education

Department:
Education
PROVINCE OF KWAZULU-NATAL

HIGH FLYERS MATERIAL

MATHEMATICS
2019
KZN DEPARTMENT OF EDUCATION

CURRICULUM GRADES 10 – 12 DIRECTORATE

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the content and give more guidance to the teachers.

TABLE OF CONTENT

FUNCTIONS AND GRAPHS

WEIGHTING: 35 ± 3

FOCUS ON:

- Definition of a function
- Types of Functions: Linear, Quadratic (parabola), Hyperbola, Exponential
- Equations of Functions: $f(x) = ax + q$, $f(x) = a(x+p)^2 + q$, $f(x) = \frac{a}{x+p} + q$, $f(x) = ab^{x+p} + q$
- Effects of parameters $a,\ p\ and\ q$
- Reflection and Translation
- Inverses of prescribed functions i.e linear ($y = ax + q$), quadratic ($y = ax^2$) and exponential ($y = b^x$); (b > 0, b ≠ 1)
- In the case of MANY-TO-ONE functions, the DOMAIN has to be restricted if the inverse is to be a function
- Shape, Domain and Range, intercepts with the axes, turning points, asymptotes, symmetry

IEB NOVEMBER 2014

QUESTION 3

		Mark	LEVEL
(a)	Given: $f(x) = \frac{x}{2} - 1$		
	The graph of $y = f(x)$ is transformed by a reflection in the x-axis followed by a translation 4 units in the positive x-direction. Determine the simplified equation of this graph.	(3)	3
(b)	Given: $g(x) = \frac{-x^2}{4}$		
	(1) Write down the domain and range of g.	(2)	1
	(2) Hence write down the domain of the inverse of g.	(1)	2
	(3) Determine the equation of the inverse of g in the form $y = ...$	(3)	2
	(4) On the same set of axes, draw the graphs of $y = g(x)$ and its inverse. So that both graphs are functions.	(4)	4
(c)	Given: $j(x) = 2x^2 - 8x + 5$		
	(1) Determine q such that $j(x) = 2(x-2)^2 + q$.	(2)	2
	(2) For which values of c, $c \in R$, will the equation $j(x) = c$ have real roots?	(3)	3
		[18]	

IEB NOVEMBER 2015

QUESTION 2

(a) Given: $f(x)=\dfrac{1}{x+1}+2$

(1) Write down the equations of the asymptotes of f. (2) 1

(2) Determine the x and y-intercepts of the graph of f. (3) 2

(3) Sketch the graph of f. Show all asymptotes and intercepts with axes. (3) 2

(b) Given: $g(x)=2.3^x-1$

(1) Determine the intercepts with axes, correct to 2 decimal digits, if necessary. (4) 2

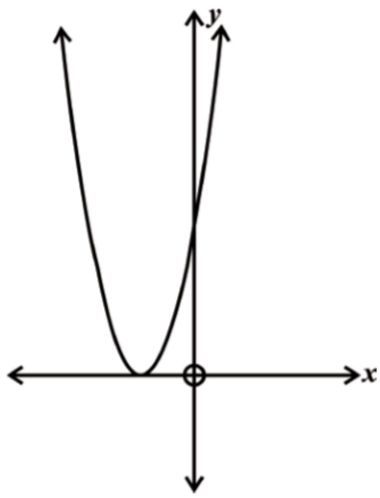
(2) Sketch the graph of g. Label clearly all asymptotes and intercepts with axes. (3) 2

[15]

QUESTION 6

LEVEL

(a) The graph of $y=ax^2+bx+c$ is sketched, where $a, b, c \in \mathbb{R}$



For each of the equations given, choose the statement ((i), (ii), or (iii)) that applies.

(1) $ax^2+bx+c=0$ (1) 1

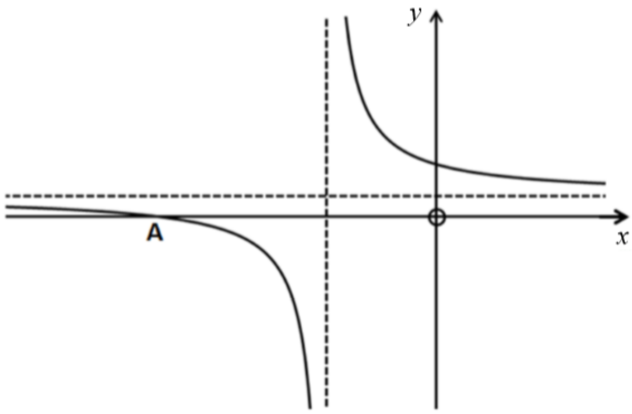
(2) $ax^2+bx+c=-2$ (1) 2

(3) $ax^2+bx+c=4$ (1) 2

(i) Roots are non-real
(ii) Roots are real and unequal
(iii) Roots are real and equal

IEB SUPPLEMENTERY EXAMINATION 2016
QUESTION 1

(c) In the diagram below, the graph of $h(x)=\dfrac{3}{x+2}+1$ has been sketched.

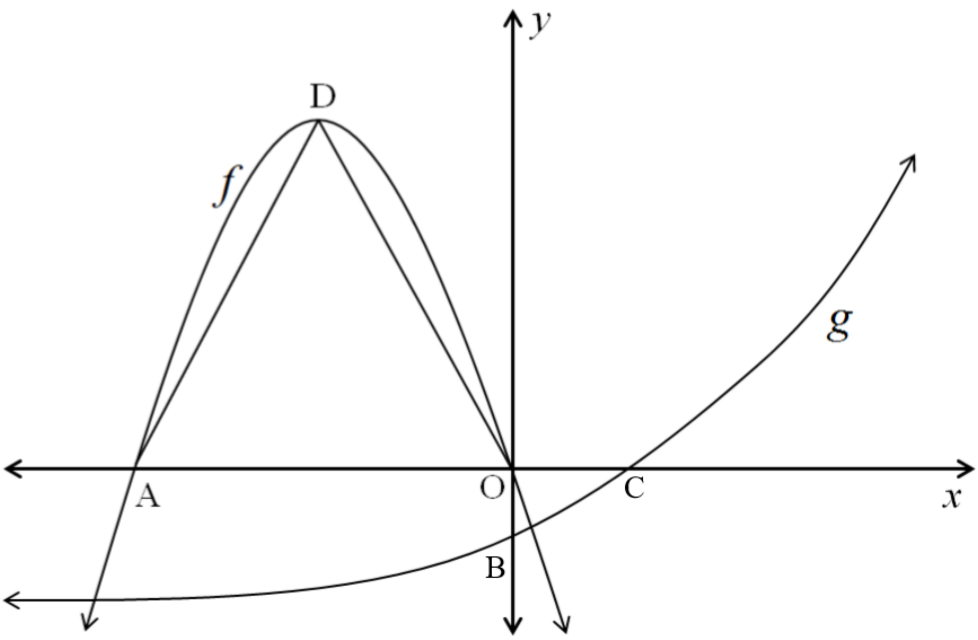


(1) Write down the equations for the asymptotes of $h(x)$. (2) 1

(2) Write down the new equation for $h(x)$ if $h(x)$ is shifted horizontally so that point A is at the origin. (3) 3

(b) In the diagram below:

- $f(x)=-x^2-4x$
- $g(x)=2^x-6$
- Point D is the turning point of f
- Points A and C are the x-intercepts of f and g
- Point B is the y-intercept of g



(1) Determine the area of ΔAOD. (6) 4

(2) For what values of k will $-x^2 - 4x = 2^x + k$ have two real roots that are opposite in sign? (2) 4

(3) For what values of p will $-(x-p)^2 - 4(x-p) = 2^x - 6$ have two real negative roots? (4) 4

[23]

QUESTION 6:

6. Given: $f(x) = \dfrac{-8}{x+2} - 3$

6.1 Write down the equations of the asymptotes for f. (2)

6.2 Calculate the x- and y-intercepts of f. (3)

6.3 Sketch the graph of f. (3)

6.4 If $y = -x + k$ the equation of the symmetry of f, determine the value of k. (2)

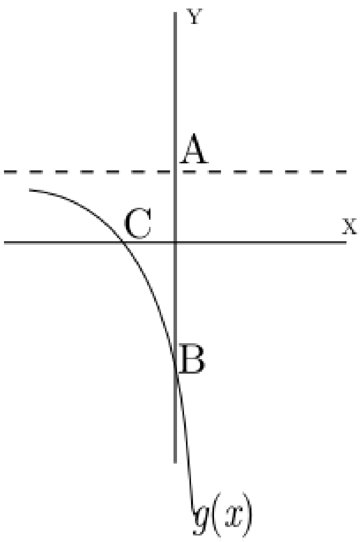
6.5 What is the domain and the range for f. (4)

6.6 Determine the equation of h where h is the reflection of f in the x-axis. (2)

[16]

QUESTION 7:

7. The diagram represents $g(x) = -4.2^{x+2} + 4$.



7.1 Determine the coordinates of A, B and C. (6)

7.2 Determine the average gradient between B and C. (4)

7.3 Find the equation of h where h is the reflection of g in the Y-axis. (2)

7.4 For which value(s) of x will $g(x) = h(x)$? (2)

7.5 Determine the x-intercept of h. (2)

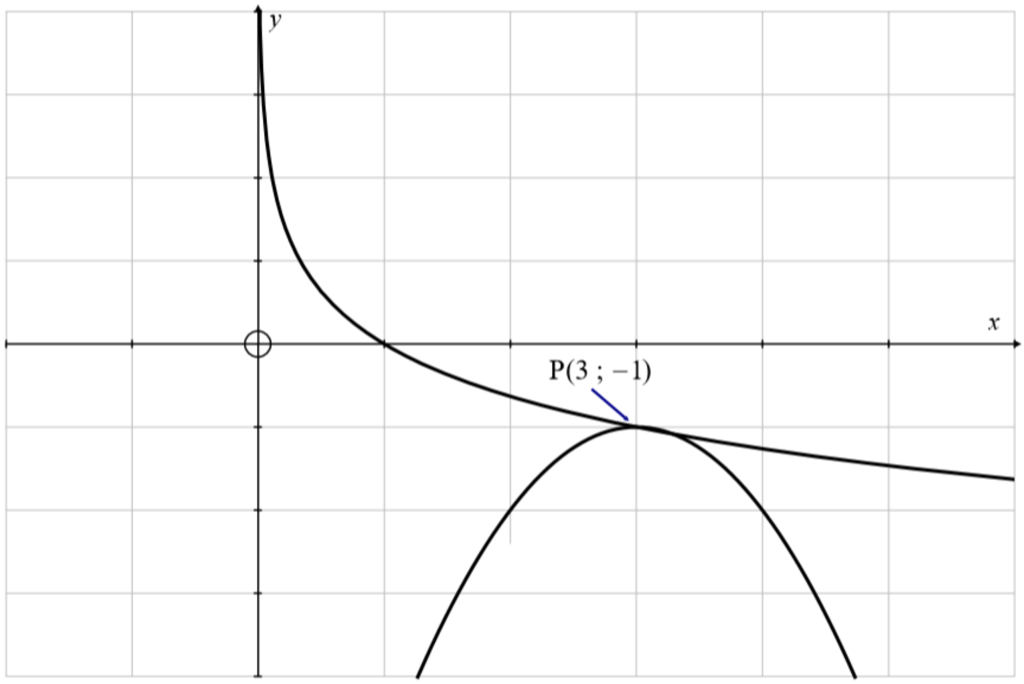
[16]

IEB EXEMPLAR 2014
QUESTION 11

Refer to the figure below.

The graphs of $y = g(x) = \log_a x$ and $y = h(x) = -(x-3)^2 - 1$ are given.

The point $P(3;-1)$ lies on the graph of both g and h.



Determine:

(a) the value of a. (2) 2

(b) the equation which defines $g^{-1}(x)$ in the form $y = ...$ (2) 2

(c) the $x-$values for which $1 \leq g^{-1}(x) \leq 3$. (2) 3

(d) a possible restriction that could be placed on $h(x)$ to ensure that $h^{-1}(x)$ is a function. (1) 2

(e) the values of x for which $g(x).h(x) < 0$. (2) 3
[9]

IEB SUPPLEMENTARY EXAMINATION 2016
QUESTION 6

(b) Given: $f(x) = \dfrac{2}{x^2} + 1$

Determine $f(x^{-1}) - x^2 f(-1)$.

Simplify your answer fully. (4) 4

(c) Given: $g(x) = x^2$ with $x \geq 0$.

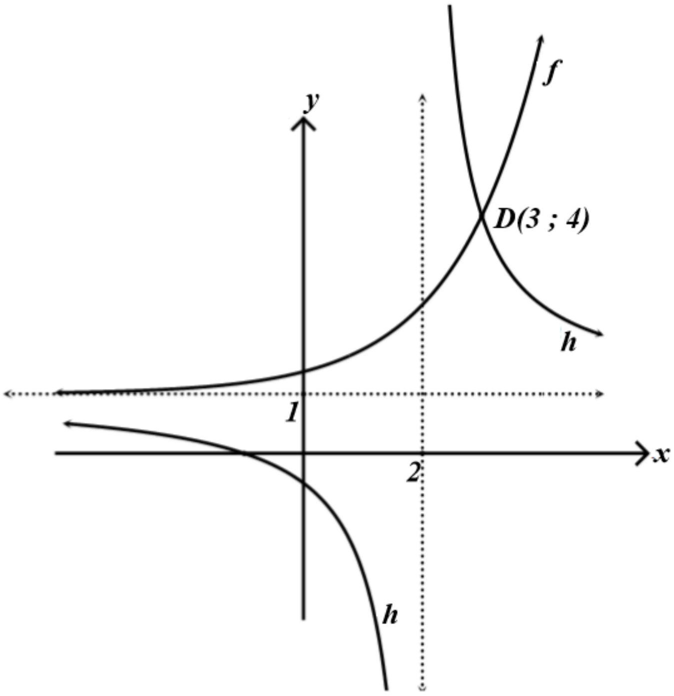
(1) Write down the range of $g^{-1}(x)$. (1) 1

(2) On the same set of axes, draw sketch graphs of $y = g(x)$ and $y = g^{-1}(x)$, clearly labelling intercepts with axes. (4) 2

LEVEL

QUESTION 7 (b) Grade 11 P1

Refer to the figure showing the graphs of a hyperbola $y = h(x)$ with asymptotes $x = 2$ and $y = 1$, with a point D(3 ; 4), which is also on an exponential graph $y = f(x) = 3.2^{x-p} + q$.



(1) Determine the equation of the hyperbola. (4) 2

(2) Determine the values of p and q in the equation of $y = f(x)$. (4) 2

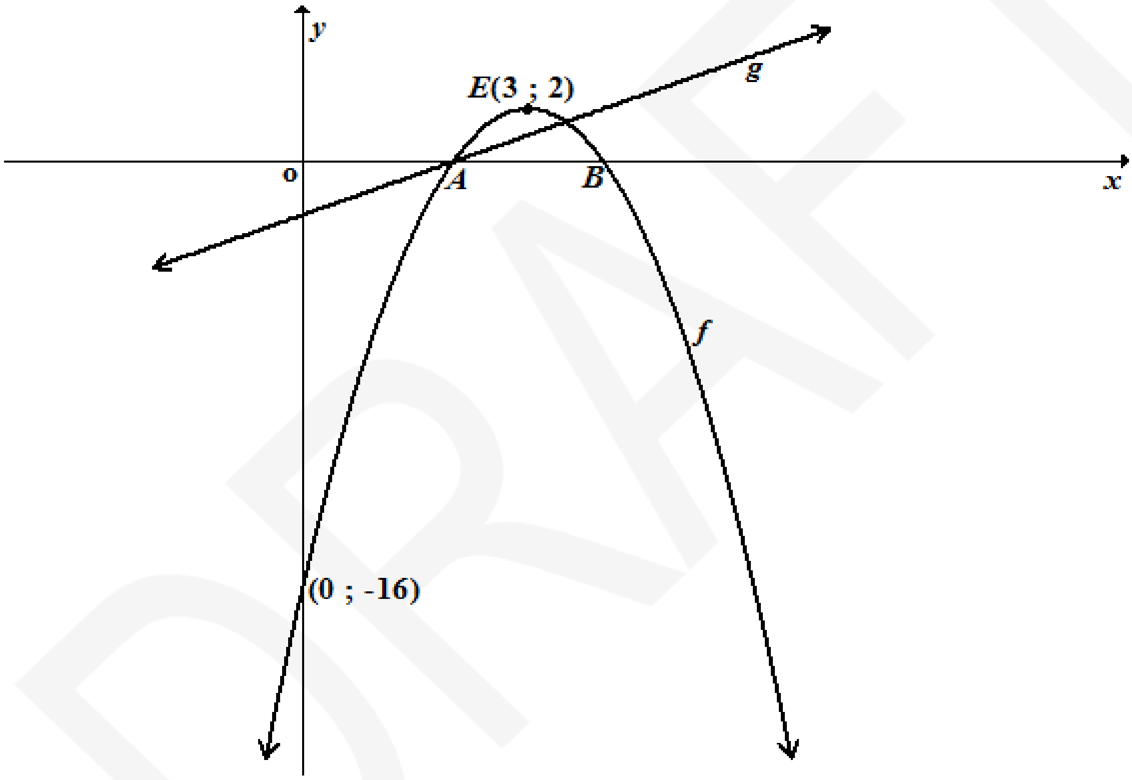
(3) Write down the values of x for which $f(x) \leq h(x)$. (2) 3

2

7

QUESTION 7

Sketched below are the graphs of $f(x) = a(x+p)^2 + q$ and $g(x) = mx + c$, where a, p, q, m and c are real constants.
$E(3;2)$ is the turning point and $(0;-16)$ is the y-intercept of f.
The graph of f intersects the x-axis at A and B.
C, a point on the y-axis, is symmetrical to A about the line $y = -x$.
The graph of g passes through points C and A.



(3) 2

2

7.2 Determine the coordinates of points A, B and C. (4) 2

7.3 Determine the equation of g. (3) 3

2

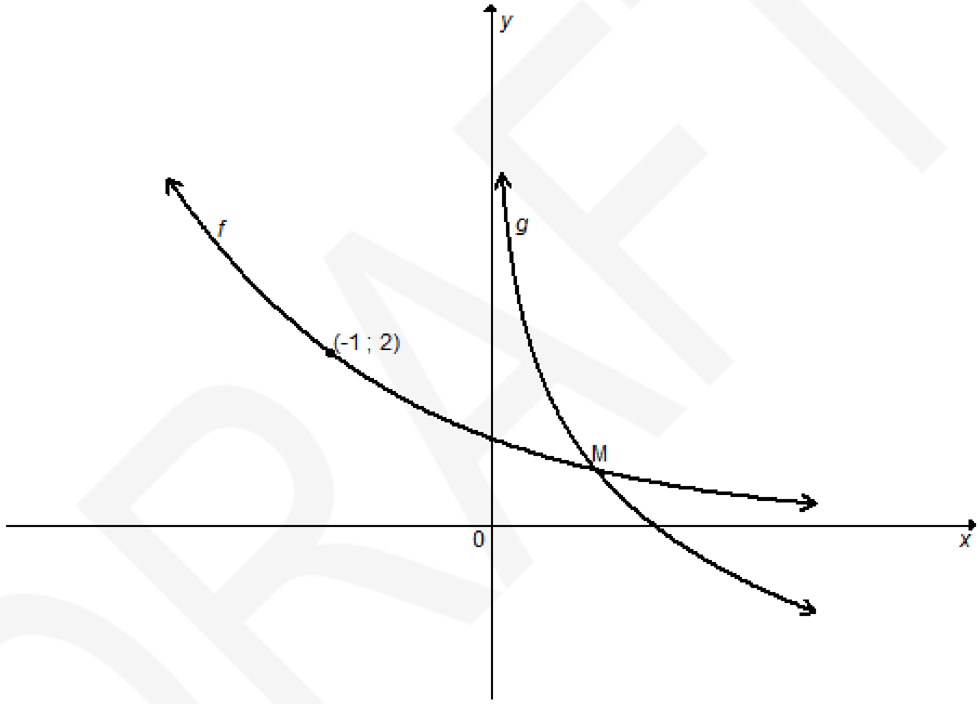
7.4 If $h(x) = \dfrac{-96}{x}$, calculate the point of intersection of h and f. (4)

7.5 Write down the turning point of p, if $p(x) = -f\left(\dfrac{x}{2}\right)$. (2)

[16]

QUESTION 8

The graphs of $f(x)=a^x$, $a>0$ and $g(x)=f^{-1}(x)$, where g is the inverse of f, are sketched below: The point $(-1;2)$ lies on f. M is the point of intersection of f and g.



1

8.2 Determine the equation of g in the form $g(x)=$ ..... (2) 2

8.3 Write down the equation of the line which divides f and g symmetrically. (1) 1

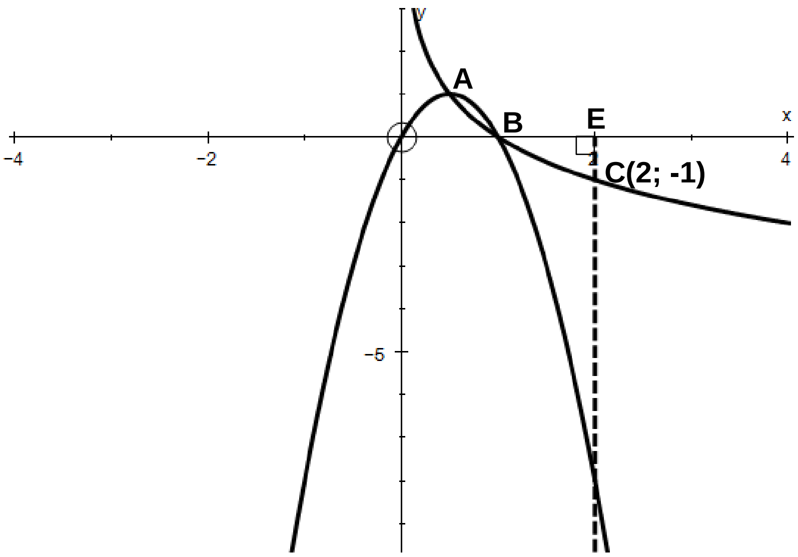
8.4 If M is the point of intersection of f and the line $y=x$, show that

$x\log\frac{1}{2}-\log x=0$. (3) 3

[7]

P1 NOV 2002 (HG)

7.1 $f(x) = \log_p x$ and $g(x) = ax^2 + bx$. A is the turning point of g, and B is a common x-intercept. ECD is perpendicular to the x-axis, with D on g. The point $C(2; 1)$ lies on f.



7.1.1 Determine the value of p. (2) 2

7.1.2 Write down the coordinates of B. (1) 2

7.1.3 Determine the coordinates of A. (3) 2

7.1.4 If A is the point $(\frac{1}{2}; 1)$, find the values of a and b. (4) 2

7.1.5 Find the length of CD. (3) 3

7.1.6 Find the equation of a function h if g and h are symmetrical about the x-axis. (2) 3

MIP 2016 pg 85

<u>March 2016</u>:

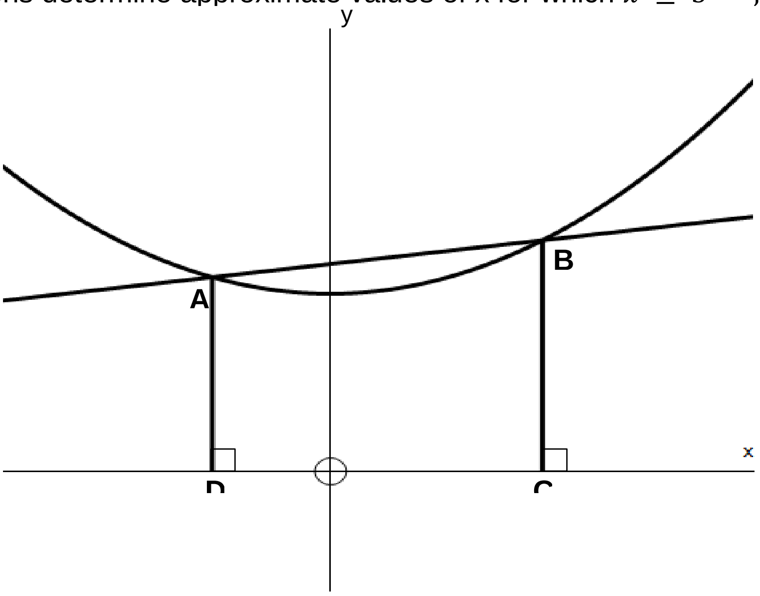
1.3 Determine the range of the function $y = x + \frac{1}{x}$, $x \neq 0$ and x is real. (6)

Question 5 B.1 Senior Certificate 2005 HG

Given: $f(x) = 3^x \text{ and } g(x) = \frac{3}{x}\,; x > 0$

5.1 Draw a sketch graph of *f* and *g* on the same set of axes. Clearly label the graphs and indicate all intercepts. (4) 2

5.2 From your graphs determine approximate values of x for which $x \geq 3^{1-x}$, using D



(i) Determine the difference in the heights above the road of the two points A and B. Give your answer to the nearest centimeter. (7)

(ii) Hence, calculate the area of quadrilateral ABCD. (3)

Question 1

1.1 On the same system of axes sketch the graphs of : $x + y = 4$; $g: x + 3y = 6$, where $x \geq 0$ and $y \geq 0$ (4)

1.2 Determine coordinate of the point when $\frac{f(x)}{g(x)} \geq 1$ (3)

1.3 Find the distance between $f(x)$ and $g(x)$ when $y = \frac{1}{2}$ (3)

Question 2

Given: $f(x) = \frac{1}{2}x^2; x \geq 0$

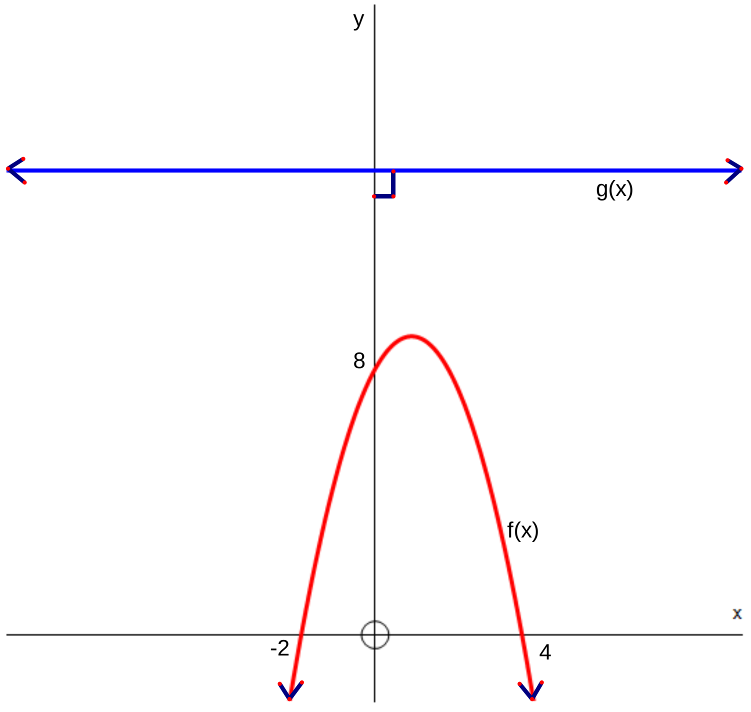
2.1 Determine the equation of $g(x)$ if $h(x)$ is a reflection of $f(x)$ in the $x - axis$ and then $h(x)$ is reflected in the $y - axis$ to create $g(x)$. (3)

2.2 On the same system of axes sketch the graphs of $f(x)$ and $g(x.)$ (4)

2.3 From your graphs determine the average gradient of $h(x)$ between $x = -4$ and $x = 4$ if $h(x)$ is a combined graph of $f(x)$ and $g(x)$ (4)

Question 3

Study the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = t^2$, where $a, b, c \in \mathbb{R}$, $a \neq 0$ and $t \in \mathbb{Z}$. Graph f cuts $x - axis$ at $x = -2$ and 4 and y – $axis$ of 8.



