MATHEMATICS

MATERIAL FOR GRADE 12

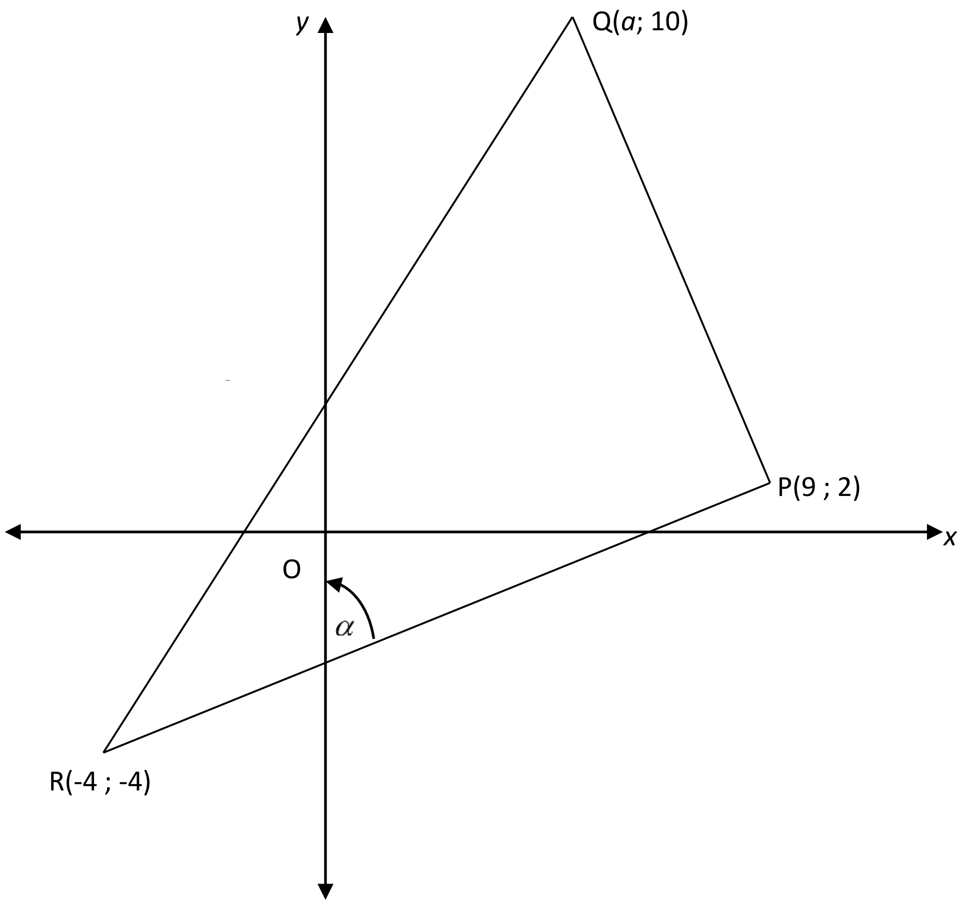
Analytical Geometry

QUESTIONS

QUESTION 1

In the diagram below, P (9; 2); Q (a; 10) and R (– 4; – 4) are the vertices of ΔPQR.

α is the angle between y-axis and the line PR.



1.1 Determine the gradient of PR. (3)

1.2 Calculate the size of α, angle between y-axis and the line PR. (4)

1.3 Show that the value of $a = 5$ if PQ = $4\sqrt{5}$ units and Q (a; 10). (6)

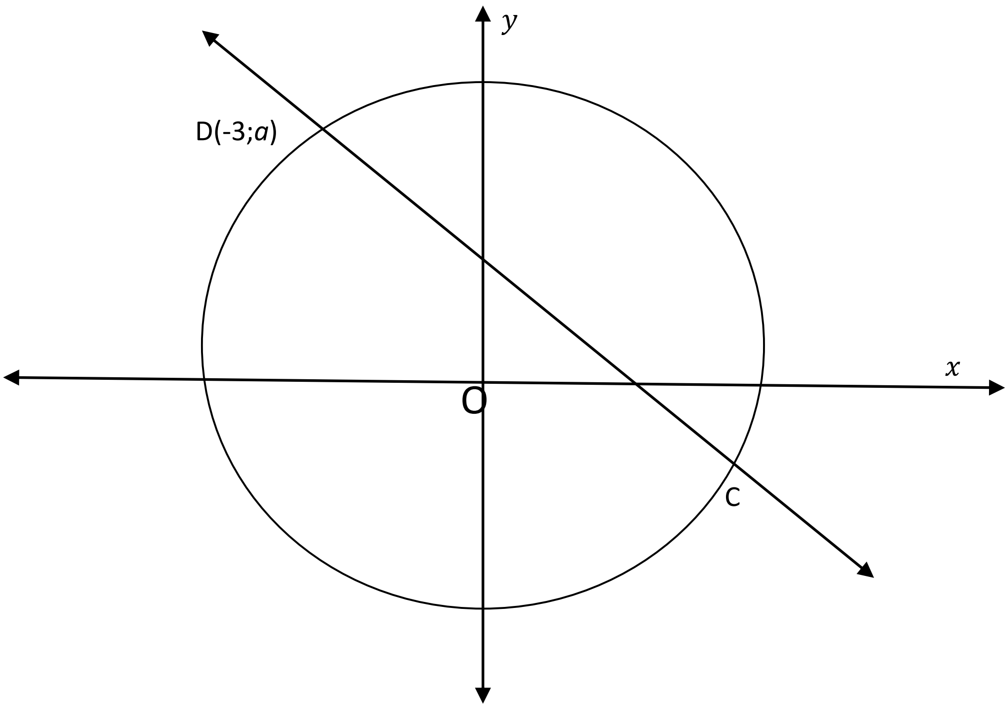
1.4 Determine the equation of a line parallel PR and passing through Q. (3)

1.5 Calculate the co-ordinates of S (x;y), if PQSR is a parallelogram and S is a point in the second quadrant. (4)

[20]

QUESTION 2

In the diagram below, the circle with centre O(0;0) cuts the straight line with equation $y = -x + 1$ at a points C and D$(-3; a)$.



