Wynberg Boys' High School

Department of Mathematics

Grade 8 – June 2012

Marks Available: 150 marks Time Available: 120 minutes

Two weeks ago South Africa was awarded an international agreement to build one of the most powerful telescopes on the planet - the Square Kilometre Array (SKA). The SKA is a next-generation radio telescope that will be powerful enough to explore the edges of the universe. The building of the SKA will only be completed and fully operational in 2024!

Unlike a normal telescope that receives light waves, this telescope will consist of a large array (grid) of receiving dishes that will receive many different types of electro-magnetic radiation. Part of the project will happen in Australia/New Zealand, but 80% of the project will be based just outside Carnarvon in the Karoo. Together all of these receiving dishes (some also in Madagascar, Botswana, Mauritius, etc.) will act as one instrument through a central computer.

The aim of the project is to try to gather clues on the very beginning of the universe, it will explore dark energy, and if there is life out there, it hopes to find it.

It will be 50 – 100 times more sensitive than the best radio-telescope currently on earth and will survey the sky 10 000 times faster. The amount of data that will move around on this project will be 10 times bigger than the total global Internet traffic today.

The director of the project, Dr Bernie Fanaroff, agrees to show you some of the mathematics that is involved in this project.

Task 1 - Numbers

The SKA project can be described in numbers. In each case below find the number in normal notation by simplifying the clue. Show all working.

a) Number of countries involved: $6+2\times7$

b) Cost of construction of SKA in Euros: €1,5 billion

c) Number of dishes at MeerKat: 2^6

d) The diameter of each telescope at SKA: $\sqrt{225}$ metres

e) The number of telescopes at SKA: 3×10^3

f) The year in which construction will start: $4^2+5\times20^2$

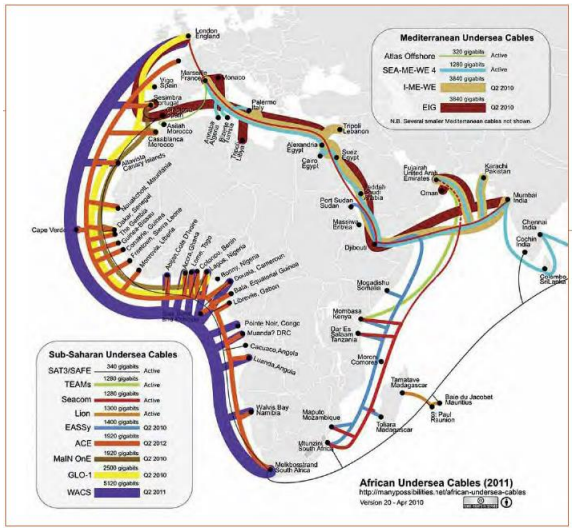
g) The year that the telescope will first be used: $2006+\sqrt[4]{81}+\sqrt{169-25}+\sqrt[3]{-\frac{1}{64}}-\sqrt{\frac{49}{16}}$

h) The diameter of the dishes at MeerKat: The HCF of 26 and 39

(8)

Task 2 – Prime Numbers and Factors

All of the data that has to move from the dishes to the central computer to the various universities around the world has to be encoded to protect it. It will move along extremely high bandwidth under-sea cables. A major amount of this code-making relies on prime numbers and factors.



a) Write the number 252 as a product of its prime factors. (4)

b) Consider the two numbers which are the distances of a satellite and the moon from the earth. These distances are in kilometres and have been shown in prime factorised form.

$$42\,300 = 2^2.3^2.5^2.47$$
$$385\,000 = 2^3.5^4.7.11$$

Find the lowest common multiple of the two numbers (do not attempt to multiply it out.) (3)

c) Fill in this table on your printed answer sheet.