3.1 Show that $a = -1$ and $b = 2$. (3)

3.2 Determine the value(s) of t for which $f(x) - g(x) = 0$ will have non-real roots. (4)

Question 4

Given: $f(x) = \frac{1}{2}x^2 - \frac{1}{2}x - 1 \; and \; g(x) = x^2 - 6x + 11$

4.1 Sketch the graphs of f and g on the same system of axes. (4)

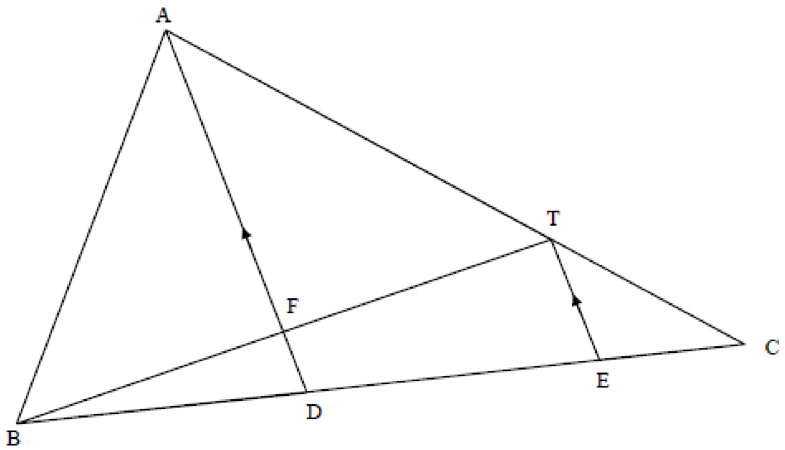
4.2 If $g'(x) = -2$

4.2.1 Determine the equation of the tangent of g. (3)

4.2.2 Hence, find the point of intersection of the tangent with $f(x)$. (3)

HIGHER ORDER – GR12 EUCLIDEAN GEOMETRY

1. In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD || TE.



1.1 Write down the numerical value of $\frac{CE}{ED}$ (1)

1.2 Show that D is the midpoint of BE. (2)

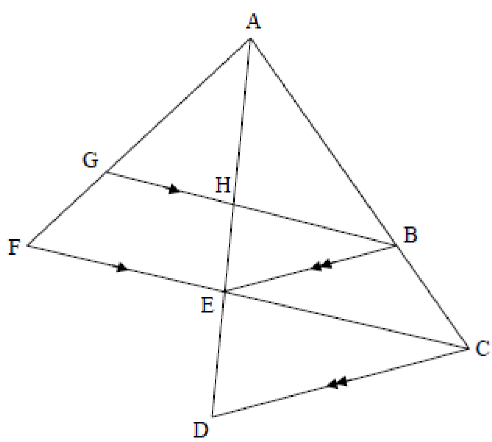
1.3 If FD = 2 cm, calculate the length of TE. (2)

1.4 Calculate the numerical value of:

1.4.1 $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (1)

1.4.2 $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

2. In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



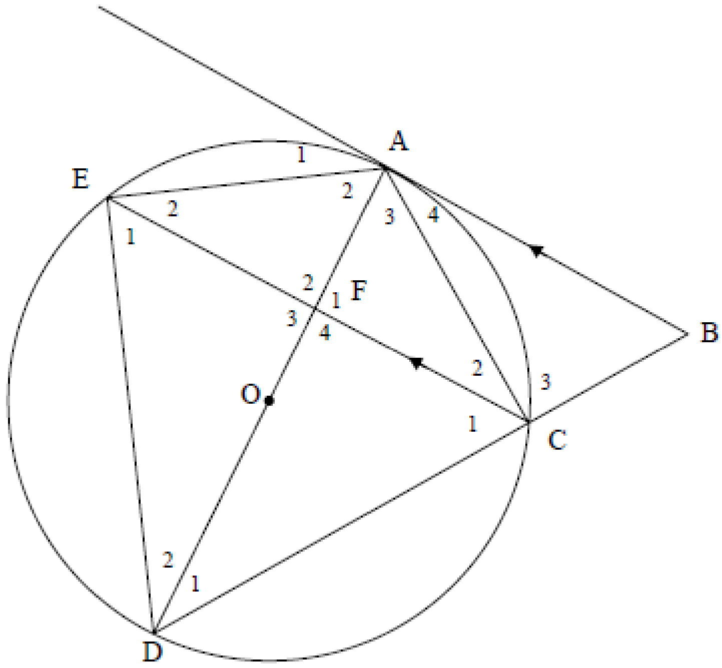
Calculate with reasons:

2.1.1 AH : ED (4)

2.1.2 $\frac{BE}{CD}$ (2)

2.2 If HE = 2 cm, calculate the value of AD × HE. (2)

3. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.

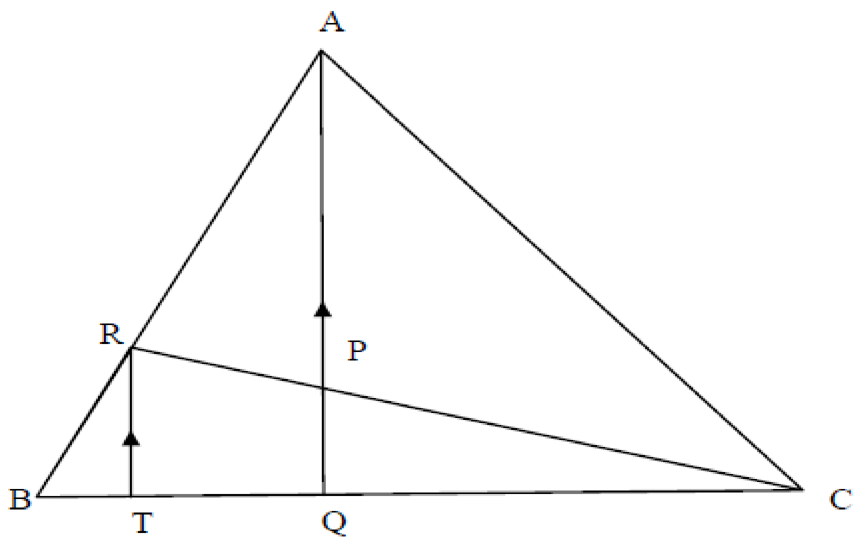


3.1 Show that $\hat{C}_2 = \hat{D}_1$ (3)

3.2 Prove that $\Delta ACF \,|||\, \Delta ADC$. (3)

3.3 Prove that $AD = 4AF$ (4)

4. In the figure AQ || RT, $\frac{BQ}{QC} = \frac{3}{5}$ and $\frac{BR}{RA} = \frac{1}{2}$



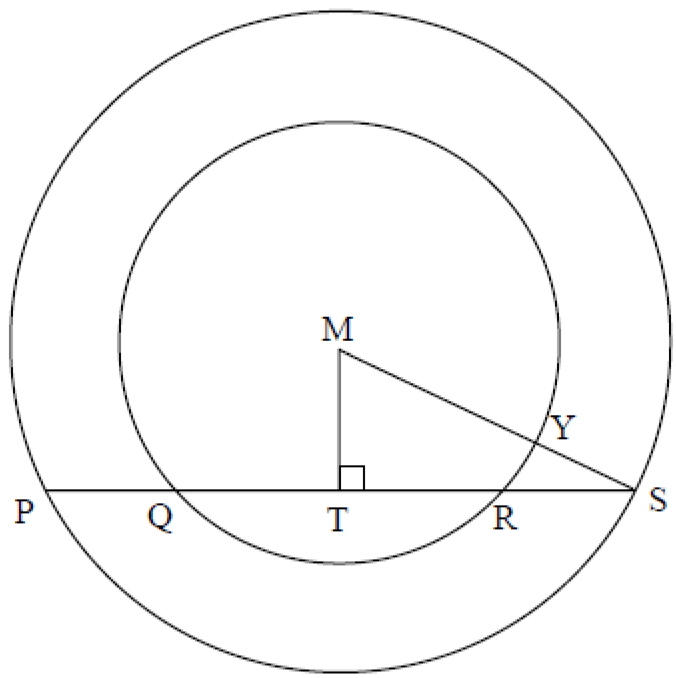
4.1. If BT = k, calculate TQ in terms of k. (2)

4.2. Hence, or otherwise, calculate the numerical value of:

4.2.1 $\frac{CP}{PR}$ (3)

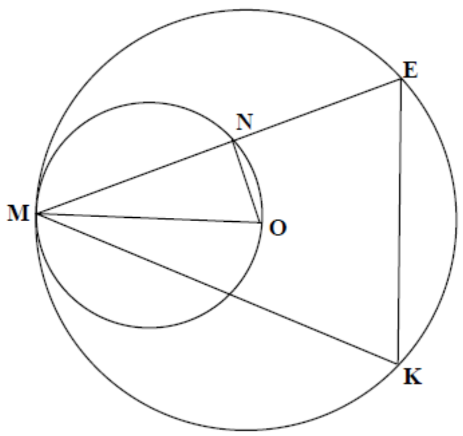
4.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC}$ (3)

5. In the diagram alongside, two concentric circles with centre at M and with radii 5 cm and 8,5 cm are given. PQRS is a chord of the larger circle cutting the smaller circle at Q and R. MYS is a straight line with Y on the smaller circle.



QR = 6 cm
Calculate, with reasons, the length of PS. (7)

6.

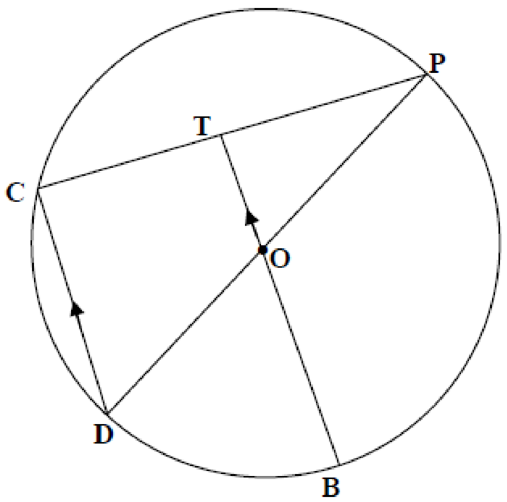


In the diagram alongside, O is the centre of circle MEK such that OM is the diameter of circle MNO. Chord EM of the larger circle cuts the smaller circle at N.
EM = $(2x^2 - 2)$ units and ON = $2x$ units

6.1 Express, with reasons, the length of the radius of circle MEK in terms of *x*. (7)

6.2 If $O\hat{M}N = \theta$ determine with reasons, the magnitude of $\hat{K}$ in terms of θ. (4)

7.



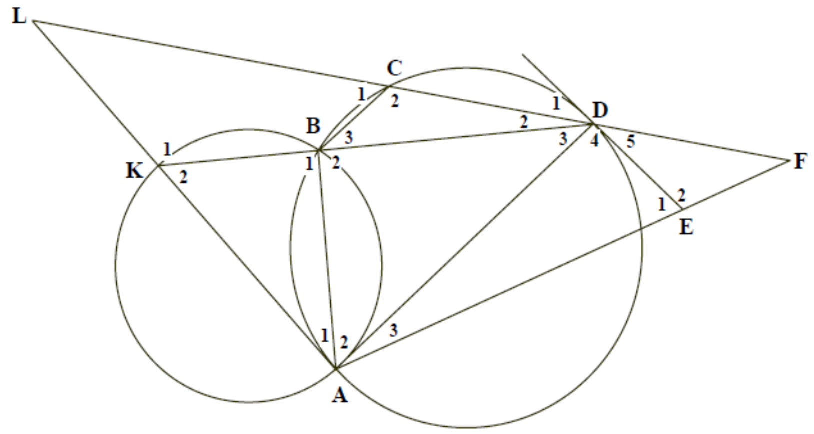
In the diagram alongside, O is the centre of circle PBDC. BOT is drawn with T on chord PC. PD and BT intersect at O. CD // TB.
PD = $10x$ units and OT = $3x$ units.

7.1 Determine TB : TO (2)

7.2 Prove that BT $\perp$ PC (3)

7.3 Express the length of PC in terms of *x* (5)

8.



In the diagram alongside, two circles intersect each other at A and B. ED is a tangent to circle ABCD. DA is a tangent to circle AKB. DBK is a straight line. AK and DC are produced to meet at L. LCD and AE are produced to meet at F. CD = DF

Prove that :

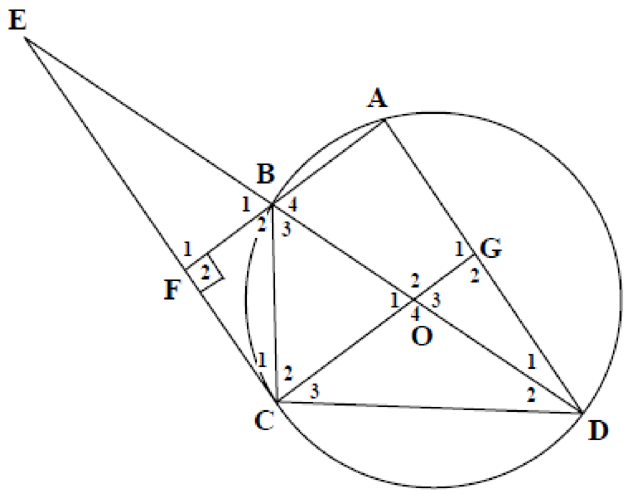
8.1 LKBC is a cyclic quadrilateral. (5)

8.2 $\hat{B}_2 = L\hat{A}D$ (3)

8.3 DE // LA (5)

8.4 CD. FA = FE . FL (4)

9.



In the diagram alongside, O is the centre of circle ABCD. AB produced meets tangent EC at F. AF ⊥ EC. CO is produced to G with G on AD. EBOD is a straight line.

9.1 Prove that FAGC is a rectangle. (7)

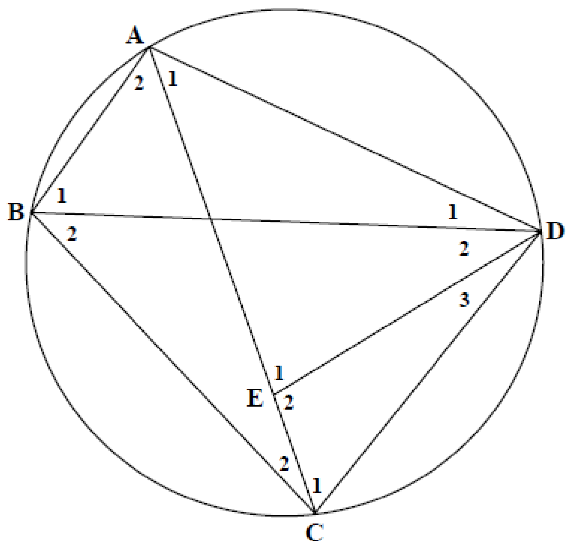
9.2 Prove that $\Delta FCB /// \Delta CBD$ (5)

9.3 Hence, complete the following statement to make it true : BC^2 = ... (1)

9.4 Prove that : CD^2 = CG . DB (5)

9.5 Hence, prove that : DB = FB + AF (5)

10.



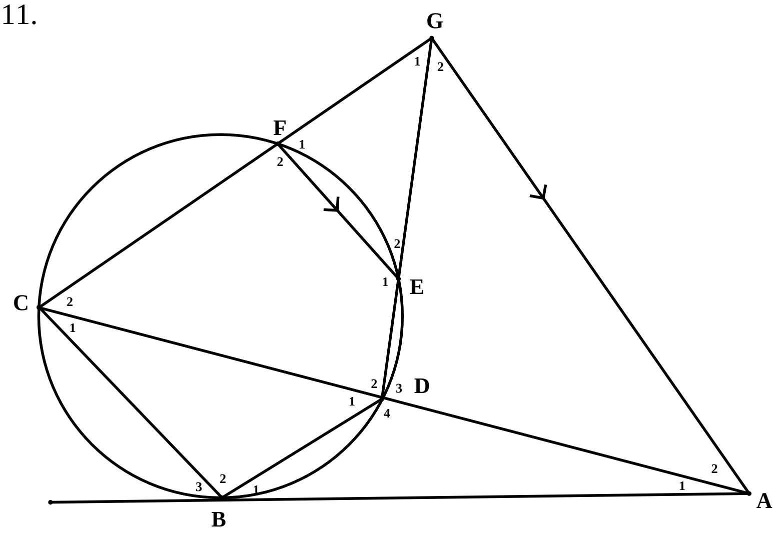
In the diagram alongside, ABCD is a cyclic quadrilateral. E is appoint on the diagonal AC such that $\hat{D}_1 = \hat{D}_3$

10.1 EC . BD = AB . CD (3)

10.2 AE . BD = BC . AD (5)

10.3 AC . BD = AB . CD + BC . AD (3)

11.

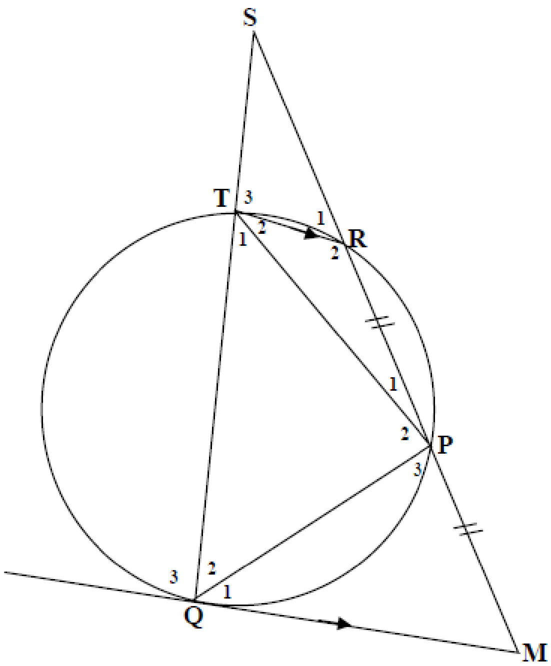


In the diagram FE // GA. BA is a tangent at B.
Prove that :

11.1 $AB^2 = AD . AC$ (3)
11.2 $\Delta ADG /// \Delta AGC$ (5)
11.3 AG = AB (2)

11.4 If AG = 6 units and AD = 4 units
Calculate:
11.4.1 The length of DC (2)
11.4.2 The length of GD correct to two decimal places, CE is the diameter of the circle. (3)

12.



In the diagram alongside, circle QPRT intersects SQ at T and SM at R and P.
QM // TR. MP = PR and 2ST = TQ
ST = *a* units and SR = *b* units

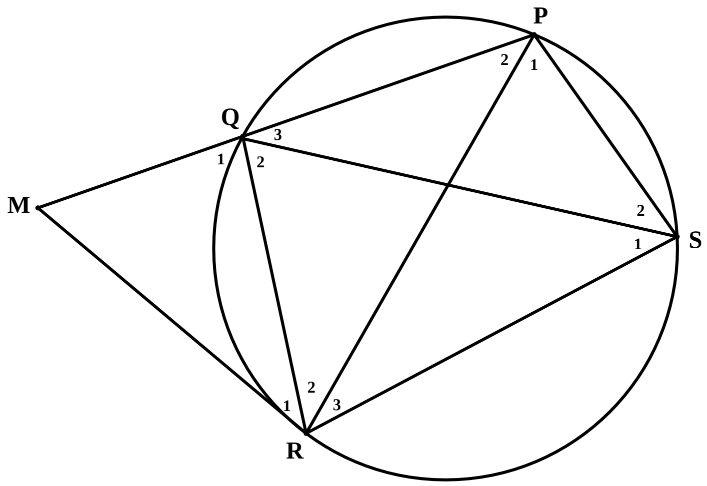
12.1 Prove that :

12.1.1 $\Delta SQP /// \Delta SMQ$ (6)

12.1.2 $\frac{QM}{QP} = \frac{3a}{2b}$ (6)

12.2 It is further given that $\Delta QPT /// \Delta MPQ$, prove :
12.2.1 QM is a tangent to circle QPRT (3)
12.2.2 $TP = \frac{4b}{3}$ (4)

13.



In the diagram PQRS is a cyclic quadrilateral.

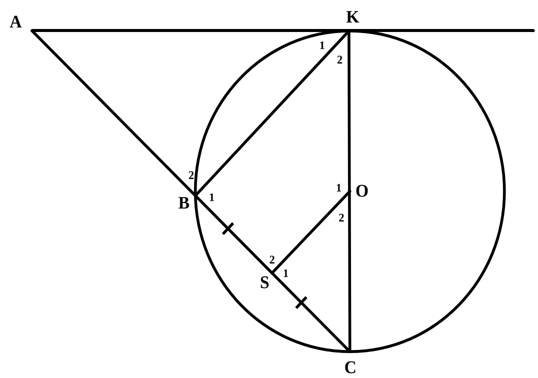
QM.PS = QR. RS and $\frac{MQ}{RS} = r$

13.1 Prove that $\frac{QR}{SP} = r$ (2)

13.2 Hence prove that $\frac{MR}{RP} = r$ (3)

13.3 Deduce that Δ MQR /// ΔRSP (2)

14. In the diagram below KOC is the diameter of circle O. KA is a tangent. AC intersects the circle at B. BS = SC. KB and OS are joined



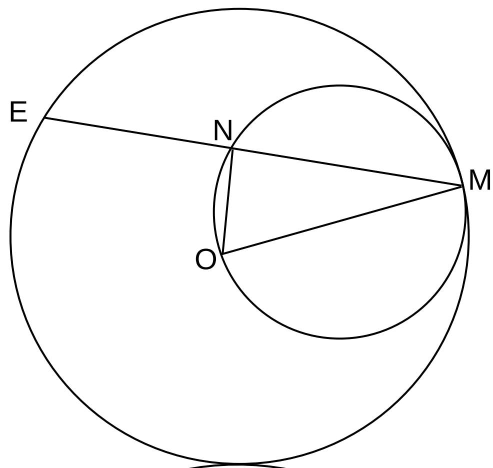
Prove :

14.1 CO : CS = CA : CK (3)

14.2 ΔCOS /// ΔKAB (4)

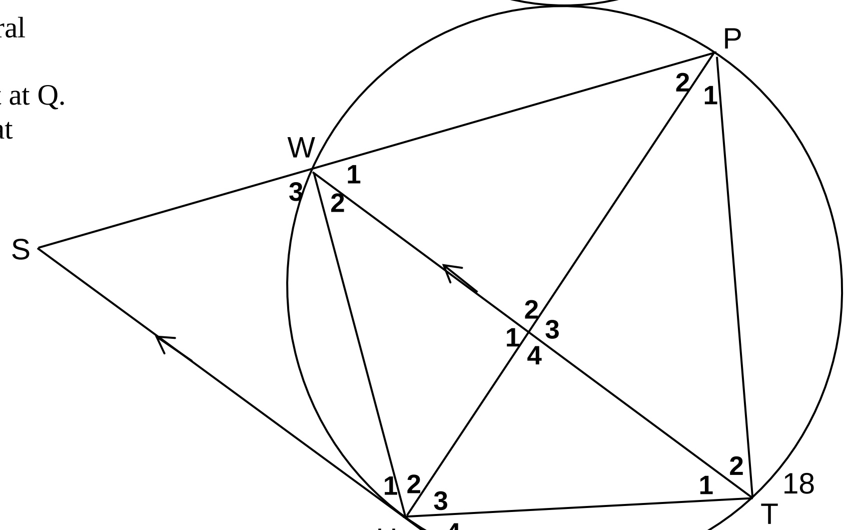
14.3 $2SO^2$ = CS . BA (3)

15. In the diagram alongside, OM is the radius of the larger circle and also the diameter of the smaller circle. Chord EM of the larger circle cuts the smaller circle at N.
If EM = $(2x^2 - 2)$ units and ON = $2x$ units, express, giving reasons, the length of the radius of the larger circle in terms of x .



(6)

16 In the diagram alongside,
PWUT is a cyclic quadrilateral with WU = TU.
Chords WT and PU intersect at Q.
PW is extended to S such that US ∥ TW.



Prove that:

16.1 US is a tangent to circle PWUT at U (5)

16.2 Δ SPU ||| Δ SUW (4)

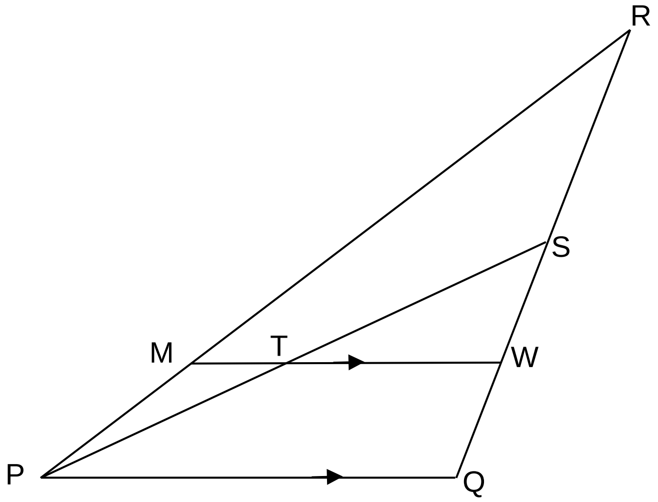
16.3 $SU^2 . QU = PU . SW^2$ (6)

17 .In the diagram alongside,

S is the midpoint of TQ in Δ PRQ.

T is the midpoint of PS

and MTW $\parallel$ PQ.

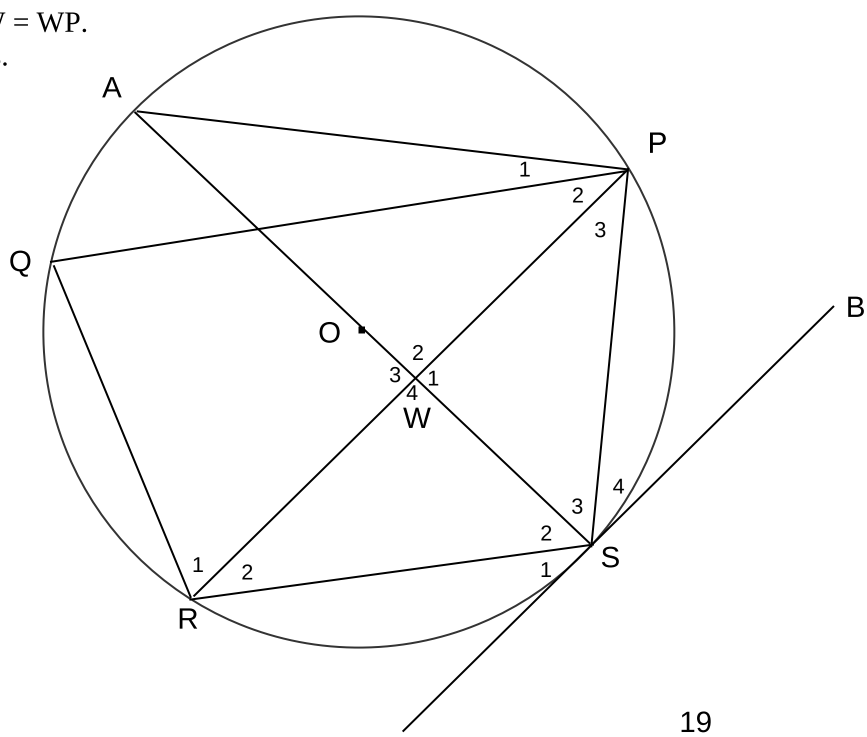


Calculate the numerical value of the following:

17.1 $\frac{RM}{RP}$ (6)

17 .2 $\frac{\text{area } \Delta \text{ RPS}}{\text{area } \Delta \text{ RMW}}$ (4)

18. In the diagram below, P, A, Q, R and S lie on the circle with centre O.
SB touches the circle at S and RW = WP.
AOWS and RWP are straight lines.

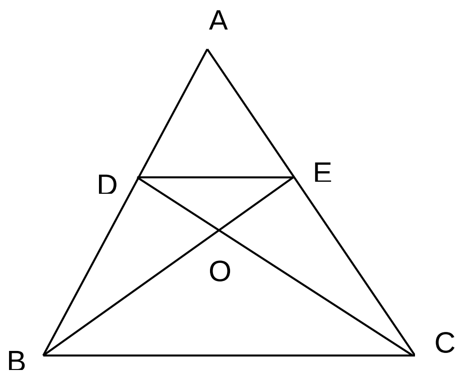


Prove that:

18.1 SB || RP (5)

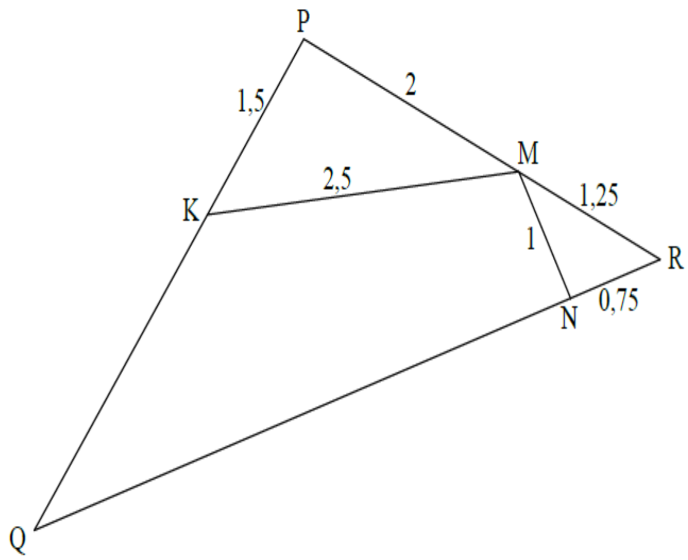
18.2 $RS^2 = WS \cdot AS$ (8)

18.3 $AS = \dfrac{RW^2}{WS} + WS$ (4)



19. In the figure D is the midpoint of AB and E is the midpoint of AC . DC and EB intersect at right angles at O.
Prove that:
$AB^2 + AC^2 = 5BC^2$

(8)



20. In the diagram alongside
K, M and N respectively are points on sides PQ, PR and QR of ΔPQR.
KP =1.5 ; PM = 2 ;KM =2.5 ;MN=1
MR=1.25 and NR= 0.75.

20.1 Prove that ΔKPM/// ΔRNM (3)

20.2 Determine the length of NQ (6)

20.3 Determine the numerical value of $\dfrac{Area\ of\ \Delta KPM}{Area\ of\ \Delta RNM}$ (2)

SEQUENCES AND SERIES [25 MARKS]

Question 1

1.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d,the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

1.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$. (4)

1.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$

$$T_3 - T_2 = 13$$

$$T_4 - T_3 = 19$$

1.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

1.3.2 Calculate the value of T_{69} if $T_{89} = 23\ 594$. (5)

[17]

Question 2

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \cdots$

2.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

2.2 Explain why this series converges. (1)

2.3 Calculate the sum to infinity of the series. (2)

2.4 What is the smallest value of n for which $s_\infty - S_n < 1$? (5)

[11]

2.5 The sequence $3\ ;\ 9\ ;\ 17\ ;\ 27$; .... is a quadratic sequence.

2.5..1 Write down the next term. (1)

2.5.2 Determine an expression for the n^{th} term of the sequence. (4)

2.5.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

Question 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove , without the use of a calculator, that the sum of the series to infinity is $16 + 8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \cdots$ to 20 terms.

3.2.1 Write the series in sigma notation. (3)

3.2.2 Calculate the value of x. (3)

[10]

Question 4

4.1 The sum to n terms of a sequence of numbers is given as : $S_n = \frac{n}{2}(5n + 9)$.

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence. Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)

[11]

Question 5

5.1 Given the sequence : $4 \ ; \ x \ ; \ 32$. Determine the value(s) of x if the sequence is :

5.1.1 Arithmetic (2)

5.1.2 Geometric (2)

5.2 Determine the value of P if

$P = \sum_{k=1}^{13} 3^{k-5}$ (4)

[8]

Question 6

The following sequence is a combination of an arithmetic and a geometric sequence:

$3 \ ; \ 3 \ ; \ 9 \ ; \ 6 \ ; \ 15 \ ; \ 12$; ....

6.1 Write down the next TWO terms. (2)

6.2 Calculate $T_{52} - T_{51.}$ (5)

6.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

[9]

Question 7

A quadratic pattern has a second term equal to 1 , a third term equal to -6 and a fifth term equal to -14.

7.1 Calculate the second difference of this quadratic pattern. (5)

7.2 Hence or otherwise , calculate the first term of the pattern. (2)

[7]

Question 8

Given the arithmetic series : $-7-3+1+\cdots+173$.