2.1 Determine the numerical value of a. (2)

2.2 Hence, show that the equation of the circle is $x^2 + y^2 = 25$. (3)

2.3 Determine the co-ordinates of C. Show ALL your working. (6)

2.4 Determine, using $D(-3;4)$:

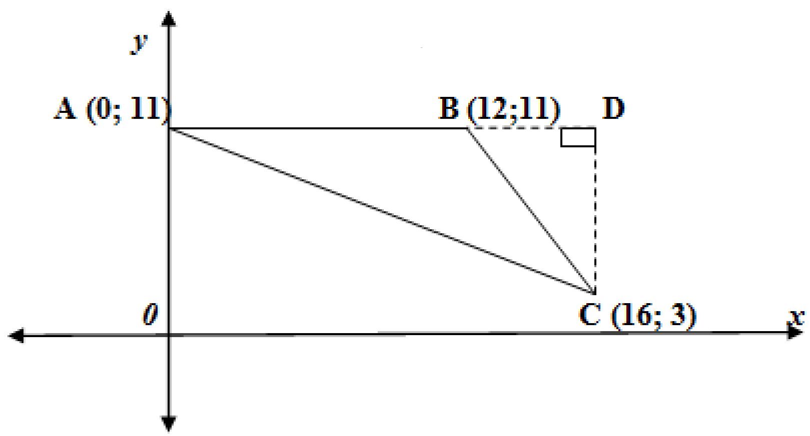
2.4.1 the gradient of OD. (2)

2.4.2 the equation of the tangent to the circle at D. (3)

[16]

QUESTION 3

In the diagram below A(0;11), B(12;11) and C(16;3) are the vertices of Δ ABC, with height CD .



3.1 Write down the equation and the length of the line AB. (3)

3.2 Write down the coordinates of the point D. (2)

3.3 Determine the coordinates of M, the midpoint of AC. (2)

3.4 Determine the equation of the perpendicular bisector of AC. (4)

3.5 Does the line in 2.5 pass through B? Justify your answer with relevant calculations. (2)

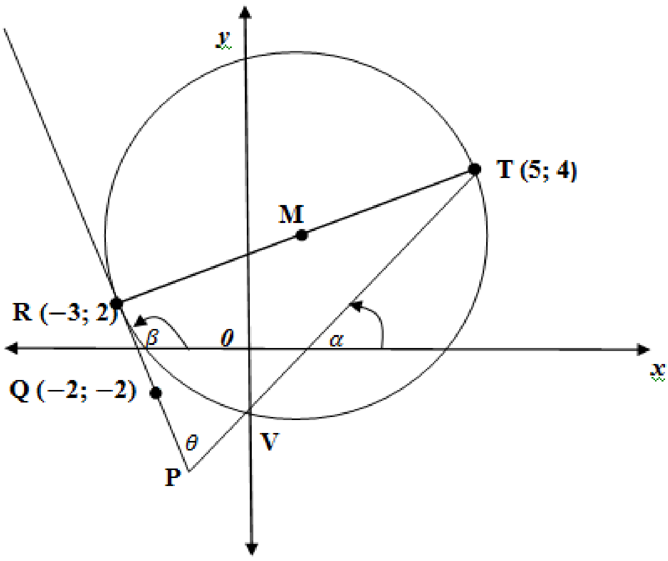
3.6 Determine the equation of the line parallel to AC, passing through D. (3)

3.7 Calculate the area of Δ ABC. (3)

[19]

QUESTION 4

In the diagram the circle with centre M passes through points V, R$(-3;2)$ and T$(5;4)$.
Q is the point $(-2;-2)$ and the lines through RQ and TV meet at P. The inclination angle of PT is α and the angle of inclination of PR is β.
V is the y-intercept of both the circle and line TP.



4.1 Determine the equation of the circle with centre M. (5)

4.2 Show, using analytical methods, that PR is a tangent to the circle at R. (3)

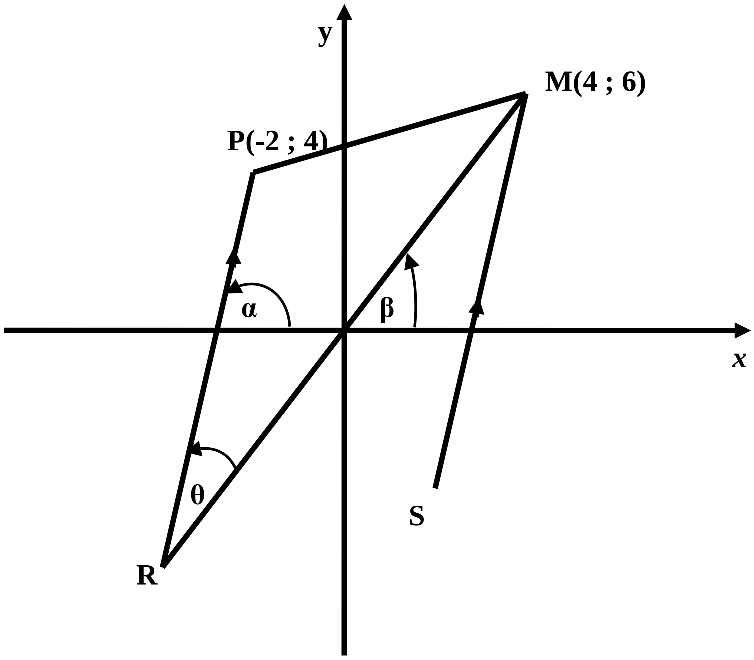
4.3 Determine the coordinates of V. (4)

4.4 If $R\hat{P}T = \theta$, calculate θ to ONE decimal place. (6)

[18]

