WTS TUTORING

2019 WTS

EUCLIDEAN GEOMETRY

GRADE : 11 AND 12

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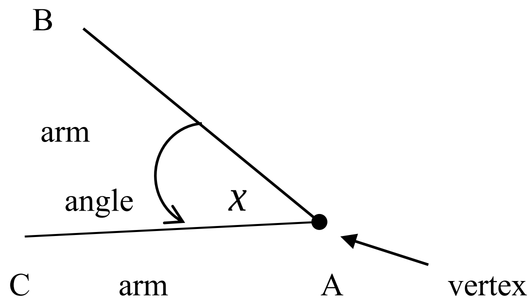
EUCLIDEAN GEOMETRY

➢ LINES AND ANGLES

- ✓ A **line** is an infinite number of points between two end points.
- ✓ Where two lines meet or cross, they form an **angle.**
- ✓ An **angle** is an amount of rotation. It is measured in **degrees**.

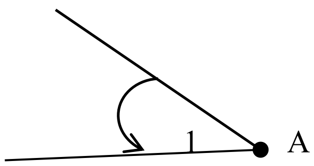
➢ ANGLE LANGUAGE:

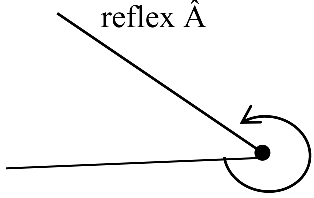


➢ LABELING ANGLES:

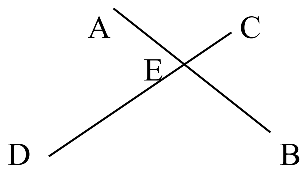
BÂC or Â or x

ALSO: $\hat{A}_1$ reflex Â





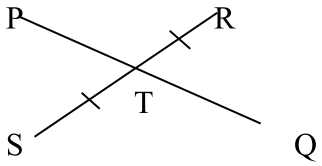
➢ INTERSECT



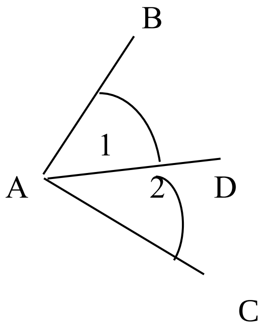
AB and CD intersect (cross or cut) at E

➢ BISECT

PQ bisects (cuts in half) RS at T

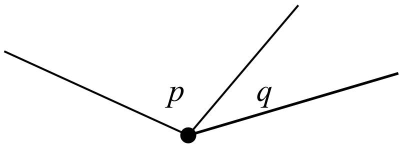


AD bisects $B\hat{A}C$ ($\hat{A}_1 = \hat{A}_2$)



➢ ADJACENT ANGLES

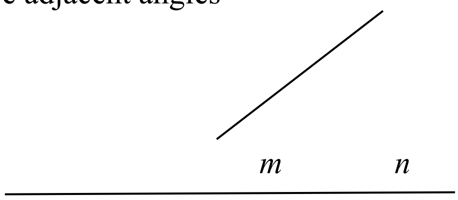
→Angles that have a common vertex and a common arm



p and *q* are adjacent angles

→Adjacent angles that lie on a straight line add up to 180°

$m + n = 180°$



➢ PERPENDICULAR LINES:

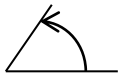
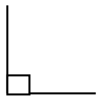
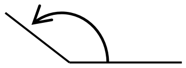
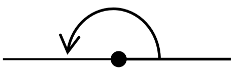
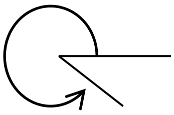
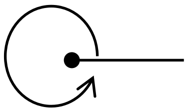
→Lines that meet or cross at 90°

$MN \perp PS$

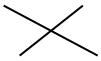
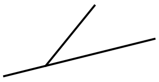
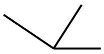
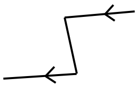
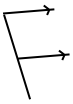
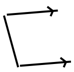
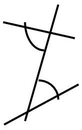
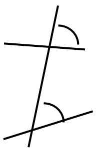
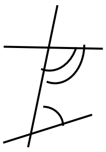
➢ TERMINOLOGY

- ✓ **Acute angle**: Greater than 0° but less than 90°
- ✓ **Right angle**: Angle equal to 90°
- ✓ **Obtuse angle**: Angle greater than 90° but less than 180°
- ✓ **Straight angle**: Angle equal to 180°
- ✓ **Reflex angle**: Angle greater than 180° but less than 360°
- ✓ **Revolution**: Sum of the angles around a point, equal to 360°
- ✓ **Adjacent angles**: Angles that share a vertex and a common side.
- ✓ **Vertically opposite angles**: Angles opposite each other when two lines intersect. They share a vertex and are equal.
- ✓ **Supplementary angles**: Two angles that add up to 180°.
- ✓ **Complementary angles**: Two angles that add up to 90°.
- ✓ **Parallel lines**: Lines that are always the same distance apart
- ✓ **A transversal line**: A line that intersects two or more parallel lines.
- ✓ **Interior angles**: Angles that lie in between the parallel lines.
- ✓ **Exterior angles**: Angles that lie outside the parallel lines.
- ✓ **Corresponding angles**: Angles on the same side of the lines and the same side of the transversal.
- ✓ **Co-interior angles**: Angles that lie in between the lines and on the same side of the transversal.
- ✓ **Alternate interior angles:** Interior angles that lie inside the line and on opposite sides of the transversal.
- ✓ **Congruent**: The same. Identical.
- ✓ **Similar**: Looks the same. Equal angles and sides in proportion.
- ✓ **Proportion**: A part, share, or number considered in comparative relation to a whole. The equality of two ratios. An equation that can be solved.
- ✓ **Ratio**: The comparison of sizes of two quantities of the same unit. An expression.
- ✓ **Area**: The space taken up by a two-dimensional polygon.
- ✓ **Theorem:** A statement that has been proved based on previously established statements
- ✓ **Converse**: A statement formed by interchanging what is given in a theorem and what is to be proved
- ✓ **Corollary**: A statement that follows with little or no proof required from an already proven statement.
- ✓ **Euclidean Geometry**: Geometry based on the postulates of Euclid. Euclidean geometry deals with space and shape using a system of logical deductions

➢ TYPES OF ANGLES

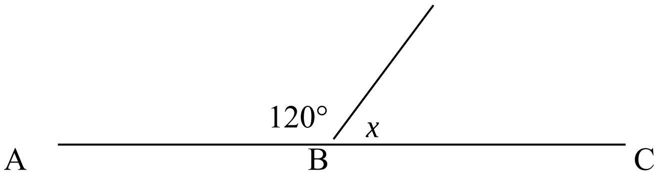
Name of angle	Example	Size of angle
Acute angle		Between 0° and 90°
Right angle		Equal to 90°
Obtuse angle		Between 90° and 180°
Straight angle		Equal to 180°
Reflex angle		Between 180° and 360°
Revolution		Equal to 360°

➢ SUMMARY OF REASONS

	vert opp < s	(are equal)
	adj < s on str line	(are supplementary)
	< s at a pt	(add up to 360°)
	alt < s _//_	(are equal)
	corr < s _//_	(are equal)
	co-int < s _//_	(are supplementary)
	alt < s =	(lines are //)
	corr < s =	(lines are //)
	co-int < s suppl	(lines are //)

➢ ADJACENT ANGLES ON A STRAIGHT LINE ARE SUPPLEMENTARY.

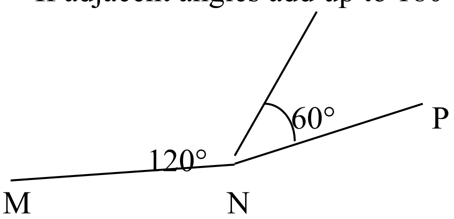
→If they are adjacent angles on a straight line, then they add up to 180°



x = 60° **reason adj <s on str line**

➢ ADJACENT SUPPLEMENTARY ANGLES ARE ON A STRAIGHT LINE.

→If adjacent angles add up to 180°, then they are on a straight line

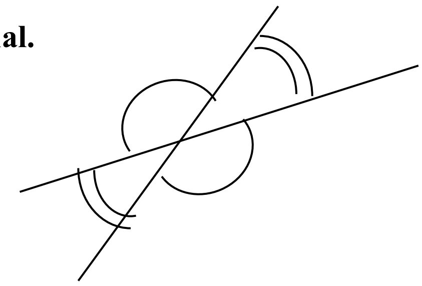


MNP is a straight line
adj supp <s

➢ VERTICALLY OPPOSITE ANGLES.

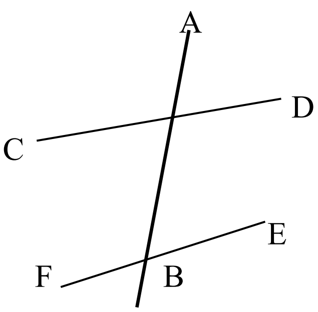
→When two straight lines intersect the angles opposite each other are called **vertically opposite angles.**

Vertically opposite angles are equal.



➢ TRANSVERSALS

→If a line cuts or touches another line, it is called a **transversal.**

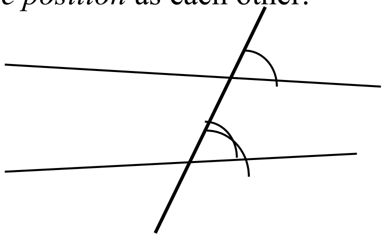


e.g. AB is a transversal because it cuts CD and EF

ANGLES AND TRANSVERSALS

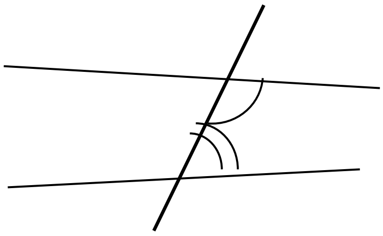
Corresponding angles are in the *same position* as each other.

***F* shape**



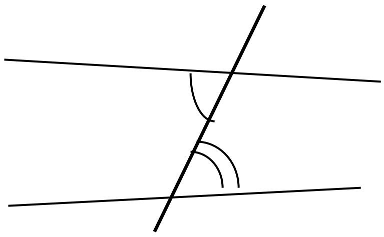
Co-interior angles are *between the lines* and *on the same side* of the transversal. They are "*inside together"*.

***C* or *U* shape**



Alternate angles are *between the lines* and on *alternate (opposite) sides* of the transversal.

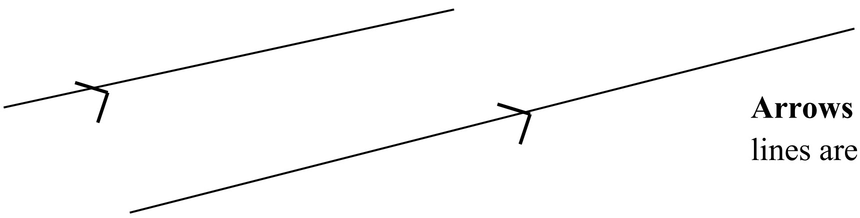
***Z* or *N* shape**



Remember the word *"FUN"{ Geometry is FUN song}*

PARALLEL LINES

Parallel lines are lines that stay the **same distance apart**, no matter how long the lines are (they are lines that never meet).



Arrows are used to indicate that lines are parallel.

PARALLEL LINES

IF LINES ARE **PARALLEL** THEN:	**REASONS:**
• The **corresponding angles are equal**	**corr <s …//…**
• The **alternate angles are equal**	**alt <s …//…**
• The **co-interior angles are supplementary**	**co-int <s …//…**

➢ TO PROVE LINES ARE PARALLEL:

Prove the corresponding angles are equal	**corr <s =**
Prove the alternate angles are equal	**alt <s =**
Prove the co-interior angles are supplementary	**co-int <s =**

➢ **Properties of the angles formed by a transversal line intersecting two parallel lines**

- **If the lines are parallel:**
  - ✓ the corresponding angles will be equal
  - ✓ the co-interior angles are supplementary
  - ✓ the alternate interior angles will be equal.

- **If the corresponding angles will be equal or the co-interior angles are supplementary or the alternate interior angles will be equal:**
  - ✓ the lines are parallel

Kwv 1

Draw the shapes of following and write their statements

a) Adjacent supplementary angles

b) Angles round a point

c) Vertically opposite angles

d) Corresponding angles

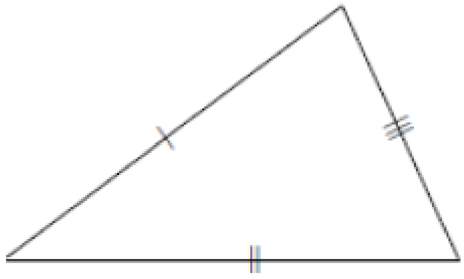
e) Alternate angles

f) Co- interior angles

TRIANGLES

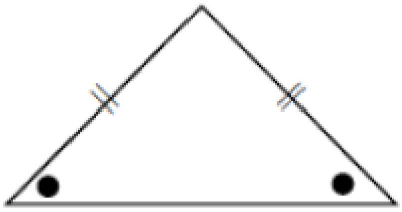
✓ **THERE ARE FOUR KINDS OF TRIANGLES**

1. **SCALENE TRIANGLE**



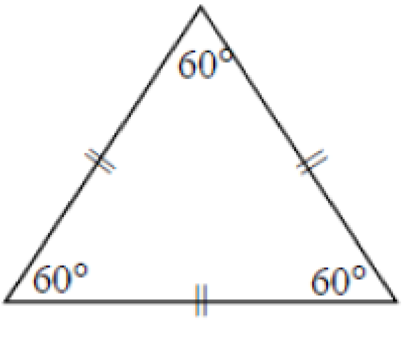
- No sides are equal in length

2. **ISOSCELES TRIANGLE**



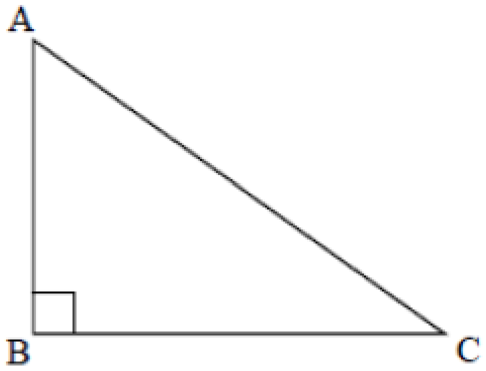
- Two sides are equal

3. **EQUILATERAL TRIANGLE**



- All three sides are equal
- All three interior angles are equal

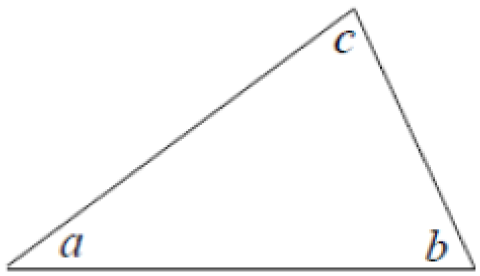
4. RIGHT-ANGLED TRIANGLE



- One interior angle is 90°

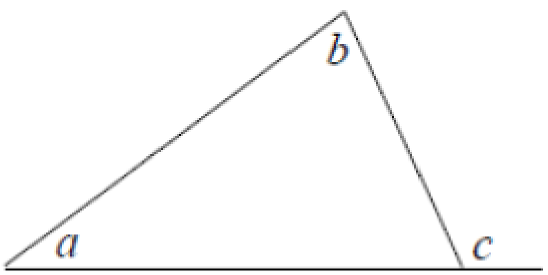
➢ RELATIONSHIP BETWEEN ANGLES

✓ SUM OF THE ANGLES OF A TRIANGLE



$a+b+c=180°$

✓ EXTERIOR ANGLE OF A TRIANGLE



$c=a+b$

 Please turn over

Kwv 1

Draw shape of the following triangles and write the conclusion of each

a) Scalene Triangle

b) Isosceles Triangle

c) Equilateral Triangle

d) Right-Angled Triangle

e) Sum of the angles of a Triangle

f) Exterior angle of a Triangle

➢ IF YOU ARE ASKED TO PROVE THAT…..

➢ **Two lines parallel**

Use the slope formula twice. (Find the slopes of the two lines.)

Determine that the slopes are equal, therefore the lines are parallel.

➢ **Two lines perpendicular**

Use the slope formula twice. (Find the slopes of the two lines.)

Determine that the slope are negative reciprocals of each other, therefore the lines are perpendicular.

➢ **A triangle is a right angle triangle**

Use the slope formula twice. (Find the slopes of the legs.)

Determine that since the slopes are negative reciprocals of each other, the lines are perpendicular, forming a right angle. This makes the triangle a right angle.

OR

Use the distance formula three times. (Find the length of the three sides.).

Determine that the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the length of the two adjacent legs, that is, use the Pythagorean Theorem ($c^2 = a^2 + b^2$).

➢ **A triangle is isosceles**

Use the distance formula twice. (Find the length of two congruent sides.)

Determine that since the lengths of two sides are equal, the triangle is isosceles.

- **A triangle is an isosceles right triangle**

Use the distance formula three times. (Find right triangle the lengths of the three sides.)
Determine that since the lengths of two sides are equal and that the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the two adjacent legs $c^2 = a^2 + b^2$, the triangle is an isosceles right triangle.

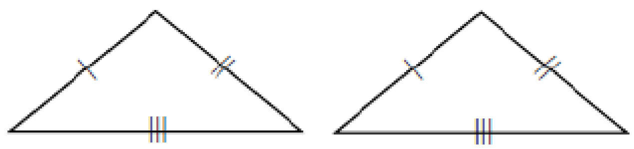
OR

Use the slope formula twice and the distance formula twice. (Find the slopes and the lengths of the two legs.)
First, prove the triangle is a right triangle (see above), and then use the distance formula to find the lengths of the two legs of the triangle. Since the lengths of two sides are equal, the triangle is isosceles. Thus, the triangle is an isosceles right triangle.

CONGRUENCY OF TRIANGLES

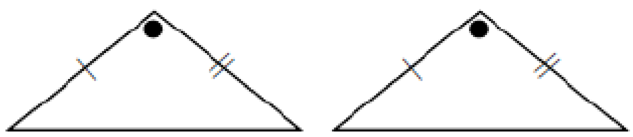
Rule 1

Two triangles are congruent if three sides of one triangle are equal in length to the three sides of the other triangle. (SSS)



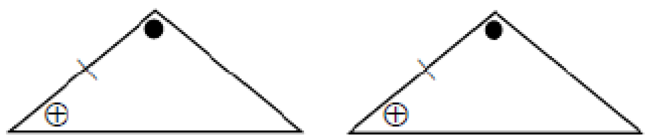
Rule 2

Two triangles are congruent if two sides and the included angle are equal to two sides and the included angle of the other triangle. (SAS)



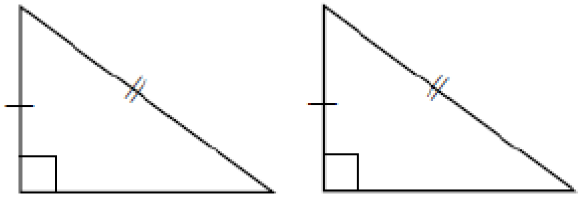
Rules 3

Two triangles are congruent if two angles and one side are equal to two angles and one side of the other triangle. (SAA)



Rule 4

Two right-angles triangles are congruent if the hypotenuse and a side of the one triangle is equal to the hypotenuse and a side of the other triangle. (RHS)



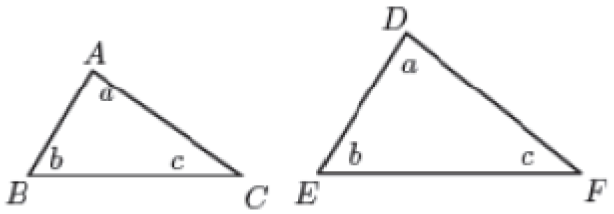
Kwv 1

Draw two triangles of each of four conditions

➢ SIMILARITY

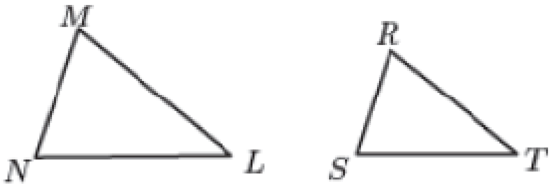
Rule 1 (AAA)

If all three pairs of corresponding angles of two triangles are equal, then the triangles are similar.



Rule 2 (SSS)

If all three pairs of corresponding sides of two triangles are in proposition, then the triangles are similar.



$$\frac{MN}{RS} = \frac{ML}{RT} = \frac{NL}{ST}$$

➢ PYTHAGORAS THEOREM

The Theorem of Pythagoras

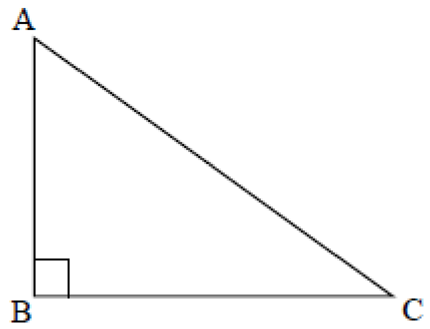
$AC^2 = AB^2 + BC^2$

or

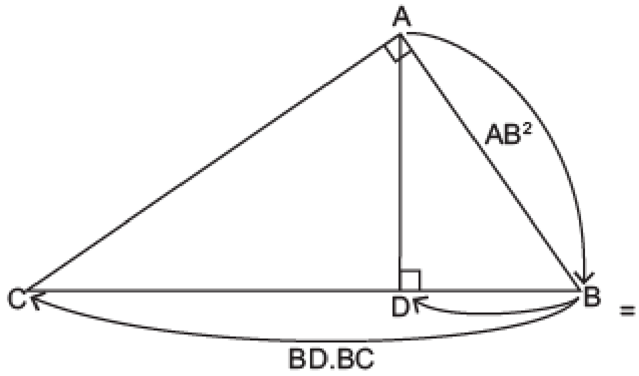
$AB^2 = AC^2 - BC^2$

or

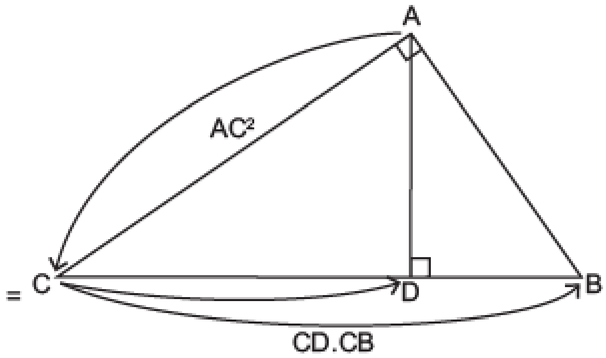
$BC^2 = AC^2 - AB^2$



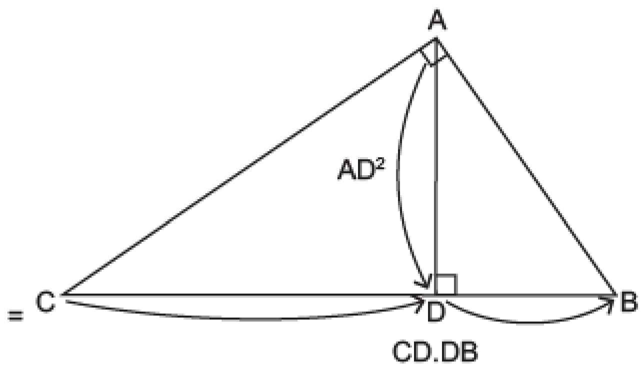
KUTHI HUUUUU!!!!!



(read right to left) $AB^2 = BD.BC$



(read right to left) $AC^2 = CD.CB$

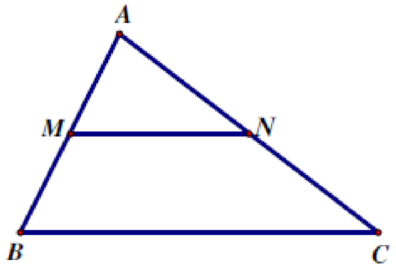


(read right to left) $AD^2 = BD.CD$

➢ THE MIDPOINT THEOREM

The line segment joining the midpoints of two sides of a triangle, is parallel to the third side of the triangle and half the length of that side.

In ΔABC, M is the mid-point of AB, N is the mid-point of AC, then $MN \,//\, BC$ and $MN = \frac{1}{2}BC$.

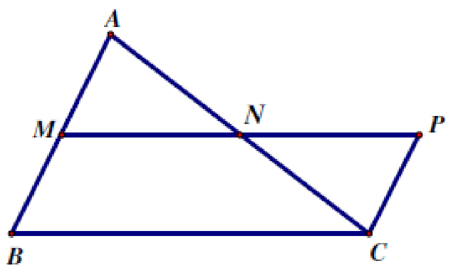


Proof: Method 1

$\frac{AM}{AB} = \frac{1}{2} = \frac{AN}{AC}$	given
$\angle MAN = \angle BAC$	Common angle
$\therefore\ \Delta AMN \sim \Delta ABC$	2 sides proportional, included angle
$MN = \frac{1}{2}BC$	ratio of sides, ~Δ's
$\angle AMN = \angle ABC$	corr. ∠s, ~Δ's
$\therefore\ MN \,//\, BC$	corr. ∠s equal

Method 2

Produce MN to P so that $MN = NP$.



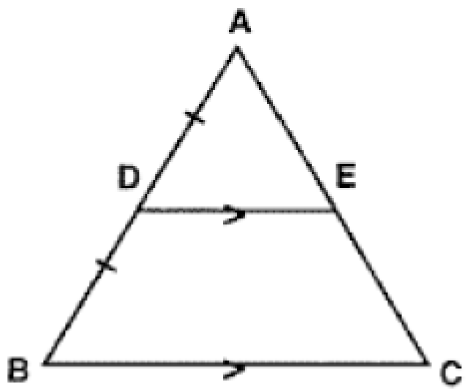
$AN = NC$	given
$MN = NP$	By construction
$\angle ANM = \angle CNP$	vert. opp. ∠s
$\therefore\ \Delta AMN \cong \Delta CPN$	S.A.S.
$CP = AM$	corr. sides, ≅ Δ's
$AM = MB$	given
$\therefore\ MB = PC$	
$\angle MAN = \angle PCN$	corr. ∠s, ≅ Δ's
$\therefore\ AM \,//\, PC$	alt. ∠s equal
$MB \,//\, PC$	
$BCPM$ is a parallelogram	opp. sides are equal and parallel
$\therefore\ MNP \,//\, BC$	Property of parallelogram
$MN \,//\, BC$	
$MN = \frac{1}{2}MP$	by construction
$\quad = \frac{1}{2}BC$	opp. sides of parallelogram
$\therefore\ MN = \frac{1}{2}BC$	

- **CONVERSE OF THE MIDPOINT THEOREM**

THEOREM

In a triangle a line draw through the midpoint of one side , parallel to another side bisects the third side.