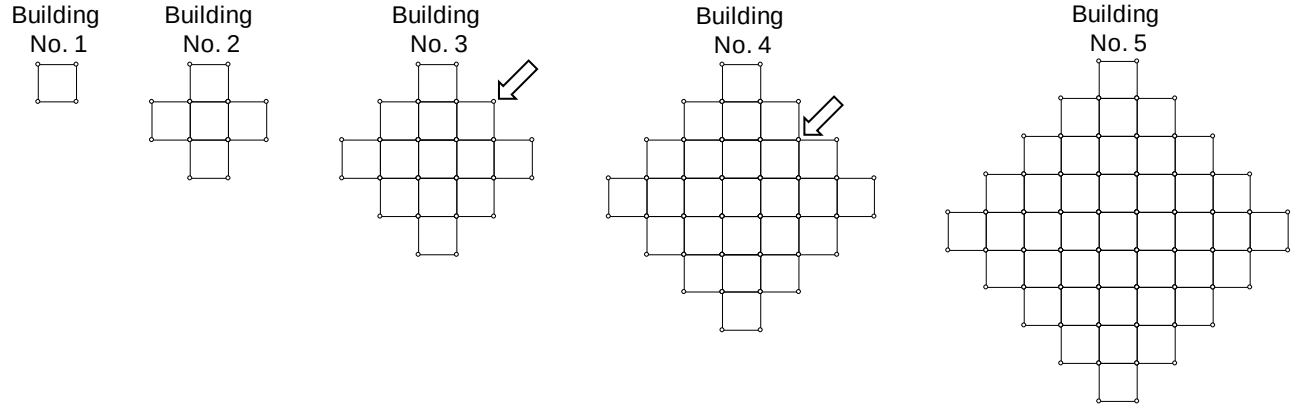
Tick or cross each block to indicate whether or not the statement is correct or not for the number on the left. Some have been filled in already. (7)

Number	Prime	Real	Non-Real	Rational	Irrational	Multiple of 2
53		✓	✗	✓	✗	
$\sqrt{-25}$	✗			✗	✗	✗
$-1,3$	✗					✗
$\sqrt{1\frac{7}{9}}$	✗	✓	✗			✗
256				✓	✗	

Task 3 - Patterns

Your first stop is at one of the buildings that will be dotted around the plateau. It has an unusual shape that follows the pattern explained below.

The buildings may be different sizes based on the pattern in the diagrams below:

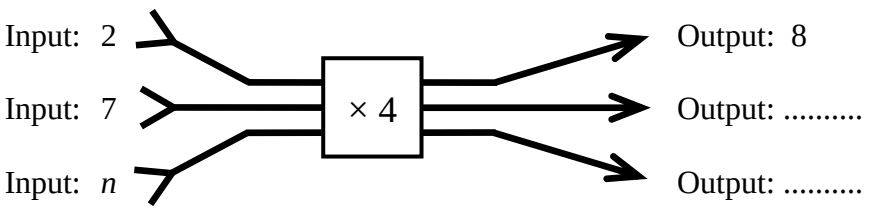


a) An example of an outward facing corner is marked with an arrow in the diagram of building 3. An example of an inward facing corner is marked with an arrow in the diagram of building 4.

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Number of outward - facing corners	4	8	12	16	A	B	
Number of inward - facing corners	0	4	8	12	C	D	

i) Find the numbers that should go in the spaces marked A to D. (4)

ii) The flow diagram for the number of outward-facing corners is shown below:



Write down the two missing outputs. (2)

iii) Find the rule for the number of inward-facing corners. (2)

b) Two further characteristics of each building in the pattern are recorded in the table below:

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Number of squares wide	1	3	5	7			
Perimeter	4	12	20	28			

Complete the table on your printed answer sheet. (6)

c) The total number of small squares in each possible building is given in the table below:

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Total number of small squares (*S*)	1	5	13	25	41	181	$2n(n-1)+1$

How many small squares would be used in building number 21? (2)

d) The number of pillars needed to construct each building (these are shown as dots on the diagram) are shown in the table. Complete this table on your printed answer sheet. (4)

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Number of pillars (dots)	4	12	24	40			

Task 4 - Substitution

Astronomers have to substitute values into formulas many times a day. Often they have to use computers to do this, due to the large number and the complexity of the calculations that they have to make.