QUESTION 5

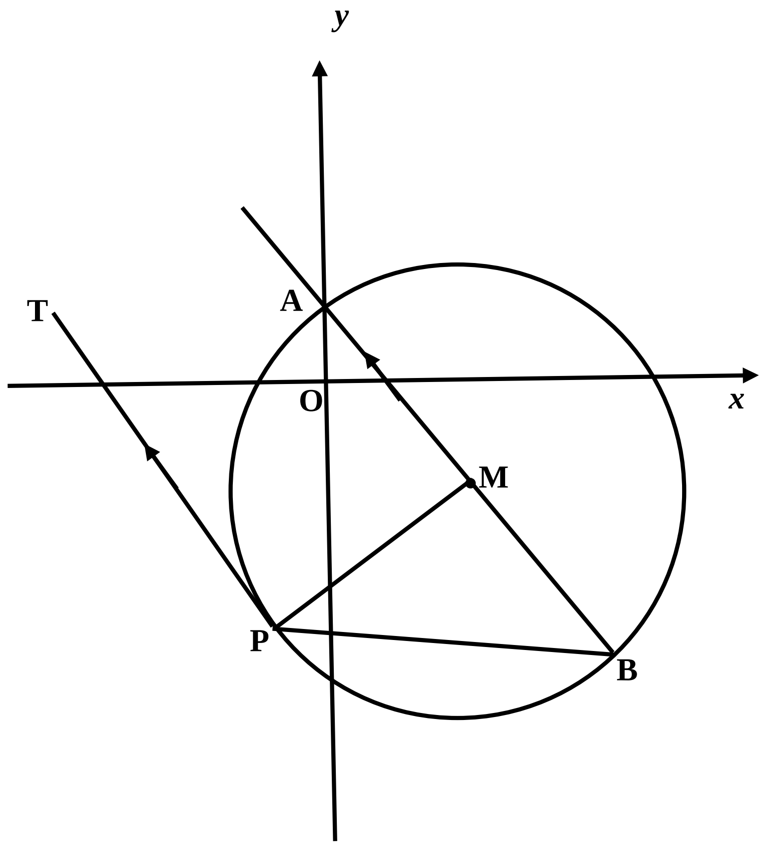
In the diagram below, ΔPMR is drawn with vertices P (–2; 4), M (4; 6) and R in the Cartesian plane. Line MR passes through the origin at O. The angle between PR and MR is θ and PR $\parallel$ MS. The equation of MS is given by $y - 5x + 14 = 0$.



5.1	Determine the equation of MR.	(3)
5.2	Determine the equation of PR.	(4)
5.3	Calculate the size of θ, rounded off to TWO decimal places.	(5)
5.4	Show that the coordinates of R is (–4; –6).	(4)
5.5	Calculate the length of MR, in simplified surd form.	(2)
5.6	Calculate the area of ΔPMR.	(5)
5.7	Write down the coordinates of S, if it is further given that PMSR is a parallelogram.	(2)
		[25]

QUESTION 6

In the diagram below, M is the centre of the circle $x^2 - 2x + y^2 + 4y - 5 = 0$.
Line AB passes through M, the centre of the circle. The equation of radius PM is $3y - x + 7 = 0$. PT is a tangent to the circle at P and PT $\parallel$ AB.



6.1 Determine the coordinates of M. (4)

6.2 Write down, with reasons, the size of $P\hat{M}B$. (3)

6.3 Determine the equation of line AB. (4)

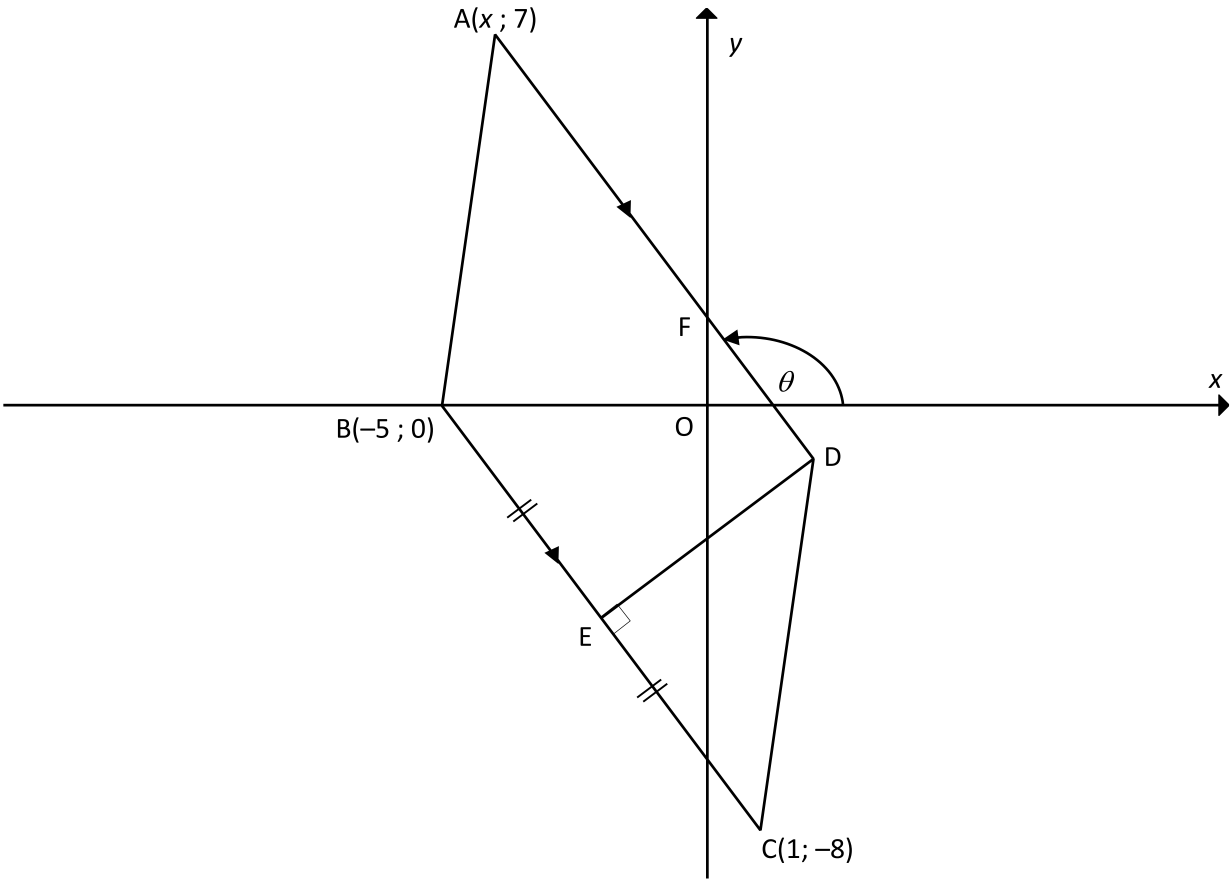
6.4 Determine the coordinates of A. (2)

6.5 If TM $= \sqrt{80}$, determine the length of the tangent PT. (3)

[16]

QUESTION 7

In the diagram, ABCD is a trapezium with AD | | BC and vertices A (x; 7), B (–5; 0), C (1; –8) and D. DE $\perp$ BC with E on BC such that BE = EC. The inclination of AD with the positive x-axis is θ and AD cuts the y-axis in F.



