education

Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P1

COMMON TEST

JUNE 2018

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

TIME: 2.5 hours

N.B. This question paper consists of 6 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Solve for x: (Answer correct to TWO decimal places where necessary)

1.1.1 $x\left(3-\frac{1}{2}x\right)=0$ (2)

1.1.2 $x^2+8x=5$ (4)

1.1.3 $2x-\sqrt{10+x}=1$ (5)

1.2 Solve the following equations simultaneously:

$y-x+1=0$ and $x^2+xy=3$ (6)

1.3 How many solutions does the equation $x \cdot 3^x - x + 33x^3 = 0$, have? Show all working. (3)
[20]

QUESTION 2

A quadratic sequence has x ; 4 ; 8 ; y ; … as its first four terms.

2.1 Determine the value(s) of x and y if its second difference is 4. (2)

2.2 Determine the n^{th} term of the sequence. (4)

2.3 Which term is equal to 12964? (3)
[9]

QUESTION 3

3.1 In an arithmetic sequence the eighth term is twice the fourth term. The sum of the first three terms is 12. Determine the first term. (5)

3.2 Evaluate: $\sum_{k=1}^{12} 3.2^{k-1}$ (3)

3.3 Determine the value(s) of x for which the series

$2+\frac{2}{3}(x+5)+\frac{2}{9}(x+5)^2+....$ will converge. (3)
[11]

QUESTION 4

Given: $g(x) = -\frac{2}{x+1} - 3$

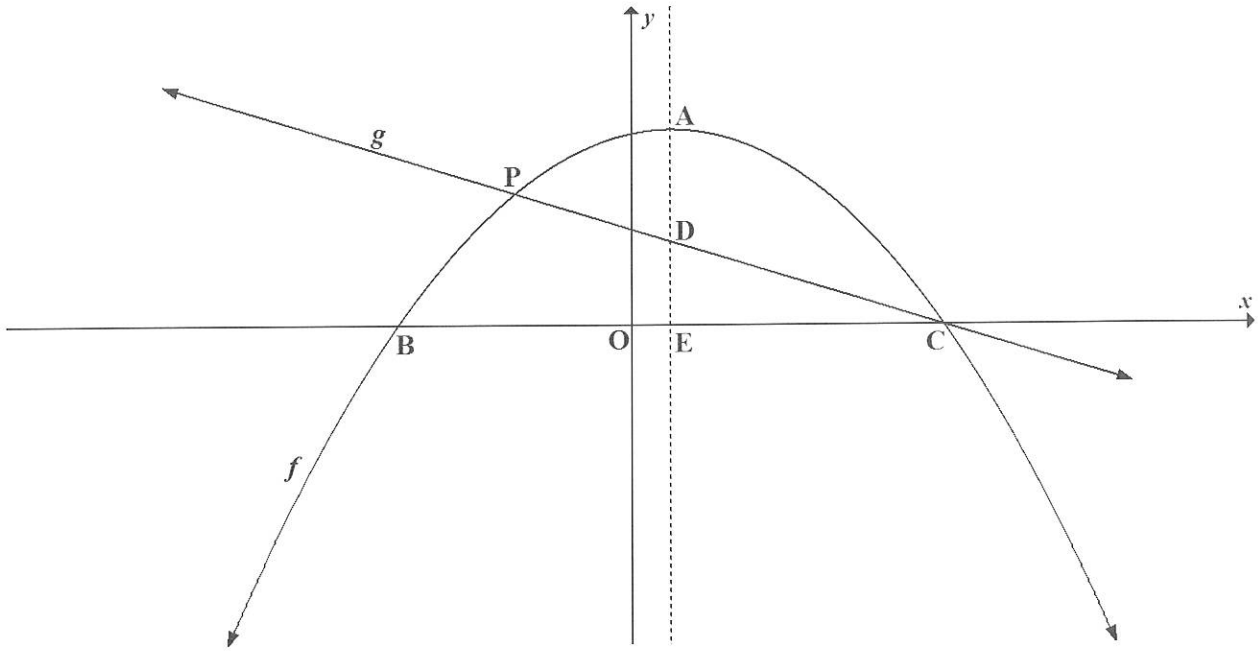
4.1 Write down the equation(s) of the vertical and horizontal asymptotes of g. (2)

