WTS TUTORING

2019 WTS MATHS P2

CROSSNIGHT

GRADE : 12

COMPILED BY : PROF KHANGELANI SIBIYA

CELL NO. : 082 672 7928

EMAIL : kwvsibiya@gmail.com

FACEBOOK P. : WTS MATHS & SCEINCE TUTORING

GROUP WHATSAP : 082 672 7928

WEBSITE : www.wtstutor.co.za

WHERE TO START MATHS & SCIENCE IS FOR THE NATION

➢ DATA HANDING

QUESTION 1

Consider the Maths test out of 50 marks for WTS finishing School classes, A and B:

A	8	8	10	12	16	19	20	21	24	25	26	
B	8	8	10	12	16	19	20	21	24	24	26	50

NB: Choose one class to answer the questions below

(a) Calculate the mean.

(b) Write down the five-number summary of the data.

(c) Draw a box and whisker diagram for the data.

(d) Refer to the box and whisker diagram and comment on the skewness of the data set.

(e) Calculate the standard deviation for this data set.

(f) How many learners and its percentage were:

(i) One standard deviation above the mean?

(ii) One standard deviation below the mean?

(iii) One standard deviation within the mean?

(v) One standard deviation outside the mean?

(g) Repeat QUESTION (f) if it is now two standard deviation of the mean

(h) Write down the range

(i) Write down the IQR

(j) Write down the semi-IQR

(k) Write down outlier interval

(l) Write down any outlier

QUESTION 2

1. The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time(hours)	**Midpoint(x)**	**Frequency(f)**	**Cumulative frequency**	$f.x$	**Points(x; y)**
$0 \leq t < 20$			25		
$20 \leq t < 40$			69		
$40 \leq t < 60$			129		
$60 \leq t < 80$			157		
$80 \leq t < 100$			166		
$100 \leq t < 120$			172		

a) Complete the above table

b) Hence calculate the mean.

c) Draw an ogive (cumulative frequency curve) to represent the above data.

d) Draw the histogram graph.

e) Hence, draw the frequency polygon.

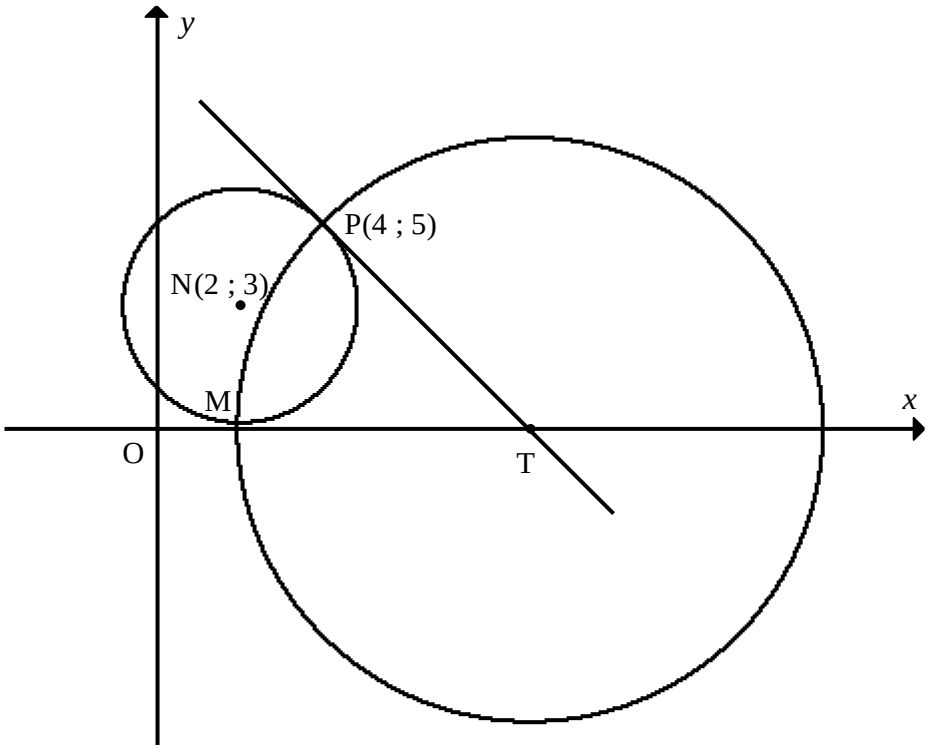
f) Write down the modal class of the data.

g) Write down the median class interval.

➢ ANALYTICAL GEOMETRY

QUESTION 1

In the figure below, T is a point on the *x*-axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.



a) Determine the equation of circle centre N in the form: $(x-a)^2+(y-b)^2=r^2$

b) Hence, rewrite the equation in the form of $Ax^2+Bx+Cy^2+Dy=E$

c) Also hence, rewrite it back to the form of $(x-a)^2+(x-b)^2=r^2$show all your working out

d) Calculate the gradient of the tangent PT

e) Hence, calculate the coordinates of T, the *x*-intercept of PT.

f) Calculate the length of PT. Leave your answer in surd form.

g) Hence, write down the equation of the circle with the center T

h) K is a point on the circumference drawn from T to form a straight line and, then calculate the following:

(i) Coordinate of K

(ii) Length of PK

(iii) Calculate the area of circle centered at T. Give your answer rounded off to the nearest integer.

i) If K $(t; -4)$ and the length of PK$= 10\sqrt{2}$, calculate the value of *t.*

j) Calculate the size of $\hat{NTP}$, correct to ONE decimal place.

k) Prove that MNPT is a kite.

l) Calculate the size of $\hat{MNP}$, correct to ONE decimal place.

m) Calculate the *y* intercept of the smaller circle.

n) Calculate the *x* intercept of the circle with center T.

