Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2018

MARKS: 75

TIME: 1½ hours

N.B. This question paper consists of 5 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 6 questions.

2.   Answer **ALL** questions.

3.   Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in
        determining your answers.

4.   Answers only will not necessarily be awarded full marks.

5.   An approved scientific calculator (non-programmable and non-graphical) may be
        used, unless stated otherwise.

6.   If necessary, answers should be rounded off to TWO decimal places, unless stated
        otherwise.

7.   Diagrams are NOT necessarily drawn to scale.

8.   Number the answers correctly according to the numbering system used in this
        question paper. Write neatly and legibly.

QUESTION 1

5 ; 12 ; 21 ; 32 ; ... is a Quadratic Sequence.

1.1   Write down the next term of the sequence.   (1)

1.2   Determine the nth term of the sequence.   (4)

1.3   Which term of the above sequence is 1152?   (4)

1.4   Prove that none of the terms in the sequence are perfect squares.   (3)

[12]

QUESTION 2

2 ; 5 ; 8 ; 11 ;... is an Arithmetic Sequence.

2.1   Determine the first term that will be greater than 2012.   (3)

2.2   Calculate the sum of the first 671 terms of the sequence.   (3)

[6]

QUESTION 3

3.1   Prove that the sum to n terms of a Geometric series is given by $S_n = \dfrac{a(1-r^n)}{1-r}$.   (4)

3.2   The first term of a geometric series is 12, the last term is $\dfrac{3}{256}$ and the sum of the series is $\dfrac{6141}{256}$. Determine the common ratio and the number of terms of the series.   (6)

[10]

QUESTION 4

The graphs below are $f(x) = \frac{1}{3}x^2$ and $g(x) = -\frac{1}{x+1} + 3$. A($a$; b) and B(c; d) are points(s) of intersection f and g.
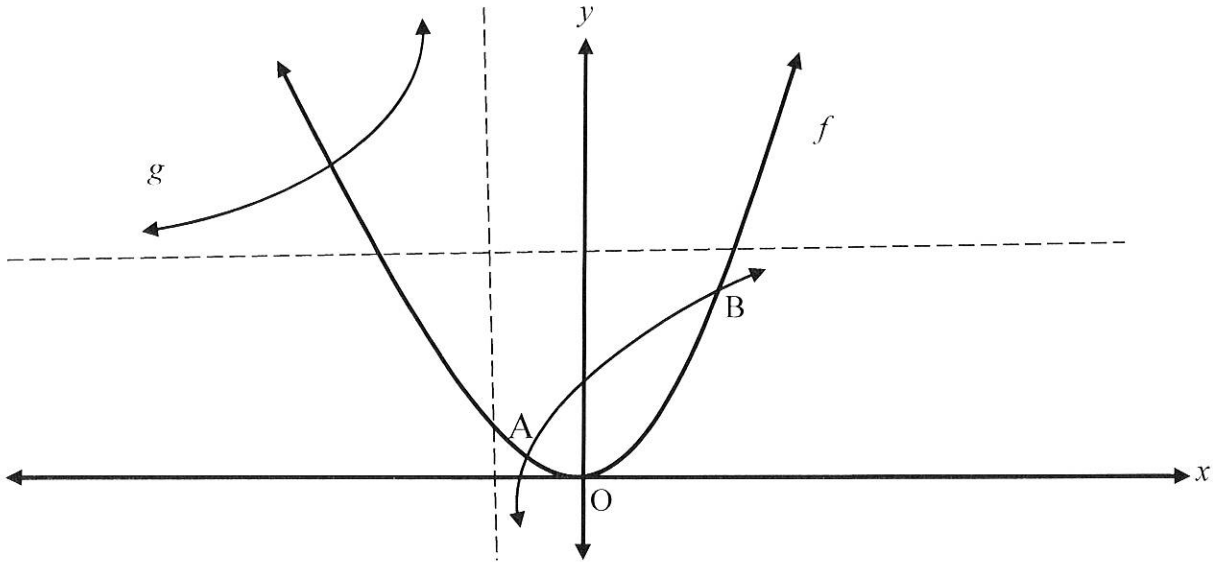
4.1   Write down the equations of the asymptotes of g.   (2)