4.2 Determine the intercepts of the graph of g with the x and y axes. (2)

4.3 Sketch the graph of g. (4)
[8]

QUESTION 5

$f(x) = -2x^2 + x + 6$ *and* $g(x) = -\frac{3}{2}x + 3$ are sketched below. f and g intersect at P and C. B and C are the x – intercepts of f. ADE is perpendicular to the x – axis, with A on f and D on g. A is the turning point of f.



5.1 Determine the co-ordinates of A, the turning point of f. (3)

5.2 Write down the range of f. (2)

5.3 Calculate the length of BC. (4)

5.4 Determine the co-ordinates of P. (5)

5.5 Determine the equation of h if $h(x) = f(x-3)$. Hence, write down the coordinates of the new turning point of h. (3)
[17]

QUESTION 6

Given $h(x) = a^x$ passes through the point $A\left(-2\,;\,\frac{1}{9}\right)$.

6.1 Calculate the value of a. (2)

6.2 Write down the equation of h^{-1}, the inverse of h, in the form $y = \ldots$ (2)

6.3 Write down the co-ordinates of any point on the graph of the inverse of h. (2)

6.4 Determine the value(s) of x for which $h^{-1}(x) \leq -2$. (2)

6.5 Write down the equation of f if $f(x) = h^{-1}\left(\frac{x}{2}\right)$. (2)

[10]

QUESTION 7

7.1 Office equipment costing R500 000 depreciates on the diminishing balance method at an interest rate of 9, 25 % p.a. Determine the value of the office equipment at the end of 7 years. (3)

7.2 A farmer has just bought a new tractor for R900 000.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time by trading the present one for R300 000. The replacement cost of the tractor is expected to increase by 8,25 % p.a. Calculate the amount that he will need to pay for the new tractor. (3)

7.2.2 One month after purchasing his present tractor, he makes monthly deposits of x rands into an account that pays interest at a rate of 12 % p.a. compounded monthly for 5 years. After 60 months he has the exact amount of money to purchase the new tractor after he trades in the present tractor. Calculate the value of x. (5)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter he withdraws R8000 from his account to pay for maintenance of the tractor. If he makes 5 such withdrawals, determine the value of the new monthly deposit. (4)

[15]

QUESTION 8

8.1 Determine the derivative of $f(x) = 3x^2 - 5x$ from first principles. (5)

8.2 Determine the derivative with respect to x if:

8.2.1 $f(x) = \dfrac{x^2 - 4}{x}$ (4)

8.2.2 $g(x) = \sqrt[3]{x^4} + (3x)^{-2}$ (4)

[13]

QUESTION 9

$f(x) = -x^3 + 3x^2 + 9x - 27 = -(x+3)(x-3)^2$ is the equation of a cubic function.

9.1 Write down the intercepts of f. (3)

9.2 Calculate the co-ordinates of the stationary points of f. (5)

9.3 Sketch the graph of f on a system of axes. (Clearly indicate the coordinates of the stationary points and the intercepts with the axes). (4)

9.4 Determine the value(s) of x for which the graph is concave down. (2)

9.5 Determine the equation of the tangent to the graph of f at $x = 0$. (3)

9.6 If $f(x) = k$ has 3 unequal real roots, determine the value(s) of k. (3)

9.7 Write down the equation of t if f is shifted 3 units horizontally to the left. (2)

[22]

TOTAL MARKS: 125

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad area\ \Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

Education

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REPUBLIC OF SOUTH AFRICA

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MEMORANDUM

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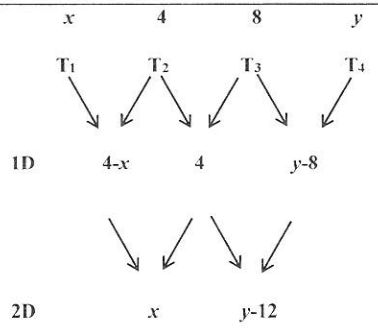
This memorandum consists of 11 pages.