8.1 How many terms are there in the series? (3)

8.2 Calculate the sum of the series. (3)

8.3 Write the series in sigma notation. (2)

[8]

Question 9

9.1 Consider the geometric sequence:

$4\ ;\ -2\ ;\ 1\ ;$ .....

9.1.1 Determine the next term of the sequence. (1)

9.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

9.1.3 Calculate the sum to infinity of series $4-2+1$ .... (2)

9.2 If x is a real number, show that the following sequence can NOT be geometric:

$1\ ;x+1\ ;x-3$ .... (3)

[10]

Question 10

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, , and has the form $d(n)=an^2+bn+c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

10.1 Determine the values of r and s. (2)

10.2 Determine the values of a ; b and c. (3)

10.3 How far is the athlete from P when $n=8$. (2)

10.4.4 Show that the athlete is moving towards P when $n<5$, and away from P when $n>5$. (3)

Question 11

11.1 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x .

11.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively. (2)

11.2.1 Calculate the 11th term of the sequence. (2)

11.2.2 The sum of the first n terms of this sequence is -560. Calculate n. (6)

[10]

Question 12

Given the geometric sequence: 27 ; 9 ; 3 ......

12.1.1 Determine a formula for T_n. (2)

12.1.2 Why does the sum to infinity for this sequence exist? (1)

12.1.3 Determine S_∞. (2)

[5]

Question 13

13.1 Given a geometric series:

$256 + p + 64 - 32 +$......

13.1.1 Determine the value of p. (3)

13.1.2 Calculate the sum of the first 8 terms of the series. (3)

13.1.3 Why does the sum to infinity for this series exist? (1)

13.1.4 Calculate S_∞. (3)

13.2 Consider the arithmetic sequence:

-8 ; -2 ; 4 ; 10 ; .....

13.2.1 Write down the next term of the sequence. (1)

13.2.2 If the n^{th} term of the sequence is 148 ; determine the value of n. (3)

13.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

13.3 Calculate $\sum_{k=1}^{30}(3k + 5)$ (3)

[22]

Question 14

Consider the sequence: 3 ; 9 ; 27 ; ...

Jacob says that the fourth term of the sequence is 81. Vusi disagrees and says that the fourth term of the sequence is 57.

14.1 Explain why Jacob and Vusi could both be correct. (2)

14.2 Jacob and Vusi continue with their number patterns. Determine a formula for n^{th} term of:

14.2.1 Jacob's sequence. (1)

14.2.2 Vusi's sequence. (4)

[7]

Question 15

The values below are consecutive terms of a sequence that behaves consistently. The 4th term is 36.

.... ; ....; ..... ; 36 ; 54 ; 75 ; 99 ; ..... ; ..... ; ......

15.1 Determine the 1st ; 2nd and 3rd terms of this sequence. (3)

15.2 Hence , determine a general formula for the nth term of this sequence. (4)

[7]

Question 16

16.1 Determine $\sum_{m=1}^{5} 4$
(1)

16.2 Consider the arithmetic series below:

$$-7 - 3 + 1 + \cdots + 173$$

16.2.1 Determine the number of terms in the series. (3)

16.2.2 Hence, rewrite the given series in sigma notation. (3)

[7]

Question 17

17.1 Prove that the sum to *n* terms of a geometric series, of which the first term is *a* and the common ratio is *r* , can be given as:

$S_n = \frac{a(r^n-1)}{r-1}$; $r \neq 1$ (4)

17.2 Grace, who weighs 52kg, desperately seeks to gain weight. She follows a specific diet and training programme, in order to obtain her goal. She gains 2kg per week for the first two weeks.

Thereafter, her weekly weight gain is 20% less than the weight gain in the previous week. Grace decides to follow this diet and fitness programme strictly, and to continue this pattern of weight gain indefinitely.

17.2.1 Write down, as a sequence, Grace's weight gained during the first FOUR weeks. (2)

17.2.2 Calculate Grace's weight after 15 weeks on this special programme. (5)

17.2.3 Calculate the maximum weight Grace will gain while following this programme. (3)

[14]

Question 18

18.1 Given the geometric sequence: 7 ; x ; 63 ; ... Determine the possible value of x. (3)

18.2 The first term of a geometric sequence is 15. If the second term is 10, calculate:

18.2.1 T_{10} (3)

18.2.2 S_9 (2)

18.3 Given : $0\ ;\ -\frac{1}{2}\ ;0\ ;\ \frac{1}{2}\ ;0\ ;\ \frac{3}{2}\ ;\ 0\ ;\ \frac{5}{2}\ ;0\ ;\ \frac{7}{2}\ ;\ 0\ ;...$ Assume that this number pattern continues consistently.

18.3.1 Write down the value of the 191st term of this sequence. (1)

18.3.2 Determine the sum of the first 500 terms of this sequence. (4)

18.4 Given : $\sum_{k=2}^{20}(4x-1)^k$ (2)

18.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x = 1$. (2)

18.4.2 For which values of x will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (3)

[18]

Question 19

19.1 Write down the next term of the number pattern: $\frac{1}{2}\ ;\ \frac{8}{9}\ ;\ \frac{27}{28}\ ;...$

19.1.1 Determine the general term. (2)

19.2 Given: $2\ ;6\ ;...$ Write down the value of k if the sequence is:

19.2.1 Arithmetic (1)

19.2.2 Geometric (1)

19.3 Evaluate the sum of the infinite series: $5{,}6\ +3{,}36\ +2{,}016\ +\ 1{,}2096\ +....$ (1)

19.4 Given: $0\ ;\ -1\ ;\ 1\ ;\ 6\ ;14$

19.4.1 Show that this sequence has a second difference. (2)

19.4.2 Determine a simplified expression for the n^{th} term of the sequence. (4)

19.4.3 Find the 30^{th} term. (2)

[15]

Question 20

The sum of the first n terms of a sequence is given by $S_n = 2^{n+2} - 4$.

20.1 Determine the sum of the first 24 terms. (2)

20.2 Determine the 24^{th} term. (3)

20.3 Prove that the n^{th} term of the sequence is 2^{n+1}. (4)

[7]

Question 21

Given the sequence: 5 ; 12 ; 21 ; 32 ; .....

21.1 Determine the formula for the n^{th} term of the sequence. (4)

21.2 Determine between which two consecutive terms in the sequence is the first difference equal to 245. (5)

21.3 Sketch a graph to represent the second differences. (2)

[11]

Question 22

22.1 Given: $16 + 8 + 4 + 2 + \cdots$

22.1.1 Determine the sum of the first forty (40) terms of the series. (3)

22.1.2 Write the given series in sigma notation. (2)

22.1.3 Explain why the series converges. (2)

22.2 Calculate: $\sum_{k=3}^{350}(1 - 3k) + \sum_{t=1}^{200}(D_x[6x]$ (6)

[13]

Question 23

The first term of a linear number pattern is 92 and the constant difference is -4.

23.1 Write down the values of the second and third terms of the number pattern. (2)

23.2 Determine an expression for the $n - th$ term of the number pattern. (2)

23.3 Determine the value of the eighteenth term. (2)

[6]

Question 24

24.1 The following number pattern has a constant second difference.

$41; 43; 47; 53; 61; 71; 83; 97; 113; 131; 151; 173; 197; 223; 251;$ ....

24.1.1 Write down the value of the constant difference. (2)

24.1.2 Determine the $n - th$ term of the number pattern. (4)

24.1.3 The first forty terms of the number pattern are all prime numbers. Determine the forty –first term and show that it is not a prime number. (3)

[9]

Question 25

25.1 Given the arithmetic series: $a + 13 + b + 27 + \cdots$

25.1.1 Show that $a = 6$ and $b = 20$ (2)

25.1.2 Calculate the sum of the first 20 terms of the series. (3)

25.1.3 Write the series in QUESTION 25.1.2 in sigma notation. (2)

25.2 Given the geometric series: $(x - 2) + (x^2 - 4) + (x^3 + 2x^2 - 4x - 8) + \cdots$

25.2.1 Determine the value(s) of x for which the series converges. (4)

25.2.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series. (4)

[15]

Question 26

The first four terms of a quadratic number pattern are $-1\ ;\ 2\ ;\ 9\ ;\ 20$.

26.1 Determine the general term of the quadratic number pattern. (4)

26.2 Calculate the value of the 48th term of the quadratic number pattern. (2)

26.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)

26.4 If the sum of the first 69 first differences in Question 3.3 equals 9 591 (that is , $S_{69} = 9\ 591$) ,which term of the quadratic number pattern has a value of 9 590? (2)

[11]

Question 27

27.1 Given the sequence: 2 ; 2 ; 5 ; 4 ; 8 ; 8 ; ....is a combination of a linear and geometric sequence.

27.1.1 If the pattern continues, then write down the next TWO terms. (2)

27.1.2 Calculate the sum of the first 40 terms of the sequence. (7)

27.2 Given the geometric series: $9x^2 + 6x^3 + 4x^4 + \cdots$

27.2.1 Determine a formula forT_n,the n^{th} term of the series. (2)

27.2.2 For which value(s) of x will the series converge? (3)

27.3 The sum to infinity of a geometric series with positive terms is 16 and the sum of the first two terms is 12. Determine the values of a and r. (8)

[22]

QUESTION 28

28.1 The twelfth term of an arithmetic sequence is 5 and the common difference of the sequence is 3.

28.1.1 Find the value of the first term (2)

28.1.2 Determine which term has a value of 47 (3)

28.2 The sum to n terms of an arithmetic series is $S_n = 4n^2 + 1$

28.2.1 Find the 15th term (3)

28.2.2 How many terms must be added to give a sum of 10001? (4)

28.3 The first term of a geometric series is 9 and the ratio of the sum of eight terms to the sum of the four terms is 97:81. Find the first three terms of the series, if it is given that all the terms of the series are positive. (8)

[20]

Question 29

29.1 Given: 0 ; -1 ; 1 ; 6 ; 14 ; ...

29.1.1 Show that this sequence has a constant second difference. (1)

29.1.2 Write down the next term of the sequence. (1)

29.1.3 Determine an expression for the n^{th} term of the sequence. (4)

29.1.4 Calculate the 30th term. (2)

29.2 In an arithmetic series: $a + 13 + b + 27 + \cdots$.

29.2.1 Prove that $a = 6$ and $b = 20$. (2)

29.2.2 Determine which term of the series will be equal to 230. (3)

29.3 For which value(s) of k will the series:

$\left(\frac{1-k}{5}\right) + \left(\frac{1-k}{5}\right)^2 + \left(\frac{1-k}{5}\right)^3 + \cdots$ converge? (3)

29.4 Given: 16 + 3 + 8 + 3 + 4 + 3 + 2 + ...

29.4.1 Determine the sum of the first 40 terms of the series, to the nearest integer. (5)

29.4.2 Write the series: 16 + 8 + 4 + 2 +....in the form $\sum_{k=\ldots}^{\ldots} T_k$ (3)

29.4.3 Determine S_∞ of the series in Question 29.4.2 (2)

[26]

Question 30

The first two terms of an arithmetic series, A, and an infinite geometric series, B, are the same.

A: $-2 + x + \cdots$ and

B: $-2 + x + \cdots$are given.

30.1 Write down in terms of x

30.1.1 The third term of the geometric series, B (2)

30.1.2 The third term of the arithmetic series, A. (2)

30.2 If the sum of the first three terms in the arithmetic series A is equal to the third term of the geometric series B, then calculate the value of x. (5)

30.3 If $x = -6$, does the geometric series B converge? Show calculations to support your answer. (3)

[12]

Question 31

Given: $\sum_{k=1}^{n} T_k = n^2 + 4n$, where T_k is the general term of a series.

31.1 Calculate $\sum_{k=1}^{250} T_k$ (2)

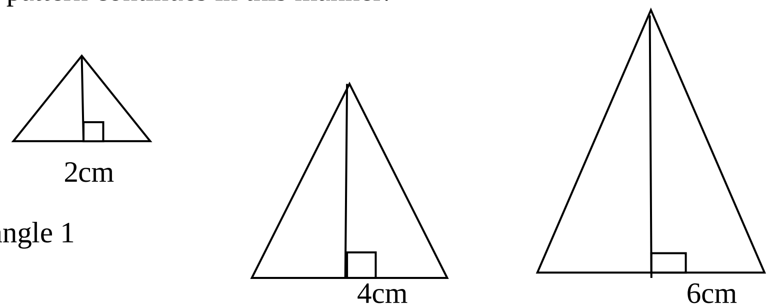
31.2 Calculate T_{100} (3)

31.3 How many terms of the sequence must be added to give a sum of 1 440? (3)

[8]

Question 32

A pattern of triangles is formed by increasing the base of the triangle by 2 cm and the perpendicular height by 1 cm, in each successive triangle. The first triangle has a base of 2cm and a height of 2cm. The pattern continues in this manner.



Triangle 1

Triangle 2

Triangle 3

32.1 Calculate the areas of the first four triangles. (2)

32.2 Calculate the area of the hundredth triangle in the pattern. (4)

[6]

Question 33

33.1 The following is a combination of a linear and a geometric series:

$$3b - 6 + 6b - 10 + 12b - 14 + \cdots$$

33.1.1 Write down the next two terms of the series. (2)

33.1.2 Determine in terms of , the sum of the first 30 terms of this series. (6)

33.2 Given the series $54 + 18 + 6 + \cdots$

33.2.1 Determine the n^{th} term of this series. (2)

33.2.2 Hence , determine the 12^{th} term of the series. (2)

33.2.3 Show that the sum to n terms of this series is $81 - 81(\frac{1}{3})^n$. (3)

33.2.4 Calculate: $\sum_{n=0}^{10} 54\left(\frac{1}{3}\right)^n$. (3)

33.2.5 Determine the maximum value of: $\sum_{n=0}^{k} 54\left(\frac{1}{3}\right)^n$. (3)

[18]

Question 34

Given the quadratic sequence: x ; y ; 16 ; 28 ; 42 ; 58 ...

34.1 Determine the values of x and , the first two terms of the sequence. (2)

34.2 Determine the 45^{th} term of this sequence. (5)

[7]

Question 35

35.1 Evaluate: $\sum_{n=3}^{20}(15-4n)$ (4)

35.2 A water tank contains 216 litres of water at the end of day 1. Because of a leak, the tank loses one-sixth of the previous day's contents each day. How many litres of water will be in the tank at the end of:

35.2.1 the 2nd day? (2)

35.2.2 the 7th day? (3)

35.3 Consider the geometric series: $2(3x-1)+2(3x-1)^2+2(3x-1)^3+\cdots$

35.3.1 For which values of x is the series convergent? (3)

35.3.2 Calculate the sum to infinity of the series if $x=\frac{1}{2}$. (4)

35.4 $2\,;x\,;12\,;y\,;$ ... are the first four terms of a quadratic sequence. If the second differences is 6, calculate the values of x and y. (5)

[19]

Question 36

Given the sequence : $x\,;x+1\,;\,-3\,;x-3\,;$ ... with a constant second difference.

36.1 Determine the value of x. (3)

36.2 Determine the n^{th} term of this sequence. (5)

[8]

Question 37

37.1 Determine the common difference and the first term of an arithmetic sequence in which the 8th term is -15 and the sum of the first eight terms is -8. (6)

37.2 Prove that the sum of the series

$2^x+2^{x+1}+3.2^x+2^{x+2}+\cdots\ (15 terms)=15.2^{x+3}$
(4)

37.3 If $(a+1)+(a-1)+(2a-5)+\cdots$are the first three terms of a convergent geometric series, calculate:

37.3.1 The value of a where $a>0$ (4)

37.3.2 The sum to infinity of the series (4)

[26]

Question 38

38.1 Consider the following quadratic sequence: $6\,;x\,;26\,;45\,;y\,;$ ...

Calculate the values of x and y. (6)

38.2 Given the following g series: $-11-4+3+\cdots+220$

38.2.1 Calculate the sum of the series. (4)

38.2.2 Write the series in sigma notation. (3)

38.3 Each time a photocopy is made from a previous photocopy, the quality of the print decreases by 11%. Determine how many times this photocopy can be done before the quality becomes less than 20% of the original. (4)

38.4 For which values of k will the series:

$$4\left(\frac{1-k}{5}\right)+8\left(\frac{1-k}{5}\right)^2+16\left(\frac{1-k}{5}\right)^3+\cdots \text{ converge?}$$ (4)

38.5 The sum of the first n terms of a sequence is: $S_n = 3^{n-5} + 2$.

Determine the 80th term. Leave your answer in the form $a.b^p$ where a ; b and p are all integers. (3)

[24]

COORDINATE GEOMETRY

LINES AND TRIANGLES

- distance formula   d = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- midpoint formula   M$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ i.e. the *average* of $(x_1; y_1)$ and $(x_2; y_2)$
- gradient of a straight line   m = $\frac{y_1 - y_2}{x_1 - x_2}$
- angle of inclination --- (angle between two lines)
- equation of a straight line   1. $y = mx + c$
-

  2. $ax + by + c = 0$

  3. $y - y_1 = m(x - x_1)$

- parallel lines have ***equal*** gradients   i.e. $m_1 = m_2$
- perpendicular lines have ***negative inverse*** gradients   i.e. $m_1 \times m_2 = -1$
- Point of intersections

1. **CIRLES AND TANGENTS**

- Circles with centre not at origin
- Touching circles
- Not touching circles
- Circles cutting each other

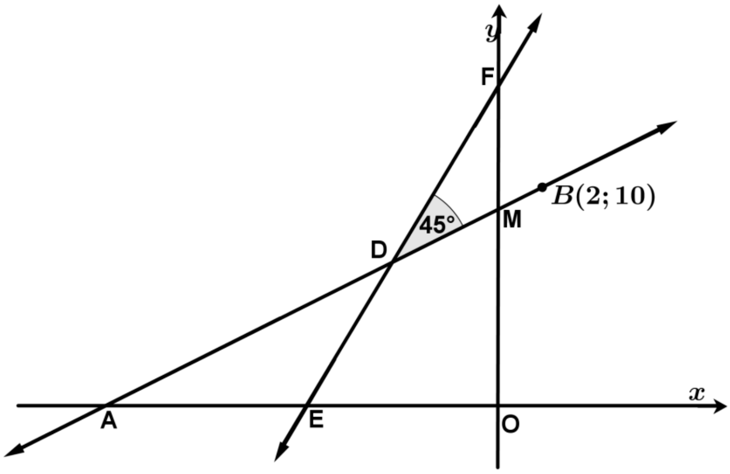
2. **PROPERTIES OF FIGURES**
3. **-** **MUST** LEARN AND KNOW

Question 1

In the given diagram, E and F are the x- and y- intercepts of the line having equation

$y = 6x + 16$. The line through B (2; 10) making an angle of 45 with EF, as shown below, has x-and y-intercepts A and M

.



1.1. Determine the coordinates of E. (2)

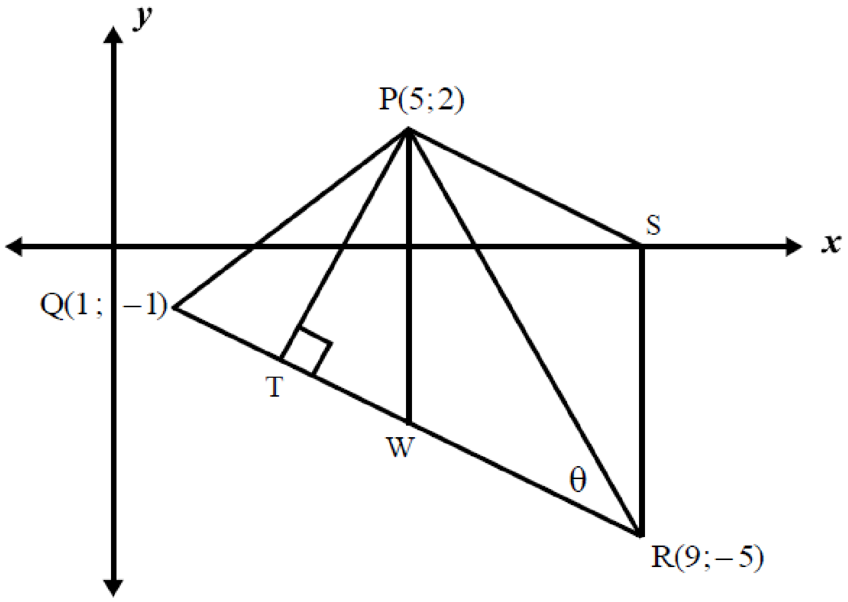
1.2. Calculate the size of $D\hat{A}E$. (3)

1.3. If AB intersect EF at D, Calculate the coordinates of D (6)

1.4. Calculate the area of quadrilateral DMOE (6)

Question 2

In the diagram, PQRS is a trapezium with vertices and S. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and P (5; 2), Q(1;1), R(9;5) $R\hat{P}S = \beta$



2.1 Determine the equation of PW if W is the mid-point of QR (2)

2.2. Determine the equation of PS (4)

2.3. Determine the equation of PS (4)

2.4. Determine the equation of PT. (3)

2.4.Show that $QT = \frac{1}{3}TR$ (5)

2.5. Calculate the size of β rounded off to two decimal places (5)

QUESTION 3

Consider the following points on a Cartesian plane: $A(1\,;2), B(3\ 1), C(-3\,;k)\ and\ D(2\,;\ -3)$
Determine k, if:

3.1. $(-1\,;3)$ is the midpoint of AC (3)

3.2. AB is parallel to CD (3)

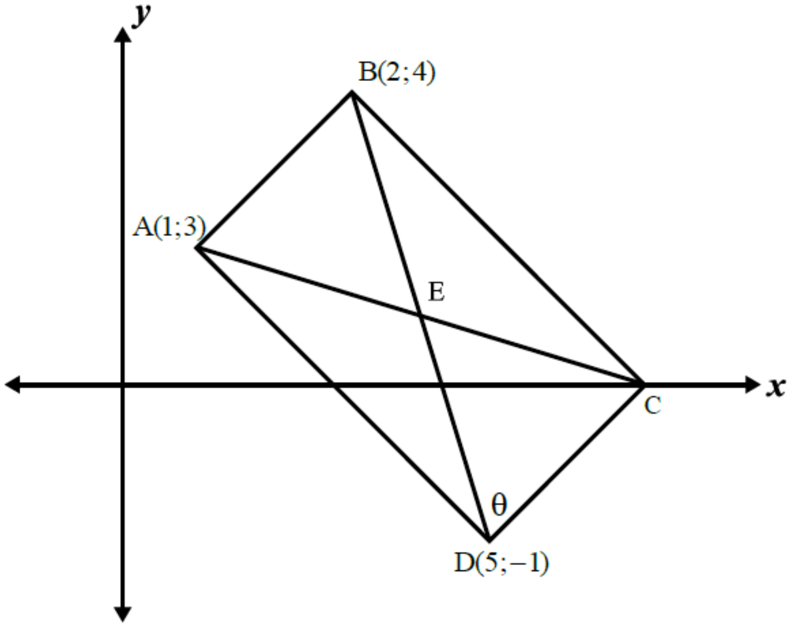
3.3. $AB \perp AC$ (3)

3.4. $A, B\ and\ C$ are collinear. (3)

3.5. $CD = 5\sqrt{2}$ (5)

QUESTION 4

ABCD is a parallelogram with vertices $A(1\,;3), B(2\,;4), C\ and\ D(5\,;\,-1)$



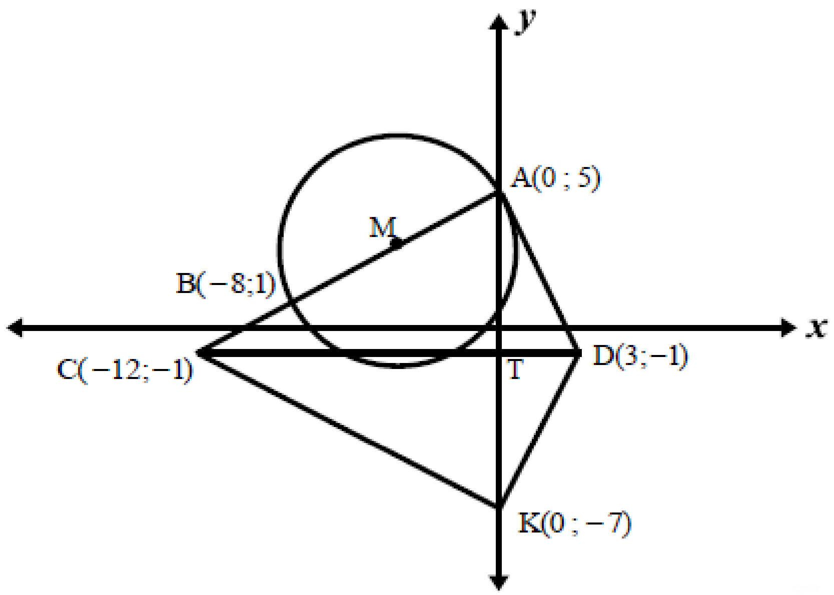
4.1. Determine the coordinates of C (1)

4.2. Show that ABCD is a rectangle. (3)

4.3. Determine the area of ABCD. (4)

Question 5

A(0 ; 5) and B(-8 ; 1) are two points on the circumference of the circle centre M, in a Catersian plane. M lies on AB. DA is a tangent to the circle at A. Points D (3 ; -1) and C (-12 ; -1) are joined. K is the point (0 ; -7). CTD is a straight line.



5.1. Determine the equation of CD. (1)

5.2. Determine the equation of the tangent AD. (4)

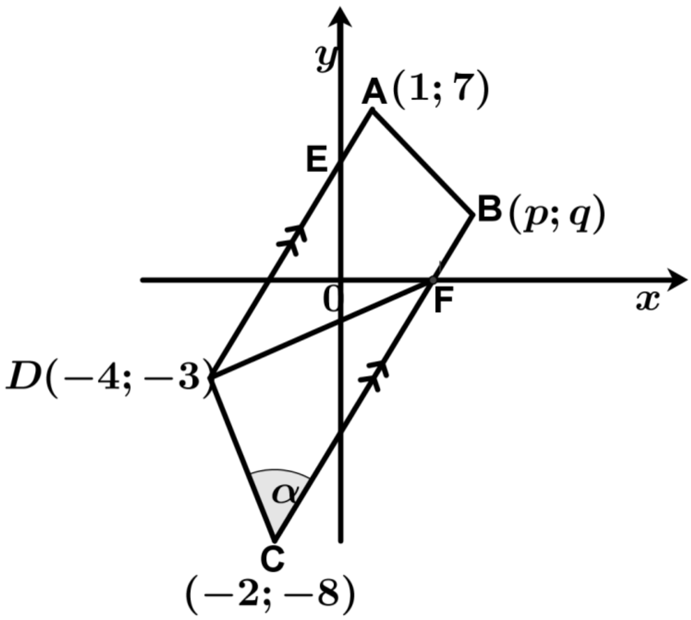
5.3. Determine the length of AM. (3)

5.4 Determine the equation of the circle centre M in the form
$ax^2 + by^2 + cx + dy + e = 0$ (4)

5.5 Quadrilateral ACKD is one of the following: parallelogram; kite, rhombus, rectangle.
Which one is it? Justify your answer. (4)

Question 6

In the diagram below, trapezium ABCD with AD// BC is drawn. The coordinates of the vertices are $A(1;7); B(p;q); C(-2;-8)\ and\ D(-4;-3)$. BC intersects the x-axis at F.$D\acute{C}$



6.1. Calculate the gradient of AD (2)

6.2. Determine the equation of BC in the form of $y = mx + c$ (3)

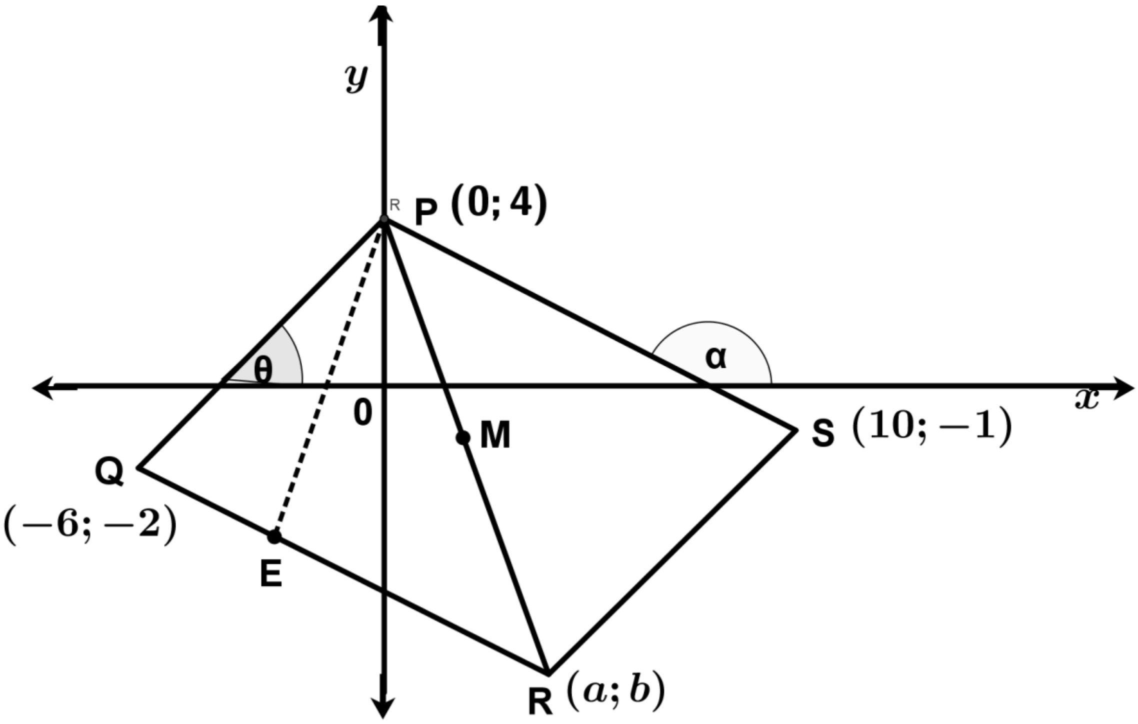
6.3. Determine the coordinates of F (2)

6.4. Show that $\alpha = 48{,}37°$ (4)

6.5. Calculate the area of $\Delta\, DCF$ (6)

Question 7

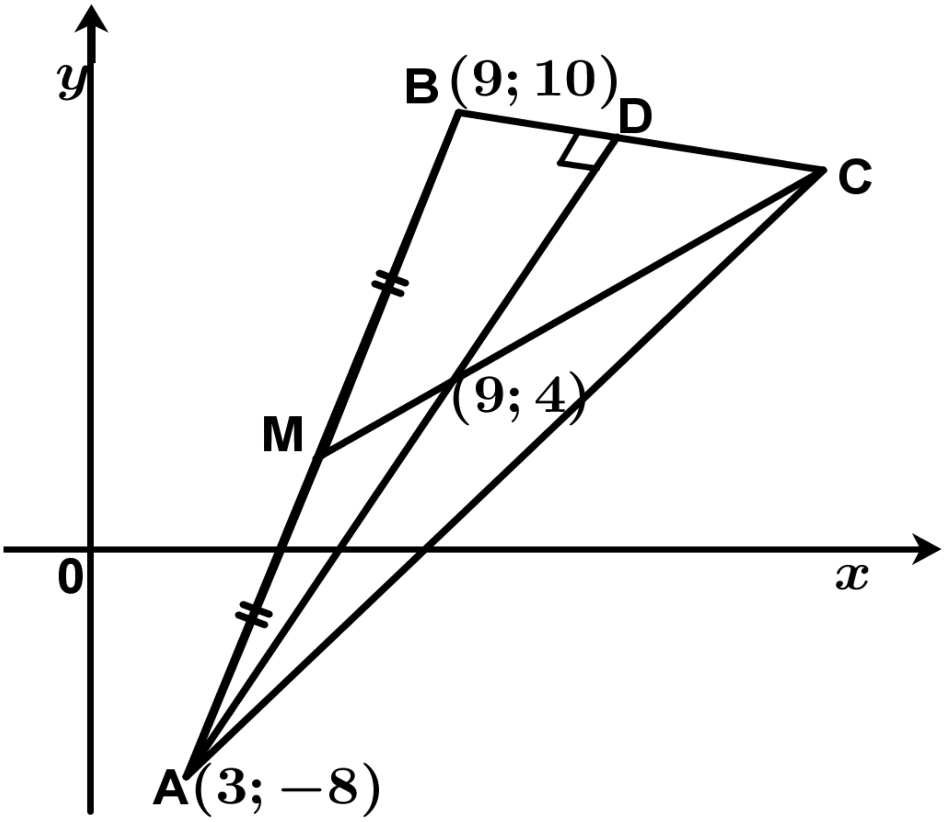
PQRS is a parallelogram with $P(0;4); S(10;-1); R(a;b)\ and\ Q(-6;-2)$ as shown below



7.1.	Determine the gradient of PQ	(2)
7.2.	Determine the equation of SR	(3)
7.3.	Determine the co-ordinates of M, the midpoint of PR	(3)
7.4.	Hence or otherwise, determine the value of a and b.	(3)
7.5.	Determine the coordinates of E if PE is perpendicular to QR with point E on QR	(7)
7.6.	Calculate the area of PQRS	(5)

Question 8

Given $B(9;10), Q(9;4); A(-3;8)$. M is the midpoint of BA; $AD \perp BC$



8.1. Find the gradient of AD. (2)

8.2. Write down the gradient of BC (2)

8.3. Determine the equation of BC (4)

8.4. Write down the coordinates of M (2)

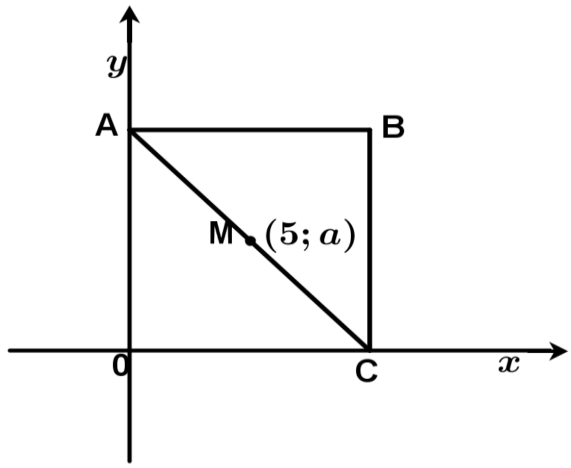
8.5. Determine the equation of CM (5)

8.6. Determine the coordinates of C. (5)

8.7. Determine the size of $B\hat{A}D$ (4)

Question 9

In the accompanying figure, ABCO is a rectangle. The length of OA is 6 units and $M(5;a)$
is the midpoint of the diagonal AC.