7.1 Calculate the gradient of BC. (2)

7.2 Calculate the coordinates of E. (2)

7.3 Determine the equation of DE in the form $y = mx + c$. (3)

7.4 Calculate the size of θ. (3)

7.5 Calculate the size of $\mathrm{O\hat{F}D}$. (2)

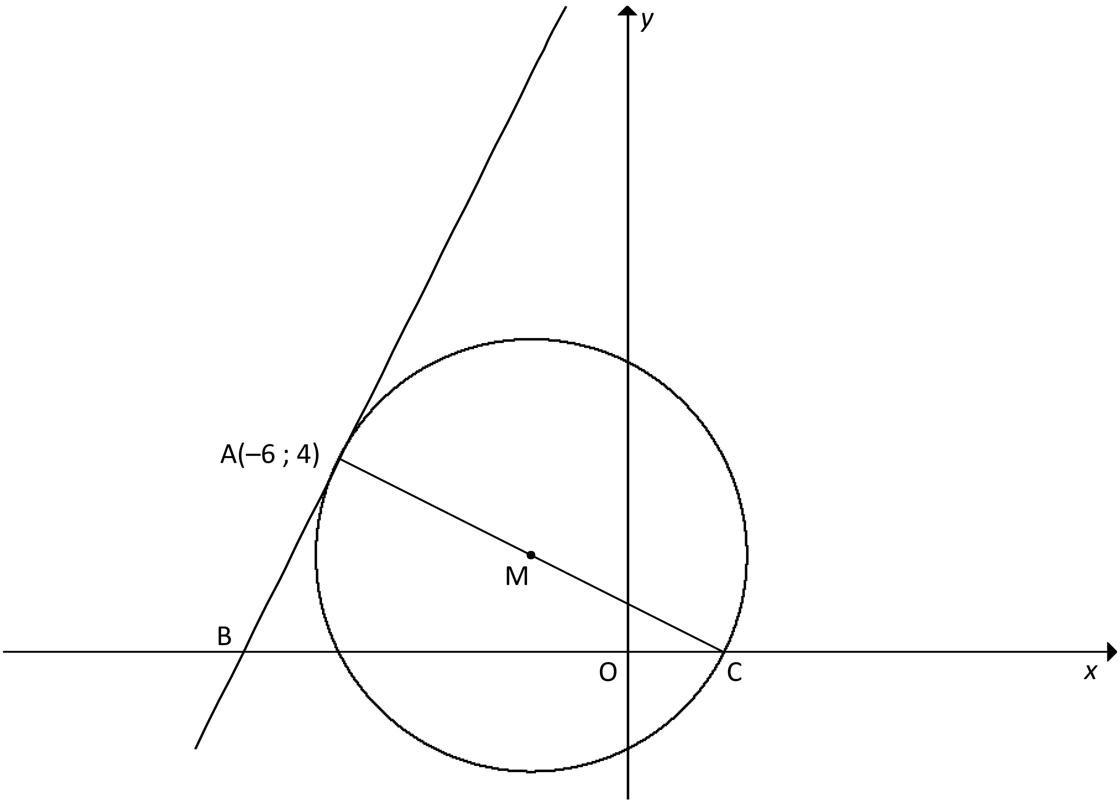
7.6 Calculate the value of x if the length of $\mathrm{AB} = 5\sqrt{2}$. (5)

7.7 Determine the equation of the circle with diameter BC in the form
$(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

QUESTION 8

In the diagram, the circle with centre M and equation $x^2 + y^2 + 4x - 4y - 12 = 0$ is drawn. C is the *x*-intercept of the circle. The tangent AB touches the circle at A(–6 ; 4) and cuts the *x*-axis at B.



8.1 Calculate the

8.1.1 coordinates of M. (2)

8.1.2 coordinates of C. (3)

8.2 Determine, giving reasons, the equation of the tangent AB in the form $y = mx + c$ if it is given that the gradient of MC is $-\frac{1}{2}$. (4)

8.3 Calculate the area of ΔABC. (5)

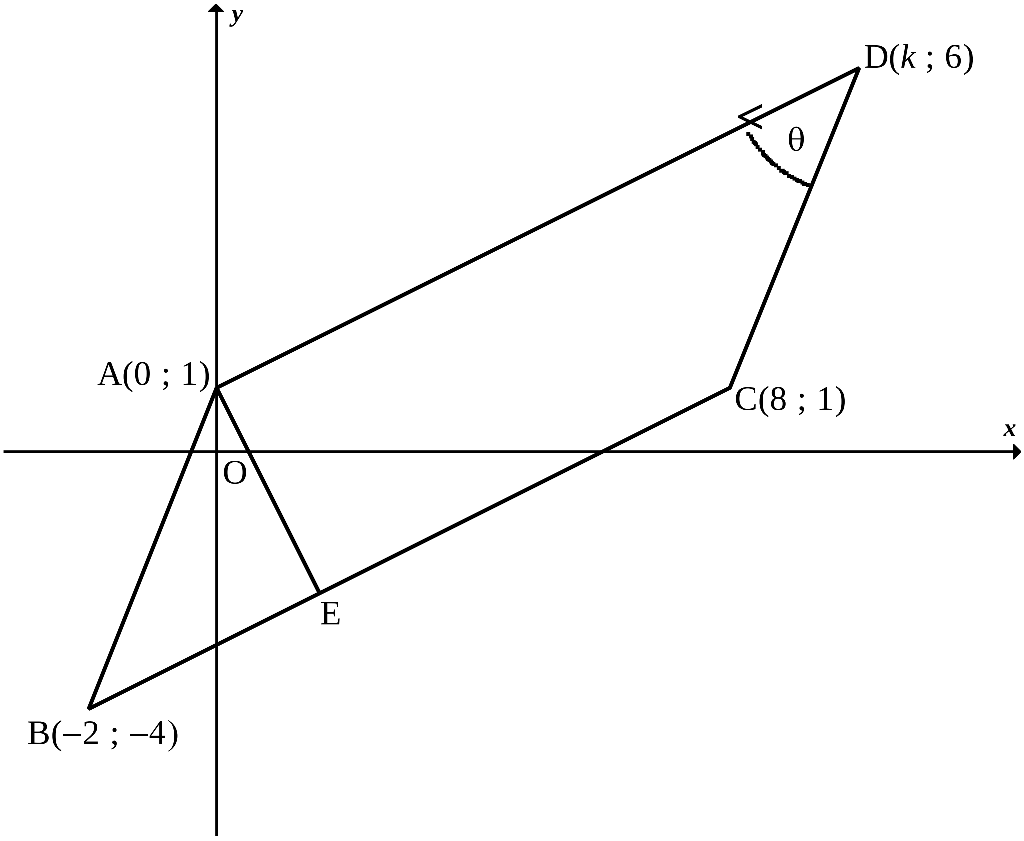
8.4 Determine for which values of k the line $y = 2x + k$ will intersect the circle at two points. (5)

