$ $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ $\left.\frac{dV}{dh}\right\|_{h=9} = \frac{1}{3}\pi(9)^2$ $= 27\pi$ or 84,82 cm^3/cm	✓ formula ✓ substitution of the value of r in terms of h ✓ simplified volume answer ✓ derivative ✓ answer (5) [7]

QUESTION/*VRAAG* 11

11.1	P(A) × P(B) = 0,2 × 0,63 = 0,126 i.e. P(A) × P(B) = P(A and B) Therefore A and B are independent/*Dus is A en B onafhanklik*	✓ 0,2 × 0,63 ✓ P(A) × P(B)=P(A and B) ✓ conclusion (3)
11.2.1	$7^7 = 823\,543$	✓✓ 7^7 (2)
11.2.2	7!=5040	✓✓ 7! (2)
11.2.3	There are 3 vowels ⇒ 3 options for first position There are 4 consonants ⇒ 4 options for last position The remaining 5 letters can be arranged in $5\times4\times3\times2\times1$ ways $3\times(5\times4\times3\times2\times1)\times4 = 1440$ *Daar is 3 klinkers ⇒ 3 opsies vir die eerste posisie* *Daar is 4 konsonante ⇒ 4 opsies vir die laaste posisie* *Die oorblywende 5 letters kan as volg gerangskik word* $5\times4\times3\times2\times1$ ways/*maniere* $3\times(5\times4\times3\times2\times1)\times4 = 1440$	✓ ×3 ✓ ×4 ✓ $5\times4\times3\times2\times1$ ✓ answer (4)

11.3	 $\text{P(Orange, Orange)} + \text{P(Yellow, Yellow)} = \frac{52}{100}$ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$ $\frac{t^2}{t^2+4t+4}+\frac{4}{t^2+4t+4}=\frac{13}{25}$ $25\left(t^2+4\right)=13\left(t^2+4t+4\right)$ $3t^2-13t+12=0$ $(3t-4)(t-3)=0$ $t=3$ There are 3 orange balls in the bag/*Daar is 3 oranje balle in die sak*	✓ $\text{P(O)} = \left(\frac{t}{t+2}\right)$ ✓ $\text{P(Y)} = \left(\frac{2}{t+2}\right)$ ✓ $\text{P(O,O)} = \left(\frac{t}{t+2}\right)^2$ ✓ $\text{P(Y,Y)} = \left(\frac{2}{t+2}\right)^2$ ✓ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$ ✓ $t = 3$ (no ca) (6) [17]
	TOTAL/TOTAAL:	**150 marks**

MATHEMATICS P2/*WISKUNDE V2*

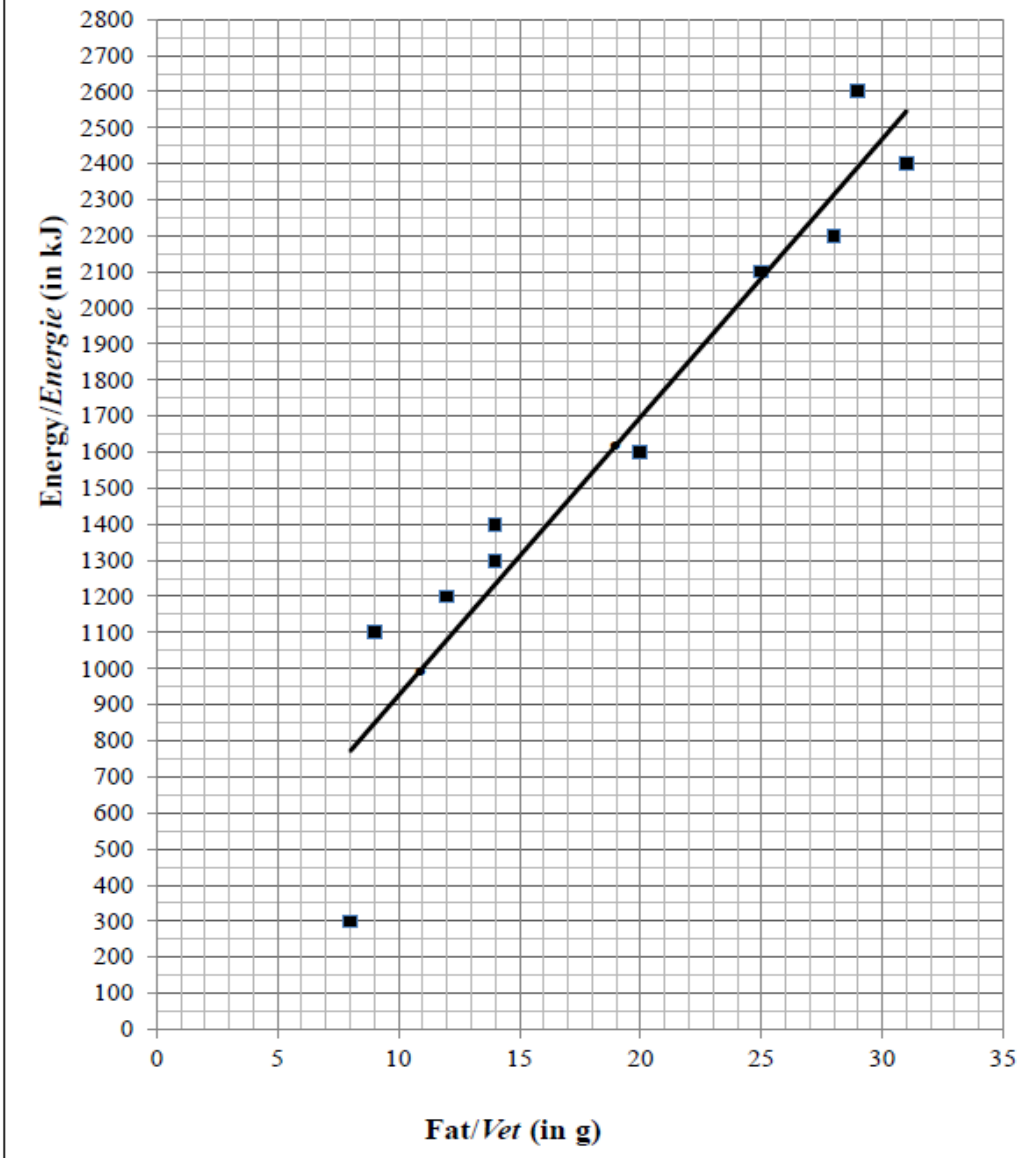
NOVEMBER 2015

MEMORANDUM

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1



1.1
no marks:
0 – 2 points correctly

✓plotting 3 – 5 points correctly

✓✓plotting 6 – 9points correctly

✓✓✓plotting all 10 points correctly

geen punte:
0 – 2 punte korrek

✓ *stip 3 – 5 pte korrek*

✓✓*stip 6 –9 pte korrek*

✓✓✓ *stip al 10 pte korrek*

(3)

1.2.2

1.2.2
✓ *y* – int close to (0 ; 150)
✓ one pt close to (25 ; 2100) or (20 ; 1700)
(2)

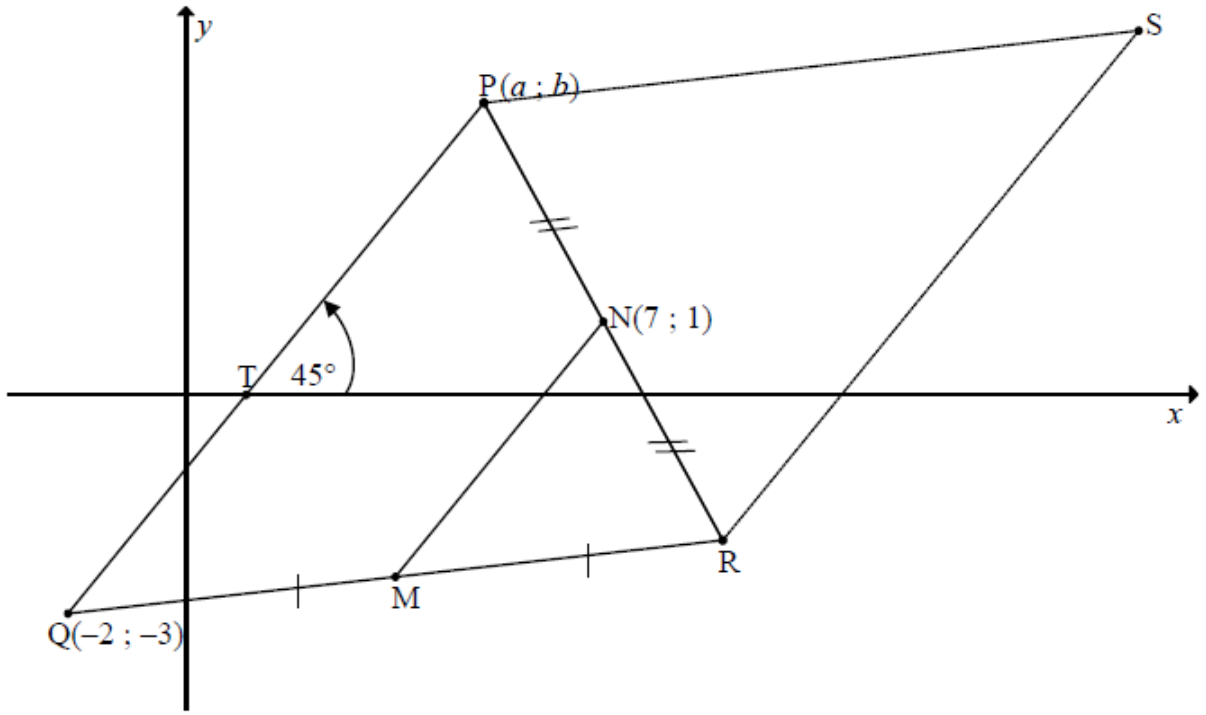
1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\,542{,}94 \approx 1\,500$ kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	$r = 0{,}9520... \approx 0{,}95$	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ *Som van die waardes op boonste vlakke*	**Frequency/ *Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1	mean/*gemiddelde* $= \frac{2(0)+3(3)+4(2)+...12(1)}{30} = \frac{202}{30}$ $= 6{,}73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $= \frac{T_{15}+T_{16}}{2} = \frac{7+7}{2} = 7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* = 2,264... ≈ 2,26	✓✓ answ/*antw* (2)
2.4	(6,73 – 2,26 ; 6,73 + 2,26) = (4,47 ; 8,99) ∴ 4 + 4 + 8 + 3 = 19 times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN | | QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore \quad y = x - 6$ **OR/*OF*** MN | | PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore \quad y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$MN = \frac{1}{2}PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

3.5	QN = NS [diag of ‖m/hoekl van ‖m] $\frac{-2+x_S}{2} = 7$ and/en $\frac{-3+y_S}{2} = 1$ $\therefore\ x_S = 16$ $\therefore\ y_S = 5$ **OR/*OF*** QN = NS [diag of ‖m/hoekl van ‖m] $\therefore$ by inspection/deur inspeksie: S(16 ; 5)	✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3) ✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)
3.6	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = b^2 + 6b + 9 + b^2 + 6b + 9$ $0 = 2b^2 + 12b - 80$ $0 = b^2 + 6b - 40$ $\therefore 0 = (b+10)(b-4)$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4) **OR/*OF*** Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = 2(b+3)^2$ $49 = (b+3)^2$ $\pm 7 = b + 3$ $\pm 7 - 3 = b$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ st form/*st vorm* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6) ✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6)

OR/*OF*

Equation of/*Vgl van PQ*: $y = x + c$

$-3 = -2 + c$

$y = x - 1 \qquad \therefore a = b + 1 \quad .....(1)$

From distance formula/*Van afstandsformule*:

$7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$

$98 = (a+2)^2 + (a-1+3)^2$

$= 2(a+2)^2$

$\therefore\ a + 2 = 7$ (since/*aangesien* $a > 0$)

$\therefore\ a = 5$

Subst $a = 4$ into (1):

$\therefore b = 5 - 1 = 4$

$\therefore$ P(5 ; 4)

✓ eq of/*vgl van* PQ

✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule*

✓ subst eq of/*vgl v.* PQ

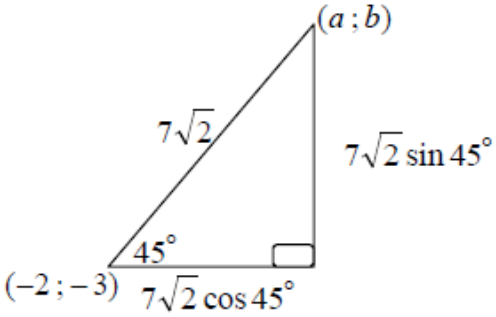
✓ simplification/ *vereenvoudig*

✓ value of/*waarde van a*

✓ value of/*waarde van b*

(6)

OR/*OF*



✓✓✓✓

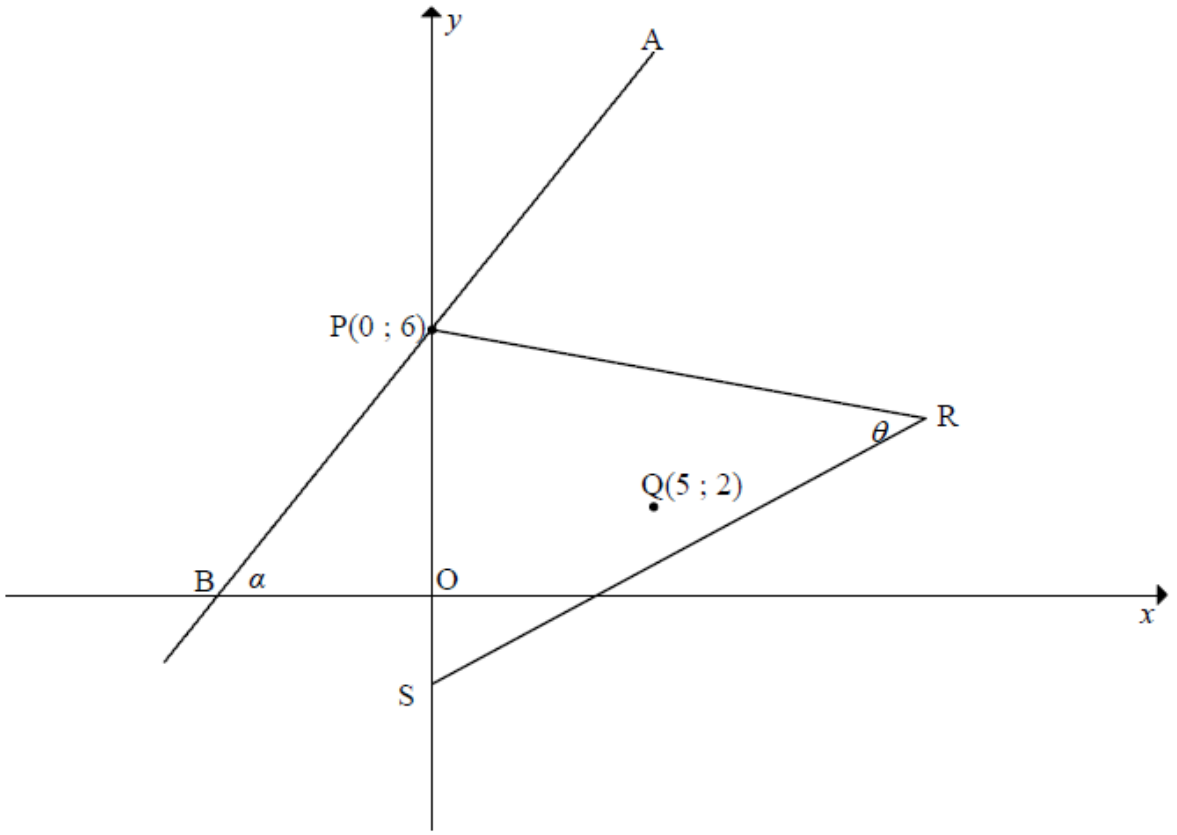
$a = -2 + 7\sqrt{2}\cos 45° = 5$ ✓

$b = -3 + 7\sqrt{2}\sin 45° = 4$ ✓

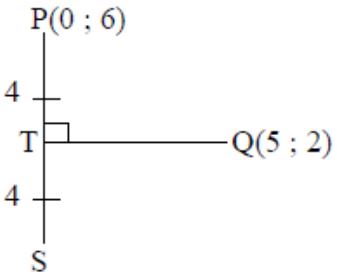
(6)

[17]

QUESTION/*VRAAG* 4

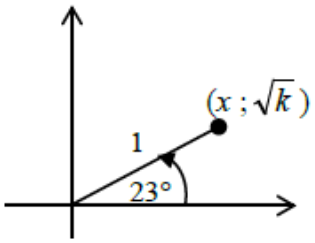


4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6$ *or / of* $y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or / *of* $y = -2$ $\therefore$ S(0 ; –2)	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3)
	OR/*OF* Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6-2 = 4$ $y_Q - y_S = 4$ $y_S = 2-4 = -2$ $\therefore$ S(0 ; –2)  	✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \frac{6-2}{0-5}$ $= -\frac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore\ m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \frac{5}{4}$ $\therefore\ \alpha = 51{,}34°$ **OR/*OF*** B(4,8 ; 0) $\therefore \tan\alpha = \frac{6}{4{,}8}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2) ✓ $\tan\alpha = \frac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$ **OR/*OF*** $PS = 8$ $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [$\angle$ at centre = 2 × $\angle$ circumf] $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}$ PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ $= 20$ sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38{,}66°$ [$\angle$at centre = 2×$\angle$ at circum/ *midpts$\angle$=2omtreks$\angle$*] $= 77{,}32°$ Area $\Delta PQS = \frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ $= 20$ sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

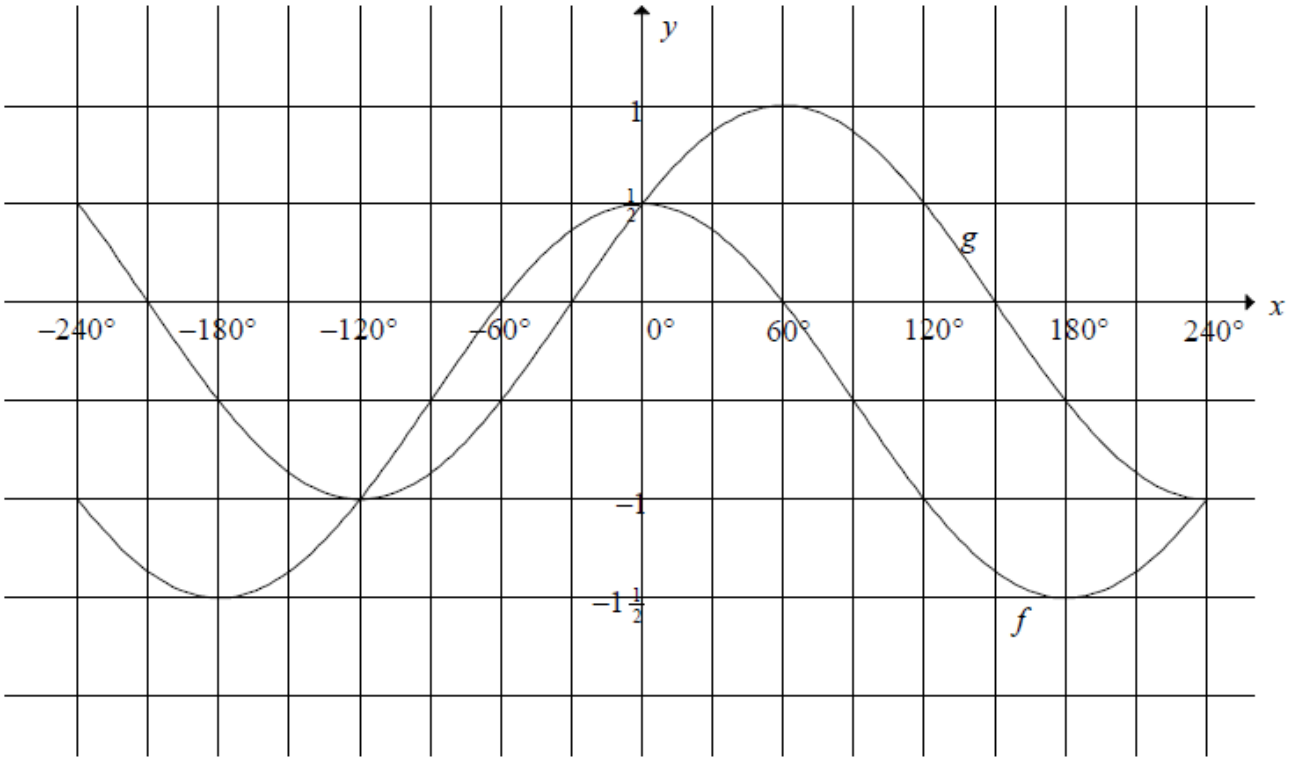
QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}$  	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3) ✓ $x^2 = 1 - k$ ✓ x ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\frac{\sin 23°}{\cos 23°}$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2) ✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\frac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \frac{-4\sin x.\cos x}{\sin 30°}$ $= \frac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

	OR/OF $\frac{4\cos x.(-\sin x)}{(\sin 30°\cos x-\cos 30°\sin x)\cos x+(\cos 30°\cos x+\sin 30°\sin x)\sin x}$ $=\frac{-4\sin x.\cos x}{(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)\cos x+(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x)\sin x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x+\sin^2 x)}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $=-8\cos x\sin x$ $=-4(2\sin x\cos x)$ $=-4\sin 2x$	✓ $\cos x$ ✓$-\sin x$ ✓ $\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$	✓ expansion/ *uitbreiding* ✓ $2\cos^2 x - 7\cos x - 4 = 0$ ✓ factors/*faktore* ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ $+ n.360°$ **OR/OF** ✓ $\pm 120°$ ✓ $+ n.360°$ (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	✓ expansion of/ *uitbreiding van* $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2\theta$ ✓ subst ✓ answ/*antw* (5) **[24]**

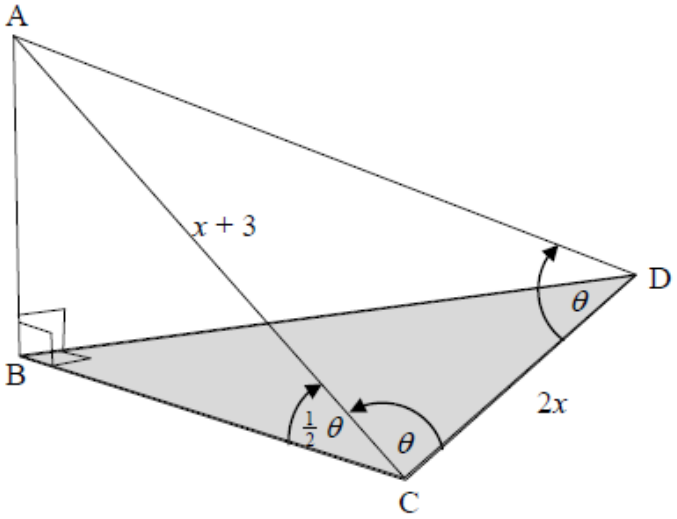
QUESTION/*VRAAG* 6

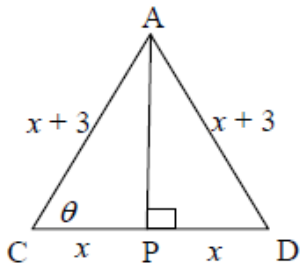


6.1	$f(x) = \cos x - \frac{1}{2}$ and/*en* $g(x) = \sin(x + 30°)$ $\therefore p = 30°$ and/*en* $q = -\frac{1}{2}$ **OR/*OF*** $\sin(60° + p) = 1$ and/*en* $\cos 0° + q = \frac{1}{2}$ $\therefore p = 30°$ $\therefore q = -\frac{1}{2}$	✓ $f(x) = \cos x - \frac{1}{2}$ ✓ $g(x) = \sin(x + 30°)$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4) ✓ $\sin(60° + p) = 1$ ✓ $\cos 0° + q = \frac{1}{2}$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4)
6.2	$x \in (-120° ; 0°)$ **OR/*OF*** $-120° < x < 0°$	✓ critical values/ *kritiese waardes* ✓ correct interval/ *korrekte interval* (2)

6.3	The graph of g has to shift 60° to the left and then be reflected about the x-axis./*Die grafiek van g moet 60° na links skuf en dan om die x-as gereflekteer word.*	✓ 60° left/*links* ✓ reflection about x-axis/*refleksie om x-as* (2)
	OR/*OF* The graph of g must be reflected about the x-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuf word.*	✓ reflection about x-axis/*refleksie om x-as* ✓ 60° left/*links* (2)
	OR/*OF* The graph of g has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuf word.*	✓ ✓ 120° right/*regs* (2)
	OR/*OF* The graph of g has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuf word*	✓ ✓ 240° left/*links* (2) **[8]**

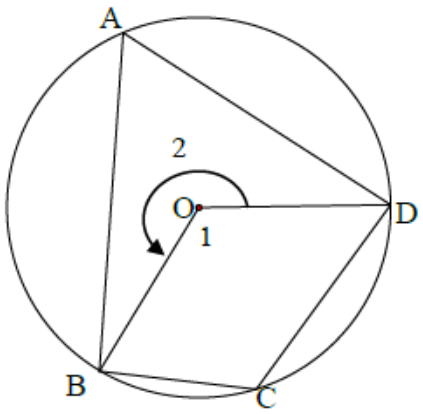
QUESTION/*VRAAG* 7



7.1	$C\hat{A}D = 180° - 2\theta$ [∠s sum of Δ/*∠e som van Δ*]	✓ answ/*antw* (1)
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$	✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ sin 2θ ✓ 2 sinθ. cos θ ✓ cos θ as subject/ *as onderwerp* (4)
	OR/*OF* AD = x + 3 [sides opp = ∠s/*sye to = ∠e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$	✓ AD = x + 3 ✓ correct subst into cosine rule/*korrekte subst in cos-reël* ✓ simplification/ *vereenvoudiging* ✓ cos θ as subject/ *as onderwerp* (4)
	OR/*OF* Draw/*Trek* AP ⊥ CD $\cos\theta = \frac{x}{x+3}$ 	✓ ✓ constr/*konstr* ✓ ✓ sketch shown/ *toon skets* (4)

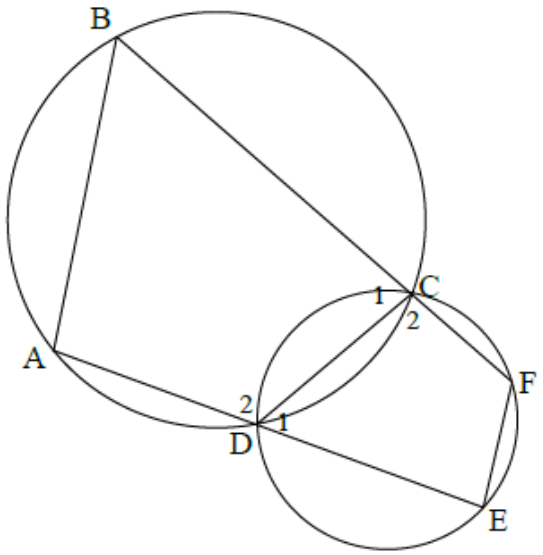
7.3	$\cos\theta = \frac{2}{5}$ $\therefore \quad \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore \quad AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5\sin\frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$	✓ $\cos\theta = \frac{2}{5}$ ✓ size of/*grootte v* θ ✓ correct ratio/ *korrekte verh* ✓ subst correctly/ *korrek* ✓ answ/*antw* (5) ✓ $AB = 5\sin\frac{\theta}{2}$ ✓ equation/*vgl* ✓ simplification/ *vereenvoudiging* ✓ value of/*waarde v* $\sin\frac{\theta}{2}$ ✓ answ/*antw* (5) **[10]**

QUESTION/*VRAAG* 8



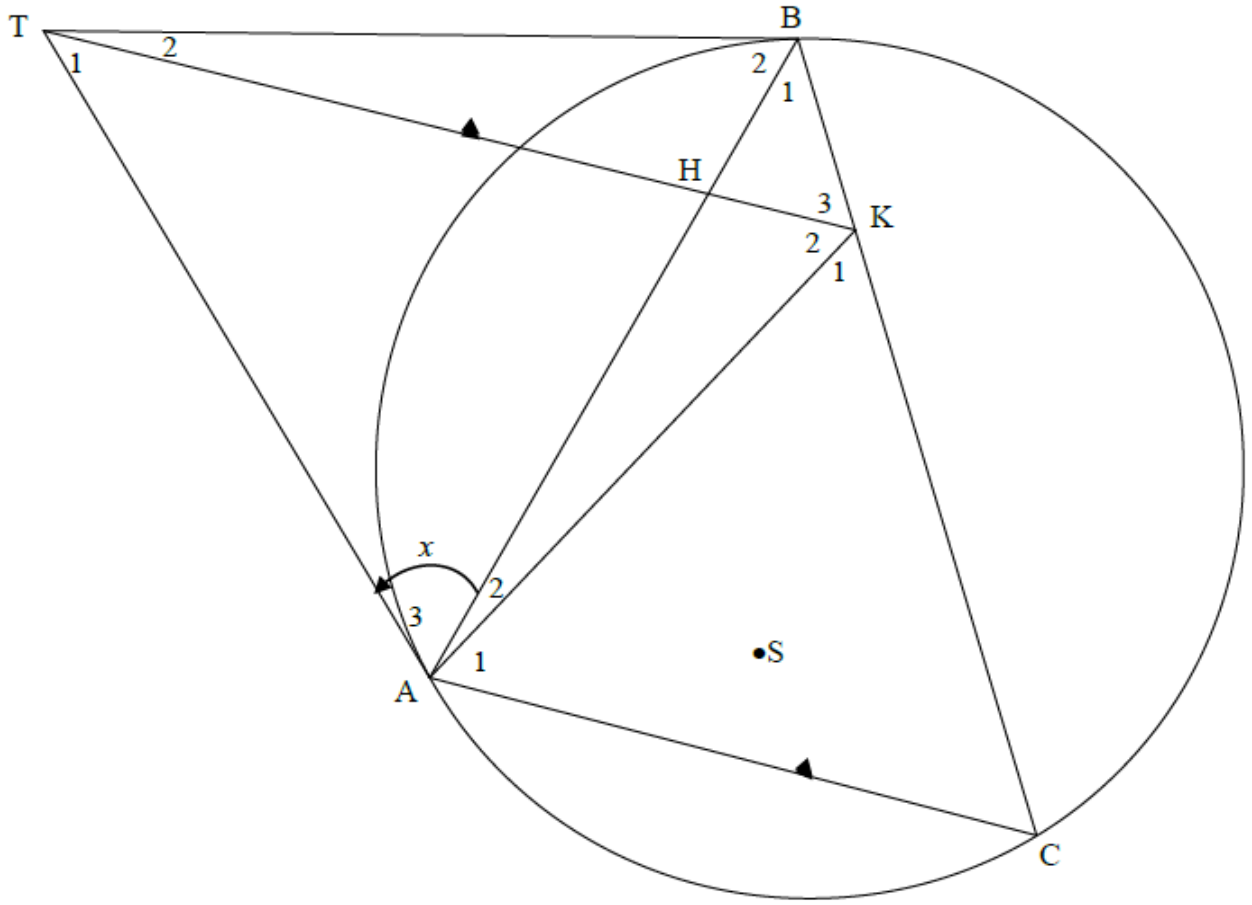
8.1.1	twice or double /*twee keer of dubbel*	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/*midpts∠=2×omtreks∠*] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/*midpts∠=2×omtreks∠*] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s in a rev/*∠e in omw of om 'n pt*] $2\hat{A} + 2\hat{C} = 360°$ $\therefore \hat{A} + \hat{C} = 180°$ **OR/OF** Let/*Gestel* $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/*midpts∠=2×omtreks∠*] $\hat{O}_2 = 360° - 2x$ [∠s in a rev/*∠e in omw of om 'n pt*] $\hat{C} = 180° - x$ [∠ at centre = 2×∠ at circ/*midpts∠=2×omtreks∠*] $\therefore \hat{A} + \hat{C} = 180°$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2



8.2	$\hat{A} = \hat{C}_2$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{E} = 180° - \hat{C}_2$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{E} = 180° - \hat{A}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5)
	OR/*OF*		
	$\hat{B} = \hat{D}_1$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{F} = 180° - \hat{D}_1$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{F} = 180° - \hat{B}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5) **[9]**

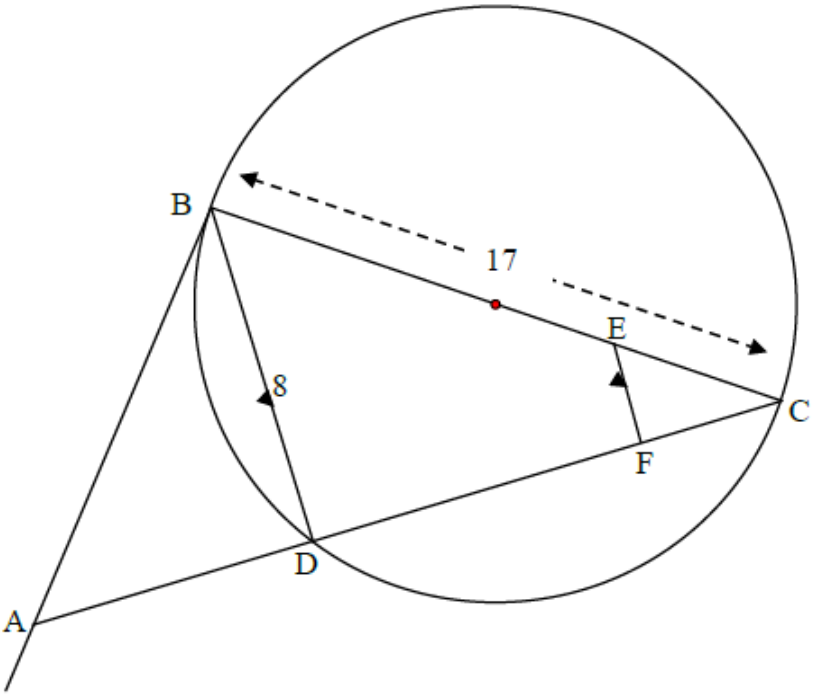
QUESTION/*VRAAG* 9



9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp ∠s/*ooreenk ∠e* ; CA| |KT] [tan-chord th/*raakl-koordst*]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ ∴ AKBT is cyc quad	[proved/*bewys* in 9.1] [line (BT) subtends equal ∠s/ *lyn (BT) onderspan gelyke ∠e*] **OR/OF** [converse ∠s in same segment/ *omgek ∠e in dies segment*]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ ∴ TK bisects/*halveer* $A\hat{K}B$ **OR/*OF*** $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/*raakl-koordst*] [∠s in the same segm/*∠e in dies segm*] [∠s in the same seg/*∠e in dies segm*] [tans from same pt; ∠s opp equal sides/ *rkle v dies pt; ∠e to gelyke sye*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

	$\therefore \quad = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/*halveer* $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/*bewys*] $\therefore$ TA tangent [converse tan chord theorem OR ∠ between line and chord/ *omgekeerde raakl-kdst **OF** ∠ tussen lyn en koord*]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/*konsiklies*] $A\hat{T}B = 180° - 2x$ [A,T,B & K concyclic/*konsiklies*] $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is ook konsiklies* [opp ∠s of quad = 180°/*tos ∠e van vierhoek=180°*] **OR/*OF*** A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. **OR/*OF*** The circle passing through points A, K and B contains the point S on the circumference (A, ,S, K and B concyclic)./*Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).* The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./*Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).* $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is konsiklies*	✓ S (both/*beide* statements/ *bewerings*) ✓ R (2) ✓ S ✓ S (2) ✓ S ✓ S (2) **[14]**

QUESTION/*VRAAG* 10



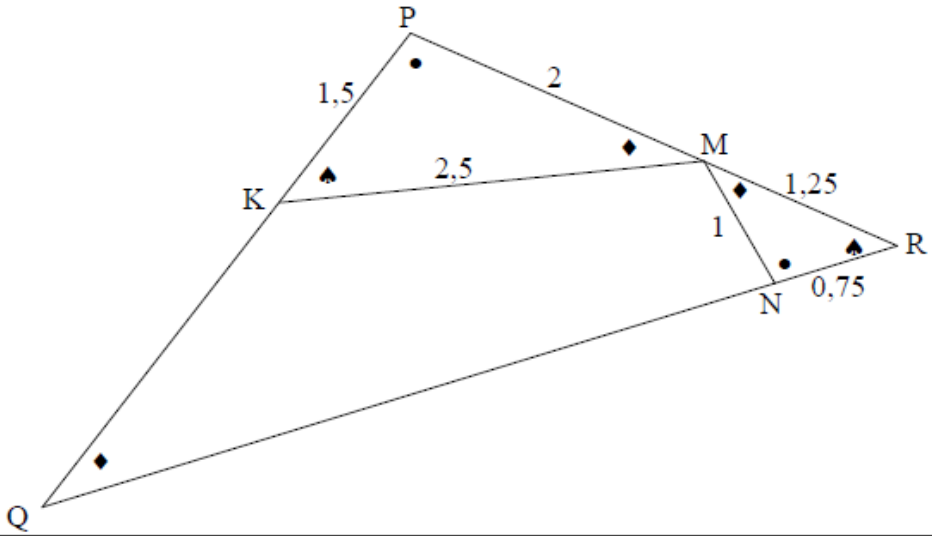
10.1	$B\hat{D}C = 90°$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$	[∠ in semi circle/∠ *in halfsirkel*] [Th of/*stelling v* Pythagoras]	✓ S ✓ using/*gebruik* Pyth *korrek*/ correctly ✓ answ/*antw* (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3{,}75$	[line || one side of Δ/*lyn* || *een sy van* Δ] **OR/OF** ΔCEF ||| ΔCBD	✓ S/R ✓ subst correctly/ *korrek* ✓ answ/*antw* (3)
10.2.2	$B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$ $\therefore$ ΔBAC ||| Δ FEC **OR/*OF*** $B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$	[∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*] [∠∠∠] [∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*]	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S

	$\hat{BAC} = \hat{FEC}$ [∠ sum in Δ/∠ *som van* Δ] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$	✓ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [ΔBAC ∣∣∣ Δ FEC] $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$	✓ length of/*lengte v* EC ✓ S ✓ subst correctly/ *korrek* ✓ answ/*antw* (4)
	OR/*OF*	
	$\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$	✓✓ correct ratios/ *korrekte verh's* ✓ subst correctly/ *korrek* ✓ answ/*antw* (4)
	OR/*OF*	
	$\Delta BCA \mid\mid\mid \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27$ or/*of* $19\frac{4}{15}$	✓ S **OR** Pyth th ✓ correct ratio ✓ subst ✓ answ/*antw* (4)
	OR/*OF*	
	$\hat{C} = A\hat{B}D$ [tan-chord theorem/*rkl-kdstelling*] $\frac{AD}{8} = \tan A\hat{B}D$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$	✓ S ✓ correct ratio ✓ subst ✓ answ/*antw* (4)