9.1 Determine the value of A. (1)

9.2 Write down the coordinates of each of the following

9.2.1. A (2)

9.2.2.C (2)

9.2.3.B (3)

9.3. The straight line with equation $px - 7y + 8 = 0$. Determine the value of p in each of the following

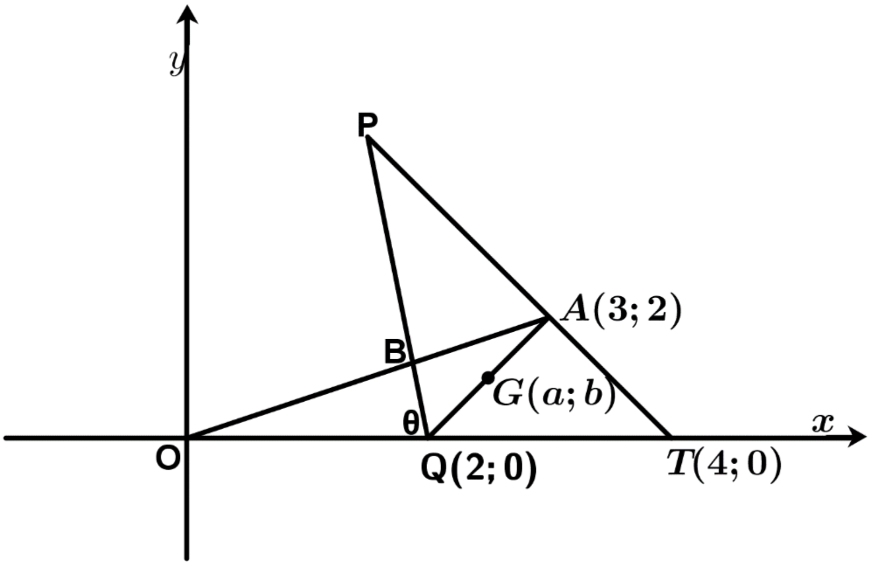
9.3.1. Passes through the point (2;2)

9.3.2. Makes an intercept of -4 on the x-axis (2)

9.3.4. Is parallel to the x-axis (3)

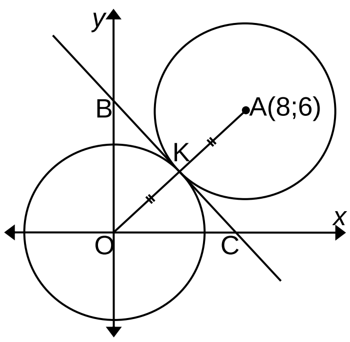
Question 10

In the diagram below, which is not drawn to scale, is the midpoint of the line segment TP. $O\hat{Q}A = \theta$ and $PQ \perp OA$. Determine



10.1	The coordinates of P	(3)
10.2	The gradient of AQ	(2)
10.3	The angle of θ , correct to ONE decimal place.	(3)
10.4.	The area of ΔAQO	(4)
10.5	$G(a; b)$ is a point of AQ. Show that $b = 2a - 4$	(3)
10.6.	The size of $Q\hat{A}T$	(3)
10.7.	the coordinates of B	(4)

Question 11



In the above diagram , O and A are the centres of the two circles with equal raddi. The two circles touch each other at point K and OKA is a straight line.

11.1.	Calculate the co-ordinates of K with KA as a radius.	(2)
11.2.	Write down the equation of circle A.	(3)

11.3. Write down the equation of circle A with AO as the radius. (3)

11.4.Find the equation of BKC, a common tangent to circle A and circle O. (3)

11.5. Does OC = OB? Explain. (2)

11.6. What kind of shape will OCAB be? Explain. (2)

Question 12

12.1. Equations of circles with centres A and B respectively, are given below.
$circle\ A: (x-2)^2 + (y-3)^2 = 9$ $circle\ B: (x-1)^2 + (y+1)^2 = 16$

Without solving for x and y, determine if the circles (show all your calculations)

12.1.1 Intersect each other at two points

12.1.2 Touch each other (2)

12.1.3. Do not intersect each other (2)

12.2. The circle defined by the equation $x^2 + y^2 - 2x + 8y + 71 = 10$ has the centre M and the circle defined by the equation $(x-2)^2 + y^2 = 5$ has centre N

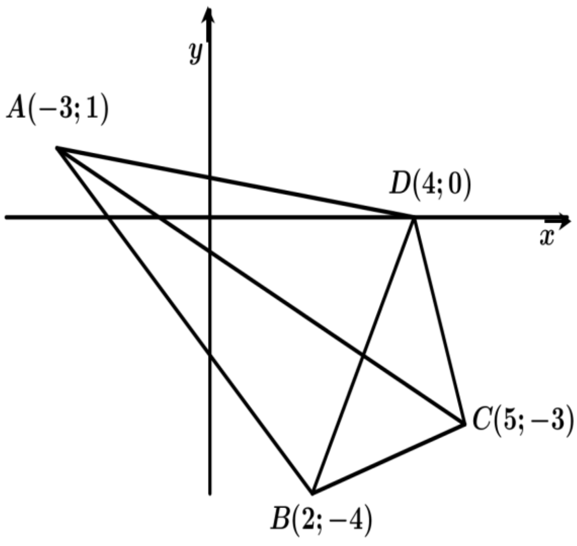
12.2.1 Determine the coordinates of the centres M and N and the radii. (6)

12.2.2 Show that the circles touch externally (4)

12.2.3 Determine the equation of a common tangent to the circles. (8)

Question 13

In the figure below, Calculate



13.1 The length of AC and leave your answer in simplest form (3)

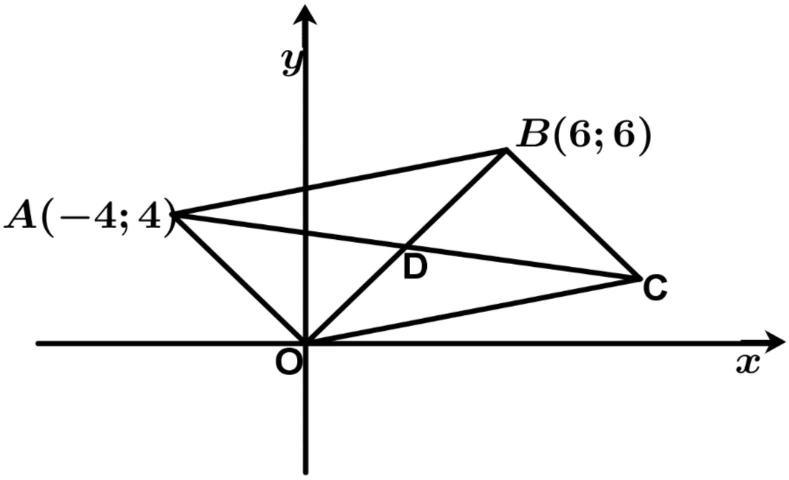
13.2 The equation of the line AC (3)

13.3 The area of the Kite ABCD (4)

13.4. The angle of inclination of AB. (3)

Question 14

In the figure OABC is a parallelogram with points $A(-4;4); B(6;6) and\ O(0;0)$ the diagonals AC and BO intersects at D. Determine,



14.1 The coordinates of D (2)

14.2 The equation of AC (3)

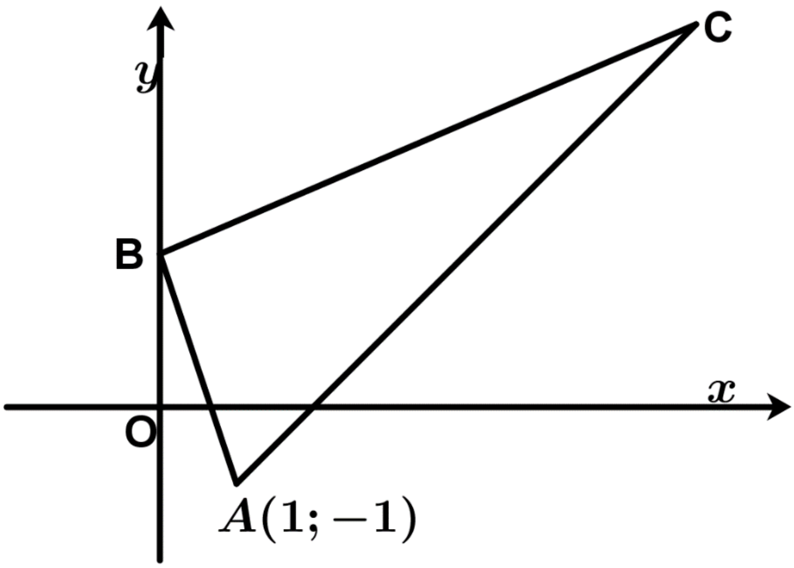
14.3 The equation of OC (3)

14.4 The coordinates of C (3)

14.5 The size $A\hat{O}C$ (5)

Question 15

Refer to the figure below. The point B is on the y-axis and the coordinates of A are (1;-1) the equations of the sides. BC and AC are $x - 3y + 6 = 0\ and\ x - y - 2 = 0$ respectively



15.1 Show that the coordinates of B are (0;2) (2)

15.2 Determine the gradient of BC (2)

15.3 Prove that $A\hat{B}C = 90°$ (3)

15.4 Determine the coordinates of C (4)

15.5 Calculate the length of AC (2)

15.6 Calculate the equation of the circle passing through points $A; B\ and\ C$ in the form of $ax^2 + bx + cy^2 + dy + c = 0$ (6)

Question 16

16.1 The circle defined by the equation $x^2 + y^2 - 20x + 8y + 21 = 0$ has centre M and the circle defined by the equation $(x-2)^2 + y^2 = 5$ has centre N.

16.1.1 Show that the two circles touch externally (10)

16.1.2 Determine the equation of the common tangent to the circles (8)

16.2 The circle defined by $(x+1)^2 + (y-1)^2 = 16$ centre A and the circle defined by $x^2 + y^2 - 2y = 8$ has centre B.

16.2.1 Show that the two circles touch each other internally (8)

16.2.2. Determine the equation of the common tangent to the circles (5)

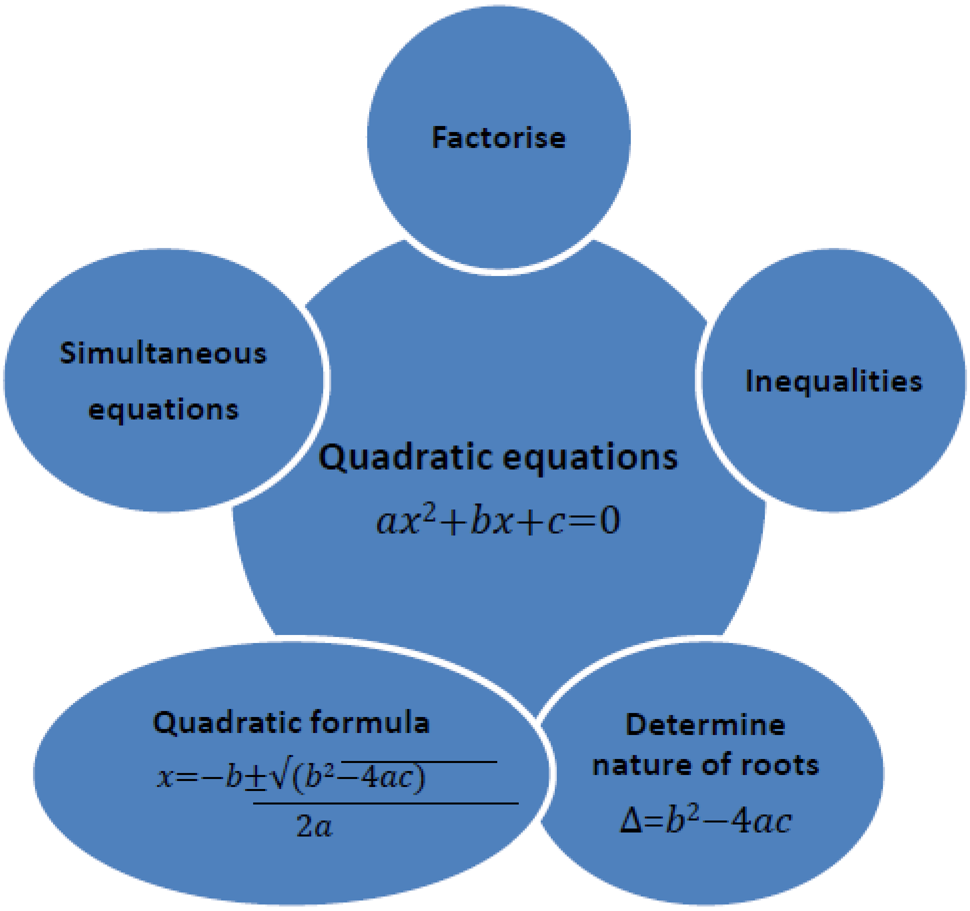
ALGEBRA: EXPRESSIONS, EQUATIONS AND INEQUALITIES

CHECKLIST

Make sure you……

- Can solve quadratic equations by:
  - Factorising
  - Using the quadratic formula
  - Using the K-substitution
- Can analyse the nature of the roots of a quadratic equation.
- Revise the surds and the laws of exponents
- Are able to solve inequalities
- Can solve simultaneous equations

Mind Map: Quadratic Equations



1. **TOPIC: EQUATIONS AND ALGEBRA**

1. **Simplify expressions using the laws of exponents for rational exponents.**
2. **)Add, subtract, multiply and divide simple surds (e.g. see curriculum statement)**
3. **Solve:**
   a. **Quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically;**
   b. **Equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically.**

HINTS TO LEARNERS: QUADRATIC EQUATIONS/INEQUALITIES

1. Simplify until one side is zero
2. Then factorise the quadratic expression
3. You will get at most two solutions.
4. You might want to check your answers.
5. When the question says correct to one decimal place. You are expected to solve the quadratic equation using the quadratic formula.
6. For the inequality, simplify so that the right hand side is zero.
7. Then use the graphic (draw the sketch of the parabola) or number line Method.
8. Don't cross multiply against an inequality sign. You will lose roots.

HINTS TO LEARNERS : SIMULTANEOUS EQUATIONS

1. Make x or y the subject of the formula in the linear equation.
2. Avoid solving for x or y if it has a co-efficient other than 1
3. Ensure after substitution you have an equation in 1 variable x or y not both.
4. Then solve the quadratic equation.
5. Substitute these values (from 4) in either equation to get the corresponding values. It is advisable to substitute into the equation obtained in 1 (changing the subject of the formula)

NB: In some cases this topic can be examined in the form of one equation.
Apply the properties of numbers and operations

HINTS TO LEARNERS: NON ROUTINE PROBLEM SOLVING

1. These are problem solving or non routine questions.
2. They require insight to strategies that are not usually taught by teachers.
3. So try using simplification and manipulation techniques.
4. Don't spend much time on these questions because you might not finish the paper.
5. Surds also feature in this part of the paper.
6. Know the laws governing exponents and surds.

SUNDAY TIMES (2009)

NC Sept 2018

Solve $3x^2 + 5x = 4$ (correct to two decimal places) (4)

KZN June 2017

Solve for x: $x - 7 - \sqrt{x-5} = 0$ (4)

Solve for x:

b) $\dfrac{x^2 - 1}{x+1} = 2$ (4)

Feb/Mar 2009
Solve for x:

c) $3x + \frac{1}{x} = 4$ (4)

EC Sept 2016

Solve for x: $3^{x^2-1} = \frac{27^{-x}}{3}$ **(4)**

GP Sept 2016

Solve for x: $3^x + 3^{-x+1} . 5 = 8$ (5)

DBE Nov 2016

Solve for x: $3^{x+3} – 3^{x+2} = 486$ (4)

Feb/Mar 2012

b) $x - \frac{2}{x} = 5$ (4)

KZN Sept 2017

Solve for x: $3^{x+1} – 4 + \frac{1}{3^x}$ **(5)**

EC Sept 2016

Given the equation: $\frac{x-\frac{1}{x}}{1+\frac{1}{x}} = 1$

a) For which values of x is the equation undefined? (2)
b) Solve for x in the equation (5)

Nov 2010

a) $4 + 5x > 6x^2$ (4)

GP September 2018

$\frac{6x^2 - 3x}{3} \leq 3x^2$ **(5)**

Western Cape (June 2015)

c) $(2x – 3)^2 \leq 4$ (4)

Last Push by Pat Tshikane

a) Solve for x: $5x^2 + 4 > 21x$

Eastern Cape (June 2014)

f) $\frac{x}{x+2} \leq 0$ (3)

Study and Master GR 11 & 12

a) Solve $\dfrac{-x^2-5}{3x-2} \geq 0$ (4)

b) For which values of k will $\dfrac{x-3}{(x-1)^2}$ = k have real roots (6)

c) The roots of the equation (x + 2) (x + k) = 2 + 3x are non real. Determine the possible values of k. (6)

Mpumalanga Sept 2017

Consider $27^{\frac{x}{3}} = 3^{y-1}$ and $2x^2 - y = 5$

a) Show that x = y – 1 (2)

b) Solve for x and y simultaneously (5)

FS Sept 2017

Consider: 5x – 3/x = 1

a) (i) Solve for x correct to two decimal places (5)

(ii) Hence, determine the value of y if $5(2y+1) - \dfrac{3}{2y+1} = 1$ **(3)**

Examplar (2014)

h) $3^x(x-5) < 0$ (2)

GP Sept 2018

$\sqrt{5-x} - x = 1$ (4)

Feb/Mar 2012

b) $\left(\sqrt[5]{\sqrt{35}+\sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35}-\sqrt{3}}\right)$ (3)

Nov 2009

Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}-\sqrt{10^{2007}}}}$$ (3)

MARITZBURG COLLEGE: Tutorial 1

d) $$\frac{\sqrt{10^{1000}+10^{1001}}}{\sqrt{11^5.10^{1000}}-21\sqrt{11.10^{1000}}}$$ (3)

Last Push by Pat Tshikane

a) Calculate a and b if $\frac{\sqrt{7^{2014}-7^{2012}}}{12}$ = a (7^b) and 'a' is not a multiple of 7

b) Solve for x: x = $\frac{a^2+a-2}{a-1}$ if a = 888 888 888 888

c) **Feb/Mar 2010**

e) If $\frac{14}{\sqrt{63}-\sqrt{28}} = a\sqrt{b}$, determine , without using a calculator, the value(s) of *a* and *b* if *a* and *b* are integers.

KZN Sept 2014

f) Solve and round off answer to TWO decimal places if necessary

$$\sqrt{\frac{x}{2}+3} = 4-x$$

(4)

KZN June 2017

g) Solve for x without using a calculator

$$\sqrt[3]{\frac{1}{x^7}} = 128$$

KZN Nov 2018

h) If 3^{9x} = 64 and $\mathbf{5^{\sqrt{p}}}$ **=** 64, Calculate without the use of a calculator, the value of: $\frac{[3^{x-1}]}{\sqrt{5}^{\sqrt{p}}}$

Feb/Mar 2014

a) $9.2^{\,x-1} = 2.3^x$ (3)

GP (September 2014)

d) $2^0 + 2^{x-2} + 2^{x+1} + 2^x = 53$ (5)

GP Sept 2018

e) $2^{x+2} + 7\sqrt{2}^x = 2$ (4)

NC Sept exam prep 2018

Solve for x and y if: $9^{x+y} = 3^{y+4}$ and $5x + 4y = 11$ (7)

EC Sept 2014

a) $2x^2 - 3xy = -4$ and $4^{x+y} = 2^{y+4}$ (7)

EC Sept 2014

Solve $3x^{2/3} - 13x^{1/3} - 10 = 0$ **(3)**

Nov 2011

c) Consider the equation: $x^2 + 5xy + 6y^2 = 0$

i) Calculate the values of the ratio $\frac{x}{y}$ (3)

ii) Hence, calculate the values of x and y if $x + y = 8$ (5)

Limpompo Sept 2014

Simplify $\frac{7^{a-2} \cdot 2^{a-2}}{14^{a-1} \cdot 2}$ (3)

Feb/Mar 2013

d) Given: $2^x + 2^{x+2} = -5y + 20$

i) Express 2^x in terms of y. (2)

ii) How many solutions for x will the equation have if $y = -4$? (2)

iii) Solve for x if y is the largest possible integer value for which

$2^x + 2^{x+2} = -5y + 20$ will have solutions. (3)

Nov 2015

e) Given: $(3x - y)^2 + (x - 5)^2 = 0$

Solve for x and y. (4)

EC Sept 2014

f) Solve for x and y:

$2x^2 - 3xy = -4$ and $4^{x+y} = 2^{y+4}$ (7)

Free State exam prep 2018

Solve without using a calculator

$\sqrt{5}.\sqrt{125}-\frac{5^x.5^{x+1}}{5^{2x}}$

g) Solve $2^{2x} - 6 \cdot 2^x = 16$

i) Solve $x - 6 + \frac{2}{x} = 0; x \neq 0$

GP Sep 2014

a) Given:
$k = (x+1)^2 - 4$, where k is a real number.
i) Solve for x if $k = 4$. (Leave the answer in the simplest surd form). (5)
ii) Write down the minimum value of k. (1)

b) Calculate the values of k, for which the equation $3x^2 + 2x - k + 1 = 0$ has real roots. (4)

Adapted from Study & Master Gr 11 & 12

j) The roots of the equation: $2x^2 - 12x + p = 0$ have a ratio 5:7. Find p and the roots (6)

k) The solution of a quadratic equation is: $x = \frac{-2 \pm \sqrt{13-2k}}{3}$ Find the largest integral value of k for which this this x value will be rational. (4)

Last Push by Pat Tshikane

a) The roots of the quadratic equation are $x = \frac{3 \pm \sqrt{13-2k}}{2}$ determine the values of k if the roots are real (4)

Feb/Mar 2014

Given: $f(x) = x^2 - 5x + c$

Determine the value of c if it is given that the solutions of $f(x) = 0$ are $\frac{5 \pm \sqrt{41}}{2}$. (3)

Feb/Mar 2012

g) The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1: 2 : 3. Calculate the length of the shortest side. (4)

TOPIC: CALCULUS

QUESTION 1

1.1 Given $f(x) = \frac{2}{x}$

1.1.1 Determine $f'(x)$ by using first principles. (5)

1.1.2 Find the equation of the tangent to *f(x)* at the point where $x = 2$. (5)

1.1.3 Determine whether $f'(1) + f'(2) = f'(1+2)$. (3)

1.2 Determine the derivative, from first principles, of $f(x) = -\frac{1}{3}x^3$. (5)

1.3 Given: $f(x) = -2x^2+1$

1.3.1 Show that the average gradient of the graph of f between the point where $x=3$ and $x=3+h$, $(h\neq 0)$, is $-12-h$. (4)

1.3.2 Use your answer in question 2.3.1 to calculate $f'(3)$ from first principles. (2)

1.3.3 Determine the numerical value f the gradient of the graph of f at $x=0$ (1)

1.4 Differentiate $f(x) = \frac{x^2}{3} + 1$ from the first principle. (4)

QUESTION 2

2.1 Determine (leaving your answers with positive indices):

2.1.1 $D_x\left[\frac{3x^3-7+x^2}{x^2}\right]$ (3)

2.1.2 $g'(x)$ if $g(x) = \frac{5x^2-5x}{x-1}$ (3)

2.1.3 $\frac{dz}{dx}$ if $z = \sqrt{\frac{4}{x}} + \frac{x}{8}$ (3)

2.1.4 $\frac{dy}{dx}$ if $\sqrt{y} = x - 2$ (4)

2.1.5 $\frac{dy}{dx}$ if $y = 7x^2 - \frac{3}{\sqrt[3]{x}} + 2^{-1}$ (4)

2.1.6 $p'(x)\ if\ p(x) = (\frac{1}{x^3} + 4x)^2$ (4)

2.2 given the following: $y = 8x^3$ and $\sqrt{a} = y^{\frac{2}{3}}$

2.2.1 $\frac{dy}{dx}$. (1)

2.2.2 $\frac{da}{dy}$ (2)

2.2.3 $\frac{da}{dx}$ (3)

2.3 given: $y = 4(\sqrt[3]{x^2}\,)$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$ (4)

QUESTION 3

3.1 $f(x) = ax^3 + bx^2 + cx + d$

The gradient at any point $(x; f(x))$ is given by $(18x^2 + 14x - 8)$, and $f(0) = -7$.
Determine the values of a, b, c and d. (5)

3.2 The following information about a cubic polynomial, $y = f(x)$ is given:

- $f(-1) = 0$
- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- If $x < -1$ then $f'(x) > 0$
- If $x > 1$ then $f'(x) > 0$

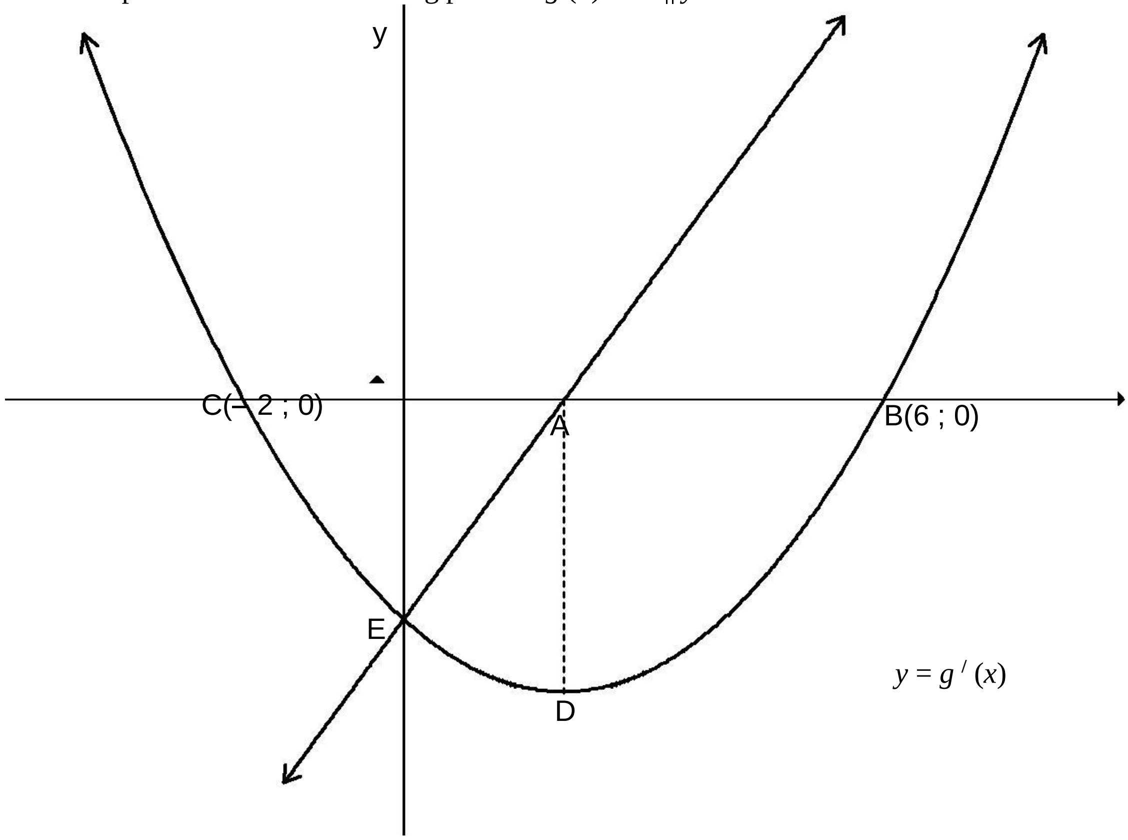
3.2.1 Use this information to draw a neat sketch graph of *f*.. (5)

3.2.2 For which value(s) of x is f strictly decreasing? (2)

3.2.3 Use your graph to determine the x-coordinate of point of inflection (2)

3.2.4 For which value(s) of *x* is *f* concave up? (2)

3.3 The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g. The graphs of h and g' have a common y-intercept at E. C(– 2 ; 0) and D(6 ; 0) are the x-intercepts of the graph of g'(x). A is the x-intercept of h and D is the turning point of g'(x) .AB || y-axis.