[19]

QUESTION 9

In the figure below, ABCD is a parallelogram with vertices:

$A(0;1)$, $B(-2;-4)$, $C(8;1)$ and $D(k;6)$. AE is perpendicular to BC.



9.1 Calculate the length of BC. Leave your answer in simplest surd form. (3)

9.2 Determine the value of k. (3)

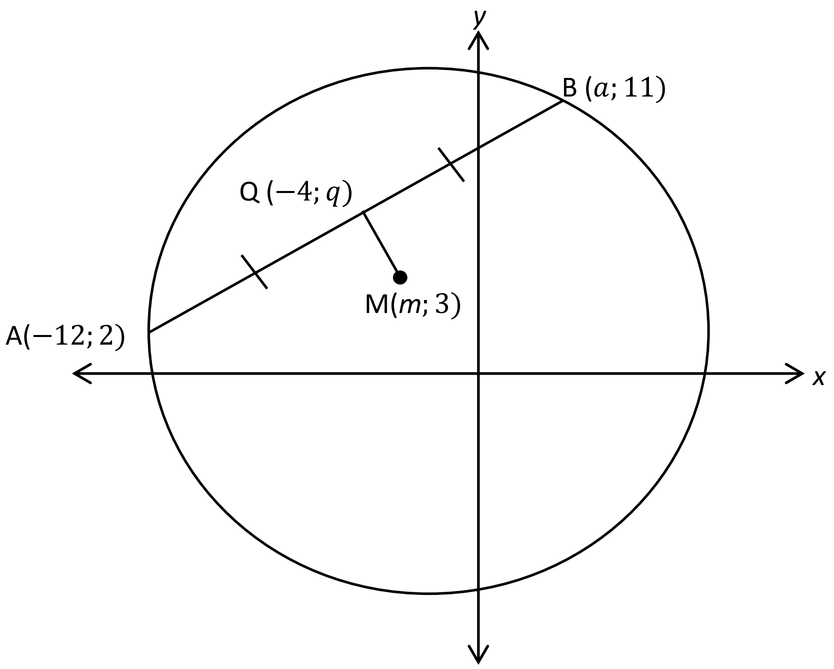
9.3 Determine the equation of AE. (4)

9.4 Calculate the size of θ rounded off to two decimal places. (6)

[16]

QUESTION 10

In the diagram below, M $(m; 3)$ is the centre of the circle. Q $(-4; q)$ is the midpoint of chord AB with A $(-12; 2)$ and B $(a; 11)$. The length of the radius of the circle is 10.



10.1 Calculate the values of a and q. (3)

10.2 Calculate the value of m. (3)

10.3 If $M(-2; 3)$, determine the equation of the circle in the form

$(x - a)^2 + (y - b)^2 = r^2$ (2)

10.4 Determine the equation of a tangent to the circle at point A. (3)

[11]

QUESTION 11

11.1 Calculate the value of k if the points $A(6;5)$, $B(3;2)$ and $C(2k;k+4)$ are collinear. (3)

11.2 The equation of circle is given: $x^2 + y^2 - 4x + 4y + 3 = 0$.

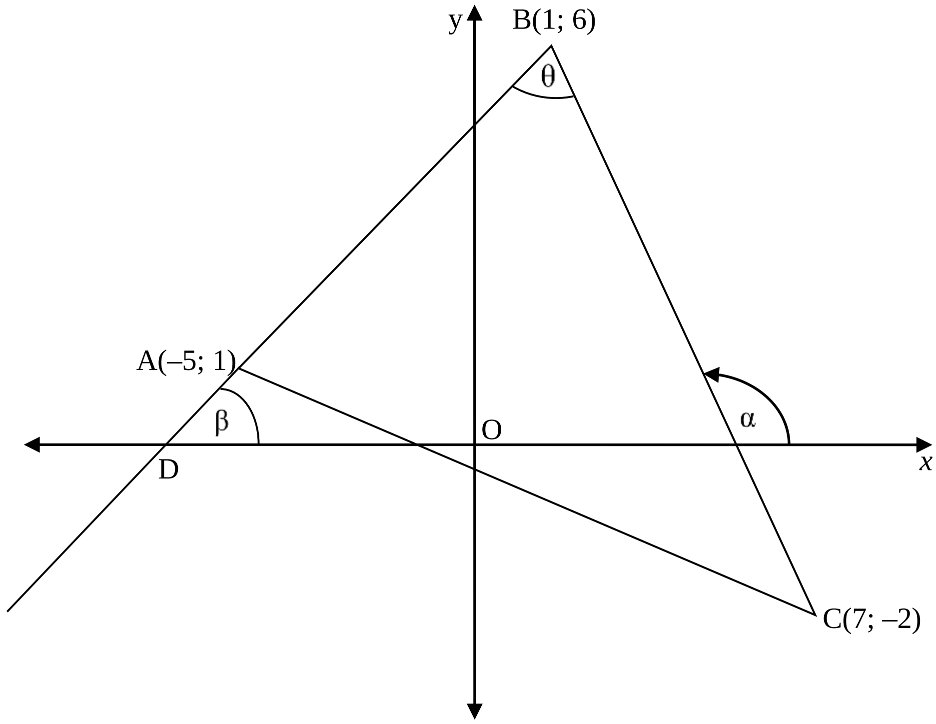
11.2.1 Determine the coordinates of the centre of the circle and the length of the radius. (4)

11.2.2 Determine whether the point $T(3;-3)$ lies inside, outside or on the circle. Show all your calculations. (2)

[9]