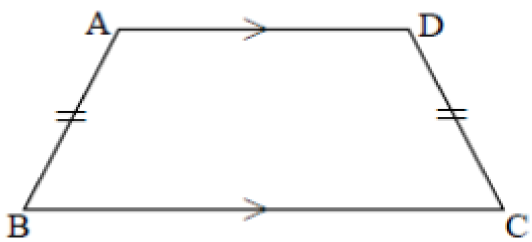
In triangle ABX, IF AD=BD and DE// BC then AE=EC



Kwv 1

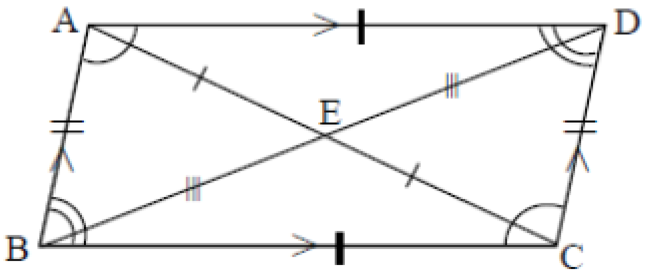
Draw the two shapes for midpoint theorem and write their statements

- **PROPERRTIES OF QUADRILATERALS**
- **TRAPEZIUM**



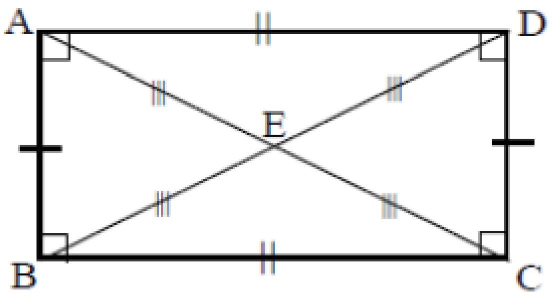
- Two sides are parallel.

PARALLELOGRAM



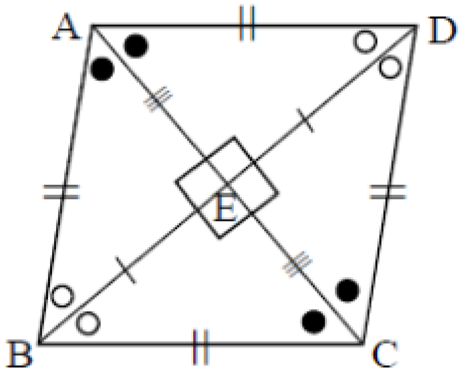
- Opposite sides parallel and equal.
- Opposite angles equal.
- Diagonals bisect each other.

RECTANGLE



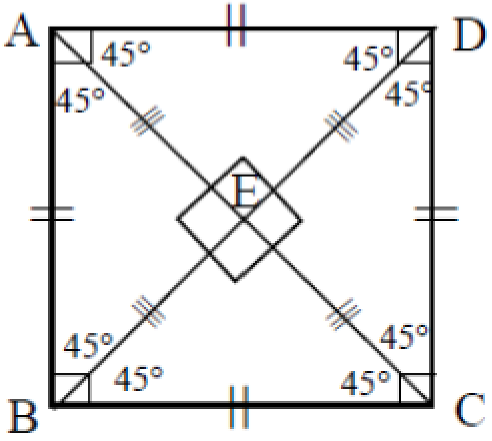
- Opposite sides parallel and equal in length.
- Diagonals are equal in length and bisect each other.
- Interior angles are right angles

RHOMBUS



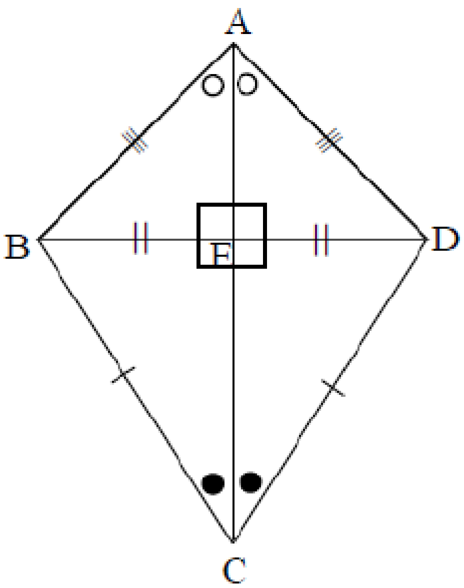
- Opposite sides are parallel.
- All sides equal in length.
- Diagonals bisect each other at the right angles.
- Diagonals bisect the opposite angles.

➢ SQUARE



- Opposite sides parallel.
- All sides equal in length.
- Diagonals are equal in length.
- Diagonals bisect each other at right angles.
- Interior angles are right angles.
- Diagonals bisect interior angles (each bisect angle equals 45˚).

➢ KITE



- Adjacent pairs of sides are equal in length.
- The longer diagonal bisects the opposite angles.
- The longer diagonal bisects the other diagonal.
- The diagonals intersect at right angles.

- **HOW TO PROVE A QUADRILATERAL**

- **PROVING A TRIANGLE IS A RIGHT TRIANGLE.**

Method 1: Show two sides of the triangle are perpendicular by demonstrating their slopes are opposite reciprocal.

Method 2: Calculate the distances of all three sides and then test the Pythagorean's theorem to show the three lengths make the Pythagorean's theorem true.

- **PROVING A QUADRILATERAL IS A PARALLELOGRAM**

Method 1: Show that the diagonals bisect each other by showing the midpoint of the diagonals are the same.

Method 2: Show both pairs of opposite sides are parallel by showing they have equal slopes.

Method 3: Show both pairs of opposite sides are equal by using distance.

Method 4: Show one pair of sides is both parallel and equal.

- **PROVING A QUADRILATERAL IS A RECTANGLE**

Prove that it is a parallelogram first, then:

Method 1: Show that the diagonals are congruent.

Method 2: Show that it has a right angle by using slope.

- **PROVING A QUADRILATERAL IS A RHOMBUS**

Prove that it is a parallelogram first, then:

Method 1: Prove that the diagonals are perpendicular.

Method 2: Prove that a pair of adjacent sides are equal.

Method 3: Prove that all four sides are equal.

- **PROVING THAT A QUADRILATERAL IS A SQUARE**

There are many ways to do this. I recommend proving the diagonals bisect each other (parallelogram), are equal (rectangle) and perpendicular (rhombus).

- **PROVING A QUADRILATERAL IS A TRAPEZOID**

Show one pair of sides are parallel (same slope) and one pair of sides are not parallel (different slopes).

- **PROVING A QUADRILATERAL IS AN ISOSCELES TRAPEZOID**

Prove that it is a trapezoid first, then:

Method 1: Prove the diagonals are congruent using distance.

Method 2: Prove that the pair of non-parallel sides are equal.

Kwv 1

In order to prove that one of the following quadrilaterals, you will need to prove at least one of the following:

a) Parallelograms

b) Rectangle

c) Rhombus

d) Square

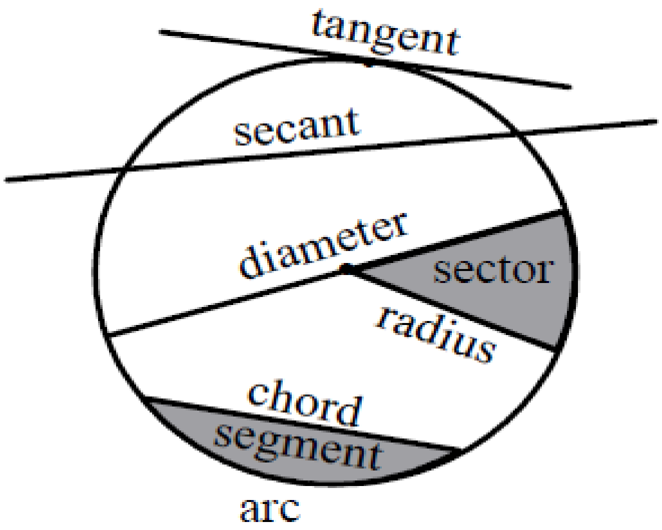
e) Trapezium

f) kite

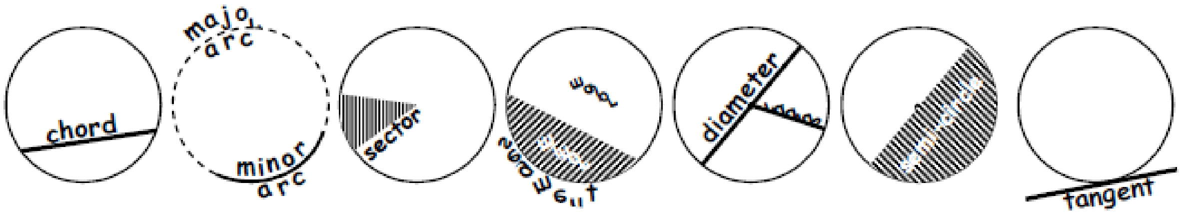
➢ CIRCLE GEOMETRY

➢ GRADE 11

KEY DEFINITION



Terminology

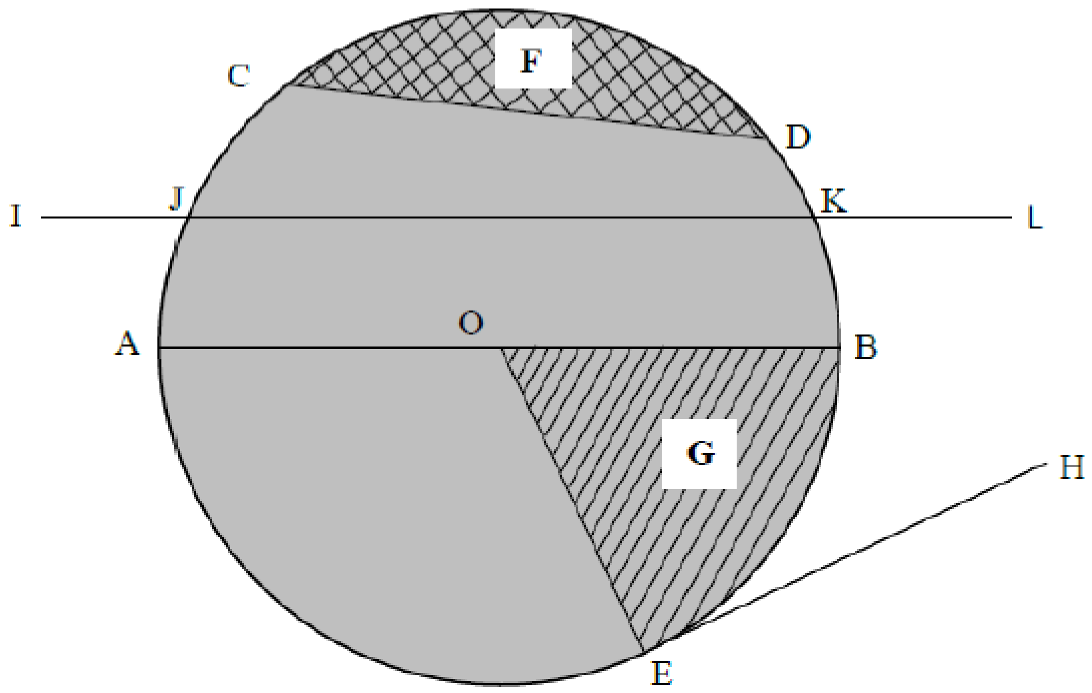


- ✓ **Radius:** a line from the centre to any point on the circumference of the circle
- ✓ **Diameter:** a line passing through the centre of the circle. It is double the length of the radius.
- ✓ **Chord:** a line with end-points on the circumference.
- ✓ **Tangent:** a line touching the circle at any one point
- ✓ **Secant:** a line passing through two points on the circle.

Kwv 1

In the diagram below, O is the centre of the circle.

Describe the following and use the figure above to write an example of each:

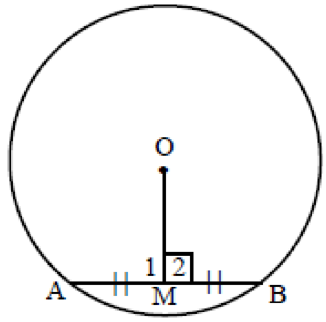


a. Diameter
b. Radius
c. Chord
d. Segment
e. Sector
f. Arc
g. Secant
h. Tangent

REVISION OF GRADE 11 THEOREMS

THEOREM

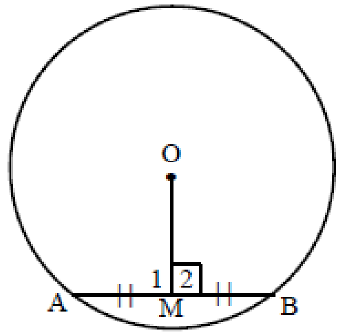
The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.



If $AM = MB$ then $OM \perp AB$ which means that $\hat{M}_1 = \hat{M}_2 = 90°$

THEOREM CONVERSE

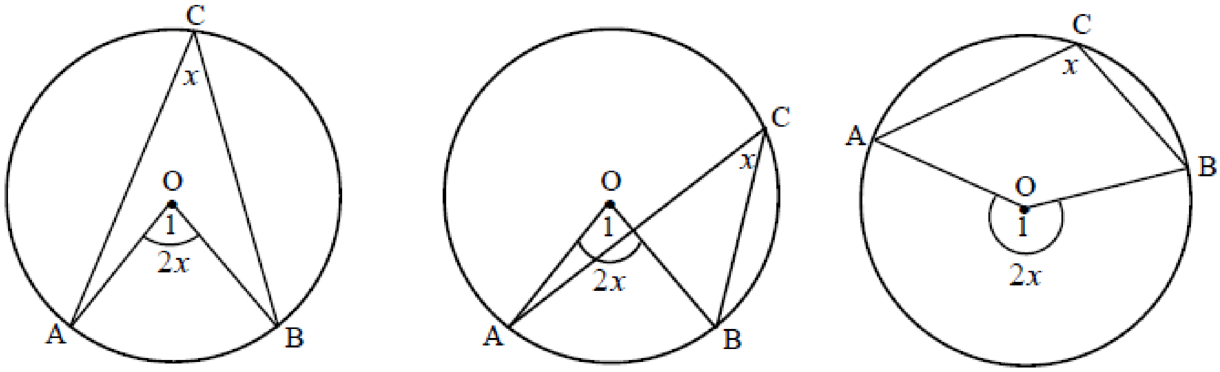
The perpendicular drawn from the centre of a circle to a chord bisects the chord.



If $OM \perp AB$ then $AM = MB$

THEOREM

The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle.

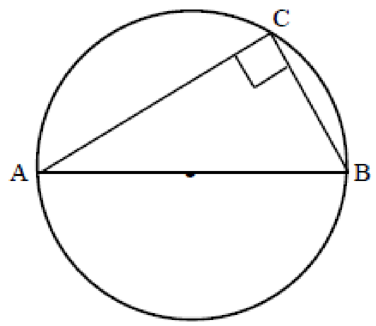


For all three diagrams: $\hat{O}_1 = 2\hat{C}$

THEOREM

The angle subtended at the circle by a diameter is a right angle. We say that the angle in a semi-circle is $90°$.

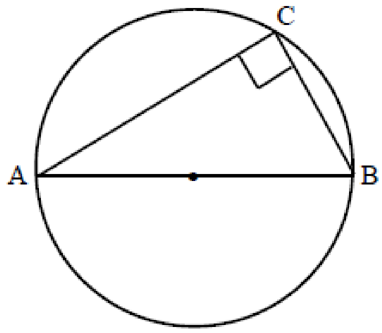
In the diagram $\hat{C} = 90°$



THEOREM CONVERSE

If the angle subtended by a chord at a point on the circle is $90°$, then the chord is a diameter.

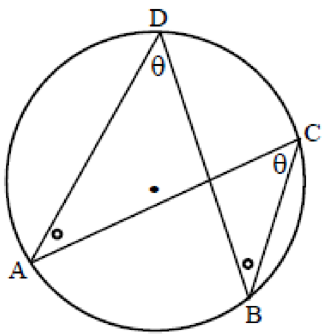
If $\hat{C} = 90°$, then the chord subtending $\hat{C}$ is a diameter.



THEOREM

An arc or line segment of a circle subtends equal angles at the circumference of the circle. We say that the angles in the same segment of the circle are equal.

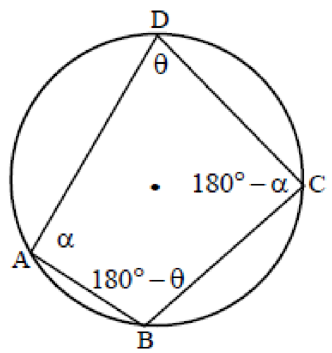
In the diagram, $\hat{A} = \hat{B}$ and $\hat{C} = \hat{D}$



THEOREM

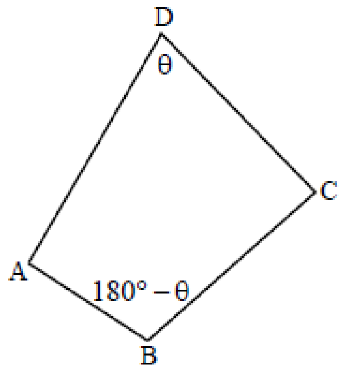
The opposite angles of a cyclic quadrilateral are supplementary (add up to $180°$)

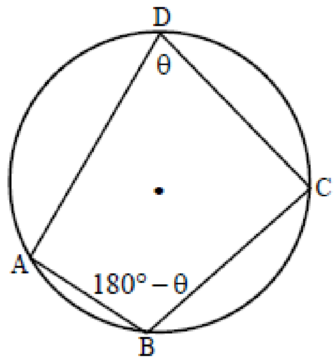
In the diagram, $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$



THEOREM CONVERSE

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic quadrilateral.

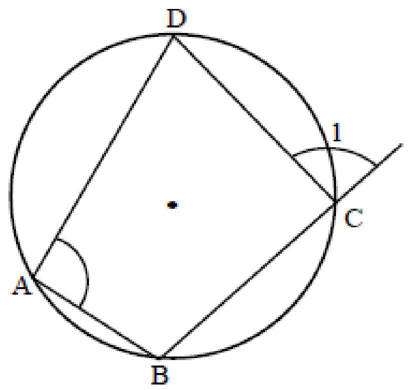




THEOREM

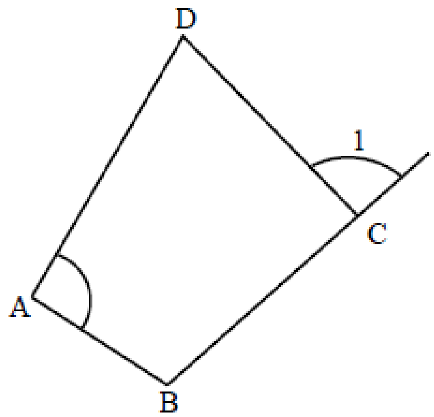
An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

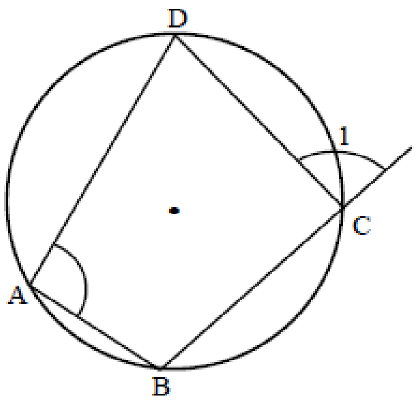
If ABCD is a cyclic quadrilateral, then $\hat{C}_1 = A$.



THEOREM CONVERSE

If an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is a cyclic quadrilateral.

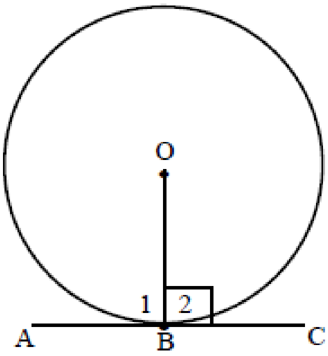




THEOREM

A tangent to a circle is perpendicular to the radius at the point of contact.

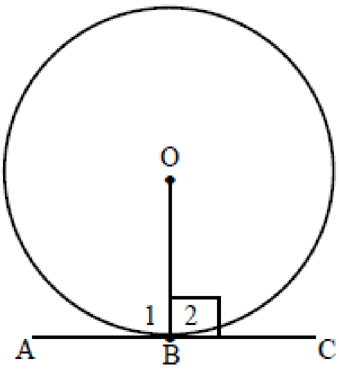
If ABC is a tangent to the circle at the point B, then the radius $OB \perp ABC$, i.e. $\hat{B}_1 = \hat{B}_2 = 90°$.



THEOREM CONVERSE

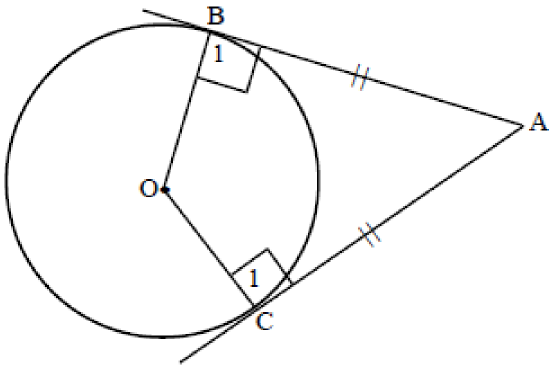
If a line is drawn perpendicular to a radius at the point where the radius meets the circle, then the line is a tangent to the circle.

If line $ABC \perp OB$ and if OB is a radius, then ABC is a tangent to the circle at B.



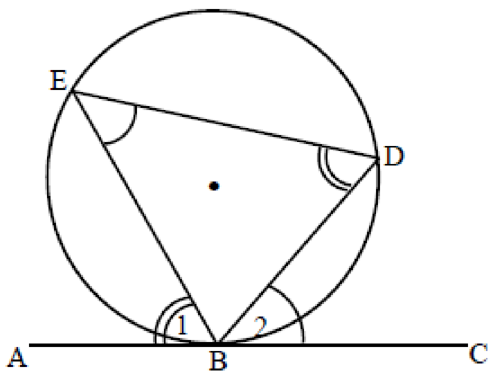
THEOREM

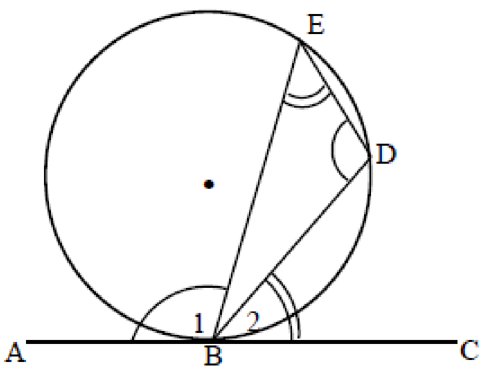
If two tangents are drawn from the same point outside a circle, then they are equal in length.



THEOREM

The angle between a tangent to a circle and a chord drawn from the point of contact in equal to an angle in the alternate segment.

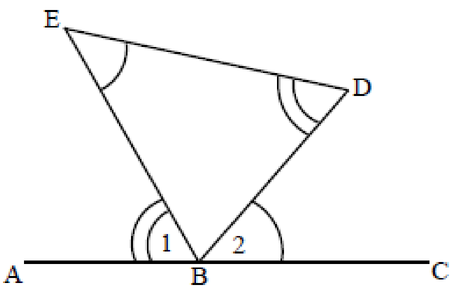


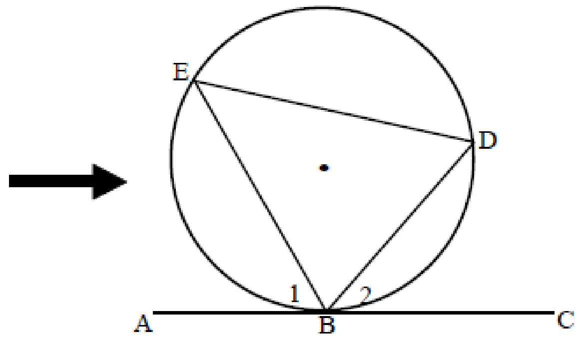


In both diagrams, $\hat{B}_2 = \hat{E}$ and $\hat{B}_1 = \hat{D}$.

THEOREM CONVERSE

If a line is drawn through the endpoint of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.





If $\hat{B}_2 = \hat{E}$ or if $\hat{B}_1 = \hat{D}$, then ABC is a tangent to the circle passing through the points B, D and E.

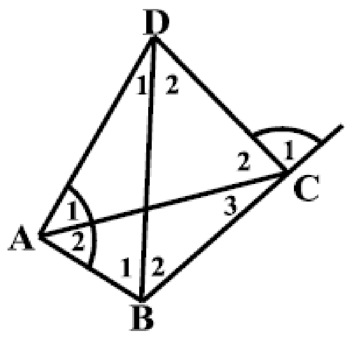
How to prove that a quadrilateral is cyclic

ABCD will be a cyclic quadrilateral if one of the following conditions is satisfied.

Condition 1
$(\hat{A}_1 + \hat{A}_2) + (\hat{C}_2 + \hat{C}_3) = 180°$ or $(\hat{B}_1 + \hat{B}_2) + (\hat{D}_1 + \hat{D}_2) = 180°$

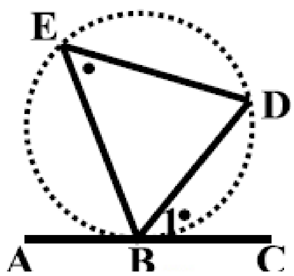
Condition 2
$\hat{C}_1 = \hat{A}_1 + \hat{A}_2$

Condition 3
$\hat{D}_1 = \hat{C}_3$ or $\hat{D}_2 = \hat{A}_2$ or $\hat{C}_2 = \hat{B}_1$ or $\hat{B}_2 = \hat{A}_1$



How to prove that a line is a tangent to a circle

ABC would be a tangent to the "imaginary" circle drawn through EBD if $\hat{B}_1 = \hat{E}$



PROOFS OF THEOREMS REQUIRED FOR EXAMS

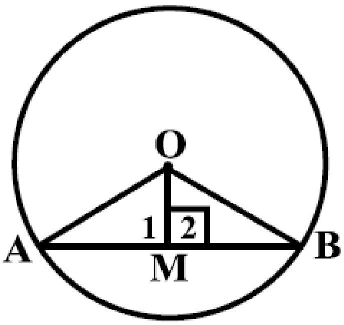
THEOREM

The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

Proof

Join OA and OB.

In ΔOAM and ΔOBM:



(a)	$OA = OB$	radii
(b)	$\hat{M}_1 = \hat{M}_2 = 90°$	given
(c)	$OM = OM$	common

$\therefore \Delta OAM \equiv \Delta OBM$ RHS

$\therefore AM = MB$

THEOREM

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).