QUESTION 1

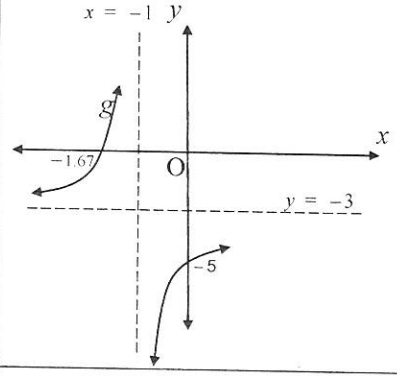
1.1.1	0 *or* 6	A✓ 0 A✓ 6	(2)
1.1.2	$x^2+8x-5=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(8)\pm\sqrt{(8)^2-4(1)(-5)}}{2(1)}$ $=0{,}58 \quad or \quad -8{,}58$	A✓formula CA✓substitution CACA✓✓answers **[Penalise for rounding off here only in the entire paper]**	(4)
1.1.3	$2x-\sqrt{10+x}=1$ $2x-1=\sqrt{10+x}$ $4x^2-4x+1=10+x$ $4x^2-5x-9=0$ $(x+1)(4x-9)=0$ $x=-1 \quad or \quad x=\frac{9}{4}$ n/a	A✓Isolating surd term CA✓ squaring both sides CA✓standard form CA✓factors CA✓answers and rejecting	(5)
1.2	$y-x+1=0$ .....(1) and $x^2+xy=3$ ....(2) From (1) : $y=x-1$.....(3) Substituti ng (3) into (2) $x^2+x(x-1)=3$ $x^2+x^2-x=3$ $2x^2-x-3=0$ $(2x-3)(x+1)=0$ $x=\frac{3}{2} \quad or \quad x=-1$ $y=\frac{1}{2} \quad or \quad y=-2$	A✓y / x the subject CA✓substitution CA✓standard form CA✓factors CA✓ x – values CA✓ y – values	(6)
1.4	$x.3^x-x+33x^3=0$ $x(3^x-1+33x^2)=0$ $x=0$ *or* $(3^x-1+33x^2)=0$ $3^x-1=-33x^2$ The graph $y=3^x-1$ has its x – intercept at $x=0$ and the graph $y=-33x^2$ has its maximum value at $x=0$. Therefore the equation has only one solution	A✓factorization A✓reasoning A✓ conclusion	(3)
			[20]

QUESTION 2

2.1	 $x = 4$ and $y = 16$	A✓ x – value A✓ y – value	(2)
2.2	$2a = 4 \quad a = 2$ $3a + b = 0 \quad b = -6$ $a + b + c = 4$ $c = 8$ $T_n = 2n^2 - 6n + 8$	CA✓ a value CA✓ b value CA✓ c value CA✓answer	(4)
2.3	$T_n = 2n^2 - 6n + 8 = 12964$ $2n^2 - 6n - 12956 = 0$ $n^2 - 3n - 6478 = 0$ $(n - 82)(n + 79) = 0$ $\therefore n = 82 \quad or \quad n \neq 79$	A✓quadratic CA✓factors CA✓ answer	(3)
			[9]