3.3.1 Write down the coordinates of E (1)

3.3.2 Determine the equation of the graph of g' in the form $y = ax^2$+bx+c (4)

3.3.3 Write down the x –coordinates of the turning points of g (2)

3.34 write down the x-coordinates of the point of inflection of the graph of g. (2)

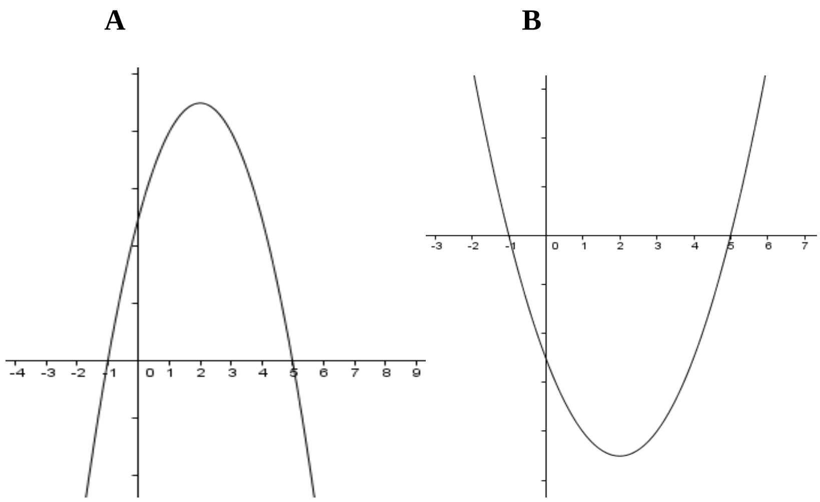
3.3.5 Explain why g has local maximum at x=-2 (3)

3.4 $f(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $a < 0$
- $d = 0$
- $f(-3) = 0$ *en* $f(8) = 0$
- $f'(-1) = 0$ *en* $f'(5) = 0$

3.4.1 Draw a sketch graph of *f* using the information given above. (5)

3.4.2 Choose from the following two graphs the one that represents $f'(x)$.
Only write A or B. (1)



3.4.3 Use your graph in 3.4.1 and your choice in 3.4.2 to determine for which values of x will $f'(x).f(x) \geq 0$. (3)

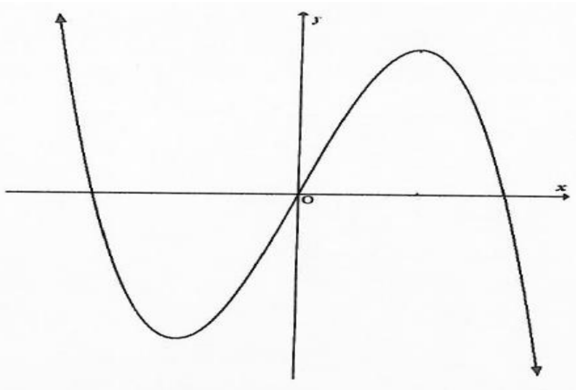
3.5 The equation of a tangent to the curve $f(x) = ax^3 + bx$ is $y - x - 4 = 0$.

If the point of contact is $(-1; 3)$, determine the value of a and b. (5)

3.6 Given: $f(x) = ax^3 + bx^2 + cx + d$

3.6.1 Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers (4)

3.7 The sketch represents the graphs of $f(x) = ax^3 + cx$. P(-1;-2) and R are turning points of *f*.



3.7.1 Calculate the value of a and c if f has a minimum value at (-1;-2) (5)

3.7.2 Determine the coordinates of point R where the function has a local maximum value (2)

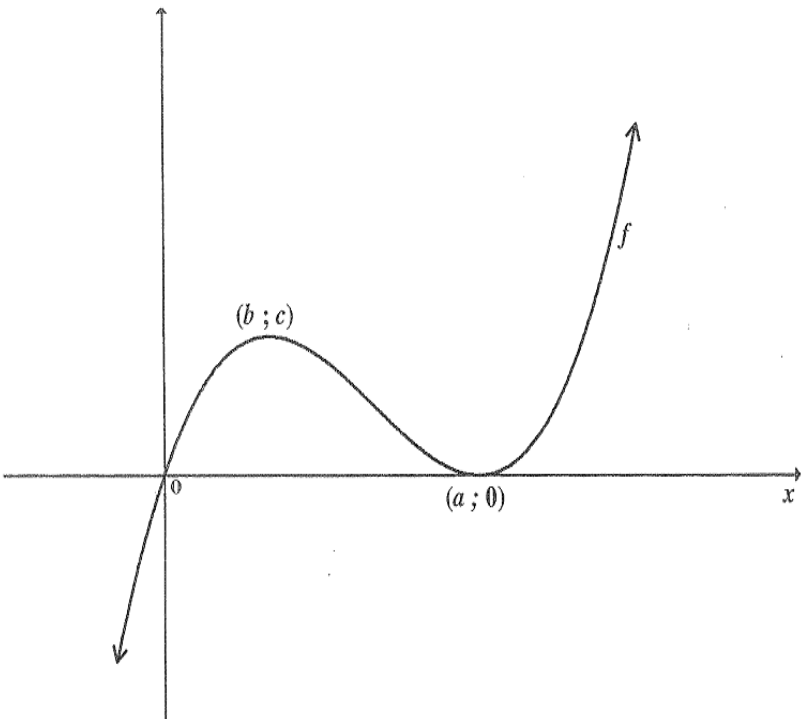
3.7.3 For which values of x will $f'(x) \geq 0$? (2)

3.8 Determine the coordinates of the points on the curve of $f(x) = x^3 - 2x^2 + 3x - 4$ such that the tangents at these points are parallel and have a gradient of 2. (5)

3.9 Write down the equation of the tangent in question 3.8.1 for which is not an integer (3)

3.10

The sketch graph below represents a cubic function f with equation $f(x) = x^3 - 4x^2 + 4x + k$. The graph passes through the origin, has a local maximum at $(b;c)$ and a local minimum at $(a;0)$.



3.9.1 Explain why $k=0$ (1)

3.9.2 using this value of k, determine the values of

3.9.2.1 a (2)

3.9.2.2 b (4)

3.9.3 The graph of g with equation $g(x)=mx$ is a tangent to f at the point (0;0) calculate the value of m (2)

3.9.4 Make use of the graph, or any other way, to determine the value of p for which $x^3 + 4x^2 + 4x + 2 = p$ (2)

QUESTION 4

4.1.1 Express the lengths of PS and RS in terms of x, the x- co-ordinate of P. (2)

41.2 Hence, show that the area, A square units, of PQRS is given by

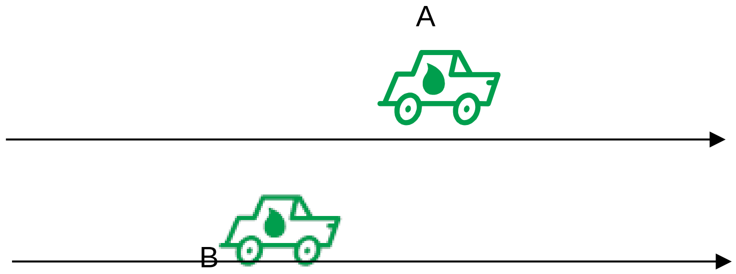
$$A = 80 - 12x - \frac{48}{x}$$ (2)

4.1.3 Find the maximum possible area of rectangle PQRS. (5)

4.3 Car A and car B are moving along parallel straight lines of a highway. Their positions at the time t are given by:

$S_A = 30t^2 + 20t + 40$; $S_B = 10t^2 + 80t$

Where S_A and S_B are the distances, in km, of car A and car B from a fixed position on the highway.



4.3.1 How far is car A ahead of car B when $t = 0$? (2)

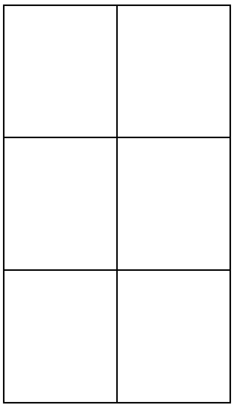
4.3.2 When are the cars next to each other? (3)

4.3.3 When are the cars travelling at the same speed? Which car is ahead at this instant in time? (5)

[10]

4.4

A window frame with dimensions y × h is illustrated below. The frame consists of six smaller frames.

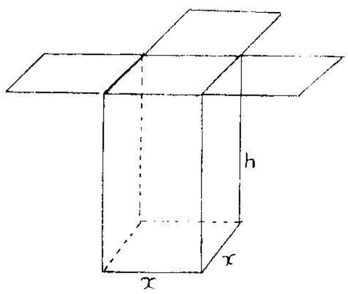


4.4.1 If 12m of material is used to make the entire frame, show that $y = \frac{1}{4}(12 - 3h)$. (2)

4.4.2 Show that the area of the window is given by A = 3h - $\frac{3}{4}h^2$. (3)

4.4.3 Find $\frac{dA}{dh}$ and hence the dimensions, h and y, of the frame so that the area of the windowis a maximum. (5)

4.5 A rectangular container has a square base and contains 1 litre of milk. The container has a folding lid that covers 3 times the area of the base. The length of the side of the square base is *x* cm and the height of the container is *h* cm. [1 litre = 1 000 cm^3]



(a) Show that the area, A, of the cardboard used to make the container can be expressed as:

$$A = 4x^2 + \frac{4000}{x}$$

(Hint: The volume of the container = area of the base × height)

(5)

(b) Determine the dimensions of the container in cm, so that the quantity of cardboard used is as small as possible.

(6)

4.6 A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.

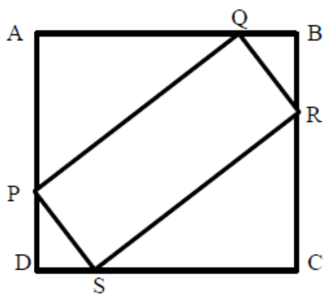
4.6.1 Determine an equation of MN in terms of a and b. (2)

4.6.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

[8]

4.7 Prove that the curve of $y = \frac{1}{x^3}$ is decreasing for all real values of x, with the exception of one value for x. Give this one value for x in your proof. (4)

4.8 ABCD is a square with sides 20mm each. PQRS is a rectangle that fits inside the square such that QB=BR=DS=DP=k mm



4.8.1 Prove that the area of PQRS= -2k(k-20) =40k-2k^2 (4)

4.8.2 Determine the value of k for which the area of PQRS is a maximum. (4)

4.9 A function $g(x) = ax^2+\frac{b}{x}$ has a minimum value at x = 4. The function value at x = 4 is 96. Calculate the values of and b (5)

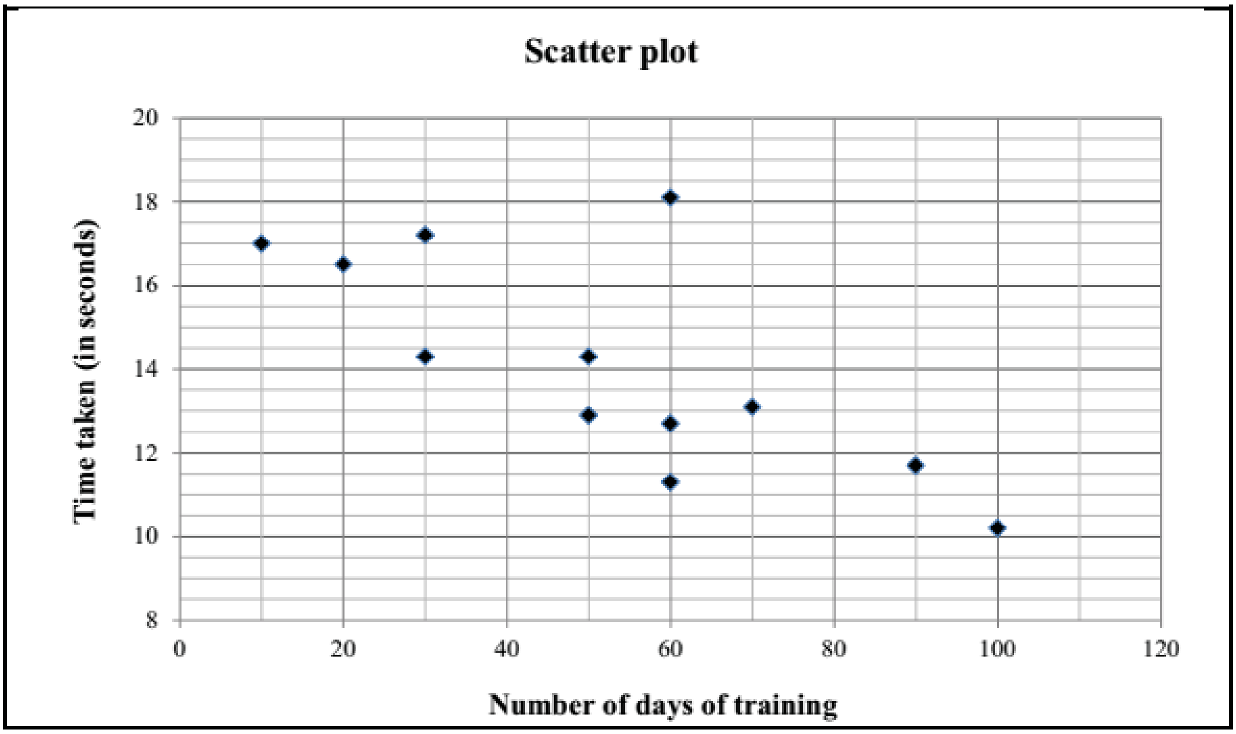
DATA HANDLING –REVISION MATERIAL

PLEASE NOTE: most of the questions hereunder are set by adaptation and although care has been taken to ensure that we retain most of the original questions, some of the questions have been beefed up and will thus not be covered by the original memoranda.

QUESTION 1 (EC SEPT 2015)

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Discuss the trend of the data collected. (1)

1.2 Identify any outlier(s) in the data. (1)

1.3 Calculate the equation of the least squares regression line. (4)

1.4 Draw the regression line on the diagram sheet provided. (2)

1.5 Use the equation of the regression line to predict the time taken to run the 100 m sprint for an athlete training for 45 days. State whether this is interpolation or extrapolation. (2)

1.6 Calculate the correlation coefficient. (2)

1.7 Comment on the strength of the relationship between the variables. (1)

1.8 The point (60;18) was wrongly captured, it was supposed to be (60;15). If this point is corrected on the scatter, what effect does it have on the value of r and give a reason for your answer. (2)

QUESTION 2 (EC SEPT 2015)

The weights (in kilogram) of the 20 boys in the hockey squad of School A are given below:

69	59	59	66	64	58	63	58	62	61
57	53	60	51	60	48	47	60	40	60

2.1 Determine the mean and variance for the weights of the School A squad. (3)

2.2 The following information was obtained from the School B boys' hockey coach, regarding the weights of the boys in his squad.

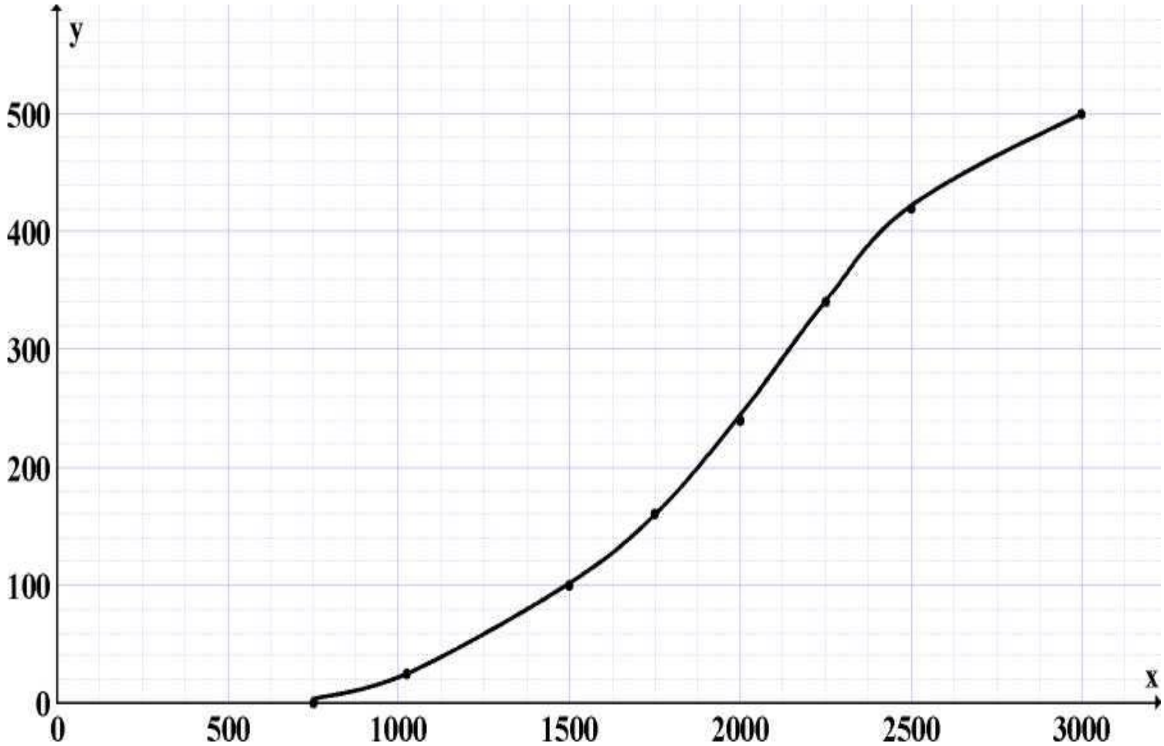
$$\sum_{n=1}^{22} x_{n=1320} \quad \text{and} \quad \sum_{n=1}^{22} (x_n - 60)^2$$

2.2.1 How many boys are in the School B squad? (1)

2.2.2 Determine the mean weight for the School B squad. (2)

2.2.3 Determine the standard deviation for the School B squad. (2)

2.2.4 If five boys of equal weight are added to the squad of School A so that the means of both schools are the same, what must be the weight of each boy? (2)



QUESTION 1 (Limpopo DoE/ Nov SUPPLEMENT 2014)

The lifetime of electric light bulbs was measured in a laboratory. The results are shown in the cumulative frequency curve below.

Lifetime of electric bulbs (in hours)

3.1 Use the above cumulative frequency curve to determine the following:

3.2 How many light bulb s were tested? (1)

3.3 The median lifetime of the electric light bulbs tested. (2)

3.4 Calculate the percentage of how many light bulbs have a lifetime of between 1500hrs to 2300hrs.

3.5 The interquartile range. (2)

3.6 Determine the modal class (2)

3.7 The number of electric light bulbs with a lifetime of between 1750 and 2000 hours. (2)

3.8 If the cost of one light bulb is R5,00, determine the amount spent on purchasing

the light bulbs that lasted longer than 2 500 hours. (2)

[9]

QUESTION 2 (Limpopo DoE / Supplement 2014)

The owner of Harvey Tours uses the following data to illustrate the relationship between the annual advertising expenditure and the annual profit of the business. (All data is in THOUSAND of Rands.)

Annual advertising expenditure	12	14	17	21	26	30
Annual Profit	60	70	90	100	100	120

4.1 Draw a scatter plot to represent the data. Use the grid provided in the answer sheet. (2)

4.2 Determine the equation of the least squares line for the data. (2)

4.3 Draw the least squares regression line on your scatterplot diagram. (1)

4.4 Predict the annual profit if the annual expenditure is R25 000. (2)

4.5 Calculate the correlation coefficient. (2)

4.6 Describe the strength of the relationship between the annual profit and the annual advertising

expenditure. (2)

[11]

QUESTION 3 (TRS RESOURCE FILE /ENRICHMENT)

The following data relates to the scores for a Maths test for Grade 12 learners at Bluebell high.

25	56	78	67	89	90	43	55	77	87	52	67	89	53	05	34
22	34	65	75	75	67	75	76	88	43	56	78	54	75	84	32

2.1 Determine the five number summary from the data above. (4)

2.2 Use the diagram sheet to draw a box and whisker diagram for the learner scores. (3)

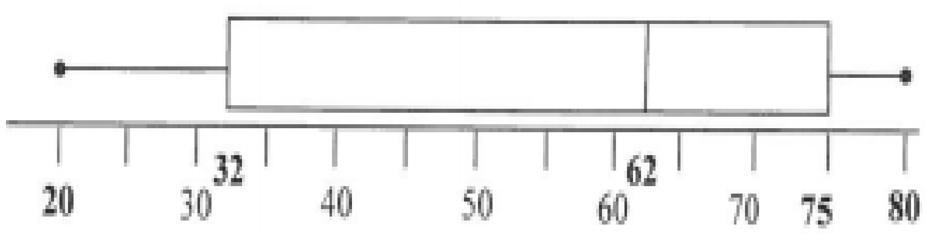
2.3 Comment on the skewness of the scores. (1)

2.4 Determine the semi-interquatile range. (2)

2.5 How many scores lie within one standard deviation from the mean. (3)

2.6 Determine the probability that a learner chosen at random scored a mark outside one standard deviation from the mean. (2)

QUESTION 4(DBE Feb/March 2016)



The box and whisker diagram above shows the marks (out of 80) obtained in a History test by a group of

1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (1)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (1)

QUESTION 2 (DBE Feb/march 2016)

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
10 <x < 20	**2**
20 <x < 30	**8**
30 < JC < 40	**5**
40 <x < 50	**10**
50 <x < 60	**12**
60 <x < 70	**18**
70 <x < 80	**3**
80 < *x* <90	**2**

2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

2.2 Determine the median class (2)

2.3 Calculate the standard deviation of the data. (2)

2.4 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

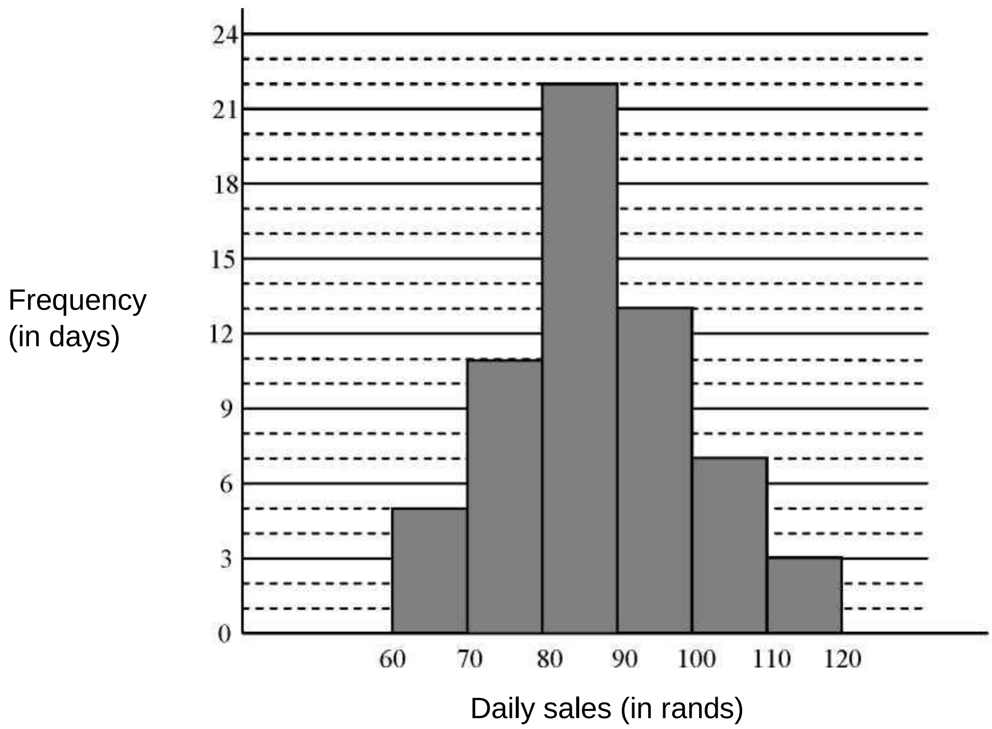
2.5 Hence, estimate the percentage number of days on which 65 or more messages were sent. (2)

[9]

QUESTION 8(DoE / NOV 2008)

A street vendor has kept a record of sales for November and December 2007.

The daily sales in rands is shown in the histogram below.



10.1 On DIAGRAM SHEET, complete the cumulative frequency table for the sales over November and December. (3)

10.2 Draw an ogive for the sales over November and December on DIAGRAM SHEET. (3)

10.3 Use your ogive to determine the median value for the daily sales. (1)

10.4 Estimate the interval of the upper 25% of the daily sales. (2)

QUESTION 11 (DoE / NOV 2009 UNUSED)

The price of 95-octane unleaded petrol in Gauteng for the period January 2007 to July 2008 is shown below. The price is in South African cents per litre.

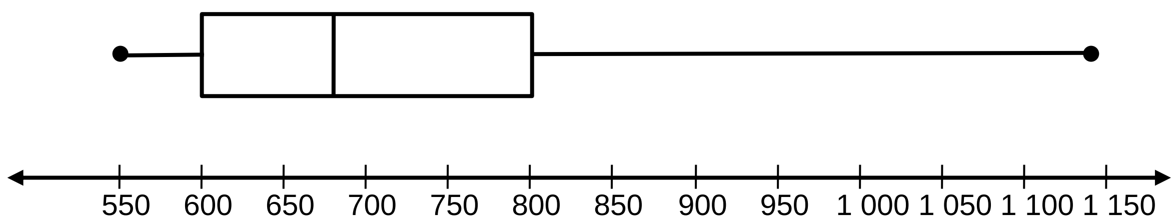
January 2007	598	February 2007	575	March 2007	599
April 2007	667	May 2007	701	June 2007	724
July 2007	716	August 2007	701	September 2007	691
October 2007	701	November 2007	704	December 2007	747
January 2008	747	February 2008	764	March 2008	825
April 2008	891	May 2008	946	June 2008	996
July 2008	1 070				

[Source: www.sasol.com]

11.1 Determine the median, lower quartile and upper quartile for the data. (4)

11.2 Draw a box and whisker diagram on DIAGRAM SHEET 2 (attached). (2)

11.3 The box and whisker diagram for the price of diesel for the same period as above is shown below. The lower quartile is 600 and the upper quartile is 800.