Help them out with some of their calculations below.

a) Complete the following table of substitutions on the table on your printed answer sheet.
You need only fill in the blocks that are unshaded. (14)

a	5	6	-1	0	$\frac{3}{4}$
b	2	8	-1	0	$\frac{5}{6}$
c	-8	9	5	-1	$\frac{8}{6}$
$3a - 2(b + 1)$					
$a - c$					
$a \times b$					
$\frac{a}{b}$					
$6b^2$					

b) One formula that is regularly used is the one that tells us how fast a star is accelerating away from us, based on its colour (measured as z). The formula is:

$$A = \frac{2(z+1)^3}{\left[(z+1)^2 + 1\right]^2}$$

Calculate the value of A if $z = 1$ (3)

c) The temperature of a star can be measured using the formula:

$$T = \frac{2897000}{W}$$

i) Is 2 897 000 divisible by 3? Explain your answer. (2)

ii) Is 2 897 000 divisible by 5? Explain your answer. (2)

iii) Calculate the value of T if $W = 500$. (1)

iv) For what value of W will the temperature be 14 485 degrees? (1)

Task 5 – Algebraic Simplification

The amount of data that will be coming in from the 3 000 dishes in SKA is enormous. So big, in fact that it will be 10 times the global Internet traffic at the moment. It is therefore essential that every algebraic formula is in its simplest form. Dr Fanaroff asks you for your help in simplifying the following.

a) $w+w$ (1)

b) $5x+5$ (1)

c) $9y-y$ (1)

d) $8z-8$ (1)

e) $9a-7a+3a$ (1)

f) $5b-6-7b+10$ (2)

g) $\frac{14c-20}{2}$ (2)

h) $3(d+2)+4(2d-3)$ (3)

i) $e\times 7+2\times e\times 4-4\div 2+3(2+3\times 2)$ (4)

j) $\frac{1}{2}(6f-18)+\frac{3}{4}(4f+8)$ (3)

He then shows you an enormous expression. Only try to simplify this once you have finished everything else.

k) $12\times m-27m\div 3-8(m-7)+\frac{64m-54m+12m}{2}+\frac{6+3\times 2}{4}-\frac{2m-10}{2}-4!-4(m+10)$ (2)

Task 6 - Equations

Solve the following equations:

a) $3a = 60$ (1)

b) $b - 12 = 10$ (1)

c) $\frac{c}{6} = 2$ (1)