4.2   If $y = x + c$ is a line of symmetry to the graph of g, calculate the value(s) of c.   (2)

4.3   Write down the range of f.   (1)

4.4   Calculate the value(s) of x for which $g(x) \geq 0$.   (4)

4.5   Write down the x – value(s) for which $f(x) = g(x)$, $x \geq -1$.   (2)

4.6   Determine the value(s) of x if $f(x) \leq g(x)$, for the interval $x \geq -1$.   (2)

4.7   If $h(x) = -f(x+2)+1$, then write down the new equation of h in the
      form $h(x) = a(x+p)^2 + q$.   (2)

4.8   Use your graph to determine the maximum value of $3^{h(x)+3}$.   (2)

4.9   Determine the value(s) of k for which $\frac{1}{3}(x+5)^2 = k$ has one root equal to 0.   (2)

                                                                                          [19]

QUESTION 5

5.1 Sketch the graph of $f(x) = 3^{-x}$ on a set of axes in your answer book. (2)

5.2 Write down the equation of the inverse of f in the form $y = \ldots\ldots$ (2)

5.3 Sketch f^{-1} on the same set of axes indicating the intercepts with the axes and the line of symmetry with the graph of f. (3)

5.4 Write down the equation of $g(x)$ if $g(x) = f^{-1}(-x)$. (2)

5.5 Determine the value(s) of x for which $f^{-1}(x) \geq -1$. (4)

[13]

QUESTION 6

6.1 How long will it take for a motor vehicle to depreciate to half its original value if the rate of depreciation is 12,35% p.a., on the diminishing balance method ? (Give your answer in years and months). (3)

6.2 On the 1 July 2017, a businessman wanted to buy a new house that costs R900 000. He took out a loan to the value of the house with a bank at an interest rate of 10,25% p.a. compounded monthly for 20 years. The bank allowed the businessman to start repayment towards the loan in three months time and indicated that his first instalment will be deducted on the 1 October 2017.

6.2.1 Calculate the value of the loan on 1 September 2017. (2)

6.2.2 Determine the monthly repayments if he now makes 238 monthly payments. (3)

6.2.3 If he does not make the 100th, 101st, and 102nd payments, calculate the new instalment that he will have to make to settle the loan in the fixed time period. (7)

[15]

TOTAL MARKS: 75

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \triangle ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

MATHEMATICS

COMMON TEST

MARCH 2018

MEMORANDUM

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 75

TIME: 1.5 hours

This memorandum consists of 9 pages.

QUESTION 1

1.1	45	A✓	(1)
1.2	 $2a = 2 \quad \therefore \quad a = 1$ $3a + b = 7 \quad \therefore b = 4$ $a + b + c = 5 \quad \therefore \quad c = 0$ $T_n = n^2 + 4n$ **OR** $T_n = T_1 + (n-1)d_1 + \dfrac{(n-1)(n-2)}{2}d_2$ $\quad = 5 + (n-1)(7) + \dfrac{(n-1)(n-2)}{2}(2)$ $\quad = 5 + 7n - 7 + n^2 - 3n + 2$ $\quad = n^2 + 4n$	A✓ a value CA✓ b value CA✓ c value CA✓ answer **OR** A✓ formula A✓ substitution into correct formula CA✓ simplifying CA✓ answer	(4) (4)
1.3	$n^2 + 4n = 1152$ $n^2 + 4n - 1152 = 0$ $(n+36)(n-32) = 0$ $n = -36 \quad or \quad n = 32$ n/a	M✓ equating to 1152 CA✓ standard form CA✓ factors CA✓ n values and rejecting	(4)
1.4	Suppose $n^2 + 4n = n^2$ $4n = 0$ $n = 0$ But $n \in N$ Therefore no such n exists and no terms are perfect squares in the sequence.	M✓ equating n^{th} term to n^2 A✓ value of n A✓ reason	(3)
			[12]