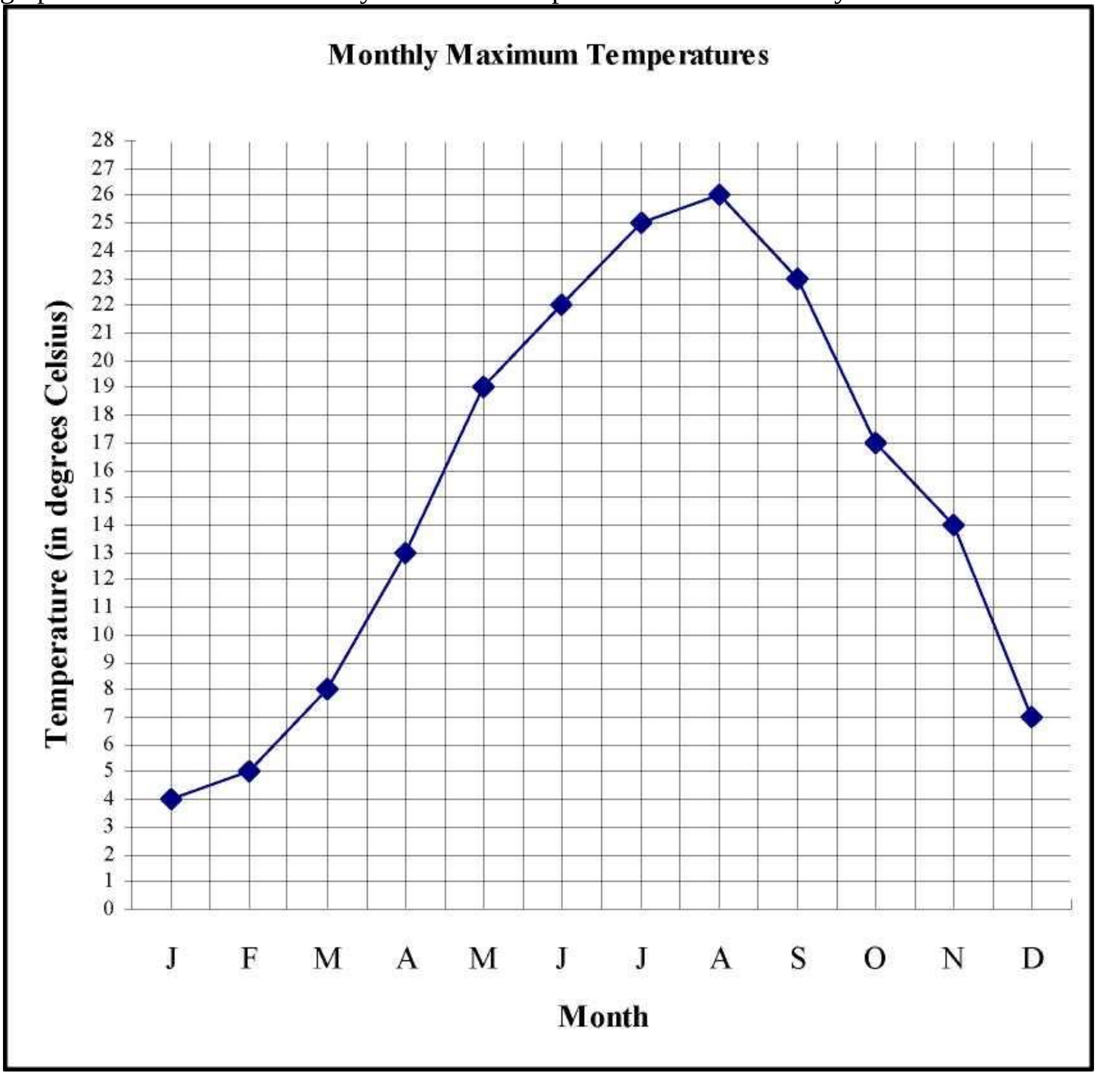


How many data points are there, strictly between 600 and 800? (2)

[8]

QUESTION 1(DoE/ FEB-MAR 2010)

The graph below shows the monthly maximum temperatures in a certain city.



1.1 What is the range of the monthly maximum temperatures? (2)

1.1 Calculate the mean monthly maximum temperature. (3)

1.2 Calculate the standard deviation of the monthly maximum temperature. (2)

1.3 It is predicted that one hundred years from now, global warming is likely to increase the city's monthly maximum temperature by 5° C in December, January and February. It will also result in an increase of 1° C in the other months of the year.

1.3.1 By how much does the mean increase? (2)

1.3.2 Describe the effect that the predicted increases in temperature will have on the the st
(2)

1.5 Learners at Phambili High School travel from three different neighbourhoods,neighbourhood A, B and C. The table below shows the number of learners from each neighbourhood, and their mean travelling times from home to school.

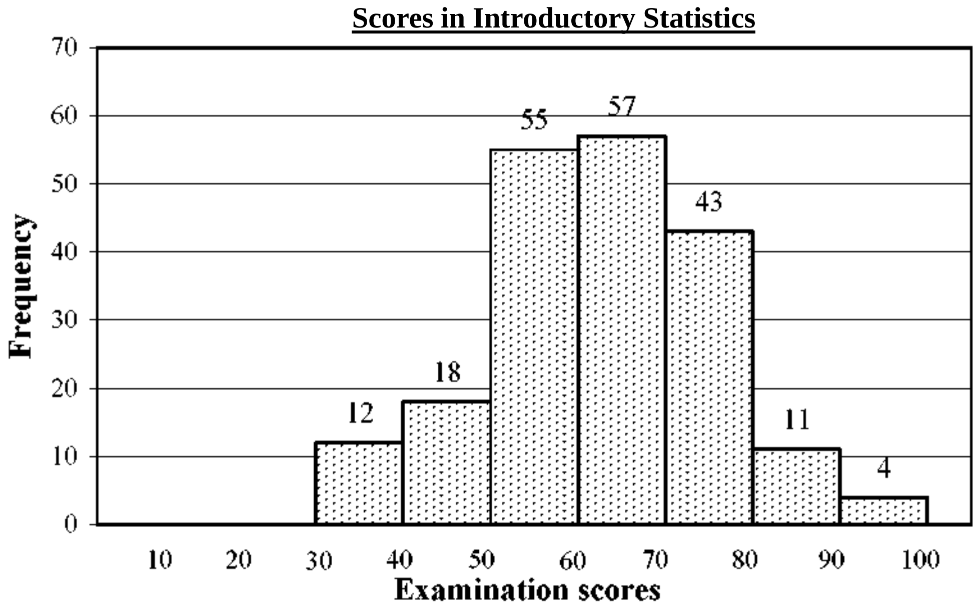
Neighbourhood	**A**	**B**	**C**
Number of learners	135	225	200
Mean travelling time (in min.)	24	32	x

The mean travelling time for learners living in neighbourhood C is the same as the mean travelling time for all 560 learners.

Calculate the mean travelling time for neighbourhood C. 4)

QUESTION 12 (DBE/ Nov 2010)

The histogram below shows the distribution of examination scores for 200 learners in Introductory Statistics.



1.6 Complete the cumulative frequency table for the above data provided on

DIAGRAM SHEET 2. (2)

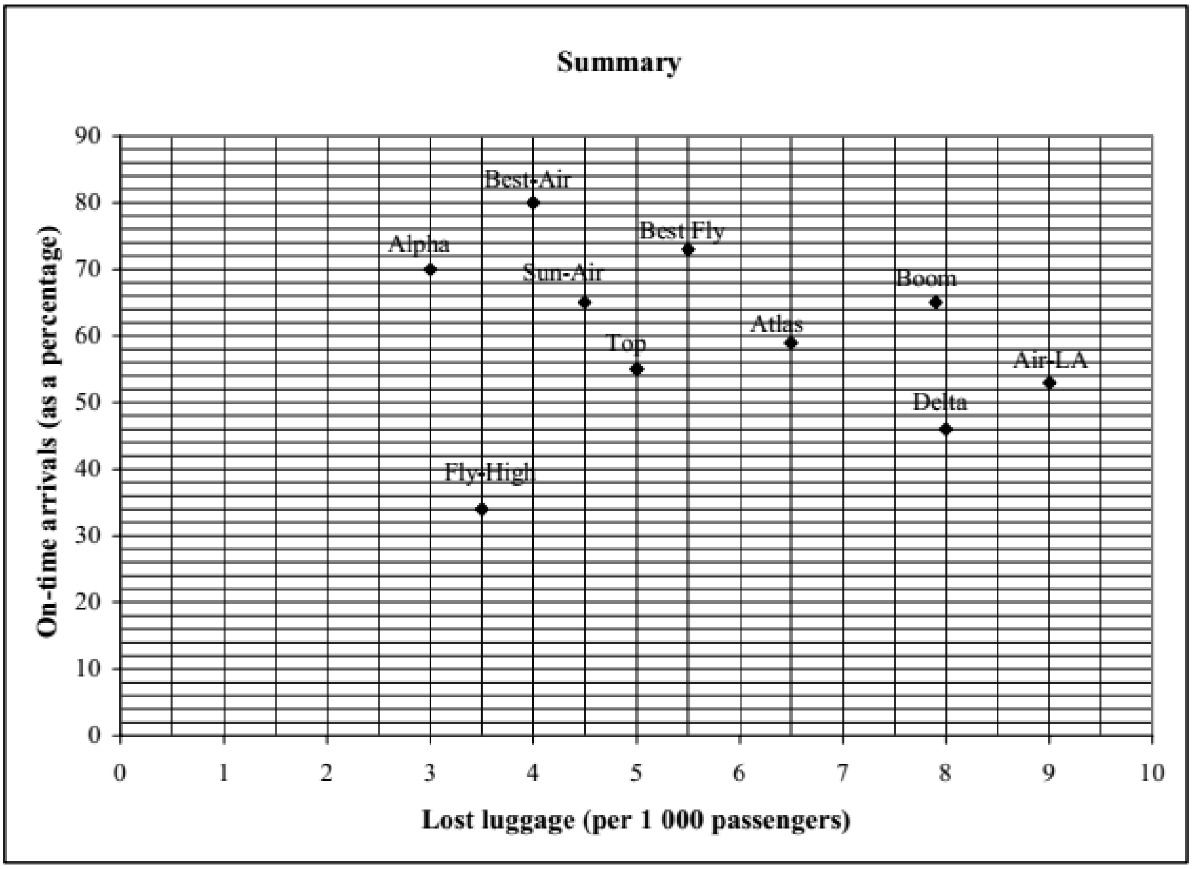
1.7 Draw an ogive of the above data on the grid provided on

1.8 DIAGRAM SHEET (5)

1.8 Use the ogive to estimate how many learners scored 75% or more for the examination.(1)

QUESTION 2 (DBE Nov 2010)

A researcher suspects that airlines, whose planes arrive on time, are less likely to lose the luggage of their passengers. Information gathered from 10 airline companies is summarised in the grid below.



Use the scatter plot to answer the following questions.

2.1 Which airline has the worst record for on-time arrivals? (1)

2.2 Is the following statement likely to be TRUE? Motivate your answer.

Of 5 120 passengers transported by Boom airlines, 40 passengers lost their luggage. (1)

2.3 Does the data confirm the researcher's suspicions? Justify your answer. (2)

2.4 Which ONE of the 10 airlines would you prefer to use? Give a reason for your answer. (2)

[6]

QUESTION 10 (DoE /NOV 2008)

A parachutist jumps out of a helicopter and his height above ground level is estimated at various times after he opened his parachute. The following table gives the results of the observations where *y* measures his height above ground level in metres and *t* represents the time in seconds after he opened his parachute.

t	2	3	4	5	6	7	8
y	500	300	200	120	70	40	20

On DIAGRAM SHEET 4, draw a scatter plot for the above information.(2)

10.3 Describe the curve of bestfit(1)

10.4 Use the scatter plot to estimate the height of the parachutist 5,5 seconds
10.5 after he had opened his parachute. (1)

[4]

QUESTION 15(ENRICHMENT)

The following marks were obtained from Mr Dlamini's 7 Mathematics learners.

It is further given that:

Range of the scores is 65

The difference between Q_1 and Q_2 is 11

Semi IQR is 17.5

The average score is 50.71

One of the learners scored a mark which is coincidentally the mean of Q_2 and Q_3

23	a	b	45	c	d	e

Find a, b, c, d and e. (7)

FINANCIAL MATHEMATICS **TYPICAL EXAM QUESTIONS**

QUESTION 1

1.1 Melokuhle invests R15 500 for *t* years at a compound interest rate of 9% p.a . compounded quarterly. At the end of *t* years, his investment is worth R40 000. Calculate the value of *t* (4)

1.2 Dela bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 18% p.a compounded monthly.

1.2.1 Calculate the annual effective interest rate of the loan. (3)

1.2.2 Calculate Dela's monthly instalments (4)

1.2.3 Dela decided to pay R12 200 each month as his repayment. Calculate the outstanding balance of the loan after 3 years. (4)

1.2.4 At the end of 3 years, the market value of Dela's car has reduced to R208 400. Calculate the rate of depreciation on the diminish value. (2)

[**17**]

QUESTION 2

2.1 How long will it take for the lump sum of money to be doubled at 4,5% p.a. interest compounded monthy? (3)

2.2 A loan of R50 000 is amortised over a period of 5 years. Payments are made monthly starting six months after the loan is granted. The interest rate is 10,5% p.a . compounded monthly.

2.2.1 Calculate the monthly repayments (6)

2.2.2 Calculate the outstanding balance after 2 years the loan was granted. (6)

[15]

QUESTION 3

Mrs Naidoo plans to buy a flat. She requires a mortgage bond of R800 000. The interest rate on the bond is 9% p.a compounded monthly. Mrs Naidoo plans to repay the loan with equal monthly payments starting one month after the loan is granted.

3.1 If Mrs Naidoo pays R6 500 per month until the bond is cleared; calculate the number of payments required to amortilise the loan. (4)

3.2 Calculate Mrs Naidoo's final payment. (5)

3.3 Determine how much interest Mrs Naidoo paid. (2)

3.4 If Mrs Naidoo want to pay R1500 per month, Decide whether the bank will allow her to takeout the bond under these conditions. (Justify your answer with calculations)(6)

[17]

QUESTION 4

A farmer buys a tractor for R2,2 million.

4.1 Determine the book value of a tractor at the end of 5 years if the depreciation is calculated at 14% p.a on a reducing balance method. (3)

4.2 Determine the expected cost of buying a new tractor in five years time if the average rate of inflation is expected to be 6% p.a (3)

4.3 The farmer decides to replace the old tractor in five years time. He will trade in the old tractor. Calculate the sinking fund. (3)

4.4 Calculate the monthly payment into the sinking fund if payments commenced one month after he bought the tractor if the interest rate is 7% per annum compounded monthly. (4)

[13]

QUESTION 5

5.1Daniel buys a house for R 450 000. He pays a 10% deposit and takes out a loan called a bond from the bank to pay off the balance. The bank charges 7,2% p.a. compounded monthly and He takes it out over a 25-year period.

5.1.1 Determine the value borrowed from the bank. (1)

5.1.2 What is his monthly repayments? (4)

5.1.3 After 11 years, He inherits money from his grandmother, and decides to pay off the rest of his bond. What is the outstanding balance that he needs to settle at the end of 11 years? (3)

5.2 At the beginning of October 2016 Lungile opened a savings account with a single deposit of R10000. She then made 24 monthly deposits of R1600 at the end of every month starting at the end of October 2016. She earns 15% p.a interest compounded monthly in her account.

Calculate the amount that should be in his savings account immediately after she makes the last deposit. (5)

[13]

QUESTION 6

6.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

6.1.1 Determine the scrap value of the machine at the end of 5 years (3)

6.1.2 After five years the machine needs to be replaced. During this time, inflation

remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

6.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000 into which equal monthly instalments must be paid, is set up. Interest on this is 8,5% per annum, compound monthly. The first payment will be made immediately, and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payment into the sinking fund. (5)

6.2 Nesta receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compound monthly.

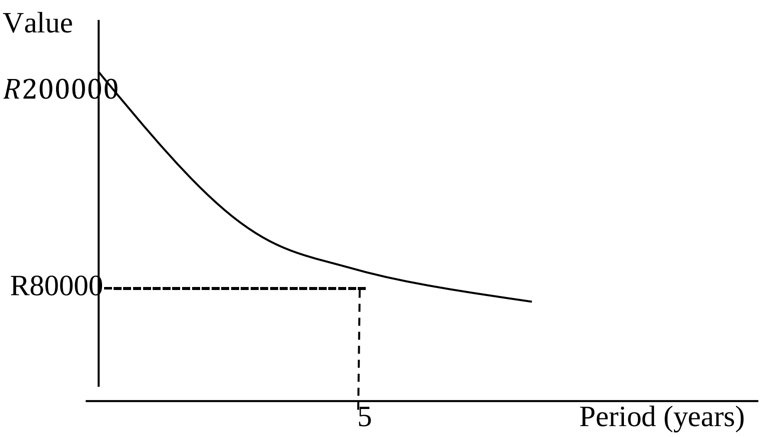
She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

For how many months will she be able to live from her investment? (6)

[17]

QUESTION 7

The sketch below represents the value of the car over a period of 5 years.



7.1 What is the initial value of a car? (1)

7.2 Calculate the interest rate. (4)

[5]

QUESTION 8

8.1 Joe invested a sum of R50 000 in a bank. The investment remained in the bank for 15 years, earning interest at a rate of 6% p.a. compounded annually. Calculate the amount at the end of 15 years. (2)

8.2 Nobuhle took a mortgage loan of R850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8% p.a. compounded monthly.

8.2.1 Show that her monthly instalment was R6 237 (4)

8.2.2 Calculate the outstanding balance on her loan at the end of the first year. (3)

8.2.3 Hence calculate how much of the R74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged. (3)

[12]

QUESTION 9

9.1 Two colleagues each receives an amount of R8 000 to invest for a period of 6 years. They invest money as follows:

- Zinhle: 7,5% p.a. simple interest. At the end of 6 years, she will receive a bonus of exactly 5% of the principal amount.
- Ntando: 7,0% p.a. compounded quarterly.

Who will have a bigger investment after 6 years? Justify your answer with appropriate calculations. (6)

9.2 How much will Thulani's investment worth at the end of 3 years, if he invests R4 million into earning interest of 6% per annum, compounded annually? (3)

9.3 Tom invests R900 000 into an account earning interest of 6,5% per annum, compounded monthly.

9.3.1 He withdraws an allowance of R20 000 per month. The first withdrawal is exactly one month after he has deposited R900 000. How many such withdrawals will Tom be able to make? (6)

9.3.2 If Tom withdraws R10 000 per month, how many withdrawals will he be able to make? (3)

[18]

QUESTION 10

Jake takes out a bank loan of R600 000 to pay for his new car. He repays the loan with monthly instalments of R9 000, starting one month after the granting of the loan. The interest rate is 13% per annum, compounded quarterly.

10.1 How many instalments of R9 000 must be paid? (5)
10.2 What will the final payment be? (5)
10.3 What did the car cost Jake by the time it is paid off? (2)

[12]

QUESTION 11

Due to load shedding, a restaurant buys a large generator for R227851. It depreciates at 23% per annum on a reducing balance. A new generator is expected to appreciate in value at a rate of 17% per annum. A new generator will be purchased in five years time.

11.1 Find the scrap value of the old generator in five years time. (3)
11.2 Find the cost of a new machine in five years. (3)

11.3 The restaurant will use the money received from the sale of the old machine (at scrap value) as part payment for the new one. The rest of the money will come from a sinking fund that was set up when the old generator was bought. Monthly payments which started one month after the purchase of the old generator, have been paid into a sinking fund account paying 11,4% per annum compounded monthly. The payments will finish three months before the purchase of a new machine.

Calculate the monthly payments into the sinking fund that will provide the required money for purchasing of the new machine. (6)

[12]

12 Mrekza takes out a loan of R450 000 at an effective interest rate of 14% p.a. in order to purchase a town house. She repays the loan with equal monthly instalments of R7500, starting one month from the granting of the loan. The interest is compounded monthly.

12.1 Show that the nominal interest rate is approximately 13,17% p.a.

Calculate:

12.1.1 The number of payments to payed up the loan. (4)

12.1.2 The value of the last payment (less than R7500). (7)

13 A loan of R180 000 is to be repaid over 20 years by means of equal monthly payments, starting 3 months after the loan is granted. The interest rate is 16% p.a. compounded monthly.

13.1 Calculate the monthly repayments. (4)

13.2 Calculate the outstanding balance after 10 years. (3)

14 A young man decides to invest money each month into a pension fund, starting on his 30^{th} birthday and ending on his 60^{th} birthday. He wants to have R1,5 million on retirement. If the interest rate is 14% p.a., compounded monthly, what will be his monthly payments? (4)

15 A company has an excavator which they have purchased for R1.5 million rand. It will depreciate on a reducing balance at 10% p.a. and it is anticipated that it will need to be replaced after 6 years. Over this period, it is predicted that inflation will run at 7% p.a.

15.1 Calculate the scrap value of the existing elevator after 6 years. (2)

15.2 Calculate the price of a new elevator in 6 years' time. (2)

15.3 Assuming that proceeds from the sale of the old excavator will be put towards the new one, determine how much money should be invested in a sinking fund today in order that the company will be able to replace the excavator in 6 years time. Assume that the sinking fund will earn 12% p.a. compounded monthly. (4

16.1 Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing- balance method. The book value of Sandile's car is currently R79 866,96.

16.1.1 How many years ago did Sandile buy the car? (3)

16.1.2 At exactly the same time that Sandile bought the car, Anile deposited R49 000 into a savings account at an interest rate of 10% p.a , compounded quarterly. Has Anil accumulated enough money in his savings account to buy Sandile's car now? (3)

16.2 Exactly 10 months ago, a bank granted Jane a loan of R800 000 at an interest rate of 10.25% p.a compounded monthly.
The bank stipulated that the loan:

- Must be repaid over 20 years
- Must be repaid by means of monthly repayments of R 7 853,15 starting one month after the loan was granted

17.1 A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method. (3)

17.2 Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly. How many months will it take Musa to repay the loan, if the monthly instalment is R1 900? (4)

17 .3 Neil set up an investment fund. Exactly 3 months later and every 3 months thereafter he deposited R3 500 into the fund. The fund pays interest at 7,5% p.a., compounded quarterly. He continued to make quarterly deposits into the fund for 6½ years from the time that he originally set up the fund. Neil made no further deposits into the fund but left the money in the same fund at the same rate of interest. Calculate how much he will have in the fund 10 years after he originally set it up. (6)

18.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate *r*, correct to ONE decimal place. (5)

18.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

18.2.1 Calculate Piet's monthly instalment. (4)

18.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

Probability

Questions 1-9 Counting Principle

Question 1

Ryan packs his suitcase for his holiday with 3caps, 5 shirts, 3 pairs of jeans and 2 pairs of takkies:

1.1 How many different outfits can he put together if when he dresses, he must wear a shirt, a pair of jeans, a pair of takkies and a cap? (2)

1.2 Ryan reaches his destination and hangs all the 5 shirts and the three pairs of jeans (each item separately) on a different hanger, on the rail in the cupboard.

(a) How many different arrangements are possible? (2)

(b) What is the probability that the shirts are all hanging together next to each other in the cupboard? (3)

1.3 While on holiday Ryan decides to buy a pair of sandals in addition to his outfit items, on a given day what is the probability that Ryan will wear a pair of sandals or a pair of takkies? (3)

1.4 Find the number of different arrangements of the letters DDD EE F G, if all the letters must be used and there are no restrictions. (3)

Question 2

Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.

2.1 How many personal identity numbers (PINs) can be made if:

2.1.1 Digits are repeated? (2)

2.1.2 Digits cannot be repeated? (2)

2.2 Suppose a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4)

Question 3

Each of the digits 1; 1; 2; 3; 4; 7 is written on a separate card. The cards are then placed next to each other to make a 6-digit number.

3.1 How many different 6-digit numbers can be formed from these digits? (2)

3.2 How many numbers start and end with the same digit? (2)

3.3 What is the probability of getting a number that starts and ends with the same digit? (1)

3.4 Find the probability that a number is 112347 or 743211 (2)

QUESTION 4

In Gauteng the number plates consists of 3 alphabets, excluding **the five vowels**, next to each other followed by 3 digits from 0 to 9. All number plates end with GP. An example: TDG 234 GP. The alphabets and digits are allowed to repeat.

4.1 Determine the number of unique number plates. (2)

4.2 Determine the probability that the number plate starts with a Y. (3)

4.3 Calculate the probability that the number plate contains an E (2)

4.4 Determine the number of number plates that will contain one 5. (3)

QUESTION 5

5.1 Consider the word "SIMPLIFY"

5.1.1 How many six letter words can be made? (2)

5.1.2 Calculate the probability of the word starting and ending with the same letter. (3)

5.2 Six cars are parked alongside each other, three are silver. How many ways can the cars be arranged if the silver cars have to be next to each other. (3)

Question 6

Three men (Peter, Jabu and Les) and two women (Liz and Kate) are to stand in a straight line to have their group photograph taken. Find the probability that Peter stands next to Liz and Jabu stands next to Kate. (5)

Question7

A music group plans a concert tour in South Africa. They will perform in cape Town, port Elizabeth, Pretoria, Johannesburg, Bloemfontein, Durban and East London.

7.1 In how many different orders can they plan their tour if there are no restrictions? (1)

7.2 In how many different orders can they plan their tour if their tour begins in Cape Town and ends in Durban? (1)

7.3 If the tour cities are chosen at random, what is the probability that their performances in Cape Town, Port Elizabeth, Durban and East London happen consecutively? Give your answer correct to 3 decimal places. (3)

Question 8

Landline telephone numbers are 10 digits long. Numbers begin with a zero followed by 9 digits chosen from the digits 0 to 9. Repetitions are allowed.

8.1 How many different phone numbers are possible? (1)

8.2 The first three digits of a number form an area code. The area code for Cape Town is 021. How many different phone numbers are available in the Cape Town area? (1)

8.3 What is the probability of the second digit being an even number? (2)

8.4 ignoring the first digit, what is the probability of a phone number consisting of only odd digits? Write your answer correct to three decimal places. (2)

Question 9

The code to a safe consists of 10 digits chosen from 0 to 9. None of the digits are repeated. Determine the probability of a code where the first digit is odd and none of the first three digits may be a zero. Write your answer as a percentage correct to two decimal places. (3)

Questions 10-13 Contingency Tables

Question 10

The data below was obtained from the financial aid office at a university.

	Receiving financial aid	**Not receiving financial aid**	**Total**
Undergraduates	4 222	3 898	8 120
Postgraduates	1 879	731	2 610
Total	6 101	4 629	10 730

10.1 Determine the probability that the student selected at random is…

10.1.1 receiving financial aid. (2)

10.1.2 a postgraduate student and not receiving financial aid. (2)

10.1.3 an undergraduate student and receiving financial aid. (2)

10.2 Are the events of being an undergraduate and receiving financial aid independent? Show ALL relevant workings to support your answer. (4)

10.3 Are the events of being an undergraduate and receiving financial aid mutually exclusive? Justify your answer (2)

Question 11

Each of the 200 employees of a company wrote a competency test. The results are indicated in the table below.

	Pass	**Fail**	**Total**
Males	46	32	78
Females	72	50	122
Total	118	82	200

11.1 Are the events Pass and Fail mutually exclusive? Explain your answer. (2)

11.2 Is passing the competency test independent of gender? Substantiate your answer with the necessary calculations. (4)

Question 12

The table summarises the results of all the language tests taken at a Language Centre in Cape Town during the first week of January.

	Male	**Female**	**Totals**
Pass	32	43	75
Fail	8	15	23
Total	40	58	98

A person is chosen at random from those who took their test during the first week of January.

12.1 Find the probability that the person was a male who failed. (2)

12.2 The person chosen is a female. Find the probability that she passed the test. (2)

Question 13

A rare kidney disease affects only 1 in 1000 people and the test for this disease has a 99% accuracy rate.

13.1 Draw a two-way contingency table showing the results, if 100 000 of the general population are tested. (4)

13.2 Calculate the probability that a person who tests positive for this rare kidney disease is sick with the disease correct to two decimal places. (2)

Questions 14-18 Tree-Diagrams

Question 14

Three cards are selected at random (without replacement) from a standard full pack of playing cards. There are 52 cards in the pack, jokers are excluded. Find the probability that the cards are all the same colour. (5)

Question 15

A study of numbers of male and female offspring in a certain population is being carried out. It is found that the first child in any family is equally likely to be male or female, but that for any subsequent offspring, the probability that they will be of the same sex as the previous child is $\frac{3}{5}$. No twins, triplets etc., are possible.

15.1 Find the probability that the first child of a family will be female. (1)

15.2 Find the probability that the first two children of a family will be female. (1)

15.3 Find the probability that a family will have two females followed by two males (in that order). Leave your answer in simplified fraction form. (2)

Question 16

There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

16.1 Calculate the probability that the first learner chosen is a boy. (1)

16.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate on your diagram ALL possible outcomes. (4)

16.3 Calculate the probability that a boy, then a girl and then another boy is chosen in that order. (3)

16.4 Calculate the probability that all three learners chosen are girls. (2)

16.5 Calculate the probability that at least one of the learners chosen is a boy. (3)

16.6 What is the probability that 5 learners chosen are of the same gender? (4)

Question 17

There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag.

(6)

Question 18

There are four black balls and $\boldsymbol{y}$ yellow balls in a bag. Thandi takes out a ball, notes its colour and then puts it back in the bag. She then takes out another ball and also notes its colour. If the probability that both balls have the same colour is $\frac{5}{8}$, determine the value of $\boldsymbol{y}$.

(6)

Questions 19-25 Venn-diagrams and Probability Rules

Question 19

At a school for boys there are 240 learners in Grade 12. The following information was gathered about participation in school sport.

122 boys play rugby (R)
58 boys play basketball (B)
96 boys play cricket (C)
16 boys play all three sports
22 boys play rugby and basketball
26 boys play cricket and basketball
26 boys do not play any of these sports
Let the number of learners who play rugby and cricket only be x.

19.1 Draw a Venn diagram to represent the above information.

19.2 Determine the number of boys who play rugby and cricket. (3)

19.3 (Leaving your answer(s) correct to THREE decimal places.)

Determine the probability that a learner in Grade 12 selected at random:

19.3.1 does not play cricket. (2)

19.3.2 participates in at least 2 of these sports (2)

Question 20

Given that:

$P(A\ only) = x$

$P(A\ and\ B\ ) = 0{,}1$

$P(B) = 0{,}4$

$P(not\ (A\ or\ B)) = y$

20.1 Represent this information in a Venn Diagram. (4)

20.2 If A and B are independent events, find the values of x and y. (5)

Question 21

Given that:

- A and B are independent events
- P(B) = 2P(A)
- P(AorB) = 0,6

Calculate P(B) (6)

Question 22

Given that: $\boldsymbol{P(A) = \frac{3}{5}\,;\, P(B) = \frac{2}{5}\,;\, P(C) = \frac{3}{10}}$

A and B are independent events

B and C are independent events

22.1 Calculate P (A or B) (2)

22.2 Calculate P(C only) (4)

Question 23

In a Physical Science quiz, two teams work independently on a problem. They are allowed a maximum of 10 minutes to solve the problem. The probabilities that each team will solve the problem are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Calculate the probability that the problem will be solved in the ten minutes allowed. (4)

Question 24

A local club has facilities that include tennis courts and a golf course.

A survey of the club members indicated that 504 regularly use the golf course and 336 regularly use the tennis courts. Some members regularly use both while 56 use neither of the facilities. The club has 700 members.

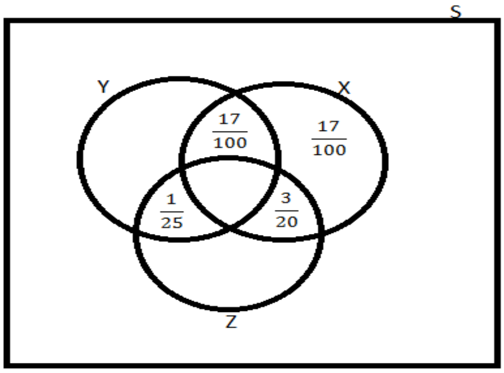
24.1 Determine the number of members that regularly use at least one of the facilities. A Venn diagram may be useful. (2)

24.2 What is the probability that a club member selected at random uses exactly (only) one facility? (3)

24.3 Given that: P (using the golf course) X P (using the tennis courts) = 0,3456. Validate statistically whether these events are independent or not. (2)

Question 25

The Venn diagram below shows probabilities of 3 events.