QUESTION 2

Consider the following**:**

a. Determine the centre and radius of the circle with $x^2 + y^2 + 8x + 4y - 38 = 0$.

b. A second circle has the equation $(x - 4)^2 + (y - 6)^2 = 26$. Calculate the distance between the centres of the two circles.

c. Hence, show that the circles described in a) and b) intersect each other.

d. Show that the two circles intersect along the line $y = -x + 4$.

e. Calculate the *x* and *y* intercept of the equation in a)

f. Calculate the *x* and *y* intercept of the equation in b)

g. Using equation in number a) check whether point $A(-2; 3)$ lies on the circle or not?

➢ TRIGONOMETRY

QUESTION 1

1. Simplify to a single trigonometric ratio of A:

(c) $\frac{2cos105°.cos15°}{\cos(45°-x).cosx-\sin(45°-x).sinx}$

(d) $\frac{tan480°.sin300°.cos14°.\sin(-135°)}{sin104°.cos225°}$

(e) $\frac{sin104°(2cos^2 15°-1)}{tan38°.sin^2 412°}$

(i) $\frac{sin33°}{sin11°}-\frac{cos33°}{cos11°}$

2. Consider the following identity:

(a) $\frac{1-2sinA.cosA}{sinA-cosA}=sinA-cosA$

i. prove the identity above

ii. For which values A of will the above identity not be defined?

(b) $\frac{1-tanA}{1+tanA}=\frac{cos2A}{1+sin2A}$

i. Calculate the values of A for which $\frac{1-tanA}{1+tanA}$ is undefined for $A \in [0°; 360°]$.

i. Hence or otherwise, calculate the value of $\frac{1-tan22,5°}{1+tan22,5°}$.

3. Prove the following:

(a) $\frac{sin2x}{cos2x+sin^2 x}=2tanx$

(b) $\frac{sin\theta}{1-cos\theta}-\frac{cos\theta}{sin\theta}=\frac{1}{sin\theta}$

4. If $\sin26°\cos16°=k$ and $\cos26°\sin16°=w$, determine interms of k and w the value of

a. $\sin42°$

b. $\text{Sin}10°$

c. $\text{Cos}10°$

5. If $cos2A = k\ and\ sin2B = w$ and then determine the following in terms of k and w

(a) $cosA$

(b) $sinA$

(c) $sinBcosB$

6. Without using a calculator, determine the following in terms of $sin36°$;

(a) $sin\ 324°$

(b) $cos\ 72°$

7. If $cos73°cos31° + sin73°sin31° = k$ then determine the value of the following in terms of k

(a) $\cos^2 21 - \sin^2 21 + 7$

(b) $\sin 42°$

8. If $sin31°cos22° + sin22°cos31° = k$ then determine the value of the following in terms of k

(a) $\sin 53°$

(b) $\cos 143°$

(c) $\sin 75°\cos 22° + \cos 75°\cos 22°$

9. If $cosA + sinA = k$, express the following in terms of k:

(a) $\cos(A - 45°)$

(b) $1 + sin2A$

QUESTION 2

1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$

5.1.2 $\sin^2 \theta$

2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio.

QUESTION 3

1. If cos20˚= k , express the following in terms of k

a. sin20˚

b. Sin40º

c. Cos40º

d. Sin10º

e. Cos10º

f. Cos50º

g. Sin80º

h. Sin10ºcos10º

2 Prove the following identity: $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$

3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$

QUESTION 4

Simplify the following:

(a) If $sinx - cosx = \frac{3}{4}$, calculate the value of $sin2x$ without using a calculator.

(b) Given that $= \frac{1}{3}$, calculate the numerical value of $sin3\alpha$, without using a calculator.

(c) If $x = 3\ sin\alpha$ and $y = 3cos\alpha$, determine the value of $x^2 + y^2$

(d) if $sinx = \frac{2n}{n^2+1}$, n > 1 and $0° < x < 90°$, prove that $\frac{1+sinx}{cosx} = \frac{n+1}{n-1}$

(e) Determine the value of $sin3x\ .cosy + cos3x\ .siny$ if 3x + y = 270°

QUESTION 5

Write down the maximum and minimum of the following:

(a) $f(x) = sinx$

(b) $f(x) = cos2x$

(c) $f(x) = 2sinx$

(d) $f(x) = 1 + sinx$

(e) $f(x) = sinxcosx$

(f) $8 - 10sinx.cox$

QUESTION 6

Determine the general solution of following:

a) $\cos 2x - 4\sin x + 5 = 0.$

b) $sin^2x + \cos 2x - cosx = 0$

c) $sinx + 2cos^2x = 1$

d) $6cosx - 5 = \frac{4}{cosx}$; $cosx \neq 0$

e) $cos3x = sinx$

f) $\cos 2x = \sin(x - 30^o)$.

s) $1 - \frac{tan^2\frac{1}{2}x}{1+tan^2\frac{1}{2}x} = \frac{1}{4}$

t) $\sin(x + 64°)\cos(x + 379°) + \sin(x + 19°)\cos(x + 244°) = \frac{1}{\sqrt{2}}$

u) $2sinx = tan\frac{1}{2}x$

v) $sin^2x - cos^2x + sinx + 1 = 0$

x) $sinx + cosx = 1$

QUESTION 7

Calculate the values of x where $x \in [-180°; 360°]$

(a) $sinx = cos2x$

(b) $sinx = \cos(x + 30°)$

(c) $2cosx = \sin(x + 30°)$

QUESTION 8

Given: $f(x) = 2cosx$ and $g(x) = tan2x$

a. Sketch the graph of f and g on the same set of axes, for $x \in [-90° ; 90°]$.

b. Solve for x if $2cosx = tan2x$ and $x \in [-90; 90]$. Show all working details.