QUESTION 2

2.1	$T_n = 3n - 1$	A✓ n^{th} term $= 3n - 1$	
	$3n - 1 > 2012 \quad or \quad 3n - 1 = 2012$	A✓ setting up inequality /equation	
	$3n > 2013 \qquad\qquad 3n = 2013$		
	$n > 671 \qquad\qquad\quad n = 671$		
		CA✓ answer	
	The 672nd term will be the first term to be greater than 2012.		(3)
2.2	$S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$	A✓ S_n formula	
	$S_{671} = \dfrac{671}{2}\big[2(2) + 670(3)\big]$	A✓ substitution of a, n and d into correct formula	
	$\qquad = 675697$	CA✓ answer	
	OR	**OR**	(3)
	$T_n = a + (n-1)d$		
	$T_{671} = 2 + (671-1)(3) = 2012$		
	$S_n = \dfrac{n}{2}\big[a + T_n\big]$	A✓ S_n formula	
	$S_{671} = \dfrac{671}{2}\big[2 + 2012\big]$	A✓ substitution of a, n and T_n into correct formula	
	$\qquad = 675697$	CA✓ answer	
			(3)
			[6]

QUESTION 3

3.1	$S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1} \rightarrow (1)$	A✓ setting up equation 1	
	$rS_n = ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n \rightarrow (2)$	A✓ setting up equation 2	
	$(1) - (2): S_n - rS_n = a - ar^n$	A✓ subtraction LHS and RHS	
	$\qquad S_n(1-r) = a(1-r^n)$	A✓ factorizing LHS and RHS	(4)
	$\qquad\quad S_n = \dfrac{a(1-r^n)}{1-r}$		

If term (2) − (1) : Continued ... then

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{FULL MARKS}$$

3.2	$a = 12$		
	$ar^{n-1} = \dfrac{3}{256}$ $12r^{n-1} = \dfrac{3}{256}$ $\rightarrow$ (1)	A✓ setting up equation	
	$S_n = \dfrac{12(1-r^n)}{1-r} = \dfrac{6141}{256}$ $\rightarrow$ (2)	A✓ setting up equation	
	From (1):		
	$\dfrac{12r^n}{r} = \dfrac{3}{256}$ $\Rightarrow 12r^n = \dfrac{3r}{256}$ $\rightarrow$ (3)	A✓ making $12r^n$ the subject	
	Substituting (3) into (2)		
	$\dfrac{12 - \dfrac{3r}{256}}{1-r} = \dfrac{6141}{256}$	CA✓ substitution of $12r^n$ into (2)	
	$12 - \dfrac{3r}{256} = \dfrac{6141}{256} - \dfrac{6141r}{256}$		
	$\dfrac{3072}{256} - \dfrac{3r}{256} = \dfrac{6141}{256} - \dfrac{6141r}{256}$		
	$6138r = 3069$		
	$r = \dfrac{1}{2}$	CA✓ r value	
	$12\left(\dfrac{1}{2}\right)^n = \dfrac{3\left(\dfrac{1}{2}\right)}{256}$		
	$\left(\dfrac{1}{2}\right)^n = \dfrac{1}{2048} = \left(\dfrac{1}{2}\right)^{11}$		
	$n = 11$	CA✓ n value	(6)
	OR	**OR**	

$a = 12$

$ar^{n-1} = \dfrac{3}{256}$ $12r^{n-1} = \dfrac{3}{256}$ $\to (1)$ A✓ setting up equation

$S_n = \dfrac{12(1 - r^n)}{1 - r} = \dfrac{6141}{256}$ $\to (2)$ A✓ setting up equation

From (1):

$\dfrac{12r^n}{r} = \dfrac{3}{256}$ $\Rightarrow r^n = \dfrac{1}{1024}r$ $\to (3)$ A✓ making r^n the subject

Substituting (3) into (2)

$\dfrac{12\left(1 - \dfrac{1}{1024}r\right)}{1 - r} = \dfrac{6141}{256}$ CA✓ substitution of r^n into (2)

$\dfrac{1 - \dfrac{1}{1024}r}{1 - r} = \dfrac{6141}{3072}$

$1 - \dfrac{1}{1024}r = \dfrac{6141}{3072} - \dfrac{6141}{3072}r$

$\dfrac{6141}{3072}r - \dfrac{1}{1024}r = \dfrac{6141}{3072} - 1$

$\dfrac{6141}{3072}r - \dfrac{3}{3072}r = \dfrac{6141}{3072} - \dfrac{3072}{3072}$

$\dfrac{6138}{3072}r = \dfrac{3069}{3072}$

$r = \dfrac{1}{2}$ CA✓ r value

$\left(\dfrac{1}{2}\right)^n = \left(\dfrac{1}{2^{10}}\right)\left(\dfrac{1}{2}\right) = \dfrac{1}{2^{11}}$