Proof

Join CO and produce.
For all three diagrams:

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ ext $\angle$ of ΔOAC

But $\hat{C}_1 = \hat{A}$ $OA = OC$, radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

Similarly, in ΔOCB $\hat{O}_2 = 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{C}_1 + \hat{C}_2)$

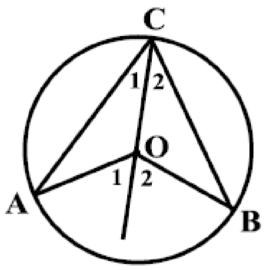
$\therefore A\hat{O}B = 2A\hat{C}B$

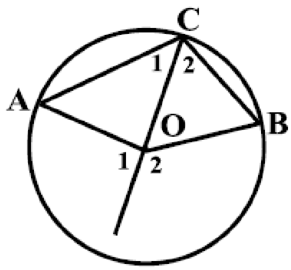
For the third diagram:

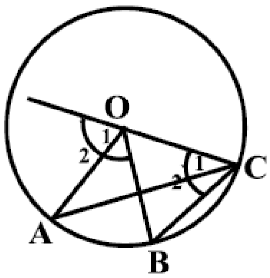
$\therefore \hat{O}_2 - \hat{O}_1 = 2\hat{C}_2 - 2\hat{C}_1$

$\therefore \hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

$\therefore A\hat{O}B = 2A\hat{C}B$



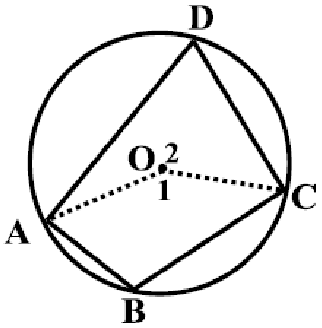




THEOREM

The opposite angles of a cyclic quadrilateral are supplementary (add up to $180°$).

Proof



Join AO and OC.

$\hat{O}_1 = 2\hat{D}$ $\quad\quad$ $\angle$ at centre $= 2\times\angle$ at circ

$\hat{O}_2 = 2\hat{B}$ $\quad\quad$ $\angle$ at centre $= 2\times\angle$ at circ

$\hat{O}_1 + \hat{O}_2 = 2\hat{D} + 2\hat{B}$

and $\hat{O}_1 + \hat{O}_2 = 360°$ $\quad\quad$ $\angle$'s round a point

$\therefore 360° = 2(\hat{D} + \hat{B})$

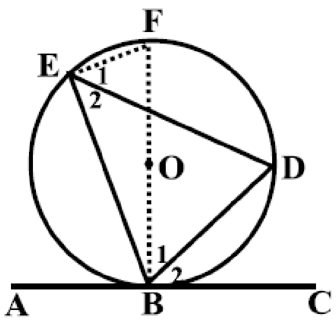
$\therefore 180° = \hat{D} + \hat{B}$

Similarly, by joining BO and DO, it can be proven that $\hat{A} + \hat{C} = 180°$

THEOREM

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment.

Proof



Draw diameter BOF and join EF

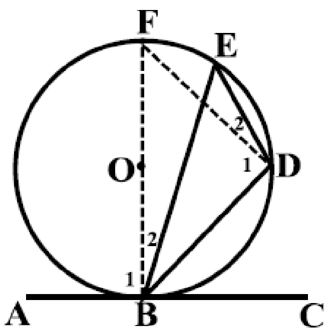
$\hat{B}_1 + \hat{B}_2 = 90°$ $\quad\quad$ tan $\perp$ rad

$\hat{E}_1 + \hat{E}_2 = 90°$ $\quad\quad$ $\angle$ in a semi-circle

But $\hat{B}_1 = \hat{E}_1$ $\quad\quad$ arc FD subt $= \angle$'s

$\therefore \hat{B}_2 = \hat{E}_2$

$\therefore C\hat{B}D = B\hat{E}D$



Draw diameter BOF and join FD

$\hat{B}_1 = 90°$ $\quad\quad$ tan $\perp$ rad

$\hat{D}_1 = 90°$ $\quad\quad$ $\angle$ in a semi-circle

$\hat{B}_2 = \hat{D}_2$ $\quad\quad$ arc FE subt $= \angle$'s

Now $\hat{B}_1 = \hat{D}_1 = 90°$ and $\hat{B}_2 = \hat{D}_2$

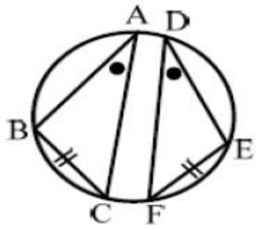
$\therefore \hat{B}_1 + \hat{B}_2 = \hat{D}_1 + \hat{D}_2$

$A\hat{B}E = B\hat{D}E$

Corollaries

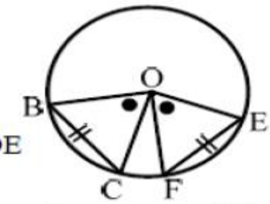
(a) Equal chords subtend equal angles at the circumference.
(equal chords ; equal $\angle$s)

$\hat{A} = \hat{D}$



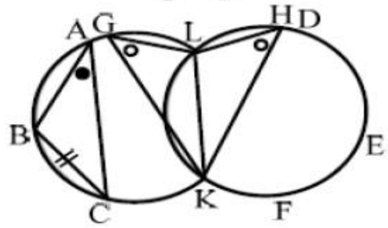
(b) Equal chords subtend equal angles at the centre.
(equal chords ; equal $\angle$s)

$B\hat{O}C = F\hat{O}E$



(c) Equal chords of equal circles subtend equal angles at the circumference.
(equal circles ; equal chords ; equal $\angle$s)

$\hat{A} = \hat{D}$ and $\hat{G} = \hat{H}$



Below are Grade 11 Theorems, Converse Theorems and their Corollaries which you must know. The proofs of the theorems marked with (**) must be studied because it could be examined.

1	**Theorem****	The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(line from centre ⊥ to chord)**
	Converse	The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. **(line from centre to midpt of chord)**
		The perpendicular bisector of a chord passes through the centre of the circle; **(perp bisector of chord)**
2	**Theorem****	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(∠ at centre = 2 ×∠ at circumference)**
	Corollary	1. Angle in a semi-circle is 90^0 **(∠s in semi circle)** 2. Angles subtended by a chord of the circle, on the same side of the chord, are equal **(∠s in the same seg)** 3. Equal chords subtend equal angles at the circumference **(equal chords; equal ∠s)** 4. Equal chords subtend equal angles at the centre **(equal chords; equal ∠s)** 5. Equal chords in equal circles subtend equal angles at the circumference of the circles. **(equal circles; equal chords; equal ∠s)**
	Corollary Converse	1. If the angle subtended by a chord at the circumference of the circle is 90^0, then the chord is a diameter. **(converse ∠s in semi circle)** 2. If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.
3	**Theorem****	The opposite angles of a cyclic quadrilateral are supplementary; **(opp ∠s of cyclic quad)**
	Converse	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is a cyclic quadrilateral. **(opp ∠s quad sup OR converse opp ∠s of cyclic quad)**
	Corollary	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral. **(ext ∠ of cyclic quad)**
	Corollary Converse	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. **(ext ∠ = int opp ∠ OR converse ext ∠ of cyclic quad)**
4	**Theorem**	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. **(tan ⊥ radius)**
	Converse	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle. **(line ⊥ radius)**
5	**Theorem**	Two tangents drawn to a circle from the same point outside the circle are equal in length. **(Tans from common pt OR Tans from same pt)**
6	**Theorem****	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. **(tan chord theorem)**
	Converse	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. **(converse tan chord theorem OR ∠ between line and chord)**

➢ ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

THEOREM STATEMENT	**ACCEPTABLE REASON(S)**
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s sup
The adjacent angles in a revolution add up to 360°.	∠s around a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	co-int ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides, then the triangle is right-angled.	converse Pythagoras **OR** converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	midpt Theorem

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2^{nd} side
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| ∆s **OR** equiangular ∆s
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	sides of ∆ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same two parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi-circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$ ⊙
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi-circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad

THEOREM STATEMENT	ACCEPTABLE REASON(S)
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	tans from common pt **OR** tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

Kwv 1

Draw the shape of all 7 theorems and write their reasons:

- **KEY WORD MIND MAP**

Kwv 1

Write all the key information if you are required to prove or given the following:

a) When parallel lines are given.

b) How to prove that lines are parallel

c) Angle or line bisectors

d) Triangle information:

- Isosceles
- Exterior
- Equilateral

e) When you must prove two sides are equal

f) Centre of a circle given

g) Diameter

h) Angles formed at the circumference

i) Chords in a circle

j) Cyclic quadrilateral given

k) How to prove that a quadrilateral is cyclic

l) Tangents to circles given

m) How to prove that a line is a tangent to a circle

➢ PROOFS

The following proofs of theorems are examinable:

Kwv 1

Prove the following theorems

a) The line drawn from the centre of a circle perpendicular to a chord bisects the chord;

b) The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);

c) The opposite angles of a cyclic quadrilateral are supplementary;

d) The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;

e) A line drawn parallel to one side of a triangle divides the other two sides proportionally;

f) Equiangular triangles are similar.

➢ **Corollaries derived from the theorems and axioms are necessary in solving riders**

• Angles in a semi-circle

• Equal chords subtend equal angles at the circumference

• Equal chords subtend equal angles at the centre

• In equal circles, equal chords subtend equal angles at the circumference

• In equal circles, equal chords subtend equal angles at the centre.

• The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.

• If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.

• Tangents drawn from a common point outside the circle are equal in length.

TIPS TO SOLVING GEOMETRY RIDERS

- **READ-READ-READ** the information next to the diagram thoroughly
- **TRANSFER** all given information to the DIAGRAM
- Look for **KEYWORDS**, e.g.
  - ✓ **TANGENT**: What do the theorems say about tangents?
  - ✓ **CYCLIC QUADRILATERAL**: What are the properties of a cyclic quad?
- **NEVER ASSUME** something!
  - ✓ Don't assume that a certain line is the DIAMETER of a circle unless it is clearly state or unless you can prove it
  - ✓ Don't assume that a point is the CENTRE of a circle unless it is clearly stated ("circle M" means "the circle with midpoint M")

NB:

The following are some forms of logic applicable in proof of theorems and riders:

- If $a = b$ and $b = c$ then $a = c$
- If $a + b = c$ and $d + b = c$ then $a + b = d + b$, so $a = d$
- If $a = b + c$ and $b = d$ then $a = d + c$

TAKE NOTE IF FOLLOWING TERMS ARE GIVEN

1. CENTRE

- ✓ Radii can be joined by chord to form an isosceles triangle.
  - $\angle$s opp equal sides
  - sides opp equal $\angle$s
- ✓ $\angle$ at centre = 2 $\times \angle$ at circumference
- ✓ line from centre to midpt of chord
- ✓ line from centre $\perp$ to chord
- ✓ perp bisector of chord

2. TANGENT (S)

- ✓ tan $\perp$ radius
  - Right angled triangle can be formed
- ✓ tan $\perp$ diameter
  - Right angled triangle can be formed
- ✓ tangents from common point or tans from same pt. **{ DEP DANCE}**
  - Isoscele
  - s triangle can be formed
- ✓ tan chord theorem **{ MODELLING STYLE}**

3. PARALLEL LINES

- ✓ If If AB || CD, then the corresponding angles are equal. **{FANELE}**
- ✓ If AB || CD, then the alternate angles are equal.{ **ZODWA WABANTU**}
- ✓ If AB || CD, then the co-interior angles are supplementary.{ **CELIWE}**

4. DIAMETER

- ✓ $\angle$s in semi-circle **OR** diameter subtends right angle **OR** $\angle$ in $\frac{1}{2}\odot$.
- ✓ tan $\perp$ diameter
- ✓ diameter = 2radii

5. FOR THE CYCLIC QUAD

- ✓ line subtends equal $\angle$s OR $\angle$s in the same seg
  - equal chords; equal $\angle$s
- ✓ opp $\angle$s of cyclic quad
- ✓ ext $\angle$ of cyclic quad

6. FOR EQUAL LINES / ANGLES

- ✓ Equal chords subtend equal angles at the circumference of the circle.
- ✓ The angles opposite the equal sides in an isosceles triangle are equal.
- ✓ The sides opposite the equal angles in an isosceles triangle are equal.

7. ANGLE BISECTOR

- ✓ The line that divides the angle into equal parts

➢ ANGLES

➢ COMPLIMENTARY ANGLES (ADD UP TO 90°)

- ✓ Angles subtended by diameter
- ✓ Radius perpendicular to tangent

➢ SUPPLIMENTARY ANGLES (ADD UP TO 180°)

- ✓ Angles on a straight line
- ✓ Sum angles of a triangle
- ✓ Opposite angles of a cyclic quad
- ✓ Co-int angles

➢ EQUAL ANGLES

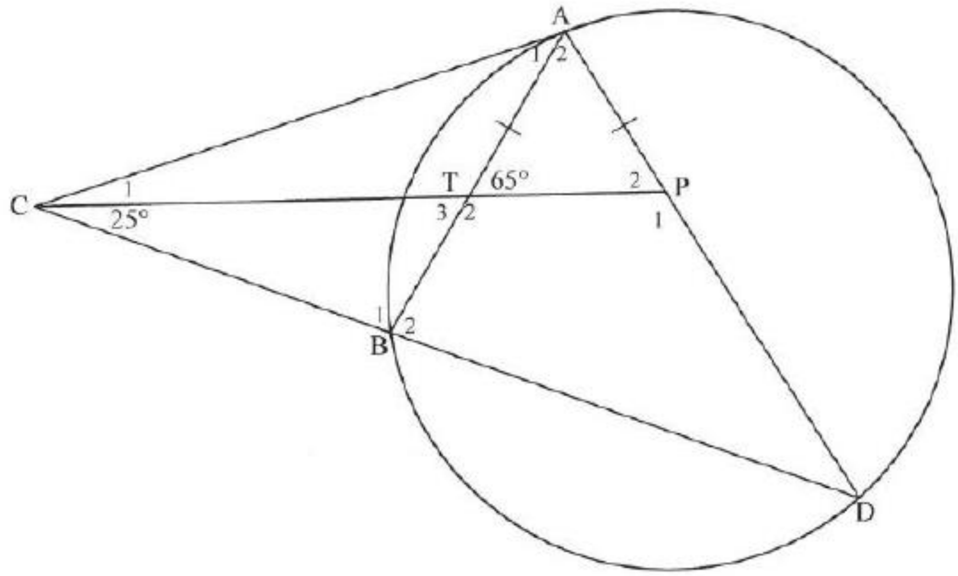
- ✓ Altanate angles
- ✓ Corresponding angles
- ✓ Tan-chord theorem
- ✓ Ext angle of a cyclic quad
- ✓ Angles substended by same chord / arc
- ✓ Equal sides oppose equal angles

➢ REVOLUTION (ADD UP TO 360°)

- ✓ Angles around the point
- ✓ Sum angles of a quad

KWV 1

In the diagram ΔACD is drawn with points A and D on the circumference of a circle. CD cuts the circle at B. P is a point on AD with CP the bisector of $A\hat{C}D$. CP cuts the chord AB at T. AT = AP, $A\hat{T}P = 65^\circ$ and $P\hat{C}D = 25^\circ$.



- **KEY WORDS**
  - **CP BISECT of A$\widehat{C}$D**

Then!!!! $\rightarrow \hat{C}_1 = \widehat{C}_2 = 25^\circ$

- **EQUAL LINES : AT =AP**

Then!!!! $\rightarrow A\widehat{T}P = \widehat{P}_2 = 65^\circ$

NB: AC is not a tangent unless it's stated.

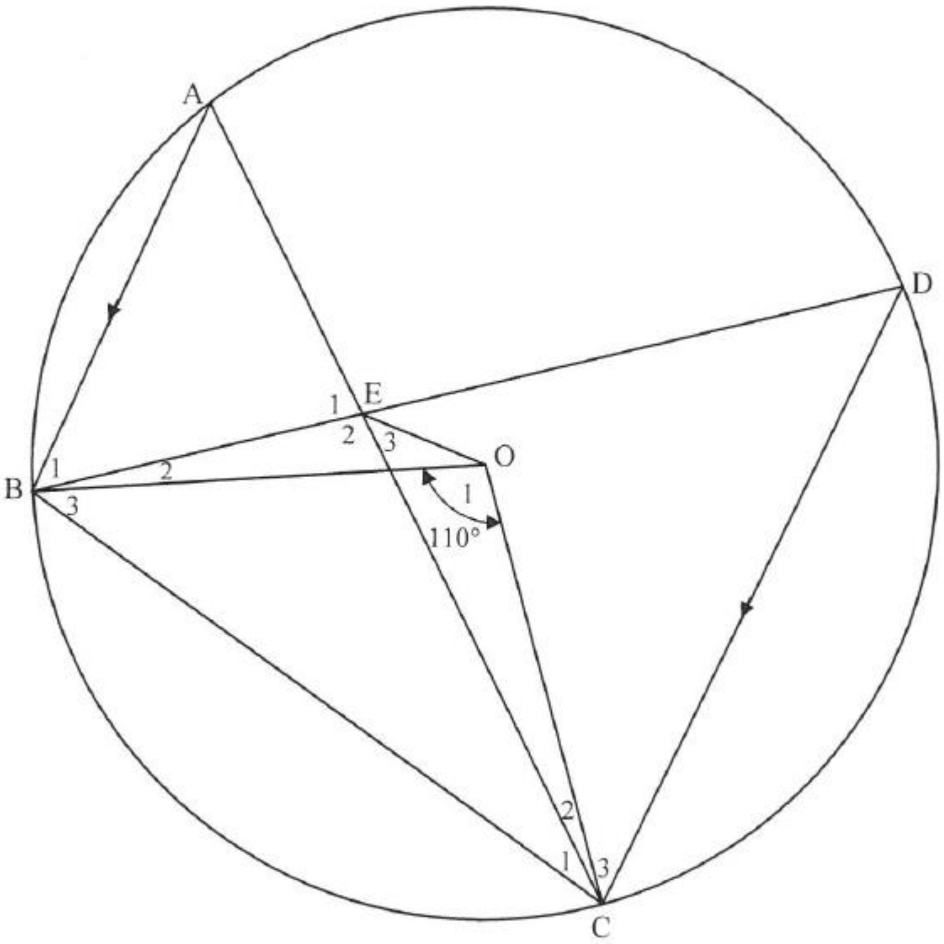
$\rightarrow$ To find $\widehat{D}$, ext $\angle$ of Δ COP can be used. $\rightarrow \widehat{D} = 40^\circ$

$\rightarrow A\widehat{T}C + A\widehat{T}P = 180^\circ$ ($\angle$'s on a str line)

$\rightarrow \hat{C}_1 + \hat{A}_1 = A\widehat{T}P$ (ext $\angle$ of Δ ACT)

KWV 2

In the diagram below, the circle with centre O passes through A, B, C and D.
AB || DC and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



- **KEY WORDS**
  - **CENTRE** O

Then!!!! $\rightarrow\ \hat{O} = 2\hat{A}$ ($\angle$ at centre = 2× $\angle$ at circ)

$\rightarrow 110° = 2\hat{A}$

OR

$\rightarrow \hat{O}_1 = 2\hat{D}$ ($\angle$ at centre = 2× $\angle$ at circ)

$\rightarrow 110° = 2\hat{D}$

- **PARALLEL LINES : AB || DC**

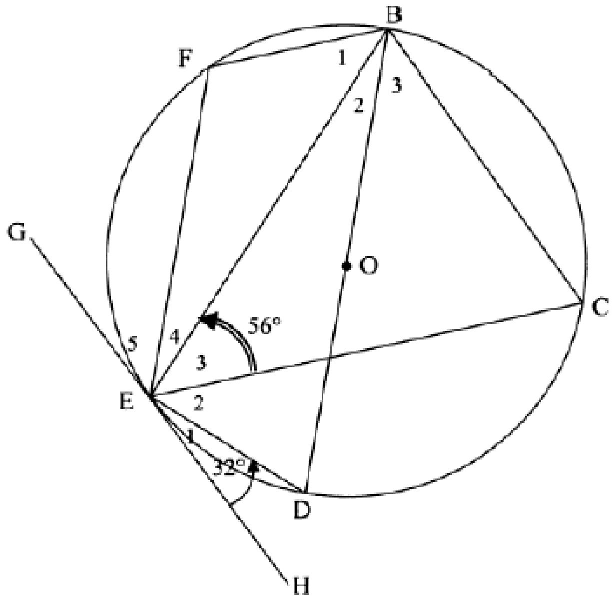
Then!!!! $\rightarrow \hat{A} = A\hat{C}D$

$\rightarrow \hat{B}_1 = \hat{D} = 55°$ (alternate $\angle$'s AB || DC)

KWV 3

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



- **KEY WORDS**
  - **CENTRE O & DIAMETER BD**

Then!!!! $\rightarrow B\hat{E}D = 90°$ ($\angle$'s in a semi-circle)

$\rightarrow \hat{E}_2 + \hat{E}_3 = B\hat{E}D$ ($\angle$'s in a semi-circle)

- **TANGENT GEH**

Then!!! $\rightarrow \hat{E}_1 + \hat{B}_2$ (tan-chord theorem)

$\rightarrow \hat{E}_5 = \hat{B}_1$ (tan-chord theorem)

$\rightarrow \hat{E}_2 = \hat{B}_3$ ($\angle$'s in the same segment)

- **CYCLIC QUAD: FBCE**

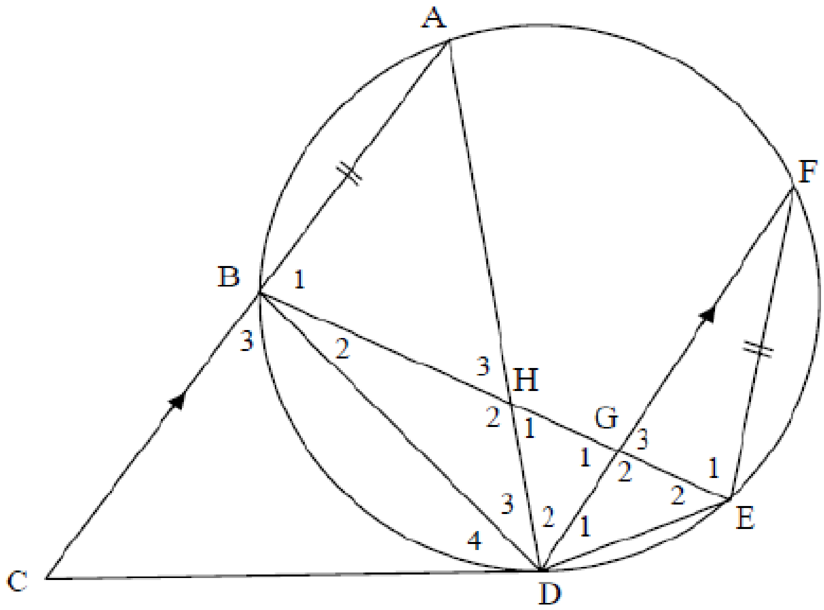
Then!!! $\rightarrow \hat{F} + \hat{D} = 180°$ (opp $\angle$'s cyclic quad)

- **CYCLIC QUAD: FBDE**

$\rightarrow \hat{F} + \hat{C} = 180°$ (opp $\angle$'s cyclic quad)

KWV 4

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. AC || FD and FE = AB. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



- **KEY WORDS**
  - **TANGENT CD**

Then!!!! $\rightarrow \widehat{D}_4 = \hat{A}$ (tan-chord theorem)

$\rightarrow \widehat{D}_4 = \hat{E}_2$ (tan-chord theorem)

  - **PARALLEL LINES : AC || FD**

Then!!!! $\rightarrow \hat{A} = \widehat{D}_2$ (alternate ∠'s AC || FD)

$\rightarrow \hat{B}_1 = \hat{G}_1$ (alternate ∠'s AC || FD)

$\rightarrow \hat{B}_1 = \hat{G}_3$ (corr ∠'s AC || FD)

  - **EQUAL LINES : FE = AB**

Then!!!! $\rightarrow \widehat{D}_3 = \widehat{D}_1$ (subt by equal chord, AB = EF)

NB: Two equal circles with equal chords, then those chords will subtend equal angles at the circumference as long as those 2 circles intersect each other.

➢ GEOMETRIC RIDERS : PROOFS

- ➢ When required to prove a geometric fact it is always good to have a thought strategy.
- ➢ The most important overriding factor to any strategies ... know your theorems.
- ➢ **One useful strategy:**
  - ✓ If required to prove something about lines or shapes
  - ✓ Ask "What if this is so?" This will usually lead on to another fact which may be easier to prove first (as long as the converse is true!).

➢ STATEMENTS AND CONVERSES:

Statement: ***If*** *today* is Tuesday ***then*** tomorrow is Wednesday.
Converse: ***If*** tomorrow is Wednesday ***then*** today is Tuesday.

Some converses are true and some are not:

Statement: *If it* is not raining *then* there are no clouds.
Converse: *If* there are no clouds *then* it is not raining.

➢ HOW TO PROVE THTA LINES ARE PARALLEL.

→If the altenate angles between two lines are equal, then the lines are parallel. (reason→alt∠'s =)

→If the corr ∠'s between two lines are equal, then the lines are parallel. (corr ∠'s =).

→If the co-interior∠'s between two lines are supplementary, then the lines are parallel. (co-int ∠'s supp)

➢ WHEN YOU MUST PROVE TWO SIDES ARE EQUAL.

→In a triangle, base angles must be equal. (sides opp equal angles)

→When lines are separated, 2 opp ∠'s on the circumfarance must be equal. (sides opp = ∠'s)

➢ DIAMETER.

→Opp ∠ in the circumfarance must be equal to 90˚.

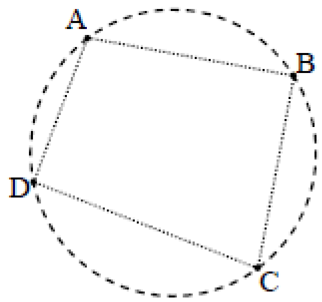
➢ HOW TO PROVE THTA A QUADRILATERAL IS CYCLIC.

Concyclic points

Points are concyclic if they lie <u>on a circle</u>.

In the figure A, B, C and D are concyclic.

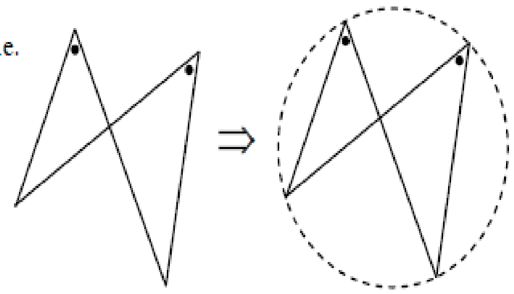
It means the same to say that ABCD is a cyclic quadrilateral.