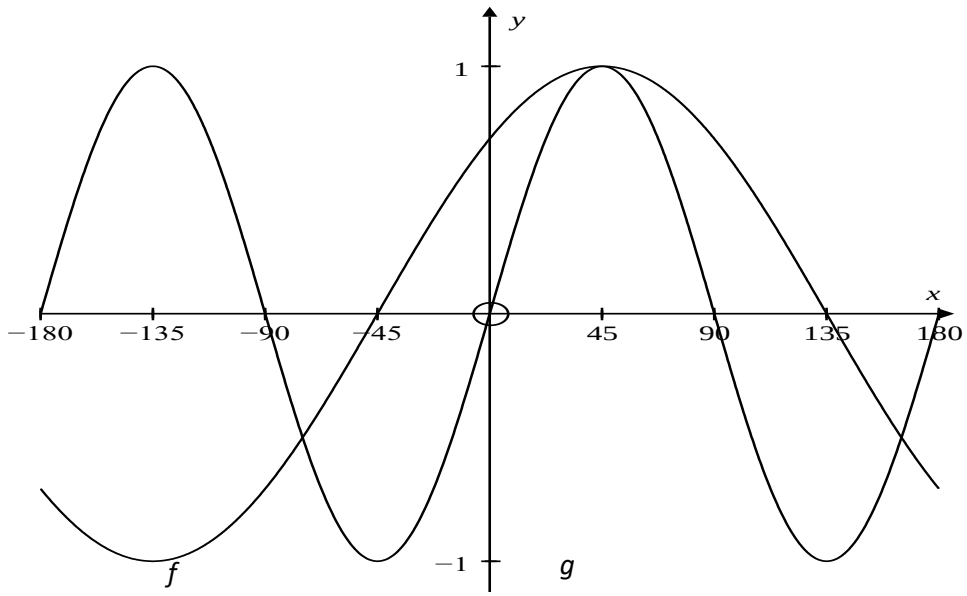
c. Use the graph to solve for x: $2cosx.tan2x > 0$.

d. Write down the equation of the asymptotes of $g(x - 25°)$, where $x \in [-90°; 90]$.

QUESTION 9

The graphs of $f(x) = \{(x; y)/y = sinax\}$and $g(x) = \{(x; y)/y = \cos(x - b)\}$for the domain

$x \in [-180°; 180°]$ are shown in the diagram below



(a) Write down the values of a and b.

(b) Write down the period of g.

(c) Determine the equation of h if $h(x) = f(x-30°)$.

(d) Explain how you would use the graphs to solve the equation: $\sqrt{2}\sin 2x = \cos x + \sin x$.

(e) Write down the amplitude and period of :

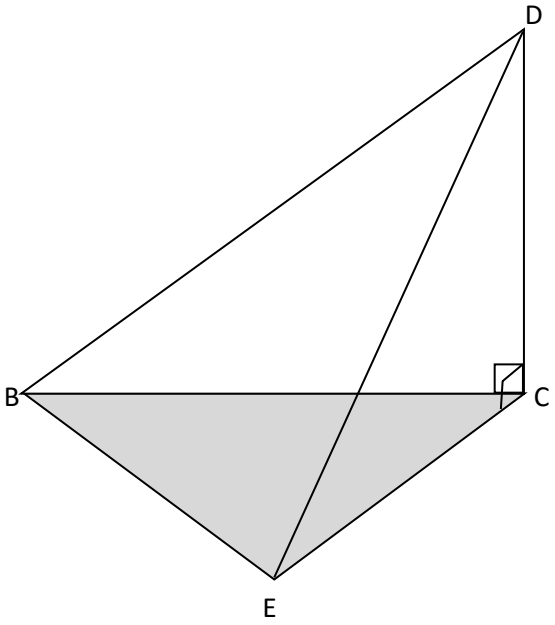
i. f

ii. $h(x) = f(\frac{2}{3}x)$

iii. $k(x) = \frac{f(x)}{2}$

QUESTION 10

In the figure below, CD is a vertical mast. The points B, C and E are in the same horizontal plane.BD and ED are cables joining the top of the mast to pegs on the ground. DE = 28,1 m and BC = 20,7 m. The angle of elevation of D from B is 43,6°. $C\hat{B}E = 63°$ and $B\hat{D}E = 35{,}7°$.



Give your answers correct to ONE decimal place in each of the following questions:

(a) Calculate the length of BD.

(b) Calculate the length of BE.

(c) Calculate the area of ΔBEC.