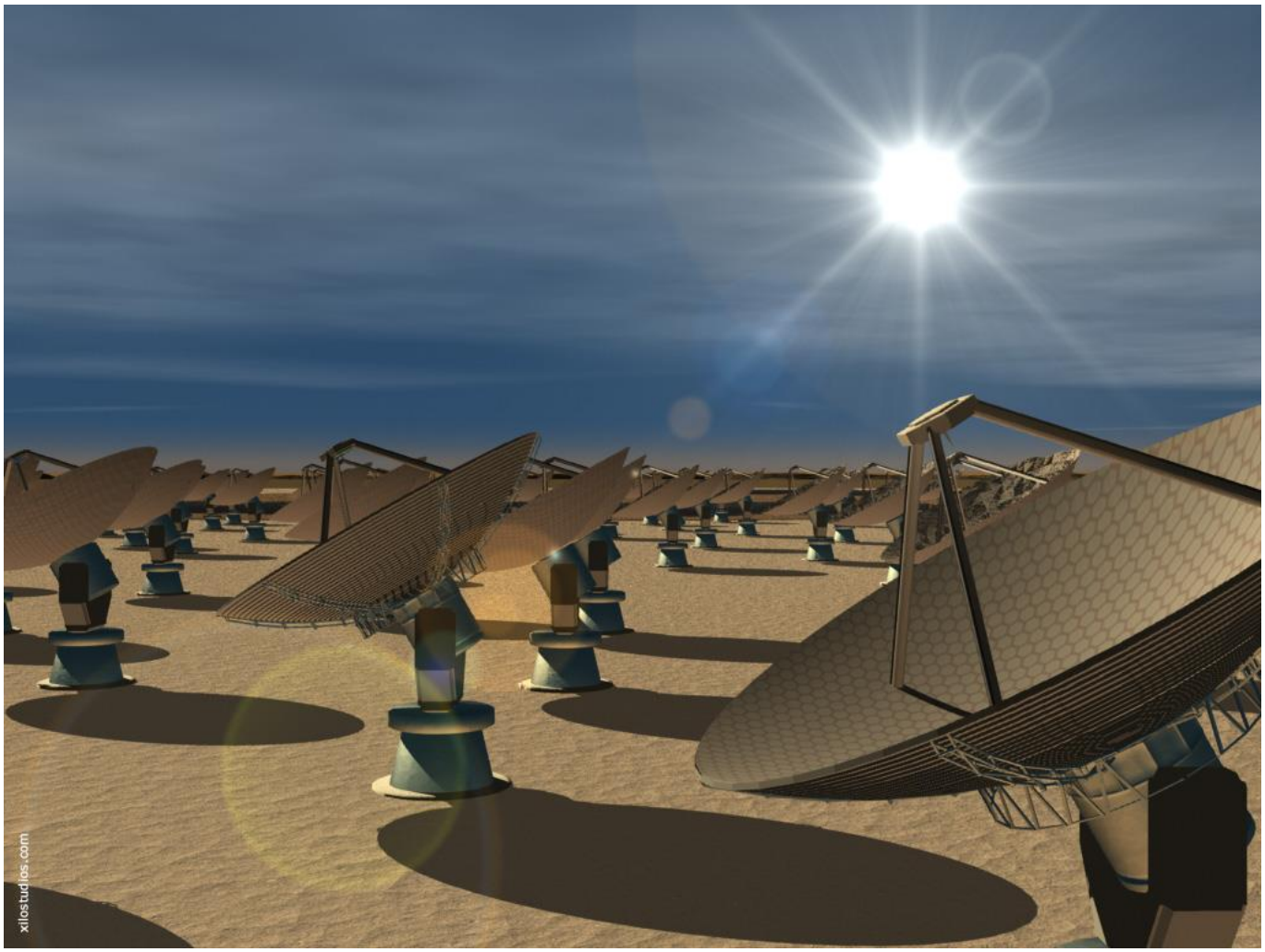
d) $4d - 15 = -3$ (2)

e) $5e + 6 = 3e - 8$ (3)

f) $\frac{2f - 7}{3} - 2 = 9$ (3)

g) $5g - 2g + 8 = 3(g + 2) + 2$ (3)