¤ **If the line segment joining two points subtends equal angles at two other points on the same side of it, then the four points are concylic.**

Reference: **conv. ∠'s in same segm.**

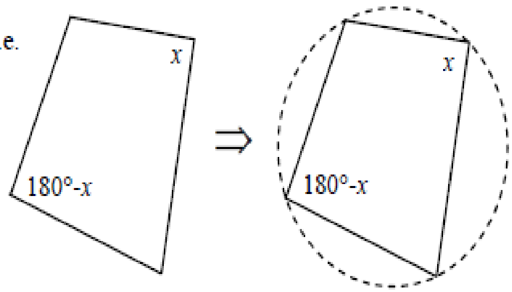
i.e.



¤ **If one pair of opposite angles of a quad-rilateral is supplementary then the quad-rilateral is cyclic.**

Reference: **conv. opp. ∠'s cyclic quad.**

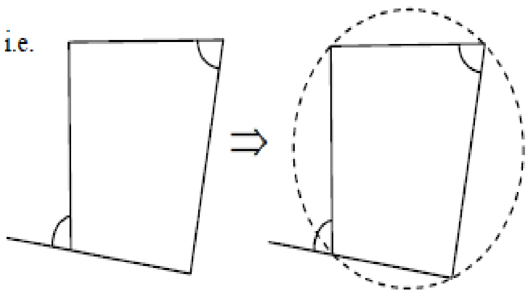
i.e.



¤ **If one side of a quadrilateral is produced and the exterior angle formed is equal to the interior opposite angle, then the quad-rilateral is cyclic.**

Reference: **conv. ext. ∠ cyclic quad.**

i.e.



→Converse ∠'s in the same segm.

→converse opp ∠'s of cyclic quad)

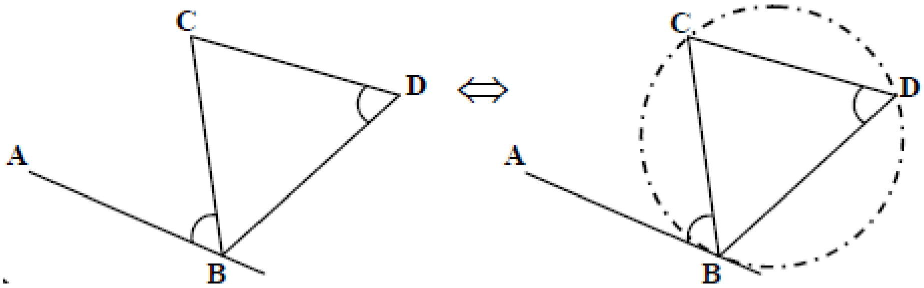
→converse ext ∠ of cyclic quad)

➢ PROVE THAT LINE IS A TANGENT TO THE CIRCLE.

¤ **Converse to alternate segment theorem**

Reference: **conv.tan.chord**

$\hat{B} = \hat{D} \Rightarrow$ AB is a tangent to circle BCD.



→Converse tan chord theorem

➢ TRIANGLE INFORMATION.

- **ISOSCELES**

→2 sides are equal

→2 angles opp = sides are equal

- **EXTERIOR**

→Ext ∠ of a Δ = the sum of the 2 int opp ∠'s of a Δ

- **EQUILATERAL**

→All sides are equal

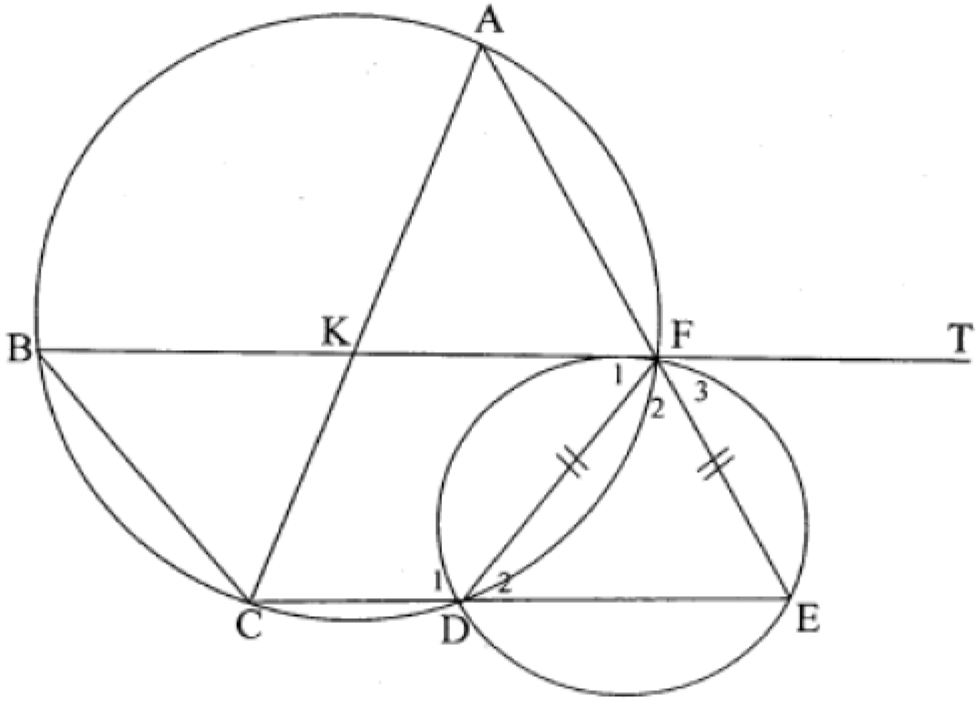
→All angles are equal

➢ ANGLE FORMED AT THE CIRCUMFERENCE.

→Two angles at the circumfarence equal, their chord(s) are also equal.

KWV 1

In the figure below, two circles cut in points F and D. BFT is a tangent to the smaller circle at F. straight line AFE is drawn so that FE = FD. CDE is a straight line and chords AC and BF cut at K.



PROVE THAT:

a) BT || CE

R.T.P.: $\hat{F}_1 = \hat{D}_2$

REASON: Alt. $\angle' s$ proved equal

b) BCEF is a parallelogram

R.T.P.: BC // FE

REASON: Corr. $\angle' s$ proved equal

c) AC = BF

R.T.P.: $\hat{E} = \hat{A}$

REASON: $\angle' s$ opposite to equal sides

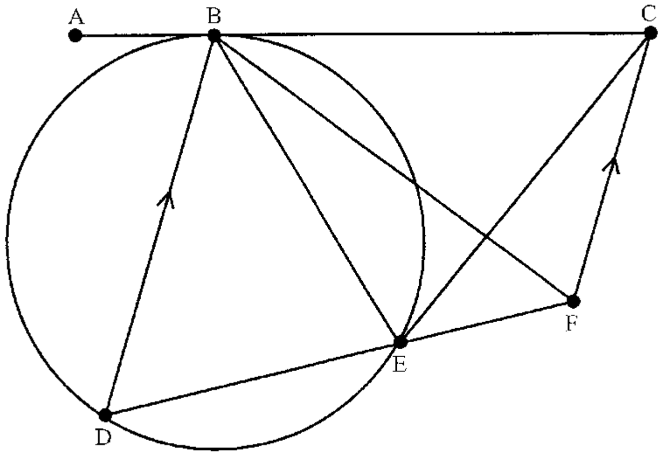
d) BF is a diameter, if it given that AF = FE

R.T.P.: BK = KF = AK = KC

REASON: = radius

KWV 2

In the figure, ABC is a tangent to the circle. Chord DE is drawn and extended to F so that BD || CF. DE, BE and BF are joined. BD bisects $A\hat{B}F$.



Prove:

a) BEFC is a cyclic quadrilateral.

R.T.P.: $D\hat{E}B = A\hat{C}F$

REASON: Exterior angle = interior opposite angle

b) BE bisects $D\hat{E}C$.

R.T.P.: $C\hat{E}B = D\hat{E}B$

REASON: BE bisects $C\hat{E}D$

c) BD is a tangent to circle BCF.

R.T.P.: $D\hat{B}F = B\hat{C}F$

REASON: Converse to tan-chord theorem

GRADE 12

PROPORTIONAL AND AREA OF TRIANGLES

RATIOS

- can be vertical or horizontal

CROSS MULTIPLICATION TECHNIQUES

- It is useful when working with **ratios**

PROPORTION THEOREM

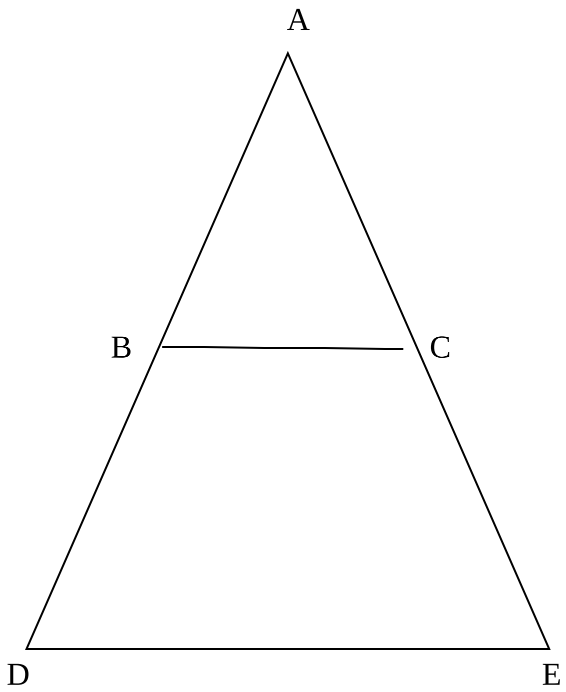
If a line is drawn parallel to one side of a triangle, it divides the other two sides in the same proportion. (Prop theorem, DE || BC)

Note:

- reason: **line // one side of Δ**
- **corollaries**

Kwv 1

Consider the diagram

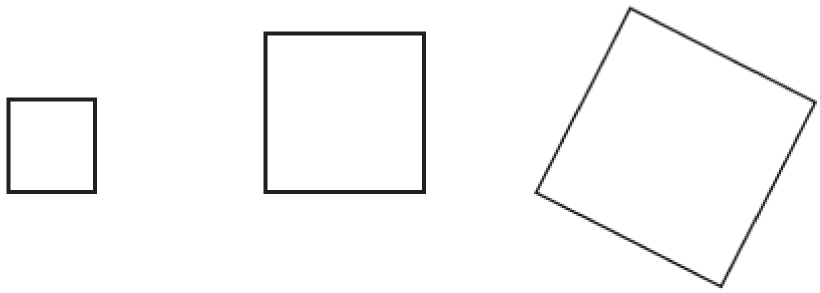


Write 4 corollaries from above diagram if BC ∥ DE

➢ SIMILAR POLYGONS

- Similar polygons have the same shape, but not necessarily the same size.

e.g. Every square is similar to every other square.



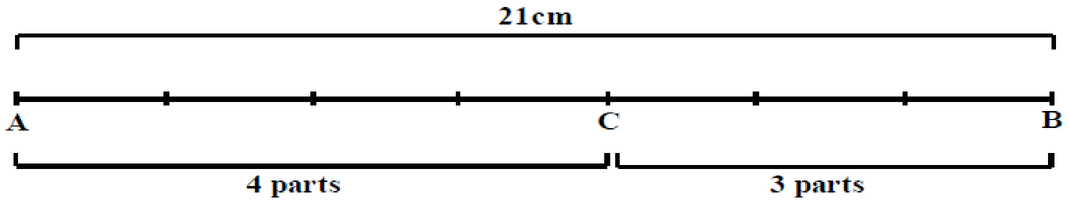
POLYGONS (WITH THE SAME NUMBER OF SIDES) ARE SIMILAR WHEN:

- All the pairs of corresponding angles are equal (They are equiangular) and
- All the pairs of corresponding sides are in the same proportion. Both of these conditions must hold at the same time.
- ||| is the symbol we use to say one polygon 'is similar to' another polygon.

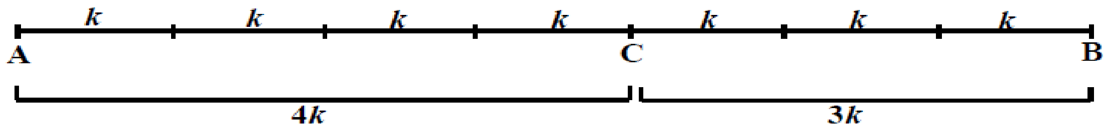
➢ TRIANGLES ARE SPECIAL POLYGONS

- If two triangles are equiangular, then their sides will always be in the same proportion, so the triangles are similar.
- If the sides of two triangles are in the same proportion, then the triangles will be equiangular, so the triangles are similar.
- equiangular Δs → similar Δs corresponding sides Δs in proportion → Δs are similar

REVISION OF THE CONCEPT OF RATIOS



Consider the line segment AB. If $AB = 21$ cm and C divides AB in the ratio $AC:CB = 4:3$, it is possible to find the actual lengths of AC and CB.
It is clear that AC doesn't equal 4cm and CB doesn't equal 3 cm because $4 + 3 \neq 21$ cm.
However, if we let each part equal k, it will be possible to find the length of AC and CB in centimetres.



The length of AC is $(4k)$cm and the length of CB is $(3k)$cm.

$\therefore 4k + 3k = 21$ cm

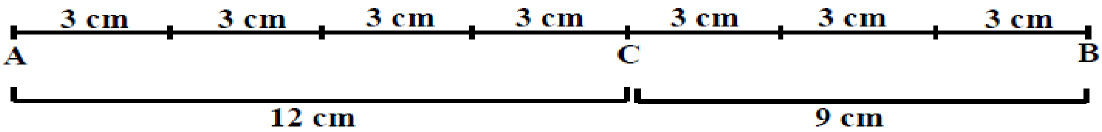
$\therefore 7k = 21$ cm

$\therefore k = 3$ cm

Each part represents 3 cm.

$\therefore AC = 4(3 \text{ cm}) = 12$ cm

and $CB = 3(3 \text{ cm}) = 9$ cm



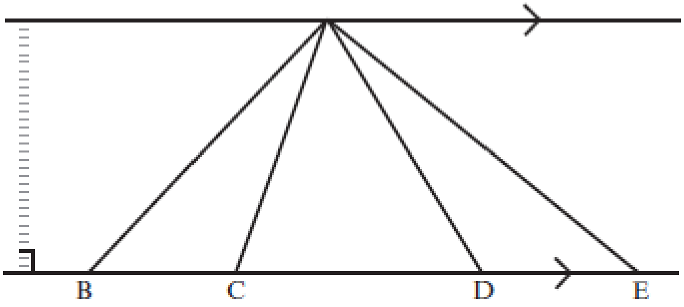
Note:

$$AC:CB = \frac{AC}{CB} = \frac{12\,\text{cm}}{9\,\text{cm}} = \frac{4}{3}$$

4:3 is the ratio of AC:CB.

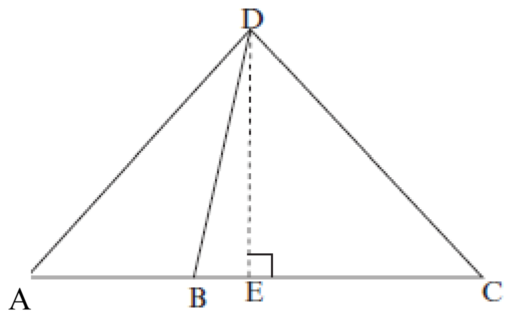
➢ AREA OF TRIANGLES

✓ If two or more triangles have a common vertex (A) and lie between the same parallel lines, they also have a common perpendicular height (altitude).



✓ The areas of triangles with equal altitudes are in the same proportion as their bases.

Remember: area Δ = ½ base × perp height

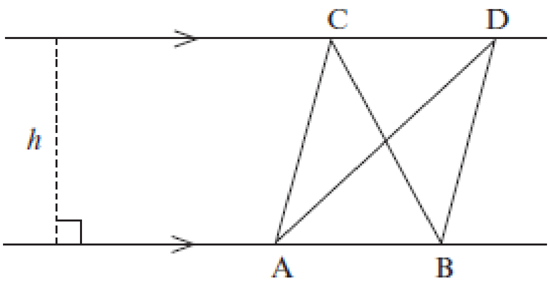


ΔADB, ΔDBC and ΔADC all have the same ⊥ height DE.

So Area ΔADB : Area ΔDBC : Area ΔADC

(½ AB × DE) : (½ BC × DE) : (½ AC × DE) AB : BC : AC

➢ CONSIDER:



✓ If two or more triangles lie between parallel lines, they have the same altitude.

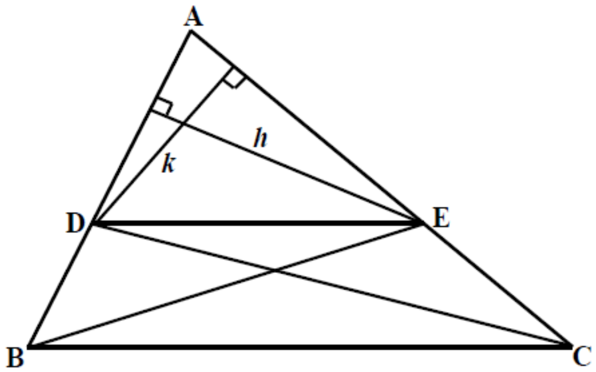
✓ Triangles on the same base (or equal bases) and between parallel lines are equal in area.

Area ΔABC = ½(AB) *h*

Area ΔADB = ½(AB) *h*

Area ΔABC = Area ΔADB

THEOREM



A line drawn parallel to one side of a triangle cuts the other two sides so as to divide them in the same proportion.

If DE||BC then $\frac{AD}{DB} = \frac{AE}{EC}$

Proof

In ΔADE, draw height h relative to base AD and height k relative to base AE.
Join BE and DC to create ΔBDE and ΔCED.

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2} . AD . h}{\frac{1}{2} . BD . h} = \frac{AD}{BD}$$

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta CED} = \frac{\frac{1}{2} . AE . k}{\frac{1}{2} . EC . k} = \frac{AE}{EC}$$

Now it is clear that
$\text{Area } \Delta BDE = \text{Area } \Delta CED$
(same base, height and
lying between parallel lines)

$$\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta CED}$$

$$\therefore \frac{AD}{BD} = \frac{AE}{EC}$$

Corollaries

(1) $\frac{AB}{AD} = \frac{AC}{AE}$ (2) $\frac{AB}{DB} = \frac{AC}{EC}$ (3) $\frac{BD}{DA} = \frac{CE}{EA}$ (4) $\frac{BD}{BA} = \frac{CE}{CA}$

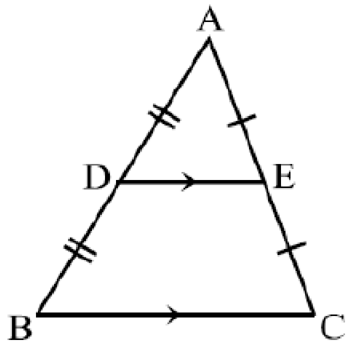
Whenever you use this theorem the reason you must give is: Line || one side of Δ

THEOREM (MIDPOINT THEOREM)

The line passing through the midpoint of one side of a triangle, parallel to another side, bisects the third side and is equal to half the length of the side it is parallel to.

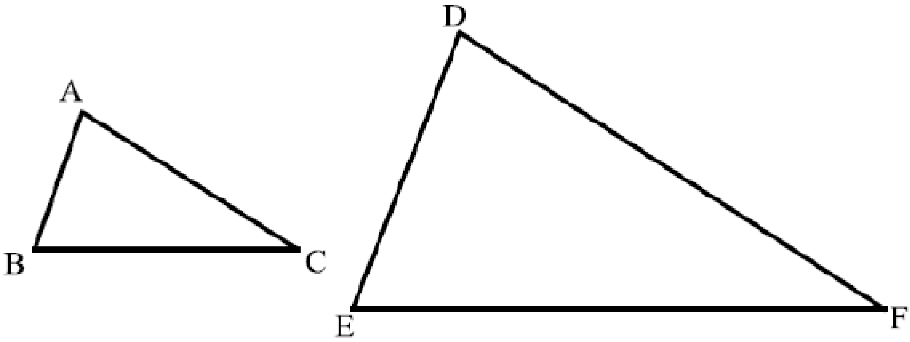
If $AD = DB$ and $DE \parallel BC$, then $AE = EC$ and $BC = 2DE$ or $DE = \frac{1}{2}BC$.

Also, if $AD = DB$ and $AE = EC$, then $DE \parallel BC$ and $BC = 2DE$ or $DE = \frac{1}{2}BC$.



SIMILARITY OF TRIANGLES

If two triangles are similar, we use the symbol ||| to indicate this.



If ΔABC is similar to ΔDEF then we write this as follows: $\Delta ABC ||| \Delta DEF$

If $\Delta ABC ||| \Delta DEF$ then the following conclusions can be made:

(a) The triangles are equiangular which means that:

$\hat{A} = \hat{D}$ $\quad \hat{B} = \hat{E}$ $\quad \hat{C} = \hat{F}$

(b) The corresponding sides are in the same proportion which means that:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Whenever two triangles are similar we can use the following diagram to match the corresponding angles and sides:

$\hat{A} = \hat{D}$ $\quad \hat{B} = \hat{E}$ $\quad \hat{C} = \hat{F}$

$$\Delta ABC \equiv \Delta DEF$$

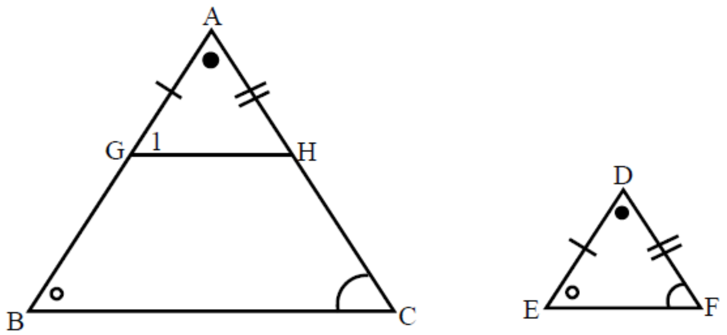
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

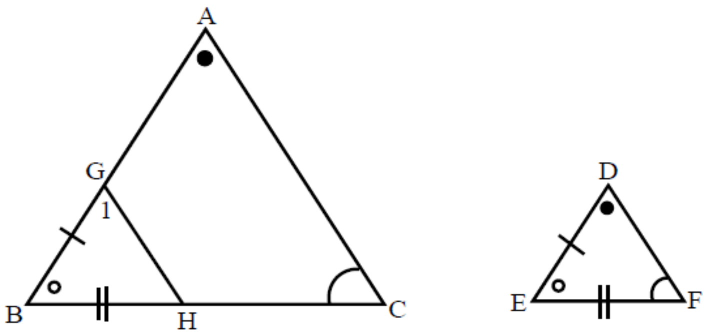
THEOREM

If two triangles are equiangular then the corresponding sides of the two triangles are in the same proportion and therefore the triangles are similar.

Proof

On AB mark off $AG = DE$.
On AC mark off $AH = DF$.
Join GH.
In ΔAGH and ΔDEF:
(1) $AG = DE$ construction
(2) $\hat{A} = \hat{D}$ given
(3) $AH = DF$ construction
$\therefore \Delta AGH \equiv \Delta DEF$ SAS
$\therefore \hat{G}_1 = \hat{E}$
But $\hat{B} = \hat{E}$ given
$\therefore \hat{G}_1 = \hat{B}$
$\therefore GH \| BC$ corr $\angle$'s equal
$\therefore \frac{AB}{AG} = \frac{AC}{AH}$
$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ $(AG = DE, AH = DF)$
Similarly, by constructing BG and BH on AB and BC respectively, it can be proved that
$\frac{AB}{DE} = \frac{BC}{EF}$
$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
Therefore the triangles are similar.





Kwv 1

If two or more triangles have a common vertex (A) and lie between the same parallel lines, they also have a common perpendicular height (altitude**), and then draw the triangle to represent the information**

Kwv 2

The areas of triangles with equal altitudes or height are in the same proportion as their bases, **and then draw the triangle to represent the information**

Kwv 3

If two or more triangles lie between parallel lines, they have the same altitude or height. And triangles on the same base (or equal bases) and between parallel lines are equal in area, **and then draw the triangle to represent the information**

- **THEOREMS**

- If two triangles are equiangular, then the corresponding sides are in proportion and therefore the triangles are similar.

NOTE:

If two triangles have 2 corresponding angles equal, then the third angles will equal each other (sum angles of a triangle = 180°) and the triangles are therefore similar and their sides will be in proportion. The shortened reason you can use is (third angle)

- If two triangles have their sides in the same proportion, then the corresponding angles will be equal and the triangles are similar.

- **CONCLUSION**

If you are required to prove similarity then:

- Two angles equal
- Obvious, other angle will be equal (3^{rd} angle of triangle)

Kwv 1

Play with the following:

a) RT = 3PN

b) $AB^2 = BC^2 + AC^2$

c) AB: CD: 3 : 5

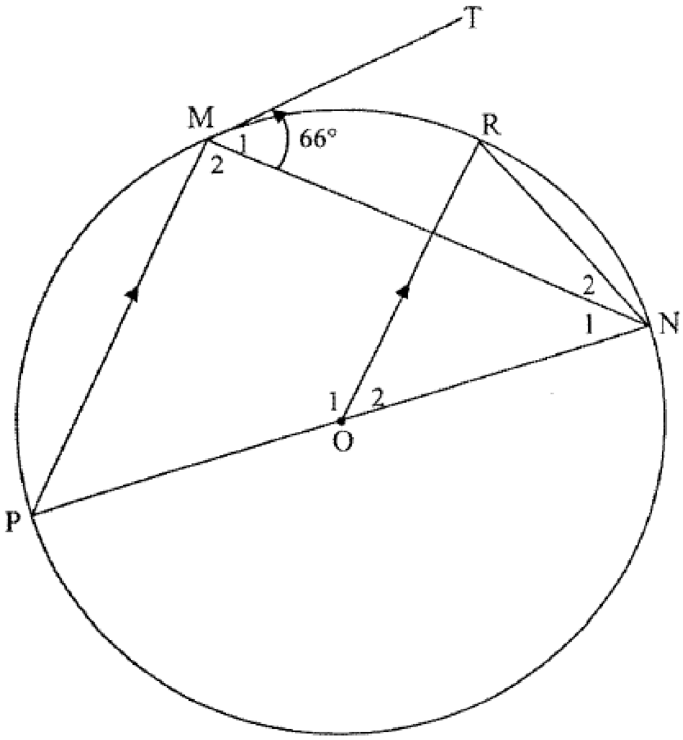
d) AB^2 = BD. BC

Kwv 1

Critical analyse the following diagrams........