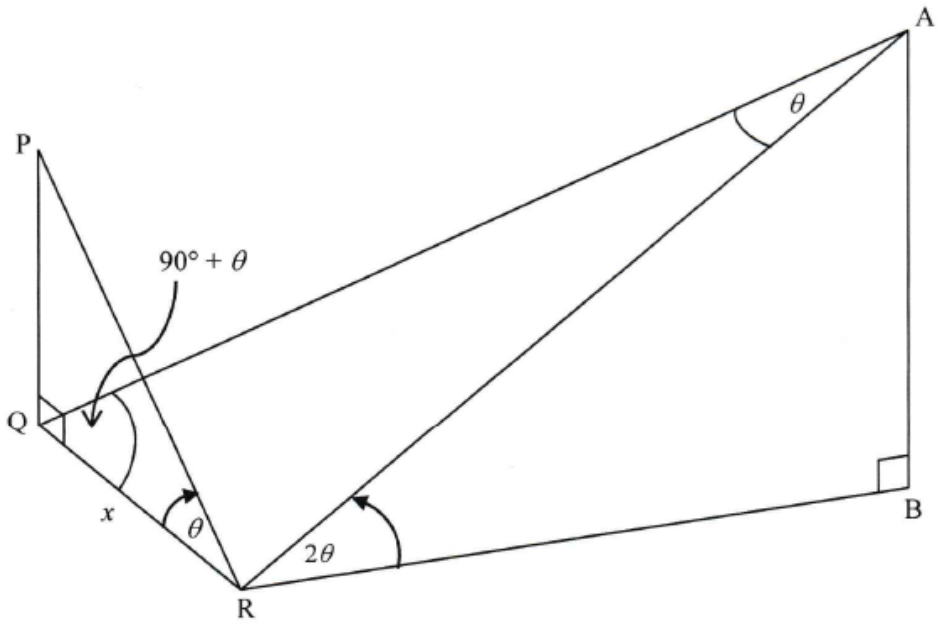
(d) Calculate the length of DC

(e) Hence, calculate angle DEC

(f) Calculate the perimeter of BDCE

QUESTION 11

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



1. Determine in terms of x and θ

   (a) QP

   (b) AR

2. Show that $AB = 2x\cos^2\theta$

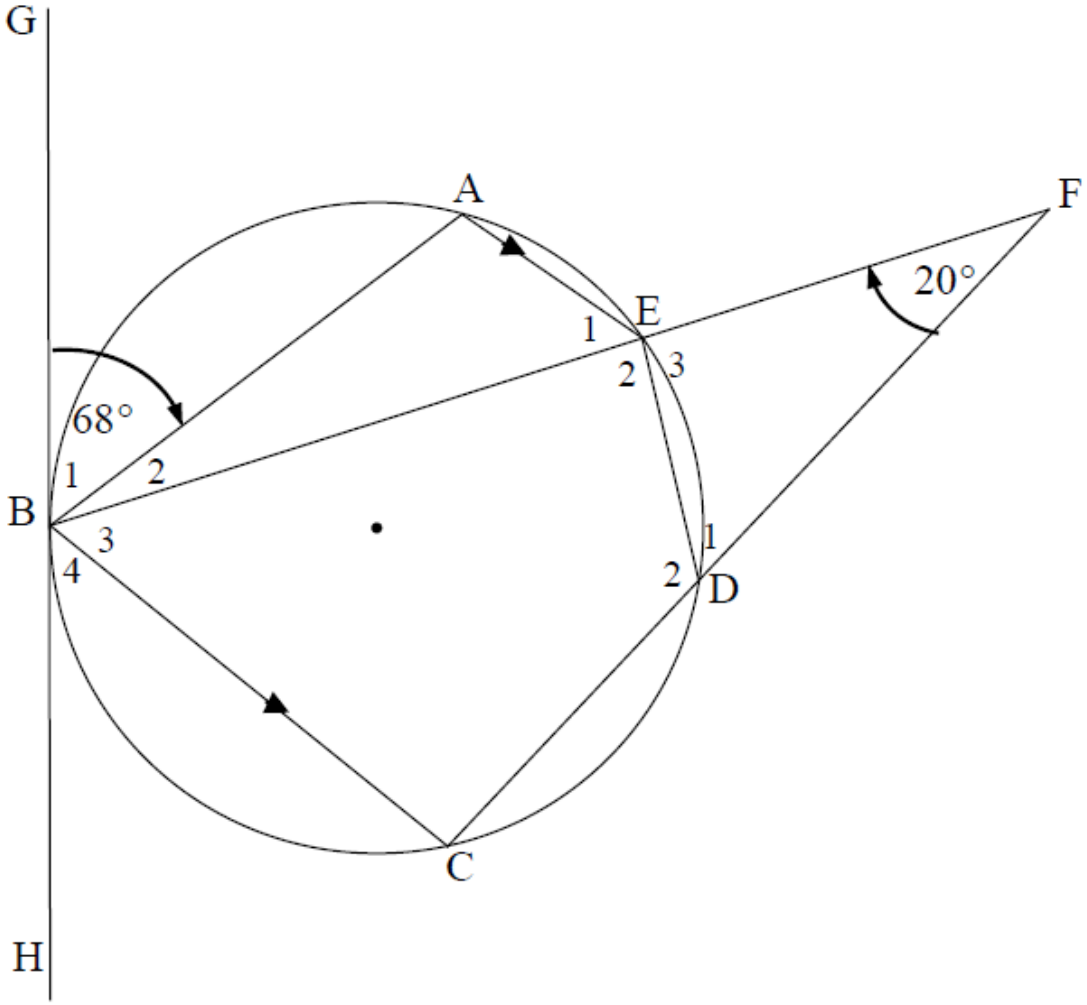
3. Determine $\frac{AB}{QP}$ $IF\ \theta = 12°$

➢ EUCLIDEAN GEOMETRY

QUESTION 1

In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B.