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° **OR** converse ∠ in semi circle] *AC is middellyn van die sirkel wat deur die punte A, B en C gaan* [*koord onderspan 90° **OF** omgek ∠ in halfsirkel*] $\therefore \text{radius} = \frac{1}{2} \times 19{,}27 = 9{,}63$ or/*of* $9\frac{19}{30}$ or/*of* $\frac{1}{2}$AC	✓ S/R ✓ answ/*antw* (2) **[17]**

QUESTION/*VRAAG* 11

11.1	equiangular or similar/*gelykhoekig of gelykvormig*	✓ answ/*antw* (1)



11.2.1	$\frac{KP}{RN} = \frac{1{,}5}{0{,}75} = 2$; $\frac{PM}{NM} = \frac{2}{1} = 2$; $\frac{KM}{RM} = \frac{2{,}5}{1.25} = 2$ $\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** $\frac{RN}{KP} = \frac{0{,}75}{1{,}5} = \frac{1}{2}$; $\frac{NM}{PM} = \frac{1}{2}$; $\frac{RM}{KM} = \frac{1{,}25}{2{,}5} = \frac{1}{2}$ $\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*]	✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓✓✓ all 3 statements/ *al 3 bewerings* (3)

11.2.2	$P\hat{K}M = \hat{R}$ [ΔKPM ||| ΔRNM]	✓ S
	$\therefore$ $\hat{P}$ is common/*gemeen*	
	$\therefore$ ΔRPQ ||| ΔKPM [∠∠∠]	✓ ΔRPQ|||ΔKPM
	$\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ ||| ΔKPM]	✓ S
	$\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$	✓ subst correctly/ *korrek*
	$\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42$ or $5\frac{5}{12}$	✓ $RQ = 5\frac{5}{12}$
	$\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$	✓ NQ = answ/*antw* (6)
	OR/OF	
	$R\hat{N}M = \hat{P}$ [ΔKPM ||| ΔRNM]	✓ S
	$\therefore$ $\hat{R}$ is common/*gemeen*	
	$\therefore$ ΔRNM ||| ΔRPQ [∠∠∠]	✓ ΔRNM ||| ΔRPQ
	$\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ]	✓ S
	$\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$	✓ subst correctly/ *korrek*
	$\therefore RQ = 5,42$ or $5\frac{5}{12}$	✓ $RQ = 5\frac{5}{12}$
	$\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$	✓ NQ = answ/*antw* (6)
	OR/*OF*	
	In ΔMNR:	
	$1,25^2 = 1^2 + 0,75^2 = 1,5625$	✓ S
	$\therefore$ $M\hat{N}R = 90°$ [converse Pyth theorem]	
	In ΔPKM:	
	$2,5^2 = 1,5^2 + 2^2 = 6,25$	
	$\therefore$ $\hat{P} = 90°$ [converse Pyth theorem]	
	In ΔMNR and ΔQPR	
	∠R is common	
	$M\hat{N}R = \hat{P} = 90°$	
	$\therefore$ ΔMNR ||| ΔQPR [∠∠∠]	✓ ΔMNR ||| ΔQPR
	$\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ]	✓ S
	$\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$	✓ subst correctly/ *korrek*
	$\therefore RQ = 5,42$ or $5\frac{5}{12}$	✓ $RQ = 5\frac{5}{12}$
	$\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$	✓ NQ = answ/*antw* (6) **[10]**

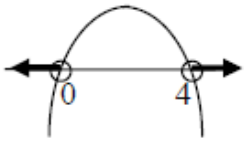
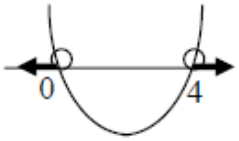
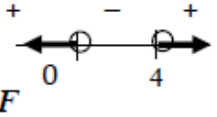
TOTAL/*TOTAAL*: 149

MATHEMATICS P1/*WISKUNDE V1*

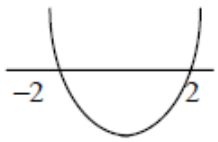
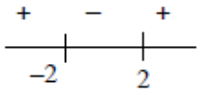
FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

MEMORANDUM

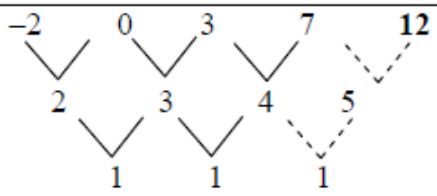
QUESTION/*VRAAG* 1

1.1.1	$x^2 - x - 12 = 0$ $(x-4)(x+3) = 0$ $x = 4 \text{ or } x = -3$ **OR/*OF*** $x^2 - x - 12 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}$ $= 4 \quad \text{or} \quad -3$	✓factors ✓✓answers (3) ✓ substitution into formula ✓✓answers (3)
1.1.2	$x(x+3) - 1 = 0$ $x^2 + 3x - 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$ $= \frac{-3 \pm \sqrt{13}}{2}$	✓standard form ✓substitution into correct formula ✓answer (3)
1.1.3	$x(4-x) < 0$ $x < 0 \text{ or } x > 4$ **OR/*OF*** $x(4-x) < 0$ $x(x-4) > 0$ $x < 0 \text{ or } x > 4$	 ✓ $x < 0$ ✓ $x > 4$ ✓or (3)  **OR/*OF***  ✓ $x < 0$ ✓ $x > 4$ ✓or (3)

1.1.4	$x = \dfrac{a^2 + a - 2}{a - 1}$ $= \dfrac{(a+2)(a-1)}{a-1}$ $= a + 2$ $= 888\,888\,888\,890$	✓ $(a+2)(a-1)$ ✓answer (check ten eights written)/*tien agtstes geskryf* (2)
1.2	$y + 7 = 2x$ $y = 2x - 7$ .....(1) $x^2 - xy + 3y^2 = 15$ substitute (1) in (2) : $x^2 - x(2x-7) + 3(2x-7)^2 = 15$ $x^2 - 2x^2 + 7x + 3(4x^2 - 28x + 49) = 15$ $x^2 - 2x^2 + 7x + 12x^2 - 84x + 147 - 15 = 0$ $11x^2 - 77x + 132 = 0$ $x^2 - 7x + 12 = 0$ $(x-3)(x-4) = 0$ $x = 3$ or $x = 4$ $y = 2(3) - 7$ $y = 2(4) - 7$ $y = -1$ $y = 1$ **OR/*OF*** $y + 7 = 2x$ $x = \dfrac{y+7}{2}$ .....(1) $x^2 - xy + 3y^2 = 15$ .....(2) substitute (1) in (2) : $\left(\dfrac{y+7}{2}\right)^2 - \left(\dfrac{y+7}{2}\right)y + 3y^2 = 15$ $\dfrac{y^2 + 14y + 49}{4} - \dfrac{y^2 + 7y}{2} + 3y^2 = 15$ $y^2 + 14y + 49 - 2y^2 - 14y + 12y^2 - 60 = 0$ $11y^2 - 11 = 0$ $y^2 - 1 = 0$ $(y-1)(y+1) = 0$ $y = -1$ $y = 1$ $x = \dfrac{-1+7}{2}$ $x = \dfrac{1+7}{2}$ $x = 3$ $x = 4$	✓ $y = 2x - 7$ ✓ substitution ✓standard form ✓factorisation ✓x-values ✓y-values (6) ✓ $x = \dfrac{y+7}{2}$ ✓ substitution ✓standard form ✓factorisation ✓y-values ✓x-values (6)

1.3	$y = x + \frac{1}{x}$ $xy = x^2 + 1$ $x^2 - xy + 1 = 0$ Since x is real, this equation has real roots./*Omdat x reëel is, het die vergelyking reële wortels.* $\Delta \geq 0$ $y^2 - 4 \geq 0$ $(y-2)(y+2) \geq 0$  **OR/** ***OF***  $y \leq -2$ or $y \geq 2$	✓ $x^2 - xy + 1 = 0$ ✓ $\Delta \geq 0$ ✓ $y^2 - 4$ ✓factors ✓ $y \leq -2$ ✓ $y \geq 2$ (6) **[23]**

QUESTION/*VRAAG* 2

2.1.1	 The next term of the sequence is 12./*Die volgende term in die ry is 12.*	✓answer (1)
2.1.2	$2a = 1$ $a = \frac{1}{2}$ $3a + b = T_2 - T_1$ $3\left(\frac{1}{2}\right) + b = 2$ $b = \frac{1}{2}$ $a + b + c = T_1$ $\frac{1}{2} + \frac{1}{2} + c = -2$ $c = -3$ $\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n - 3$ **OR/*OF***	✓value of a ✓ $3\left(\frac{1}{2}\right) + b = 2$ ✓value of b ✓ $\frac{1}{2} + \frac{1}{2} + c = -2$ ✓ value of c (5)

$2a = 1$

$a = \frac{1}{2}$ ✓value of a

$T_n = an^2 + bn + c$

$-2 = \frac{1}{2} + b + c \ldots\ldots\ldots\ldots T_1$ ✓ $-2 = \frac{1}{2} + b + c$

$b + c = -\frac{5}{2} \ldots\ldots\ldots\ldots$ line 1

$0 = 2 + 2b + c \ldots\ldots\ldots\ldots T_2$ ✓ $0 = 2 + 2b + c$

$2b + c = -2 \ldots\ldots\ldots\ldots$ line 2

line 2 – line 1:

$b = \frac{1}{2}$ ✓value of b

substitute in line 1 or substitute in line 2

$\frac{1}{2} + c = -\frac{5}{2}$ $2\left(\frac{1}{2}\right) + c = -2$

$c = -3$ ✓ value of c

$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n - 3$ (5)

OR/*OF*

$T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$

$= -2 + (n-1)(2) + \frac{(n-1)(n-2)}{2}(1)$ ✓formula

$= -2 + 2n - 2 + (n^2 - 3n + 2)(\frac{1}{2})$ ✓substitution

$= -2 + 2n - 2 + \frac{1}{2}n^2 - \frac{3}{2}n + 1$

$= \frac{1}{2}n^2 + \frac{1}{2}n - 3$ ✓ value of a
✓value of b
✓ value of c

OR/*OF* (5)

$2a = 1$

$a = \frac{1}{2}$ ✓value of a

$3a + b = T_2 - T_1$

$3\left(\frac{1}{2}\right) + b = 2$ ✓ $3\left(\frac{1}{2}\right) + b = 2$

$b = \frac{1}{2}$ ✓value of b

$T_0 = c = -3$ ✓ $T_0 = c$

$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n - 3$ ✓ value of c

OR/*OF* (5)

	Since $T_2 = 0, (n-2)$ is a factor of T_n $T_n = an^2 + bn + c$ $= a(n-2)(n-k)$ $T_1 = -2 = a(1-2)(1-k)$ $-2 = -a(1-k)$ $a = \frac{2}{1-k}$ $T_3 = 3 = a(3-2)(3-k)$ $3 = a(3-k)$ $a = \frac{3}{3-k}$ $\frac{2}{1-k} = \frac{3}{3-k}$ $2(3-k) = 3(1-k)$ $6-2k = 3-3k$ $k = -3$ $a = \frac{1}{2}$ $T_n = \frac{1}{2}(n-2)(n+3)$ $= \frac{1}{2}n^2 + \frac{1}{2}n - 3$	✓ $T_n = a(n-2)(n-k)$ ✓ $-2 = a(1-2)(1-k)$ ✓ $3 = a(3-2)(3-k)$ ✓value of k ✓value of a (5)
2.1.3	$\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ $n^2 + n - 6 = 644$ $n^2 + n - 650 = 0$ $n = \frac{-1 \pm \sqrt{1^2 - 4(1)(650)}}{2}$ $n = 25$ or $n = -26$ The 25th term has a value of 322./*Die 25ste term se waarde is 322.* **OR/*OF***	✓ $\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ ✓standard form ✓substitution into quadratic formula ✓answer (4)
	$\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ $n^2 + n - 6 = 644$ $n^2 + n - 650 = 0$ $(n-25)(n+26) = 0$ $n = 25$ or $n = -26$ The 25th term has a value of 322./*Die 25ste term se waarde is 322.* **OR/*OF***	✓ $\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ ✓standard form ✓factors ✓answer (4)

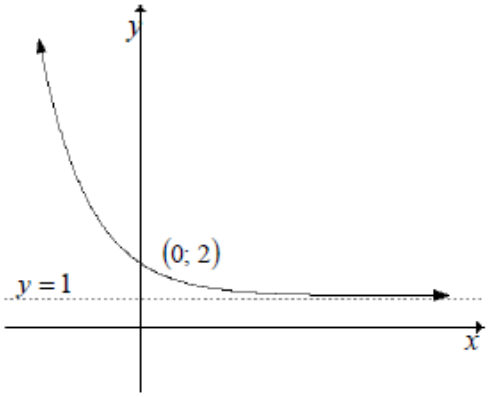
	$\frac{1}{2}n^2+\frac{1}{2}n-3=322$ $n^2+n-6=644$ $(n+3)(n-2)=23\times 28$ $n-2=23$ $n=25$	✓ $\frac{1}{2}n^2+\frac{1}{2}n-3=322$ ✓ $(n+3)(n-2)$ ✓ 23×28 ✓ answer (4)
2.2.1	$T_2: \quad a+d=8$ $T_5: \quad a+4d=10$ $T_5-T_2: \quad 3d=2$ $d=\frac{2}{3}$	✓ $a+d=8$ ✓ $a+4d=10$ ✓ answer (3)
2.2.2	$T_1=T_2-d$ $=8-\frac{2}{3}$ $=\frac{22}{3}$ $T_n=a+(n-1)d$ $=\frac{22}{3}+(n-1)\frac{2}{3}$ $=\frac{2n+20}{3}$ $S_{50}=\sum_{n=1}^{50}\left(\frac{22}{3}+(n-1)\frac{2}{3}\right)$ **OR/*OF*** $S_{50}=\sum_{n=1}^{50}\left(\frac{2n+20}{3}\right)$	✓ $T_1=\frac{22}{3}$ ✓ answer (2) (2)
2.2.3	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{50}=\frac{50}{2}\left[2\left(\frac{22}{3}\right)+(50-1)\left(\frac{2}{3}\right)\right]$ $=\frac{3550}{3}$	✓ correct substitution into correct formula ✓✓ answer (3) **[18]**

QUESTION/*VRAAG* 3

3.1	$r = \frac{70}{100}$ $= \frac{7}{10}$	✓value of r
	$T_n = ar^{n-1}$ $11{,}76 = 100\left(\frac{7}{10}\right)^{n-1}$	✓substitution in formula for T_n
	$\left(\frac{7}{10}\right)^{n-1} = \frac{11{,}76}{100}$ $n-1 = \log_{\frac{7}{10}}\left(\frac{11{,}76}{100}\right)$	✓use of logarithms
	$n-1 = 6$ $n = 7$ During the 7th year/*In die 7de jaar*	✓answer (4)
	OR/*OF*	
	$r = \frac{70}{100}$ $= \frac{7}{10}$	✓value of r
	$T_n = ar^{n-1}$ $11{,}76 = 100(0{,}7)^{n-1}$ $0{,}7^{n-1} = \frac{11{,}76}{100}$ $= 0{,}1176$	✓substitution in formula for T_n
	$(n-1)\log 0{,}7 = \log 0{,}1176$ $n-1 = \frac{\log 0{,}1176}{\log 0{,}7}$ $n-1 = 6$ $n = 7$ During the 7th year/*In die 7de jaar*	✓use of logarithms ✓answer (4)
3.2	$h(n) = 130 + (100 + 70 + 49 + \ldots \text{to } n \text{ terms})$ $= 130 + \frac{100\left(1-(0{,}7)^n\right)}{1-0{,}7}$ $= 130 + \frac{100\left(1-(0{,}7)^n\right)}{0{,}3}$	✓130 ✓ $100 + 70 + 49 + \ldots$ to n terms ✓answer (3)

3.3	Eventual height of the tree/*Uiteindelike hoogte van die boom* $= 130 + \frac{100}{1-0,7}$ $= 463,33$ mm OR $\frac{1390}{3}$ mm	✓✓$130 + \frac{100}{1-0,7}$ ✓answer (3) **[10]**

QUESTION/*VRAAG* 4

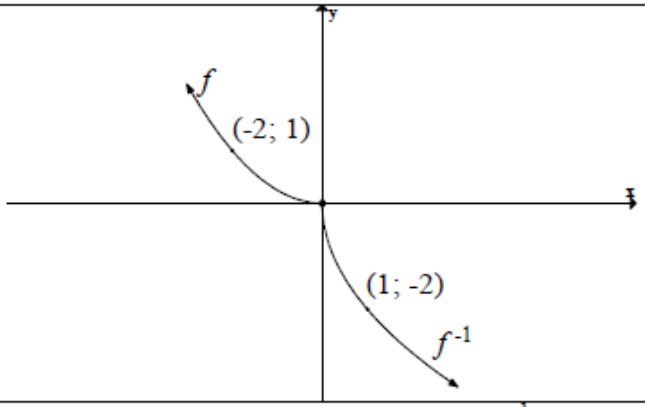
4.1	$(0\ ;2)$	✓answer (1)
4.2		✓shape ✓$(0;2)$ ✓asymptote (3)
4.3	$f(-2)=5$ $f(1)=2^{-1}+1=\frac{3}{2}$ Average gradient $=\frac{f(1)-f(-2)}{1-(-2)}$ $=\frac{\frac{3}{2}-5}{3}$ $=-\frac{7}{6}$	✓ $f(-2)=5$ ✓ $f(1)=\frac{3}{2}$ ✓ answer (3)
4.4	Since the asymptote of f is $y=1$, the asymptote of $h(x)=3f(x)$ will be $y=3$. *Omdat die asimptoot van* f $y=1$ *is,* *sal die asimptoot van* $h(x)=3f(x)$ $y=3$ *wees.*	✓answer (1) **[8]**

QUESTION/*VRAAG* 5

5.1	$y = a(x+p)^2 + q$ Turning point $(1\,;-8)$: $y = a(x-1)^2 - 8$ Substitute $(0\,;-4)$: $-4 = a(0-1)^2 - 8$ $-4 = a - 8$ $a = 4 \quad p = -1 \quad q = -8$ $y = 4(x-1)^2 - 8$	✓ $y = a(x-1)^2 - 8$ ✓ substitute $(0;-4)$ ✓ $a = 4$ ✓ p and q values (4)
5.2	Asymptote is $y = -2 \quad \Rightarrow \quad d = -2$ Substitute $(1;-8)$: $-8 = \frac{k}{1+r} - 2$ $k = -6(1+r)$ $k = -6 - 6r$ .....line 1 Substitute $(0;-4)$: $-4 = \frac{k}{r} - 2$ $\frac{k}{r} = -2$ $k = -2r$ .....line 2 Equating lines 1 and 2: $-6 - 6r = -2r$ $-4r = 6$ $r = -\frac{3}{2}$ Substituting into line 2 or line 1: $k = (-2)\left(-\frac{3}{2}\right) = 3$ $\quad$ $k = -6 - 6\left(-\frac{3}{2}\right) = 3$	✓ $d = -2$ ✓ $k = -6 - 6r$ ✓ $k = -2r$ ✓ $-6 - 6r = -2r$ ✓ value of r ✓ value of k (6)
5.3	$g(x) \geq f(x)$ $\therefore 0 \leq x \leq 1$	✓ $0 \leq x$ ✓ $x \leq 1$ (2)
5.4	The line $y = k$ must pass through f twice on the positive side of the x-axis./*Die lyn $y = k$ moet twee keer deur f aan die positiewe kant van die x-as sny.* $-8 < k < -4$	✓ $-8 < k$ ✓ $k < -4$ (2)

5.5	$y=-x+c$ Substitute the intersection point of the asymptotes, i.e. $\left(\frac{3}{2};-2\right)$: *Vervang die snypunt van die asimptote, m.a.w.* $\left(\frac{3}{2};-2\right)$: $-2=-\frac{3}{2}+c$ $c=-\frac{1}{2}$ $y=-x-\frac{1}{2}$ **OR/*OF*** $y=-x$ is translated $\frac{3}{2}$ units right and 2 units down/ $y=-x$ *transleer* $\frac{3}{2}$ *eenhede na regs en 2 eenhede na onder* $\Rightarrow$ $y=-\left(x-\frac{3}{2}\right)-2$ $y=-x-\frac{1}{2}$	✓ $y=-x+c$ ✓ $-2=-\frac{3}{2}+c$ ✓ answer (3) ✓ $y=-x$ ✓ $y=-\left(x-\frac{3}{2}\right)-2$ ✓ answer (3)
5.6	By symmetry, $Q=\left(\frac{3}{2}+8-2;-2+\frac{3}{2}-1\right)$ $=\left(\frac{15}{2};-\frac{3}{2}\right)$	✓ $x=\frac{15}{2}$ ✓ $y=-\frac{3}{2}$ (2) **[19]**

QUESTION/*VRAAG 6*

6.1	$f: \quad y=\frac{1}{4}x^2$ $f^{-1}: \quad x=\frac{1}{4}y^2$ $y^2=4x$ $y=\pm\sqrt{4x}$ $f^{-1}(x)=-\sqrt{4x}$ **OR**/OF $f^{-1}(x)=-2\sqrt{x}$	✓interchanging x and y ✓ $y^2=4x$ ✓answer (3)
6.2		✓both graphs pass through (0 ; 0) ✓shape for both ✓one additional point on both graphs (3)
6.3	Yes. No value of x in the domain of f^{-1} maps onto more than one y-value. *Ja. Geen waarde van x in die definisieversameling van f^{-1} assosieer met meer as een y-waarde nie.* **OR/*OF*** Yes. One to one function./*Ja. Een-tot-een-funksie.* **OR/*OF*** Yes. Vertical line test holds./*Ja. Die vertikale lyntoets werk.*	✓yes ✓reason (2) ✓yes ✓reason (2) ✓yes ✓reason (2) **[8]**

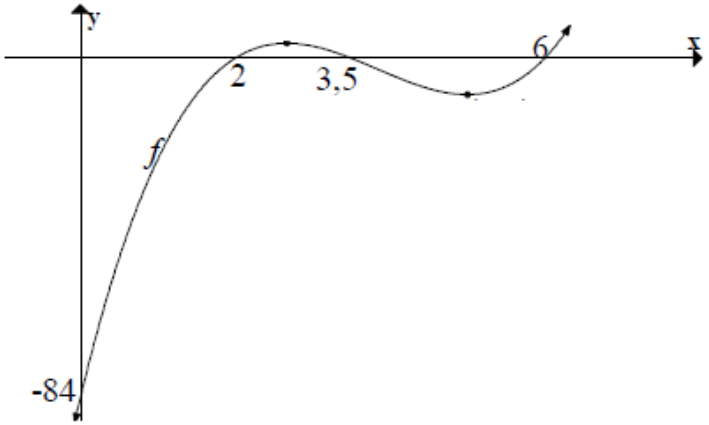
QUESTION/*VRAAG* 7

7.1.1	Quarterly interest rate/*Kwartaallikse rentekoers* $= \frac{10\%}{4}$ $= 2{,}5\%$	✓answer (1)
7.1.2	$A = P(1+i)^n$ $= 5000\left(1+\frac{2{,}5}{100}\right)^{2\times4}$ $= \text{R}6\,092{,}01$	✓$n = 8$ ✓$5000\left(1+\frac{2{,}5}{100}\right)^{2\times4}$ ✓answer (3)
7.2.1	$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $800\,000 = \frac{10\,000\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\frac{0{,}14}{12}}$ $\frac{800\,000}{10\,000}\times\frac{0{,}14}{12} = 1-\left(1+\frac{0{,}14}{12}\right)^{-n}$ $\left(1+\frac{0{,}14}{12}\right)^{-n} = 1-\frac{800\,000}{10\,000}\times\frac{0{,}14}{12}$ $-n = \frac{\log\left[1-\frac{800000\times\frac{0{,}14}{12}}{10000}\right]}{\log\left(1+\frac{0{,}14}{12}\right)}$ $n = 233{,}4699962$ Motloi can make 233 withdrawals of R10 000./*Motloi kan 233 onttrekkings van R10 000 maak.*	✓$i = \frac{0{,}14}{12}$ ✓substitute into present value formula ✓$\left(1+\frac{0{,}14}{12}\right)^{-n} = 1-\frac{800\,000}{10\,000}\times\frac{0{,}14}{12}$ ✓ use of logs ✓233 (5)
7.2.2 (a)	$\text{A - F}_\text{v} = 800\,000\left(1+\frac{0{,}14}{12}\right)^{48} - \frac{10\,000\left[\left(1+\frac{0{,}14}{12}\right)^{48}-1\right]}{\frac{0{,}14}{12}}$ $= 1\ 396\ 005{,}54 - 638\ 577{,}36$ $= \text{R}757\,428$ **OR/*OF***	✓$n = 48$ in both formulae ✓$i = \frac{0{,}14}{12}$ in both formulae ✓ substitution into both formulae ✓answer (4)

	$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{10000\left[1-\left(1+\frac{0,14}{12}\right)^{-185,4699962...}\right]}{\frac{0,14}{12}}$ $= \text{R}757\,428$	✓ $n = -185,46996...$ ✓ $i = \frac{0,14}{12}$ ✓ $\frac{10000\left[1-\left(1+\frac{0,14}{12}\right)^{-185,4699962...}\right]}{\frac{0,14}{12}}$ ✓ answer (4)
7.2.2 (b)	Let the purchase price of the house be y./*Laat die koopprys van die huis y wees.* $\frac{757\,428}{y} = 30\%$ $757\,428 = 0,3y$ $y = \frac{757\,428}{0,3}$ $= \text{R}2\,524\,760$ **OR/*OF*** Let the purchase price of the house be y./*Laat die koopprys van die huis y wees.* $y = \frac{757\,428}{30} \times 100$ $= \text{R}2\,524\,760$	✓ answer (1) ✓ answer (1) **[14]**

QUESTION/*VRAAG* 8

8.1	$f(x+h)=-(x+h)^2+4=-(x^2+2xh+h^2)+4$ $=-x^2-2xh-h^2+4$ $f(x+h)-f(x)=-2xh-h^2$	✓ finding $f(x+h)$ ✓ $-2xh-h^2$
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $=-2x$	✓formula ✓factorisation ✓answer (5)
	OR/*OF*	
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-(x+h)^2+4-(-x^2+4)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-x^2-2xh-h^2+4+x^2-4}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h))}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $=-2x$	✓formula ✓ finding $f(x+h)$ ✓ $-2xh-h^2$ ✓factorisation ✓answer (5)
8.2.1	$y=3x^2+10x$ $\dfrac{dy}{dx}=6x+10$	✓ $6x$ ✓ 10 (2)
8.2.2	$f(x)=\left(x-\dfrac{3}{x}\right)^2$ $=x^2-6+\dfrac{9}{x^2}$ $=x^2-6+9x^{-2}$ $f'(x)=2x-18x^{-3}$	✓ $x^2-6+\dfrac{9}{x^2}$ ✓ $9x^{-2}$ ✓ $2x-18x^{-3}$ (3)

8.3.1	$f(2) = 2(2)^3 - 23(2)^2 + 80(2) - 84$ $= 0$ $\therefore (x-2)$ is a factor	✓substitution of 2 into f ✓value of 0 (2)
8.3.2	$f(x) = 2x^3 - 23x^2 + 80x - 84$ $= (x-2)(2x^2 - 19x + 42)$ $= (x-2)(2x-7)(x-6)$	✓ $2x^2 - 19x + 42$ ✓ $(x-2)(2x-7)(x-6)$ (2)
8.3.3	$f'(x) = 6x^2 - 46x + 80$ $6x^2 - 46x + 80 = 0$ $3x^2 - 23x + 40 = 0$ $(3x-8)(x-5) = 0$ $x = \frac{8}{3}$ or $x = 5$	✓ $f'(x) = 6x^2 - 46x + 80$ ✓ $f'(x) = 0$ ✓factors ✓x-values (4)
8.3.4		✓x-intercepts ✓y-intercept ✓shape (3)
8.3.5	$6x^2 - 46x + 80 = 40$ $6x^2 - 46x + 40 = 0$ $3x^2 - 23x + 20 = 0$ $(3x-20)(x-1) = 0$ $x = \frac{20}{3}$ or $x = 1$ But x must be an integer, so $x = 1$ at the point where tangent touches f /*x moet heelgetal wees so x = 1 by punt waar die raaklyn f raak:* $y = f(1) = 2(1)^3 - 23(1)^2 + 80(1) - 84 = -25$ $y = mx + c$ $-25 = 40(1) + c$ $-65 = c$ $(0;-65)$	✓ $6x^2 - 46x + 80 = 40$ ✓factors ✓$x = 1$ ✓ y-value ✓ $-25 = 40(1) + c$ ✓answer (6) **[27]**

QUESTION/*VRAAG* 9

9.1	$340 = \pi r^2 h$ $\therefore h = \frac{340}{\pi r^2}$	✓substitution into volume formula ✓answer (2)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\frac{340}{\pi r^2}\right)$ $= 2\pi r^2 + 680r^{-1}$	✓formula ✓substitution of h (2)
9.3	$A(r) = 2\pi r^2 + 680r^{-1}$ $A'(r) = 4\pi r - 680r^{-2}$ $4\pi r - 680r^{-2} = 0$ $4\pi r = \frac{680}{r^2}$ $r^3 = \frac{680}{4\pi}$ $r = \sqrt[3]{\frac{680}{4\pi}}$ cm or 3,78 cm	✓ $4\pi r$ ✓ $-680r^{-2}$ ✓ $r^3 = \frac{680}{4\pi}$ ✓answer (4) **[8]**

QUESTION/*VRAAG* 10

10.1.1	160	✓answer (1)
10.1.2	$P(M) = \frac{60}{160}$ $= \frac{3}{8}$ $= 0{,}375$	✓60 ✓answer (2)
10.1.3	$P(Male) \times P(Coffee) = P(Male\ and\ Coffee)$ $P(Manlik) \times P(Koffie) = P(Manlik\ en\ Koffie)$ $\frac{3}{8} \times \frac{80}{160} = \frac{b}{160}$ $\frac{3}{16} = \frac{b}{160}$ $16b = 480$ $b = 30$	✓formula ✓ $\frac{80}{160}$ ✓ $\frac{b}{160}$ ✓ answer (4)

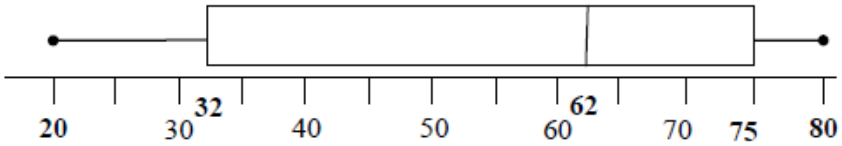
10.2.1	$6!$ $= 6\times5\times4\times3\times2\times1$ $= 720$	✓6! ✓answer (2)
10.2.2	number of ways Xoliswa sits next to Anees/ *getal maniere waarop Xoliswa langs Anees sit* $= 5!\times2$ $= 240$ **OR/*OF*** Regard Xoliswa and Anees as a single entity/*Beskou Xoliswa en Anees as een* Number of ways in which 5 passengers can be arranged = 5! *Getal maniere waarop 5 passasiers gerangskik kan word* = 5! So 5! different arrangements for XA and 5! different arrangements for AX *So 5! verskillende rangskikkings vir XA en 5! verskillende rangskikkings vir AX* number of ways Xoliswa sits next to Anees *getal maniere waarop Xoliswa langs Anees sit* $= 5!\times2$ $= 240$	✓ $5!\times2$ ✓answer (2) ✓ $5!+5!$ ✓answer (2)
10.2.3	number of ways Mary is at an end of the row on the left $= 1\times5!$ number of ways Mary is at an end of the row on the right $= 5!\times1$ total number of arrangements $= 6!$ $P(\text{Mary is at an end of the row}) = \frac{5!\times1+1\times5!}{6!}$ $= \frac{1}{3}$ *getal maniere waarop Mary aan die einde van die ry links is* $= 1\times5!$ *getal maniere waarop Mary aan die einde van die ry regs is* $= 5!\times1$ *totale getal rangskikkings* $= 6!$ $P(\text{Mary is } \textit{aan einde van die ry}) = \frac{5!\times1+1\times5!}{6!}$ $= \frac{1}{3}$	✓both LHS and RHS ways ✓6 ! ✓setting up probability ✓answer (4) **[15]**
	TOTAL/*TOTAAL*:	**150**

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

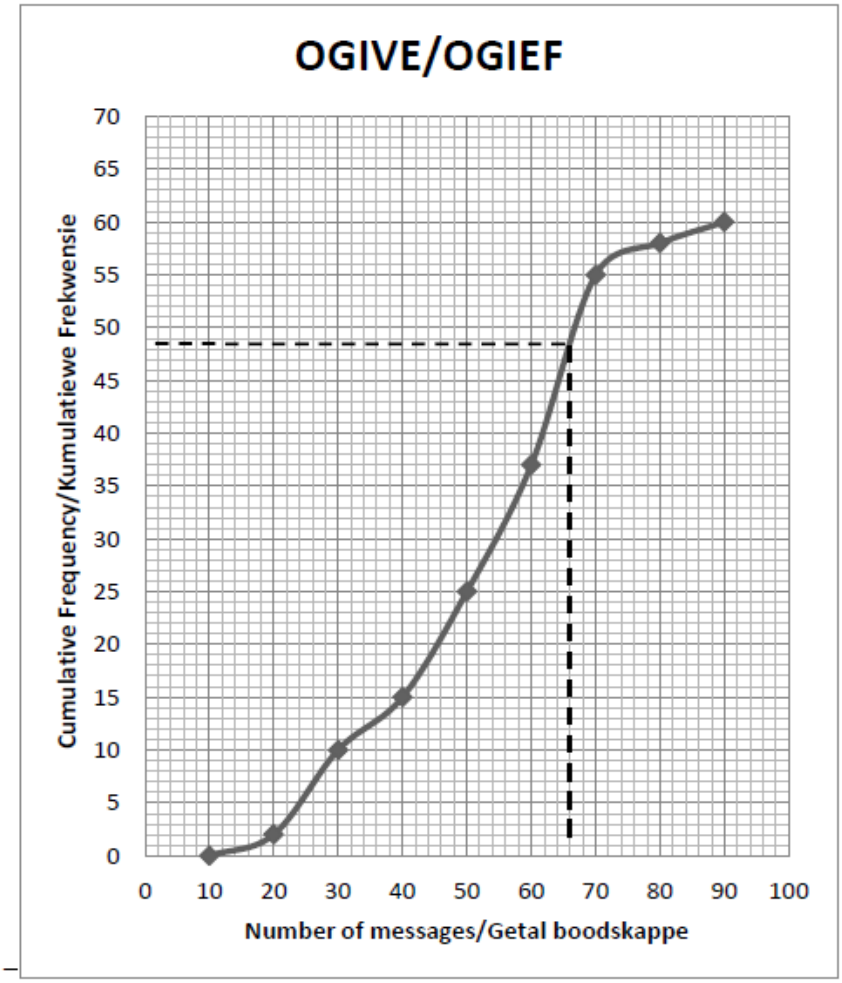
MEMORANDUM

QUESTION/*VRAAG* 1

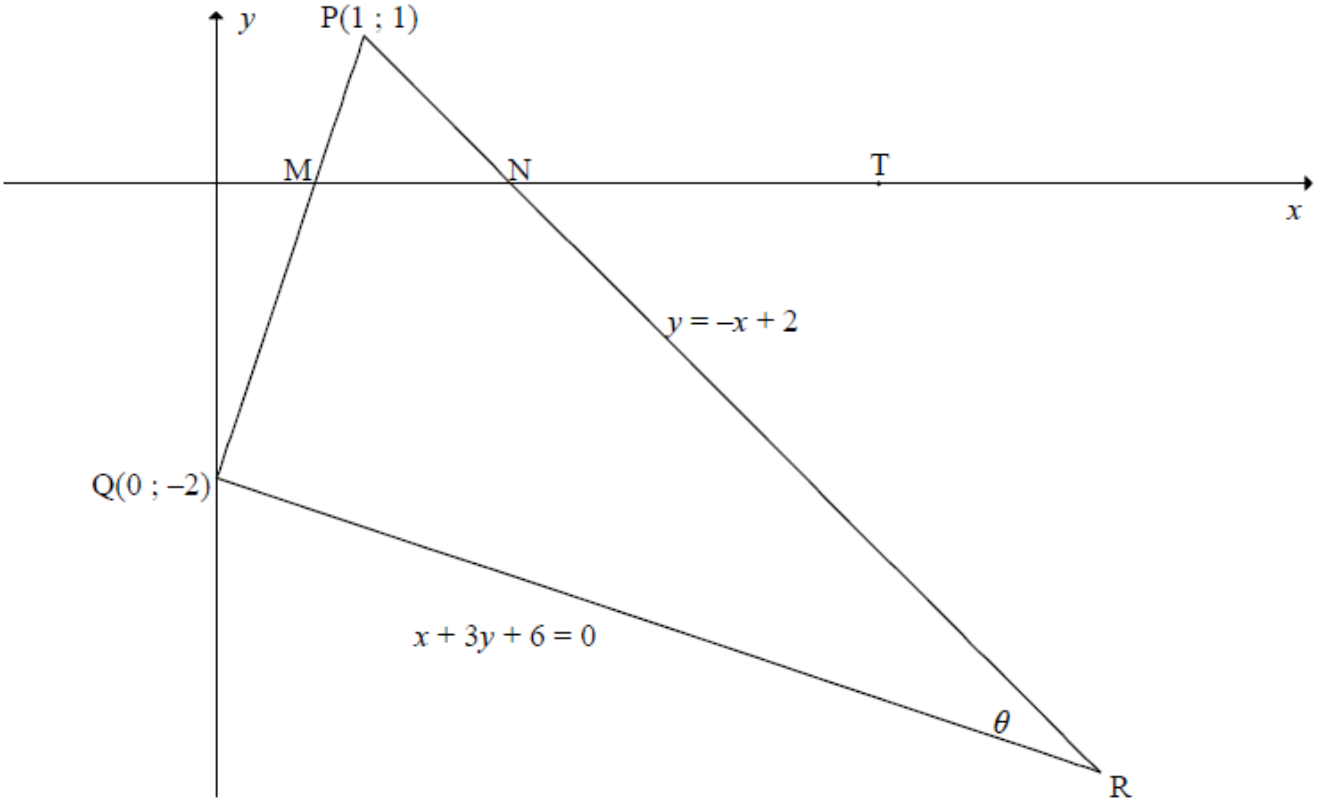


1.1	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** The data is negatively skewed/*Die data is negatief skeef.*	✓answ/*antw* ✓answ/*antw* (1)
1.2	Range/*Omvang* = 80 – 20 = 60	✓ max. – min. ✓answ/*antw* (2)
1.3	25% of the learners failed/*van die leerders het gedruip*	✓ ✓answ/*antw* (2)
1.4	$54 = \frac{445 + T_4}{9}$ $T_4 = 41$ **20** 28 36 **41** **62** 69 **75** **75** **80**	✓ 20 ✓✓ 41 ✓ 62 ✓ 75 ✓ 80 (6) **[11]**