$n = 11$ CA✓ n value   (6)

[10]

Very long underline

QUESTION 4

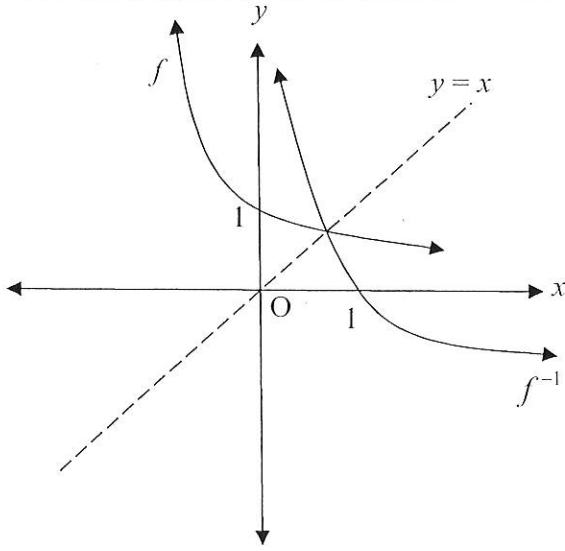| 4.1	$x = -1$ $y = 3$ — DON'T accept ($p = -1$ $q = 3$)	A✓ vertical asymptote A✓ horizontal asymptote	(2)
4.2	$y = x + c$ $3 = -1 + c$ $c = 4$	A✓ substituting $(-1 ; 3)$ CA✓ c – value	(2)
4.3	$y \geq 0$ or $y \in [0 ; \infty)$	A✓ answer	(1)
4.4	$\dfrac{-1}{x+1} + 3 \geq 0$ $\dfrac{-1 + 3x + 3}{x+1} \geq 0$ $\dfrac{3x + 2}{x+1} \geq 0$ $x < -1 \quad or \quad x \geq -\dfrac{2}{3}$ **OR** $-\dfrac{1}{x+1} + 3 = 0$ $3 = \dfrac{1}{x+1}$ $3x + 3 = 1$ $3x = -2$ $x = -\dfrac{2}{3}$ $x < -1 \quad or \quad x \geq -\dfrac{2}{3}$ *If a candidate cross multiplies by $(x+1)$ only – 0/4 B/D.*	A✓ determining LCD and numerator CA✓ simplification CA✓CA✓ answers **OR** *Using x-int approach / then, a graph only* A✓ setting up equation to calculate x – intercept CA✓ x – value CA✓CA✓ answers	(4) (4)
4.5	$x = a \quad or \quad x = c$ *If Candidates solved simultaneously - have irrational roots - full marks*	A✓A✓ answers Equating / $x^3 + x^2 - 9x = 6$ Full marks without solving	(2)
4.6	$a \leq x \leq c$ *If Candidates solved simultaneously - have irrational roots - full marks* Remove Oust	AA✓✓ critical values, inequality	(2)

4.7 $h(x) = -\frac{1}{3}(x+2)^2 + 1$ ✓ A (2)

4.8 Max value of $h(x) = 1$ ✓CA
∴ $3^{h(x)+3} = 3^4 = 81$ ✓CA (2)

4.9 $\frac{1}{3}(x+5)^2 = k$
$\frac{1}{3}(0+5)^2 = k$ A
OR $k = \frac{25}{3}$ ✓A
(ans only 2 marks)

4.7	$h(x) = -\dfrac{1}{3}(x+2)^2 + 1$	AA✓✓ answer	(2)
4.8	Maximum value of $h(x) = 1$ Therefore maximum value of $3^{h(x)+3}$ $\qquad\qquad = 3^4 = 81$	CA✓ 1 CA✓ 81	(2)

4.9	$\frac{1}{3}(x+5)^2 = k$	Answer only	A✓ substituting $k = 0$	
	$\frac{1}{3}(0+5)^2 = k$	full marks	A✓ answer	
	$\frac{25}{3} = k$			(2)
				[19]