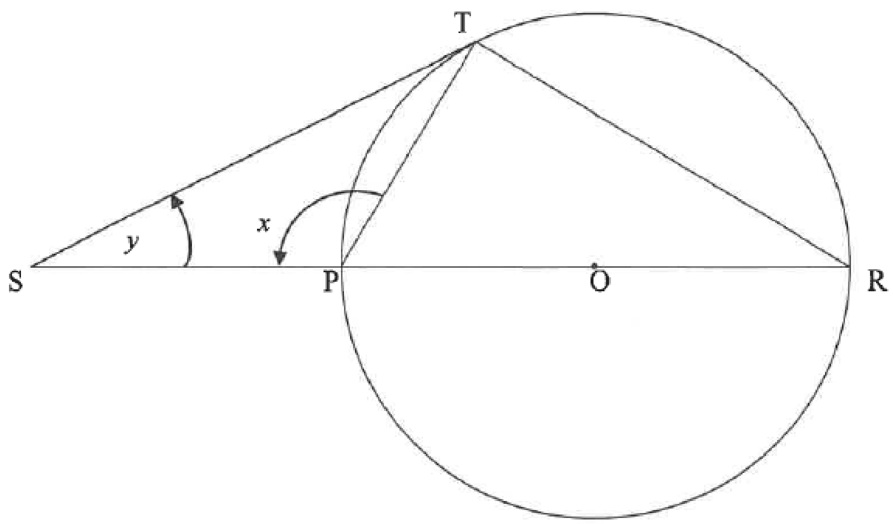
a.

PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



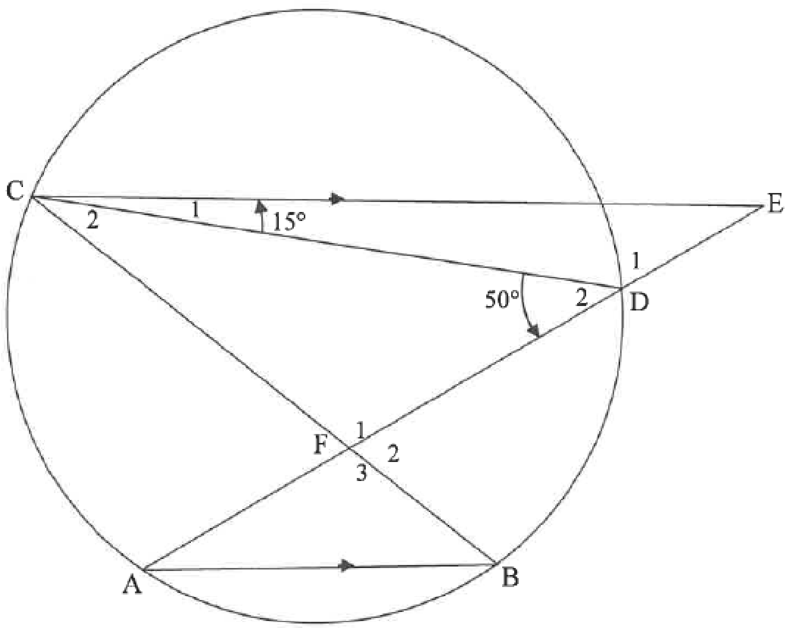
b.

In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.



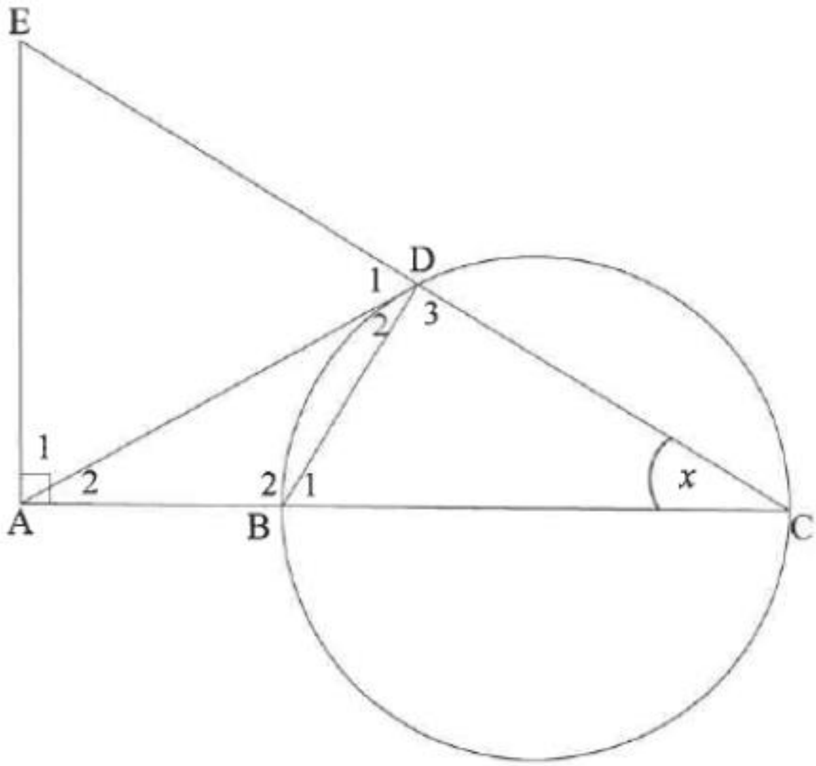
c.

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



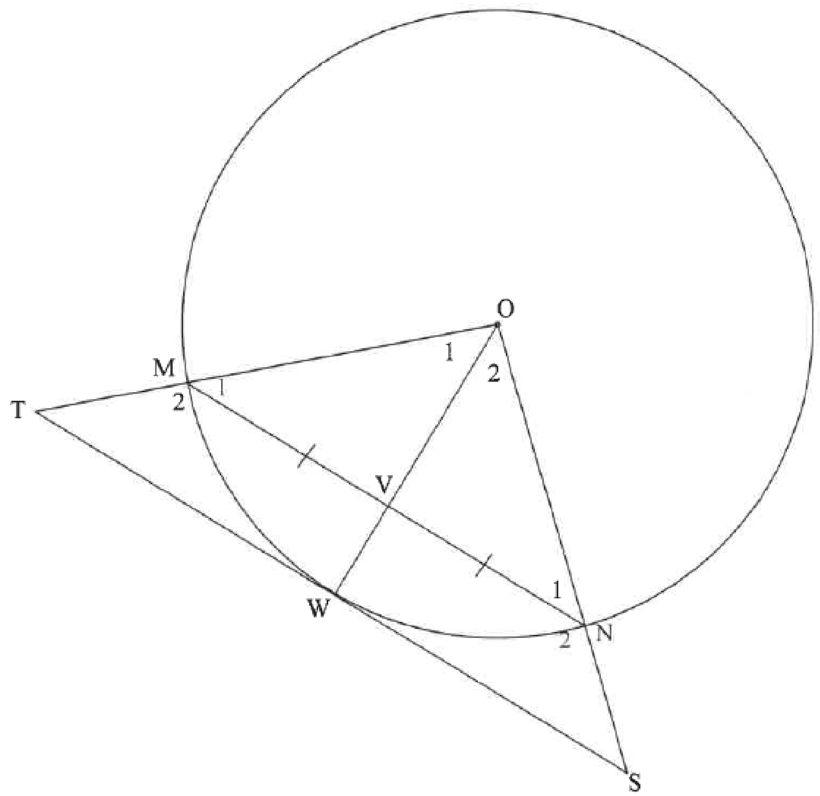
d.

In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.
Let $\hat{C} = x$.



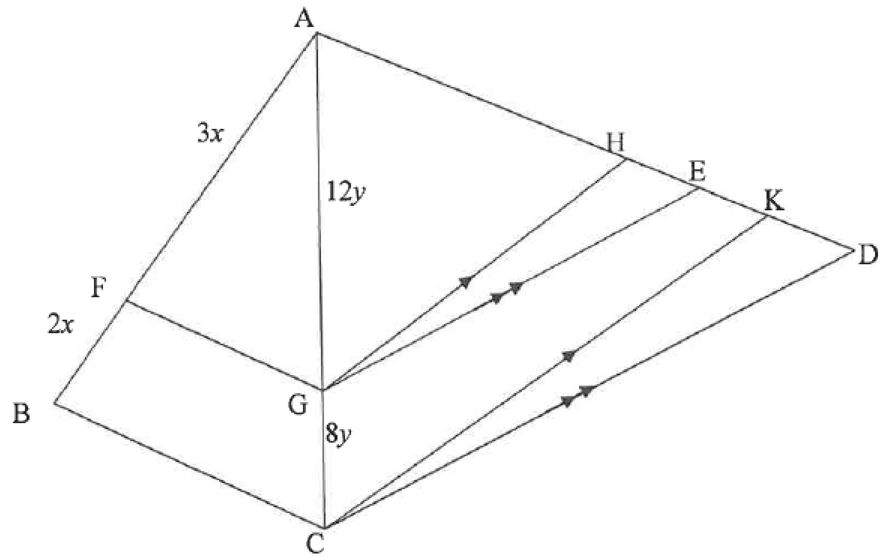
e.

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



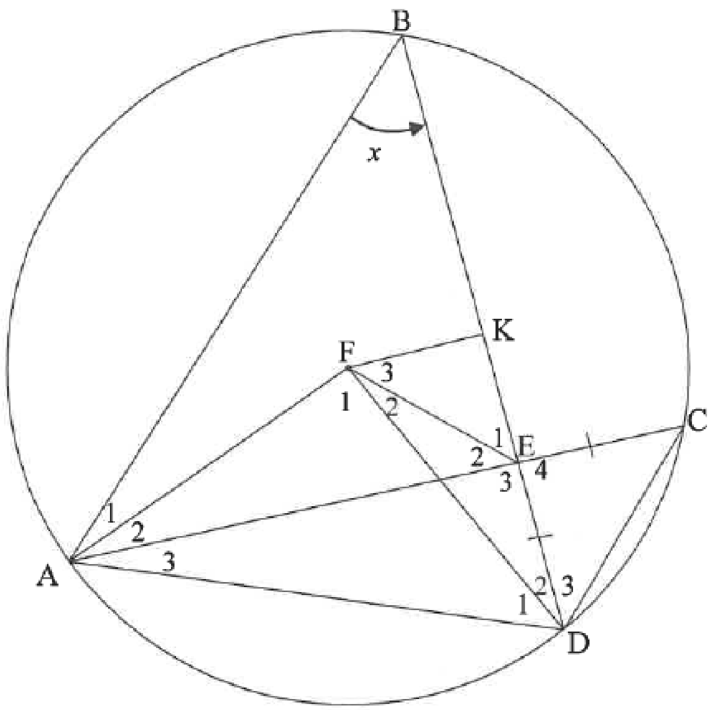
f.

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that $AF = 3x$, $FB = 2x$, $AG = 12y$ and $GC = 8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



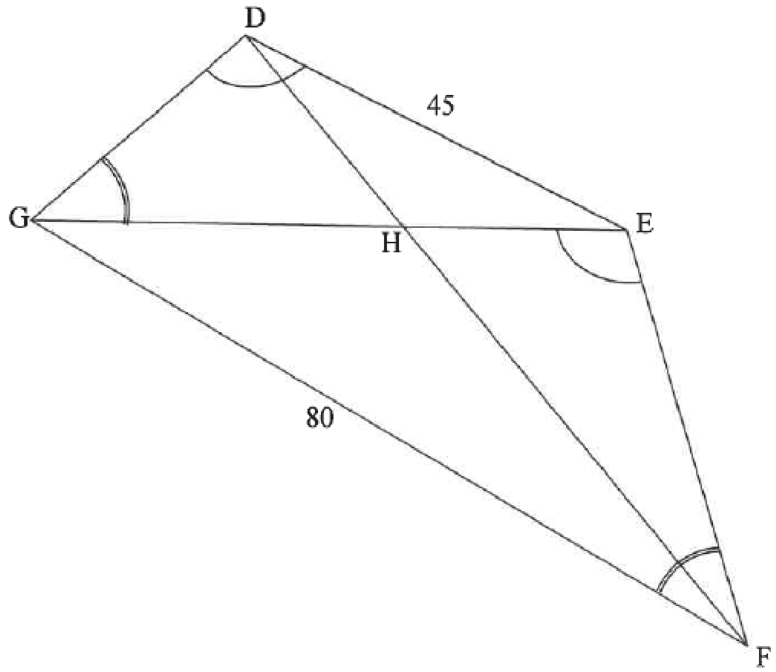
g.

In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



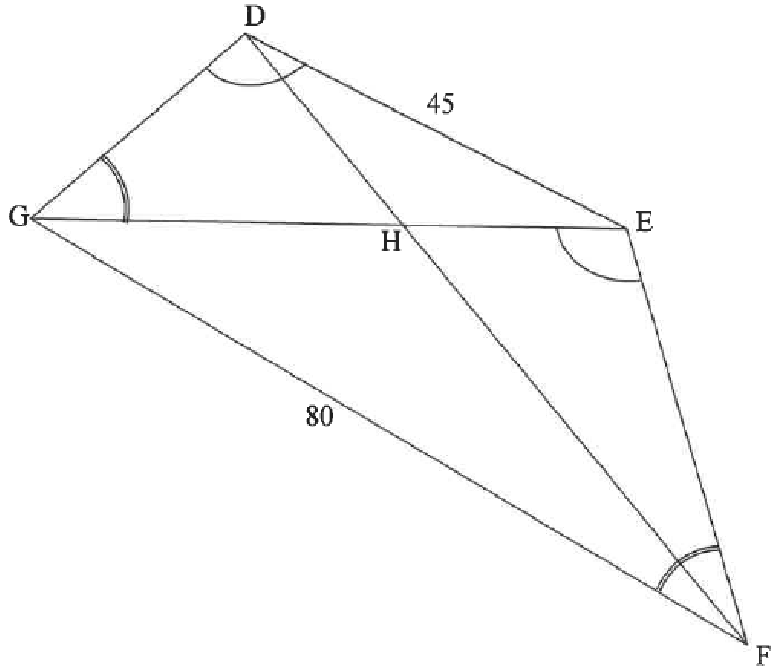
h.

In the diagram, DEFG is a quadrilateral with DE = 45 and GF = 80. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



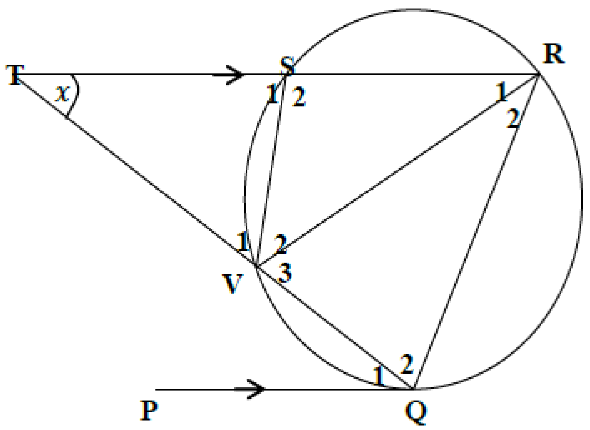
i.

In the diagram, DEFG is a quadrilateral with $DE = 45$ and $GF = 80$. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



IDENTIFYING TRIANGLES

In the diagram below, PQ is a tangent to the circle at Q. TSR is a line which cuts the circle at S such that TR//PQ. QV is produced to meet RST at T. $\hat{T} = x$.



Prove that TS.TR = TV. TQ

- If you are required to prove, any products, while you are not asked to prove any similarity
- Follow the procedure:

Prove that

TS .TR = TV. TQ

KEY:

TS → line of the first triangle

TR →line of the second triangle

TV →line of the first triangle

TQ →line of the second triangle

And thus complete the triangles

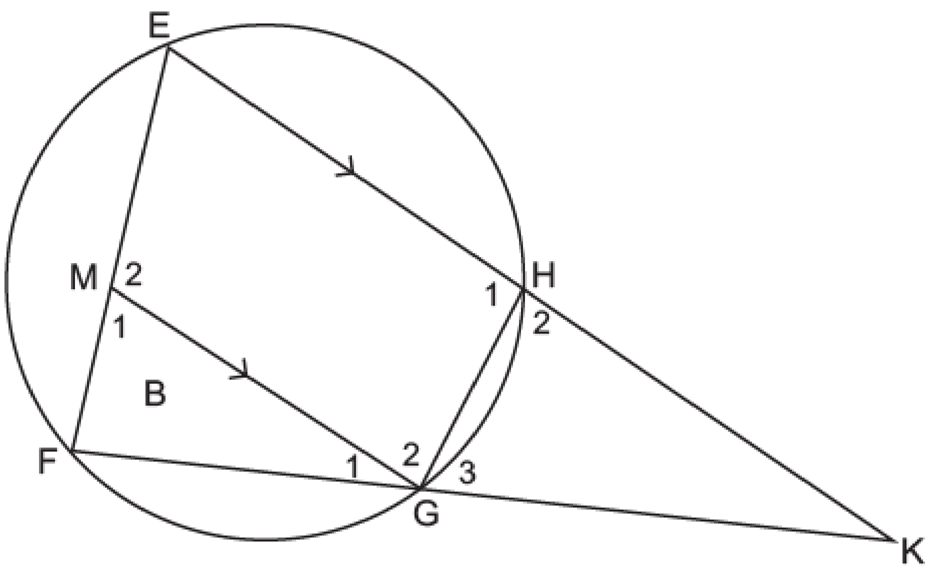
- **WHEN REQUIRED TO PROVE THAT : $AB^2 = CD.AC$**

  - **Note that they are two conditions:**

1. AB either common in both triangles
2. AB either equal to another line in the triangles

KWV 1

In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that MG||EK. Also, KG = EF.



1. Prove that:

a) ΔKGH /// ΔKEF

In ΔKGH and ΔKEF

$\hat{K} = \hat{K}$	common
$\hat{H}_2 = \hat{F}$	ext < cyclic quad
$\therefore \hat{G}_3 = \hat{E}$	int <'s of Δ
$\therefore \Delta KGH /// \Delta KEF$	AAA

b) $EF^2 =$ KE.GH

$\frac{KG}{KE} = \frac{GH}{EF}$	$\Delta KGH /// \Delta KEF$
but $KG = EF$	given
$\therefore \frac{EF}{KE} = \frac{GH}{EF}$	
$\therefore EF^2 = KE.GH$	

➢ EUCLIDEAN GEOMETRY QUESTIONS

QUESTION 1

Complete the following statement:

a. The line drawn from the centre of a circle perpendicular to a chord………………………..……

b. The opposite angles of a cyclic quadrilateral are ……………………………………..…….…

c. The angle between the tangent and a chord is …………………………………………..…..

d. If two triangle are equiangular, the corresponding sides are…………………………………..…

e. The opposite angles of a cyclic quadrilateral are … ………………………………………………

f. The angle subtended by an arc at the centre of the circle is ……………………………………….

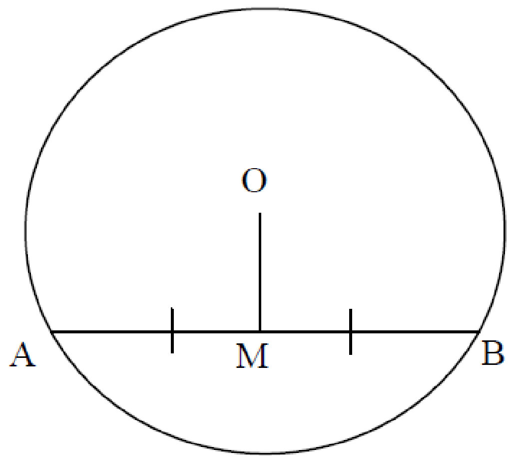
g. The angle between a tangent to a circle and a chord is ……………………………………………

h. The angle subtended by a chord at the centre of a circle is … ……………………………………

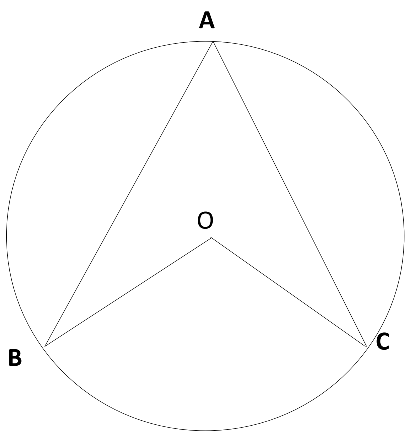
i. If two triangles are equiangular, then the corresponding sides are …………………………….

QUESTION 2

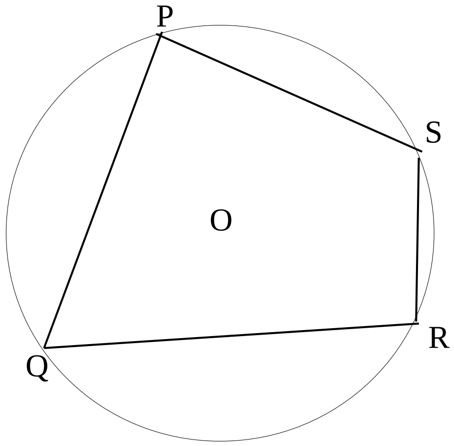
(a) In the diagram below, AB is a chord of the circle with centre O. M is the midpoint of AB. Prove the theorem that states $OM \perp AB$.



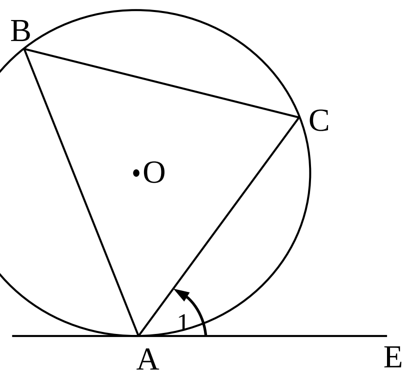
(b) In the figure Below o is the centre of the circle and A,B and C are three points on the circumference of the circle. Use the figure and prove the theorem that states that $B\hat{O}C = 2\hat{A}$



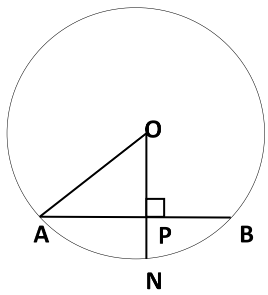
(c) In the diagram below, O is the centre of the circle. Use the diagram to prove the theorem which states that: If PQRS is a cyclic quadrilateral then $P\hat{Q}R + P\hat{S}R = 180^{\circ}$.



(d) In the diagram below, O is the centre of the circle passing through A, B and C. EA is a tangent to the circle at A. Use this diagram to prove the theorem which states that $E\hat{A}C = A\hat{B}C$.



QUESTION 3



O is the centre of the circle. AB is a chord and

OP is extended and intersects the circle at N.

AB = 16 cm and PN = 2 cm.

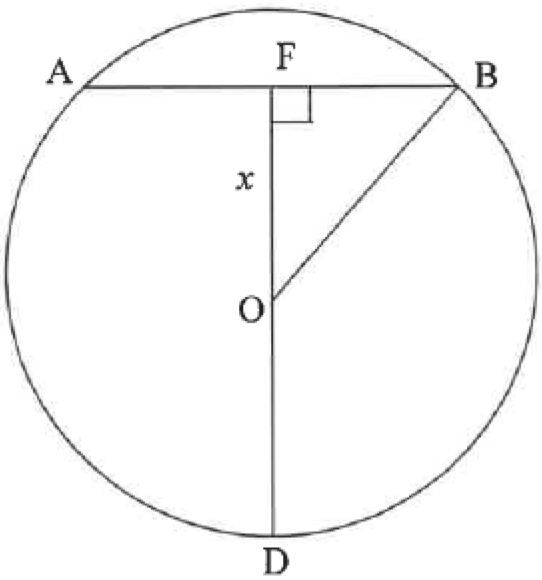
a. Calculate the length of AP.

b. Calculate the length of the radius of the circle.

c. Hence calculate the length of ON

QUESTION 4

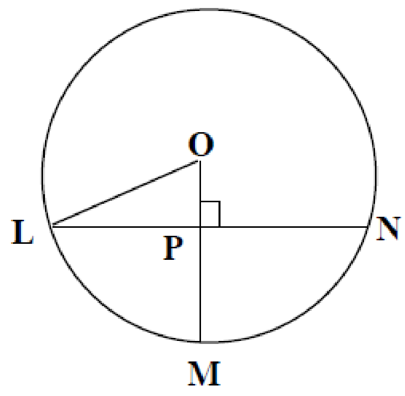
In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$. AB = FD = 8 cm and OF = x cm.



Determine the length of the radius of the circle.

QUESTION 5

In the accompanying diagram alongside, LN is a chord of the circle with centre O. OP is drawn perpendicularly onto LN and meets the circle in M.

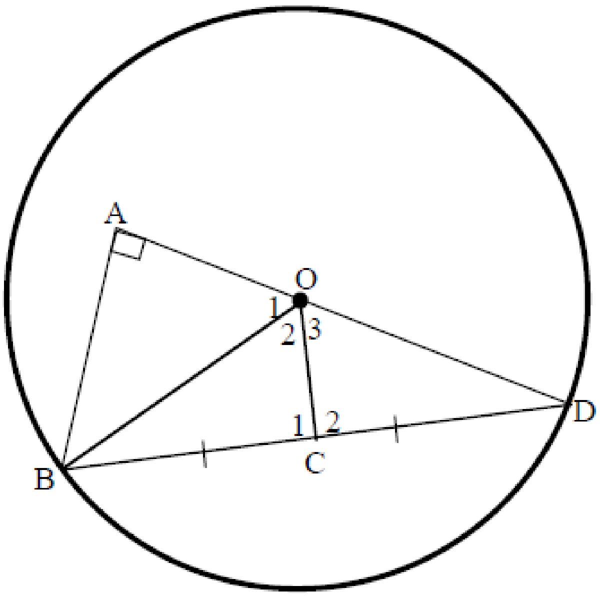


(a) Prove that $MN^2 = 2OM.PM$

(b) If OL = 9 units and OP = 1 unit, calculate the length of MN

QUESTION 6

In the diagram below, O is the centre of the circle. C is the midpoint of chord BD. Point A lies within the circle such that $BA \perp AOD$.