QUESTION/*VRAAG* 2

2.1	Mean/*Gemiddelde* $= \frac{2(15)+8(25)+...2(85)}{60} = \frac{3080}{60}$ = 51,33 messages per day/*boodskappe per dag*	✓ 3 080 ✓ $\frac{3080}{60}$ ✓answ/*antw* (3)
2.2		✓grounding at (10 ; 0) ✓plotting at upper limits ✓plotting. cumulative *f* ✓smooth shape of curve ✓*geanker by* (10 ; 0) ✓*stip by boonste limiete* ✓*plot kumulatiewe f* ✓*gladde vorm van kurwe* (4)
2.3	Number of days/*Getal dae* = 60 – 46 (see on graph above/*sien op grafiek hierbo*) = 14 days/*dae* **OR/*OF*** Number of days/*Getal dae* = $2 + 3 + \frac{1}{2} \times 18$ = 14 days/*dae*	✓46 (accept 45 – 49) ✓answ/*antw* (accept 11 – 15) (2) ✓add correct values/*tel korrekte waardes by* ✓answ/*antw* (2) **[9]**

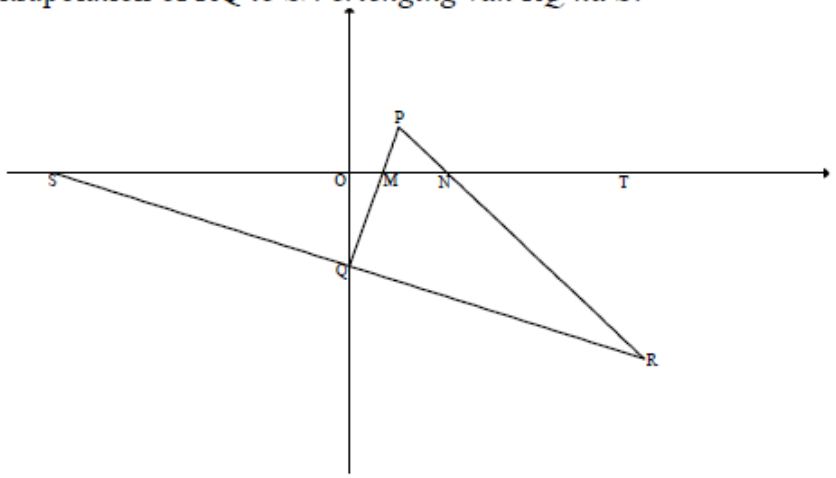
QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore$ R(6 ; –4)	✓equating/*gelyk stel* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (3)
3.4	$PR=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $PR^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/*'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2};-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore\left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2};-1\frac{1}{2}\right)$ ✓*r*-value/*waarde* ✓answ/*antw* (6)
3.6	*m* of/*van* radius = –1 $\therefore$ *m* of/*van* tangent/*raaklyn* = 1 Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ **OR/*OF*** $y=x+c$ $y-1=x-1$ **OR/*OF*** $1=1+c$ $\therefore y=x$ **OR/*OF*** $y=x$	✓*m* of tang/*rkl* ✓subst *m* & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan P\hat{N}T=m_{PR}=-1$ $\therefore P\hat{N}T=135°$ $\tan P\hat{M}T=m_{PQ}=3$ $\therefore P\hat{M}T=71{,}57°$ $\hat{P}=63{,}43°$ [ext ∠ of Δ/*buite ∠v* Δ] $\therefore\theta=26{,}57°$ [sum of ∠s in Δ/*som v ∠e in* Δ] **OR/*OF***	✓ $\tan P\hat{N}T=-1$ ✓ $P\hat{N}T=135°$ ✓ $P\hat{M}T=71{,}57°$ ✓ $\hat{P}=63{,}43°$ ✓answ/*antw* (5)

Extrapolation of RQ to S/*Verlenging van RQ na S*:



$\tan \hat{PNT} = m_{PR} = -1$
$\therefore \hat{SNR} = 135°$
$\tan \hat{NSR} = m_{RS} = -\frac{1}{3}$
$\therefore \hat{NSR} = 18{,}43°$
$\theta = 180° - (135° + 18{,}43°)$ [sum of ∠s in Δ/*som v ∠e in* Δ]
$= 26{,}57°$

✓ $\tan \hat{PNT} = -1$
✓ $\hat{SNR} = 135°$
✓ $\tan \hat{NSR} = -\frac{1}{3}$
✓ $\hat{NSR} = 18{,}43°$
✓answ/*antw*
(5)

OR/*OF*

$PQ^2 = 1^2 + 3^2 = 10$
$PQ = \sqrt{10}$
$\therefore \sin\theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$
$\therefore \theta = 26{,}57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

$QR^2 = 6^2 + 2^2 = 40$
$QR = 2\sqrt{10}$
$\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$
$\therefore \theta = 26{,}57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

$$\tan\theta = \frac{m_{RQ} - m_{PR}}{1 + m_{RQ}.m_{PR}}$$

$$= \frac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$$

$$= \frac{1}{2}$$

$\therefore\ \theta = 26{,}57°$

✓correct formula/ *korrekte formule*

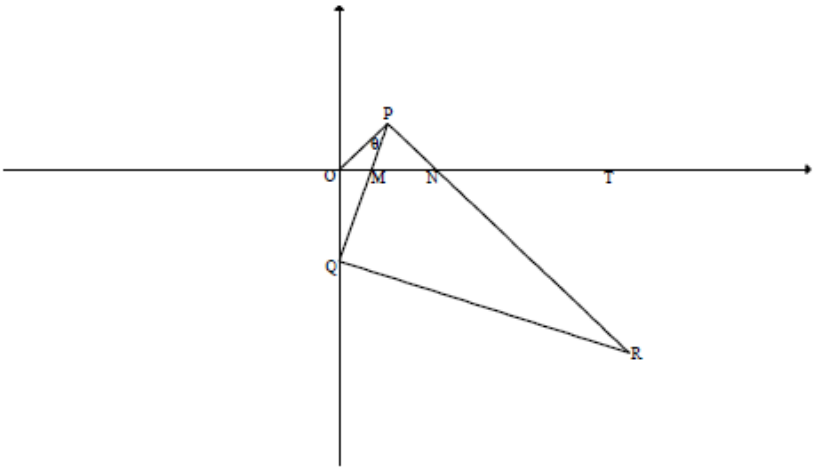
✓ $m_{RQ} = -\frac{1}{3}$

✓correct subst/ *subst korrek*

✓ $\tan\theta = \frac{1}{2}$

✓ $\theta = 26{,}57°$

(5)



tangent OP goes through the origin/*raakl OP gaan deur oorsprong*

$\hat{POM} = 45°$

$\hat{OPM} = \theta = \hat{P}$ [tan-chord theorem/*raakl-kdst*]

$\tan \hat{PMT} = m_{PQ} = 3$

$\therefore \hat{PMT} = 71{,}57°$

$\therefore\ \theta + 45° = 71{,}57°$ [ext ∠ of Δ/*buite-∠ v Δ*]

$\therefore\ \theta = 26{,}57°$

✓ $\hat{POM} = 45°$

✓R

✓ $\hat{PMT} = 71{,}57°$

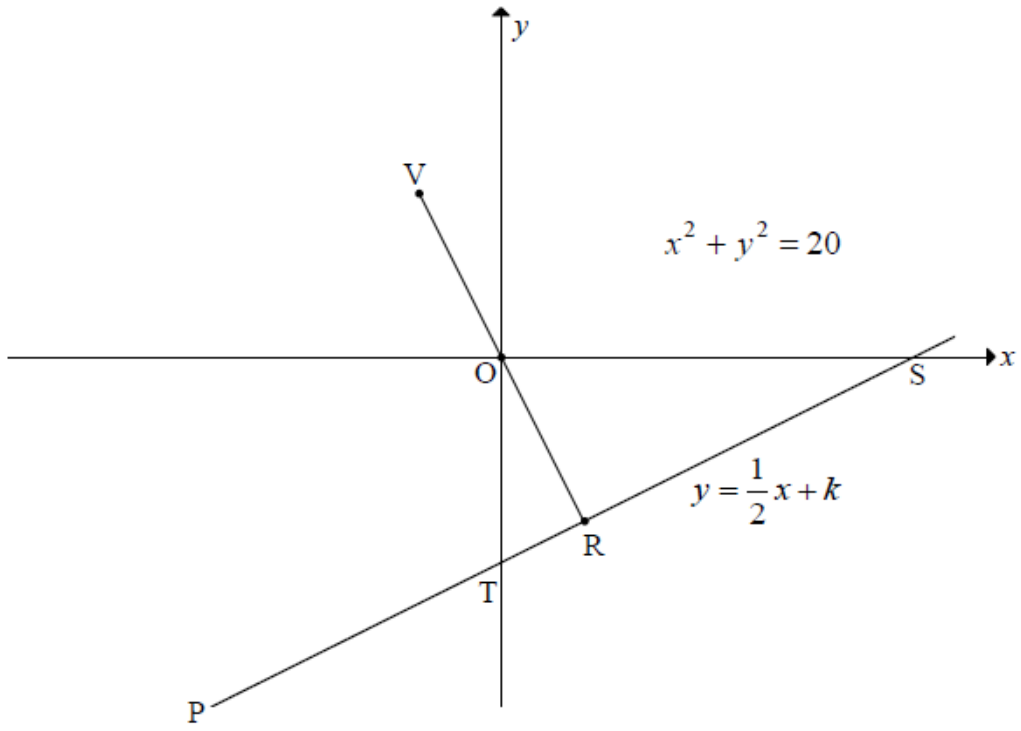
✓S

✓ $\theta = 26{,}57°$

(5)

[23]

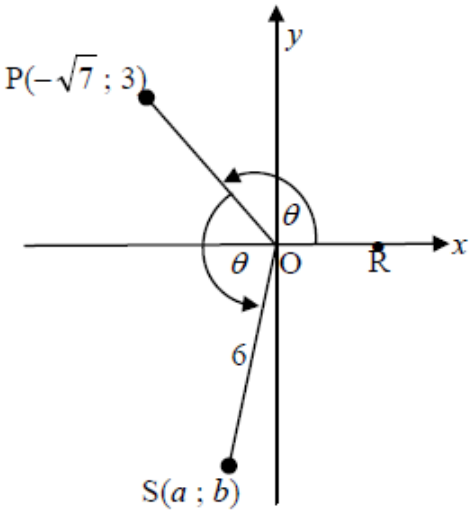
QUESTION/*VRAAG* 4



4.1	OR ⊥ TR [radius ⊥ tangent/*raakl*] $\therefore\ m_{TR} \times m_{OR} = -1$ $\therefore\ m_{OR} = -2$ $\therefore\ y = -2x$	✓S/R ✓*m* of/*van* OR ✓equation/*vgl* (3)
4.2	$x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 - 20 = 0$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $\therefore x = 2$ $y = -2(2) = -4$ $\therefore$ R(2 ; –4)	✓subst eq of OR into circle eq/ *subst vgl OR in sirkelvgl* ✓st. form/*st. vorm* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (4)

4.3	Subst R(2 ; –4) into the equation of/*in vgl van* PRS: $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore$ OT = 5 $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore$ OS = 10 Area/*Oppervlakte* $= \frac{1}{2}$ OS . OT $= \frac{1}{2}(10)(5)$ = 25 sq units/*vk eenh*	✓correct subst/ *korrekte subst* ✓value of k ✓$y = 0$ ✓x-intercept/*afsnit* ✓correct subst into area form/ *subst korrek in opp-formule* ✓answ/*antw* (6)
4.4	$0 = \frac{x_V + 2}{2}$ and/*en* $0 = \frac{y_V - 4}{2}$ $\therefore$ V(–2 ; 4) T(0 ; –5) .... from/*van* 4.3 VT $= \sqrt{(-2-0)^2 + (4-(-5))^2}$ $= \sqrt{4+81}$ $= \sqrt{85}$	✓x-value/*waarde*V ✓y-value/*waarde*V ✓subst of points V and T into distance formula/ *subst punte V en T in afst-form* ✓answ/*antw* (4) **[17]**

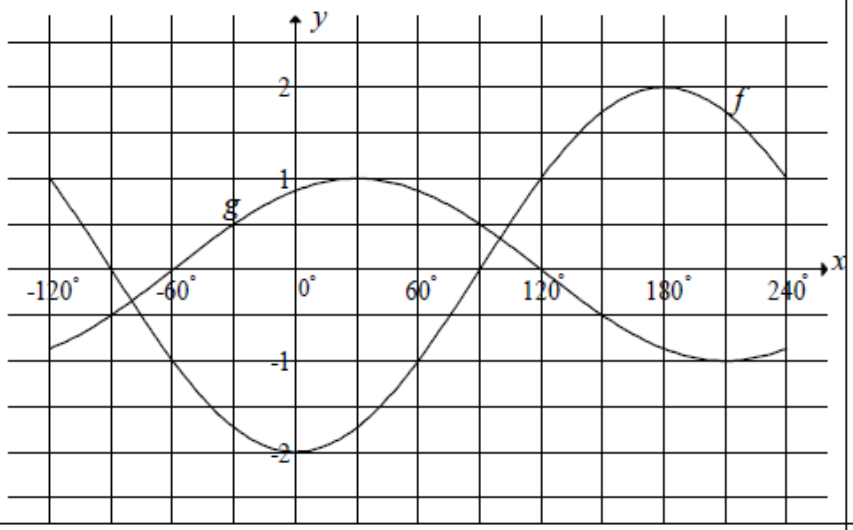
QUESTION/*VRAAG* 5



5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓reduction/ *reduksie* ✓ OP = 4 ✓answ/*antw* (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ **OR/*OF*** $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2\theta - \sin^2\theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	✓ trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ & $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4)
5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x - 1} = \frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $= \frac{2\sin 2x}{-\cos 2x}$ $= -2\tan 2x$	✓2sin 2x ✓–cos 2x ✓answ/*antw* (3)
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15° - 1} = -2\tan 2(15°)$ $= -2\tan 30°$ $= -2\left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}}$ **OR/*OF*** $-\frac{2\sqrt{3}}{3}$	✓ $-2\tan 2(15°)$ ✓answ/*antw* (2) **[13]**

QUESTION/*VRAAG 6*

6.1	$\sin(x+60°)+2\cos x=0$ $\sin x\cos 60°+\cos x\sin 60°+2\cos x=0$ $\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x+2\cos x=0$ $\frac{1}{2}\sin x=-2\cos x-\frac{\sqrt{3}}{2}\cos x$ $\sin x=-4\cos x-\sqrt{3}\cos x$ $\sin x=\cos x(-4-\sqrt{3})$ $\frac{\sin x}{\cos x}=\frac{\cos x(-4-\sqrt{3})}{\cos x}$ $\therefore \tan x=-4-\sqrt{3}$	✓expansion/*uitbreiding* ✓special angle values/ *spesiale ∠-waardes* ✓simpl/*vereenv* ✓ $\sin x=\cos x(-4-\sqrt{3})$ (4)
6.2	$\tan x=-4-\sqrt{3}$ $\tan x=-(4+\sqrt{3})$ *ref* $\angle = 80{,}10°$ $x=-80{,}1°$ or/*of* $99{,}9°$	✓80,10° ✓99,90° ✓–80,1° (3)
6.3.1		✓(30° ; 1) ✓(–60° ; 0) ✓shape/*vorm* (3)
6.3.2	$\therefore \sin(x+60°)>-2\cos x$ $x\in(-80{,}10°\ ;\ 99{,}90°)$ **OR/*OF*** $-80{,}10°<x<99{,}90°$	✓✓critical values/ *kritiese waardes* ✓notation/*notasie* (3) **[13]**

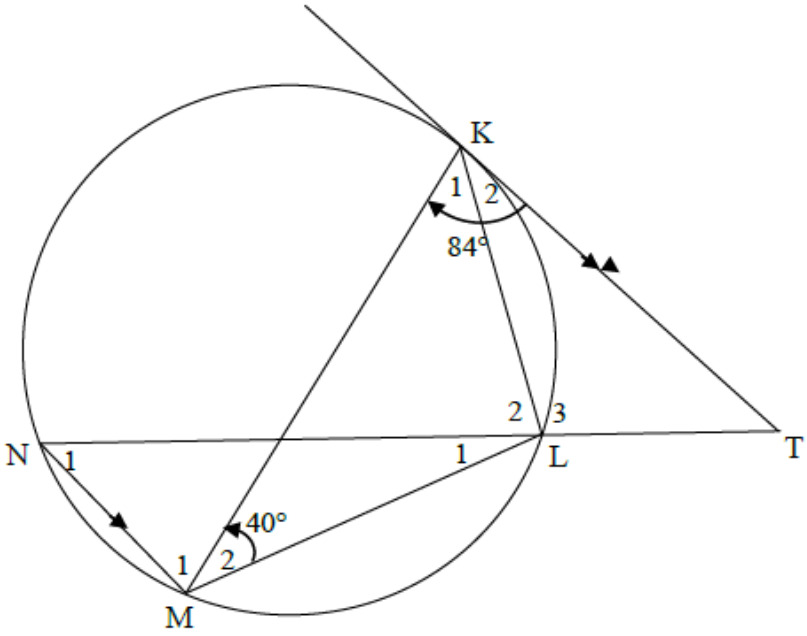
QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2}PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore\ x_{max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5 **OR/*OF*** $x_{max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5 **OR/*OF*** $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore\ x = 0$ or 5 $\therefore\ x_{max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓$(\text{Area } \Delta PQR)' = 0$ ✓$5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3) ✓formula/*e* ✓ subst ✓answ/*antw* (3) ✓*x*-intercepts/ *x-afsnitte* ✓subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore\ RP = 9{,}01$	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito *h* and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓sin $(90° - \beta)$ = cos β ✓answer ito *h* (7)
	OR/*OF*	
	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 (1 - 2\sin^2\beta)$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$	✓AB ito *h* and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito *h* (7)
	OR/*OF*	
	Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then $\Delta ABC \equiv \Delta BAE$ (AB = AC, $A\hat{B}E = B\hat{A}C = \beta$, *h*) $\therefore AE = ED = BC = h$ $\therefore AD = 2h$	(7)
		[15]

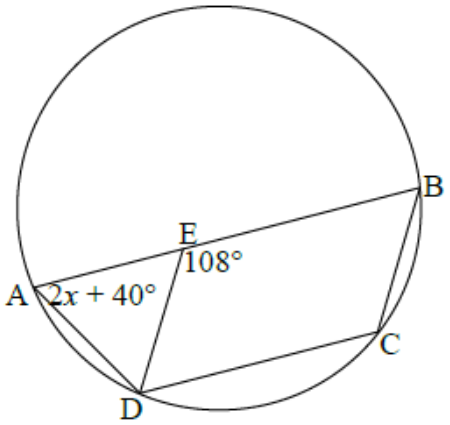
QUESTION/*VRAAG* 8

8.1



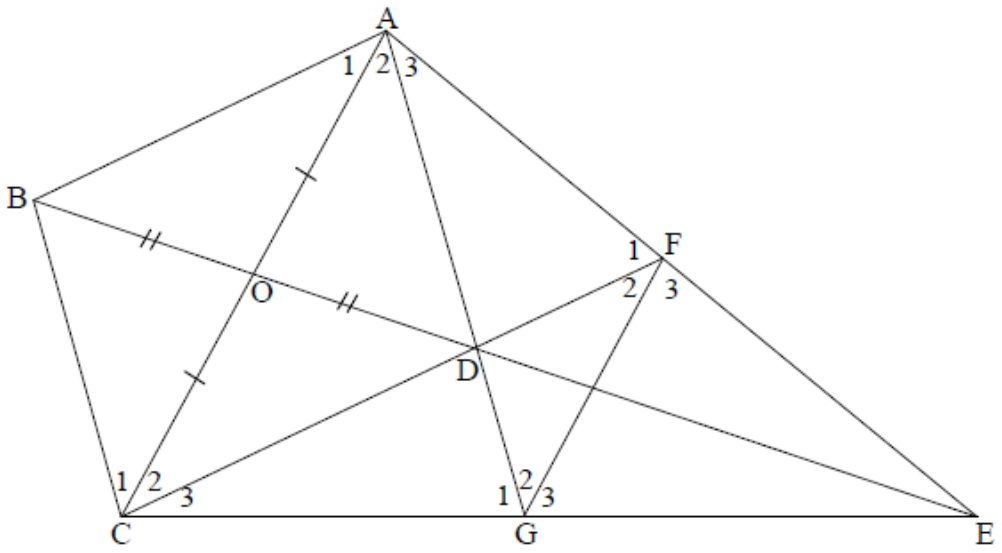
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40°$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/*∠e in dies segm*] $\hat{K}_1 = 84° - 40° = 44°$ $\therefore\ \hat{N}_1 = 44°$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44°$ [alt/*verw* ∠s/*e*; KT || NM]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite ∠ v Δ*] $= 40° + 44°$ $= 84°$	✓R ✓S (2)
8.1.5	In ΔKLM: $44° + 84° + 40° + \hat{L}_1 = 180°$ [∠s sum in Δ/*∠e som in Δ*] $\therefore \hat{L}_1 = 12°$	✓S (1)

8.2



8.2	$\hat{C} = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x + 40° + 108° = 180°$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5)
		OR/*OF*	
	$\hat{C} = 180° - (2x + 40°)$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$180° - (2x + 40°) = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5)
			[15]

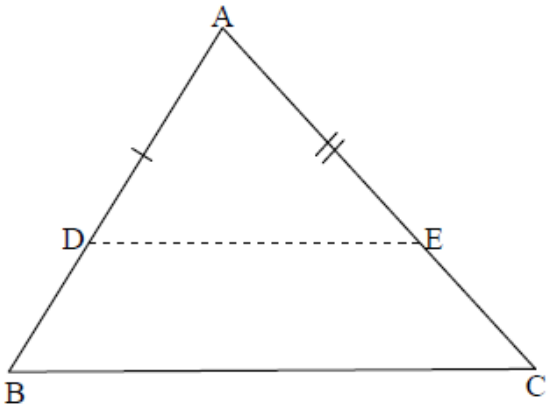
QUESTION/*VRAAG* 9

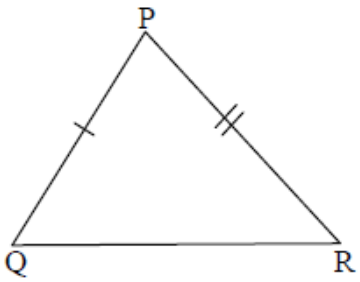


9.1	ABCD is a ‖m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB}=\frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF ‖ BA] $\frac{ED}{DB}=\frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG ‖ BC]	✓S ✓R ✓S ✓R (4)
9.3	$\frac{FE}{AF}=\frac{GE}{CG}$ [proved/*bewys*] ∴AC ‖ FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2=\hat{F}_2$ [alt/*verw* ∠s/*e*; AC ‖ FG] $\hat{A}_1=\hat{C}_2$ [alt/*verw* ∠s/*e*; AB ‖ CD] ∴ $\hat{A}_1=\hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1=\hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2=\hat{F}_2$ [$\hat{A}_1=\hat{F}_2$] ∴ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2=\hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2=\hat{G}_2$ [alt/*verw*-∠s/*e*; AC ‖ FG] ∴ $\hat{C}_2=\hat{G}_2$ ∴ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

QUESTION/*VRAAG* 10

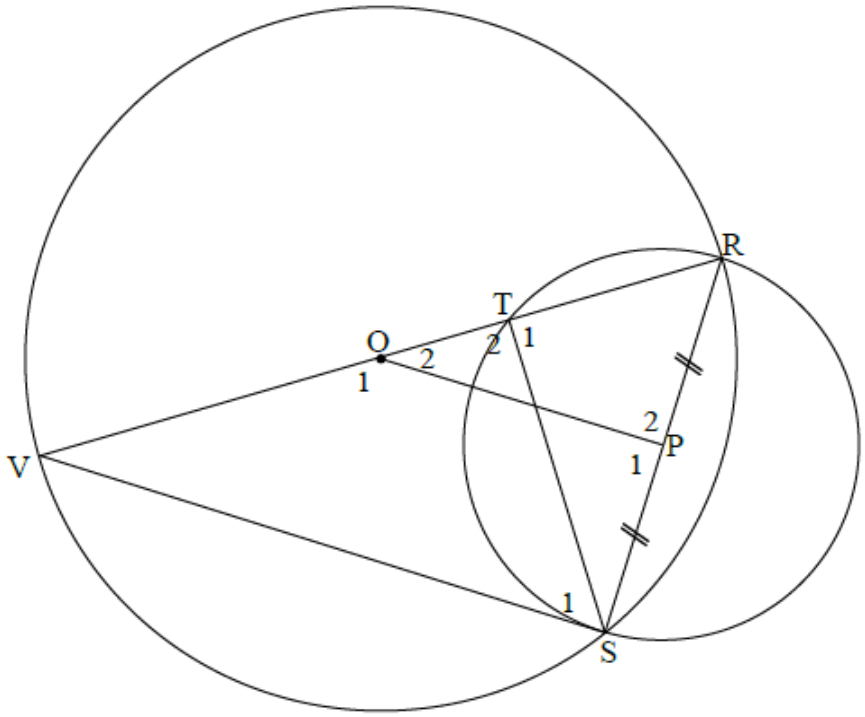
10.1





10.1.1	In ΔADE and/*en* ΔPQR: AD = PQ [construction/*konstr*] $\hat{A} = \hat{P}$ [given/*gegee*] AE = PR [construction/*konstr*] ∴ΔADE ≡ ΔPQR [S∠S]	✓all/*al* 3 S's/*e* ✓reason/*rede* (2)
10.1.2	$A\hat{D}E = \hat{Q}$ [Δs ≡ ∴ corres/*ooreenk* ∠s/*e* =] But $\hat{B} = \hat{Q}$ [given/*gegee*] ∴ $A\hat{D}E = \hat{B}$ ∴ DE || BC [corres/*ooreenk* ∠s/*e* =]	✓ $A\hat{D}E = \hat{Q}$ ✓ $A\hat{D}E = \hat{B}$ ✓reason/*rede* (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/*Eweredigh st*; DE || BC] But/*Maar* AD = PQ and/*en* AE = PR [construction/*konstr*] ∴ $\frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

10.2



10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] ∴ΔROP ||| ΔRVS [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] ∴ΔROP ||| ΔRVS [∠,∠,∠]	✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ ∴ΔRVS ∣∣∣ ΔRST [∠,∠,∠]	✓S ✓R ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [∠ s on straight line/∠*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ ∴ΔRTS ∣∣∣ ΔSTV [∠,∠,∠] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓ΔRTS ∣∣∣ ΔSTV ✓ratio/*verh* (6)
		[21]

TOTAL/*TOTAAL*: 150

MATHEMATICS P1/*WISKUNDE V1*

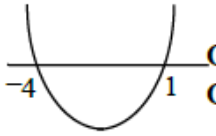
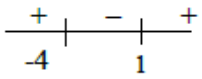
NOVEMBER 2016

MEMORANDUM

QUESTION/*VRAAG* 1

1.1.1	$x(x-7)=0$ $x=0$ or $x=7$	✓ $x=0$ ✓ $x=7$ (2)
1.1.2	$x^2-6x+2=0$ $x=\frac{6\pm\sqrt{(-6)^2-4(1)(2)}}{2(1)}$ $x=\frac{6\pm\sqrt{28}}{2}$ $x=0{,}35$ or $x=5{,}65$ **OR/OF** $x^2-6x+2=0$ $x^2-6x+9=-2+9$ $(x-3)^2=7$ $x-3=\pm\sqrt{7}$ $x=3\pm\sqrt{7}$ $x=0{,}35$ or $x=5{,}65$	✓correct substitution into correct formula ✓ $x=0{,}35$ ✓ $x=5{,}65$ (3) ✓ $(x-3)^2=7$ ✓ $x=0{,}35$ ✓ $x=5{,}65$ (3)
1.1.3	$\sqrt{x-1}+1=x$ $\sqrt{x-1}=x-1$ $x-1=x^2-2x+1$ $x^2-3x+2=0$ $(x-2)(x-1)=0$ $x=2$ or $x=1$ Both answers are valid **OR/OF**	✓ isolate $\sqrt{x-1}$ ✓ x^2-2x+1 ✓ standard form ✓ factors ✓ both answers (5)

	$\sqrt{x-1}+1=x$ $\sqrt{x-1}=x-1$ Let $x-1=k$ $\sqrt{k}=k \quad k\geq 0$ $k=k^2$ $k^2-k=0$ $k(k-1)=0$ $(x-1)(x-2)=0$ $x=2$ or $x=1$; $x\geq 1$ Both answers are valid **OR/OF** $\sqrt{x-1}+1=x$ $\sqrt{x-1}=x-1$ By inspection : $x-1=0$ or $x-1=1$ $x=2$ or $x=1$	✓ isolate $\sqrt{x-1}$ ✓ k^2 ✓ standard form ✓ factors ✓ both answers (5) ✓ isolate $\sqrt{x-1}$ ✓ $x-1=0$ ✓ $x-1=1$ ✓ $x=2$ ✓ $x=1$ (5)
1.1.4	$3^{x+3}-3^{x+2}=486$ $3^x3^3-3^x3^2=486$ $3^x(3^3-3^2)=486$ $3^x=27$ $3^x=3^3$ $x=3$ **OR/OF** $3^{x+3}-3^{x+2}=486$ $3^{x+2}(3^1-1)=486$ $3^{x+2}=243$ $3^{x+2}=3^5$ $x+2=5$ $x=3$	✓ expansion ✓ common factor ✓ $3^x=27$ ✓ $x=3$ (4) ✓ common factor ✓ (3^1-1) ✓ $3^{x+2}=243$ ✓ $x=3$ (4)
1.2.1	$f(x)=x^2+3x-4$ $0=(x+4)(x-1)$ $x=-4$ or $x=1$	✓ factors ✓ both answers (2)

1.2.2	$x^2+3x-4<0$ $(x+4)(x-1)<0$  **OR/ OF**  $-4<x<1$ **OR/*OF*** $x\in(-4\,;1)$	✓ ✓ $-4<x<1$ (2)
1.2.3	$2x+3\geq 0$ $x\geq -\frac{3}{2}$ $f'(x)\geq 0$ when f is increasing The turning point occurs at $x=\frac{-4+1}{2}$ $x\geq -\frac{3}{2}$	✓ $2x+3$ ✓ $x\geq -\frac{3}{2}$ (2) ✓ $x=\frac{-4+1}{2}$ ✓ $x\geq -\frac{3}{2}$ (2)
1.3	$x=2y$ and $x^2-5xy=-24$ $(2y)^2-5(2y)(y)=-24$ $4y^2-10y^2=-24$ $-6y^2=-24$ $y^2=4$ $y=-2$ or $y=2$ $x=-4$ or $x=4$ **OR/OF** $x=2y$ and $x^2-5xy=-24$ $y=\frac{x}{2}$ $x^2-5(x)\left(\frac{x}{2}\right)=-24$ $x^2-\frac{5}{2}x^2=-24$ $-\frac{3}{2}x^2=-24$ $x^2=16$ $x=-4$ or $x=4$ $y=-2$ or $y=2$ **OR/OF**	✓ substitution of $2y$ ✓ $-6y^2=-24$ ✓ both y – values ✓ both x– values (4) ✓ substitution of $\frac{x}{2}$ ✓ $-\frac{3}{2}x^2=-24$ ✓ both x – values ✓ both y – values (4)

	$x = 2y$ and $x^2 - 5xy = -24$ $y = \frac{x}{2}$ $y = \frac{-x^2 - 24}{-5x}$ $\frac{x}{2} = \frac{x^2 + 24}{5x}$ $5x^2 = 2x^2 + 48$ $3x^2 = 48$ $x^2 = 16$ $x = -4$ or $x = 4$ $y = -2$ or $y = 2$	✓ equating $\frac{x}{2} = \frac{x^2 + 24}{5x}$ ✓ $3x^2 = 48$ ✓ both x – values ✓ both y – values (4)
		[24]

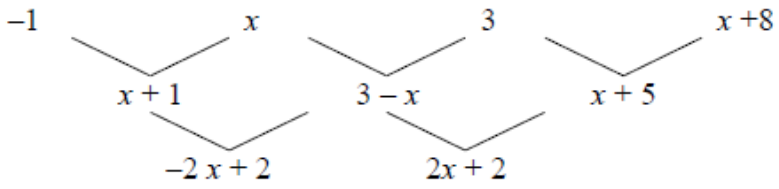
QUESTION/*VRAAG* 2

2.1	$T_4 = -7$	✓ -7 (1)
2.2	$T_n = a + (n-1)d$ $-87 = 5 + (n-1)(-4)$ $-87 = 5 - 4n + 4$ $4n = 96$ $n = 24$ **OR/OF** $-4n + 9 = -87$ $-4n = -96$ $n = 24$	✓ $a = 5$ and $d = -4$ ✓ $-87 = 5 + (n-1)(-4)$ ✓ $n = 24$ (3) ✓ $-4n + 9$ ✓ $-4n + 9 = -87$ ✓ $n = 24$ (3)
2.3	$-3; -7; \ldots\ldots; -87$ $S_n = \frac{n}{2}[a + T_n]$ $S_{22} = \frac{22}{2}[-3 - 87]$ $= -990$ **OR/OF**	✓ $n = 22$ ✓ $a = -3$ ✓ answer (3)

	$-3;-7;.....;-87$ $S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{22} = \frac{22}{2}[2(-3)+(22-1)(-4)]$ $= -990$ **OR/OF** All negative terms can be written down and added to get the answer of –990./*Alle negatiewe terme kan neergeskryf word en dan bymekaar getel word om –990 te kry.* **OR/OF** Sum $= S_{24} - (5+1)$ $= \frac{24}{2}[5-87]-6$ $= -990$	✓ $n = 22$ ✓ $a = -3$ ✓ answer (3) ✓ $a = -3$ ✓✓ answer (3) ✓ $\frac{24}{2}[5-87]$ ✓ -6 ✓ answer (3)
2.4	$5;-15;-35.....$ $d = -20$ $T_n = -20n+25$ Last term in the sequence divisible by 5 is:/*Laaste term in die ry deelbaar deur 5 is:* $-4187+4(3)$ $= -4175$ $T_n = -20n+25$ $-4175 = -20n+25$ $20n = 4200$ $n = 210$ There will be 210 terms in the sequence that is divisible by 5./*Daar is 210 terme in die ry deelbaar deur 5.* **OR/OF**	✓ $d = -20$ ✓ $T_n = -20n+25$ ✓ $-4175 = -20n+25$ ✓ $n = 210$ (4)

	$5\ ;\ 1\ ;\ -3\ ;\ \ldots\ ;\ -83\ ;\ -87;\ \ldots\ldots;\ -4187$ $T_n = -4n + 9$ $-4187 = -4n + 9$ $4n = 4196$ $n = 1049$ There are 1049 terms in the sequence./*Daar is* 1049 *terme in die ry.* $T_1\ ;\ T_6\ ;\ T_{11}\ ;\ T_{16}$ ... are divisible by 5./*is deelbaar deur* 5. The largest integer value of k such that $5k - 4 \le 1049$ $5k \le 1053$ $k \le 210{,}6$ $k = 210$ **OR/*OF*** **5** ; 1 –3 ; –7 ; … ; **–4175**; –4179 ;–4183 ; –4187 $T_n = a + (n-1)d$ $-4175 = 5 + (n-1)(-4)$ $-4180 = -4(n-1)$ $n = 1046$ Number of terms divisible by 5 $= \frac{1046 - 1}{5} + 1$ $= 210$	✓ $-4n + 9 = -4187$ ✓ $n = 1049$ ✓ $5k - 4 \le 1049$ ✓ $k = 210$ (4) ✓ $d = -4$ ✓ $-4175 = -4n + 9$ ✓ 1046 ✓ $n = 210$ (4) **[11]**

QUESTION/*VRAAG* 3

3.1.1	$-1 \;;\; x \;;\; 3 \;;\; x+8 \;;\; \ldots$  $-2x+2 = 2x+2$ $4x = 0$ $x = 0$	✓ $x+1;3-x$ and $x+5$ ✓ calculating second differences ✓ $-2x+2 = 2x+2$ ✓ $x = 0$ (4)
3.1.2	First differences/*Eerste verskille*: 1 ; 3 ; 5 ; …… $S_n = \frac{n}{2}[2(1)+(n-1)(2)]$ $= n^2$ $250 < n^2$ $n > \sqrt{250}$ $\therefore n > 15{,}8$ The sum of the 16 first differences will be greater than 250. Therefore the 17th term of the quadratic number pattern is the first satisfying this condition./*Die som van* 16 *eerste verskille sal groter as* 250 *wees. Gevolglik sal die* 17*de term van die kwadratiese getalpatroon die eerste wees wat aan die voorwaarde voldoen.*	✓ $S_n = n^2$ ✓ $S_n > 250$ ✓ $n > 15{,}8$ ✓ $n = 17$ (4)
3.2.1	$21 + 21(0{,}85) + 21(0{,}85)^2 + \ldots\ldots$ $T_n = ar^{n-1}$ $T_{10} = (21)(0{,}85)^9$ $= 4{,}86$ cm	✓ $n = 10$; $r = 0{,}85$ or $\frac{17}{20}$ ✓ substitution into correct formula ✓ answer (3)
3.2.2	$S_n = \frac{a(1-r^n)}{1-r}$ $S_{15} = \frac{21(1-(0{,}85)^{15})}{1-0{,}85}$ $= 127{,}77$ Area of the page = 30 x 21 = 630 Percentage of paper covered in grey ink: $= \frac{127{,}77}{630} \times 100\%$ $= 20{,}28\%$	✓ $n = 15$ ✓ 127,77 ✓ 630 ✓ 20,28 (4) **[15]**