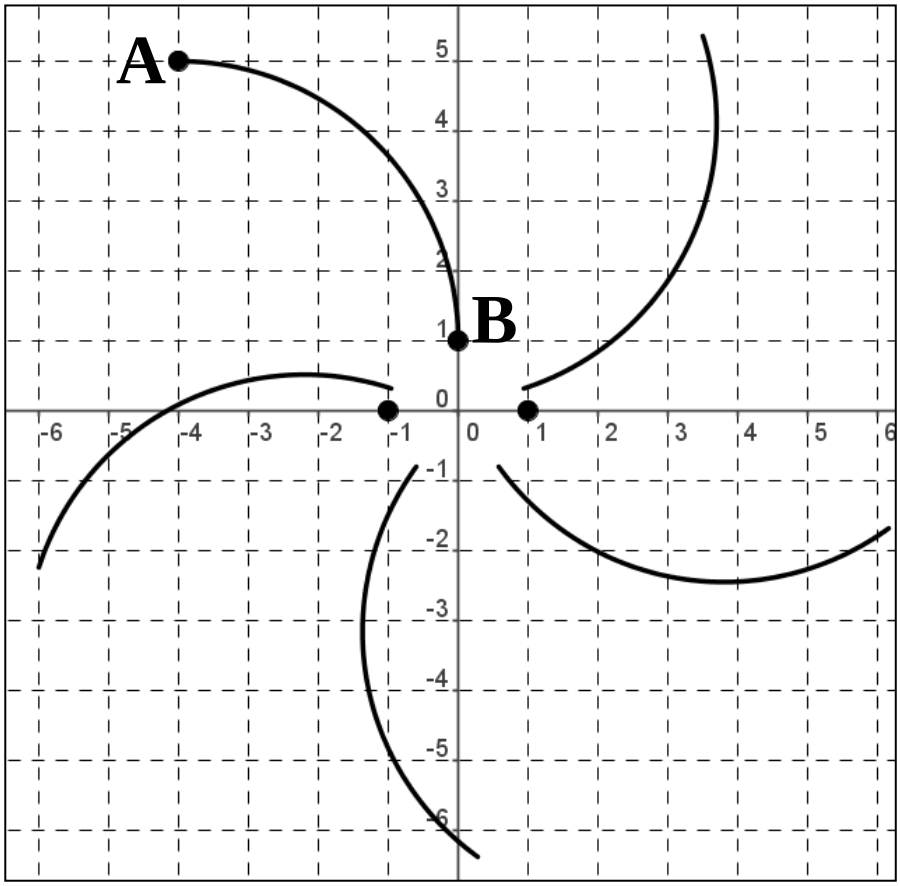
h) $2(h - 5) - (h + 1) = 3 + 2(4h - 1)$ (5)



Task 7 – Cartesian Plane

The receiving dishes for the telescope will be laid out in a spiral pattern, with many near the centre, and fewer further along.

Look at the diagram of that pattern below and answer the questions.



a) Give the co-ordinates of point A. (2)

b) Give the co-ordinates of point B. (2)

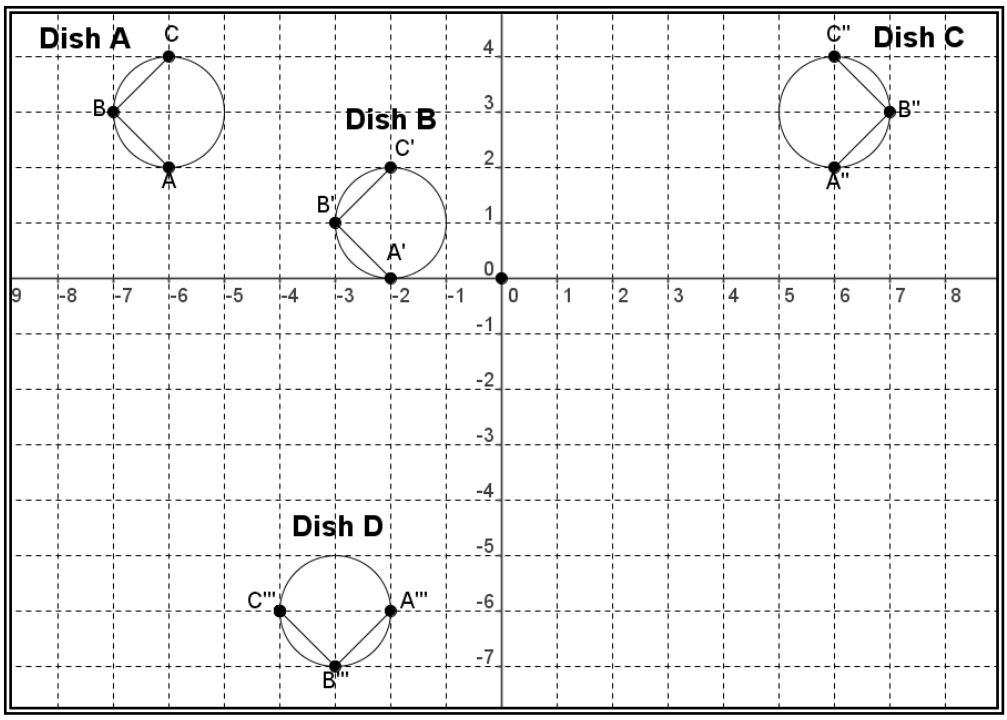
c) The astronomers need to place a new dish at the point (3 ; 2).