QUESTION 12

In the diagram below, A (–5; 1) , B(1; 6) and C(7; –2) are vertices of Δ ABC with AB produced to D. BD forms an angle, β, with the negative x – axis and BC forms an angle, α, with the positive x – axis. $A\hat{B}C = \theta$



Determine:

12.1 the length AC (2)

12.2 the equation of line BC (3)

12.3 $A\hat{B}C$ (5)

12.4 the midpoint P of AB (2)

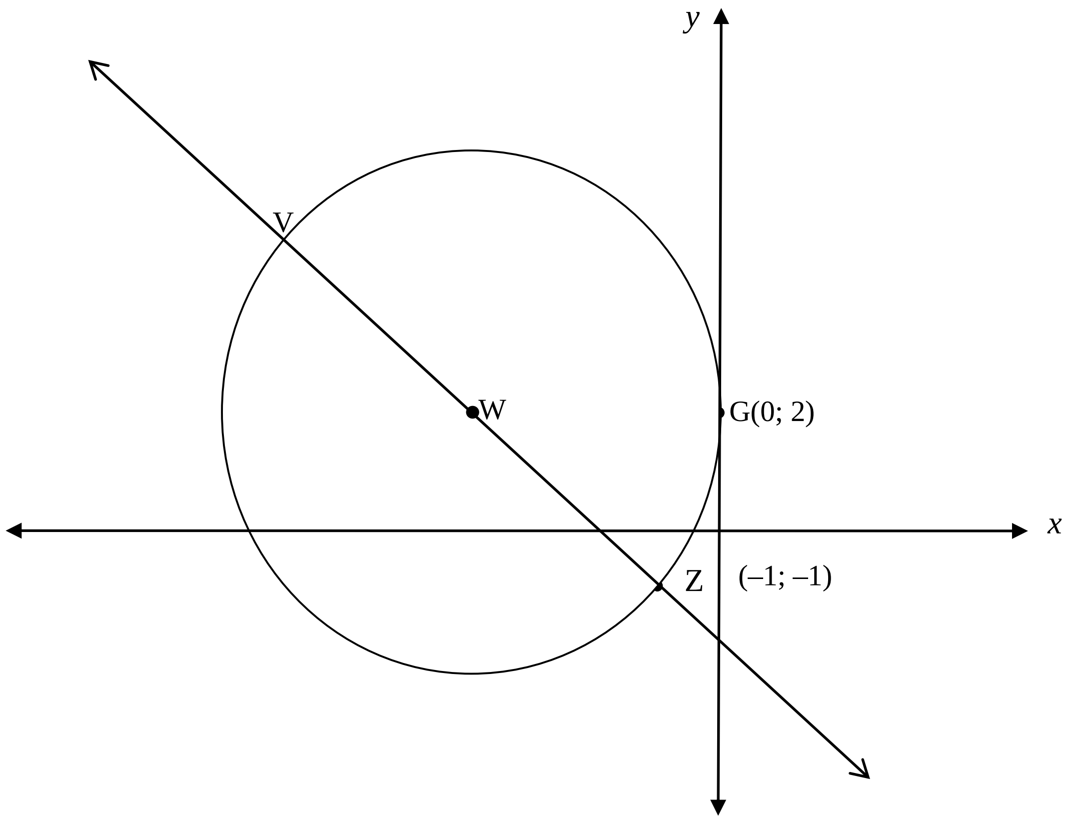
12.5 the equation of the line parallel to AC and passing through the point (–1; 3) (3)

12.6 Show that AB is perpendicular to $6x + 5y = 18$ (3)

[18]

QUESTION 13

In the diagram below, centre W of the circle lies on the straight line $3x + 4y + 7 = 0$
The straight line cuts the circle at V and Z(–1; –1). The circle touches the y-axis at G(0; 2)



13.1.1 Determine the equation of the circle in the form $(x – a)^2 + (y – b)^2 = r^2$. (5)

13.1.2 Determine the length of diameter VZ (1)

13.1.3 Calculate the gradient of GZ. (2)

13.1.4 Write down the coordinates of the midpoint of GZ. (1)

13.1.5 Determine the equation of the line that is the perpendicular bisector of GZ. (3)

13.1.6 Show that the line in QUESTION 4.1.5 and straight line VZ intersect at W. (2)

13.2 The circle defined by $(x + 2)^2 + (y – 1)^2 = 25$ has centre M, and the circle defined by $(x –1)^2 + (y – 3)^2 = 9$ has centre N.

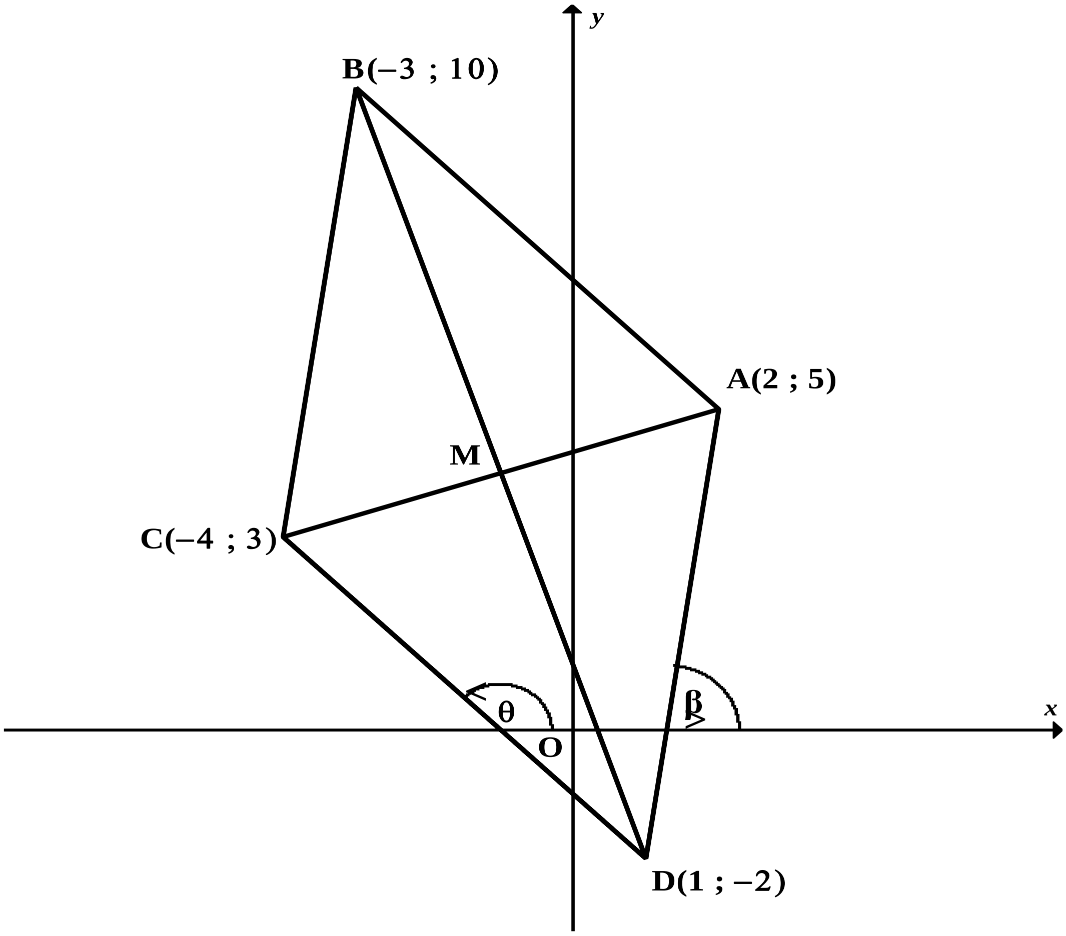
13.2.1 Show that the circles intersect each other at two distinct points. (6)