QUESTION/*VRAAG* 4

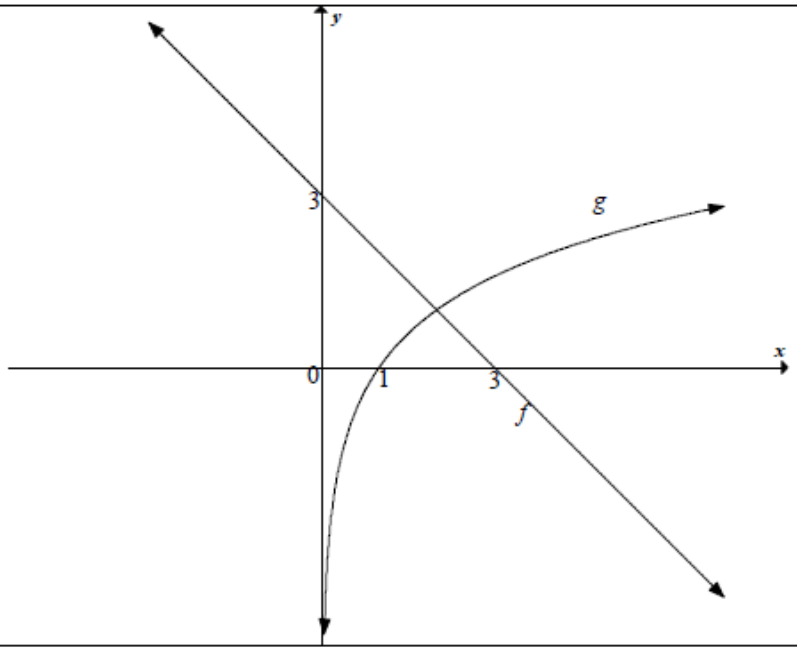
4.1	$y = 0$	✓ $y = 0$ (1)
4.2	R(0 ; 1)	✓ answer (1)
4.3	$y = a^x$ $9 = a^2$ $\therefore a = 3$	✓ substitution ✓ $a = 3$ (2)
4.4	DP = $2 - b$ $y = 3^x$ $\frac{1}{81} = 3^b$ $3^{-4} = 3^b$ $b = -4$ DP = $2 - (-4)$ = 6 units	✓ $\frac{1}{81} = 3^b$ ✓ 3^{-4} or use of logs ✓ $b = -4$ ✓ DP = 6 units (4)
4.5	$h(x+2) + k = 0$ $h(x+2) = -k$ $0 < -k < \frac{1}{81}$ $-\frac{1}{81} < k < 0$	✓✓ $-k < \frac{1}{81}$ or $k > -\frac{1}{81}$ ✓ $-\frac{1}{81} < k < 0$ (3)
		[11]

QUESTION/*VRAAG* 5

5.1	$f(x)=-x^2+4x-3$ $f'(x)=0$ or $x=-\frac{4}{2(-1)}$ $-2x+4=0$ $x=2$ $x=2$ $y=-(2)^2+4(2)-3$ $=1$ $B(2;1)$ **OR/OF** $-x^2+4x-3=0$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3$ or $x=1$ $x=\frac{3+1}{2}$ $x=2$ $y=-(2)^2+4(2)-3$ $=1$ B(2 ; 1)	✓ $-2x+4=0$ or $x=-\frac{4}{2(-1)}$ ✓ $y=-(2)^2+4(2)-3$ (2) ✓ $x=\frac{3+1}{2}$ ✓ $y=-(2)^2+4(2)-3$ (2)
5.2	Range/*Waardeversameling* : $y\le 1$ **OR/OF** Range/*Waardeversameling* : $y\in(-\infty;1]$	✓ $y\le 1$ (1) ✓ $(-\infty;1]$ (1)
5.3	$x\le -1$ or $x>2$ **OR/OF** $(-\infty;-1]\cup(2;\infty)$	✓ critical values ✓ $x\le -1$ or $x>2$ (2) ✓ critical values ✓ $x\le -1$ or $x>2$ (2)
5.4	$(x-p)(y+t)=3$ Vertical asymptote of h(x) /*vertikale asimptoot* at $x=2$ Translation 4 units to the left / *Translasie 4 eenhede links* $x=2-4=-2$ is the equation of the vertical asymptote of h(x + 4) $x=2-4=-2$ *is die vergelyking van die vertikale asimptoot* **OR/OF**	✓ $x=-2$ (1)

	OR/OF $h(x)=\frac{3}{x-2+4}+1$ $=\frac{3}{x+2}+1$ $x=-2$ is the equation of the vertical asymptote / *is die vergelyking van die vertikale asimptoot*	✓ $x=-2$ (1)
5.5	$(x-p)(y+t)=3$ $(y+t)=\frac{3}{(x-p)}$ $y=\frac{3}{x-p}-t$ $B(2;1)$ Point of intersection of the asymptotes *Snypunt van die asimptote* $p=2$ $-t=1$ $t=-1$	✓ $\frac{3}{x-p}$ ✓ $-t$ ✓ $p=2$ ✓ $t=-1$ (4)
5.6	x-intercepts of f / *x-afsnitte van f*: $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or $x=3$ $g'(x)<0$ for $x\in R;\ x\neq 2$ Hence $f(x)<0$ $x\leq 1$ or $x\geq 3$ **OR/OF** $(-\infty;1]\cup[3;\infty)$	✓ both critical values ✓ $x\leq 1$ ✓ or ✓ $x\geq 3$ (4) **[14]**

QUESTION/*VRAAG 6*

6.1		*g*: ✓shape: increasing curve ✓(1 ; 0): only on log graph *f*: ✓(3 ; 0) ✓(0 ; 3) (4)
6.2	$y = \log_2 x$ $g^{-1}: \ x = \log_2 y$ $y = 2^x$	✓ interchange x and y ✓ $y = 2^x$ (2)
6.3	$\log_2(3-x) = x$ $2^x = 3 - x$ $2^x = -x + 3$ Reflect the graph of g about the line $y = x$ to obtain g^{-1} and determine the point of intersection of f and g^{-1} / *Reflekteer die grafiek van g om die lyn* $y = x$ *en bepaal die snypunt van* f and g^{-1}	✓✓ $2^x = -x + 3$ ✓ point of intersection of f and g^{-1} (3)
6.4	$x = 1$	✓ answer (1) **[10]**

QUESTION/*VRAAG* 7

7.1	$A = P(1+i)^n$ $= 250000\left(1+\frac{0,15}{12}\right)^2$ $= R256\ 289,06$	✓substituting i and n values in correct formula ✓ answer (2)
7.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $256\ 289,06 = \frac{x\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $3203,6133 = x\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]$ $x = R\ 7\ 359,79$ per month **OR/*OF*** $250\,000 = \frac{x\left(1+\frac{0,15}{12}\right)^{-2}\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $x = R\ 7\ 359,79$	✓ $i = \frac{0,15}{12}$ ✓ $n = 46$ ✓ substitution into correct formula ✓ answer (4) ✓ $i = \frac{0,15}{12}$ ✓ $n = 46$ ✓ substitution into correct formula ✓ answer (4)
7.3	$256\ 289,06 = \frac{9\,000\left[1-\left(1+\frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\left(1+\frac{0,15}{12}\right)^{-n} = 0,6440429722$ $-n\log\left(1+\frac{0,15}{12}\right) = \log 0,6440429722$ $n = 35,41872568$ months/ *maande* $\therefore$ 36 payments are required $\therefore$ 36 *paaiemente moet betaal word* $\therefore$ Thabiso will pay his loan off 10 months sooner./Thabiso *los sy lening 10 maande vroeër af.* **OR/*OF***	✓$x = 9\,000$ ✓ substitute into correct formula ✓ use of logs ✓ $n = 35,42$ ✓ 10 months (5)

	$256289{,}06\left(1+\frac{0{,}15}{12}\right)^n = \frac{9000\left[\left(1+\frac{0{,}15}{12}\right)^n - 1\right]}{\frac{0{,}15}{12}}$ $3203{,}61325\left(1+\frac{0{,}15}{12}\right)^n = 9000\left(1+\frac{0{,}15}{12}\right)^n - 9000$ $9000 = 5796{,}38675\left(1+\frac{0{,}15}{12}\right)^n$ $n = \log_{\left(1+\frac{0{,}15}{12}\right)} 1{,}5523691425$ $n = 35{,}41872568$ $\therefore$ 36 payments are required $\therefore$ 36 *paaiemente moet betaal word* $\therefore$ Thabiso will pay his loan off 10 months sooner./Thabiso *los sy lening 10 maande vroeër af.*	✓ 9 000 ✓ substitute into correct formula ✓ use of logs ✓ n =35,42 ✓ 10 months (5)
7.4	The balance of his loan after the 35th payment was made: *Die balans van sy lening nadat die 35ste paaiement betaal is:* $\text{Balance} = 256289{,}06\left(1+\frac{0{,}15}{12}\right)^{35} - \frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{35} - 1\right)}{\frac{0{,}15}{12}}$ $= \text{R } 3\,735{,}45$ $\text{Final instalment} = 3\,735{,}45\left(1+\frac{0{,}15}{12}\right)$ $= \text{R } 3\,782{,}14$	✓ $256289{,}06\left(1+\frac{0{,}15}{12}\right)^{35}$ ✓ $\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{35} - 1\right)}{\frac{0{,}15}{12}}$ ✓ $3\,735{,}45\left(1+\frac{0{,}15}{12}\right)$ ✓ answer (4)
	OR/OF $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ Final instalment $= \frac{9\,000\left[1-\left(1+\frac{0{,}15}{12}\right)^{-0{,}41872568}\right]}{\frac{0{,}15}{12}}\left(1+\frac{0{,}15}{12}\right)$ $= \text{R } 3\,782{,}14$	✓ 0,41872568 ✓ $\frac{9\,000\left[1-\left(1+\frac{0{,}15}{12}\right)^{-0{,}41872568}\right]}{\frac{0{,}15}{12}}$ ✓ $\times\left(1+\frac{0{,}15}{12}\right)$ ✓ answer (4)
	OR/OF	

	$\text{Balance} = 256289{,}06\left(1+\frac{0{,}15}{12}\right)^{36} - \frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{36}-1\right)}{\frac{0{,}15}{12}}$ $= \text{R } -5\,217{,}86$ $\text{Final payment} = 9\,000 - 5217{,}86$ $= \text{R } 3\,782{,}14$	✓ $256289{,}06\left(1+\frac{0{,}15}{12}\right)^{36}$ ✓ $\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{36}-1\right)}{\frac{0{,}15}{12}}$ ✓ $9\,000 - 5217{,}86$ ✓ answer (4) **[15]**

QUESTION/*VRAAG* 8

8.1	$f(x+h) = 3(x+h)^2$ $= 3(x^2 + 2xh + h^2)$ $= 3x^2 + 6xh + 3h^2$ $f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 - 3x^2$ $= 6xh + 3h^2$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{6xh+3h^2}{h}$ $= \lim_{h\to 0}\frac{h(6x+3h)}{h}$ $= \lim_{h\to 0}(6x+3h)$ $= 6x$	✓ $3(x+h)^2$ ✓ $6xh + 3h^2$ ✓ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓ $\lim_{h\to 0}(6x+3h)$ ✓ $6x$ (5)
	OR/OF $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{3(x+h)^2 - 3x^2}{h}$ $= \lim_{h\to 0}\frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$ $= \lim_{h\to 0}\frac{6xh+3h^2}{h}$ $= \lim_{h\to 0}\frac{h(6x+3h)}{h}$ $= \lim_{h\to 0}(6x+3h)$ $= 6x$	✓ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓ $3(x+h)^2 - 3x^2$ ✓ $6xh + 3h^2$ ✓ $\lim_{h\to 0}(6x+3h)$ ✓ $6x$ (5)

8.2	$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$ $g(x)=\sqrt{x}$ $a=4$	✓ answer ✓ answer (2)
8.3	$y=\sqrt{x^3}-\frac{5}{x^3}$ $y=x^{\frac{3}{2}}-5x^{-3}$ $\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+15x^{-4}$	✓ $x^{\frac{3}{2}}$ ✓ $-5x^{-3}$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ ✓ $15x^{-4}$ (4)
8.4	$f(x)=x^3+ax^2+bx+18$ $f'(x)=3x^2+2ax+b$ At $x=1,\ m_{tan}=-8$ $f'(1)=-8$ $3(1)^2+2a(1)+b=-8$ $3+2a+b=-8$ $2a+b=-11$ .....(1) $y=f(1)$ $=g(1)$ $=-8(1)+20$ $=12$ $1+a+b+18=12$ $a+b=-7$ .....(2) $a=-4$ $b=-3$	✓ $3x^2+2ax+b$ ✓ $f'(1)=-8$ or $3(1)^2+2a(1)+b=-8$ ✓ $1+a+b+18=12$ ✓ $a=-4$ ✓ $b=-3$ (5) **[16]**

QUESTION/*VRAAG* 9

9.1	$f'(x) = 3x^2 + 8x - 3 = 0$ $(3x-1)(x+3) = 0$ $x = \frac{1}{3}$ *or* $x = -3$	✓ equating derivative to zero ✓ factors ✓ x – values (3)
9.2	$f''(x) = 6x + 8$ $6x + 8 < 0$ $x < -\frac{4}{3}$ **OR** $x = \frac{\frac{1}{3} - 3}{2}$ $= \frac{4}{3}$ $\therefore x < -\frac{4}{3}$	✓ $6x+8$ ✓✓ $x < -\frac{4}{3}$ (3) ✓ $\frac{\frac{1}{3} - 3}{2}$ ✓✓ $x < -\frac{4}{3}$ (3)
9.3	$x \le -3$ or $x \ge \frac{1}{3}$ **OR/OF** $[-\infty;-3] \cup \left[\frac{1}{3};\infty\right]$	✓ $x \le -3$ ✓ $x \ge \frac{1}{3}$ (2) ✓ $[-\infty;-3]$ ✓ $\left[\frac{1}{3};\infty\right]$ (2)
9.4	$f(0) = -18$ $d = -18$ $f(x) = ax^3 + bx^2 + cx - 18$ $f'(x) = 3ax^2 + 2bx + c$ $f'(x) = 3x^2 + 8x - 3$ $3a = 3 \quad 2b = 8$ $a = 1 \quad b = 4 \quad c = -3$ $f(x) = x^3 + 4x^2 - 3x - 18$ **OR/OF** $f'(x) = 3x^2 + 8x - 3$ By integration/*Deur integrasie* $f(x) = x^3 + 4x^2 - 3x + d$ $f(0) = d = -18$ $a = 1$ $b = 4$ $c = -3$	✓ $d = -18$ ✓ $f'(x) = 3ax^2 + 2bx + c$ ✓ $a = 1$ ✓ $b = 4$ ✓ $c = -3$ (5) ✓ $f(x) = x^3 + 4x^2 - 3x + d$ ✓ $d = -18$ ✓ $a = 1$ ✓ $b = 4$ ✓ $c = -3$ (5)
		[13]

QUESTION/*VRAAG* 10

10.1	$M(t) = -t^3 + 3t^2 + 72t$ $M(3) = -(3)^3 + 3(3)^2 + 72(3)$ $= 216$ 216 molecules/*molekules*	✓ $M(3) = -(3)^3 + 3(3)^2 + 72(3)$ ✓ 216 (2)
10.2	$M(t) = -t^3 + 3t^2 + 72t$ $M'(t) = -3t^2 + 6t + 72$ $M'(2) = -3(2)^2 + 6(2) + 72$ $= 72$ 72 molecules per hour/*molekules per uur*	✓ $M'(t) = -3t^2 + 6t + 72$ ✓ $M'(2)$ ✓ 72 (3)
10.3	$M(t) = -t^3 + 3t^2 + 72t$ $M'(t) = -3t^2 + 6t + 72$ $M''(t) = 0$ $-6t + 6 = 0$ $t = 1$ Maximum rate of change of the number of molecules of the drug in the bloodstream is after 1 hour./*Maksimum tempo van verandering van die getal molekules in die bloedstroom is na 1 uur*	✓ $M''(t)$ ✓ $M''(t) = 0$ ✓ answer (3) **[8]**

QUESTION/*VRAAG* 11

11.1

	Watches TV during exams	Do not watch TV during exams	Total
Male	80	a	$80+a$
Female	48	12	60
Total	b	32	160

$a+12=32$

$a=20$

$b=80+48$

$=128$

✓$a=20$

✓$b=128$ (2)

11.2

No

$P(\text{M and not watching TV})=\frac{20}{160}\neq 0$

✓No

✓reason (2)

11.3.1

$P(\text{watching TV})=\frac{128}{160}=\frac{4}{5}=0,8=80\%$

✓128
✓160 (2)

11.3.2

$P(\text{female and not watching TV})=\frac{12}{160}=\frac{3}{40}=0,075=7,5\%$

✓ 12
✓160 (2)

[8]

QUESTION/*VRAAG* 12

12.	We want to create codes that are even numbers greater than 5000. The digit 6 can be used in one of two places in these codes and therefore this presents two scenarios. *Ons wil kodes kry wat ewe getalle groter as* 5000 *is. Die syfer* 6 *kan in twee posisies in die kode gebruik word en twee opsies is moontlik:* CASE 1: The first digit is a 6./*Die eerste syfer is'n* 6. **6** ____ × ____ × ____ × **2** / **4** 1 × 5 × 4 × 2 Number of codes starting with 6./*Getal kodes wat met* 6 *begin.* $= 1 \times 5 \times 4 \times 2 = 40$ CASE 2: The first digit is a 5 or 7./*Die eerste syfer is'n* 5 *of* 7. **5** / **7** ____ × ____ × ____ × **2** / **4** / **6** 2 × 5 × 4 × 3 Number of codes not starting with 6./*Getal kodes wat nie met* 6 *begin* $= 2 \times 5 \times 4 \times 3 = 120$ Therefore total number of possible codes./*Die totale getal moontlike kodes* = 40 + 120 = 160. **OR/OF** $(3\times5\times4\times1) + (3\times5\times4\times1) + (2\times5\times4\times1)$ $= 60 + 60 + 40$ $= 160$ **OR/OF** $(3\times5\times4\times3) - (1\times5\times4\times1)$ $= 180 - 20$ $= 160$	✓ $1 \times 5 \times 4 \times 2$ ✓ 40 ✓ $2 \times 5 \times 4 \times 3$ ✓ 120 ✓ 160 **[5]** ✓ $(3\times5\times4\times1)$ ✓ $(3\times5\times4\times1)$ ✓ $(2\times5\times4\times1)$ ✓✓ 160 **[5]** ✓✓ $(3\times5\times4\times3)$ ✓✓ $(1\times5\times4\times1)$ ✓ 160 **[5]**

TOTAL/*TOTAAL*: 150

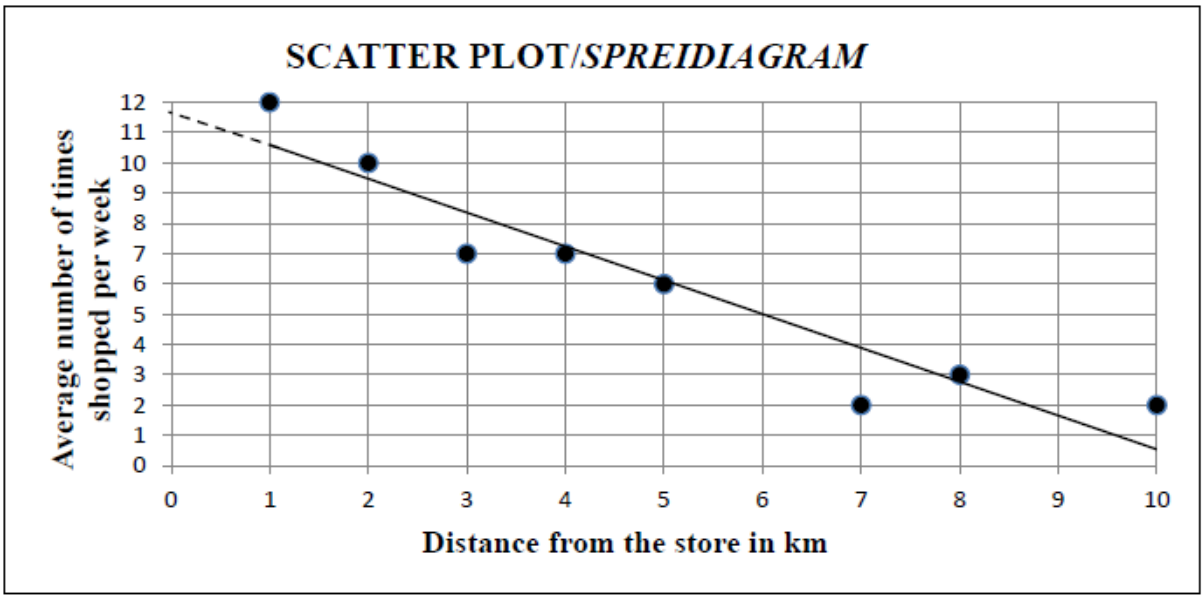
MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

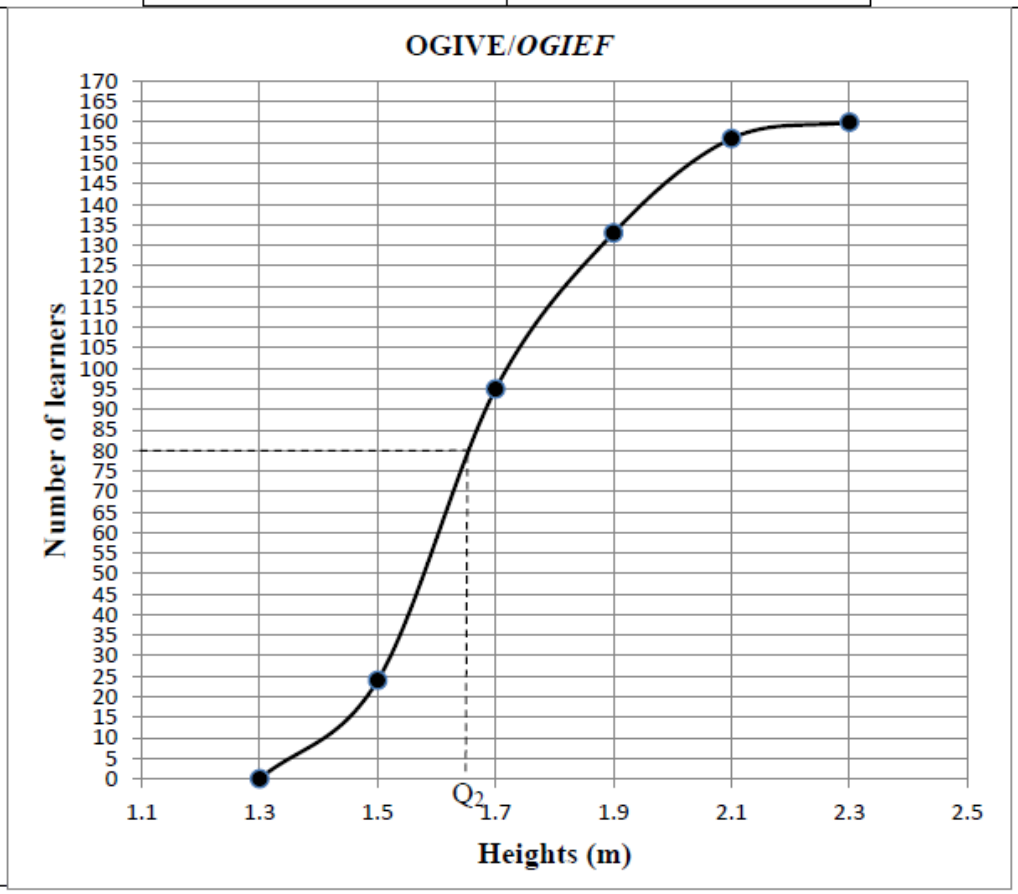
QUESTION/*VRAAG* 1

Distance from the store in km *Afstand vanaf die winkel in* **km**	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers die winkel per week besoek*	12	10	7	7	6	2	3	2

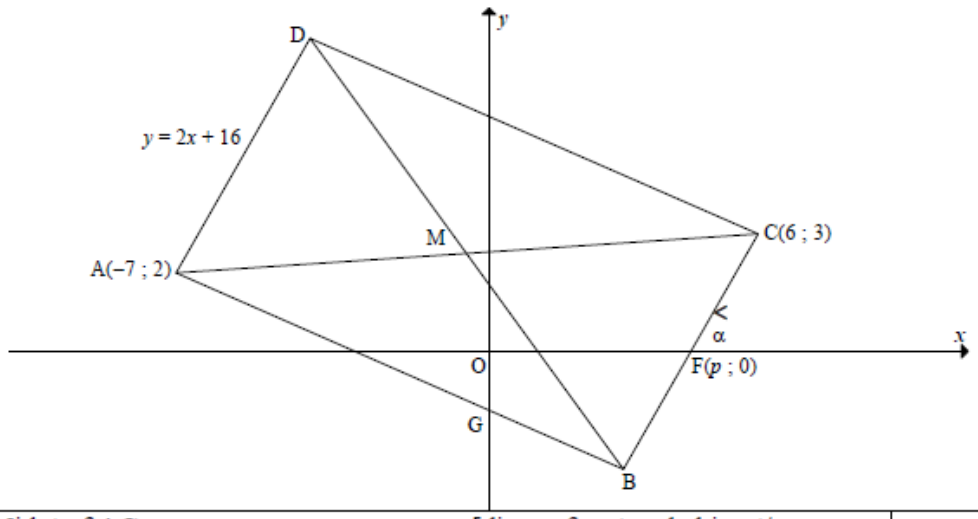


1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71$ (11,7132…) $b = -1{,}12$ (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of a ✓ value of b ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ = 5 times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

QUESTION/*VRAAG* 2

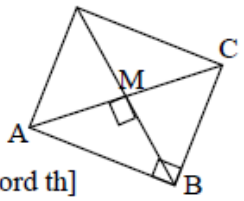
2.1	Positively skewed **OR** skewed to the right/*positief skeef **OF** skeef na regs*	✓ answer (1)
2.2	Range/*Omvang* = 2,21 – 1,39 = 0,82 m	✓ subtract values ✓answer (2)
2.3	**Intervals** *Klasse* / **Cumulative frequency** *Kumulatiewe frekwensie*: $1,3 \le x < 1,5$: 24; $1,5 \le x < 1,7$: 95; $1,7 \le x < 1,9$: 133; $1,9 \le x < 2,1$: 156; $2,1 \le x < 2,3$: 160	✓95 , 133, 156 ✓160 (2)
2.4		✓ upper limits / *boonste limiete* ✓ cum *f*/ *kum f* ✓ shape/ *vorm* ✓ grounded *geanker* (4)
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)	✓ method ✓ answer (2)
2.6.1	The mean would change by 0,1 *m* *Die gemiddelde sal met 0,1 m verander*	✓ answer (1)
2.6.2	No influence/change as there is no difference in variation of data./*Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.*	✓ answer (1) **[13]**

QUESTION/*VRAAG 3*

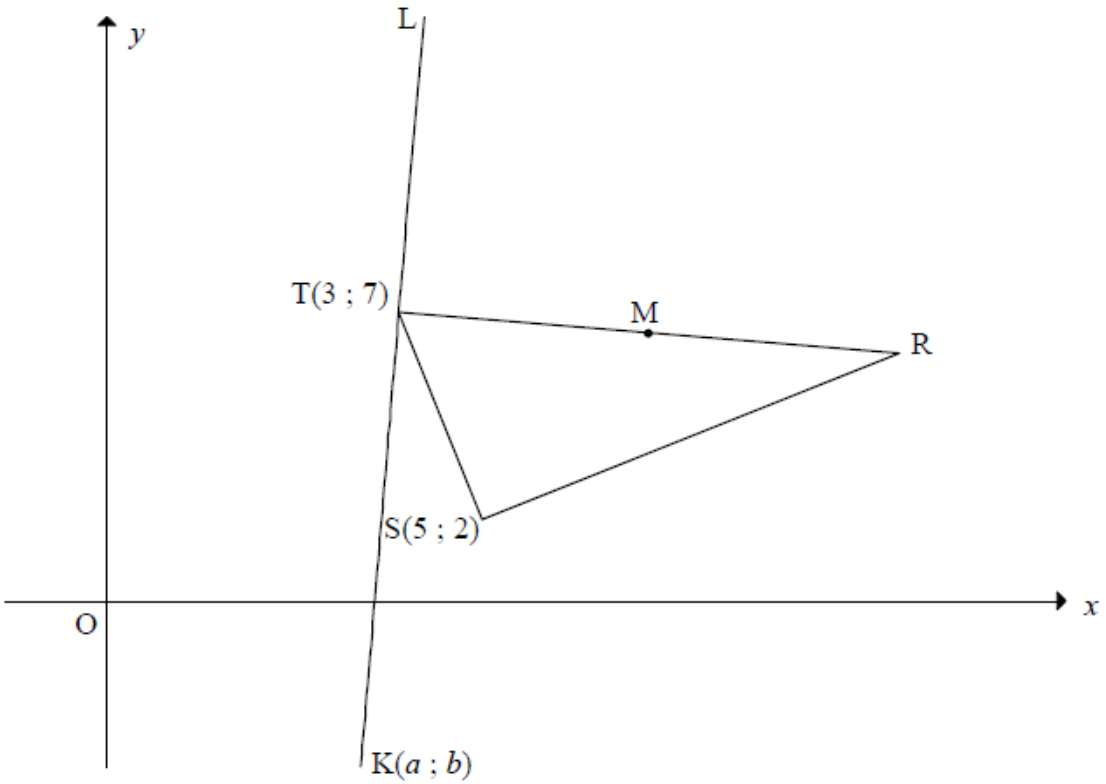


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= \text{M}\left(\frac{-7+6}{2}\,;\,\frac{2+3}{2}\right)$ $= \text{M}\left(-\frac{1}{2}\,;\,\frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{AD} = m_{BC}$ [AD || BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6;3)$ $y - 3 = 2(x - 6)$ $\therefore y = 2x - 9$ *but* $y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{BC} = 2$ ✓ equating ✓answer (3) ✓ $m_{BC} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \quad \therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst ∠e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som ∠e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42{,}5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$C\hat{B}M = B\hat{A}M = 45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF*** $A\hat{M}B = 90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th] 	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION/*VRAAG 4*



4.1	∠ in semi circle/ ∠ at centre = 2∠ on circle *∠ in halfsirkel /∠ by middelpt = 2∠ op sirkel*	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

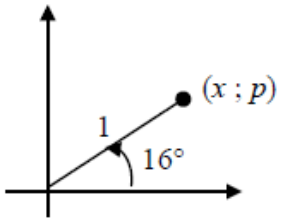
	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓ substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore$ R(15 ; 6) Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9+6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)

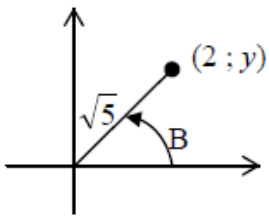
	$m_{TM} = \dfrac{9-3}{6\frac{1}{2}-7} = -\dfrac{1}{12}$ $TM: \quad 7 = -\dfrac{1}{12}(3) + c \quad y = -\dfrac{1}{12}x + \dfrac{29}{4} \quad .....(1)$ $SR: \quad y = \dfrac{2}{5}x \quad .....(2)$ $\dfrac{2}{5}x = -\dfrac{1}{12}x + \dfrac{29}{4}$ $\dfrac{29}{60}x = \dfrac{29}{4}$ $\therefore x = 15$ $\therefore y = \dfrac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $ST = \sqrt{(5-3)^2 + (2-7)^2}$ $ST = \sqrt{4+25} = \sqrt{29}$ $\sin R = \dfrac{TS}{TR} = \dfrac{\sqrt{29}}{\sqrt{145}} or \dfrac{\sqrt{5}}{5} or \dfrac{1}{\sqrt{5}} or \ 0{,}45$ **OR/OF** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ area of $\Delta TSR = \dfrac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$ $29 = \dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \dfrac{\sqrt{5}}{5} or \dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ **OR/OF** $m_{TR} = \dfrac{7-6}{3-15} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $m_{KTL} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x - 3)$ $y = 12x - 29$ substitute $K(a;b)$: $b = 12a - 29$ **OR/OF**	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a - 29$ **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓ substitution (3 ; 7) & (a ; b) (3) ✓ subst into Pyth ✓ multiplication ✓ simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a - 29 - 7)^2 = 145$ $(a-3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2 \text{ or } a = 4$ $\therefore b = 12(2) - 29 \quad \text{or} \quad b = 12(4) - 29$ $= -5 \qquad\qquad = 19$ $\therefore$ K(2 ; –5) **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	TK = TR $\sqrt{(a-3)^2+(b-7)^2} = \sqrt{145}$ $(a-3)^2+(b-7)^2 = 145$ Substitute $b = 12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2 = 145$ $(a-3)^2+(12a-36)^2 = 145$ $(a-3)^2+144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ $a = 2$ or 4 $\therefore b = 12(2)-29$ or $b = 12(4)-29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5) **OR/*OF*** $KR^2 = TR^2+TK^2$ $(a-15)^2+(b-6)^2 = 145+145$ $(a-15)^2+(12a-29-6)^2 = 290$ $(a-15)^2+(12a-35)^2 = 290$ $a^2-30a+225+144a^2-840a+1225 = 290$ $145a^2-870a+1160 = 0$ $a^2-6a+8 = 0$ $\therefore (a-2)(a-4) = 0$ $a = 2$ or $a = 4$ $\therefore b = 12(2)-29$ or $b = 12(4)-29$ $= -5$ $= 19$ K(2 ; – 5)	✓ substitution into distance formula ✓ substitution of $b = 12a-29$ ✓ $(a-3)^2 = 1$ ✓ ± 1 ✓ values of a ✓ value of b (6) ✓ substitution ✓ substitution of $b = 12a-29$ ✓standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

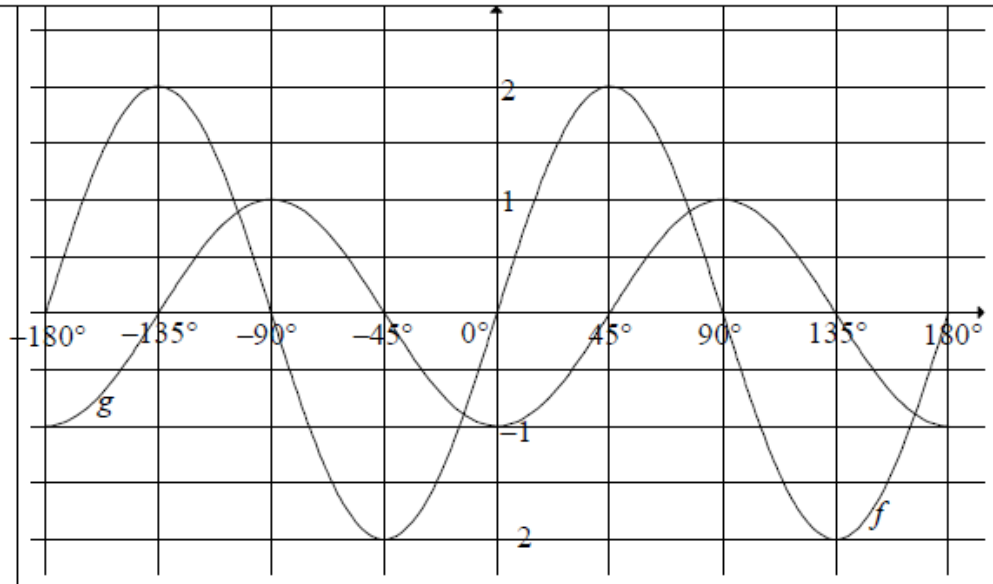
QUESTION/*VRAAG 5*

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF*** $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\frac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ $= \frac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \frac{\sin 2A}{\cos A.(-\sin A)}$ $= \frac{2\sin A\cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\frac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90° + A)} = \frac{\sqrt{1-(2\cos^2 A - 1)^2}}{\cos A.-\sin A}$ $= \frac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A.-\sin A}$ $= \frac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$ $= \frac{2\cos A.\sin A}{\cos A.-\sin A}$ $= -2$ **OR/OF**	✓$\sqrt{\sin^2 2A}$ ✓cosA ✓$-\sin A$ ✓$2\sin A\cos A$ ✓ answer (5) ✓$2\cos^2 A - 1$ ✓cosA ✓$-\sin A$ ✓identity ✓ answer (5)

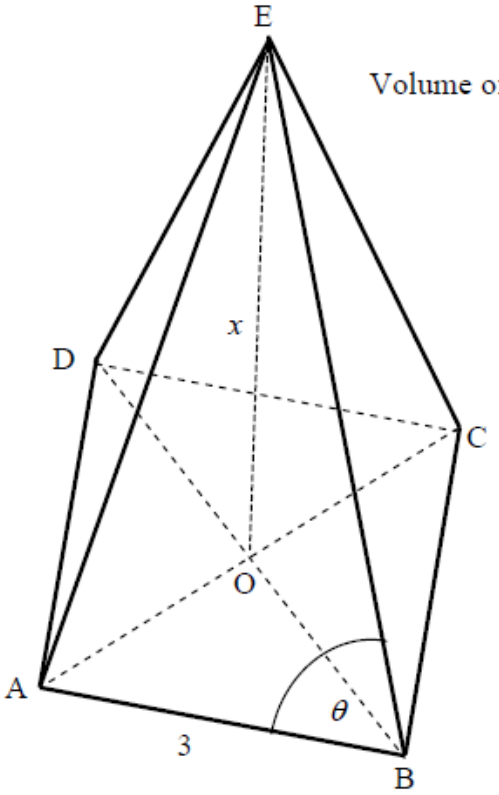
	$\frac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A.-\sin A}$ $=\frac{\sqrt{1-(1-4\sin^2 A+4\sin^2 A)}}{\cos A.-\sin A}$ $=\frac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A.-\sin A}$ $=\frac{2\sin A\sqrt{\cos^2 A}}{\cos A.-\sin A}$ $=-2$	✓ $1-2\sin^2 A$ ✓ cosA ✓ $-$ sinA ✓ identity ✓ answer (5)
5.4.1	$\cos 2B=\frac{3}{5}$ $2\cos^2 B-1=\frac{3}{5}$ $\cos^2 B=\frac{4}{5}$ $\therefore \cos B=\sqrt{\frac{4}{5}}$ *or* $\frac{2}{\sqrt{5}}$ *or* $\frac{2\sqrt{5}}{5}$ $[0°\le B\le 90°]$ **OR/*OF*** $\cos B=\frac{\sqrt{\cos 2B+1}}{2}$ $=\frac{\sqrt{\frac{3}{5}+1}}{2}$ $=\frac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2 B$ ✓ answer (3) ✓ $=\frac{\sqrt{\cos 2B+1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer (3)
5.4.2	$\sin^2 B=1-\cos^2 B$ $=1-\left(\frac{2}{\sqrt{5}}\right)^2$ $=\frac{1}{5}$ $\therefore \sin B=\frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2+y^2=(\sqrt{5})^2$ $4+y^2=5$ $y^2=1$ $y=1$ $\therefore \sin B=\frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$  	✓ $\sin^2 B=\frac{1}{5}$ ✓ answer (2) ✓ $y=1$ ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
	OR/*OF* $\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