$\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

a. $\angle E_1$

b. $\angle B_3$

c. $\angle D_1$

d. $\angle E_2$

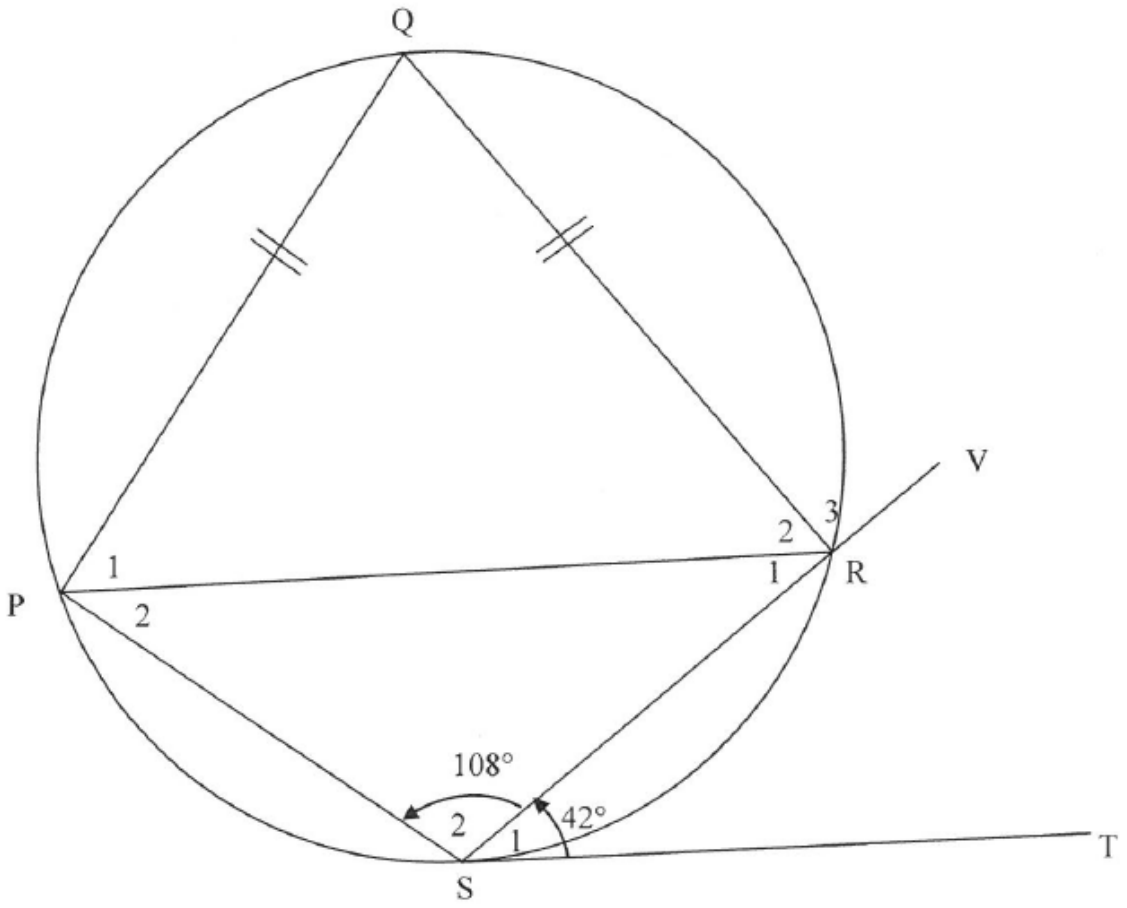
e. $\angle E_3$

f. $\angle C$

g. $\angle D_2$

QUESTION 2

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

a. $\angle Q$

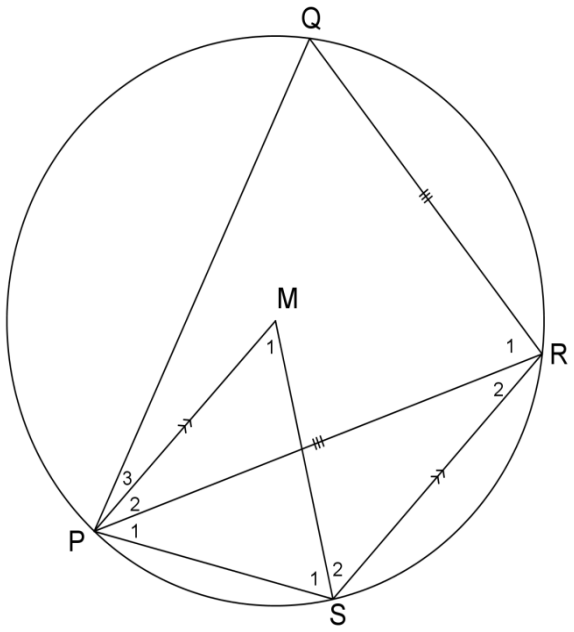
b. $\angle R_2$

c. $\angle P_2$

d. $\angle R_3$

QUESTION 3

In the diagram alongside, M is the centre of circle PQRS. PM ∥ RS , QR = PR and $\hat{R}_2 = 28^0$



Determine, giving reasons, the size of the following angles:

a. $\hat{S}_2$

b. $P\hat{S}R$

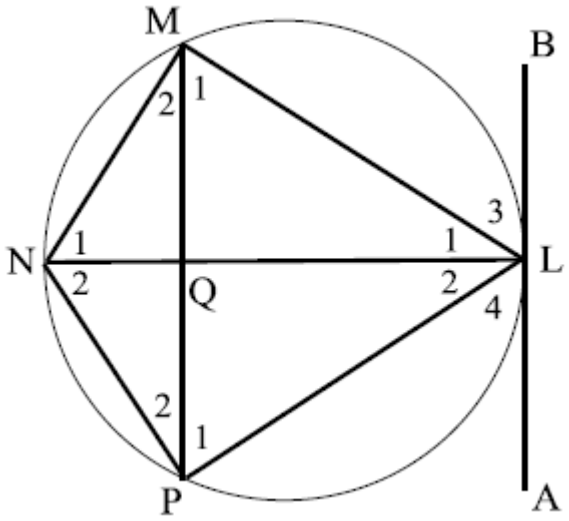
c. $\hat{Q}$

d. $\hat{P}_3$

e. $\angle R_1$

QUESTION 4

ALB is a tangent to circle LMNP. ALB || MP.