13.2.2 Determine the equation of the common chord. (3)

[23]

QUESTION 14

ABCD is a quadrilateral with vertices A(2 ; 5), B(−3 ; 10); C(−4 ; 3) and D(1 ;−2).

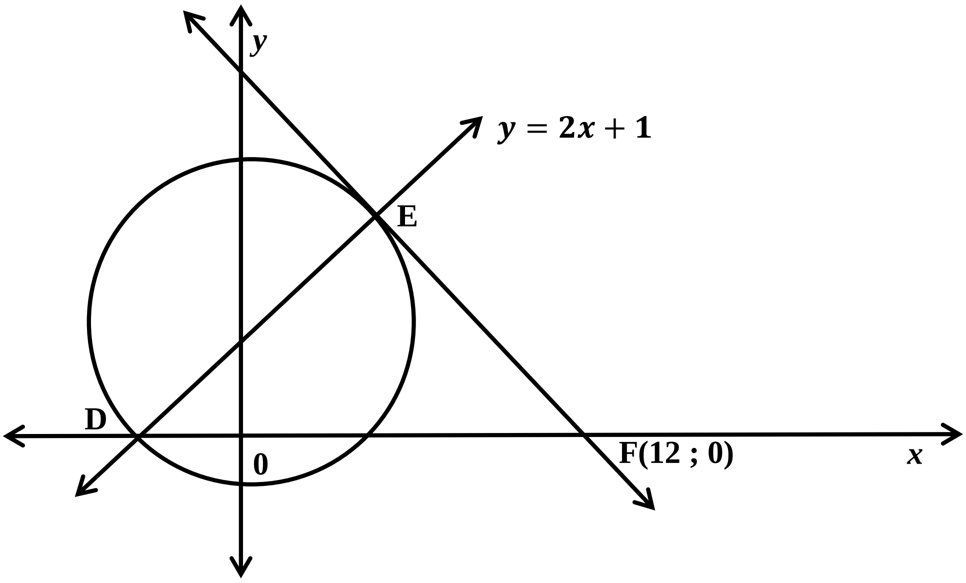


14.1	Calculate the length of AC. (Leave the answer in simplest surd form.)	(2)
14.2	Determine the coordinates of M, the midpoint of AC.	(2)
14.3	Show that BD and AC bisect each other perpendicularly.	(5)
14.4	Calculate the area of ΔABC.	(4)
14.5	Determine the equation of DC.	(3)
14.6	Determine θ, the angle of inclination of DC.	(3)
14.7	Calculate the size of $A\hat{D}C$.	(3)

[22]

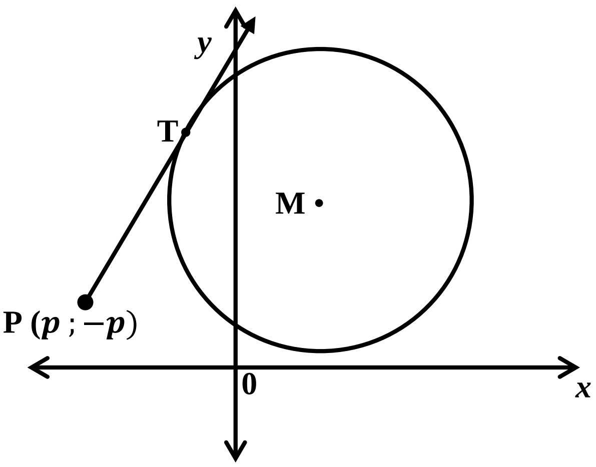
QUESTION 15

15.1 A circle has a diameter with equation $y = 2x + 1$. The tangent to the circle at point E intersects the x-axis at F(12 ; 0).



Determine the coordinates of E. (6)

15.2 M is the centre of the circle defined by $x^2 + y^2 - 2x - 4y + 1 = 0$.
P$(p\ ; -p)$ is any point on the tangent to the circle at T.