QUESTION/*VRAAG 6*

6.1		✓ x- intercepts/ *afsnitte* ✓ y- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ $\therefore$ maximum value $=2-3=-1$	✓ ✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z$ *or* $x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°$ *or* $333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°$ *or* $166{,}72°+k.180°;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or $-26{,}56°$ ✓ $76{,}72°$ or $-13{,}28°$ ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & $333{,}43°$ ✓ $76{,}72°$ & $166{,}72°$ ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

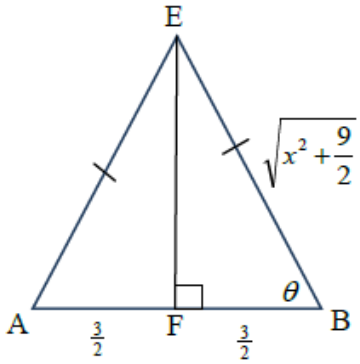
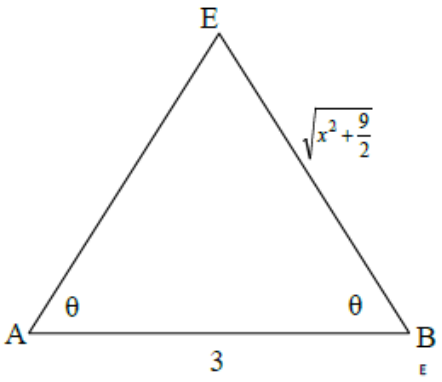
QUESTION/*VRAAG* 7



$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

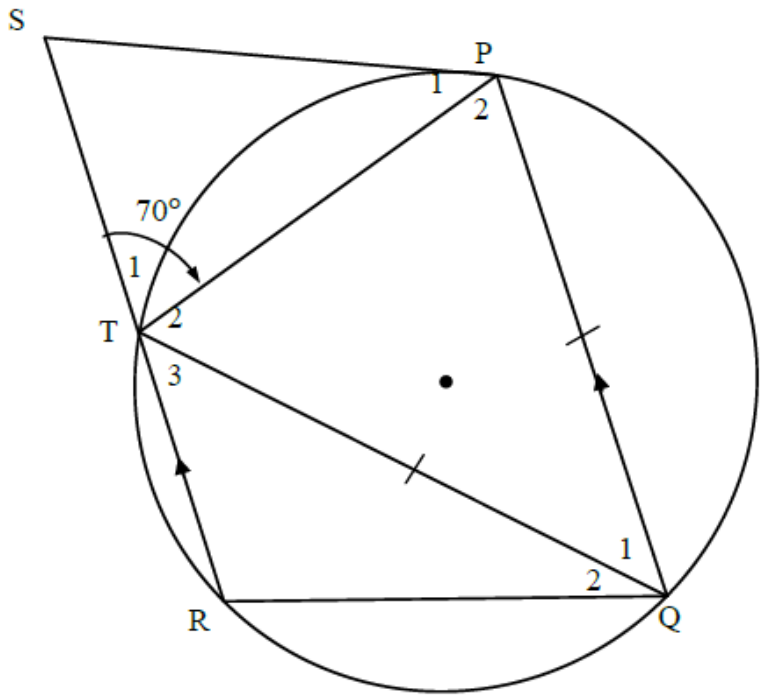
7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12 **OR/*OF*** $\sin 45° = \frac{OB}{3}$ $OB = 3\sin 45°$ $OB = \frac{3\sqrt{2}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* 2,12 **OF/OR** $\cos 45° = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)

	OR/OF $A\hat{O}B = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ cos θ as subject ✓ simplification (5)
	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right).\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	s ✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ substituting ✓ cos θ as subject (5)

	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  **OR/OF** $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\sin\theta\cos\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\cos\theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5) ✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	$\text{Volume} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$ $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

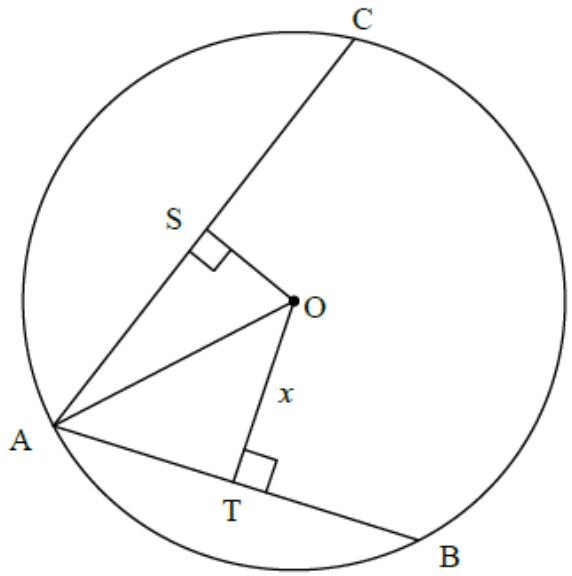
QUESTION/*VRAAG 8*

8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ∥ SR		✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ $\therefore \hat{Q}_1 = 180° - 2(70°)$ $= 40°$	[∠s opp = sides/*∠e teenoor = sye*] [∠s/*e* Δ = 180°]	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$	[tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

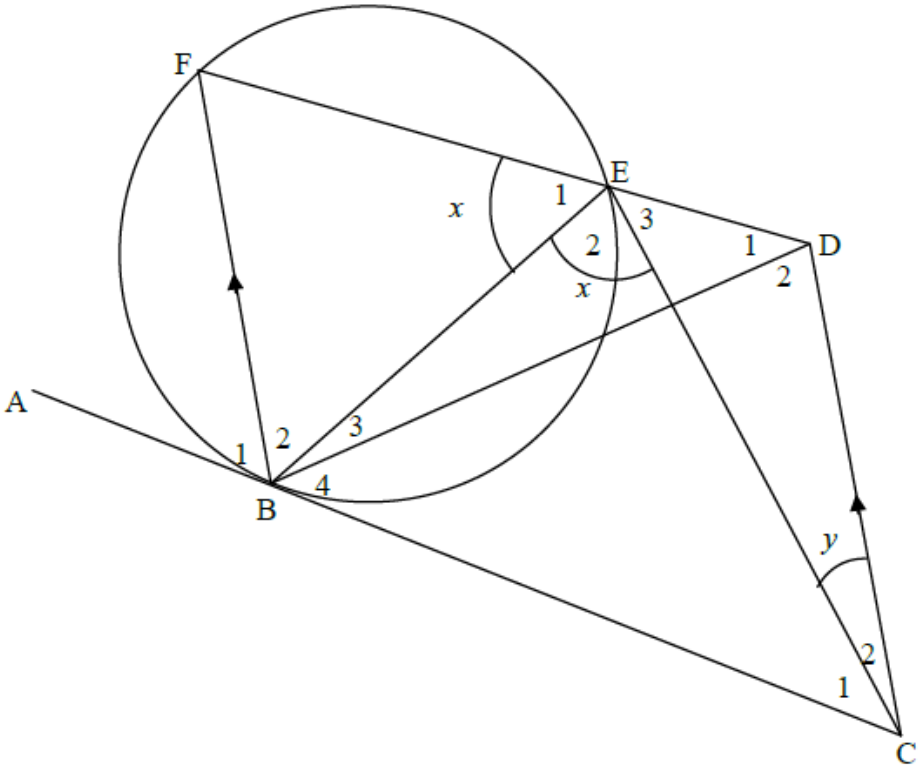
8.2



8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore OA = 25$ **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] Let OT = 15x. Then OS = 7x But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ equating ✓ AS = 24 ✓ substitution ✓ $x = 1$ ✓ radius (5) ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth ✓ radius (5) **[12]**

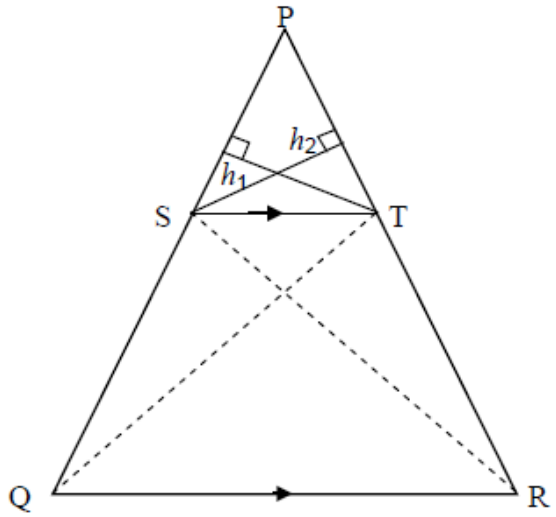
QUESTION/*VRAAG 9*



9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige ∠s/e*; FB ∥ DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ ∴ BCDE = cyclic quad [converse ext ∠ cyc quad/*omgek: buite∠kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [*∠s* in the same segment/*∠e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt *∠s*, BF ∥CD/*verwiss ∠e,BF∥CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [∠s in the same segment/*∠e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [$= x$] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] ∴ ΔBFE///ΔCBE [∠,∠,∠] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

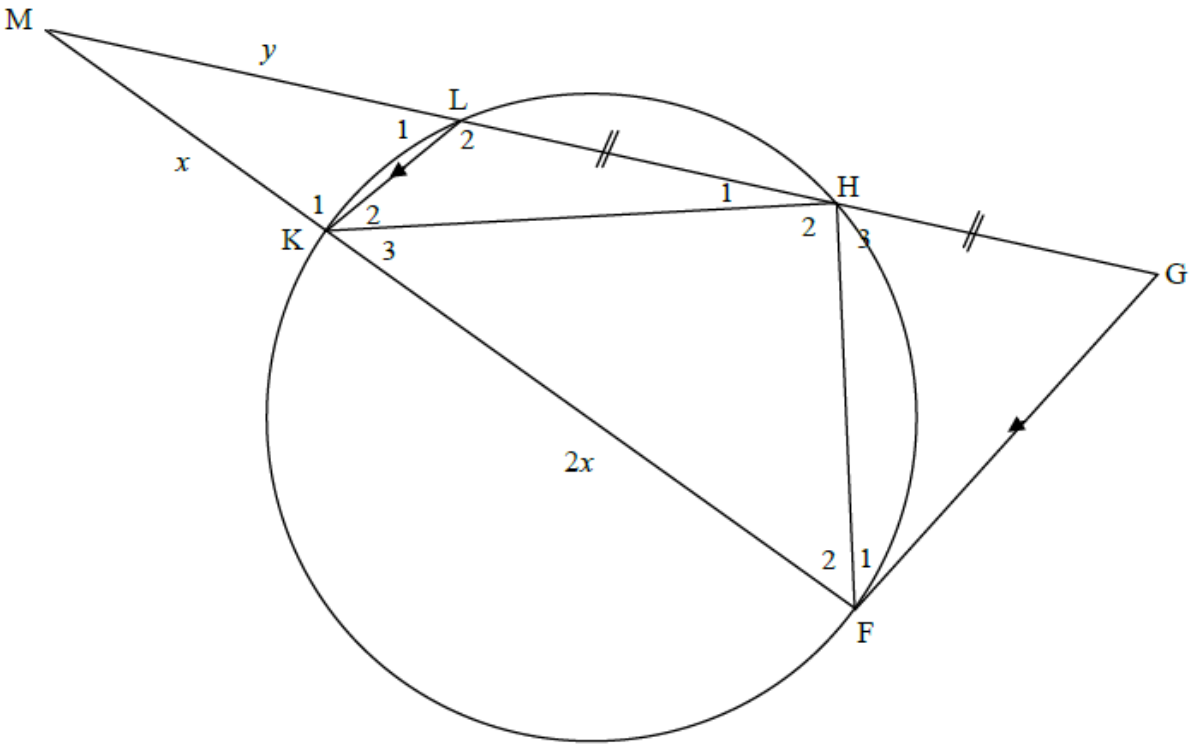
QUESTION/*VRAAG* 10

10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S ⊥ PT and h_2 from T ⊥ PS/ *Verbind SR en TQ en trek h_1 van S ⊥ PT en h_2 van T ⊥ PS*] Proof : $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$ equal altitudes $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes area ΔPST = area ΔPST [common] But area ΔQST = area ΔSTR [same base, height; ST ∥ QR] $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ constr/*konstruksie* ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST}$ $= \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2}$ ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$ ✓ S ✓ R ✓ S (6)

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF ‖ LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM}=\frac{FK}{KM}$ OR $\frac{GL}{y}=\frac{2x}{x}$ [prop theorem/*eweredighst*; GF ‖ LK] $\frac{2GH}{y}=\frac{2x}{x}$ [LH = HG] ∴ GH = *y*	✓S ✓ R ✓ GL = 2GH (3)

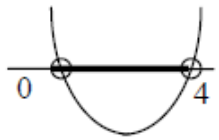
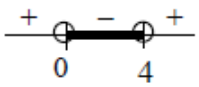
10.2.2(b)	$\hat{K}_1 = G\hat{F}M$	[corresponding/*ooreenkomst* ∠ s; GF || LK]	
	$L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$	[ext ∠ cyclic quad/*buite∠koordevh*]	✓S ✓ R
	$M\hat{H}F = G\hat{F}M$		✓ S
	In ΔMFH and ΔMGF:		
	$\hat{M} = \hat{M}$	[common/*gemeen*]	✓ S
	$M\hat{H}F = G\hat{F}M$	[proven/*bewys*]	
	∴ ΔMFH ||| ΔMGF	[∠∠∠]	✓ R (5)
	OR/*OR*		
	$\hat{K}_1 = G\hat{F}M$	[corresponding/*ooreenkomst* ∠ s; GF || LK]	
	$L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$	[ext ∠ cyclic quad/*buite∠koordevh*]	✓S ✓ R
	$M\hat{H}F = G\hat{F}M$		✓ S
	In ΔMFH and ΔMGF:		
	$\hat{M} = \hat{M}$	[common/*gemeen*]	✓ S
	$M\hat{H}F = G\hat{F}M$	[proven/*bewys*]	
	$\hat{F}_2 = \hat{G}$	[∠s of Δ = 180°]	✓ S (5)
	∴ ΔMFH ||| ΔMGF		
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$	[||| Δs]	✓S ✓R
	$= \frac{3x}{2y}$		(2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$	[||| Δs]	✓ S
	$\frac{3x}{2y} = \frac{3y}{3x}$	[from 10.2.2(*c*)]	✓substitution
	$\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$		✓ simplification
	$\frac{y}{x} = \sqrt{\frac{3}{2}}$		(3) **[20]**
		TOTAL MARKS	**150**

MATHEMATICS P1/*WISKUNDE V1*

MARCH 2017

MEMORANDUM

QUESTION/*VRAAG* 1

1.1.1	$(x-3)(x+1)=0$ $x=3$ or $x=-1$	✓ answer ✓ answer (2)
1.1.2	$\sqrt{x^3}=512$ $x^{\frac{3}{2}}=512$ $\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}=\left(8^3\right)^{\frac{2}{3}}$ $x=64$ **OR** $\sqrt{x^3}=512$ $x^3=262144$ $x^3=2^{18}$ $x=2^6$ $x=64$	✓ $x^{\frac{3}{2}}$ ✓ $\left(8^3\right)^{\frac{2}{3}}$ ✓ answer (3) ✓ squaring both sides ✓ $x^3=2^{18}$ ✓ answer (3)
1.1.3	$x(x-4)<0$  **OR / OF**  $0<x<4$ **OR / OF** $x\in(0;4)$	✓ critical values ✓ inequality or interval (2)

1.2.1	$x^2-5x+2=0$ $x=\dfrac{5\pm\sqrt{(-5)^2-4(1)(2)}}{2(1)}$ $x=\dfrac{5\pm\sqrt{17}}{2}$ $x=0{,}44$ or $x=4{,}56$ **OR** $x^2-5x+2=0$ $x^2-5x=-2$ $x^2-5x+\left(-\dfrac{5}{2}\right)^2=-2+\left(-\dfrac{5}{4}\right)^2$ $\left(x-\dfrac{5}{2}\right)^2=\dfrac{17}{4}$ $x=\dfrac{5+\sqrt{17}}{2}$ or $x=\dfrac{5-\sqrt{17}}{2}$ $x=0{,}44$ or $x=4{,}56$	✓ subst correct formula ✓ answer ✓ answer (3) ✓ $\left(x-\dfrac{5}{2}\right)^2=\dfrac{17}{4}$ ✓ answer ✓ answer (3)
1.2.2	$f(x)=x^2-5x+2$ $x^2-5x+2=c$ $x^2-5x+2-c=0$ $b^2-4ac<0$ $(-5)^2-4(1)(2-c)<0$ $25-8+4c<0$ $4c<-17$ $c<-\dfrac{17}{4}$	✓ standard form ✓ $b^2-4ac<0$ ✓ substitution ✓ answer (4)
1.3	$x=2y+2$ $x^2-2xy+3y^2=4$ $(2y+2)^2-2y(2y+2)+3y^2=4$ $4y^2+8y+4-4y^2-4y+3y^2=4$ $3y^2+4y=0$ $y(3y+4)=0$ $y=0$ or $y=-\dfrac{4}{3}$ $x=2$ $x=-\dfrac{2}{3}$	✓ substitution ✓ simplification ✓ standard form ✓ factors ✓ $y=0$; $y=-\dfrac{4}{3}$ ✓ x-values (ca on both x-values) (6)

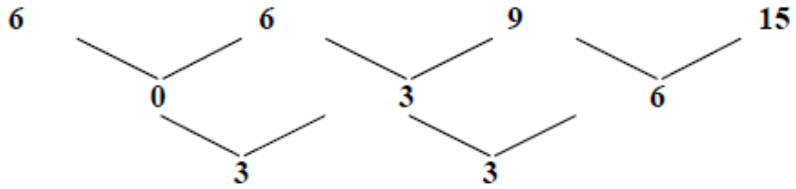
	OR / OF $x = 2y + 2$ $y = \frac{1}{2}x - 1$ $x^2 - 2xy + 3y^2 = 4$ $x^2 - 2x\left(\frac{1}{2}x - 1\right) + 3\left(\frac{1}{2}x - 1\right)^2 = 4$ $x^2 - x^2 + 2x + 3\left(\frac{1}{4}x^2 - x + 1\right) = 4$ $2x + \frac{3}{4}x^2 - 3x + 3 = 4$ $3x^2 - 4x - 4 = 0$ $(3x + 2)(x - 2) = 0$ $x = 2 \quad \text{or} \quad x = -\frac{2}{3}$ $y = 0 \qquad y = -\frac{4}{3}$	✓ substitution ✓ simplification ✓ standard form ✓ factors ✓ $x = 2$; $x = -\frac{2}{3}$ ✓ y-values (ca on both y-values) (6)
1.4	$S = \frac{6}{x^2 + 2}$ For S to be a maximum the denominator needs to be at a minimum. *Vir* S *om 'n maksimum waarde te hê, moet die deler 'n minimum waarde h* Minimum of $x^2 + 2$ is 2 Maximum of $S = \frac{6}{x^2 + 2}$ $= \frac{6}{2}$ $= 3$	✓ Minimum of $x^2 + 2$ is 2 ✓3 (2)
		[22]

QUESTION/*VRAAG* 2

2.1	For geometric: $-\frac{1}{4}\ ;\ b\ ;-1;\ .....$ $\frac{b}{-\frac{1}{4}}=-\frac{1}{b}$ $b^2=\frac{1}{4}$ $b=\pm\frac{1}{2}$ **OR** $b=\pm\sqrt{\left(-\frac{1}{4}\right)(-1)}$ $b=\pm\frac{1}{2}$	✓ $\frac{b}{-\frac{1}{4}}=-\frac{1}{b}$ ✓ $b=\frac{1}{2}$ ✓ $b=-\frac{1}{2}$ (3) ✓ $b=\pm\sqrt{\left(-\frac{1}{4}\right)(-1)}$ ✓ $b=\frac{1}{2}$ ✓ $b=-\frac{1}{2}$ (3)
2.2	$-\frac{1}{4};\ \frac{1}{2};-1;\ .....$ $r=-2$ $T_{19}=ar^{18}$ $=\left(-\frac{1}{4}\right)(-2)^{18}$ $=\left(-\frac{2^{18}}{2^2}\right)$ $=-2^{16}$ $=-65536$ **OR / OF** $T_{19}=ar^{18}$ $=\left(-\frac{1}{4}\right)(-2)^{18}$ $=\left(-2^{-2}\right)\left(2^{18}\right)$ $=-2^{16}$ $=-65536$	✓$r=-2$ ✓subst. into correct formula ✓-65536 / -2^{16} (3) ✓$r=-2$ ✓subst. into correct formula ✓-65536 / -2^{16} (3)

2.3	The series is: $-\frac{1}{4};\frac{1}{2};-1;2;-4;8;.....$ The new positive term series: $\frac{1}{2};\ 2\ ;\ 8\ ;\ 32\ ;\ 128\ ;$ ..... $a=\frac{1}{2}\qquad r=4$ $\sum_{n=1}^{20}\left(\frac{1}{2}\right)(4)^{n-1}$ **OR/*OF*** $\sum_{p=0}^{19}\left(\frac{1}{2}\right)(4)^{p}$ **etc.**	✓ $a=\frac{1}{2}$ ✓ $r=4$ ✓ $\sum_{n=1}^{20}$ or $\sum_{p=0}^{19}$ ✓ correct formula (4)
2.4	No, the series is not convergent / *Nee, die reeks konvergeer nie* $r=4$ and for convergence $-1<r<1$ $r=4$ *en vir konvergering* $-1<r<1$	✓no ✓reason (2)
		[12]

QUESTION/*VRAAG* 3

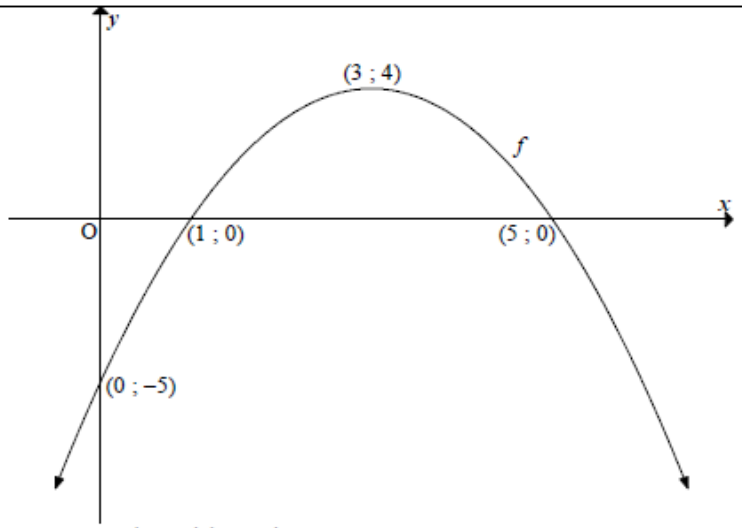
3.1.1	24	✓ 24 (1)
3.1.2	 $2a = 3$ $3a + b = 0$ $a + b + c = 6$ $a = \frac{3}{2}$ $b = -\frac{9}{2}$ $c = 9$ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ **OR/OF** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 6 + (n-1)(0) + \frac{(n-1)(n-2)(3)}{2}$ $= 6 + \frac{n^2 - 3n + 2}{1}\left(\frac{3}{2}\right)$ $= 6 + \frac{3}{2}n^2 - \frac{9}{2}n + 3$ $= \frac{3}{2}n^2 - \frac{9}{2}n + 9$	✓ $a = \frac{3}{2}$ ✓ $b = -\frac{9}{2}$ ✓ $c = 9$ ✓ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ (4) ✓formula ✓substitution ✓simplifying ✓ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ (4)
3.1.3	$\frac{3}{2}n^2 - \frac{9}{2}n + 9 = 3249$ $3n^2 - 9n + 18 = 6498$ $3n^2 - 9n - 6480 = 0$ $n^2 - 3n - 2160 = 0$ $(n + 45)(n - 48) = 0$ $n \neq -45$ *or* $n = 48$	✓ equating general term to 3249 ✓ standard form ✓ factors ✓ $n \neq -45$ or $n = 48$ (4)
3.2	$-1\ ;\ 2\sin 3x\ ;\ 5; \ldots\ldots$ $2\sin 3x + 1 = 5 - 2\sin 3x$ $4\sin 3x = 4$ $\sin 3x = 1$ $3x = 90°$ $x = 30°$	✓ $2\sin 3x + 1 = 5 - 2\sin 3x$ ✓ $\sin 3x = 1$ ✓ $3x = 90°$ ✓ $x = 30°$ (4) **[13]**

QUESTION/*VRAAG* 4

4.1	$U(1;0)$	✓ $(1;0)$ (1)
4.2	$x=1$ $y=1$	✓ $x=1$ ✓ $y=1$ (2)
4.3	$\frac{2}{x-1}+1=0$ $2=-x+1$ $x=-1$ $T(-1;0)$	✓ $y=0$ ✓ $x=-1$ (2)
4.4	$f(x)=\log_5 x$ $h:\ x=\log_5 y$ $y=5^x$	✓ change x and y ✓ $y=5^x$ (2)
4.5	$y=0$	✓ answer (1)
4.6	$V(\sqrt{2}+1;\sqrt{2}+1)$ $V(2{,}41;2{,}41)$ **OR / OF** $x=\frac{2}{x-1}+1$ $x^2-x=2+x-1$ $x^2-2x-1=0$ $x=\frac{2\pm\sqrt{4-4(1)(-1)}}{2}$ $=\frac{2\pm\sqrt{8}}{2}$ $=\frac{2\pm2\sqrt{2}}{2}$ $=1\pm\sqrt{2}$ $V(1+\sqrt{2}\ ;1+\sqrt{2})$ **OR / OF** $x-1=\frac{2}{x-1}$ $(x-1)^2=2$ $x=1\pm\sqrt{2}$ $V(1+\sqrt{2}\ ;1+\sqrt{2})$	✓✓ $\sqrt{2}+1$ ✓✓ $\sqrt{2}+1$ (4) ✓ $x=\frac{2}{x-1}+1$ ✓subs into correct formula ✓ $x=\sqrt{2}+1$ ✓ $y=\sqrt{2}+1$ (4) ✓ $x-1=\frac{2}{x-1}$ ✓ $(x-1)^2=2$ ✓ $x=\sqrt{2}+1$ ✓ $y=\sqrt{2}+1$ (4)
4.7	$T'(3;2)$	✓ $x=3$ ✓ $y=2$ (2) **[14]**

QUESTION 5

5.1.1	C(0 ; –3)	✓ C(0 ; –3) (1)
5.1.2	$f(x)=x^2-2x-3$ $(x-3)(x+1)=0$ $x=-1$ or $x=3$ AB = 3 – (–1) AB = 4 units	✓ factors ✓ x-value ✓ other x-value ✓ answer (4)
5.1.3	$x=\frac{2}{2(1)}$ or $2x-2=0$ or $x=\frac{-1+3}{2}$ $=1$ $y=(1)^2-2(1)-3$ $=-4$ D(1 ; –4)	✓ $x=1$ ✓ y value (2)
5.1.4	C(0 ; –3) D(1 ; – 4) Average gradient / *Gemiddelde gradiënt* $=\frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ $=-1$	✓ $\frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ ✓ – 1 (2)
5.1.5	OC = OB = 3 $O\hat{C}B$= 45° isosceles right angled triangle *Gelykbenige reghoekige driehoek* **OR / OF** $\tan\beta = m_g$ $\tan\beta = 1$ $\beta = 45°$ $O\hat{B}C = 45°$ $O\hat{C}B = 45°$	✓equal lengths ✓ 45° (2) ✓ $\tan\beta=1$ ✓ 45° (2)
5.1.6	$-4 < k < -3$ **OR** (–4 ; –3)	✓–4 ✓ –3 ✓ notation (3)
5.1.7	$f'(x)\,.\,f''(x) > 0$ $(2x-2).2 > 0$ $2x-2>0$ $x>1$	✓ $2x-2$ ✓ 2 ✓ $x>1$ (3)

5.2	 $f(x)=a(x-1)(x-5)$ $4=a(3-1)(3-5)$ $4=-4a$ $a=-1$ $f(x)=-x^2+6x-5$	TP ✓ $x=3$ ✓ $y=4$ ✓ x – intercepts ✓ y-intercept ✓ shape (5)
		[22]

QUESTION/*VRAAG 6*

6.1.1	$A=150\,000(1-0,2)^2$ $=R96\,000$	✓ $n=2$ ✓ 150 000 in correct formula ✓ 96 000 (3)
6.1.2	$150\,000(1-0,2)^n=49\,152$ $(0,8)^n=\frac{1024}{3125}$ $n\log(0,8)=\log\frac{1024}{3125}$ $n=5$ The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word* **OR / OF** $150\,000(1-0,2)^n=49\,152$ $(0,8)^n=\frac{1024}{3125}$ $n=\log_{0,8}\frac{1024}{3125}$ $n=5$ The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word*	✓ $150\,000(1-0,2)^n=49\,152$ ✓ $n\log(0,8)=\log\frac{1024}{3125}$ ✓ $n=5$ ✓ 2020 (4) ✓ $150\,000(1-0,2)^n=49\,152$ ✓ $n=\log_{0,8}\frac{1024}{3125}$ ✓ $n=5$ ✓ 2020 (4)

6.1.3	R280 000 – R49 152 =R230 848 $230\ 848 = \dfrac{x\left[\left(1+\dfrac{0,085}{4}\right)^{20}-1\right]}{\dfrac{0,085}{4}}$ $x = \text{R}9\,383,26$	✓ R230 848 ✓ $i = \dfrac{0,085}{4} = 0,02125$ and $n = 20$ ✓ subs into correct formula ✓ R 9 383,26 (4)
6.2	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $= \dfrac{9\,000\left[1-\left(1+\dfrac{0,11}{12}\right)^{-180}\right]}{\dfrac{0,11}{12}}$ = R791 837,43 Lerato qualifies for a loan of R 791 000 under the given conditions / *Lerato kwalifiseer vir 'n lening van* R 791 000 *gegewe die kondisies*	✓ $i = \dfrac{0,11}{12}$ ✓ $n = 180$ ✓ substitution correct formula ✓ R791 837,43 ✓ R791 000 (5) **[16]**

QUESTION/*VRAAG* 7

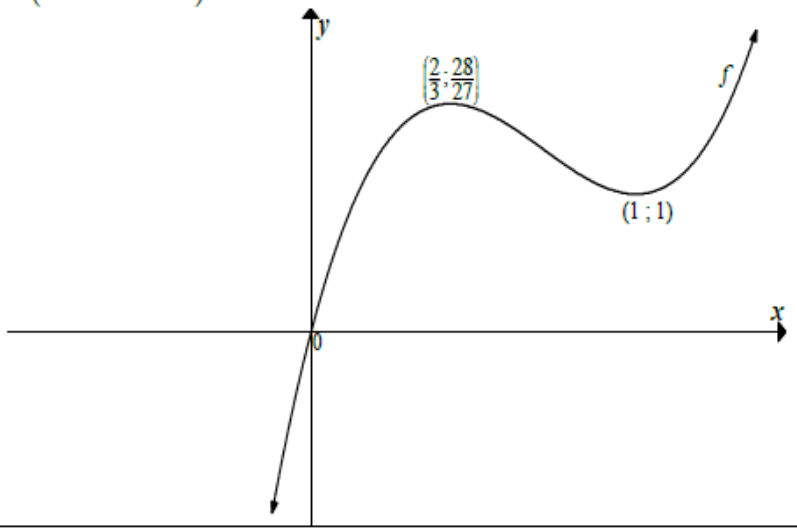
PENALISE ONLY ONCE for incorrect notation in this question.

7.1	$f(x+h)=(x+h)^2-5=(x^2+2xh+h^2)-5$ $=x^2+2xh+h^2-5$ $f(x+h)-f(x)=x^2+2xh+h^2-5-(x^2-5)$ $=2xh+h^2$	✓simplifying
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$	✓formula
	$=\lim\limits_{h\to 0}\dfrac{2xh+h^2}{h}$	✓subst. into formula
	$=\lim\limits_{h\to 0}\dfrac{h(2x+h)}{h}$	✓factorisation
	$=\lim\limits_{h\to 0}(2x+h)$ $=2x$	✓answer (5)
	OR/OF	
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$	✓formula
	$=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2-5-(x^2-5)}{h}$	✓subst. into formula
	$=\lim\limits_{h\to 0}\dfrac{2xh+h^2}{h}$	✓simplifying
	$=\lim\limits_{h\to 0}\dfrac{h(2x+h)}{h}$	✓factorisation
	$=\lim\limits_{h\to 0}(2x+h)$ $=2x$	✓answer (5)
7.2	$g(x)=5x^2-\dfrac{2x}{x^3}$ $=5x^2-2x^{-2}$	✓ $5x^2-2x^{-2}$
	$g'(x)=10x+4x^{-3}$	✓ $10x$
	$=10x+\dfrac{4}{x^3}$	✓ $4x^{-3}$ or $\dfrac{4}{x^3}$ (3)

7.3	$h(x)=ax^2, x>0$ $h^{-1}: x=ay^2 \quad y>0$ $y=\sqrt{\frac{x}{a}}$ $h^{-1}(8)=\sqrt{\frac{8}{a}}$ $h'(x)=2ax$ $h'(4)=2a(4)$ $=8a$ $\sqrt{\frac{8}{a}}=8a$ $64a^2=\frac{8}{a}$ $a^3=\frac{1}{8}$ $a=\frac{1}{2}$	✓ $y=\sqrt{\frac{x}{a}}$ ✓ $\sqrt{\frac{8}{a}}$ ✓ $h'(4)=8a$ ✓ $\sqrt{\frac{8}{a}}=8a$ ✓ $a^3=\frac{1}{8}$ ✓ $a=\frac{1}{2}$ (6)
		[14]

QUESTION/*VRAAG* 8

8.1	$f'(x)=0$ $6x^2-10x+4=0$ $3x^2-5x+2=0$ $(3x-2)(x-1)=0$ $x=\frac{2}{3}$ or $x=1$ $y=2\left(\frac{2}{3}\right)^3-5\left(\frac{2}{3}\right)^2+4\left(\frac{2}{3}\right)$ $y=2(1)^3-5(1)^2+4(1)$ $y=\frac{28}{27}$ or $y=1$ Turning points are $\left(\frac{2}{3};\frac{28}{27}\right)$ and $(1;1)$	✓ derivative ✓ derivative $=0$ ✓ factors ✓ x-values ✓ y-values (5)

8.2	$2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad x = \frac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$ $= \frac{5 \pm \sqrt{-7}}{4}$ No real roots / *Geen reële wortels* **OR / OF** $2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad b^2 - 4ac = 25 - 4(2)(4)$ $= -7 < 0$ No real roots / *Geen reële wortels*	✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $\frac{5 \pm \sqrt{-7}}{4}$ (3) ✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $b^2 - 4ac < 0$ (3)
8.3	$f(x) = 2x^3 - 5x^2 + 4x$ $x(2x^2 - 5x + 4) = 0$  	✓ (0 ; 0) ✓ turning points ✓ shape (3)

8.4	$f(x)=2x^3-5x^2+4x$ $f'(x)=6x^2-10x+4$ $f''(x)=12x-10$ $f''(x)>0$ $12x-10>0$ $x>\frac{5}{6}$	✓ $12x-10$ ✓ $f''(x)>0$ ✓ answer (3)
	OR Point of inflection: $x=-\frac{b}{3a}$ $x=-\frac{(-5)}{3(2)}$ $x=\frac{5}{6}$ The function is concave up for $x>\frac{5}{6}$ since $a>0$	✓ $x=-\frac{(-5)}{3(2)}$ ✓ $x=\frac{5}{6}$ ✓ $f''(x)>0$ (3)
	OR Point of inflection: $x=\frac{\frac{2}{3}+1}{2}$ $x=\frac{5}{6}$ The function is concave up for $x>\frac{5}{6}$ since $a>0$	✓ $x=\frac{\frac{2}{3}+1}{2}$ ✓ $x=\frac{5}{6}$ ✓ $f''(x)>0$ (3) **[14]**

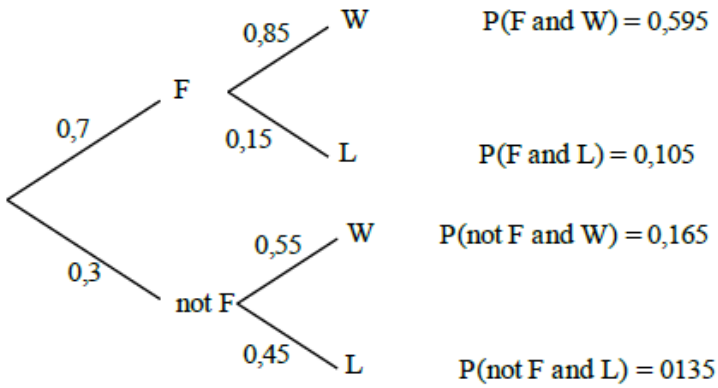
QUESTION/*VRAAG* 9

9.	Length of one side of the square / *lengte van sy van vierkant* $= \frac{x}{4}$ Length of the rectangle / *lengte van die reghoek* : $2l + x + \frac{x}{4} = 6$ $l = \frac{6 - \frac{5x}{4}}{2}$ $= \frac{24 - 5x}{8}$ $A = \left(\frac{x}{4}\right)^2 + \frac{x}{4}\left(\frac{24 - 5x}{8}\right)$ $= \frac{x^2}{16} + \frac{24x - 5x^2}{32}$ $= \frac{24x - 3x^2}{32}$ $A = \frac{24x - 3x^2}{32}$. For minimum area / *Vir min imum oppervlaktte* $\frac{dA}{dx} = 0$ $\frac{dA}{dx} = \frac{24 - 6x}{32}$ $6x = 24$ $x = 4$	✓ $\frac{x}{4}$ ✓ $\frac{6 - \frac{5x}{4}}{2}$ or $\frac{24 - 5x}{8}$ ✓ $\left(\frac{x}{4}\right)^2$ ✓ $\frac{x}{4}\left(\frac{24 - 5x}{8}\right)$ ✓ $\frac{dA}{dx} = 0$ ✓ $\frac{24 - 6x}{32}$ ✓ $x = 4$ (7)
		[7]