Prove that:

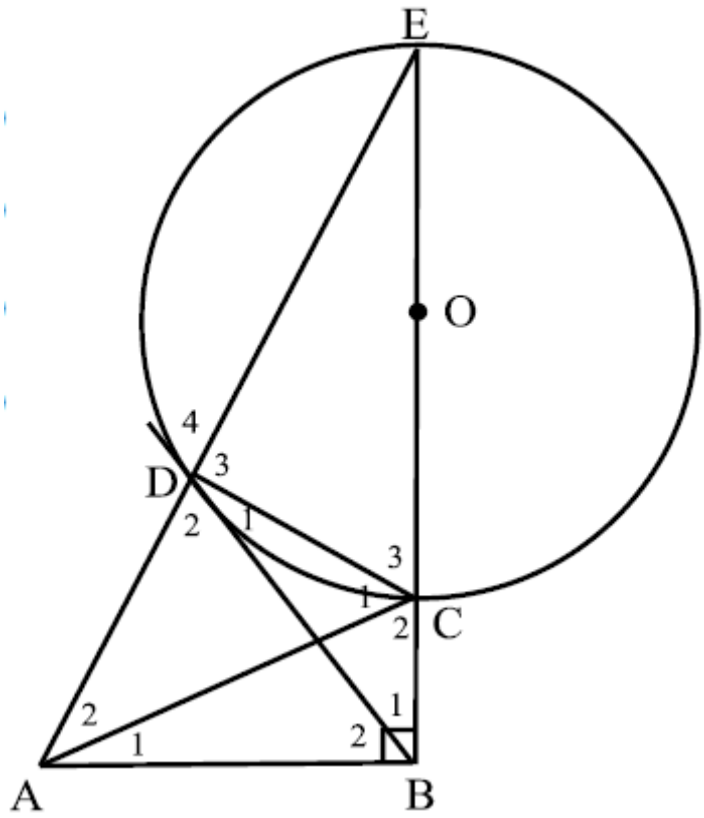
a. LM = LP

b. LN bisects angle MNP

c. LM is a tangent to circle MNQ

QUESTION 5

EC is a diameter of circle DEC. EC is produced to B. BD is a tangent at D. ED is produced to A and AB⊥ BE .



Prove that:

a. ABCD is a cyclic quadrilateral.

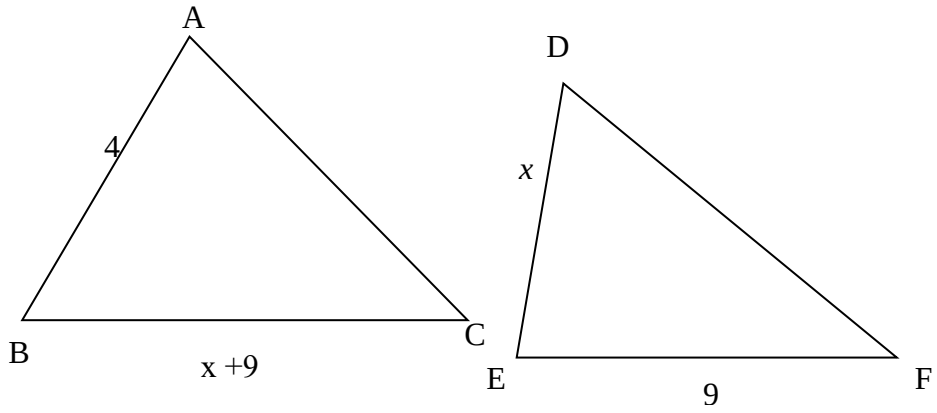
b. $\widehat{A_1} = \hat{E}$

c. BD = BA

d. $\widehat{C_2} = \widehat{C_3}$

QUESTION 6

In the diagram below, ΔABC and ΔDEF are drawn. AB = 4 units, BC = (x + 9) units, DE = x units and EF = 9 units.

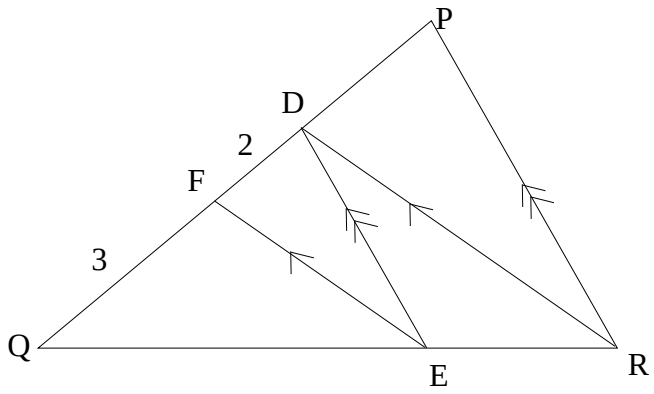


a) If $\Delta ABC \;|||\; \Delta DEF$, calculate the value of x.

b) Hence, write down the length of BC

QUESTION 7

In the diagram below, DE || PR, FE || DR, QF = 3 cm, FD = 2 cm.



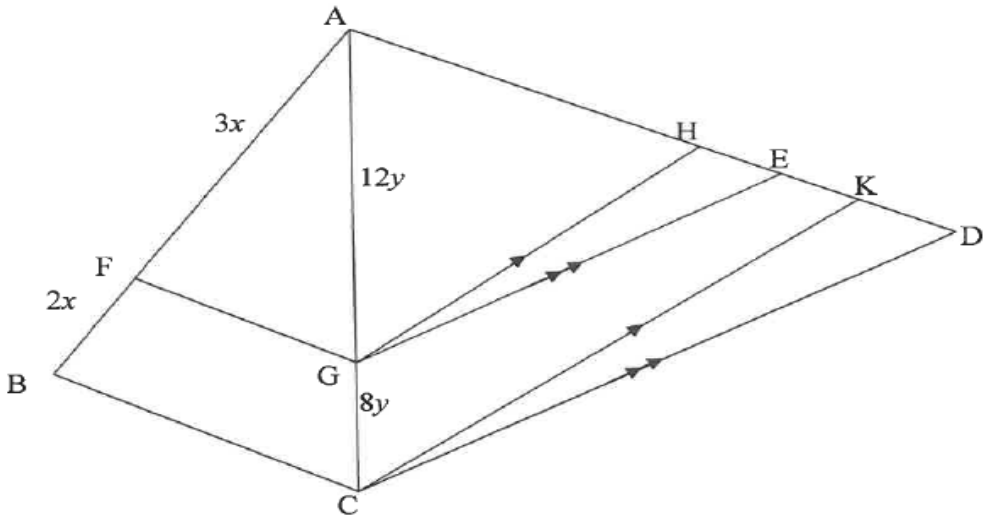
a. Determine the value of $\frac{QE}{QR}$.