Complete the Venn diagram using the additional information provided.

- P(Z and(not Y)) = $\frac{31}{100}$
- P(Y and X) =$\frac{23}{100}$
- P(Y) =$\frac{39}{100}$

After completing the Venn diagram, compute P(Z and not (X or Y)) (4)

Question 26

26.1 N and M are wo events .P(N) =0,3;P(M) =0,4 and P(M or N)=0,6.

26.1.1 Sketch a Venn-diagram to represent the events. (5)

26.1.2 Are the events N and M independent? Motivate your answer by showing all relevant calculations . (5)

26.2 A five-digit code is created by using digits 0 to 9.Digits may not be repeated. How many different codes are possible if the code must be a multiple of 5 and the code must start with an 8 ? (4)

Question 27

27.1 The digits 0 to 9 are used to form codes.

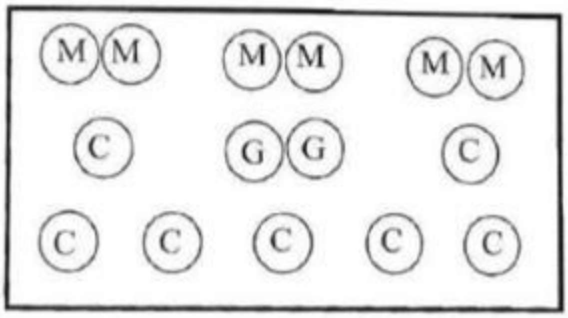
27.1.1 Determine the number of different 6-digit codes that can be formed If repetition of digits is allowed. (1)

27.1.2 Determine the number of 6-digit codes that can be formed that starts with a 9 and ends with a 2 if repetition of digits is not allowed. (2)

27.2 The digits 0 to 9 are used to form10-digit codes. Determine the number of 10-digit codes that can be formed if the 2 and the 3 may not appear next to each other and if repetition of digits is not allowed. (3)

Question 28

The Ngcobo family takes family photos.The photographer arranges three married couples,seven children and two grandparents as follows:
The couples stand husband and wife together at the back ,the grandparents in the middle and the children in the other positions as shown in the diagram below.



M	Married Couples
G	Grandparents
C	Children

How many different ways can the Ngcobo family be arranged for the photo?

Question 29

Events A,B and C occur as follows where A and B are independent events:

- P(A) = 0.38
- P(B)= 0.42
- P(A ∩ B) =0.1596
- P(C) =0.28
- There are 456 people in event A

29.1 Are A and B mutually exclusive events? Motivate your answer. (2)
29.2 By using an appropriate formula,show that the value of P(A U B) =0.64 (2)
29.3 Calculate the number of people in the sample space (2)
29.4 Determine n(C). (2)

Question 30

Five boys and four girls go to the movies. They are all seated next to each other in the same row

30.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)
30.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)

Question 31

31.1. Four digits codes (not beginning with 0), are to be constructed from the set of digits {1;3;4;6;7;8;0} (2)
31.2 How many four - digit codes can be constructed, if repetition of digits is allowed? (2)
31.3 How many four - digit codes can be constructed, if repetition of digits is not allowed? (2)
31.4 Calculate the probability of randomly constructing a four-digit code which is divisible by 5 if repetition of digits is allowed. (3)

Question 32

A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	Total
HAIR COLOUR	Light	*a*	*b*	20
	Dark	*c*	*d*	40
	Total	48	12	60

The survey concluded that the 'hand used for writing' and 'hair colour' are independent (5) events. Calculate the values of *a, b* and *c*.

Question 33

The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny ,the probability that Pitso plays soccer is $\frac{4}{5}$. If it is not sunny , the probability that Pitso plays soccer is $\frac{2}{5}$ .Determine the probability that Pitso does not play soccer. (5)

Question 34

There are t orange balls and 2 yellow balls in a bag.Craig randomly selects one ball from the bag,records his choice and returns the ball to the bag.He then randomly selects a second ball from the bag,records his choice and returns it to the bag.It is known that the probability that Craig will select two balls of the same colour from the bag is 52 %.

Calculate how many orange balls are in the bag . (6)

TRIGONOMETRY

(Questions include Integration)

Concepts covered.

1. **GENERAL TRIGONOMETRY**

2. **PROVING IDENTITIES**

3. **GENERAL SOLUTION, SOLVING TRIG. EQUATIONS, GIVEN DOMAIN**

4. **TRIGONOMETRIC GRAPHS**

5. **2D AND 3D TRIGONOMETRY**

6. **INTEGRATION**

1. GENERAL TRIGONOMETRY

- ***Using a Sketch***
- ***Reduction Formulae, (180°-x); (180°+x); (360°-x); (90°-x); (90°+x); (-x),***
- ***Special Angles, 0°; 30°; 45°; 60°; 90°***
- ***Compound and double angles, in disguise, Quadratic Trig form.***

1.1 Show that the value of the following expression is independent of the value of A:
$\sin(\mathrm{A}+40°)\cos(\mathrm{A}+30°)-\cos(\mathrm{A}+40°)\sin(\mathrm{A}+30°)$

1.2Given: $\sin \mathrm{A}\cos \mathrm{A}=k$ and k is acute.

(a) Determine the value of $\tan \mathrm{A}+\dfrac{1}{\tan \mathrm{A}}$ in terms of k.

(b) Prove that $\sin \mathrm{A}+\cos \mathrm{A}=\sqrt{1+2k}$.

1.3 Given that P and Q are both acute, solve for P and Q if:
$\sin \mathrm{P}\sin \mathrm{Q}-\cos \mathrm{P}\cos \mathrm{Q}=\frac{1}{2}$ and $\sin(\mathrm{P}-\mathrm{Q})=\frac{1}{2}$.

1.4 Determine the value of the following: $[\sin(22\frac{1}{2})°+\cos(22\frac{1}{2})°]^2$

1.5 Simplify: $\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$

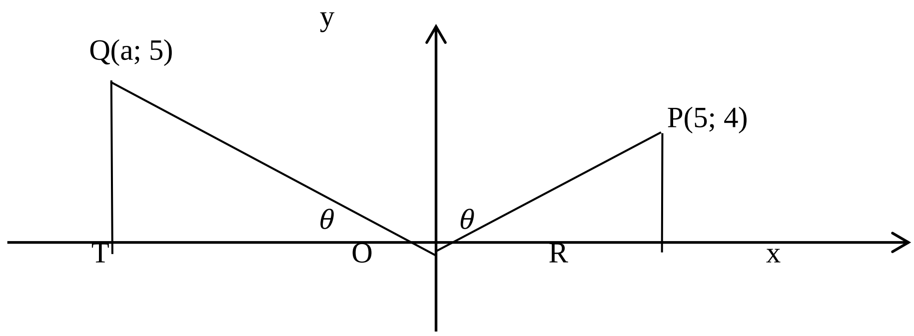
1.6 Simplify: $\dfrac{\cos(-x).\sin(x-180°).\tan x}{\sin 960°.\cos^2(x-90°).\sin 270°}$

1.7 If $\tan\theta = 1,5$ and $90° \leq \theta \leq 360°$, calculate without using a calculator and by using a diagram, the value of $\sin 2\theta$.

1.8 Simplify: $\dfrac{\sin 6\mathrm{A}}{\sin 2\mathrm{A}} - \dfrac{\cos 6A}{\cos 2A}$

1.9 If $\cos\beta + \sin\beta = T$, express $\frac{\cos 2\beta}{\sin(\beta - 45°)}$ in terms of T.

1.10 In the diagram below, similar triangles ΔOPR and ΔOQT are drawn.
O is the origin. R and T are points on the x-axis.



Determine, leaving answers in surd form if necessary:

a) $\cos(90° + \theta)$

b) The value of a

1.11 Simplify the following expression as far as possible.

a) $$\frac{\sin(180° - \theta).\cos(90° - \theta) - 1}{\cos(-\theta)}$$

b) Hence determine for which value(s) of $\theta \in [0°;\ 360°]$,

$$\sqrt{\frac{\sin(180° - \theta).\cos(90° - \theta) - 1}{\cos(-\theta)}}$$

will be real.

1.12 Given $\tan\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta < 0$. Without the use of a calculator, express each of the following in its simplest form:

a) $\sin\theta$ and $\cos\theta$

b) $\sin 2\theta$

c) $cos^2(90° + \theta)$

1.13 Simplify fully

$$\frac{\tan(180° - \theta).\cos(-\theta).\sin 390°}{(\cos 300°.\sin\theta) - \cos 450°}$$

1.14 If $\cos A + \sin A = k$, express the following in terms of k:

a) $\cos(A - 45°)$
b) $1 + \sin 2A$

1.15 $4\tan\theta + 5 = 0$ and $\theta \in [0°;\ 180°]$.
Determine, without the use of a calculator, the value of
$\sqrt{41}\ \cos\theta\ -\ 4\sin(-150°).\cos 180°$

1.16 If $\tan 50° = k,$ evaluate

$$\frac{4\ cos^2 25° - 2}{2\sin 25°.\cos 25°}$$

in terms of k.

1.17 Given the expression

$$\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A}$$

a) If $A \in [0°;\ 360°]$, for what value(s) of A is the expression undefined?

b) Prove that

$$\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A} = 2\cos A - 1$$

1.18 Given,
$\sin(A + B) = sinAcosB + cosAsinB$, $\cos(A + B) = cosAcosB - sinAsinB$,
and $tan\theta = y$, then determine

a) $\sin 2\theta$
b) $\cos 2\theta$

1.19 Simplify:

$$\frac{\cos 10°.\cos 340° - \sin 190°.\sin(-20°)}{\sin 80°.\cos 20° + \cos 100°.\cos 70°}$$

1.20 a) Prove $\cos(A - B) - \cos(A + B) = 2\sin A.\sin B$

b) Hence find without the use of a calculator the value of
$\cos 15° - \cos 75°$

1.21 Prove without the use of a calculator: $\cos 80° + \cos 40° = \cos 20°$

1.22 If $\sin\frac{x}{2} = p$, express the following in terms of p:

a) $\sin x$

b) $\cos x$

c) $\tan x$

1.23 If $\sin 2A = \frac{2\sqrt{6}}{5}$ where $A > 45°$, determine with the aid of a sketch, the value of $\sin A$

1.24 1.24.1 Deduce that $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

1.24.2 Use 1.24 (a) to simplify $\frac{\sin 3\theta - \sin\theta}{cos^2\theta - sin^2\theta}$

1.25 Given: $\cos D = 2p$ and $\cos 2D = 7p$

1.25.1 Calculate the value(s) of p

1.25.2 If $\hat{D} \in [0°;\ 360°]$, calculate the values of $\hat{D}$.

1.26 If $q\sin 61° = p$, express the following in terms of p and q

1.26.1 $\cos 151°$

1.26.2 $\cos 1°$

1.26.3 $\sin 122°$

1.26.4 $\cos 40°.\cos 8° + \sin 40° \sin 8°$

2. PROVING IDENTITIES

$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

2.1 Prove that $\cos(A + 45°) = \frac{\cos A - \sin A}{\sqrt{2}}$.

Then solve for A if $\cos A - \sin A = \frac{1}{\sqrt{2}}$ and $0° \le A \le 90°$.

2.2 Prove that $\sqrt{3}\sin(x + 60°) - \sin(x + 30°) = \cos x$.

2.3 Prove the following identities:

a) $1+\sin 2B=(\sin B+\cos B)^2$

b) $\cos A-\sin A=\dfrac{\sin 2A-1}{\sin A-\cos A}$.

2.4 a) Prove the identity

$$\frac{2\tan\theta}{1+tan^2\theta} = \sin 2\theta$$

b) Hence, or otherwise, determine the maximum value of

$$\frac{(1+\tan\theta)^2}{1+tan^2\theta}$$

2.5 Prove the following identity

$$\tan\theta\,.\sin\theta + \cos\theta = \frac{1}{\cos\theta}$$

2.6 Prove that

$$\frac{\cos(A-45°)}{\cos(A+45°)} = \frac{1+\sin 2A}{\cos 2A}$$

2.7 Prove the following identity:

$$\frac{\sin 3\alpha}{\sin\alpha} = 3 - 4\,sin^2\alpha$$

2.8 Prove that:

$$\frac{\cos 3x}{\cos x} = 2\cos 2x - 1$$

2.9 Prove that

$$\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A} = 2\cos A - 1$$

2.10 Prove that $\sin 2x+2\sin^2(45°-x)=1$ and hence deduce, without the use of a calculator, that $\sin^2 15°=\frac{2-\sqrt{3}}{4}$.

2.11 Given the following identity:

$$\frac{\cos x - \sin x\sin 2x}{\cos 2x} = \cos x$$

a) Prove the identity.

b) For which values of x is the identity undefined?
Give your answer in general solution form.

2.12 Prove:

$$\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{1}{\tan x}$$

3. <u>GENERAL SOLUTION, SOLVING TRIG EQUATIONS, GIVEN DOMAIN</u>

- **Compound and double angles, in disguise, quadratic trig form,**
- ***Sin x* and *cos x*, period is 360^0 and 180^0 (*tan x*), so k.180 or k.360**
- **When a rational function is undefined**
- **Where 2 trig functions intersect graphically.**

3.1 If $\cos\theta = 2\sin 75°\sin 15°$; $\theta \in [-360°;360°]$, determine θ without using a calculator.

3.2 Solve for A if $\tan A = \tan 135°$ and

a) $180° < A < 360°$

b) $360° < A < 720°$

3.3 Determine the general solution to

$$3\sin\theta.\sin 22° = 3\cos\theta.\cos 22° + 1$$

3.4 Determine the general solution to

$$\tan\theta.\sin\theta + \cos\theta = \frac{3}{\sin\theta}$$

3.5 Determine the general solution to

$$\frac{\sin 3\alpha}{\sin\alpha} = 2$$

3.6 Consider $cos6x + cos2x = 2cos4x.cos2x$

a) Show that $cos6x + cos2x = 2cos4x.cos2x$

b) Hence otherwise, write down the general solution of the equation
$cos6x + cos2x + cos4x = 0$

3.7 If $A \in [0°;\ 360°]$, for what value(s) of A is the expression below undefined?

$$\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A}$$

3.8 Calculate the values of x if $4sin^2x + 6\sin x.\cos x - 2\sin x - 3\cos x = 0$,

for $-360° \leq x \leq 0°$. Round off the answer to 2 decimal digits, if necessary.

3.9 Determine the general solution of the equation $2\sin A.\cos A - 0{,}8 = 0$

3.10 If $\theta \in [-180°;180°]$, determine the value(s) of θ:

(a) $\sin 5\theta \cos 20° - \cos 5\theta \sin 20° = 1$

(b) $2\cos 3\theta \cos 30° - 2\sin 3\theta \sin 30° = 1$

3.11 Calculate the value of x between 0° and 360° if: $\cos 2x + \sin x = 0$.

3.12 Determine the general solution:

(a) $\sin x = 2\cos^2 15° - 1$

(b) $\cos 2x = \sin x - 2$

(c) $\sin 3x \cos x - \cos 3x \sin x = \sin x$

(d) $\sin x \cos 320° + \cos x \sin 320° = -1$

(e) $2\sin 2x + \cos 2x + 2 = 0$ and $\tan 71{,}6° = 3$

3.13 Find the values of x between $-180°$ and 180° if: $7\sin(x - 30°) + 2 = 0$

3.14 Prove for any angles A and B:

3.14.1) $\frac{\sin A}{\sin B} - \frac{\cos A}{\cos B} = \frac{2\sin(A-B)}{\sin 2B}$

3.14.2) Hence show without using a calculator

a) $\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = 4\cos 2B$

b) $\frac{1}{\sin 18°} = 4\cos 36°$

c) $\sin 18°$ is a solution of the cubic equation $8x^3 - 4x + 1 = 0$

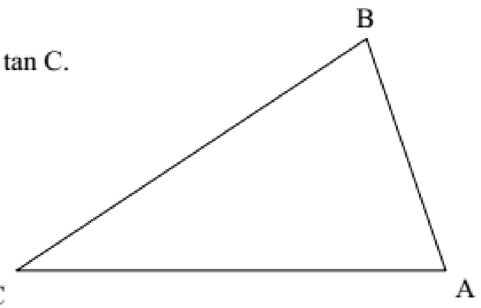
3.15.1 Using the expansions for sin(A + B) and cos(A + B), prove the identity of:

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$ (3)

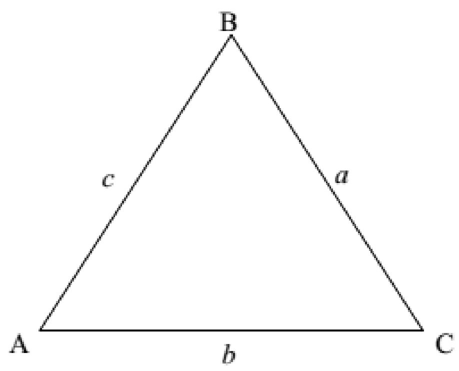
3.15.2 If $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$, prove in any ΔABC that

tan A.tan B.tan C = tan A + tan B + tan C. (4)

[7]



3.16 Triangle ABC is isosceles with AB = BC.



Prove that $\cos B = 1 - \frac{b^2}{2a^2}$ (4)

3.17 Prove the following identity: $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$ (3)

3.18 Given that $cos 314° = t$

Calculate, with the aid of a sketch:

3.18.1 $sin 46°$ (3)

3.18.2 $tan 88°$ (4)

3.18.3 $\cos 134°$ (2)

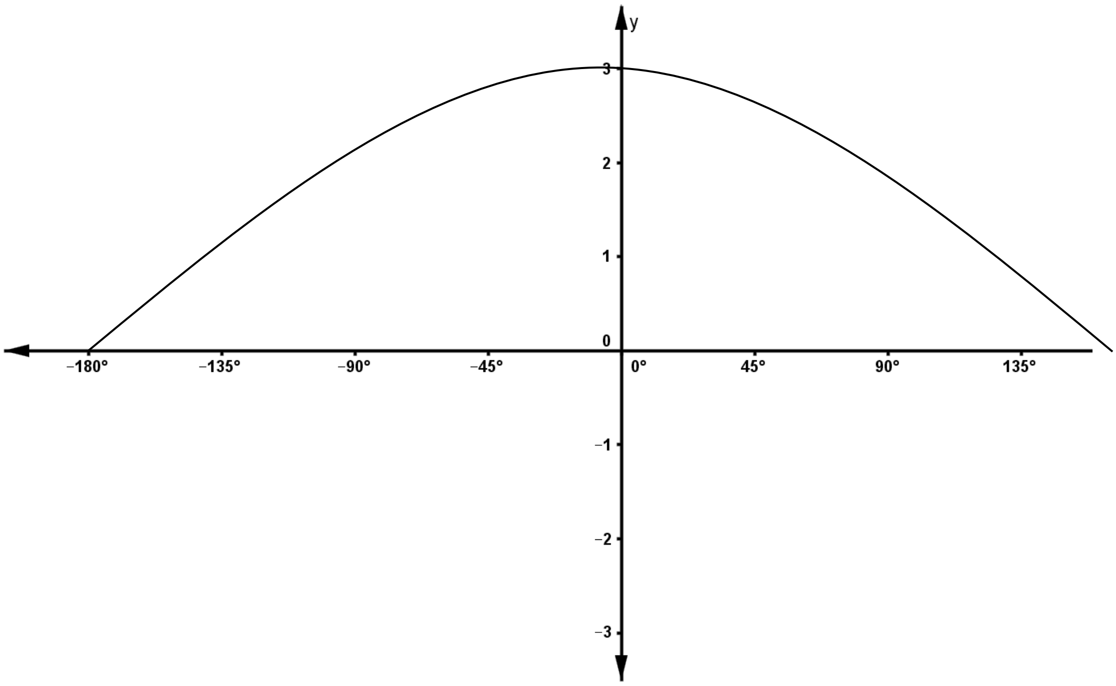
3.19 Determine, without using a calculator, the numerical value of:

$tan1° \times tan2° \times tan3° \times tan4° \times \ldots\ldots..\times tan87° \times tan88° \times tan89°$ (4)

4. <u>TRIGONOMETRIC GRAPHS</u>

- **Domain**
- **Range**
- **Determine equations**
- **Amplitude**
- **Intersection between TWO graphs**
- **Increasing and decreasing graphs**
- **Inequalities**
- **Distance between curves**
- **Transformation of functions**

4.1 On the axes the graph of $f(x) = a\cos bx$ for $-180° \leq x \leq 180°$ is sketched.



a) Write down the values of a and b.

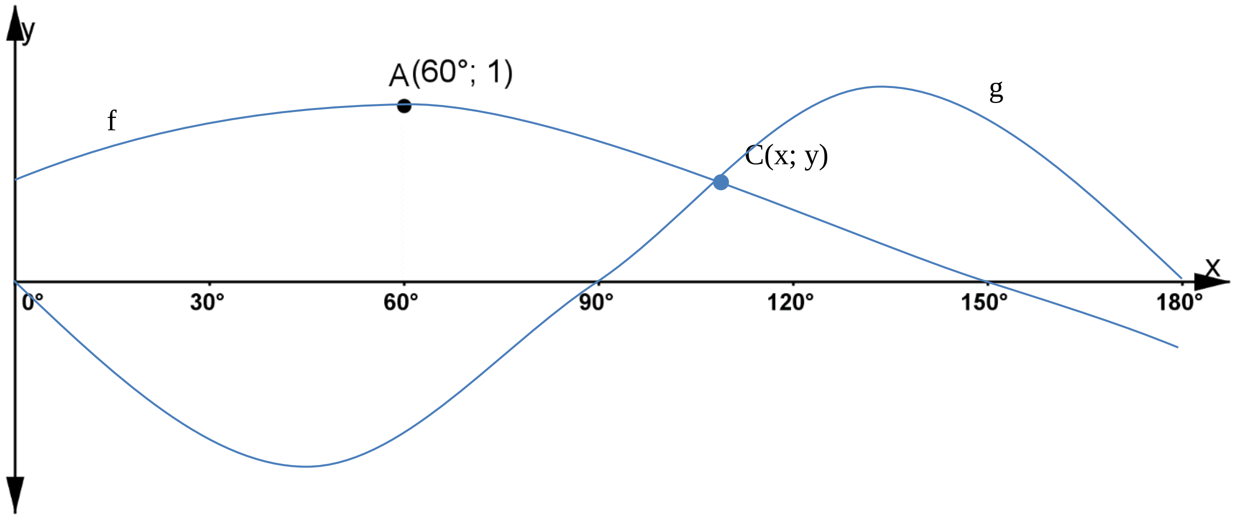
b) Write down the period of f.

c) Show on the x-axis where you would read off the solution to
$a\ \cos bx\ = 2,$ for $-180° \leq x \leq 180°$.
(Use letters A and B on the x-axis.)

d) On the same set of axes, draw a second graph which would allow you to read off the solution of the equation

$$a\ \cos bx\ = 1 + \sin(x - 45°); \quad -180 \leq x \leq 180°$$

4.2 The figure shows the graph $f(x) = \cos(x + \theta)$ and $g(x) = -\sin 2x$ for $x \in [0°;\ 180°]$.



a) Write down the range of g.

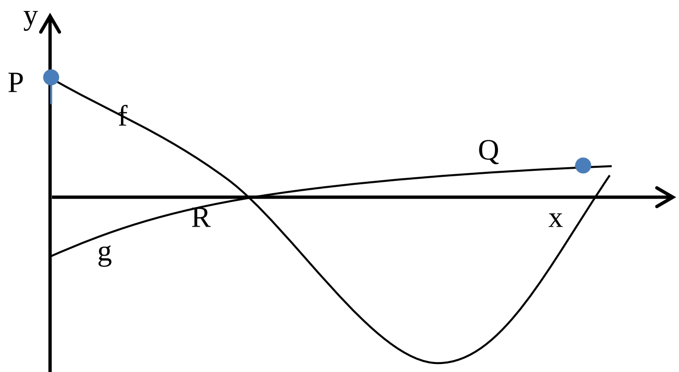
b) Determine the value of θ

c) $C(x; y)$ is the point of intersection of the two graphs. Solve for x.

d) For which values of x is $f(x).g(x) > 0$?

e) For which values of x is $f'(x).g'(x) > 0$?

4.3 The diagram shows part of the graphs of $f(x) = 4\cos 3x$ and $h(x) = \sin(x + b)$.
Points P and Q are the respective maximum points on these graphs.
The graphs intersect on the x axis at R.



a) Write down the coordinate of R.

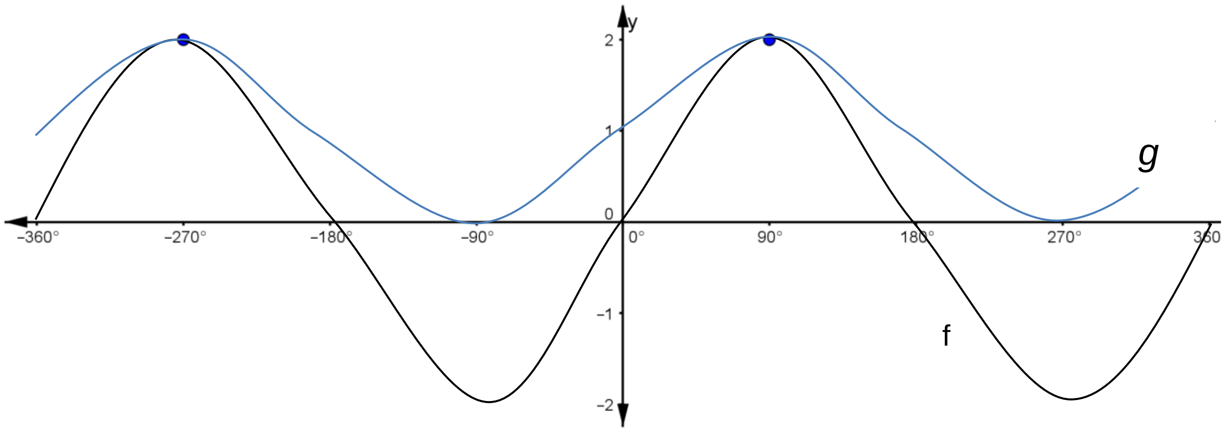
b) Write down the value of b

c) Write down the period of f.

d) If f is shifted horizontally by 30^0 to the right, write down the resulting graph.

e) If h is shifted vertically by 2 units, write down the amplitude of the resulting graph.

4.4 The sketch shows the curves of $f = \{(x; y)/\ y = a \sin x\}$ and g for $x \in [-360°;\ 360°]$.



Answer the following questions with the aid of the graph:

a) The maximum value of f is . . .

b) The value of a is . . .

c) The amplitude of g is . . .

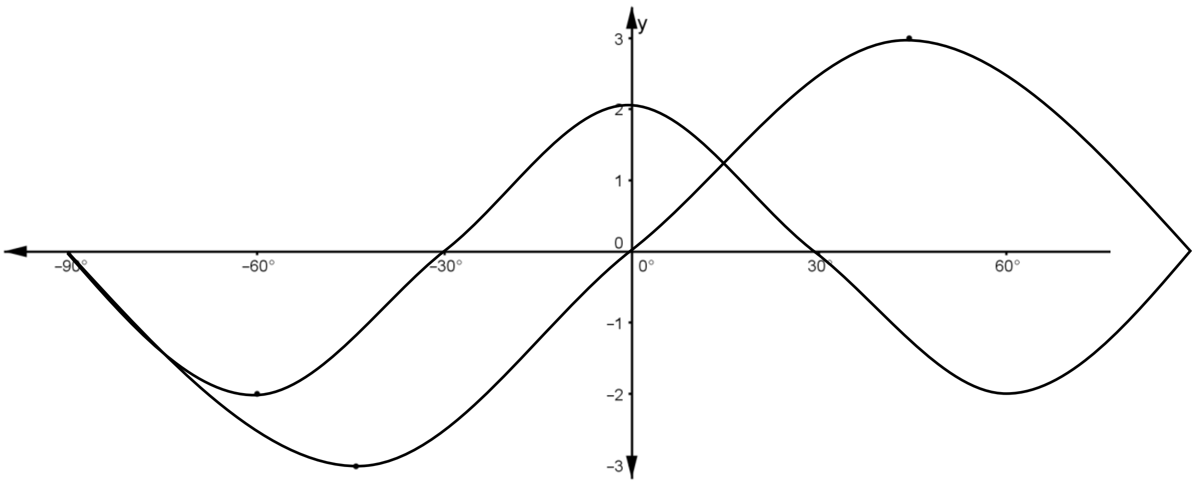
d) The equation of g is . . .

e) Write down two values of x for which $\sin x = \frac{1}{2}\sin x + \frac{1}{2}$

f) For which negative values of x will g decrease if x increase?

g) For which values of x is $\frac{f(x)}{g(x)}$ undefined?

4.5 In the figure are sketch graphs of the functions
$y = n\ \sin 2x$ and $y = 2\cos mx$ for $x \in [-90°;\ 90°]$.



Use the sketch graphs to answer the following questions.

4.5.1 Determine the value of m and n.

4.5.2 Write down the range of $\{(x; y/2\cos mx\ \leq y\ \leq\ n \sin 2x)\}$ for $x \in [-90°;\ 90°]$.

4.6 Given: $f(x) = \sin(x + 30°)$ and $g(x) = \cos 2x$

4.6.1 Draw neat sketch graphs of $f(x)$ and $g(x)$ on the same set of axes for $x \in [-180°;\ 180°]$.

4.6.2 For which value(s) of x is $f(x).g(x) > 0$ for $x \in [-180°;\ -30°]$.

4.6.3 Write down the period of $h(x) = g\left(\frac{x}{2}\right)$.

4.6.4 Write down the new equations of the transformations if f is moved 60^0 to the right and g is moved 2 units up.
Give your answers in the form $f'(x) = \ .\ .\ .$ and $g'(x) = \ .\ .\ .$

4.7 Given: $f(x) = \cos 2x$ and $g(x) = -\sin x$, for $x \in [-180°;\ 180°]$

4.7.1 Calculate the values of x for which $f(x) = g(x)$ for $x \in [-180°;\ 180°]$

4.7.2 Sketch, on the same set of axes, the graphs of f and g showing all intercepts with the axes as well as the turning points for $x \in [-180°;\ 180°]$.