QUESTION/*VRAAG* 10

10.1.1	$P(S \text{ and } T) = P(S) \times P(T)$ $\frac{1}{6} = \left(\frac{1}{4}\right) \times P(T)$ $P(T) = \frac{2}{3}$	✓ $P(S \text{ and } T) = P(S) \times P(T)$ ✓ $P(T) = \frac{2}{3}$ (2)
10.1.2	$P(S \text{ or } T) = P(S) + P(T) - P(S \text{ and } T)$ $= \left(\frac{1}{4}\right) + \left(\frac{2}{3}\right) - \frac{1}{6}$ $= \frac{3}{4}$	✓ $\left(\frac{1}{4}\right) + \left(\frac{2}{3}\right) - \frac{1}{6}$ ✓ $\frac{3}{4}$ (2)
10.2.1	$5!$ $= 120$	✓ 5 ✓ 5! or 120 (2)
10.2.2	5^5 $= 3125$	✓ 5^5 or 3 125 (1)
10.3	$n(E) = 5! \times 2! \times 2!$ $n(S) = 7!$ $P(E) = \frac{5! \times 2! \times 2!}{7!}$ $= \frac{2}{21}$	✓ 5! ✓ $2! \times 2!$ ✓ $\frac{5! \times 2! \times 2!}{7!}$ ✓ $\frac{2}{21}$ (4)
		[11]

QUESTION/*VRAAG* 11

11	 P(F and W) = 0,595 P(F and L) = 0,105 P(not F and W) = 0,165 P(not F and L) = 0135	✓ 0,3
	P(Win) = P(F and W) + P(not F and W) = 0,7 × 0,85 + 0,3 × 0,45 = 0,595 + 0,165 = 0,76 = 76% $= \frac{19}{25}$	✓ P(F and W) = 0,7 × 0,85 = 0,595 ✓ P(not F and W) = 0,3 × 0,45 = 0,165 ✓ 0,595 + 0,165 ✓ 0,76 / 76% / $\frac{19}{25}$ (5)
		[5]

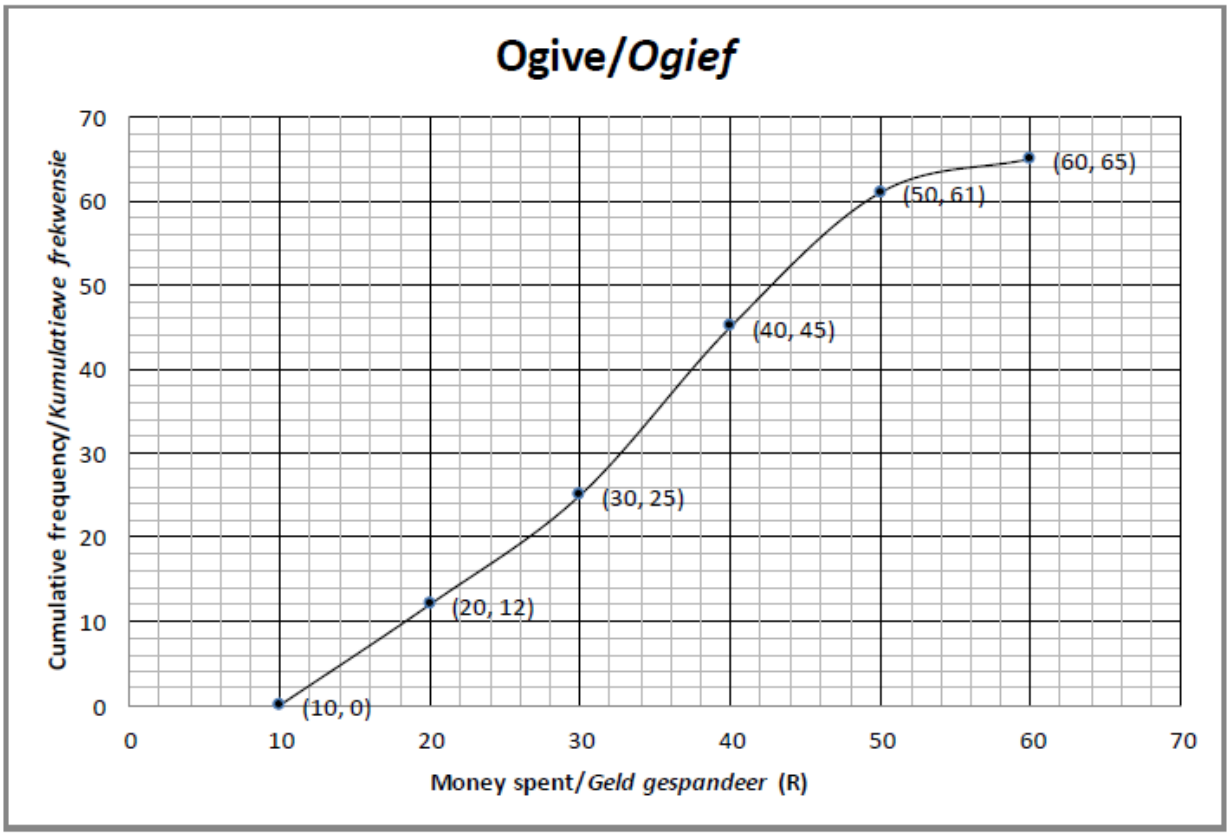
TOTAL/*TOTAAL*: 150

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2017

MEMORANDUM

QUESTION/*VRAAG* 1

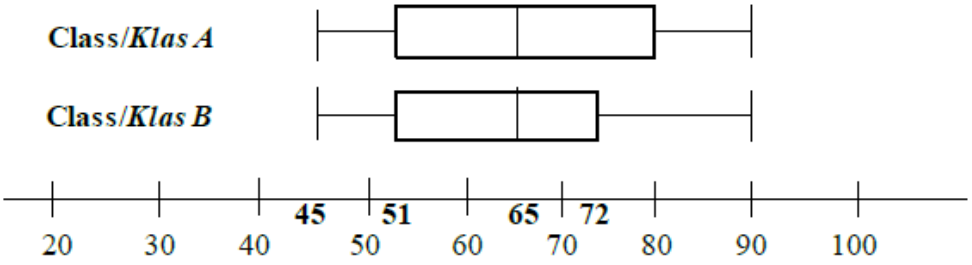


Amount of money/ *Bedrag geld* (in R)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency *Frekwensie*	*a*	13	20	*b*	4

1.1	65 learners/*leerders*	✓ answer (1)
1.2	Modal class/*Modale klas*: $30 \le x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/*Aantal leerders* = 65 – 54 **OR/*OF*** 65 – 55 = 11 = 10 Answer only: full marks	✓ 54 or 55 ✓ 11 or 10 (2) **[6]**

QUESTION/*VRAAG* 2

2.1



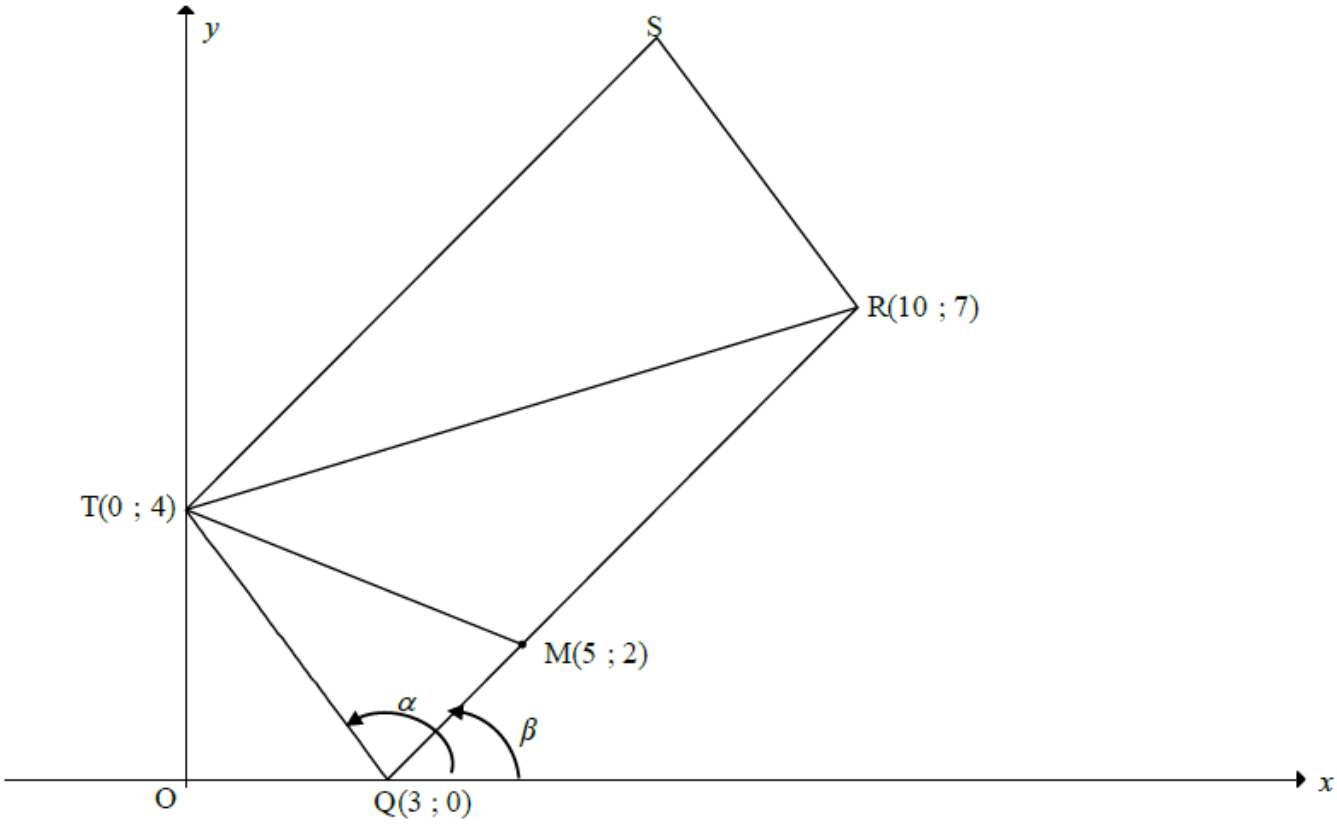
2.1.1	IQR of Class B/*IKV van Klas B* = $Q_3 - Q_1$ = 72 – 51 = 21 marks/*punte*	✓72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid.* **OR/OF** Although the boxes contain the same number of data points, the marks for Class B are more clustered./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.*	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/*Mediane is dieselfde* Ranges are the same **OR** Maximum and minimum values are the same/*Variasiewydtes is dieselfde **OF** die maksimum en minimum waarde is dieselfde* 75% of both classes obtained 51 and above/*75% van albei klasse behaal 51 en meer.*	✓ ✓ any TWO of the 3 reasons mentioned (2)

2.2

COUPLE/*PAAR*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
JUDGE 1/ *BEOORDELAAR 1*	18	4	6	8	5	12	10	14
JUDGE 2/ *BEOORDELAAR 2*	15	6	3	5	5	14	8	15

2.2.1	$a = -0,03$ $b = 0,93$ $\hat{y} = -0,03 + 0,93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0,03 + 0,93(15)$ = 13,92 **OR/*OF*** 13,85 ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes **OR** they are consistent, because $r = 0,9$. ($r = 0,89567\ldots$)/*Ja OF hulle is konsekwent, want $r = 0,9$. ($r = 0,89567\ldots$)*	✓ statement ✓ $r = 0,9$ (2) **[13]**

QUESTION/*VRAAG* 3



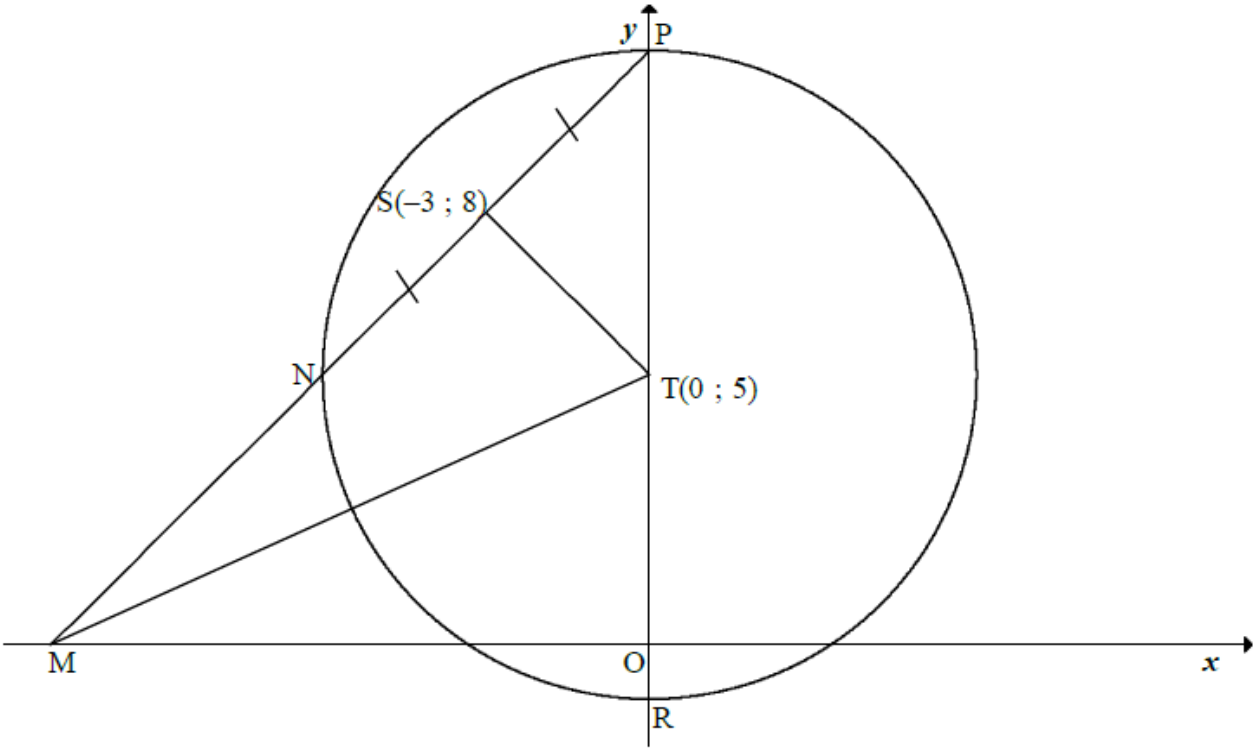
3.1	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	✓ answer (1)
3.2	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $RQ = \sqrt{(10-3)^2+(7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓ substitution/*substitusie* ✓ answer in surd form (2)
3.3	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3}$ $4k-12 = 24$ $k = 9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\frac{-8-4}{k-0} = -\frac{4}{3}$ $-36 = -4k$ $k = 9$ **OR/*OF*** Equation of TQ: $y = -\frac{4}{3}x+4$ $-8 = -\frac{4}{3}k+4$ $k = 9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4) ✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of $(k\,;-8)$ /*substitusie van* $(k\,;-8)$ ✓ answer (4)

3.4	Using transformation/*Gebruik transformasie*: $\therefore$ S(7 ; 11)	✓ ✓ *x*–value/*waarde* ✓ ✓ *y*–value/*waarde* (4)
	OR/*OF* Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR = $(5 ; \frac{11}{2})$ $\frac{x_S+3}{2}=5$ and $\frac{y_S+0}{2}=\frac{11}{2}$ $\therefore x_S=7$ and $y_S=11$ $\therefore$ S(7 ; 11)	✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4)
	OR/*OF* Equation of TS: $y=\left(\frac{7-2}{10-5}\right)x+4=x+4$ Equation of RS: $y-7=-\frac{4}{3}(x-10)$ $y=-\frac{4}{3}x+\frac{61}{3}$ $x+4=-\frac{4}{3}x+\frac{61}{3}$ $7x=49$ $x=7$ $\therefore y=11$ $\therefore$ S(7 ; 11)	✓ equations of TS and RS/*vgls van TS en RS* ✓ equating / *gelykstel* ✓ *x*–value/*waarde* ✓ *y*–value/*waarde* (4)
3.5	$T\hat{S}R = T\hat{Q}R$ [opp ∠s of ||m/*teenoorst ∠e* ||*m*] $T\hat{Q}R=\alpha-\beta$ $tan\,\alpha=m_{TQ}=-\frac{4}{3}$ $\therefore \alpha = 180°-53{,}13°=126{,}87°$ $tan\,\beta=m_{RQ}=\frac{7}{7}=1$ $\therefore \beta=45°$ $T\hat{Q}R=126{,}87°-45°$ $=81{,}87°$ $T\hat{S}R=81{,}87°$	✓ $T\hat{Q}R=\alpha-\beta$ ✓ $\tan\alpha=m_{TQ}$ ✓ α ✓ $\tan\beta=m_{RQ}$ ✓ β ✓ answer (6)
	OR/*OF*	

	$TQ = SR = 5$ $TR = \sqrt{100+9} = \sqrt{109}$ $RQ = TS = \sqrt{49+49} = \sqrt{98}$ $\cos R\hat{Q}T = \cos T\hat{S}R = \frac{TQ^2 + RQ^2 - TR^2}{2.TQ.RQ}$ $= \frac{25+98-109}{2(5)(\sqrt{98})}$ $= 0{,}141...$ $R\hat{Q}T = T\hat{S}R = 81{,}87°$	✓ length of TQ **OR** SR ✓ length of TR ✓ length of RQ **OR** TS ✓ correct subst into cosine rule ✓ simplification ✓ answer (6)
3.6.1	$MQ = \sqrt{(5-3)^2 + (2-0)^2}$ $MQ = \sqrt{8}$ $\frac{MQ}{RQ} = \frac{\sqrt{8}}{\sqrt{98}}$ Answer only: full marks $= \frac{2}{7}$ or 0,29	✓substitution/*substitusie* ✓ $MQ = \sqrt{8} = 2\sqrt{2}$ ✓ answer (3)
3.6.2	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [⊥h same/*dieselfde*] $= \frac{QM}{QR} = \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\text{area of } \Delta TQM}{2 \times \text{area of } \Delta TQR}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{QM}{QR}$ $= \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\text{area of } \Delta TQM}{2\text{area of } \Delta TQR}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$ **OR/*OF***	✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm $RQTS = 2$area ΔTQR ✓ answer (3) ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm $RQTS = 2$area ΔTQR ✓ answer (3)

	$\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2\text{area of } \Delta QTR}$ $= \frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR\sin(\alpha-\beta)]}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$	✓ $\frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ ✓ $\frac{1}{2}\left(\frac{2}{7}\right)$ ✓ answer (3) ✓ area parm $RQTS = 2$area ΔTQR ✓ $\frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR\sin(\alpha-\beta)]}$ ✓ answer (3) **[23]**

QUESTION/*VRAAG* 4



4.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore\ m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [*y*-intercept of chord NP] ∴ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the *x*-axis/ *Vgls van die raaklyne aan die sirkel || aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(–11 ; 0) [*x*–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

4.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek ∠in $\frac{1}{2}$ sirkel*] radius $= \frac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* $= \left(\frac{-11}{2}; \frac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x+\frac{11}{2}\right)^2 + \left(y-\frac{5}{2}\right)^2 = \frac{146}{4}$ **OR/*OF*** $\left(x+5\frac{1}{2}\right)^2 + \left(y-2\frac{1}{2}\right)^2 = \frac{73}{2} = 6{,}04$	✓ radius of circle ✓ *x* value of M ✓ *y* value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION/*VRAAG* 5

5.1	$a = -1$ $b = 2$	✓ answer ✓ answer (2)
5.2	$f(3x) = -\sin 3x$ Period of $f(3x) = \frac{360°}{3}$ $= 120°$ Answer only: Full marks	✓ $\frac{360°}{3}$ ✓ answer (2)
5.3	$x \in [90° ; 135°) \cup \{180°\}$ **OR/*OF*** $90° \leq x < 135°$ or $x = 180°$	✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct brackets (3) ✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct inequalities (3) **[7]**

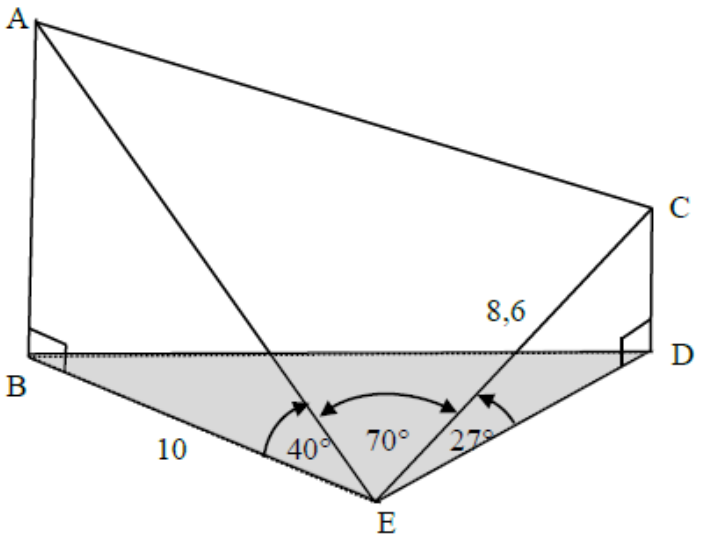
QUESTION/*VRAAG 6*

6.1.1	$\sin(360° - 36°) = -\sin 36°$	✓ answer (1)
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$ Answer only: Full marks	✓ double angle/*dubbelhoek* ✓answer (2)
6.2	R.T.P.: $1 - \frac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $LHS = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{1}{\cos^2\theta}}$ $= \cos^2\theta$ = RHS **OR/*OF*** $LHS = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\cos^2\theta}{1}$ $= \cos^2\theta$ = RHS **OR/*OF***	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity/ *kwosiëntidentiteit* ✓ denominator as a single fraction / *Noemer as enkelbreuk* ✓ square identity/*vierkantidentiteit* (4) ✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity / *kwosiëntidentiteit* ✓ $\times \frac{\cos^2\theta}{\cos^2\theta}$ ✓ square identity/*vierkantidentiteit* (4) ✓ quotient identity/

	$\text{LHS} = 1 - \left(\frac{\sin^2\theta}{\cos^2\theta} \div \left(1 + \frac{\sin^2\theta}{\cos^2\theta} \right) \right)$ $= 1 - \left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta} \right)$ $= 1 - \left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1} \right)$ $= 1 - \sin^2\theta$ $= \cos^2\theta$ $= \text{RHS}$	*kwosiëntidentiteit* ✓ writing as a single fraction/ *skryf as enkelbreuk* ✓ square identity/*vierkantidentiteit* ✓ simplification/*vereenvoudiging* (4)
6.3	$\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = 60° + k.360°$ or $\frac{1}{2}x = 300° + k.360°$ or $\frac{1}{2}x = 120° + k.360°$ or $\frac{1}{2}x = 240° + k.360°$ $x = 120° + k.720°$ or $x = 600° + k.720°$ or $x = 240° + k.720°$ or $x = 480° + k.720°;\ k \in Z$ **OR/*OF*** $\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = \pm 60° + k.360°$ or $\frac{1}{2}x = \pm 120° + k.360°$ $x = \pm 120° + k.720°$ or $x = \pm 240° + k.720°;\ k \in Z$	✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°;\ k \in Z$ (6) ✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ ±60° ✓±120° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°\ k \in Z$ (6)

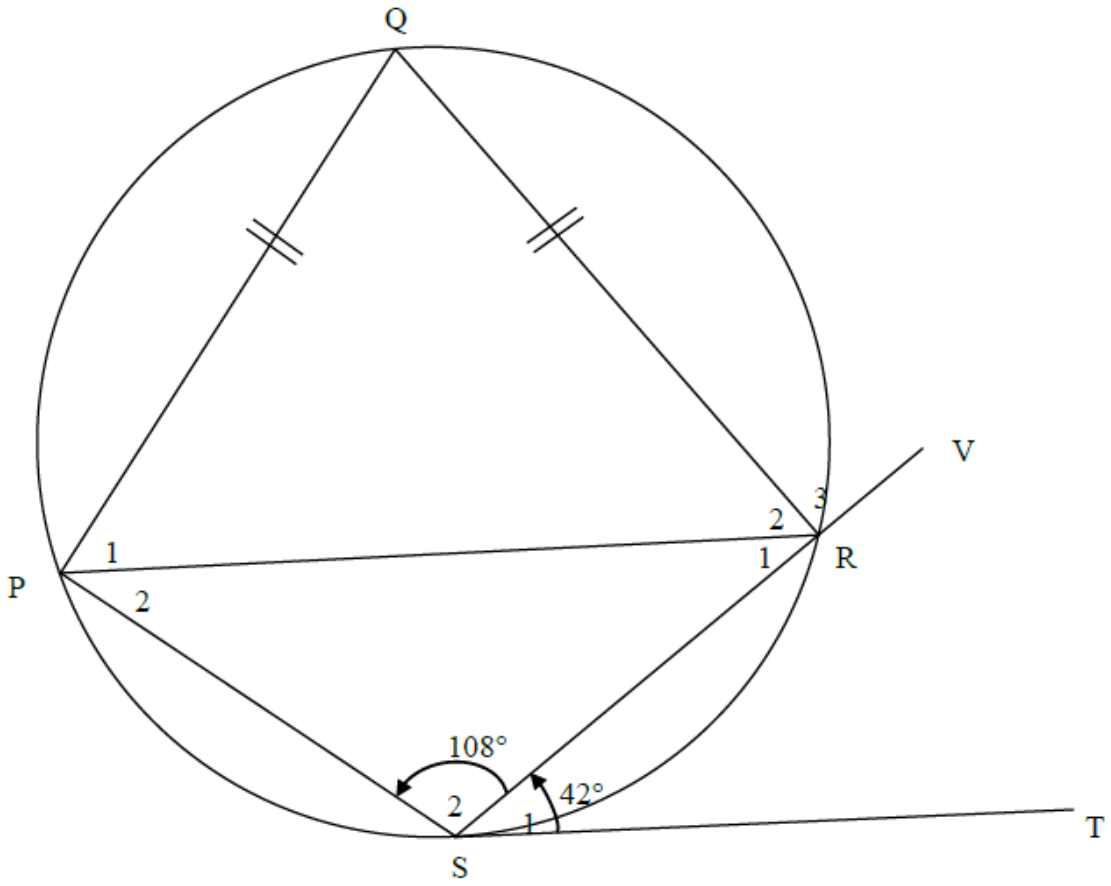
6.4.1	$\sin(A-B) = \cos[90° - (A-B)]$ $= \cos[(90° - A) - (-B)]$ $= \cos(90° - A)\cos(-B) + \sin(90° - A)\sin(-B)$ $= \sin A\cos B + \cos A(-\sin B)$ $= \sin A\cos B - \cos A\sin B$ **OR/*OF*** $\sin(A-B) = \cos[90° - (A-B)]$ $= \cos[(90° + B) - A]$ $= \cos(90° + B)\cos A + \sin(90° + B)\sin A$ $= -\sin B\cos A + \cos B\sin A$ $= \sin A\cos B - \cos A\sin B$	✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4) ✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4)
6.4.2	$\sin(x+64°)\cos(x+379°) + \sin(x+19°)\cos(x+244°)$ $= \sin(x+64°)\cos(x+19°) + \sin(x+19°)[-\cos(x+64°)]$ $= \sin(x+64°)\cos(x+19°) - \cos(x+64°)\sin(x+19°)$ $= \sin[x+64° - (x+19°)]$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$	✓ $\cos(x+379°) = \cos(x+19°)$ ✓✓ $\cos(x+244°) = -\cos(x+64°)$ ✓✓compound formula identity/ *saamgestelde identiteit* ✓ $\sin 45°$ (6) **[23]**

QUESTION/*VRAAG* 7



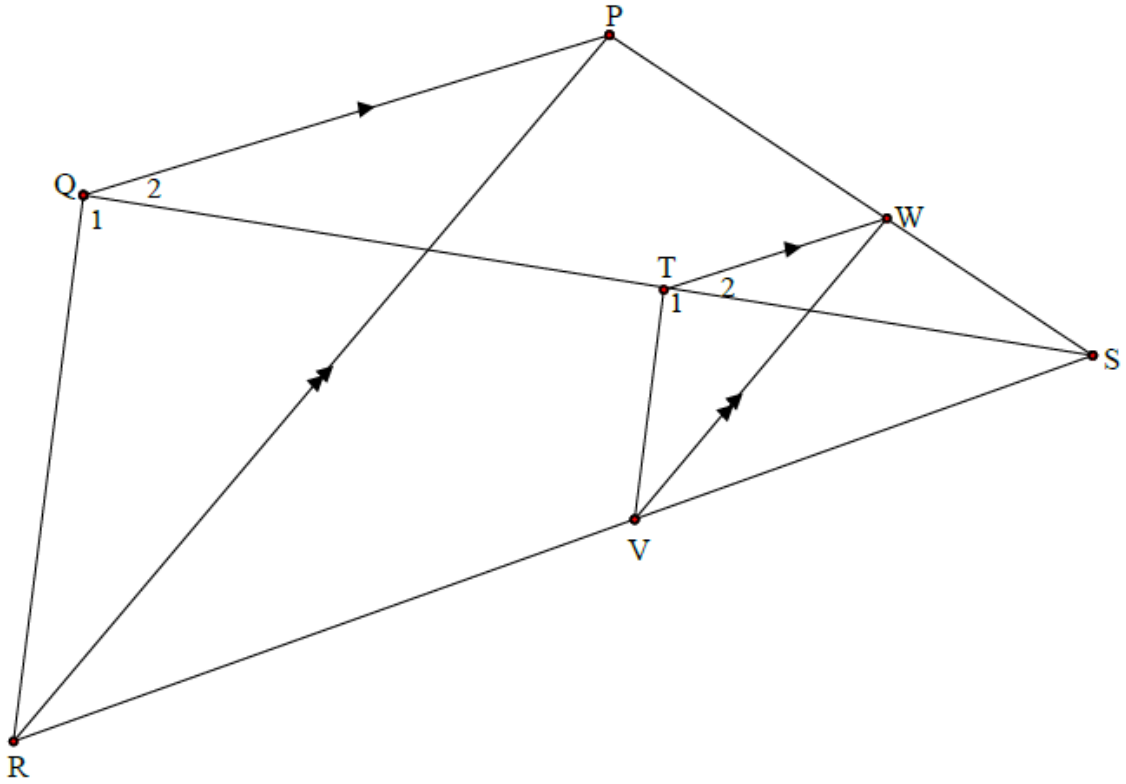
7.1	$\sin 27° = \frac{CD}{8,6}$ $CD = 8,6\sin 27°$ $CD = 3,90\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.2	$\cos 40° = \frac{10}{AE}$ $AE = \frac{10}{\cos 40°}$ $AE = 13,05\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$ $= 167,49$ $AC = 12,94\text{ m}$	✓ correct use of cosine rule in ΔACE/ *korrekte gebruik van reel in* ΔACE ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) **[8]**

QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72°$ [opp ∠s of cyclic quad/*teenoorst ∠e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{R}_2 = \frac{180° - 72°}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*] $= 54°$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42°$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= 54° + 42°$ $= 96°$ **OR/*OF*** $\hat{R}_1 = 180° - 108° - 42° = 30°$ [sum of/*som van*∠s/e in Δ] $\hat{R}_3 = 180° - \hat{R}_1 - \hat{R}_2$ [∠s on str line/*∠e op reguitlyn*] $= 180° - 30° - 54°$ [sum of/*som van*∠s/e in Δ] $= 96°$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30°$ ✓ S (2) **[8]**

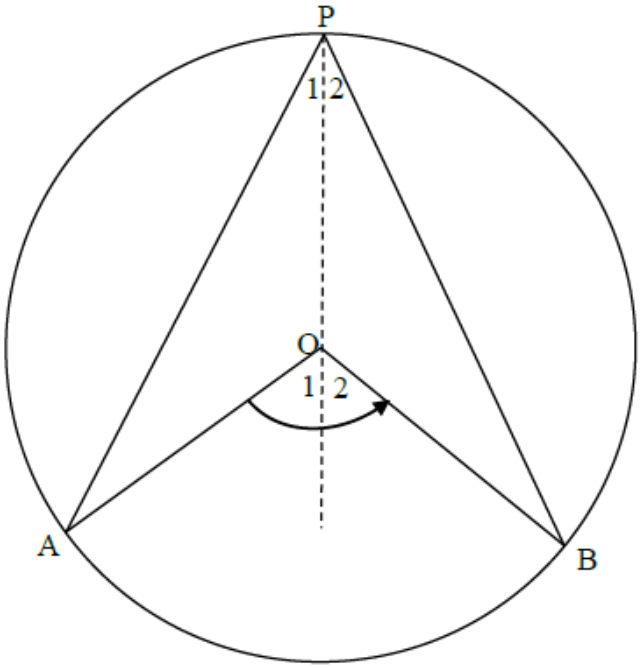
QUESTION/*VRAAG 9*



9.1.1	$\frac{ST}{TQ}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ‖ QP] $=\frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ‖ RP] $=\frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ}=\frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] $\therefore$ TV ‖ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1=\hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ‖ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| ΔRPS	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR}=\frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $=\frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR}=\frac{SV}{SR}$ [ΔVWS ||| Δ RPS] $=\frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

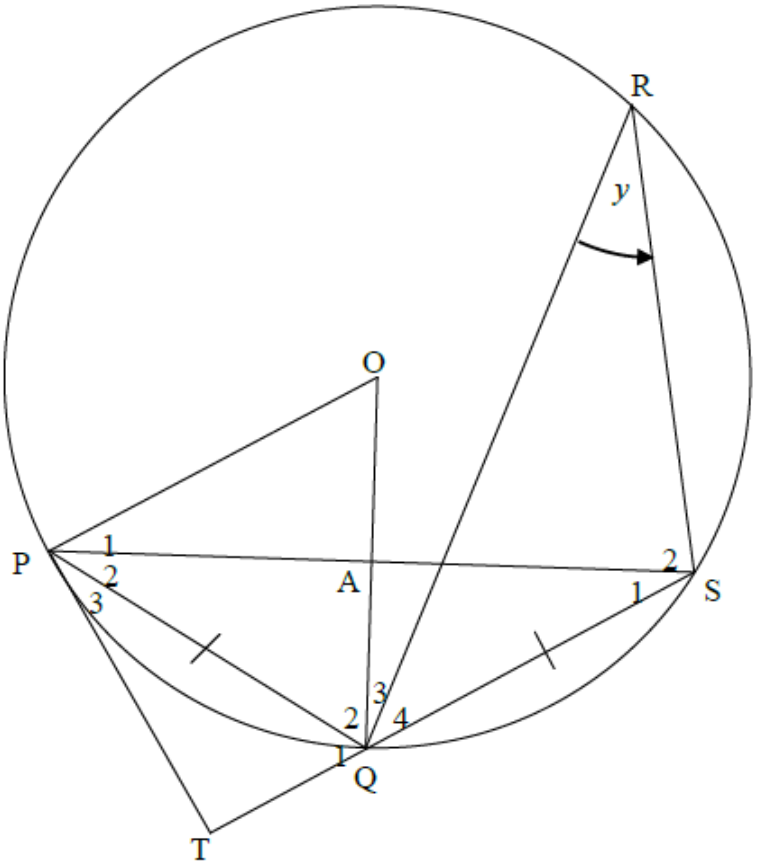
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [∠s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext ∠ of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

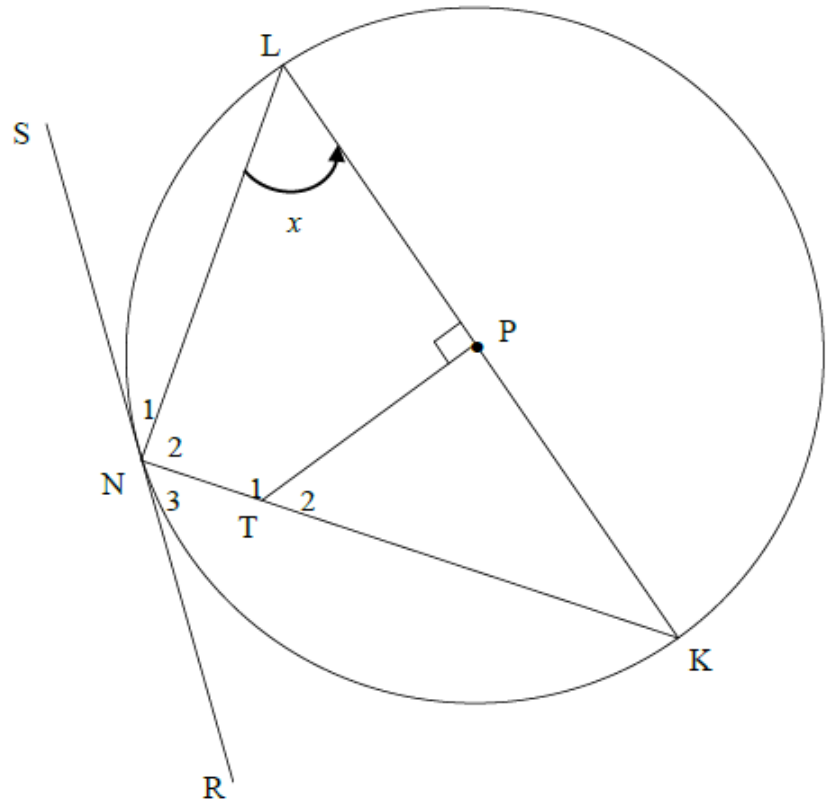
10.2



10.2.1	∠s in the same segment/ *∠e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/*∠e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore\ \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre =2×∠at circ/*midpts∠= 2×omtreks∠*]	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore\ T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