Indicate this on the grid on your printed answer sheet. (2)

d) What type of transformation has been used to create this pattern? (1)

Task 8 - Transformations

The positioning of all the dishes requires a good understanding of transformations.



a) Consider the transformation of Dish A to Dish B.

i) Find the co-ordinates of A and A'. (4)

ii) Describe, in words, the transformation that has mapped Dish A onto Dish B. (2)

iii) Describe the transformation algebraically in the form $(x; y) \rightarrow (.....;.....)$ (2)

b) Consider the transformation of Dish A to Dish C.

i) Find the co-ordinates of A". (2)

ii) Describe, in words, the transformation that has mapped Dish A onto Dish C. (2)

iii) Describe the transformation algebraically in the form $(x; y) \rightarrow (.....;.....)$ (2)

c) Consider the transformation of Dish A to Dish D.

i) Find the co-ordinates of A'". (2)

ii) Describe, in words, the transformation that has mapped Dish A onto Dish D. (2)

iii) Describe the transformation algebraically in the form $(x; y) \rightarrow (.....;.....)$ (2)

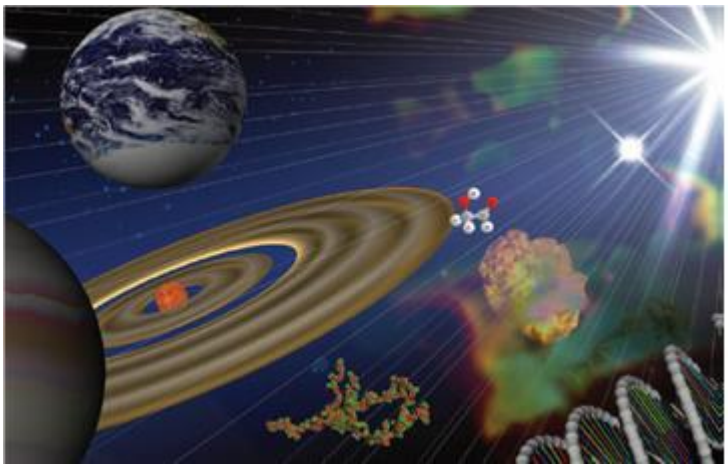
d) Do this question on the printed answer sheet.

Draw Dish E which is obtained by transforming Dish A using the transformation below:

$$(x; y) \rightarrow (x + 3; -y)$$ (2)

Task 9 – Scientific Notation

Astronomers continually work with very large numbers. Scientific notation is very useful for them to calculate distances to stars and galaxies and the speeds at which they are moving. It is hoped that SKA will help us to decide which came first – stars or galaxies.



a) A square kilometre is 1 km wide and 1 km long.
The number of square centimetres in this area is:

$$(1\times10^{5})\times(1\times10^{5})$$

Simplify this, giving your answer in scientific notation. (1)

b) The amount of data that will have to move along fibre optic cables from each of the 3 000 dishes is 160 Gigabytes per second. This can be written as:

160×10^{9} bytes

Write this number in scientific notation. (1)

c) The speed of light is: 299 792 kilometres per second

Write this number in scientific notation, rounded off to three decimal places. (2)

d) The number of seconds in a year is: $3{,}156\times10^{7}$.