b. Calculate the length of DP.

c. Determine the value of $\frac{QE}{ER}$

QUESTION 8

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



1. Prove that:

   (a) FG | | BC

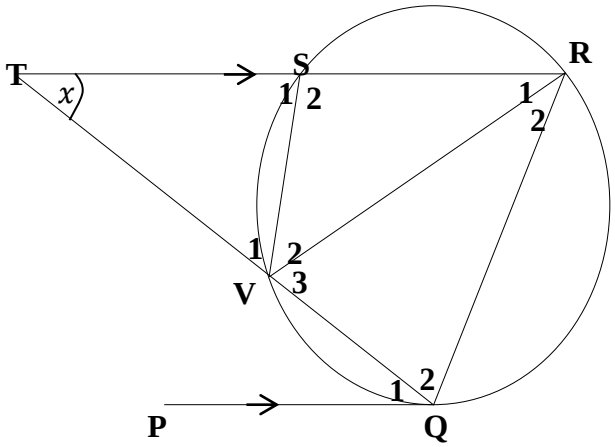
   (b) $\frac{AH}{HK} = \frac{AE}{ED}$

2. If it is future given that AH= 15 and ED= 12, Calculate the length of EK.

3. Calculate the area of ΔABC: ΔAFG

QUESTION 9

In the diagram below, PQ is a tangent to the circle at Q. TSR is a line which cuts the circle at S such that TR//PQ. QV is produced to meet RST at T. $\hat{T} = x$.



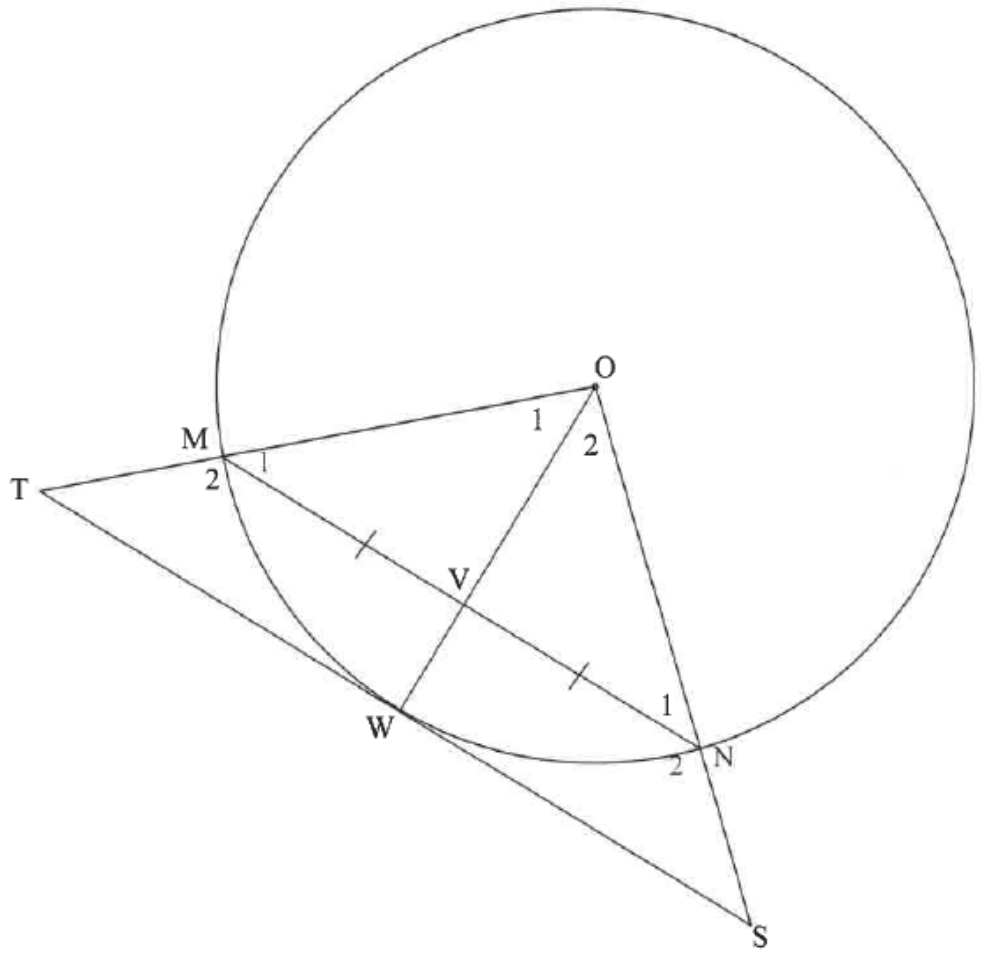
a) Write, down with reasons, TWO other angles each equal to x .

b) Prove that TSV /// RQV.

c) Prove that TS.TR = TV. TQ

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.