10.2.5	$O\hat{P}Q + O\hat{Q}P = 180° - 2y$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$\therefore\ O\hat{Q}P = 90° - y$ [∠s opp equal sides/∠*e to = sye;* OP = OQ]	✓ S ✓ R
	In ΔPAQ:	
	$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180°$	
	$90° - y + y + Q\hat{A}P = 180°$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$Q\hat{A}P = 90°$	✓ S
	$\therefore\ O\hat{A}P = 90°$ [∠s/*e* on straight line/*op reguitlyn*]	(5)
	OR/*OF*	
	$O\hat{P}T = 90°$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$\therefore\ \hat{P}_1 = 90° - 2y$	✓ S
	$\hat{P}_1 + \hat{O} + O\hat{A}P = 180°$ [sum of/*sum v* ∠s/*e* in Δ]	
	$(90° - 2y) + 2y + O\hat{A}P = 180°$	✓ S
	$\therefore\ O\hat{A}P = 90°$	✓ S (5)
	OR/*OF*	
	POSQ is a kite/'*n vlieër*	
	$\therefore$ OQ ⊥ PS [diag of a kite/*hoeklyne v vlieër*]	✓✓✓ S ✓✓ R
	$\therefore\ O\hat{A}P = 90°$	(5)
	OR/*OF*	
	In ΔOAP and ΔOAS	
	OP = OS (radii)	✓ S
	OA is common	✓ S
	$P\hat{O}A = 2y$	
	$= 2\hat{P}_2$	
	$= Q\hat{O}S$	✓ S
	ΔOAP ≡ ΔOAS (SAS)	✓ R
	$O\hat{A}P = O\hat{A}S$ (≡Δs)	
	$O\hat{A}P = O\hat{A}S = 90°$ (∠s on str line)	✓ S (5) **[19]**

QUESTION/*VRAAG* 11



11.1	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad/ 'n *koordevh* [opp ∠s of quad is suppl/ *teenoor∠e v vh is suppl*] **OR** $\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)
11.2	$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*] $\hat{N}_2 = 90°$ [∠in semi circle/ *halfsirkel*] $\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ R ✓ S✓ R (3) ✓ R ✓ S✓ R (3) ✓ R ✓ S ✓ S (3)

11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore$ Δ KTP ||| Δ KLN [∠∠∠] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of ∠s in Δ] $\therefore$ Δ KTP ||| Δ KLN	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [||| Δs] $\therefore KT \cdot KN = KP \cdot KL$ But KL = 2KP [radii: PK = LP] $\therefore KT \cdot KN = KP \cdot 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

TOTAL/*TOTAAL*: 150

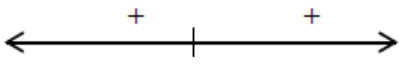
MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2017

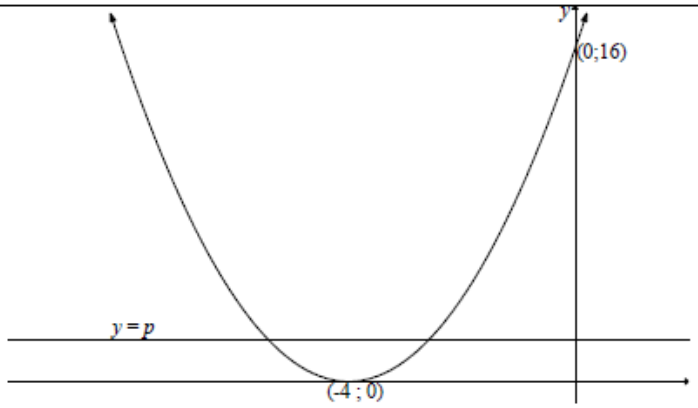
MARKING GUIDELINES/*NASIENRIGLYNE*

QUESTION/*VRAAG* 1

1.1.1	$x^2+9x+14=0$ $(x+7)(x+2)=0$ $x=-7$ or $x=-2$	✓ factors ✓ $x=-7$ ✓ $x=-2$ (3)
1.1.2	$4x^2+9x-3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-9\pm\sqrt{9^2-4(4)(-3)}}{2(4)}$ $=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$ **OR/*OF*** $x^2+\frac{9}{4}x+\frac{81}{64}=\frac{3}{4}+\frac{81}{64}$ $\left(x+\frac{9}{8}\right)^2=\frac{129}{64}$ $x+\frac{9}{8}=\pm\frac{\sqrt{129}}{8}$ $x=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$	✓ substitution ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ **OR/*OF*** ✓ for adding $\frac{81}{64}$ on both sides ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ (4)
1.1.3	$\sqrt{x^2-5}=2\sqrt{x}$ $x^2-5=4x$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$ $x=5$	✓ $x^2-5=4x$ ✓ standard form ✓ both answers ✓ select $x=5$ (4)

1.2	$3x-y=4$ $y=3x-4$ $x^2+2xy-y^2=-2$ $x^2+2x(3x-4)-(3x-4)^2=-2$ $x^2+6x^2-8x-(9x^2-24x+16)=-2$ $7x^2-8x-9x^2+24x-16=-2$ $-2x^2+16x-14=0$ $x^2-8x+7=0$ $(x-7)(x-1)=0$ $x=1$ or $x=7$ $y=3(1)-4$ $y=3(7)-4$ $y=-1$ or $y=17$ **OR/*OF*** $3x-y=4$ $x=\frac{y+4}{3}$ $x^2+2xy-y^2=-2$ $x^2+2xy-y^2=-2$ $\left(\frac{y+4}{3}\right)^2+2\left(\frac{y+4}{3}\right)y-y^2=-2$ $y^2+8y+16+6y^2+24y-9y^2=-18$ $-2y^2+32y+34=0$ $y^2-16y-17=0$ $(y-17)(y+1)=0$ $y=-1$ or $y=17$ $x=\frac{-1+4}{3}$ $x=\frac{17+4}{3}$ $x=1$ or $x=7$	✓ y subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓ x-values ✓ y-values **OR/*OF*** ✓ x subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓ y-values ✓ x-values (6)
1.3.1	$x^2+8x+16>0$ $(x+4)(x+4)>0$ $x\in R, x\neq -4$ **or** $x\in(-\infty;-4)$ or $x\in(-4;\infty)$ **or** $x<-4$ or $x>-4$ **OR/*OF*** $x^2+8x+16>0$ $(x+4)(x+4)>0$ + +  -4 The function values remain positive $x\in R, x\neq -4$	✓ $(x+4)(x+4)$ ✓✓ any one of the solutions **OR/*OF*** ✓ $(x+4)(x+4)$ ✓✓ any one of the solutions (3)

1.3.2



✓ 0
✓ 16

For two negative unequal roots:

$0 < p < 16$

✓✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 = p$

$x^2 + 8x + 16 - p = 0$

$0 < 16 - p < 16$

$-16 < -p < 0$

$0 < p < 16$

✓ 0
✓ 16

✓✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 - p = 0$

$x = \dfrac{-8 \pm \sqrt{64 - 4(16 - p)}}{2}$

$0 < 64 - 4(16 - p) < 64$

$0 < 4p < 64$

$0 < p < 16$

✓ 0
✓ 16

✓✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 = p$

$x^2 + 8x + 16 - p = 0$

Roots are real and unequal:

$8^2 - 4(16 - p) > 0$

$4p > 0$

$p > 0$

Roots are: $\dfrac{-8 \pm \sqrt{4p}}{2}$

For both roots to be negative:

$\sqrt{4p} < 8$

$4p < 64$

$p < 16$

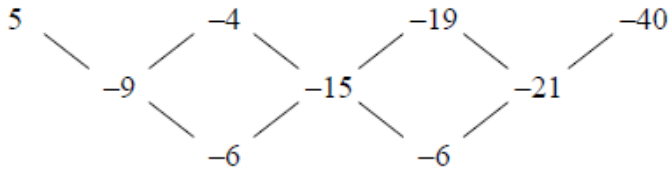
$0 < p < 16$

✓ 0
✓ 16

✓✓ $0 < p < 16$ (4)

[24]

QUESTION/*VRAAG* 2

2.1.1	 first differences: −9; −15; −21 second difference = − 6	✓ first differences ✓ − 6 (2)
2.1.2	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ **OR/*OF*** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \frac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ **OR/*OF*** ✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ (4)
2.1.3	$-3n^2 + 8 = -25\,939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93 \quad or \quad n = 93$ The 93rd term has a value of −25 939	✓ $T_n = -25\,939$ ✓ $n^2 = 8649$ ✓ answer (3)

2.2.1	$2k-7\ ;\ k+8$ and $2k-1$ $k+8-(2k-7)=2k-1-(k+8)$ $-k+15=k-9$ $2k=24$ $k=12$ $2k-7;k+8$ and $2k-1$ $17\ ;20\ ;23.....$ $d=3$ $T_{15}=17+14(3)$ $=59$	✓ $k+8-(2k-7)=2k-1-(k+8)$ ✓$k=12$ ✓17 ✓ $d=3$ ✓ $T_{15}=59$ (5)
2.2.2	Sequence is 17 ; 20 ; 23 ; 26 ; 29 ; 32 ..... Every alternate term of the sequence will be even / *Elke tweede term van die ry sal ewe wees* $20+26+32+.....$ $S_{30}=\frac{30}{2}[2(20)+(29)(6)]$ $=15[40+174]$ $=3210$ **OR/*OF*** $T_{30}=20+29(6)$ $=94$ $S_{30}=\frac{30}{2}(20+194)$ $=3210$	✓ $20+26+32+.....$ ✓ $a=20\ \ d=6$ ✓subst into correct formula ✓ answer (4) ✓ $a=20\ \ d=6$ ✓ $T_{30}=94$ ✓ $S_{30}=\frac{30}{2}(20+194)$ ✓ answer (4) **[18]**

QUESTION/*VRAAG* 3

3.1	$a+ar=2$ $a(1+r)=2$ $a=\frac{2}{1+r}$	✓ $a+ar=2$ ✓ $a=\frac{2}{1+r}$	(2)
	OR/*OF* $\frac{a}{1-r}-2=\frac{1}{4}$ $4a-8(1-r)=1-r$ $4a-8+8r=1-r$ $4a=9-9r$ $a=\frac{9-9r}{4}$	✓ $\frac{a}{1-r}-2=\frac{1}{4}$ ✓ $a=\frac{9-9r}{4}$	(2)
	OR/*OF* $S_n=\frac{a(r^n-1)}{r-1}$ $2=\frac{a(r^2-1)}{r-1}$ $2=\frac{a(r-1)(r+1)}{r-1}$ $2=a(r+1)$ $a=\frac{2}{r+1}$	**OR/*OF*** ✓ $2=\frac{a(r^2-1)}{r-1}$ ✓ $a=\frac{2}{1+r}$	(2)
	OR/*OF* $\frac{ar^2}{1-r}=\frac{1}{4}$ $a=\frac{1-r}{4r^2}$	**OR/*OF*** ✓ $\frac{ar^2}{1-r}=\frac{1}{4}$ ✓ $a=\frac{1-r}{4r^2}$	(2)

3.2	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$	
	$S_\infty = 2 + \frac{1}{4}$	✓ $S_\infty = 2 + \frac{1}{4}$
	$\frac{a}{1-r} = 2 + \frac{1}{4}$	
	$\frac{a}{1-r} = \frac{9}{4}$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$	✓ substitution of a into the correct formula
	$\frac{2}{1-r^2} = \frac{9}{4}$	
	$8 = 9 - 9r^2$	
	$9r^2 = 1$	✓ $9r^2 = 1$
	$r = \frac{1}{3}$	✓ $r = \frac{1}{3}$
	$a = \frac{3}{2}$	✓ $a = \frac{3}{2}$
		(6)
	OR/*OF*	**OR/*OF***
	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$	
	$S_\infty = 2 + \frac{1}{4}$	✓ $S_\infty = 2 + \frac{1}{4}$
	$\frac{a}{1-r} = 2 + \frac{1}{4}$	
	$\frac{a}{1-r} = \frac{9}{4}$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$4a = 9 - 9r$	
	$r = \frac{9-4a}{9}$	✓ $r = \frac{9-4a}{9}$
	$a + a\left(\frac{9-4a}{9}\right) = 2$	
	$9a + 9a - 4a^2 = 18$	
	$2a^2 - 9a + 9 = 0$	✓ substitution of a into the correct formula
	$(a-3)(2a-3) = 0$	
	$a = \frac{3}{2}$ or $a = 3$	
	$r = \frac{1}{3}$ or $r = -\frac{1}{3}$	
	N/A	✓ $a = \frac{3}{2}$

OR/*OF*

$r = \frac{2-a}{a}$

$\frac{ar^2}{1-r} = \frac{1}{4}$

$4ar^2 = 1-r$

$4a\left(\frac{2-a}{a}\right)^2 = 1-\frac{2-a}{a}$

$16-16a+4a^2 = 2a+2$

$2a^2-9a+9 = 0$

$(2a-3)(a-3) = 0$

$a = \frac{3}{2}$ $\quad a \neq 3$

$r = \frac{1}{3}$ $\quad r \neq -\frac{1}{3}$

✓ $r = \frac{1}{3}$

(6)

OR/*OF*

✓ $r = \frac{2-a}{a}$

✓ $\frac{ar^2}{1-r} = \frac{1}{4}$

✓ substitution of a

✓ $(2a-3)(a-3) = 0$

✓ $a = \frac{3}{2}$

✓ $r = \frac{1}{3}$

(6)

OR/*OF*

$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$

$S_\infty = 2 + \frac{1}{4}$

$\frac{a}{1-r} = 2 + \frac{1}{4}$

$\frac{a}{1-r} = \frac{9}{4}$

$\left(\frac{1-r}{4r^2}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$

$\frac{1}{4r^2} = \frac{9}{4}$

$4 = 36r^2$

$9r^2 = 1$

$r = \frac{1}{3}$

$a = \frac{3}{2}$

OR/*OF*

✓ $S_\infty = 2 + \frac{1}{4}$

✓ $\frac{a}{1-r} = \frac{9}{4}$

✓ substitution of a

✓ $9r^2 = 1$

✓ $r = \frac{1}{3}$

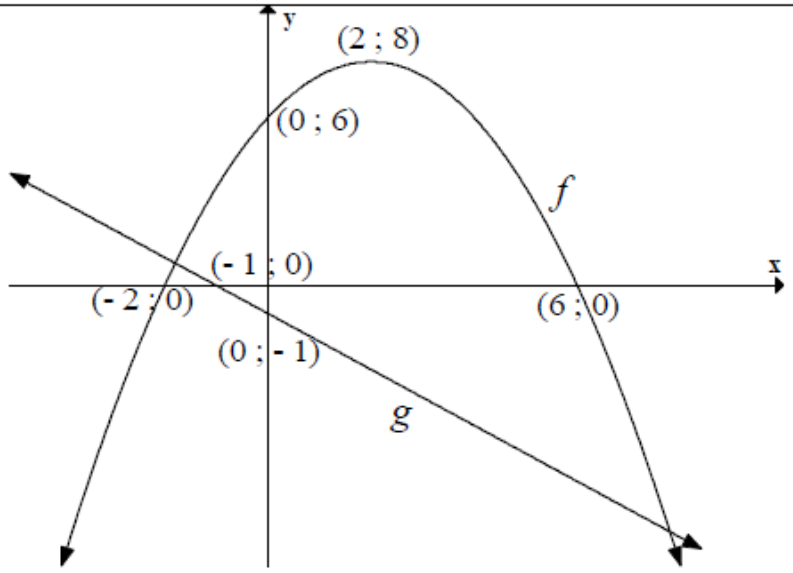
✓ $a = \frac{3}{2}$

(6)

[8]

QUESTION/*VRAAG* 4

4.1	$f(x)=-ax^2+bx+6$ $f'(x)=-2ax+b$ $-2ax+b=3$ at $x=-1$ $2a+b=3$ [1] $f(-1)=\frac{7}{2}$ $-a-b+6=\frac{7}{2}$ $-2a-2b+12=7$ $2a+2b=5$ [2] [2]−[1] $b=2$ $2a+2=3$ $a=\frac{1}{2}$ **OR/*OF*** $f'(x)=-2ax+b$ $3=2a+b$ $b=3-2a$ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ $a+3=\frac{7}{2}$ $a=\frac{1}{2}$ $b=2$	✓$-2ax+b$ ✓✓$2a+b=3$ ✓ $-a-b+6=\frac{7}{2}$ ✓solve simultaneously (5) ✓$-2ax+b$ ✓✓$2a+b=3$ ✓ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ ✓solve simultaneously (5)
4.2	$f(x)=-\frac{1}{2}x^2+2x+6$ $x-$intercepts: $-\frac{1}{2}x^2+2x+6=0$ $-x^2+4x+12=0$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $(-2;0)$ $(6;0)$	✓ $-\frac{1}{2}x^2+2x+6=0$ ✓$(-2;0)$ ✓ $(6;0)$ (3)

4.3	$f(x)=-\frac{1}{2}x^2+2x+6$ $f'(x)=0$ or $x=-\frac{b}{2a}$ or $x=\frac{-2+6}{2}$ $-x+2=0$ $x=-\frac{2}{2.\left(-\frac{1}{2}\right)}$ $x=2$ $x=2$ $x=2$ $y=-\frac{1}{2}(2)^2+2(2)+6$ $=-2+4+6$ $=8$ TP $(2;8)$ **OR/OF** $y=-\frac{1}{2}(x^2-4x-12)$ $=-\frac{1}{2}\left[(x-2)^2-4-12\right]$ $=-\frac{1}{2}(x-2)^2+8$ TP $(2;8)$	✓ $-x+2$ / $-\frac{2}{2.\left(-\frac{1}{2}\right)}$ / $\frac{-2+6}{2}$ ✓ $x=2$ ✓ $y=8$ **OR/*OF*** ✓ $-\frac{1}{2}(x-2)^2+8$ ✓ $x=2$ ✓ $y=8$ (3)
4.4 4.6		4.4: *f*: ✓shape ✓*x*- intercepts ✓*y*- intercept ✓(2 ; 8) (4) 4.6: *g*: ✓ *x*- intercept ✓ *y*- intercept (2)
4.5	$0<x<4$ or $(0;4)$	✓ 4 ✓✓ $0<x<4$ (3)
4.7	$x\leq -2$ or $-1\leq x\leq 6$ **OR/OF** $(-\infty;-2]$ or $[-1;6]$	✓ $x\leq -2$ ✓✓ $-1\leq x\leq 6$ (3) **[23]**

QUESTION/*VRAAG* 5

5.1	$y \in R; y \neq -1$ **OR/*OF*** $y < -1$ or $y > -1$ **OR/*OF*** $y \in (-\infty; -1)$ or $y \in (-1; \infty)$ **OR/*OF*** $R - \{-1\}$	✓✓ answer (2)
5.2	$D(2; -1)$ $g(x) = \frac{2}{x-2} - 1$	✓ $D(2; -1)$ ✓ $\frac{2}{x-2} - 1$ (2)
5.3	$f(x) = \log_3 x.$ $\log_3 t = 1$ $t = 3$ **OR/*OF*** $g(x) = \frac{2}{x-2} - 1$ $1 = \frac{2}{t-2} - 1$ $2 = \frac{2}{t-2}$ $2t - 4 = 2$ $t = 3$	✓correct substitution of A ✓✓ $t = 3$ (3)
5.4	$x = \log_3 y$ $y = 3^x$	✓interchange x and y ✓ $y = 3^x$ (2)
5.5	$3^x < 3^1$ $x < 1$ **OR/*OF*** $3^x < 3^1$ $x \in (-\infty; 1)$	✓ $3^x < 3^1$ ✓ $x < 1$ (2) ✓ $3^x < 3^1$ ✓ $x \in (-\infty; 1)$ (2)
5.6	Equation of the axis of symmetry: $y = -x + 1$ x-intercept of the axis of symmetry is at $x = 1$ f has an x-intercept at B(1 ; 0) which is the same as the x-intercept of the axis of symmetry Point of intersection: B $(1; 0)$ **OR/*OF*** Since BE = ED = 1 and D lies on the axis of symmetry and the gradient of the axis of symmetry is –1, B will also lie on the axis of symmetry. But B also lies on f. Therefore B(1 ; 0) is the point of intersection between f and the axis of symmetry with a negative gradient./ *Omdat* BE = ED = 1 *en* D *op die simmetrie-as lê en die simmetrie-as se gradiënt* –1 *is, sal* B *ook op die simmetrie-as lê. Maar* B *lê ook op f. Dus is* B(1 ; 0) *die snypunt van f en die simmetrie-as met negatiewe gradiënt.*	✓✓ equation of axis of symmetry ✓B or $(1; 0)$ **OR/*OF*** ✓✓ BE = ED = 1 ✓ B or $(1; 0)$ (3) **[14]**

QUESTION/*VRAAG 6*

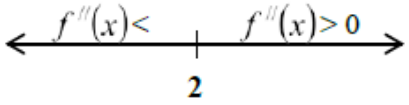
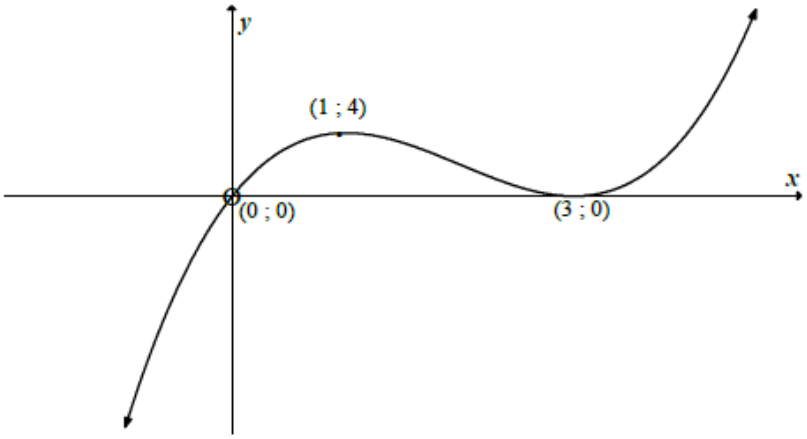
6.1	$A = P(1+i)^n$ $12\ 146{,}72 = 10\ 000\left(1+\frac{r}{12}\right)^{36}$ $\left(1+\frac{r}{12}\right)^{36} = 1{,}214672$ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ $= 1{,}005416$ $\frac{r}{12} = 0{,}005416$ $r = 0{,}06500$ $r = 6{,}5\%$	✓ $\frac{r}{12}$ ✓ $n = 36$ ✓ correct substitution into formula ✓ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ ✓ 6,5% (5)
6.2.1	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $235\ 000 = \frac{x\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}{\frac{0{,}11}{12}}$ $x = \frac{235\ 000\times\frac{0{,}11}{12}}{\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}$ $= \text{R}5\ 536{,}95$ His monthly instalment is R 5 536,95	✓ $i = \frac{0{,}11}{12}$ ✓ $n = 54$ ✓ correct substitution in P ✓ answer (4)
6.2.2	Amount paid for the year : $(5\ 536{,}95\times 12) = \text{R}66\ 443{,}40$ $\text{Balance} = 235\ 000\left(1+\frac{0{,}11}{12}\right)^{12} - \frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ $= 192\ 296{,}17$ $\text{Interest} = (5\ 536{,}95\times 12)-(235\ 000-192\ 296{,}17)$ $= 66\ 443{,}40 - 42\ 703{,}83$ $= 23\ 739{,}57$ **OR/*OF***	✓ R66 443,40 ✓ $235\ 000\left(1+\frac{0{,}11}{12}\right)^{12}$ ✓ $\frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 **OR/*OF***

	Total amount paid in first year $= R\,5\,536.95 \times 12$ $= R66\,443{,}40$	
	Balance on loan after 1 year = P of remaining installments	✓ R66 443,40
	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$	
	$= \frac{5\ 536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-42}\right]}{\frac{0{,}11}{12}}$	✓ $n = -42$ ✓ substitution into correct formula
	$= R192\,296{,}20$	✓ R192 296,20
	Amount paid off in the first year: R235 000 – R192 296,20 = R42 703,80	✓ R42 703,80
	Amount of interest = R66 443,40 – R42 703,80 = R23 739,60	✓ R23 739,60 (6)
	OR/*OF*	**OR/*OF***
	$P = \frac{5536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-12}\right]}{\frac{0{,}11}{12}}$	
	$= R\ 62\,648{,}18$	✓ R62 648,18
	235 000 – 62 648,18 = R172 351,82	✓ R172 351,82
	After 12 months, money owed on house is	
	$172\ 351{,}82\left(1+\frac{0{,}11}{12}\right)^{12}$	
	= 192 296,17	✓ R192 296,17
	Amount paid after 12 months is	
	5 536,95 × 12 = R 66 443, 40	✓ R66 443,40
	Amount of interest paid:	
	R 66 443, 40 – (235 000 – 192 296,17)	✓ 235 000 – 192 296,17
	= R 23 739, 57	✓ R23 739,57 (6) **[15]**

QUESTION/*VRAAG* 7

7.1	$f(x+h)=2(x+h)^2-(x+h)$ $=2(x^2+2xh+h^2)-x-h$ $=2x^2+4xh+2h^2-x-h$ $f(x+h)-f(x)=2x^2+4xh+2h^2-x-h-2x^2+x$ $=4xh+2h^2-h$ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{4xh+2h^2-h}{h}$ $=\lim_{h\to0}\frac{h(4x+2h-1)}{h}$ $=\lim_{h\to0}(4x+2h-1)$ $=4x-1$ **OR/*OF*** $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{2(x+h)^2-(x+h)-(2x^2-x)}{h}$ $=\lim_{h\to0}\frac{2x^2+4xh+2h^2-x-h-2x^2+x}{h}$ $=\lim_{h\to0}\frac{4xh+2h^2-h}{h}$ $=\lim_{h\to0}\frac{h(4x+2h-1)}{h}$ $=\lim_{h\to0}(4x+2h-1)$ $=4x-1$	✓ $2x^2+4xh+2h^2-x-h$ ✓ $4xh+2h^2-h$ ✓ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ ✓ subst. into formula ✓ $\lim_{h\to0}(4x+2h-1)$ ✓ $4x-1$ **OR/*OF*** ✓ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ ✓ subst. into formula ✓ $2x^2+4xh+2h^2-x-h$ ✓ $4xh+2h^2-h$ ✓ $\lim_{h\to0}(4x+2h-1)$ ✓ $4x-1$ (6)
7.2.1	$D_x[(x+1)(3x-7)]$ $=D_x(3x^2-4x-7)$ $=6x-4$	✓ $3x^2-4x-7$ ✓ $6x-4$ (2)
7.2.2	$y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ $y=x^{\frac{3}{2}}-5x^{-1}+\frac{1}{2}\pi$ $\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+5x^{-2}$	✓ $x^{\frac{3}{2}}-5x^{-1}$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ ✓ $+5x^{-2}$ ✓ derivative of $\frac{1}{2}\pi$ is 0 (4) **[12]**

QUESTION/*VRAAG* 8

8.1	$f(x)=x^3-6x^2+9x$ $f'(x)=3x^2-12x+9$ $f''(x)=6x-12=0$ $x=2$ $f''(0)=6(0)-12$ $=-12$ $f''(3)=6(3)-12$ $=6$  Point of inflection at $x=2$	✓ x^3-6x^2+9x ✓ $3x^2-12x+9$ ✓ $6x-12$ ✓ $6x-12=0$ ✓ explanation (5)
8.2	 	✓shape ✓ (0 ; 0) ✓ (3 ; 0) as TP ✓(1 ; 4) (4)
8.3	f concave up for $x>2$ $y=-f(x)$ will be concave down for $x>2$	✓✓ $x>2$ (2)
8.4.1	$(3;7)$	✓ 3 ✓7 (2)
8.4.2	Do not agree with Claire as her statement is incorrect. Between $x=1$ and $x=3$ the graph of f is decreasing. Therefore at $x=2$ the gradient will have a negative value. *Stem nie saam met Claire nie, want haar stelling in verkeerd.* *Die grafiek van f is dalend/afnemend tussen $x=1$ en $x=3$.* *By $x=2$ moet die gradiënt dus 'n negatiewe waarde hê.* **OR/*OF*** $f'(2)=3(2)^2-12(2)+9$ $=-3$ $\neq 1$	✓no ✓justification (2) **[15]**

QUESTION/*VRAAG* 9

	$y = x^2 + 2$ $P(x\,;x^2+2)$ $B(0\,;3)$	
	$PB^2 = (x-0)^2 + (x^2+2-3)^2$	✓ $(x-0)^2 + (x^2+2-3)^2$
	$= x^2 + x^4 - 2x^2 + 1$	
	$= x^4 - x^2 + 1$	✓ $x^4 - x^2 + 1$
	PB will be a minimum if PB^2 is a minimum	
	$\frac{d(PB^2)}{dx} = 4x^3 - 2x$	✓ $4x^3 - 2x$
	$4x^3 - 2x = 0$	✓ $\frac{d(PB^2)}{dx} = 0$
	$x(2x^2 - 1) = 0$	
	$x = 0$ or $x^2 = \frac{1}{2}$	
	$x = \frac{1}{\sqrt{2}}$	✓ $x = \frac{1}{\sqrt{2}}$
	$PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$	✓ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$
	$= \frac{1}{4} - \frac{1}{2} + 1$	
	$= \frac{3}{4}$	
	$PB = \frac{\sqrt{3}}{2} = 0{,}87$	✓ answer
	OR/*OF*	**OR/*OF***

Gradient of tangent to curve $=2x$ ✓ $=2x$

Gradient of line joining B and the curve $=\dfrac{x^2+2-3}{x-0}$

$=\dfrac{x^2-1}{x}$ ✓ $=\dfrac{x^2-1}{x}$

Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.

$\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$ ✓ $\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$

$2x(x^2-1)=-x$

$2x^3-2x=0$ ✓ $2x^3-2x=0$

$x(2x^2-1)=0$

$x=0 \quad \text{or} \quad x^2=\dfrac{1}{2}$

$x=\dfrac{1}{\sqrt{2}}$ ✓ $x=\dfrac{1}{\sqrt{2}}$

$\text{PB}^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$ ✓ $\text{PB}^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$

$=\dfrac{1}{4}-\dfrac{1}{2}+1$

$=\dfrac{3}{4}$

$\text{PB}=\dfrac{\sqrt{3}}{2}=0{,}87$ ✓ answer

OR/*OF* **OR/*OF***

$\text{P}\left(k;k^2+2\right)$ and B(0; 3) ✓ $\text{P}\left(k;k^2+2\right)$

BP ⊥ tangent passing through $y=x^2+2$ at P.

$m_{\text{tangent at P}}=2k$ ✓ $m_{\text{tangent at P}}=2k$

$m_{\text{BP}}=-\dfrac{1}{2k}$ ✓ $m_{BP}=-\dfrac{1}{2k}$

Equation of BP: $y=\left(-\dfrac{1}{2k}\right)x+3$ ✓ $y=\left(-\dfrac{1}{2k}\right)x+3$

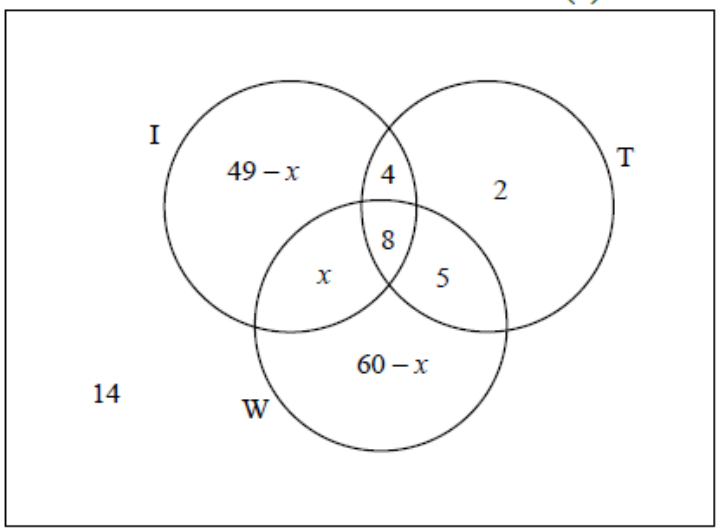
$y_{\text{P}}=\left(-\dfrac{1}{2k}\right)(k)+3=2{,}5$ ✓ value of y at P

$\Rightarrow k^2+2=2{,}5$ and so $k=\sqrt{0{,}5}$ and $\text{P}\left(\sqrt{0{,}5};2{,}5\right)$ ✓value of k

$\text{BP}=\sqrt{\left(\sqrt{0{,}5}-0\right)^2+\left(2{,}5-3\right)^2}=\sqrt{\dfrac{3}{4}}=\dfrac{\sqrt{3}}{2}=0{,}87$ ✓ answer

[7]

QUESTION/*VRAAG* 10

10.1	n(S) = 100  	8 values need to be placed in correct position: 2 or 3 correct: 1 mark 4 or 5 correct: 2 marks 6 or 7 correct: 3 marks 8 correct: 4 marks (4)
10.2	$(49-x)+x+8+4+5+2+(60-x)+14=100$ $-x+142=100$ $x=42$	✓ setting up equation ✓ answer (2)
10.3	P (use only one application) $=\frac{7+2+18}{100}$ $=\frac{27}{100}$ or 27%	✓ $\frac{7+2+18}{100}$ ✓answer (2) **[8]**

QUESTION/*VRAAG* 11

11.1	5 x 5 x 10 x 9 = 2250	✓ 5 x 5 ✓ 10 x 9 ✓ 2250 (3)
11.2	<table><tr><th>No of digits used</th><th>Letters</th><th>Digits</th><th>Total</th></tr><tr><td>1</td><td>5 x 5</td><td>10</td><td>250</td></tr><tr><td>2</td><td>5 x 5</td><td>10 x 9</td><td>2 250</td></tr><tr><td>3</td><td>5 x 5</td><td>10 x 9 x 8</td><td>18 000</td></tr><tr><td>4</td><td>5 x 5</td><td>10 x 9 x 8 x7</td><td>126 000</td></tr><tr><td>5</td><td>5 x 5</td><td>10 x 9 x 8 x7 x 6</td><td>756 000</td></tr></table> Codes of two letters and five digits will ensure unique numbers for 700 000 clients.	✓ 5 x 5 x 10 x 9 x 8 x 7 x 6 ✓✓ five digits (3) **[6]**

TOTAL/*TOTAAL*: 150

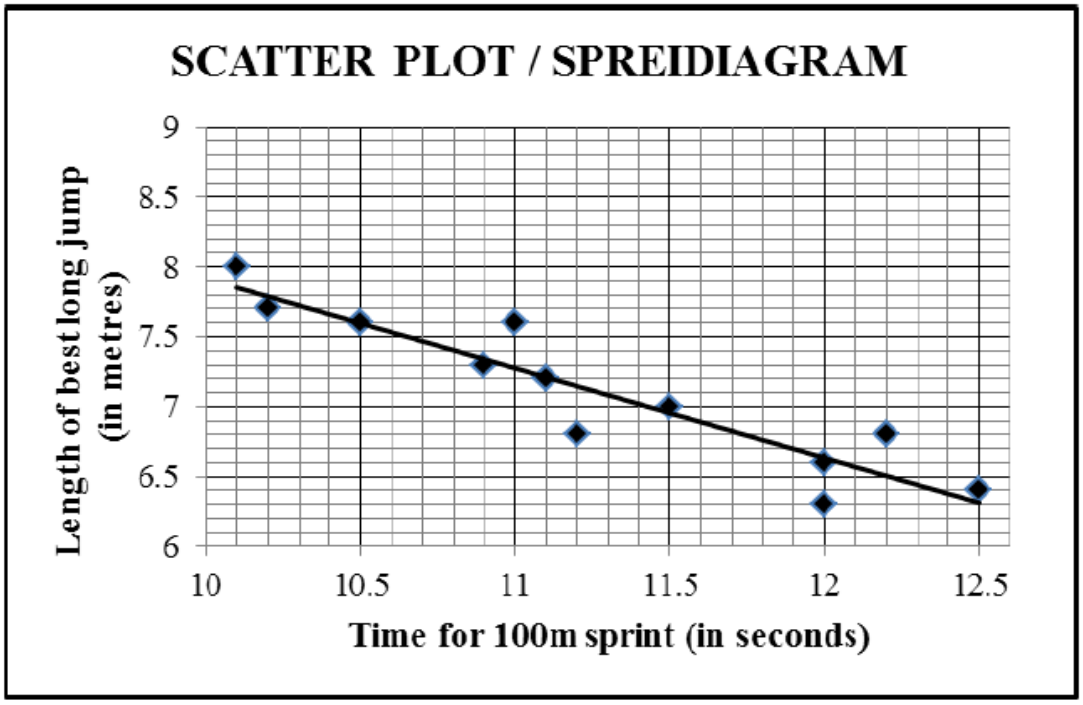
MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

QUESTION/*VRAAG* 1

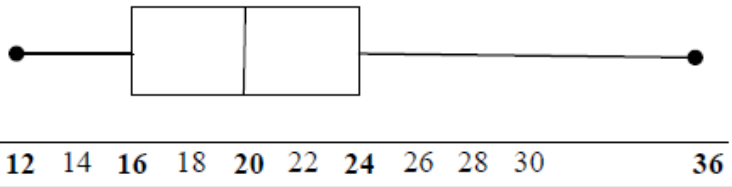
Time for 100 m sprint (in seconds) *Tyd vir 100 m-naelloop (in sekondes)*	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) *Afstand van beste sprong in verspring (in meter)*	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