Write this number in ordinary notation. (1)

e) The total amount of data that will be collected by the 3 000 dishes of SKA in one year will be:

$$(160\times10^{9})\times(3000)\times(3{,}156\times10^{7})=1{,}51488\times10^{22}$$

i) How many zeros will this number have if it is written out in full? (1)

ii) If this data was burned onto CDs it would need 2×10^{13} disks.

If you stack a million CDs on top of each other, the stack would be about 1 km tall.

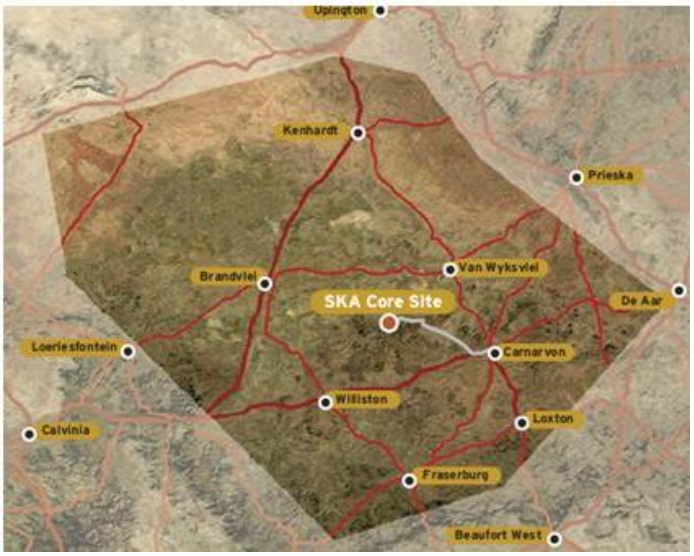
How many kilometres tall would the stack of CDs be for the SKA data for one year?

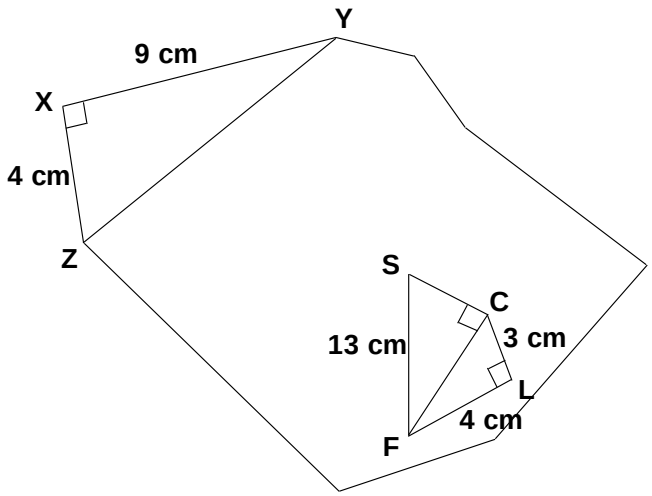
Write your answer in normal notation. (1)

Task 10 – Theorem of Pythagoras

The proposed SKA site is near Carnarvon in the Karoo, at the centre of a radio-signal free reserve. The towns that surround the SKA site are shown on the map.

A simplified version of the map is shown below with a number of distances (in centimetres) indicated.





a) The distances from Loxton (L) to Canarvon (C) and Loxton to Fraserburg (F) are shown. Calculate the length of FC. (2)

b) The distance from Fraserburg to the SKA site (S) is shown. Now find the length of SC. (3)

c) In the top left hand corner, various measurements are given for ΔXYZ .
Find the length of ZY. (3)

Task 11 – Problems

DO NOT ATTEMPT THIS QUESTION UNTIL YOU HAVE FINISHED EVERYTHING ELSE.