QUESTION 3

3.1	$a ; (a + d) ; (a + 2d) ; \ldots\ldots$ $a + 7d = 2(a + 3d)$ $a + 7d = 2a + 6d$ $a - d = 0 \quad \rightarrow (1)$ $a + a + d + a + 2d = 12$ $3a + 3d = 12$ $a + d = 4 \quad \rightarrow (2)$ $(1) + (2) : 2a = 4 \quad \Rightarrow a = 2$	A✓ $a + 7d = 2(a + 3d)$ CA✓$a - d = 0$ A✓ $a + a + d + a + 2d = 12$ CA✓ $a + d = 4$ CA✓a – value	(5)
3.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{12} = \frac{3(2^{12} - 1)}{2 - 1}$ $= 12285$	A✓correct formula A✓substitution CA✓ answer	(3)
3.3	$-1 < r < 1$ $-1 < \frac{x + 5}{3} < 1$ $-3 < x + 5 < 3$ $-8 < x < -2$	A✓condition A✓substitution of r value CA✓answer	(3)
			[11]

4.1	$x=-1$ *and* $y=-3$	A✓ $x=-1$ A✓ $y=-3$	(2)
4.2	y – intercept : $y=-5$ x – intercept : $x=-\frac{5}{3}$	A✓ y – intercept value A✓ x – intercept value	(2)
4.3		A✓ shape CA✓ x – intercept CA✓ y – intercept CA✓ asymptotes	(4)
			[8]

5.1	$y=-2\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)+6=6\frac{1}{8}$ $A\left(\frac{1}{4};6\frac{1}{8}\right)$	A✓ axis if symmetry value CA✓ y – value CA✓ answer [coordinate form]	(3)
5.2	$y\le 6\frac{1}{8}$	CACA✓✓ answer	(2)
5.3	$-2x^2+x+6=0$ $2x^2-x-6=0$ $(2x+3)(x-2)=0$ $x=-\frac{3}{2}$ *or* $x=2$ BC $=3\frac{1}{2}$ units	A✓ equation equal to 0 CA✓ factors CA✓ x – values CA✓ answer	(4)
5.4	$-2x^2+x+6=-\frac{3}{2}x+3$ $-4x^2+2x+12=-3x+6$ $4x^2-5x-6=0$ $(4x+3)(x-2)=0$ $x=-\frac{3}{4}$ *or* $x=2$ $P\left(-\frac{3}{4};4\frac{1}{8}\right)$	A✓ equating CA✓ standard form CA✓ factors CA✓ x – values CA✓ answer	(5)
5.5	$h(x)=-2(x-3)^2+(x-3)-3$ $=-2\left(x^2-6x+9\right)+x-6$ $=-2x^2+13x-24$ New turning point $A\left(3\frac{1}{4};6\frac{1}{8}\right)$	A✓ substituting $(x-3)$ CA✓ x – value CA✓ y – value	(3)
			[17]

QUESTION 6

6.1	$h(x) = a^x$ $\frac{1}{9} = a^{-2}$ $3^{-2} = a^{-2}$ $a = 3$	A✓ substitution of the point A CA✓ answer	(2)
6.2	$y = \log_3 x$	CACA✓✓ answer	(2)
6.3	$\left(\frac{1}{9}; -2\right)$	AA✓ ✓answer or any other point that satisfies the equation.	(2)
6.4	$h^{-1}(x) \leq -2$ $\log_3 x = -2$ $x = 3^{-2} = \frac{1}{9}$ $0 < x \leq \frac{1}{9}$	A✓ x – value CA✓ answer	(2)
6.5	$f(x) = h^{-1}\left(\frac{x}{2}\right)$ $= \log_3\left(\frac{x}{2}\right)$	CACA✓✓ answer	(2)
			[10]