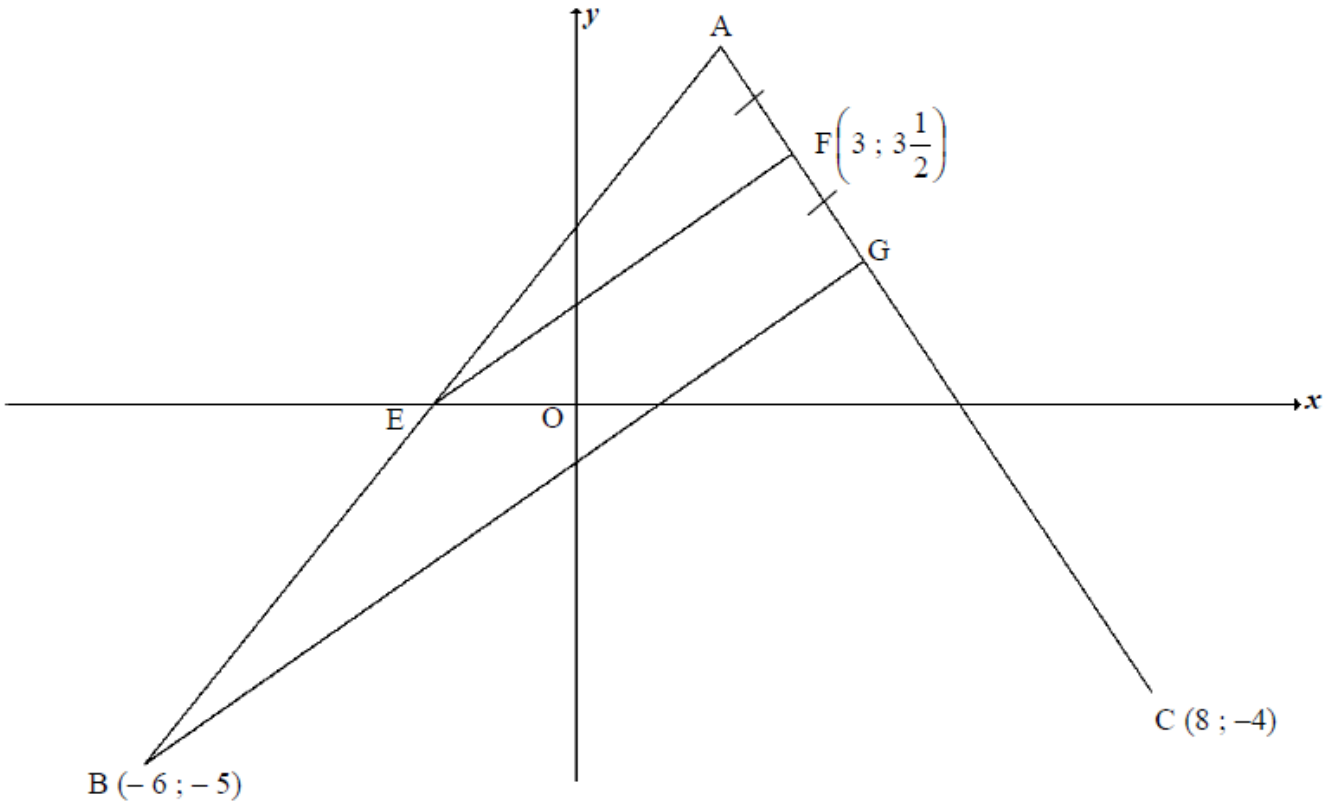
1.1	$a = 14{,}343\ldots = 14{,}34$ $b = -0{,}642\ldots = -0{,}64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. /*Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) [7]

QUESTION/*VRAAG* 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* = $Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ $\therefore$ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3	 **12** 14 **16** 18 **20** 22 **24** 26 28 30 **36**	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een** van die seuns **nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

QUESTION/*VRAAG* 3



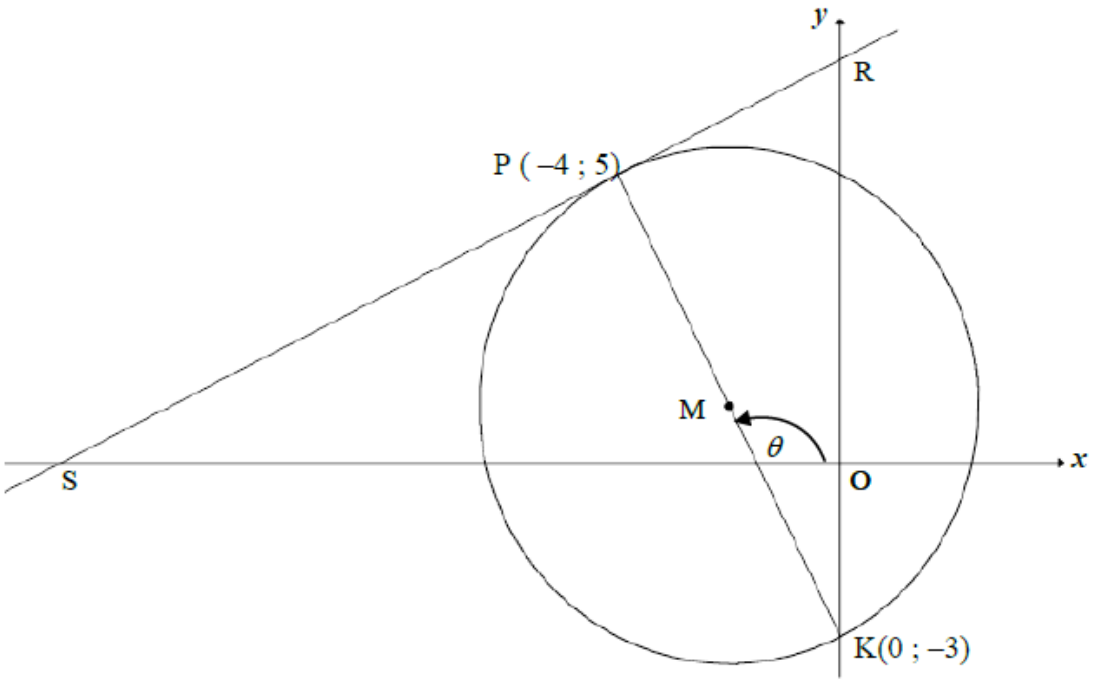
3.1.1	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ $y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$ $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c$ **OR/*OF*** $(y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $\qquad$ $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$ $\qquad$ $y = -\frac{3}{2}x + 8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3\,;\,3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and (8 ; –4) ✓ equation of AC (4)

	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - \left(3\frac{1}{2}\right)}{8 - 3}$ $= -\frac{3}{2}$ $y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $\qquad$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ $c = 8$ $\qquad$ **OR/*OF*** $\qquad$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$ $\qquad$ $y = -\frac{3}{2}x + 8$	✓ substitution of (8 ; –4) & $\left(3; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3; 3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$ $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2) **OR/*OF*** BG: $7x - 10y = 8$ $\quad \therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8$ [CA from 3.1.1] $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2)	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3)
3.2	$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$ $\therefore$ A(2 ; 5) **OR/*OF*** by translation/*deur translasie*: $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ $\therefore$ A(2 ; 5)	✓ equation ito x ✓ equation ito y (2) ✓ equation ito x ✓ equation ito y (2)

3.3	The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ **But the *y*-coordinate of E is 0** **∴ E(–2 ; 0) is the midpoint of AB** ∴ EF ‖ BG [midpoint theorem/*middelpuntst* ***OR/OF*** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] **OR/*OF*** The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ $AE=\sqrt{(-2-2)^2+(0-5)^2}=\sqrt{41}$ $EB=\sqrt{(-2-(-6))^2+(0-(-5))^2}=\sqrt{41}$ ∴ In ΔABG: **AE = EB** and AF = FG ∴ EF ‖ BG [midpoint theorem/*middelpuntst*] **OR/*OF*** Equation of AB: $y-(-5)=\left(\frac{5-(-5)}{2-(-6)}\right)(x-(-6))$ $y+5=\frac{10}{8}x+\frac{15}{2}\qquad\therefore y=\frac{5}{4}x+\frac{5}{2}$ x-intercept of AB: $0=\frac{5}{4}x+\frac{5}{2}\quad\therefore x=-2$ ∴ E(–2 ; 0) $m_{EF}=\frac{3\frac{1}{2}-0}{3-(-2)}=\frac{7}{10}$ $m_{EF}=m_{BG}=\frac{7}{10}$ ∴ EF ‖ BG BG: $7x-10y=8$ $\therefore\ y=\frac{7}{10}x-\frac{8}{10}$ $\therefore\ m_{BG}=\frac{7}{10}$	✓ subst A & B into midpt formula ✓ *y* coordinate = 0 ✓ E = midpt ✓ Reason (4) ✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4) ✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)

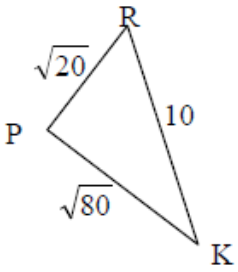
3.4	Midpoint of AC $= \left(5 ; \frac{1}{2}\right)$ $\frac{x_D + (-6)}{2} = 5$ and $\frac{y_D + (-5)}{2} = \frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5 ; \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC} = \frac{-5-(-4)}{-6-8} = \frac{1}{14}$ and $m_{AB} = \frac{5-(-5)}{2-(-6)} = \frac{5}{4}$ AD: $y - 5 = \frac{1}{14}(x-2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$ CD: $y + 4 = \frac{5}{4}(x-8) \Rightarrow y = \frac{5}{4}x - 14$ $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ $\therefore \quad x = 16$ $y = 6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

QUESTION/*VRAAG* 4



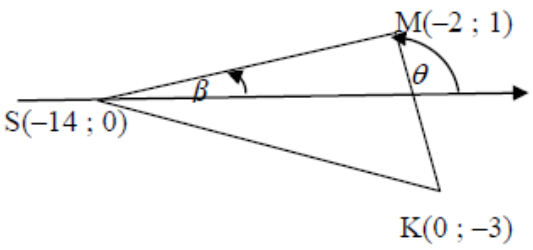
4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK ⊥ SR [radius ⊥ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK ⊥ SR **OR** r ⊥ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of m and P ✓ equation (2)

4.1.3	$M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore$ M(–2 ; 1) $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore$ M(–2 ; 1) $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore$ M(–2 ; 1) $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$	✓ *x* value of M ✓ *y* value of M ✓ $r^2=20$ ✓ equation (4) ✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4) ✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)

4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext ∠of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF*	
		
	In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ $RK = 10$	✓ lengths of PK, PR & RK
	$\cos P\hat{K}R = \frac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ correct values into cos rule ✓ answer (3)
	OR/*OF*	
	$\sin P\hat{K}R = \frac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \frac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF*	
	$\tan P\hat{K}R = \frac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

4.1.5	RS ∥ tangent at K(0 ; −3) $\therefore m_{PS} = m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{tang} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3 ; 7)$ **OR/*OF*** $-3 < t < 7$	✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ ∴ S(−14 ; 0) $SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK = $\frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) ✓ coordinates of S

$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ ✓ length of SM

$\tan\beta = \frac{1-0}{-2-(-14)} = \frac{1}{12} \quad \therefore \beta = 4{,}76°$

$\therefore S\hat{M}K = 116{,}57° - 4{,}76°$ [ext ∠ of Δ]

$= 111{,}81°$ ✓ size of/*grootte v* $S\hat{M}K$

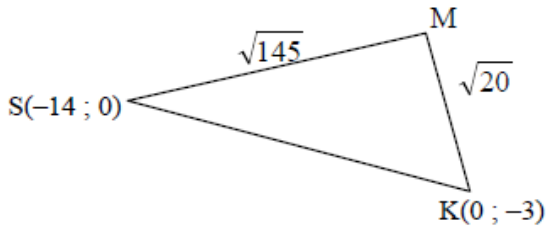
$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units ✓ answer

(5)

OR/*OF*



RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) ✓ coordinates of S

$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$ ✓ length of SK

$\cos S\hat{M}K = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$S\hat{M}K = 111{,}80°$ ✓ size of /*grootte v* $S\hat{M}K$

$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units ✓ answer

(5)

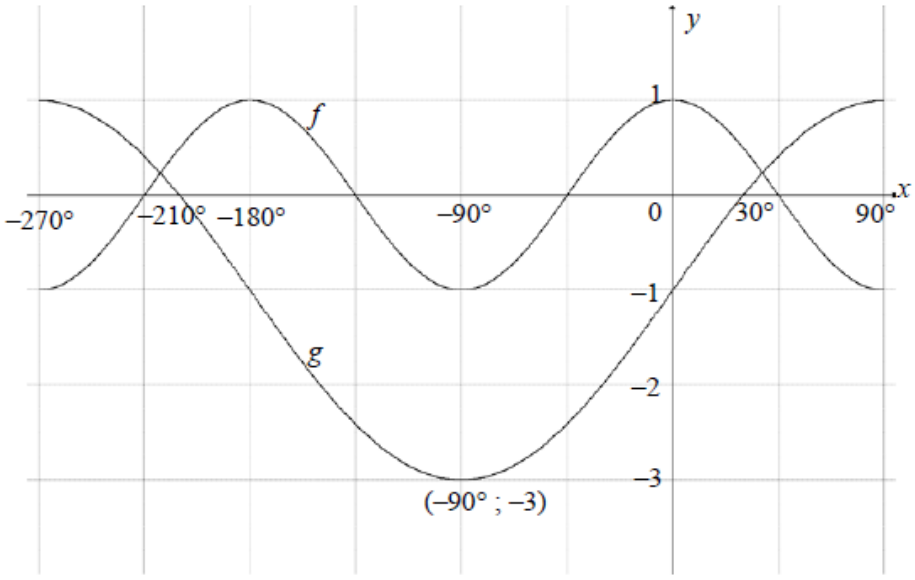
	OR/*OF*	
	Produce KS to T	
	RS: $y = \frac{1}{2}x + 7$ $\quad \therefore$ S(–14 ; 0)	✓ coordinates of S
	$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$	✓ length of SK & SM
	$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$	
	$m_{SK} = -\frac{3}{14} \Rightarrow T\hat{S}O = 167{,}91°$	
	$m_{SM} = \frac{1}{12} \Rightarrow M\hat{S}O = 4{,}76°$	
	$M\hat{S}K = 180° - 167{,}91° + 4{,}76° = 16{,}85°$	✓ size of /*grootte v* $M\hat{S}K$
	Area $\Delta SMK = \frac{1}{2}(SM)(SK).\sin M\hat{S}K$	
	$= \frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$	✓ correct substitution into area rule
	$= 24{,}9985 = 25$ square units	✓ answer
		(5)

QUESTION/*VRAAG* 5

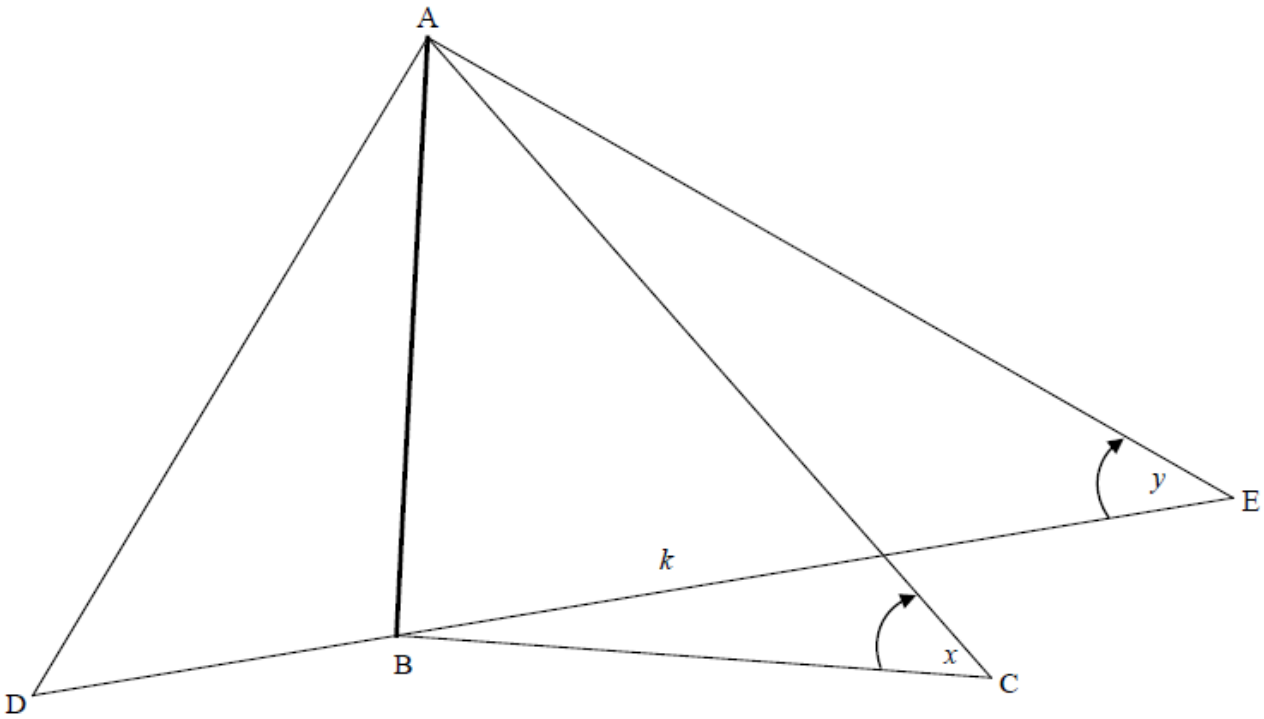
5.1	$\frac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$ $=\frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $=\frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $=\cos A$	✓ sin A ✓ –sin A ✓ sin A ✓ –tan A ✓ $\tan A=\frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$t^2=(\sqrt{34})^2-(3)^2$ $\therefore t=-5$	✓ substitution ✓ answer (2)
5.2.2	$\tan\beta=\frac{-5}{3}$	✓ correct ratio (1)
5.2.3	$\cos 2\beta = 2\cos^2\beta-1$ $=2\left(\frac{3}{\sqrt{34}}\right)^2-1$ $=2\left(\frac{9}{34}\right)-1$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=1-2\sin^2\beta$ $=1-2\left(-\frac{5}{\sqrt{34}}\right)^2$ $=1-2\left(\frac{25}{34}\right)$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=\cos^2\beta-\sin^2\beta$ $=\left(\frac{3}{\sqrt{34}}\right)^2-\left(-\frac{5}{\sqrt{34}}\right)^2$ $=\frac{9}{34}-\frac{25}{34}$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)

5.3.1	LHS = sin(A + B) – sin(A – B) = sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B) = sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B = 2cos A.sin B = RHS	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= 2\cos 60°.\sin 17°$ $= 2 \times \frac{1}{2} \times \sin 17°$ $= \sin 17°$ **OR/*OF*** $\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= (\sin 60° \cos 17° + \cos 60° \sin 17°) -$ $(\sin 60° \cos 17° - \cos 60° \sin 17°)$ $= \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17° - \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17°$ $= \sin 17°$	✓ 60° + 17° ✓ 60° – 17° ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ 60° + 17° ✓ 60° – 17° ✓ expansion ✓ $\frac{1}{2}$ (4) **[19]**

QUESTION/*VRAAG 6*

6.1		✓ (–90° ; –3) ✓ (0 ; –1) ✓ x – intercepts: –210° & 30° ✓ shape (4)
6.2	$\cos 2x = 2\sin x - 1$ $1 - 2\sin^2 x = 2\sin x - 1$ $2\sin^2 x + 2\sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \dfrac{-1+\sqrt{5}}{2}$, since $\sin x = \dfrac{-1-\sqrt{5}}{2} < -1$ has no solution	✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula (4)
6.3	$\sin x = \dfrac{-1+\sqrt{5}}{2} = 0{,}618...$ Reference $\angle = 38{,}17°$ $\therefore x = 38{,}17° + k.360°$ or $x = 141{,}83° + k.360°$; $k \in Z$ $\therefore x = 38{,}17°$ or $-218{,}17°$ $y = 0{,}24$ $\therefore$ Points of intersection/*snypunte*: $(38{,}17° ; 0{,}24)$ and $(-218{,}17° ; 0{,}24)$	✓ 38,17° ✓ 141,83° ✓ –218,17° ✓ 0,24 (4) **[12]**

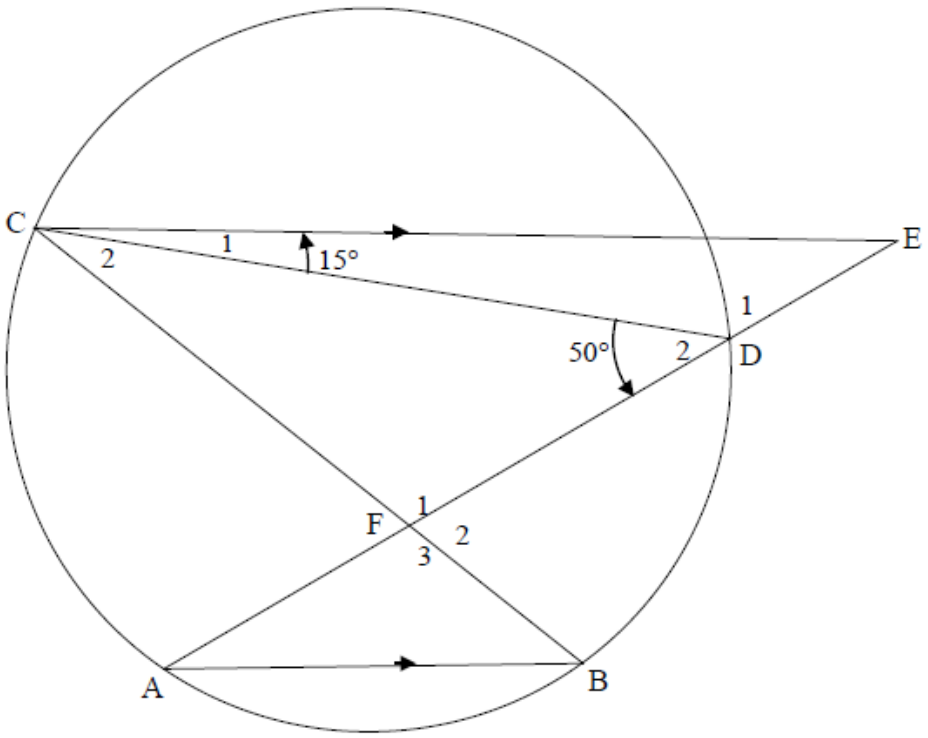
QUESTION/*VRAAG* 7



7.1	$A\hat{B}C = 90°$	✓ answer (1)
7.2	In Δ ABE: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In Δ ABC: $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution (4)

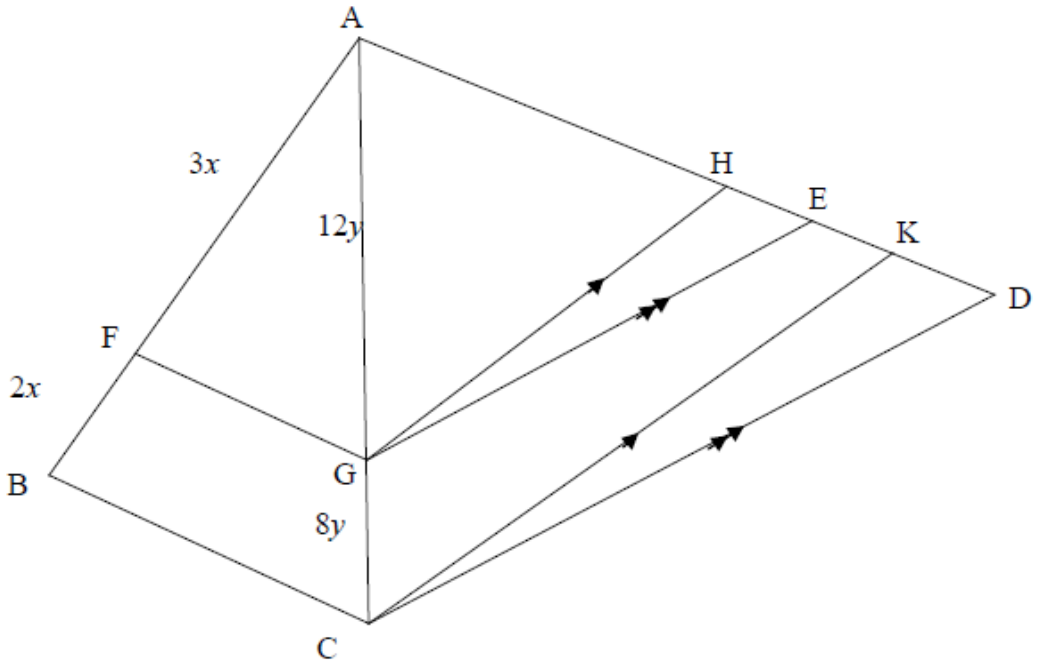
7.3	$A\hat{D}C = A\hat{C}D = \frac{180° - 2x}{2} = 90° - x$	✓ $90° - x$
	$\frac{DC}{\sin 2x} = \frac{AC}{\sin(90° - x)}$	✓subst into sine rule
	$\frac{DC}{2\sin x \cos x} = \frac{AC}{\cos x}$	✓ $2\sin x \cos x$
	$DC = \frac{AC(2\sin x \cos x)}{\cos x}$	✓ $\cos x$
	$= \frac{k\tan y}{\sin x} \cdot \frac{2\sin x \cos x}{\cos x}$	✓ substitution
	$= 2k\tan y$	(5)
	OR/*OF*	
	$DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$	✓substitution into cos rule
	$= AC^2 + AC^2 - 2AC^2\cos 2x$	
	$= 2AC^2(1 - \cos 2x)$	✓ factorisation
	$= 2AC^2(1 - 1 + \sin^2 x)$	✓ $1 - 2\sin^2 x$
	$= 4AC^2\sin^2 x$	✓ DC ito AC and $\sin x$
	$DC = 2AC.\sin x$	
	$= 2\left(\frac{k.\tan y}{\sin x}\right).\sin x$	✓ substitution
	$= 2k.\tan y$	(5)
	OR/*OF*	
	$DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$	✓correct cos rule
	$= 2\left(\frac{k\tan y}{\sin x}\right)^2 - 2\left(\frac{k\tan y}{\sin x}\right)^2\cos 2x$	✓ substitution
	$= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x}(1 - 2\sin^2 x)$	✓ $1 - 2\sin^2 x$
	$= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x} + 4k^2\tan^2 y$	✓ squaring and multiplication
	$DC = \sqrt{4k^2\tan^2 y}$	✓ $\sqrt{4k^2\tan^2 y}$
	$= 2k\tan y$	(5) **[10]**

QUESTION/*VRAAG* 8



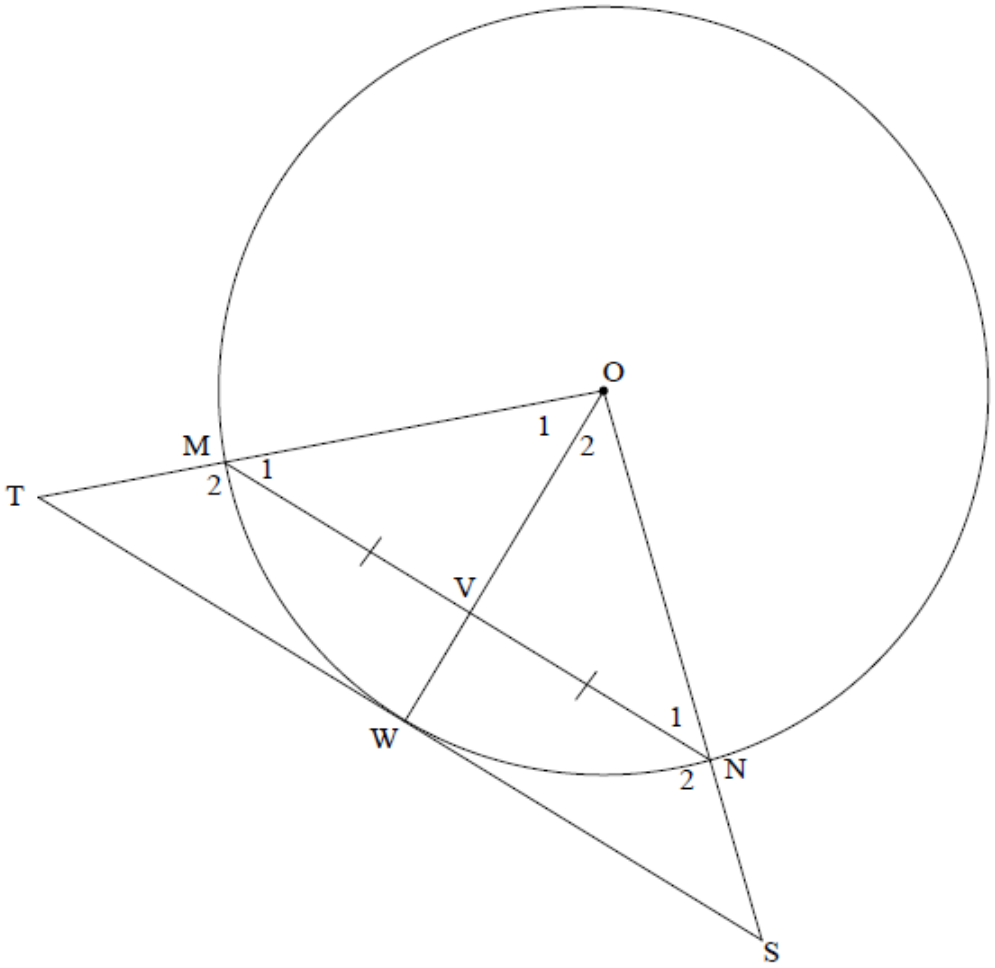
8.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite ∠ van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{E} = 180° - (130° + 15°) = 35°$ [str line; ∠s of Δ/*rt lyn;∠e van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{B} = 50°$ [∠s in same segment/*∠e in dieselfde segment*] $\hat{C}_2 + 15° = 50°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] $\therefore \hat{C}_2 = 35°$ $\hat{A} = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3)
8.1.2	$\hat{C}_2 = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓ R (2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] ∴ CF is a tangent to the circle [converse tan chord theorem] ∴ *CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]*	✓ S ✓ R (2) **[7]**

QUESTION/*VRAAG* 9



9.1.1	$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$ $\therefore \frac{AF}{BF}=\frac{AG}{CG}$ $\therefore$ FG ∥ BC [conv prop th/*omg eweredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v Δ in dies verh*]	✓ $\frac{AF}{BF}=\frac{AG}{CG}$ ✓R (2)
9.1.2	$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*; GH ∥ CK **OR** line ∥ to 1 side of Δ/*lyn* ∥ *1 sy van Δ*] $\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*; GE ∥ CD] $\therefore \frac{AH}{HK}=\frac{AE}{ED}$	✓S ✓ R ✓ S (3)
9.2	$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$ $\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$ $\therefore$ AE = 18 and HK = 10 $\therefore$ HE = AE – AH = 18 – 15 = 3 $\therefore$ EK = HK – HE = 10 – 3 = 7 **OR/*OF*** AD = 30 KD = AD – AH – HK = 30 – 15 – 10 = 5 EK = ED – KD = 12 – 5 = 7	✓ use of ratios ✓ AE = 18 ✓ HK = 10 ✓ HE = 3 **or** KD = 5 ✓ EK = 7 (5) **[10]**

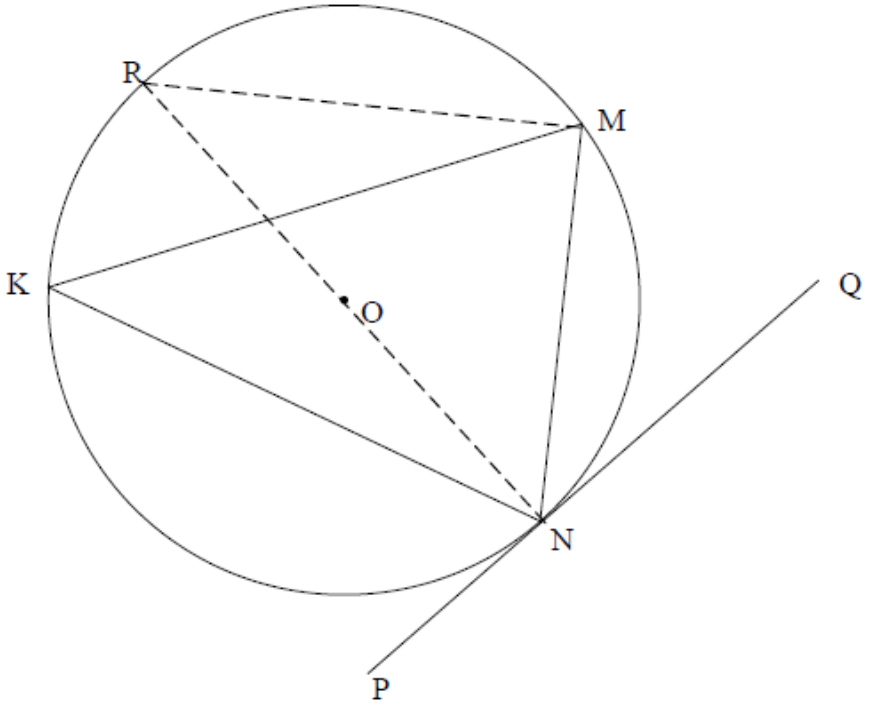
QUESTION/*VRAAG* 10



10.1	Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*	✓ R (1)
10.2.1	$O\hat{W}T = O\hat{W}S = 90°$ [radius ⊥ tangent/*raaklyn*] ∴ MN ‖ TS [corresp ∠s =/*ooreenkomstige ∠e* = **OR** co-int ∠s 180°/*ko-binne∠e 180°* **OR** alternate ∠s/*verwiss ∠e*]	✓ R ✓ R (2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{M}_1 = \hat{T}$ [corresp ∠s/*ooreenk ∠e*; MN ‖ TS] $\therefore \hat{N}_1 = \hat{T}$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*] **OR/*OF*** $\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{N}_1 = \hat{S}$ [corresp ∠s/*ooreenk ∠e*; MN ‖ TS] $\therefore \hat{S} = \hat{M}_1$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*]	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)

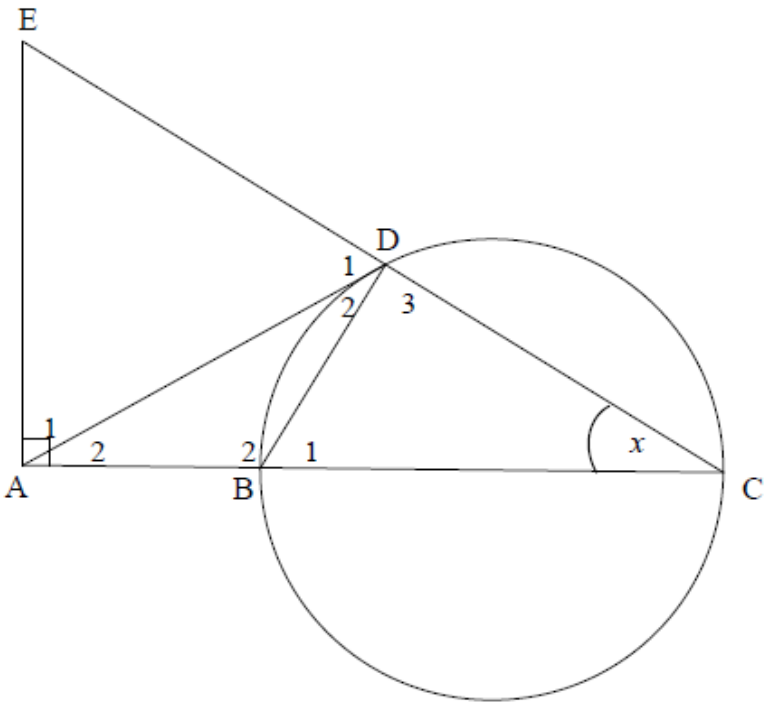
10.2.3	In ΔOVN and ΔOWS $\hat{O}_2 = \hat{O}_2$ [common/*gemeenskaplik*] $O\hat{V}N = O\hat{W}S = 90°$ [from 10.1] $O\hat{N}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] ∴ΔOVN ||| ΔOWS [∠, ∠, ∠] $\therefore \frac{VN}{WS} = \frac{ON}{OS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ ∴OS.MN = 2ON. WS **OR/*OF*** In ΔOVM and ΔOWS $O\hat{V}M = O\hat{W}S = 90°$ [from 10.1] $O\hat{M}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] ∴ΔOVM ||| ΔOWS [∠, ∠, ∠] $\therefore \frac{OM}{OS} = \frac{VM}{WS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{OM}{OS}$ ∴OS.MN = 2ON. WS [VM = VN] **OR/*OF*** If any other 2 Δs are used, first need to prove that TW = WS by proving ΔOWT ≡ ΔOWS In ΔOVM and ΔOWT $\hat{O}_1 = \hat{O}_1$ [common/*gemeenskaplik*] $O\hat{V}M = O\hat{W}T = 90°$ [from 10.1] $O\hat{M}V = O\hat{T}W$ [sum ∠s Δ/*som ∠e Δ*] ∴ΔOVM ||| ΔOWT [∠, ∠, ∠] $\therefore \frac{VM}{WT} = \frac{OM}{OT}$ But $VN = VM = \frac{1}{2}MN$ [given] and WT = WS and OT = OS [ΔOWT ≡ ΔOWS] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ ∴OS.MN = 2ON. WS	✓ S; S; S **OR** S; S; R ✓ΔOVN ||| ΔOWS ✓ $\frac{VN}{WS} = \frac{ON}{OS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5) ✓ S; S; S **OR** S; S; R ✓ΔOVM ||| ΔOWS ✓ $\frac{OM}{OS} = \frac{VM}{WS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5) ✓✓ similarity ✓✓ congruency ✓ $VN = VM = \frac{1}{2}MN$ (5) **[12]**

QUESTION/*VRAAG* 11



11.1	Construction: Draw diameter NR and draw RM *Konstruksie: Trek middellyn NR en verbind RM* $O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*] $\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ] $= M\hat{N}Q$ but $M\hat{R}N = M\hat{K}N$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$ **OR/*OF***	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)

11.2



11.2.1(a)	Angle in a semi circle/ *Hoek in halfsirkel*	✓ R (1)
11.2.1(b)	Exterior ∠of quad = opp interior ∠/ *Buite ∠ van vierh = teenoorst binne ∠* **OR/*OF*** Opp ∠s of quad supplementary/ *Teenoorst ∠e van vierh is supplementêr*	✓ R (1)
11.2.1(c)	tangent chord theorem/ *raaklyn koord stelling*	✓ R (1)
11.2.2(a)	In ΔAEC $\hat{E} = 180° - (90° + x)$ [sum ∠s Δ] $= 90° - x$ $\hat{D}_1 = 180° - (90° + x)$ [∠s on a straight line] $= \hat{E} = 90° - x$ ∴ AD = AE [sides opp = ∠s/ *sye teenoor = ∠e*]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum $\angle^e$ Δ] ∴ΔADB ||| ΔACD **OR/*OF*** In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] ∴ΔADB ||| ΔACD [∠, ∠, ∠]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

11.2.3(a)	$\frac{AD}{AC} = \frac{AB}{AD}$ [||| Δs] $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution (2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)] $AB = r$ and $BC = 2r$ $\therefore AC = 3r$ <u>In ΔACE:</u> $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [∠s of Δ = 180°] ∴ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC} = \frac{DB}{CD}$ [||| Δs] $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ ∴ In ΔBDC: $x = 30°$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [∠s of Δ = 180°] ∴ ΔADE is equilateral/*is gelyksydig*	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 ∠s = 60° (4) ✓ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ ✓ $\frac{1}{\sqrt{3}} = \tan x$ ✓ $x = 30°$ ✓ all 3 ∠s = 60° (4)
	$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ ∴ ED = EC – DC $= \sqrt{3}r$ ∴ ED = EA = AD ∴ ΔADE is equilateral/*is gelyksydig*	✓ $EC = 2\sqrt{3}r$ ✓ ED = EA = AD (4) **[20]**

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