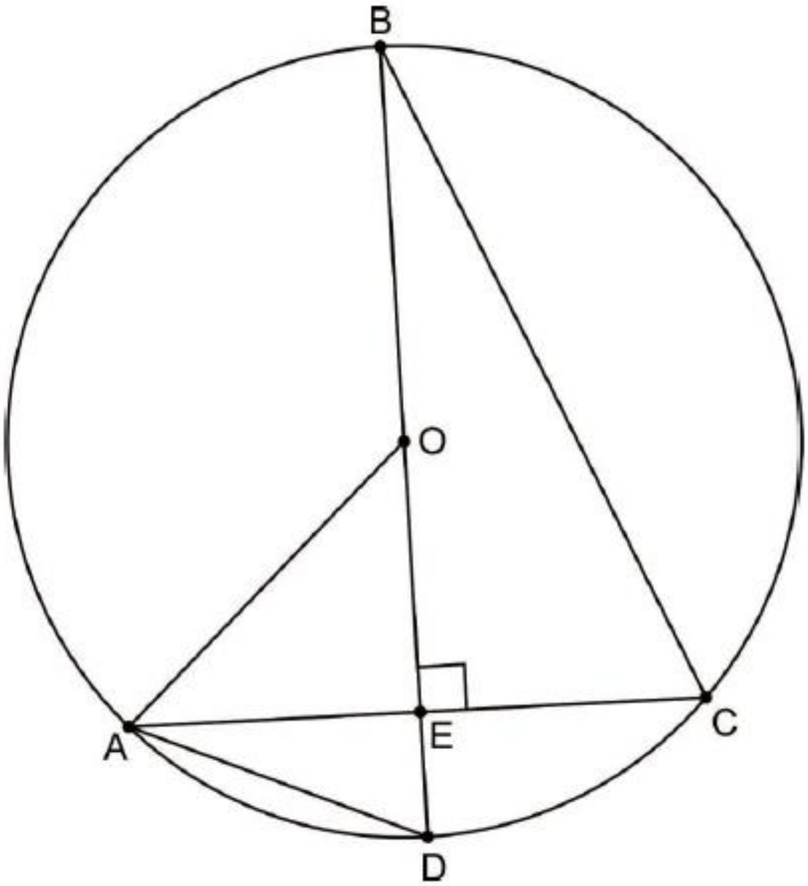
a. Show that $DA.OD = OD^2 + OD.OA$

b. Prove that $2DC = OD^2 + OD.OA$

QUESTION 7

(a) In the diagram below, a circle with centre O is drawn.

- $OD \perp AC$ and OD and AC intersect at E.
- A, B, C and D lie on the circumference of the circle.

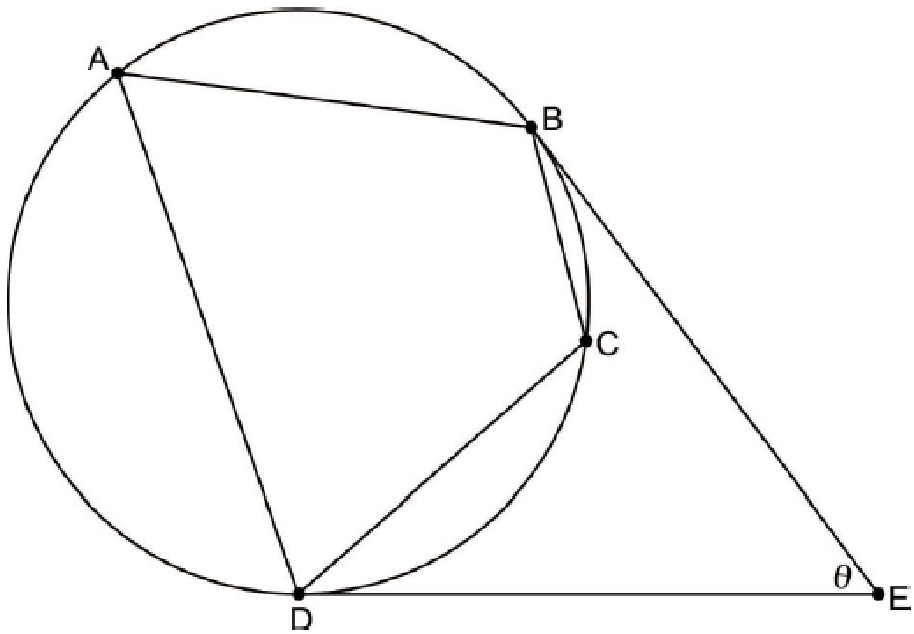


(1) Determine the length of BE in terms of AO and ED.

(2) Prove that $(2AO - ED)^2 = BC^2 - AE^2$.

(b) In the diagram below, a circle is drawn passing through A, B, C and D.

- $B\hat{E}D = \theta$.
- BE and ED are tangents at B and D respectively.



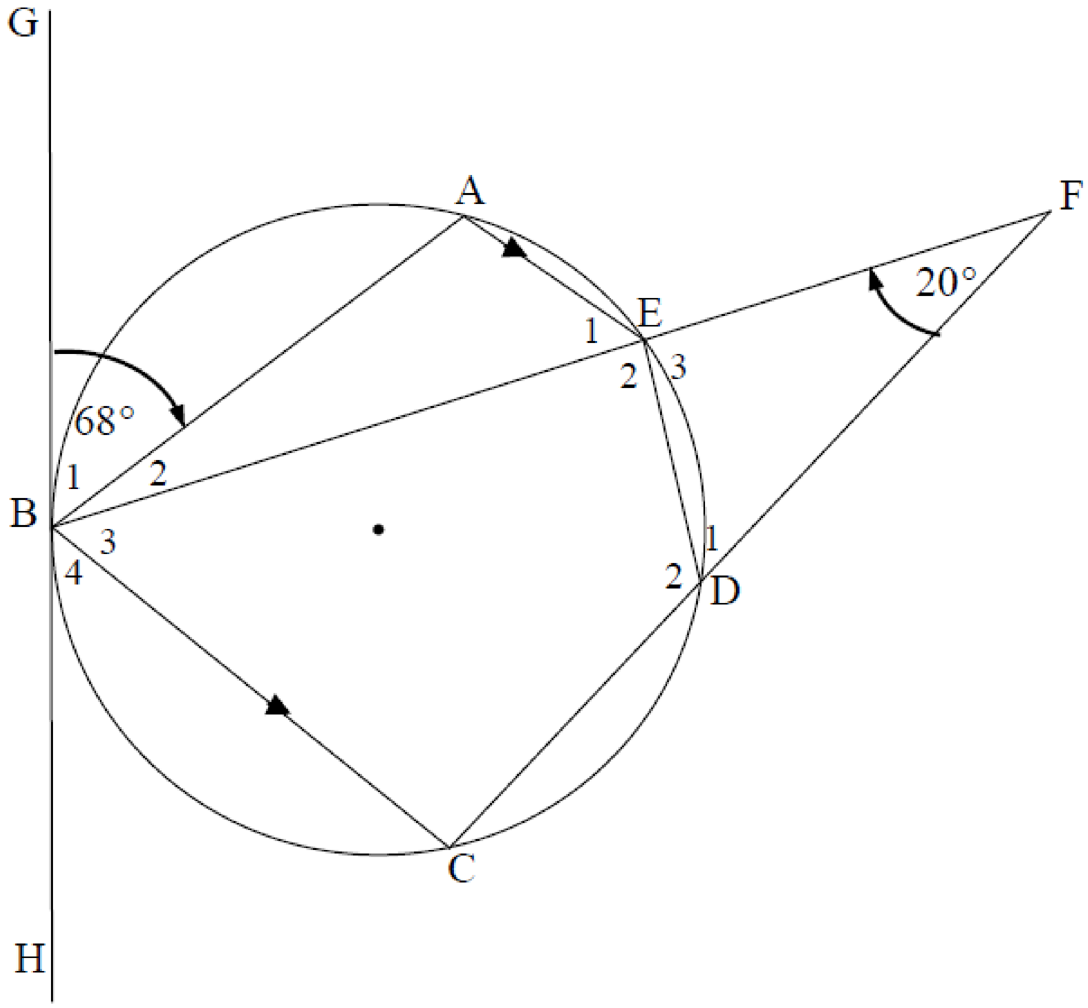
Prove that $B\hat{C}D = 90° + \frac{\theta}{2}$.

QUESTION 8

In the diagram, A, B, C, D and E are points on the circumference of the circle such that

AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B.

$\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

c. $\angle E_1$

d. $\angle B_3$

e. $\angle D_1$

f. $\angle E_2$

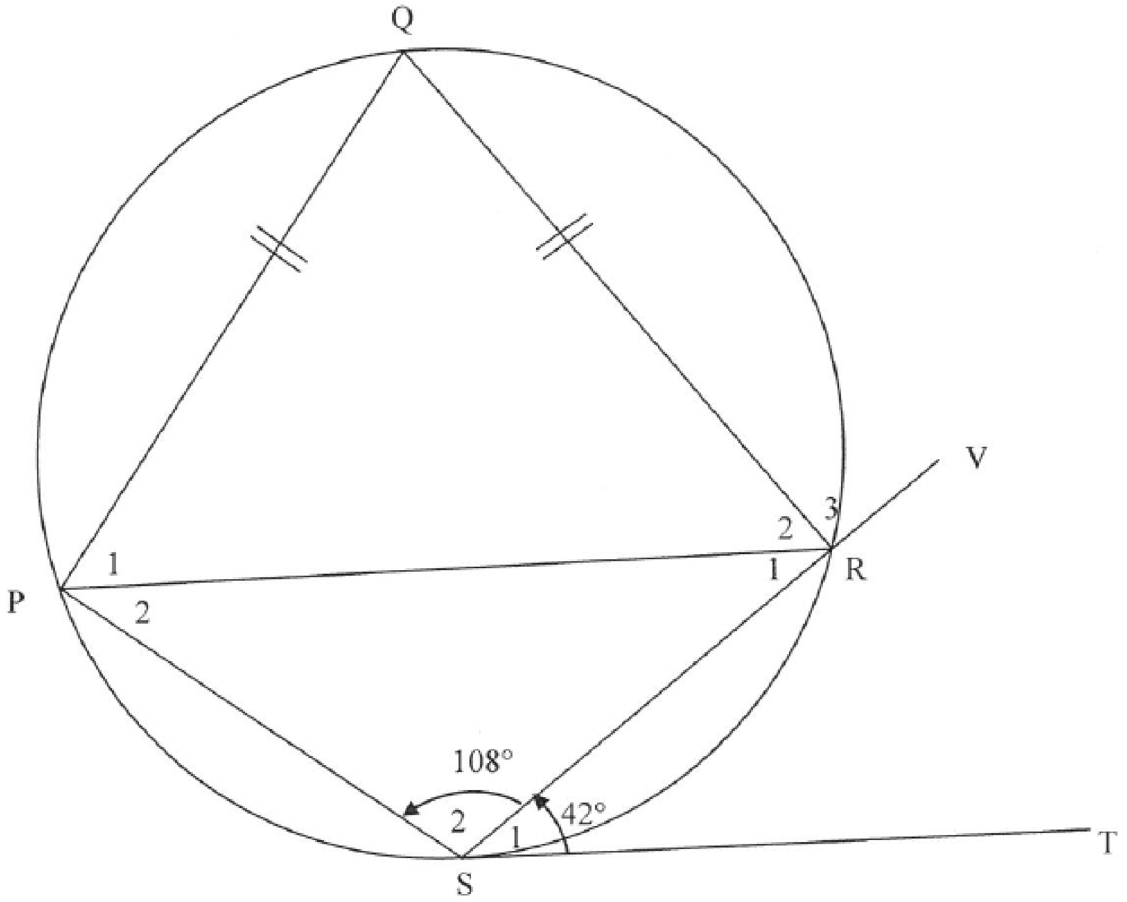
g. $\angle E_3$

h. $\angle C$

i. $\angle D_2$

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

a. $\angle Q$

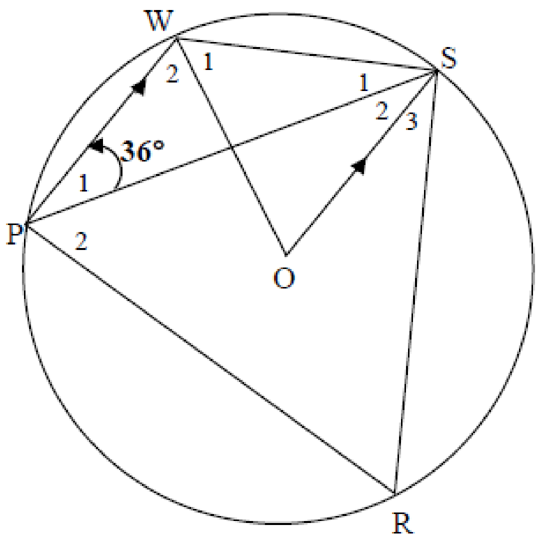
b. $\angle R_2$

c. $\angle P_2$

d. $\angle R_3$

QUESTION 10

In the diagram, O is the centre of the circle. PWSR is a cyclic quadrilateral. PS, WO and OS are drawn. PW || OS and $\hat{P}_1 = 36°$.



Calculate the sizes of the following angles:

a. $\angle$ SOW

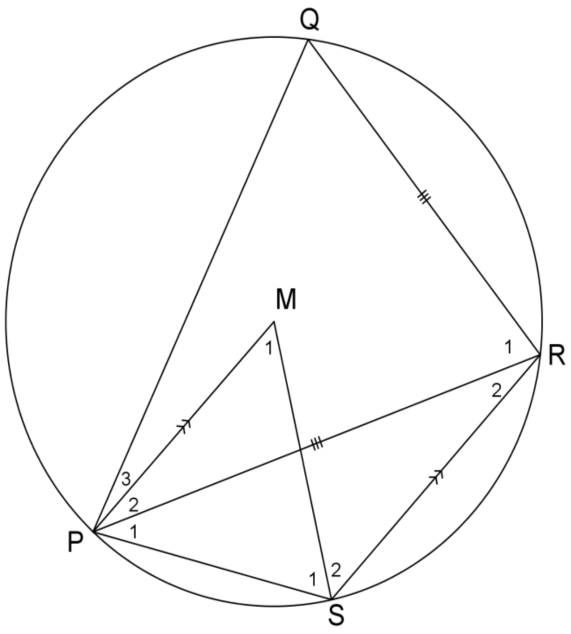
b. $\angle W_2$

c. $\angle$ OSW

d. $\angle$ R

QUESTION 11

In the diagram alongside, M is the centre of circle PQRS. PM $\parallel$ RS , QR = PR and $\hat{R}_2 = 28^0$



Determine, giving reasons, the size of the following angles:

a. $\hat{S}_2$

b. $P\hat{S}R$

c. $\hat{Q}$

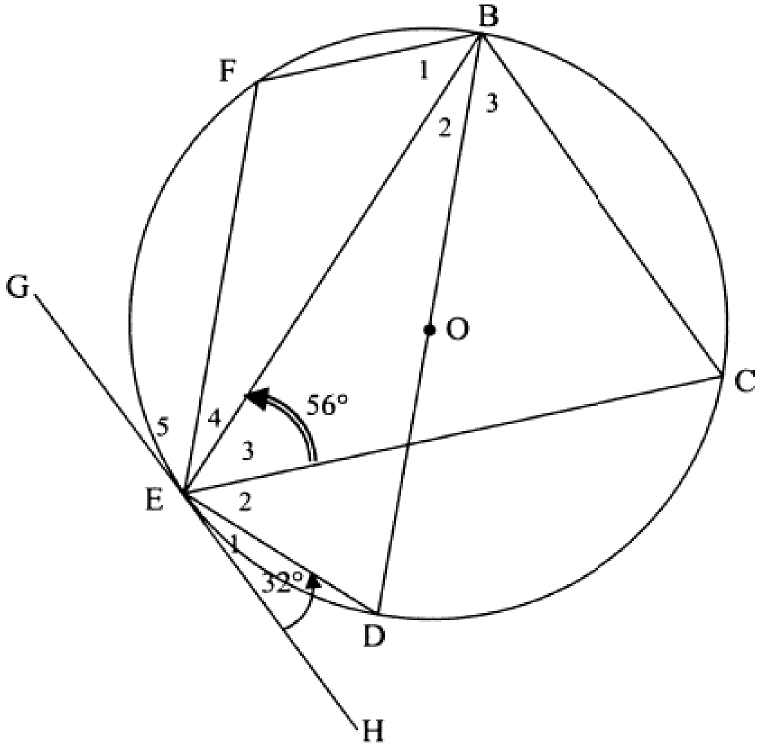
d. $\hat{P}_3$

e. $\angle R_1$

QUESTION 12

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



Calculate, with reasons, the values of:

a. $\angle E_2$

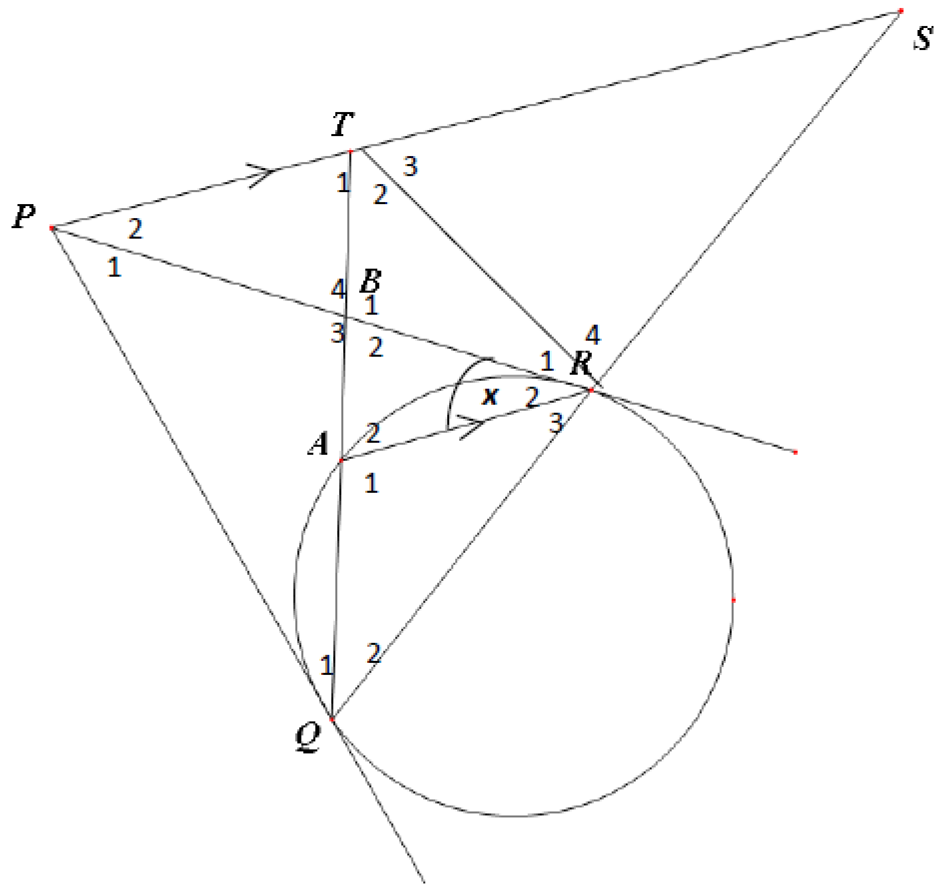
b. $\angle EBC$

c. $\angle F$

QUESTION 13

In the figure PR and PQ are two tangents drawn from point P to circle AQR. The straight line drawn through P parallel to AR meets QR produced at S, and QA produced at T.

The tangent PR cuts QT at B.



Let $\hat{R}_2 = x$

a. Prove that PTRQ is a cyclic quadrilateral.

b. If it is further given that QA = RA, prove that:

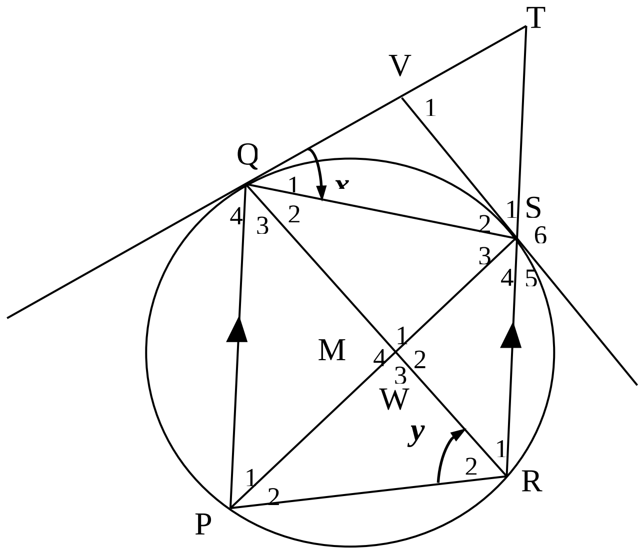
i) $\hat{S} = x$

ii) PQ = RS

iii) PTS is a tangent to circle TAR.

QUESTION 14

In the diagram below, PQ and RS are chords of the circle such that PQ || RS. The tangent to the circle at Q meets RS produced at T and the tangent a meets QT at V. PS and QR intersect at W. QS and PR are drawn. Let $\hat{Q}_1 = x$ and $\hat{R}_2 = y$.



a) Write down a reason why QV = VS.

b) Write down the following angles in terms of x:

(i) $\hat{S}_2$

(ii) $\hat{R}_1$

(iii) $\hat{V}_1$

c) Show that $\hat{R}_1 = \hat{S}_4$

d) Prove that QVSW is a cyclic quadrilateral.

e) Write down the following angles in terms of y:

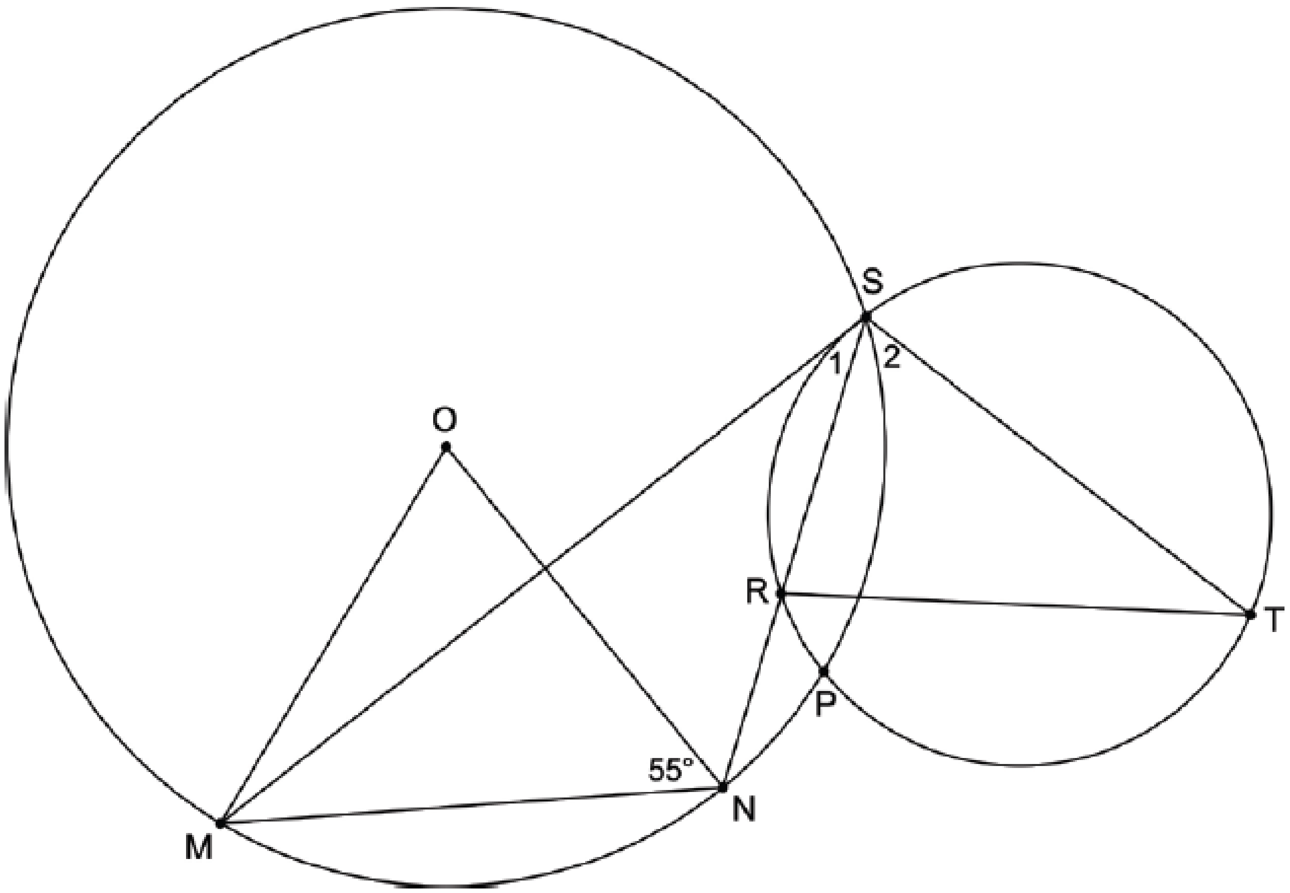
(i) $\hat{Q}_4$

(ii) $\hat{T}$

QUESTION 15

In the diagram below, two circles intersect at S and P:

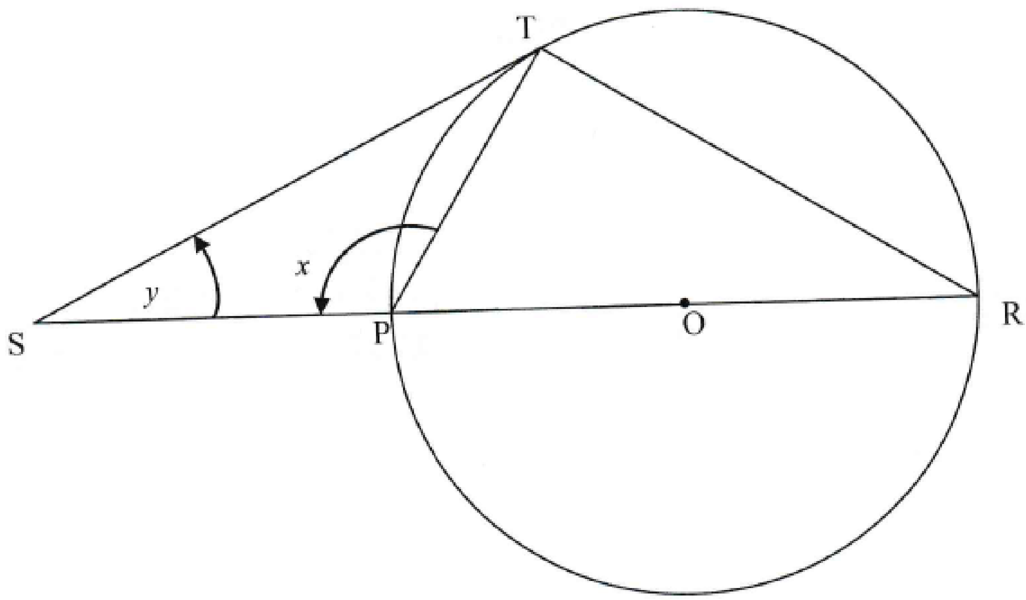
- O is the centre of the large circle.
- MS is a tangent to the smaller circle at point S.
- $M\hat{N}O = 55°$.
- M and N are points on the larger circle.
- R and T are points on the smaller circle.
- SRN is a straight line.