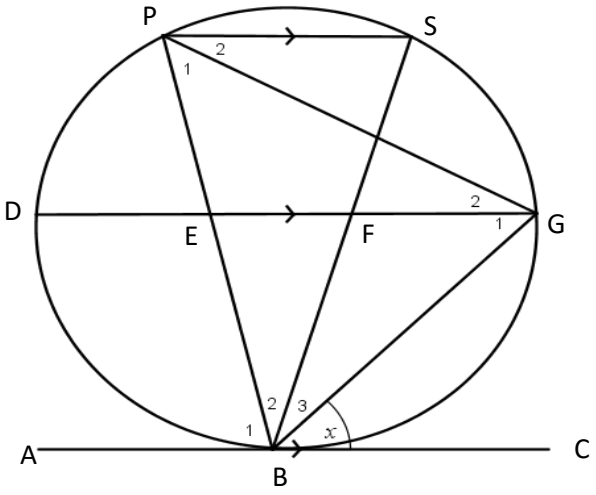


1. Give a reason why OV ⊥ MN.
2. Prove that:
   (a) MN | | TS
   (b) TMNS is a cyclic quadrilateral
   (c) OS . MN = 2ON . WS

QUESTION 11

In the diagram, P, S, G, B and D are points on the circumference of the circle such that

PS || DG || AC. ABC is a tangent to the circle at B.



a. Give a reason why $\widehat{G_1} = x$

b. Prove that:

i. $BE = \frac{BP.BF}{BS}$

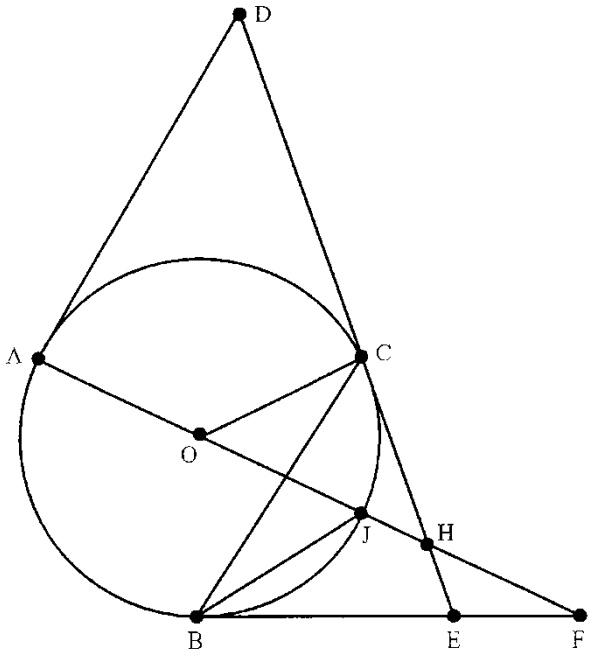
ii. ΔBGP ||| ΔBEG

iii $BG^2 = BE.BP$

iv. $\frac{BG^2}{BP^2} = \frac{BF}{BS}$

QUESTION 12

In the figure, AD, DC and BE are tangents to the circle. CO is a radius and chord BC is drawn. Radius AO is drawn and extended to cut the circle at J and BE is extended at F.



a. Prove $\Delta DAH \,|||\, \Delta OCH$

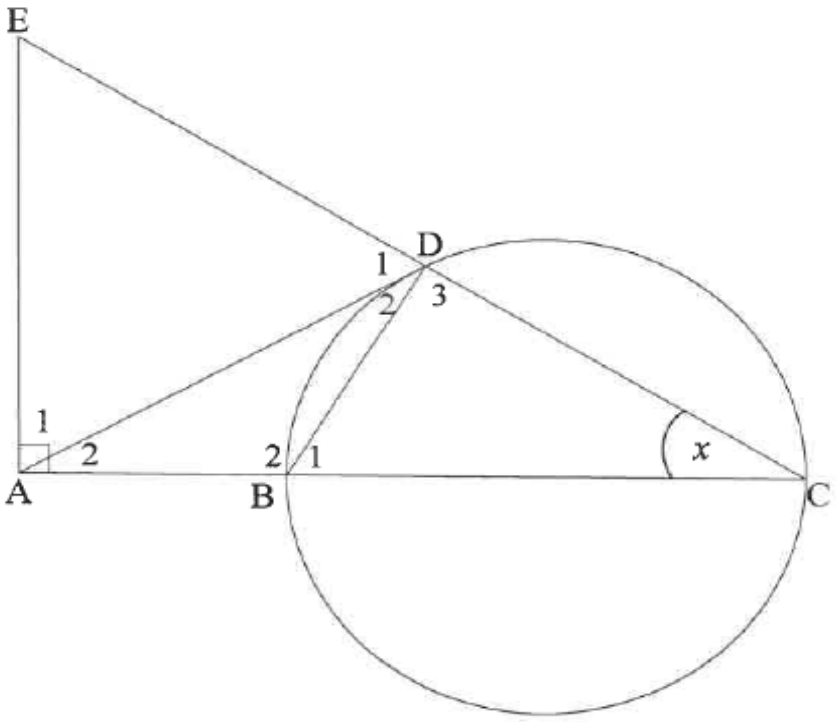
b. Prove $OH = \dfrac{AO.DH}{DC}$

c. Prove $\Delta JBF \,|||\, \Delta BAF$

d. Prove $BF^2 = JF.AF$

QUESTION 13

In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA $\perp$ AC. BD is drawn.
Let $\hat{C} = x$.



1 Give a reason why:

(a) $\hat{D}_3 = 90°$

(b) ABDE is a cyclic quadrilateral

(c) $\hat{D}_2 = x$

2 Prove that:

(a) AD = AE

(b) $\Delta ADB \,|||\, \Delta ACD$

3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$

(b) Hence, prove that ΔADE is equilateral.

WTS TUTORING WINTER CAMP

SUBJECTS : MATHS, MATHS LIT, PHYSCS,

ACCOUNTING & LIFE SCIENCES

PROVINCE : KWAZULU NATAL

VENUE : EMZINGAZI PRIMARY SCHOOL

GRADE : 08 TO 12

DATE : 28 JUNE TO 07 JULY 2019

TIME : 08:00 TO 21:30 DAILY

PRICE : R2 000

WE ALSO PROVIDE ONE ON ONE, CROSSNIGHTS AND WEEKENDS LESSONS

FOR MORE INFORMATION CONTACT: PROF KWV SIBIYA: 082 6727 928