QUESTION 5

5.1		f: A✓ shape A✓ y – intercept = 1	(2)
5.2	$y = \log_{\frac{1}{3}} x = -\log_3 x = \log_3 \frac{1}{x}$	AA✓✓ answer	(2)
5.3	see graph in 5.1 to allocate marks	f^{-1}: CA✓ shape A✓ x – intercept at $1/(1\,;\,0)$ A✓ $y = x$ line	(3)
5.4	$g(x) = \log_{\frac{1}{3}}(-x)\ ;\ x < 0$ OR $g(x) = -\log_3(-x);\ x < 0$ OR $g(x) = \log_3\left(\frac{1}{-x}\right);\ x < 0$	✓✓CACA answer [any form]	(2)

5.5	$f^{-1}(x) \geq -1$ $\log_{\frac{1}{3}} x \geq -1$ or $\log_{\frac{1}{3}} x = -1$ $x \leq 3$ $x = 3$ $0 < x \leq 3$	CA✓ setting up inequality/equation CA✓ x – value = 3 CACA✓✓ critical values and inequality	(4)
			[13]

QUESTION 6

| 6.1 | $A = P(1-i)^n$ $\frac{1}{2}P = P(1-0,1235)^n$ [If $P = \frac{1}{2}P(1-0,1235)^n$ Breakdown $-0/3$] $\frac{1}{2} = (0,8765)^n$ $n = \log_{0,8765} \frac{1}{2}$ $n = 5,258342252\, years$ *Time period will be 5 years and 4 months* **OR** $A = P(1-i)^n$ $\frac{1}{2}P = P(1-0,1235)^n$ $\frac{1}{2} = (0,8765)^n$ $\log \frac{1}{2} = \log(0,8765)^n$ $n = \dfrac{\log 0,8765}{\log \frac{1}{2}}$ $n = 5,258342252\, years$ *Time period will be 5 years and 3 months* | A✓ substitution into correct formula M✓ use of logs CA✓ answer **OR** A✓ substitution into correct formula M✓ use of logs CA✓ answer | (3) (3) |
| 6.2.1 | $A = P(1+i)^n$ $= 900\,000\left(1 + \dfrac{0,1025}{12}\right)^2$ $= R\,915440,66$ | A✓ substitution into correct formula CA✓ answer | (2) |

6.2.2	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $915440,66 = \dfrac{x\left[1-\left(1+\dfrac{0,1025}{12}\right)^{-238}\right]}{\dfrac{0,1025}{12}}$ $x = R9009,43$	A✓ substitution of i and n value into correct formula CA✓ substitution of P into correct formula CA✓ answer	(3)
6.2.3	Balance outstanding after the 99th payment : $P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $= \dfrac{9009,43\left[1-\left(1+\dfrac{0,1025}{12}\right)^{-139}\right]}{\dfrac{0,1025}{12}}$ $= R731384,7351$ Value of the loan after 3 months of non - payment : $A = 73138,7351\left(1+\dfrac{0,1025}{12}\right)^{3} = R750287,0104$ The new monthly instalment to settle the loan : $750287,0104 = \dfrac{x\left[1-\left(1+\dfrac{0,1025}{12}\right)^{-136}\right]}{\dfrac{0,1025}{12}}$ $x = R9349,11$	A✓ substitution of $n=139$ into correct formula CA✓ substitution of x into correct formula CA✓ $R731384,7351$ CA✓ substitution into correct formula CA✓ $R750287,0104$ A✓ substitution $n=136$ into correct formula CA✓ answer	(7)
			[15]

Total Marks : 75

6.2.3. Out. Bal $= A - Fv$

$$= P(1+i)^{99} - \frac{x\left[(1+i)^{99}-1\right]}{i}$$

$$= 915440,66\left(1+\frac{10,25\%}{12}\right)^{99} - \frac{9009,43\left[\left(1+\frac{10,25\%}{12}\right)^{99}-1\right]}{\frac{10,25\%}{12}}$$

$$= 2124813,805 - 1393428,915$$

$$= R731384,8896$$

Value of Loan for 3 months of Non-payment
$$= 731384,8896\left(1+\frac{10,25\%}{12}\right)^{3}$$

$$= 750287,1688$$

New Monthly payment $x = \dfrac{750287,1688 \times \frac{10,25\%}{12}}{\left[1-\left(1+\frac{10,25\%}{12}\right)^{-136}\right]}$

$$= 9349,108527$$

$$= R9349,11$$