Determine the size of angle $S\hat{T}R$.

QUESTION 16

In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.

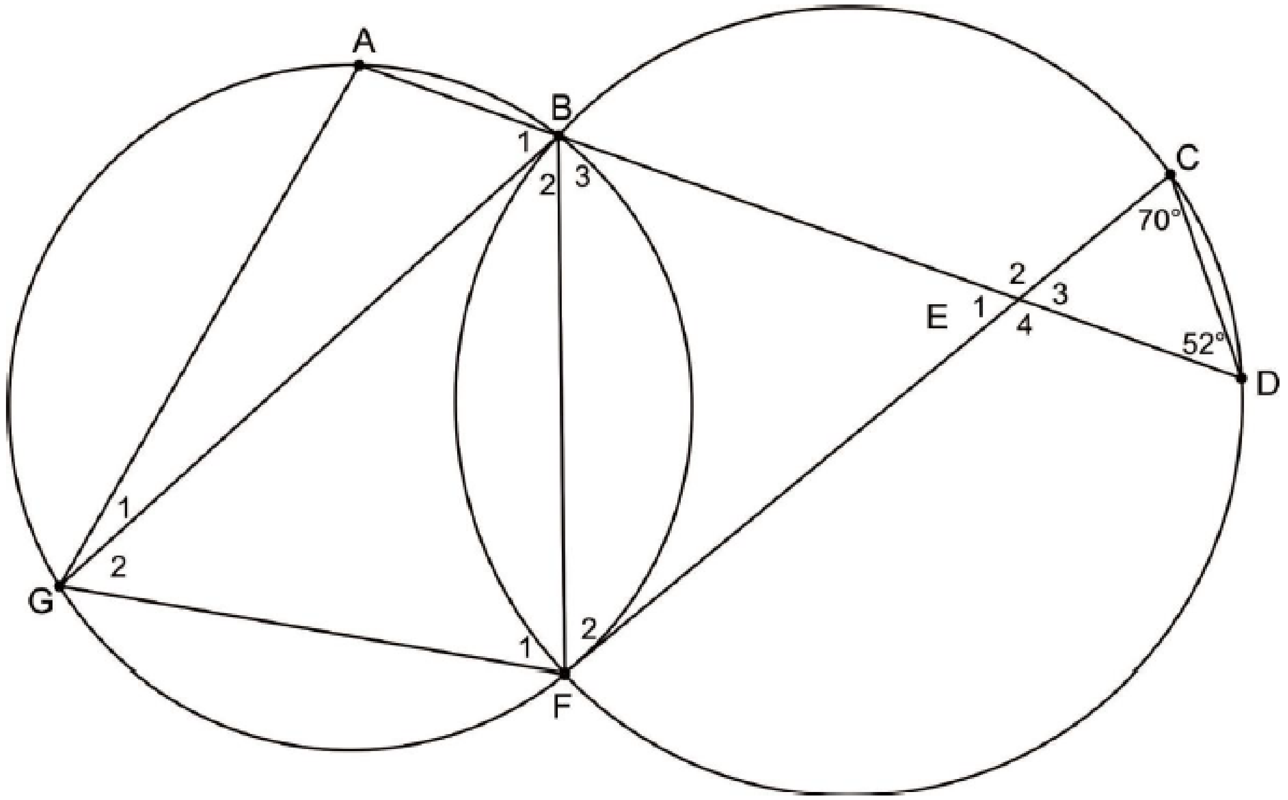


Determine, with reasons, y in terms of x.

QUESTION 17

In the diagram below, two circles are drawn intersecting at B and F.

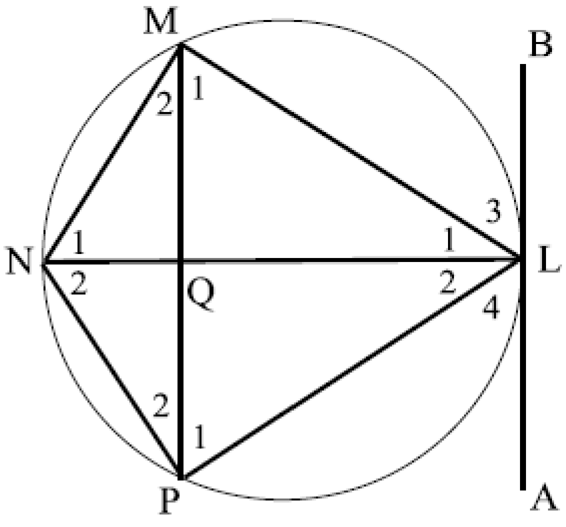
- CF is a tangent to the smaller circle at F.
- A and G are points on the circumference of the smaller circle.
- Chords FC and BD of the larger circle intersect at E.
- ABD is a straight line.
- $\hat{C} = 70°$ and $\hat{D} = 52°$.



Determine the size of $\hat{G}_1$.

QUESTION 18

ALB is a tangent to circle LMNP. ALB || MP.



Prove that:

a. LM = LP

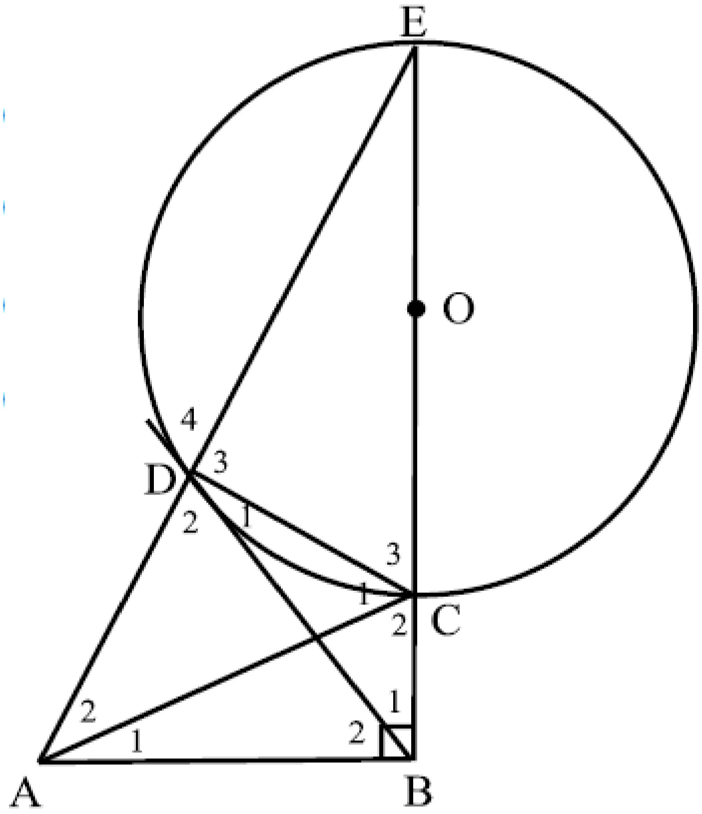
b. LN bisects angle MNP

c. LM is a tangent to circle MNQ

QUESTION 19

EC is a diameter of circle DEC. EC is produced to B. BD is a tangent at D. ED is produced to A and

AB⊥ BE .



Prove that:

a. ABCD is a cyclic quadrilateral.

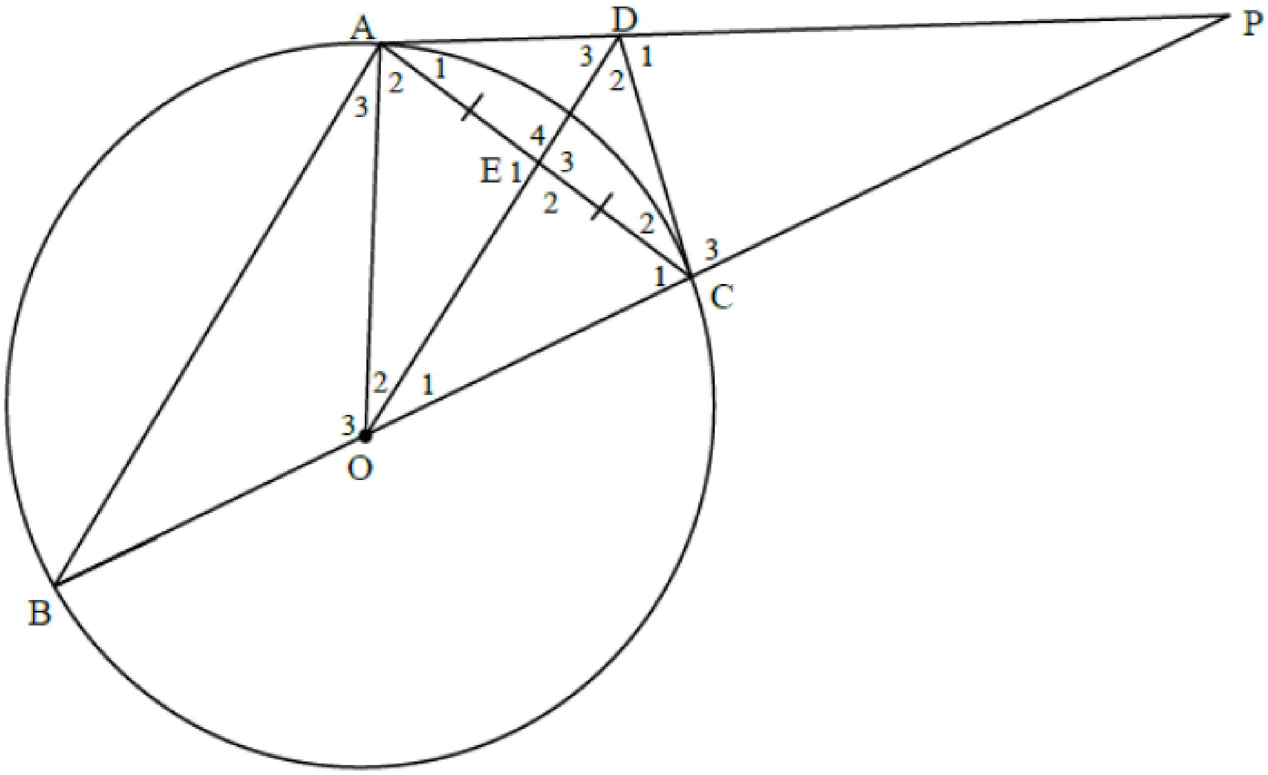
b. $\widehat{A_1} = \hat{E}$

c. BD = BA

d. $\widehat{C_2} = \widehat{C_3}$

QUESTION 20

In the diagram below, BOC is a diameter of the circle. AP is a tangent to the circle at A and AE = EC.



Prove that:

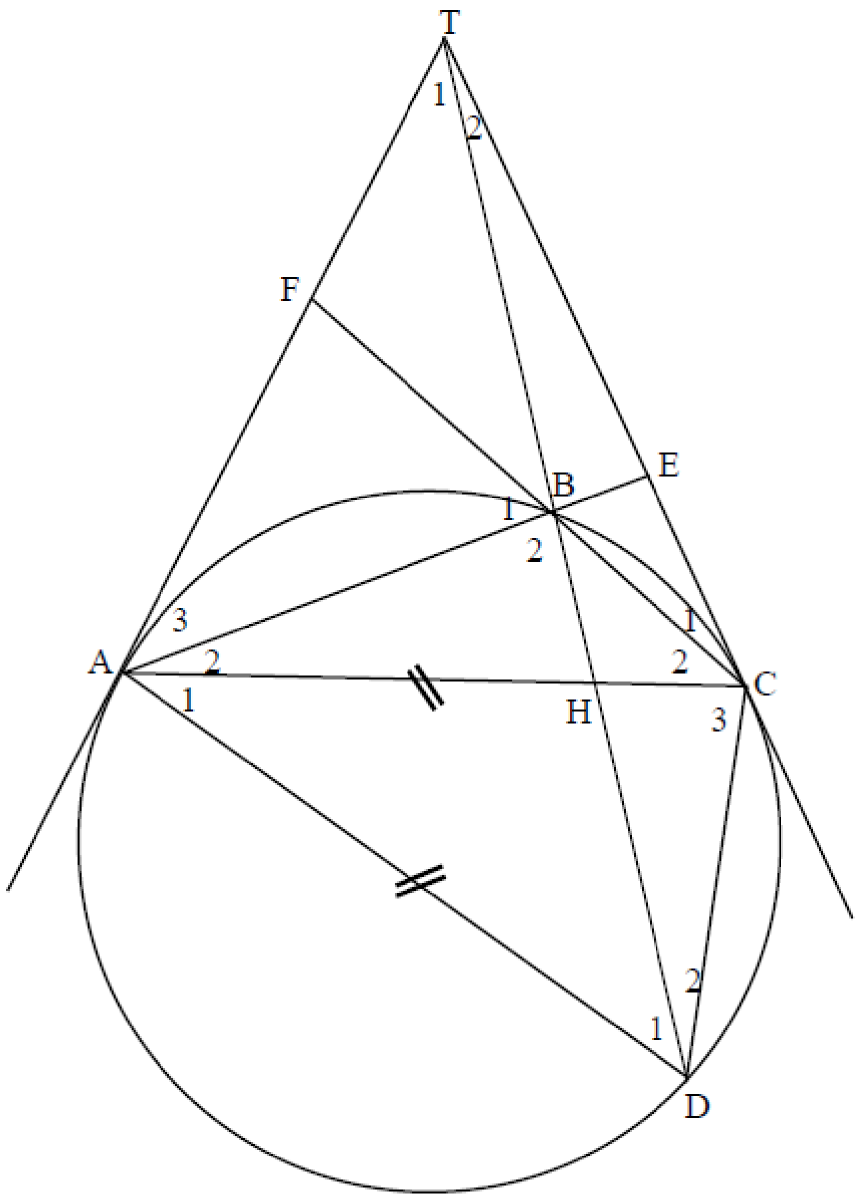
a. BA | | OD

b. AOCD is a cyclic quadrilateral

c. DC is a tangent to the circle at C

QUESTION 21

In the diagram below, ABCD is a cyclic quadrilateral with AC = AD. Tangents AT and CT touch the circle at A and C respectively. FBC, ABE, AHC and DBT are straight lines.



Prove:

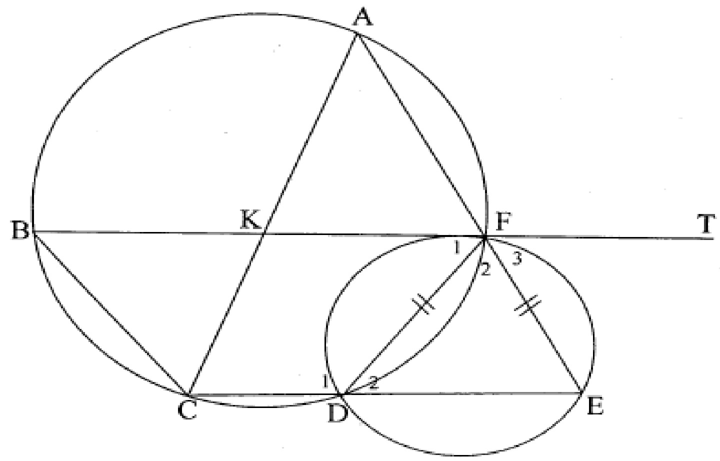
a. $\angle B_1 = \angle B_2$

b. BECH is a cyclicquadrilateral.

c. CA is a tangent to the circle passing through points A, B and T.

QUESTION 22

In the figure below, two circles cut in points F and D. BFT is a tangent to the smaller circle at F. Straight line AFE is drawn so that FE = FD. CDE is a straight line and chords AC and BF cut at K.



Prove that:

a. BT || CE.

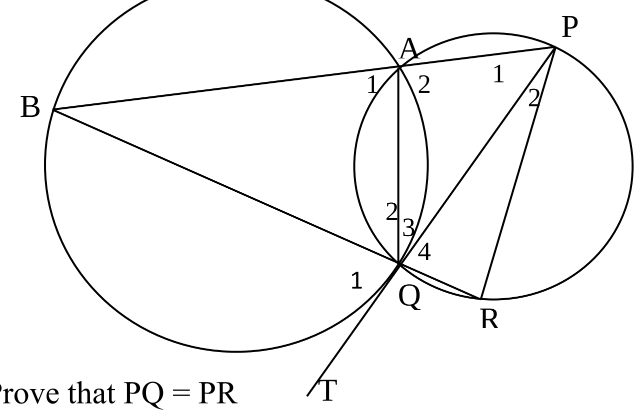
b. BCEF is a parallelogram.

c. AC = BF.

d. BF is a diameter if it is given that AF = FE.

QUESTION 23

In the diagram below, PQT is a tangent to the larger circle ABQ at Q. A smaller circle intersects the larger circle at A and Q. BAP and BQR are straight lines with P and R on the smaller circle. AQ and PR are drawn.



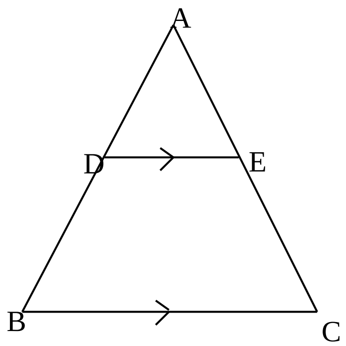
a) Prove that PQ = PR

b) Prove that Δ PBQ ||| Δ PQA.

c) Prove that the lengths of PA, PR and PB (in this order) form a geometric sequence.

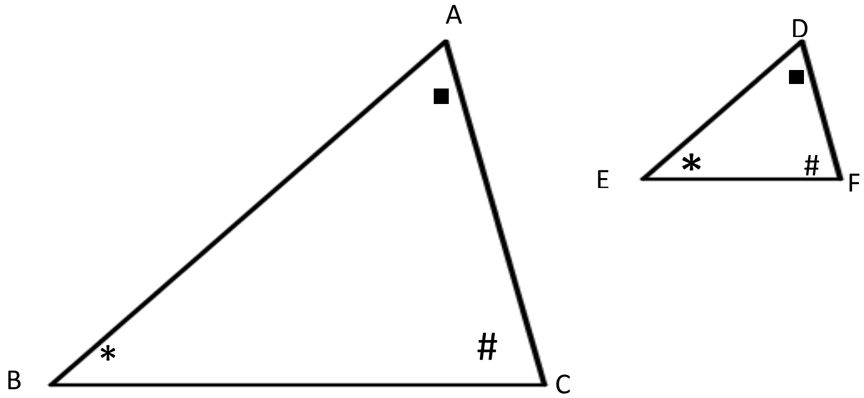
QUESTION 24

a. Given Δ ABC with $DE \parallel BC$as shown in the figure below:



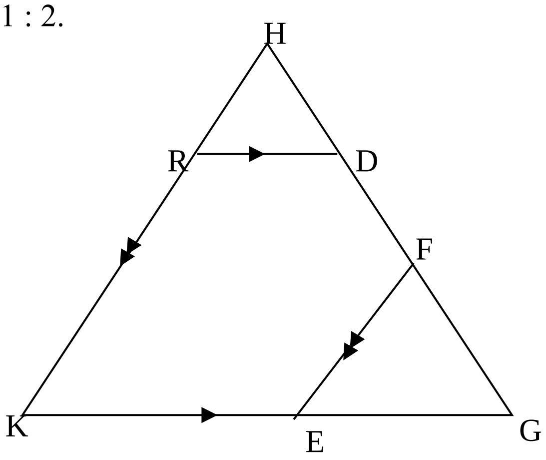
Prove that: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

b. In the diagram below, ABC and DEF are given with. Use the diagram in the ANSWER BOOK to prove the theorem that states that $\dfrac{DE}{AB} = \dfrac{DF}{AC}$.



QUESTION 25

In the diagram below, ΔGHK is drawn having the point R on KH and the points D and F on GH such that RD || KG and EF || KH. It is also given that RH = 3 units, RK = 9 units, HD = 2 units and GE : EK = 1 : 2.



Calculate the length / ratio of:

a. $\frac{GF}{HF}$

b. $\frac{GF}{GH}$

c. $\frac{GH}{HF}$

d. $\frac{HD}{GD}$

e. $\frac{HG}{DG}$

f. DG

g. FD

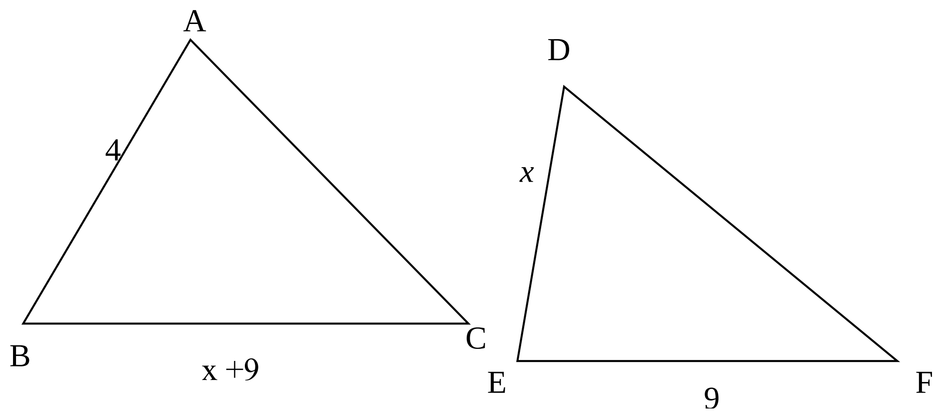
h. Calculate the value of $\frac{area\ of \Delta KHG}{area\ of \Delta EFG}$

i. Calculate the value of $\frac{area\ of \Delta RHD}{area\ of \Delta KHG}$

j. Calculate the ratio of $area\ of\ \Delta RHD : area\ of\ \Delta KHG$

QUESTION 26

In the diagram below, ΔABC and ΔDEF are drawn. AB = 4 units, BC = (x + 9) units, DE = x units and EF = 9 units.

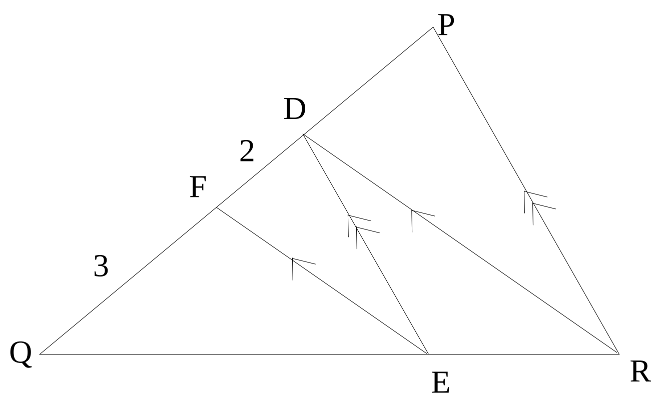


a) If $\Delta ABC \mid\mid\mid \Delta DEF$, calculate the value of x.

b) Hence, write down the length of BC

QUESTION 27

In the diagram below, DE || PR, FE || DR, QF = 3 cm, FD = 2 cm.



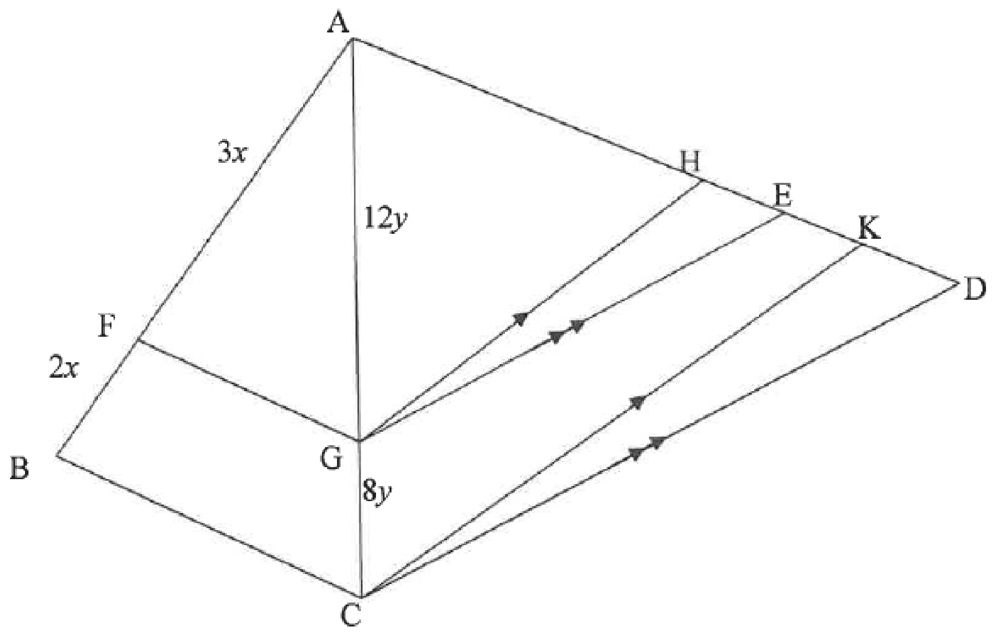
a. Determine the value of $\frac{QE}{QR}$.

b. Calculate the length of DP.

c. Determine the value of $\frac{QE}{ER}$

QUESTION 28

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



1. Prove that:

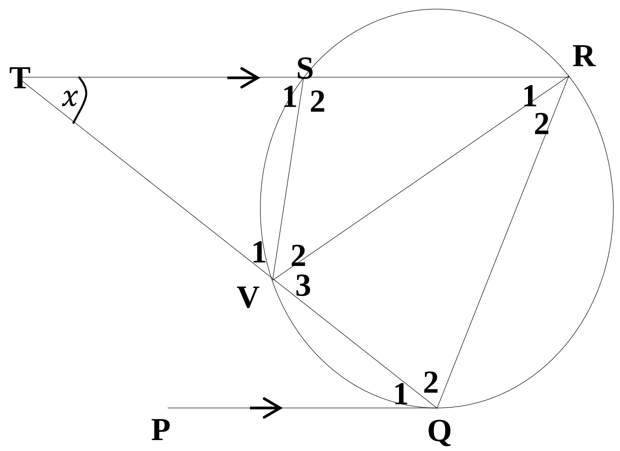
   (a) FG | | BC

   (b) $\frac{\text{AH}}{\text{HK}} = \frac{\text{AE}}{\text{ED}}$

2. If it is future given that AH= 15 and ED= 12, Calculate the length of EK.

QUESTION 29

In the diagram below, PQ is a tangent to the circle at Q. TSR is a line which cuts the circle at S such that TR//PQ. QV is produced to meet RST at T. $\hat{T} = x$.