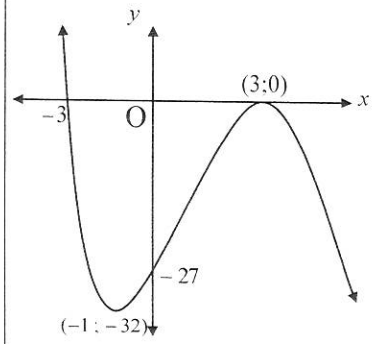
QUESTION 7

7.1	$A = P(1-i)^n$ $= 500\ 000(1-0{,}0925)^7$ $= R253\ 452{,}42$	A✓formula A✓substitution into formula CA✓answer	(3)
7.2.1	$A = P(1+i)^n$ $= 900\ 000(1+0{,}0825)^5$ $= R133\ 7771{,}79$ He will pay R1337771,79 – R300 000 =R1037771,79	A✓substitution into formula CA✓value CA✓answer	(3)
7.2.2	$F_v = \frac{x[(1+i)^n - 1]}{i}$ $1037771{,}79 = \frac{x\left[\left(1+\frac{0{,}12}{12}\right)^{60} - 1\right]}{\frac{0{,}12}{12}}$ $= R12706{,}94$	A✓formula A✓ substitution of F A✓substitution of i A✓substitution of n CA✓answer	(5)
7.2.3	$F_v = \frac{x[(1+i)^n - 1]}{i}$ $8000 = \frac{x\left[\left(1+\frac{0{,}12}{12}\right)^{12} - 1\right]}{\frac{0{,}12}{12}}$ $= R630{,}79$ New monthly deposit = R 13337,73	A✓formula A✓substitution of F and i values A✓substitution of n value CA✓ R630,79	(4)
			[15]

QUESTION 8 [Penalise once only in questions 8 and 9 for notational error]

8.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{3(x+h)^2-5(x+h)-(3x^2-5x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{3x^2+6xh+3h^2-5x-5h-3x^2+5x}{h}$ $=\lim\limits_{h\to 0}\dfrac{6xh+3h^2-5h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h-5)}{h}$ $=6x-5$	A✓formula A✓substitution CA✓ simplification of numerator CA✓factors CA✓answer	(5)
8.2.1	$f(x)=\dfrac{x^2-4}{x}$ $=x-4x^{-1}$ $f'(x)=1+4x^{-2}$	A✓x A✓$-4x^{-1}$ CA✓answer CA✓answer	(4)
8.2.2	$g(x)=\sqrt[3]{x^4}+(3x)^{-2}$ $=x^{\frac{4}{3}}+\dfrac{1}{9}x^{-2}$ $g'(x)=\dfrac{4}{3}x^{\frac{1}{3}}-\dfrac{2}{9}x^{-3}$	A✓$x^{\frac{4}{3}}$ A✓$\dfrac{1}{9}x^{-2}$ CA✓answer CA✓answer	(4)
			[13]

QUESTION 9

9.1	$x=-3$ *or* $x=3$ $y=-27$	AA✓✓x – intercepts / point form A✓y – intercept /point form	(3)
9.2	$f(x)=-x^3+3x^2+9x-27$ $f'(x)=-3x^2+6x+9=0$ $x^2-2x-3=0$ $(x+1)(x-3)=0$ $x=-1$ *or* $x=3$ $y=-32$ *or* $y=0$ $(-1;-32)$ $(3;0)$	A✓derivative and equal to 0 CA✓standard form CA✓factors CA✓x – values CA✓y – values	(5)
9.3		A✓shape CA✓ x – intercepts CA✓ y – intercept CA✓ turning points	(4)
9.4	$f'(x)=-3x^2+6x+9$ $f''(x)=-6x+6=0$ $x=1$ $x>1$	CA✓second derivative CA✓answer	(2)

9.5	$f'(x) = -3x^2 + 6x + 9$ $f'(0) = 9$ $y = 9x - 27$	A✓substituting $x = 0$ into derivative CA✓gradient value CA✓equation	(3)
9.6	$-32 < k < 0$	CA✓-32 CA✓0 A✓inequality	(3)
9.7	$t(x) = f(x+3)$ $= -(x+3)^3 + 3(x+3)^2 + 9(x+3) - 27$	A✓$f(x+3)$ CA✓substitution of $(x+3)$ into f	(2)
			[22]

Total Marks : 125