4.7.3 Write down the period of f

4.7.4 Determine the values of x for which $f(x) - g(x) \leq 0$ for $x \in [-180°;\ 180°]$.

4.7.5 Hence, determine the maximum value of $\cos 2x + \sin x$ on the interval $[-180°;\ 180°]$.

4.7.6 $g(x)$ is reflected about the x-axes and then shifted 1 unit down to $h(x)$.
Write down the equation of $h(x)$.

4.8 Given: $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30°)$

4.8.1 On the same set of axes, draw sketch graphs of the curves of f and g for $x \in [-120°;\ 120°]$. Show clearly all intercepts with the axes, coordinates of all turning points and coordinates of all end points of both curves.

4.8.2 Use the graphs drawn in 4.8.1 to determine for which value(s) of $x \in [-120°;\ 60°]$ is:

a) $\cos(60° - x) < 0$

b) $f(x) - g(x) > 0$

c) $\frac{f(x)}{g(x)}$ undefined ?

4.9.1 Given: $f(x) = \sin(x + a)$
Give one possible value of a if the graph of f has a maximum value at $x = 60°$

4.9.2 Given: $g(x) = \sin x + b$
For what value of b will g have a maximum value of 3 ?

4.9.3 On the same set of axes, sketch the graphs of $f(x) = \cos(x - 22{,}5°)$ and $g(x) = \tan 2x$ for $x \in [-90°;\ 135°]$.
Ensure that all x-intercepts, maximum/minimum points and asymptotes are clearly shown.

4.9.4 Answer the following questions with the help of your graphs ABOVE:

a) For which value(s) of x, in the given domain, is $f(x) = 0$?

b) What is the period of $g(\frac{3x}{2})$?

c) For what positive values of x are $f(x)$ and $g(x)$ both positive in the given domain?

d) If the y-axis is shifted $22{,}5^0$ to the left, what is the equation, and the corresponding domain, of $f(x)$ relative to the new axes?

4.10 a) Determine the general solution of $\sin 2x = \cos(x + 60°)$

b) Hence, solve for x if $\sin 2x = \cos(x + 60°)$ and $x \in [-90°;\ 180°]$.

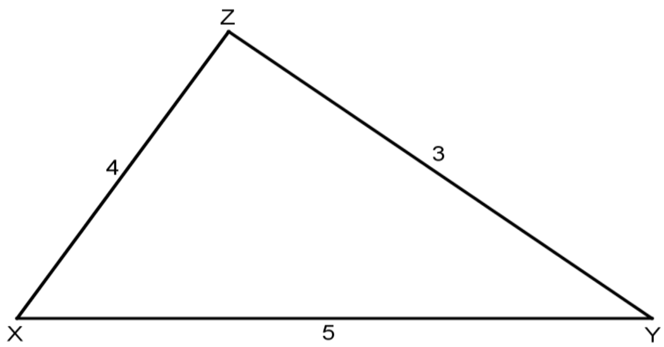
4.11 a) Draw on the same set of axes sketch graphs of $f(x) = \sin 2x$ and $g(x) = \cos(x + 60°)$ for $x \in [-90°;\ 180°]$. Show clearly all the intercepts on the axes and the coordinates of the turning points.

b) Write down the range of g.

c) Let $h(x) = g(-x)$.
Describe the transformation of g onto h.

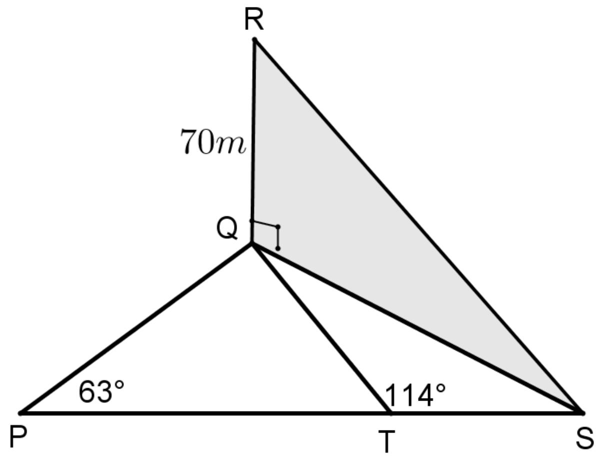
5. <u>2D AND 3D TRIGONOMETRY</u>

5.1 ΔXYZ has lengths 4, 5 and 6 as shown in the diagram.



Using the Cosine rule, show that $\cos \hat{Y} + \cos \hat{Z} = \frac{7}{8}$

5.2 The diagram represents a triangular car park PQS and a building RQ of height 70 metres. T is a point on PS such that PT : TS = 5 : 3
PS = 144 metres, $Q\hat{P}T = 63°$ and $S\hat{T}Q = 114°$.



a) Show that QT = 103 metres rounded to the nearest whole number.

b) Determine the angle of elevation of R from S. Round off your final answer to the nearest whole number.

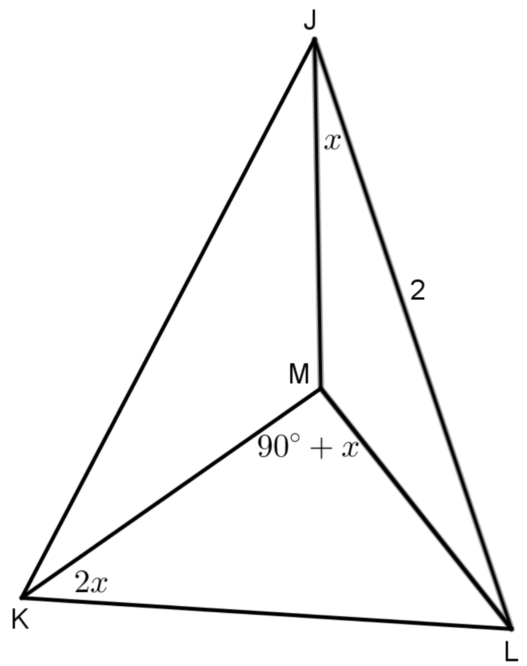
5.3 JM is a vertical tower and points K and L are in the same horizontal plane as point M, the foot of the tower.

$M\hat{J}L = x$

$K\hat{M}L = 90° + x$

$M\hat{K}L = 2x$

$JL = 2\ units$

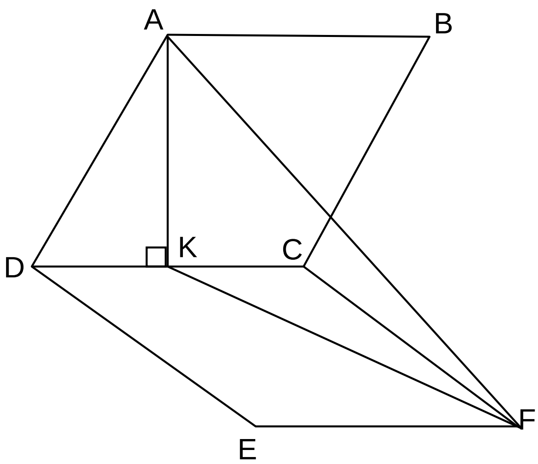


a) Show that KL = 1

b) Show that $MK = 2\cos 2x - 1$.

c) Find the values of x for which MK exists.

5.4 Two identical rhombuses ABCD and EFCD are placed at right-angles against each other.
$A\hat{D}C = 60°$, AD = h units and $AK \perp DC$,



Calculate the following lengths in terms of h

(a) AK

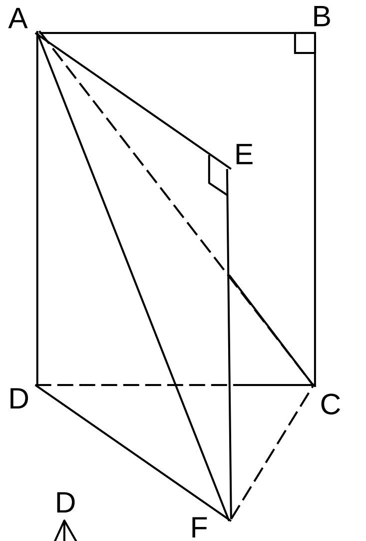
(b) DK

(c) KF

If the two rhombuses are pressed against each other so that the angle between them is < 90°
the area of ΔAKF becomes $\frac{\sqrt{21}\,h^2}{16}$

(d) Calculate the angle between the two rhombuses.

5.5 The cover of a book EABCDF stands upright as in the figure. AC and AF are the diagonals of identical rectangles ABCD and AEFD respectively. AB = p units and CF = q units.

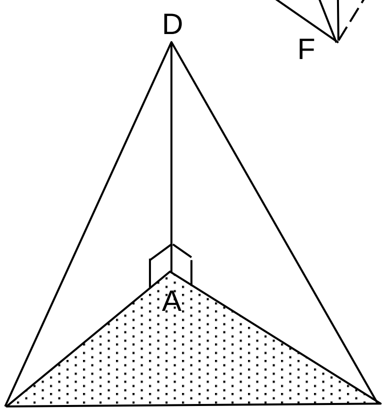


(a) Show that $\cos C\hat{D}F = 1 - \frac{q^2}{2p^2}$ A

(b) If p = 15 and q = 12, calculate

(i) the size of $C\hat{D}F$
(ii) the length of AC if $F\hat{A}C = 27,8°$

5.6 A, B and C are three points in the same horizonal plane.
DA is a vertical cliff.
The angle of elevation from B to the top of the cliff is θ.
$A\hat{B}C = A\hat{C}B = \alpha$.
The distance between B and C is k metres.
Prove that $AD = \frac{k\tan\theta}{2\cos\alpha}$

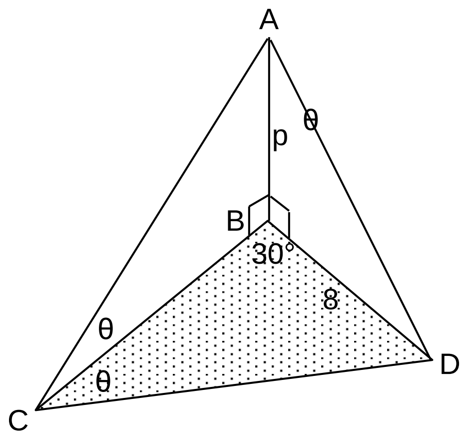


5.7 B, C and D are three points in the same horizontal plane.
AB is a vertical pole of length p metres.

The angle of elevation of A from C is θ.

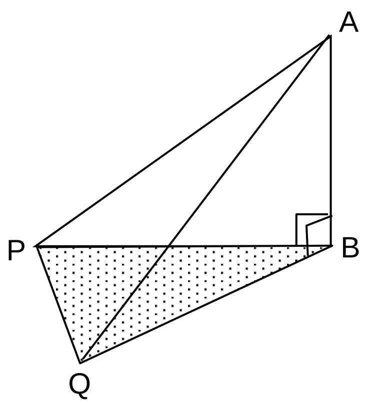
$B\hat{C}D = \theta \qquad C\hat{B}D = 30° \qquad BD = 8\text{ m}$

Prove, without using a calculator that $p = 4\left(1+\sqrt{3}\tan\theta\right)$ metres.



5.8 AB is a vertical corner post and AP and AQ are two cables.
P, Q and B are in the same horizontal plane.
The height of the corner post AB is x metres.
PQ = 2x metres
The angle of elevation of A from Q is θ.

$A\hat{P}Q = 90° - \theta \qquad P\hat{A}Q = \alpha$



a) Determine α in terms of θ.

b) If $\alpha = 2\theta$ show that the area of ΔAPG = $\dfrac{x^2}{\tan\theta}$

5.9 B, C and D are three points in the same horizontal plane.

BC = CD = d and $C\hat{B}D = x$

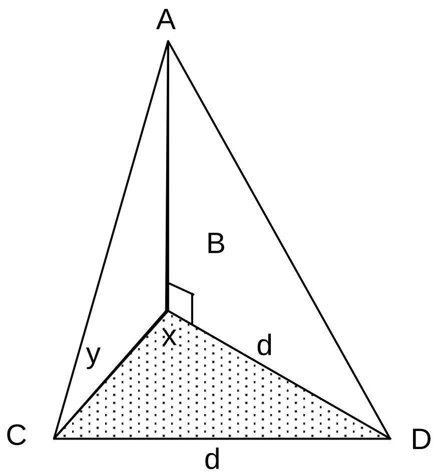
AB is perpendicular to the plane.

The angle of elevation of A from C is y.

Answer the questions below:

(a) Prove that $AB = 2d\cos x\tan y$

(b) Given that $d = \sqrt{2}$ units, x = 75° and y = 30°, calculate the length of AB without using a calculator.



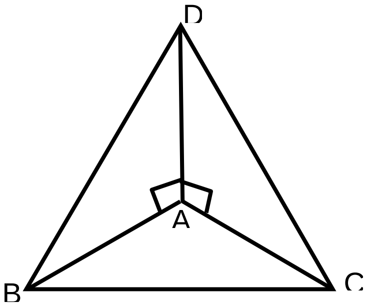
5.10 DB is a vertical tower h metres high. A, B and C are in the same horizontal plane.

From A and C, the angles of elevation to D are both α .

$A\hat{B}C = 2\alpha$

Prove AC = 2h cos α

5.11 ABC is a triangle in the horizontal plane.



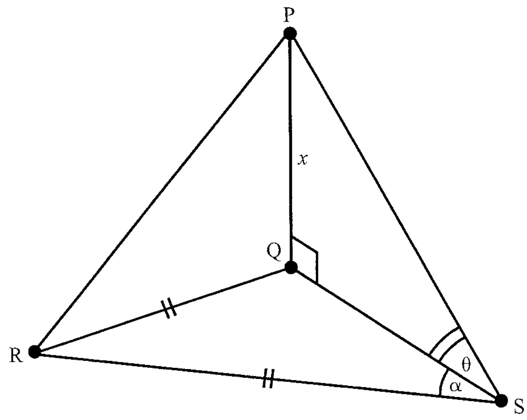
AD is a vertical post which subtends $x°$ at B
BD and DC are equal in length and

$D\hat{C}B = y^0$.

AB = k

Prove that $BC = \dfrac{2k\cos y}{\cos x}$

5.12 PQ is a vertical flagpole of length x metres, with Q at the foot of the flagpole. R, Q and S are three points on the same horizontal surface. If RQ = RS, $Q\hat{S}R = \alpha$ and $P\hat{S}Q = \theta$:

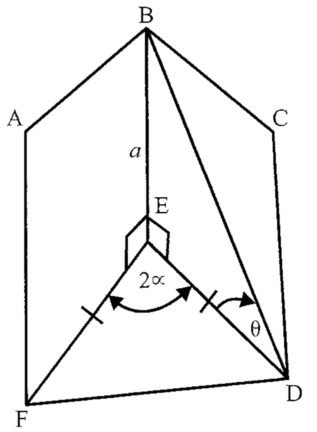


a. Show that: $QS = \dfrac{x}{\tan\theta}$

b. Prove that: $RS = \dfrac{x}{2\tan\theta\cos\alpha}$

c. If $\theta = 45°$ and $\alpha = 60°$ and x = 4 metres, calculate the length of RS.

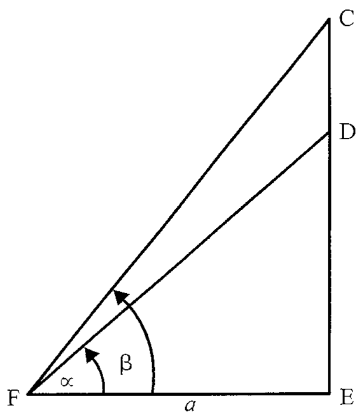
5.13 The figure shows a book standing open at an angle of 2α, where the height of the book is *a* cm and $E\hat{D}B = \theta$



a. Prove that FD = $\dfrac{2a\sin\alpha}{\tan\theta}$

b. Determine the height of the book if $\alpha = 43°$ and $\theta = 57°$ and FD = 30 cm (round off to the nearest cm).

5.14 CD is a flagstaff on the top of a building, ED. The angle of elevation from point F, *a* metres away from the base of the building, to the top of the building is α. The angle of elevation from F to the top of the flagstaff is β.

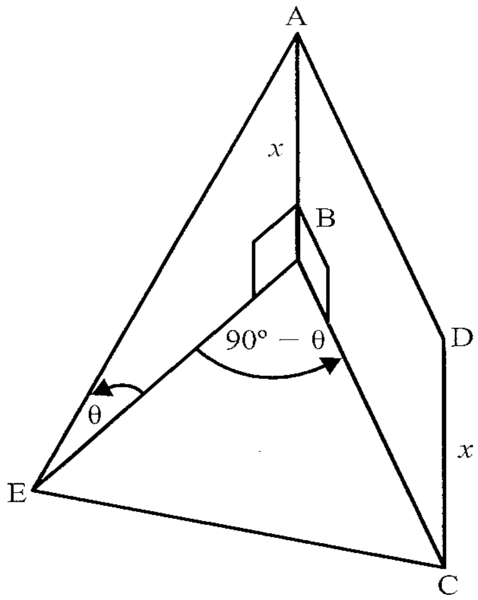


a. Determine $F\hat{C}E$.

b. Prove: $CD = \dfrac{a\sin(\beta - \alpha)}{\cos\alpha\cos\beta}$

c. If $a = 5$ metres, $\alpha = 30°$ and $\beta = 60°$, determine without a calculator, the length of the flagstaff.

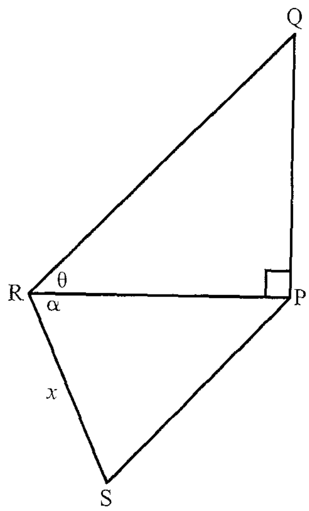
5.15 The rectangular wall ABCD has a length that is twice as long as its height. Let the height equal a length of x units and $E\hat{B}C = 90° - \theta$. The angle of elevation of A from E is θ.



a. Find the length of EB in terms of x and θ.

b. Prove that: $EC = x\sqrt{\dfrac{1}{\sin^2\theta} - 4\cos\theta + 3}$

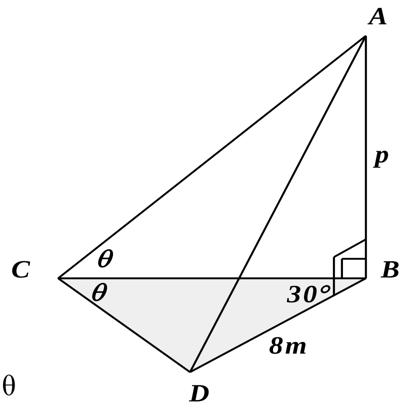
5.16 Points R, P and S are three points in the same horizontal plane, where P is the foot of a tower PQ. The angle of elevation of R to Q is θ, while $P\hat{R}S = \alpha$ and RS is x metres in length. The area of triangle PRS is A m^2.



a. Prove that: $PQ = \dfrac{2A\tan\theta}{x\sin\alpha}$

b. If it is given that PQ is 4 metres, x is 3 metres, θ is 30° and α is 60°, determine without using a calculator, the area of triangle PRS.

5.17 In the sketch below, B, C and D are three points in the same horizontal plane. AB is a vertical pole p metres high. The angle of elevation of A from C is θ, $B\hat{C}D = \theta$, $C\hat{B}D = 30^0$ and $BD = 8$ metres.



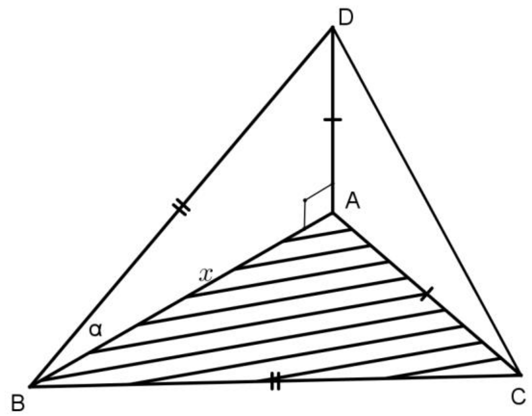
a. Express $C\hat{D}B$ in terms of θ.

b. Express BC in terms of p and a trigonometric ratio of θ

c. Hence or otherwise, show that $P = 4(1 + \sqrt{3}tan\theta)$

.

5.18 In the diagram, DA represents a vertical tower. B and C are two points in the same horizontal plane as A, the foot of the tower. The angle of elevation of D, as measured from B, is equal to α and $B\hat{A}C = 90°$.
It is further given that $BD = BC,\ AD = AC$ and $AB = x$ units.

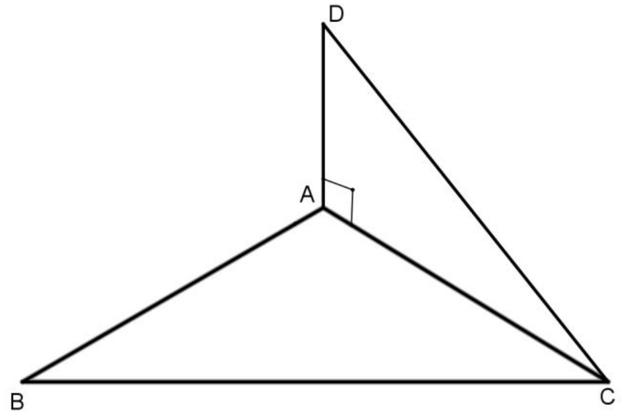


a) Express AC and BC in terms of x and α

b) Express CD^2 in terms of x and α

c) Hence prove that $\cos C\hat{B}D = cos^2\alpha$

5.19 In the figure, A, B and C are three points in the same horizontal plane. DA is perpendicular to the horizontal plane at A and D is joined to C.
$AB = \frac{1}{2}BC = a \ \ and \ \ A\hat{C}D = \frac{1}{2}A\hat{B}C = \alpha.$



a) Show that $AD = a.tan\alpha.\sqrt{1 + 8sin^2\alpha}$

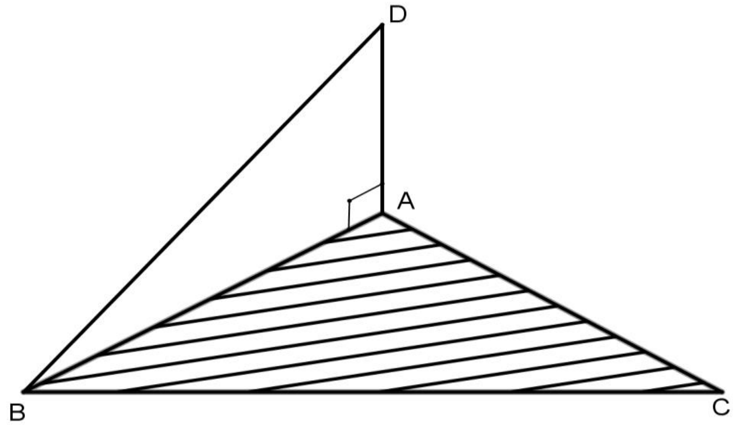
b) Hence calculate the value of AD if $a = 89\ mm$ and $\alpha = 35°$.
(Round the answer off to one decimal digit.)

5.20
a. Prove in any ΔABC that $a^2 = b^2 + c^2 - 2bc\,.\cos A$.

b. In triangle ABC the $\hat{C}$ is obtuse.

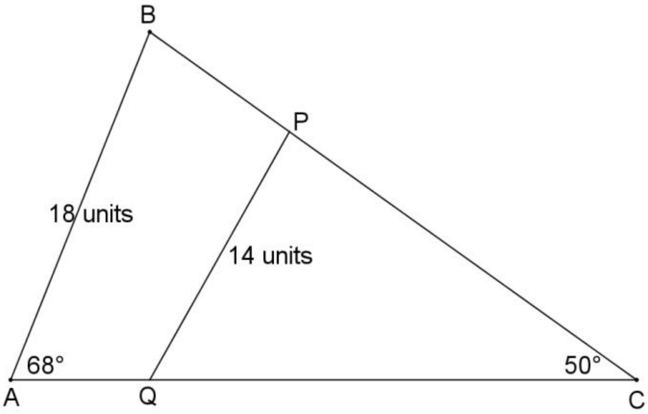
Prove that: $\frac{b}{sinB} = \frac{c}{sinC}$

5.21 In the figure, B and C are points in the same horizontal plane as A, the foot of a vertical tower DA.
$D\hat{B}A = B\hat{A}C = \alpha$ and $DB = AC = x$
Prove that $BC = x\sqrt{cos^2\alpha - cos2\alpha}$



5.22 In the diagram below, an acute-angled triangle ABC is drawn:

- A line PQ is drawn, where P lies on the line BC and Q lies on the line AC.
- The length of PQ is 14 units and the length of AB is 18 units.
- $\hat{A} = 68°$ and $\hat{C} = 50°$.

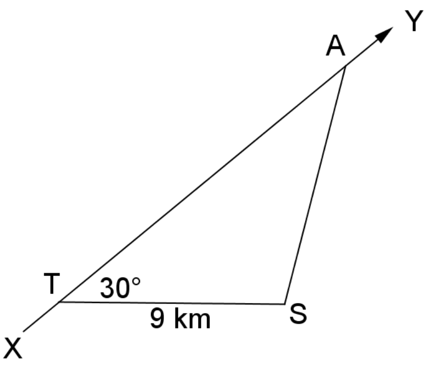


If the ratio of BP : PC is 2 : 3 determine the size of $P\hat{Q}C$.

5.23 In the figure, S represents the position of a stationary submarine which is involved in target practice. A target vessel is steering a straight course along the path XY. When the target vessel is at T, it is 9 kilometres from S. The submarine is armed with torpedoes which have a maximum range of 7,5 kilometres. $S\hat{T}Y = 30°$.

5.23.1 If A is the furthest point along XY that can be reached by a torpedo fired from the submarine at S, calculate the size of $T\hat{A}S$ to the nearest degree.

5.23.2 Hence calculate the total length of the path XY that can be brought under fire from the submarine at S.



5.24 Consider ΔABC in which AB = 10 units, AC = 5 units and $B\hat{A}C$ is a variable angle. Which of the statements (if any) are always true? Give reasons for answers.

5.24.1 As the size of $B\hat{A}C$ increase, so BC increases in length.

5.24.2 As the size of $B\hat{A}C$ increase, so the area of ΔABC increases.

5.25 Using the information in 5.24 , answer the following questions, rounding off answers to one decimal digit:

5.25.1 What is the length of BC if $B\hat{A}C = 31{,}5°$?

5.25.2 What is the size of $B\hat{A}C$ if the length of BC is double the length is was when $B\hat{A}C$ was $31{,}5°$?

5.26 In ΔABC with C an obtuse angle, $c^2 = a^2 + b^2 - 2ab\cos C$

5.26.1 Use the formula and prove that:

$$1 - \cos C = \frac{(c - a + b)(c + a - b)}{2ab}$$

5.26.2 If $b = c \ \ and \ \ a^2 = 7b^2,$ explain why ΔABC cannot be constructed.

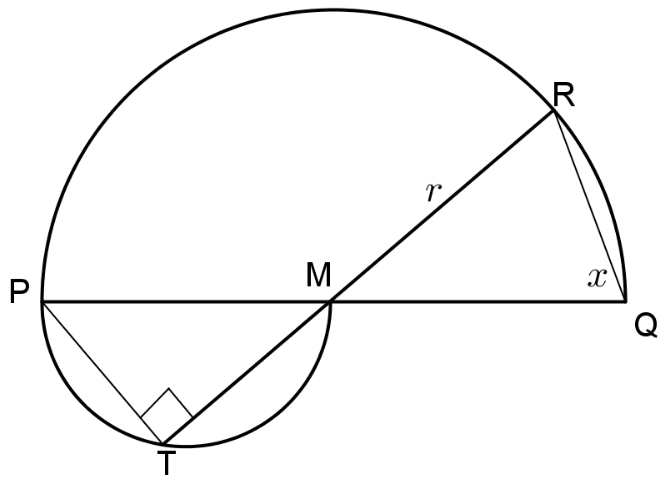
5.27 Prove that for any $\Delta PQR,$ its area A, is given by

$$A = \frac{p^2 \sin Q \sin R}{2 \sin P}$$

5.28 In the figure, M is the centre of semicircle PRQ and r is the radius.

PM is the diameter of semicircle PTM.

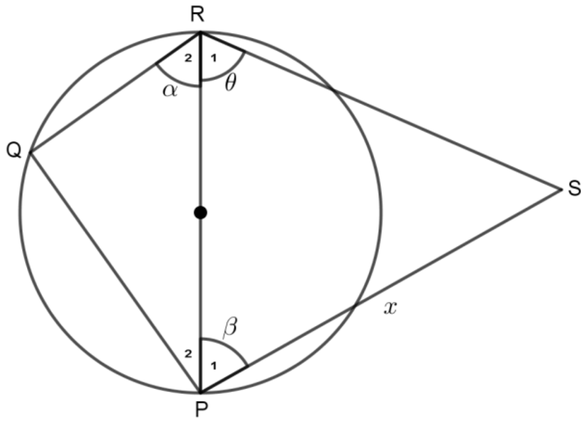
$\hat{Q} = x.$



5.28.1 Determine RQ in terms of r and x and simplify the expression as far as possible.

5.28.2 Determine the area of ΔPTM in terms of r and x .

5.29 In the circle below, PR is a diameter of the circle, passing through P, Q and R. S is a point outside of the circle. RS and PS are drawn. $PS = x$. $P\hat{R}S = \theta$. $R\hat{P}S = \beta$ and $P\hat{R}Q = \alpha$.



5.29.1 Prove that

$$PR = \frac{x\sin(\theta + \beta)}{\sin\theta}$$

5.29.2 Prove that

$$QR = \frac{x\cos\alpha\,.\sin(\theta + \beta)}{\sin\theta}$$