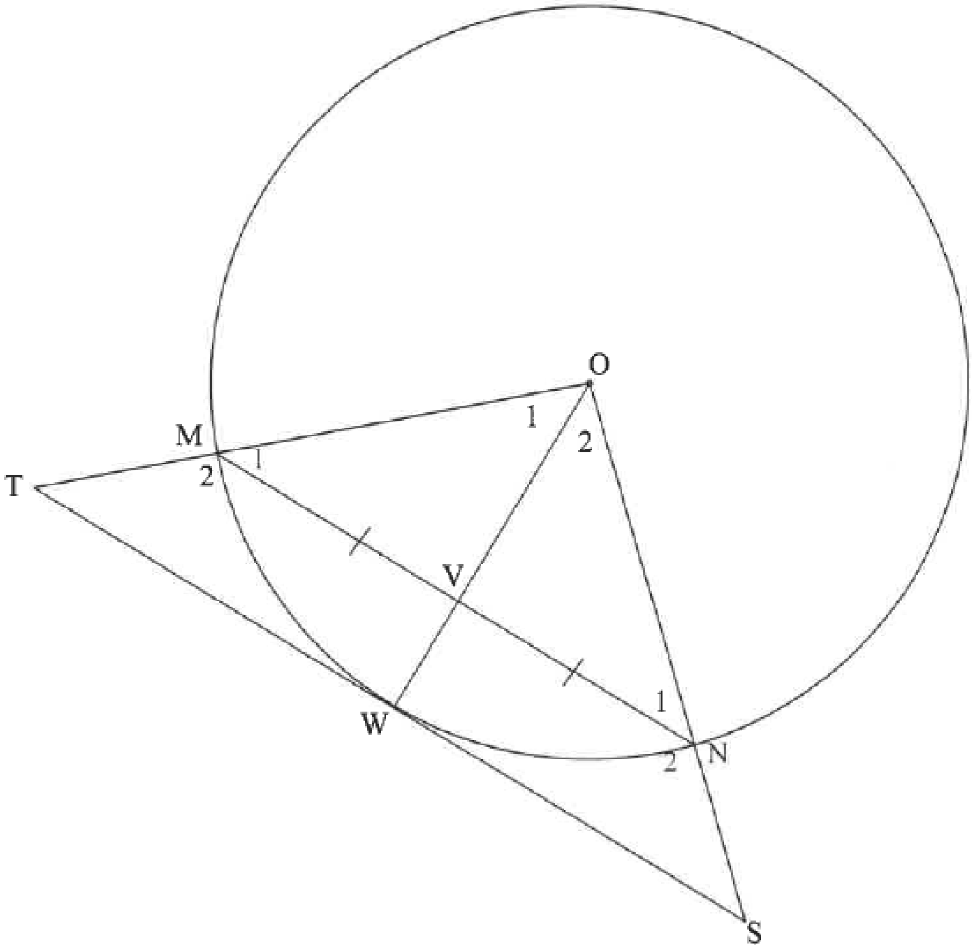
a) Write, down with reasons, TWO other angles each equal to x .

b) Prove that TSV /// RQV.

c) Prove that TS.TR = TV. TQ

QUESTION 30

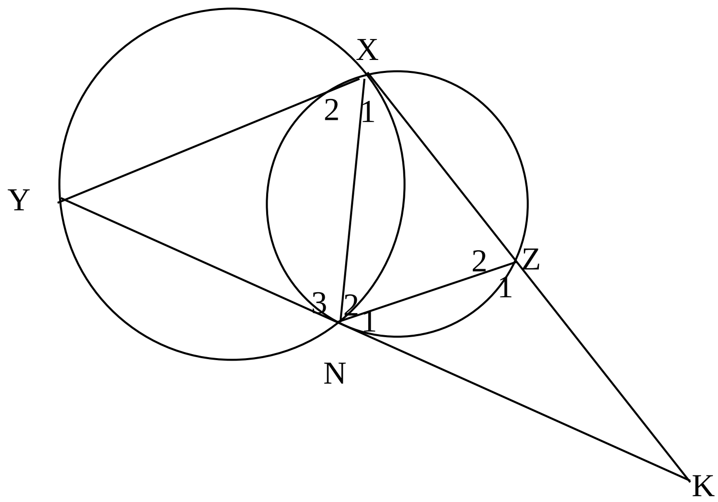
In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



1. Give a reason why OV ⊥ MN.
2. Prove that:
   (a) MN | | TS
   (b) TMNS is a cyclic quadrilateral
   (c) OS . MN = 2ON . WS

QUESTION 31

Two circles intersect at X and N. KX is a tangent to the larger circle XYN at X and KX cuts the smaller circle at Z. KNY is a tangent to the circle XZN at N. XN, XY and NZ are drawn.



a) Prove that YX//NZ.

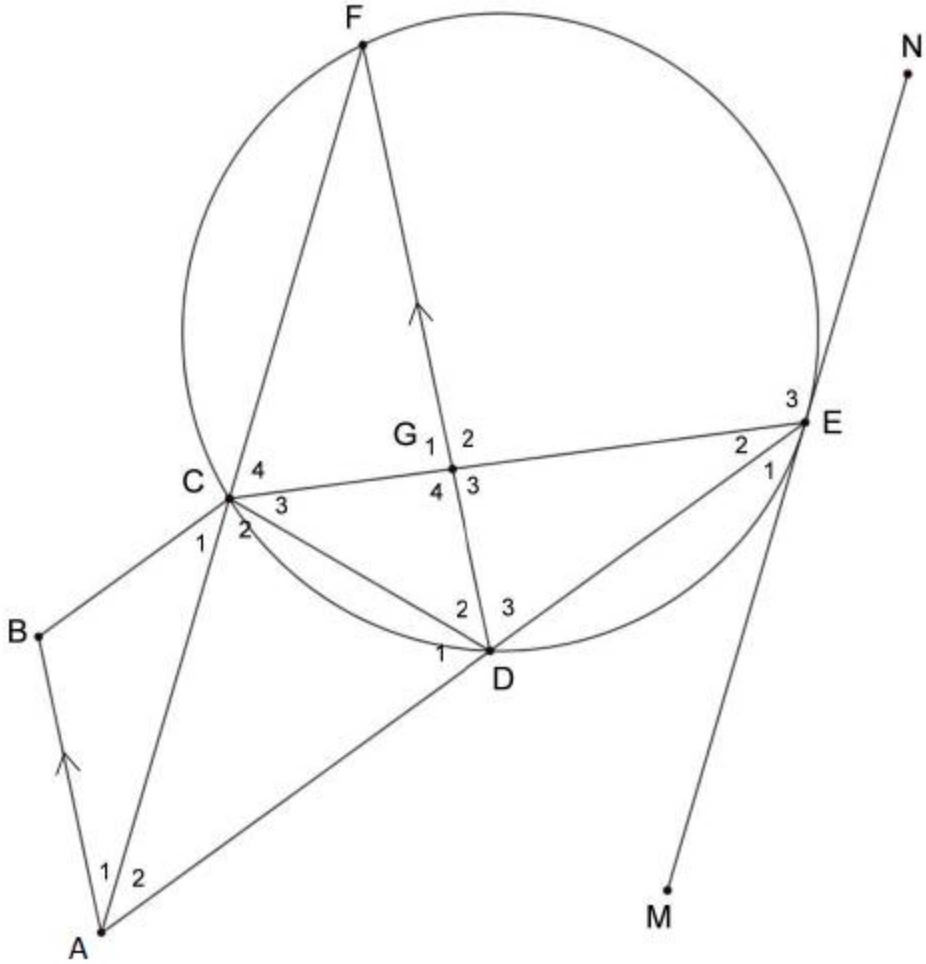
b) Write down two triangles similar to ΔXYK.

c) Prove that $ZK = \dfrac{XK^3}{KY^2}$.

QUESTION 32

In the diagram below:

- C, D, E and F are points on a circle.
- Line MN is a tangent to the circle at E.
- Line AF intersects the circle at C.
- AB//DF.
- $\hat{C}_1 = \hat{C}_3$.



(a) Prove that $\triangle CBA \,|||\, \triangle CDE$.

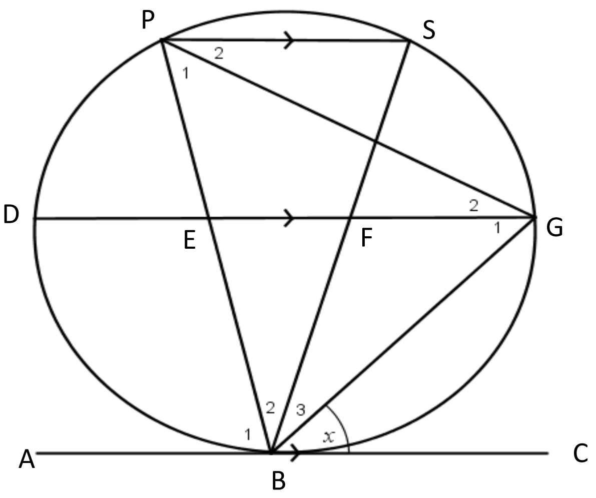
(b) Prove that ABCD is a cyclic quad.

(c) Prove that $\hat{E}_3 = \hat{A}_2 + \hat{C}_2$.

QUESTION 33

In the diagram, P, S, G, B and D are points on the circumference of the circle such that

PS || DG || AC. ABC is a tangent to the circle at B.



a. Give a reason why $\widehat{G_1} = x$

b. Prove that:

i. $BE = \frac{BP.BF}{BS}$

ii. ∆BGP ||| ∆BEG

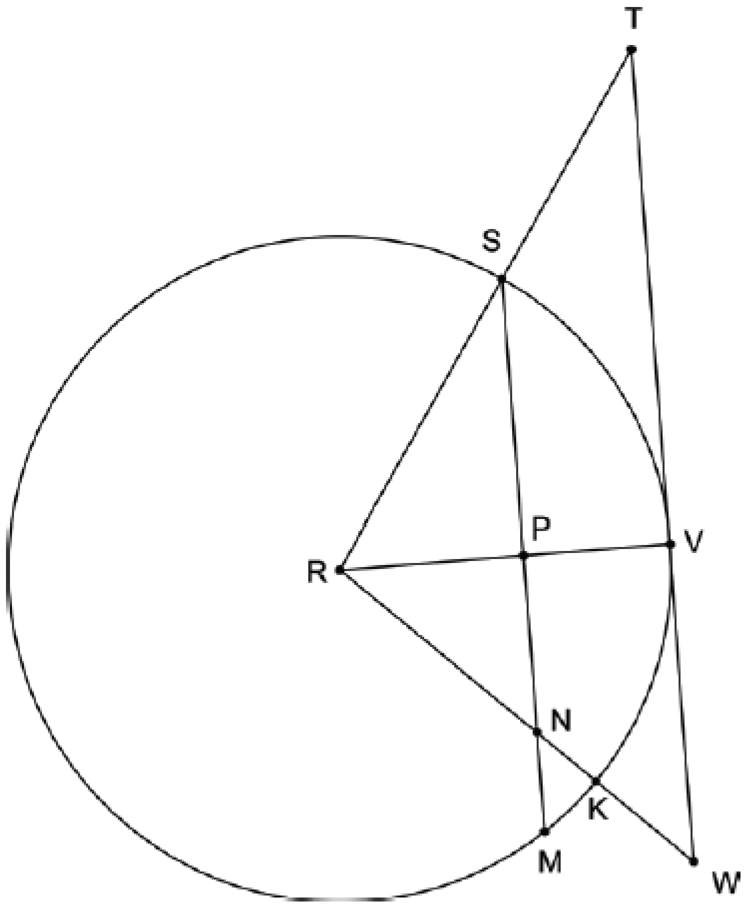
iii $BG^2 = BE.BP$

iv. $\frac{BG^2}{BP^2} = \frac{BF}{BS}$

QUESTION 34

In the diagram below:

- TW is a tangent to circle centre R at point V.
- Radius RV intersects chord SM at P and MP = PS.
- The circle has a radius of 10 units.
- RST and RKW are straight lines.
- RW intersects the circle at K and chord SM at N.
- ST = 7 and NW = 6 units.



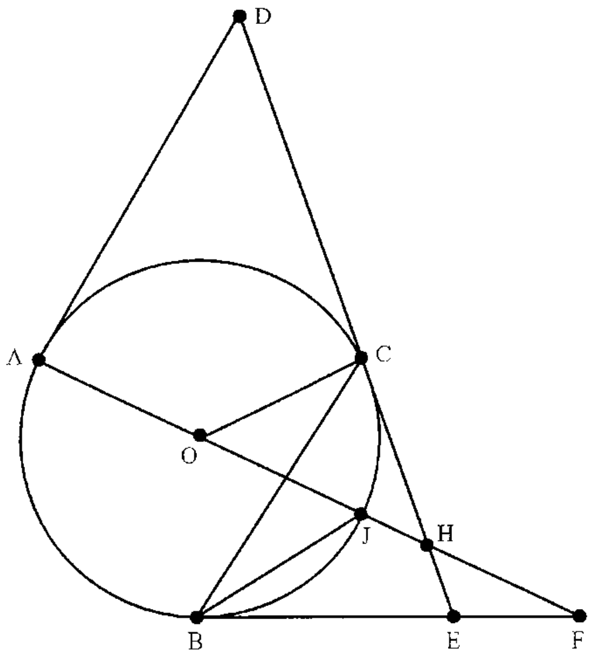
(a) Prove that TW//SN.

(b) Determine the length of NK.

(c) Calculate the length of PN.

QUESTION 35

In the figure, AD, DC and BE are tangents to the circle. CO is a radius and chord BC is drawn. Radius AO is drawn and extended to cut the circle at J and BE is extended at F.



a. Prove $\Delta DAH \parallel\!\mid \Delta OCH$

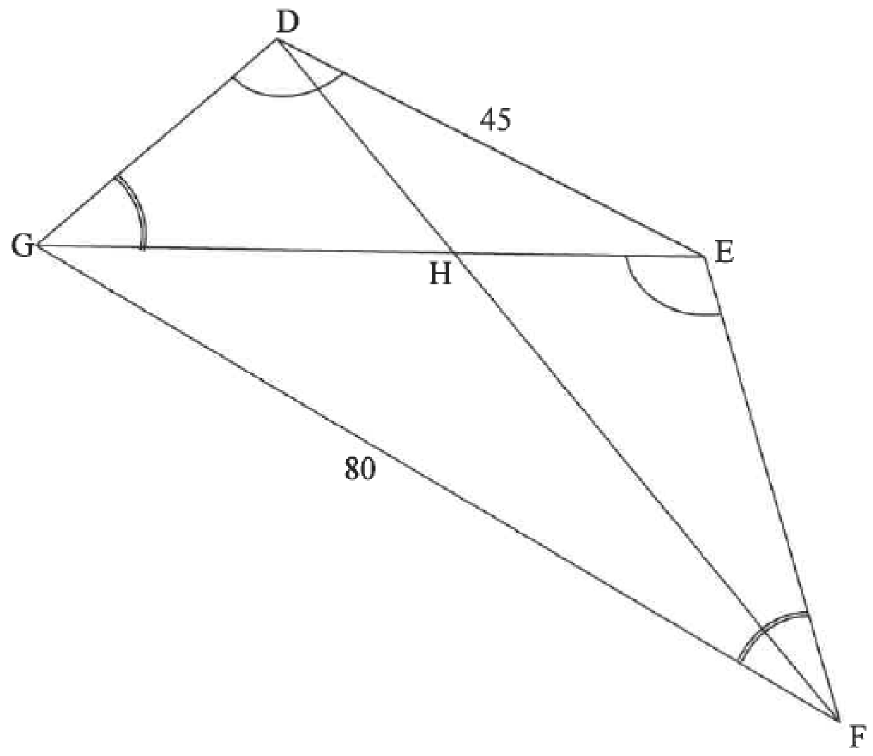
b. Prove $OH = \frac{AO.DH}{DC}$

c. Prove $\Delta JBF \parallel\!\mid \Delta BAF$

d. Prove $BF^2 = JF.AF$

QUESTION 36

In the diagram, DEFG is a quadrilateral with $DE = 45$ and $GF = 80$. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



a. Give a reason why ΔDEG||| ΔEGF.

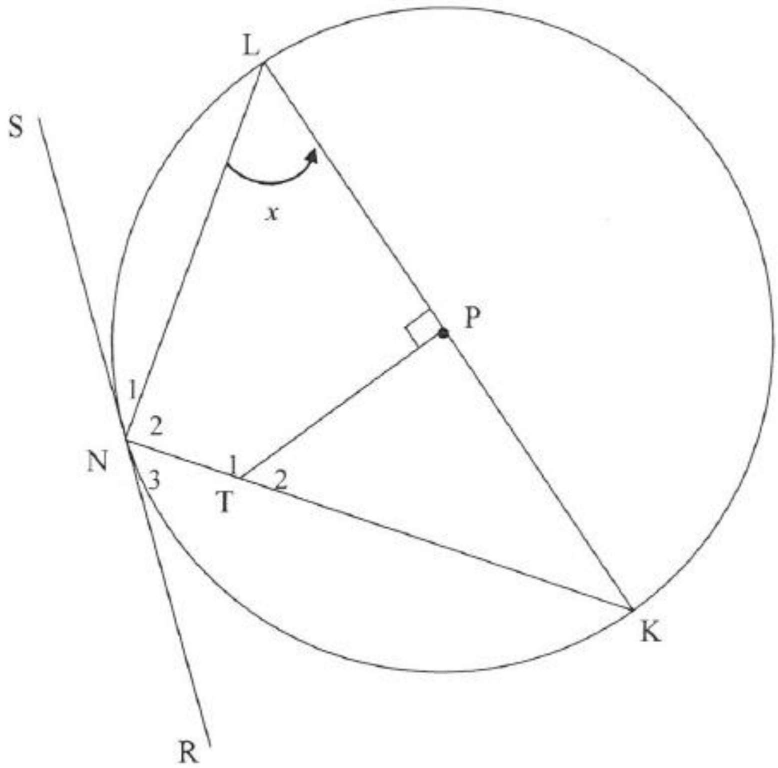
b. Calculate the length of GE.

c. Prove that ΔEDH ||| ΔFGH.

d. Hence, calculate the length of GH.

QUESTION 37

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and TP ⊥ KL. $P\hat{L}N = x$.



a. Prove that TLPN is a cyclic quadrilateral.

b. Determine , giving reasons, the size of $\angle N_1$ in terms of x

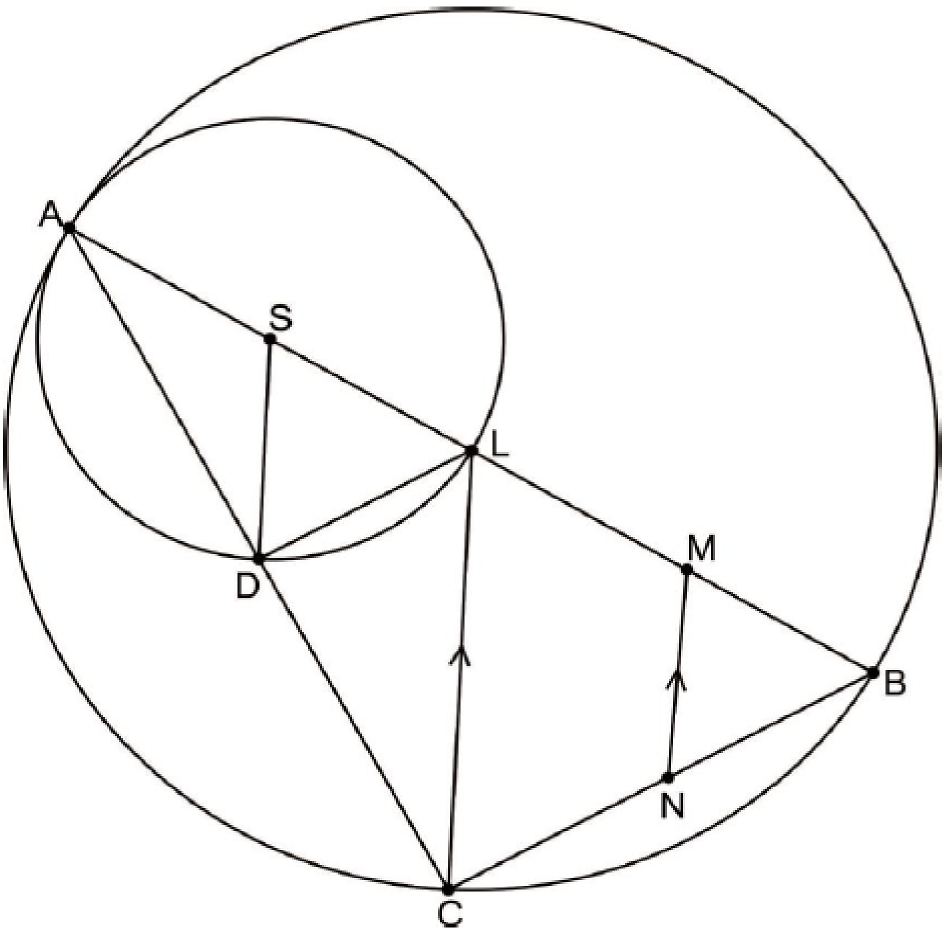
c. Prove that:

1. ΔKTP ||| ΔKLN
2. KT.KN = $2KT^2 - 2TP^2$

QUESTION 38

In the diagram below, two circles touch internally at A.

- AB is the diameter of the larger circle and AL is the diameter of the smaller circle.
- S and L are the centres of the circles.
- D is a point on the smaller circle and C is a point on the larger circle. ADC is a straight line.
- M is a point on LB so that MN || LC.



(a) Prove that DL || CB.

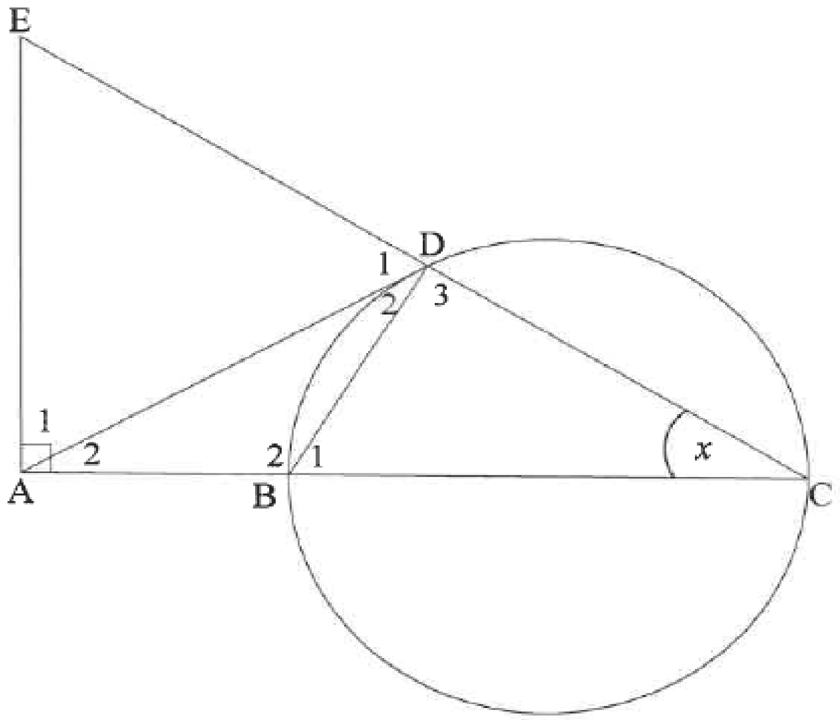
(b) Prove that 2SD = LC.

(c) Determine the value of $\frac{SL}{AB}$.

(d) If AB = 30 units and $\frac{BN}{NC} = \frac{7}{9}$, then determine the length of LM.

QUESTION39

In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA $\perp$ AC. BD is drawn.
Let $\hat{C} = x$.



1 Give a reason why:

(a) $\hat{D}_3 = 90°$

(b) ABDE is a cyclic quadrilateral

(c) $\hat{D}_2 = x$

2 Prove that:

(a) AD = AE

(b) ΔADB ||| ΔACD

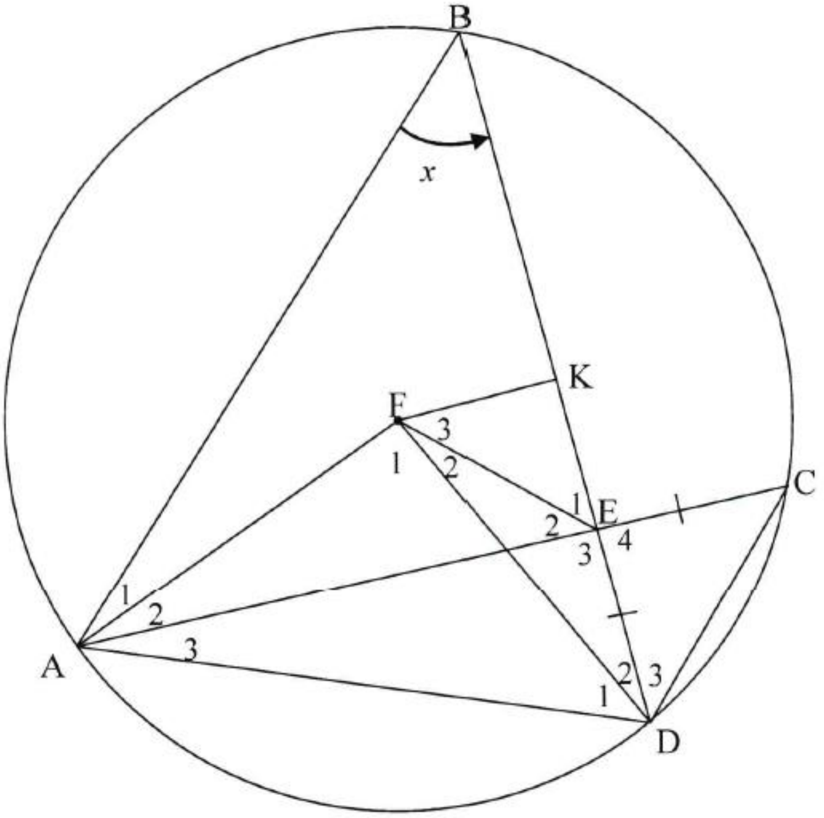
3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$

(b) Hence, prove that ΔADE is equilateral.

QUESTION 40

In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$

(b) $\hat{C}$

2 Prove, with reasons, that AFED is a cyclic quadrilateral.

3 Prove, with reasons, that $\hat{F}_3 = x$.

4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\dfrac{AE}{ED}$.

INFORMATION SHEET: MATHEMATICS

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) - P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

MERCY!!!!!

WHERE TO START MATHS AND SCIENCE TUTORING

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ACKNOWLEDGEMENTS

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I THANK YOU!!!!!!!!!!!!!!

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