Dr Fanaroff is one of those people who is forever spotting relations between numbers. He has noticed, for instance, that the number of his house and those of two neighbouring professors form three consecutive prime numbers whose product is equal to his telephone number. Dr Fanaroff lived in the middle house, and had a five-digit telephone number whose first digit was 6. Find the number of Dr Fanaroff's house and of his telephone.

(1)

Answer Sheet Name: . . . . . .

Task 2c)

Number	Prime	Real	Non-Real	Rational	Irrational	Multiple of 2
53		✓	✗	✓	✗	
$\sqrt{-25}$	✗			✗	✗	✗
$-1{,}3$	✗					✗
$\sqrt{1\frac{7}{9}}$	✗	✓	✗			✗
256				✓	✗	

Task 3b)

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Number of squares wide	1	3	5	7			
Perimeter	4	12	20	28			

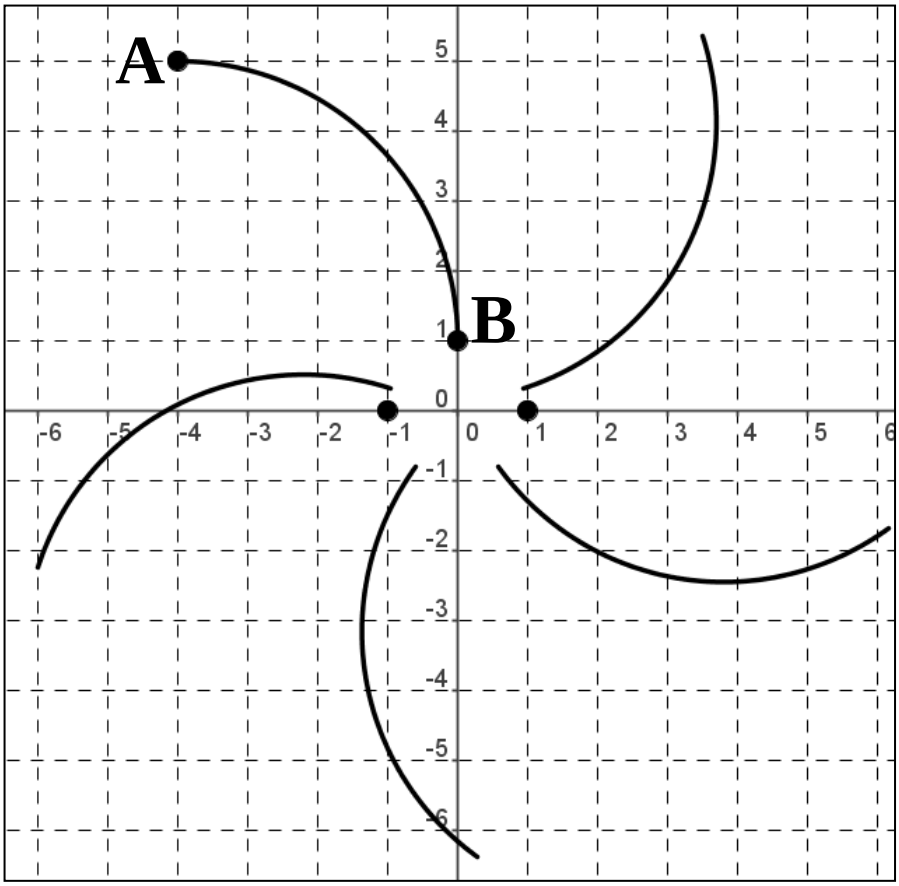
Task 3d)

Building Number	**1**	**2**	**3**	**4**	**5**	**10**	***n***
Number of pillars (dots)	4	12	24	40			

Task 4a)

a	5	6	-1	0	$\frac{3}{4}$
b	2	8	-1	0	$\frac{5}{6}$
c	-8	9	5	-1	$\frac{8}{6}$
$3a-2(b+1)$					
$a-c$					
$a\times b$					
$\frac{a}{b}$					
$6b^2$					

Task 7c)



Task 8d)