15.2.1 Show, by calculation, that the coordinates of M are (1; 2). (3)

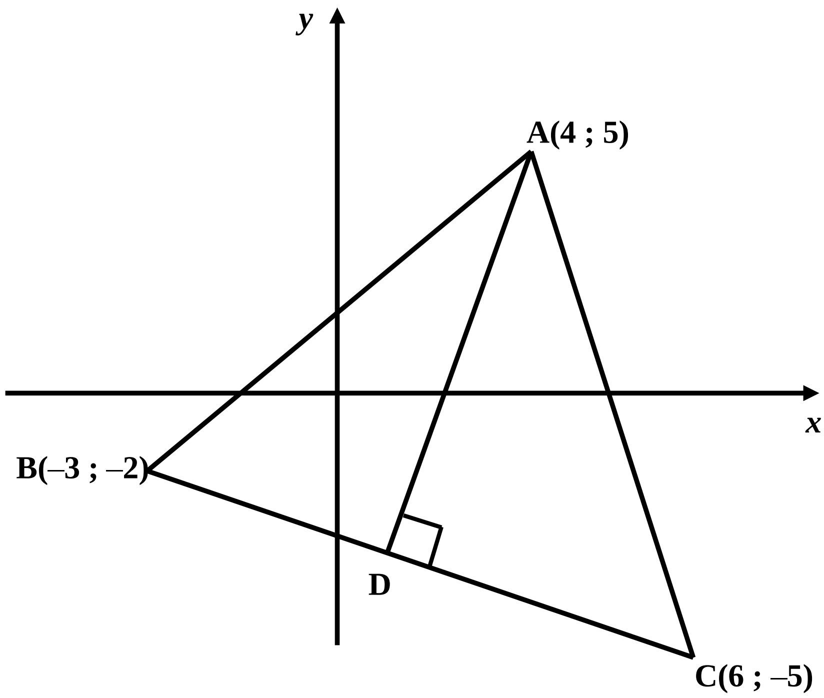
15.2.2 Prove that the length of PT = $\sqrt{2p^2 + 2p + 1}$ (3)

15.2.3 Calculate the coordinates of P where P is as close as possible to T and hence calculate the minimum length of PT. (5)

[17]

QUESTION 16

In the diagram below, A (4; 5), B (–3; –2) and C (6; –5) are the vertices of ΔABC. AD is drawn perpendicular to BC.



16.1 Calculate the length of BC. (2)

16.2 Determine the equation of BC. (3)

16.3 Determine the equation of AD. (3)

16.4 Determine the coordinates of D. (3)

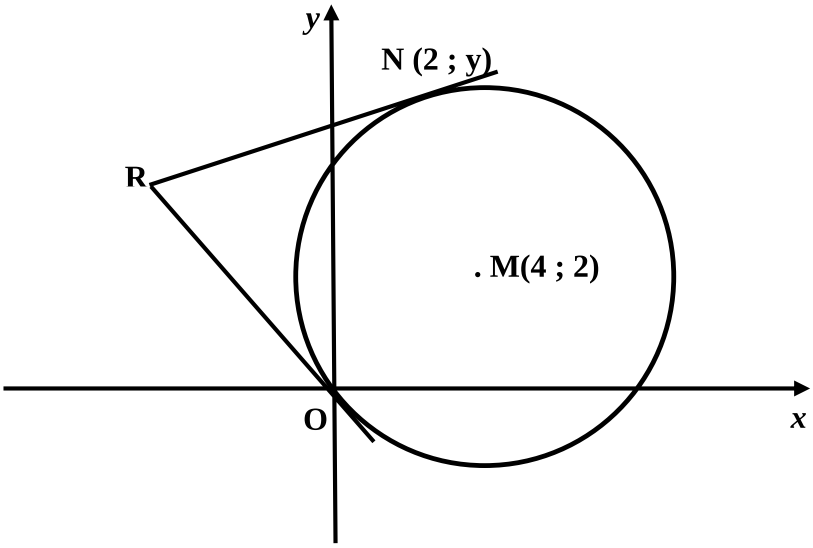
16.5 Calculate the size of $B\hat{A}D$. (5)

16.6 Calculate the coordinates of a point E if the area of ΔEBC = area of ΔABC and E is a point on the positive *x* axis. (4)

[20]

QUESTION 17

In the diagram below, O (0; 0) and N (2; y) are two points on the circumference of a circle with centre M (4; 2). The tangents at O and N meet at R.

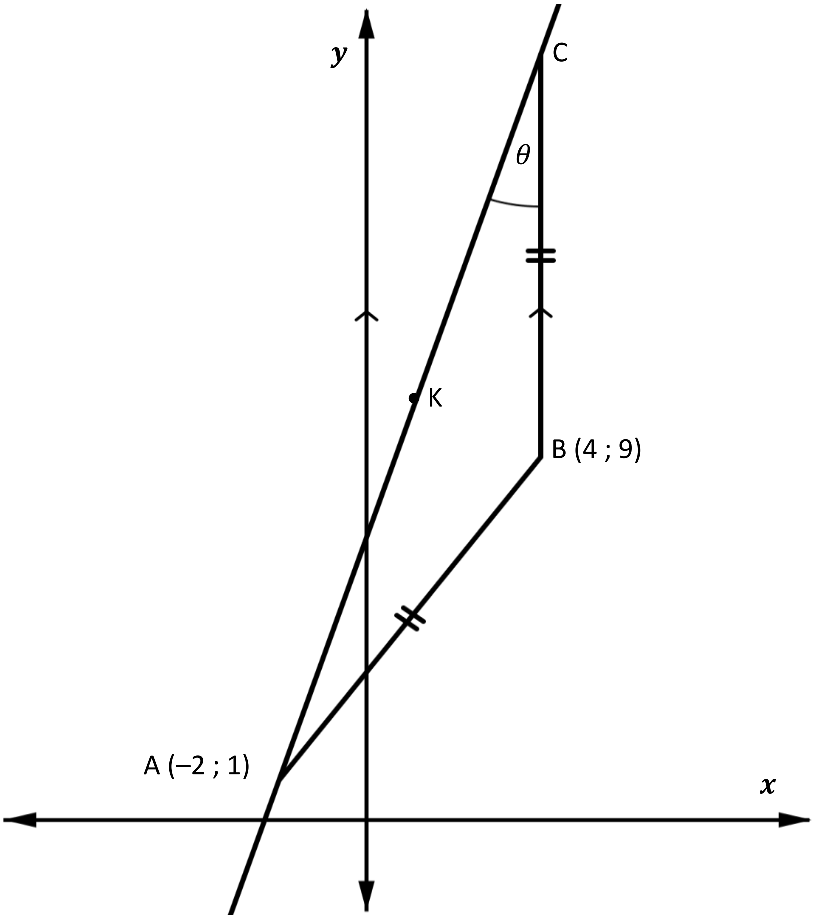


17.1	Determine the equation of the circle.	(3)
17.2	Calculate the value of y	(4)
17.3	Determine the equation of OR.	(3)
17.4	Calculate the coordinates of R.	(6)
17.5	Determine, with a reason, the type of quadrilateral represented by MNRO.	(2)

[18]

QUESTION 18

In the diagram below, ABC is an isosceles triangle with A(–2;1) and B(4 ; 9). AB = BC and BC is parallel to the y-axis.

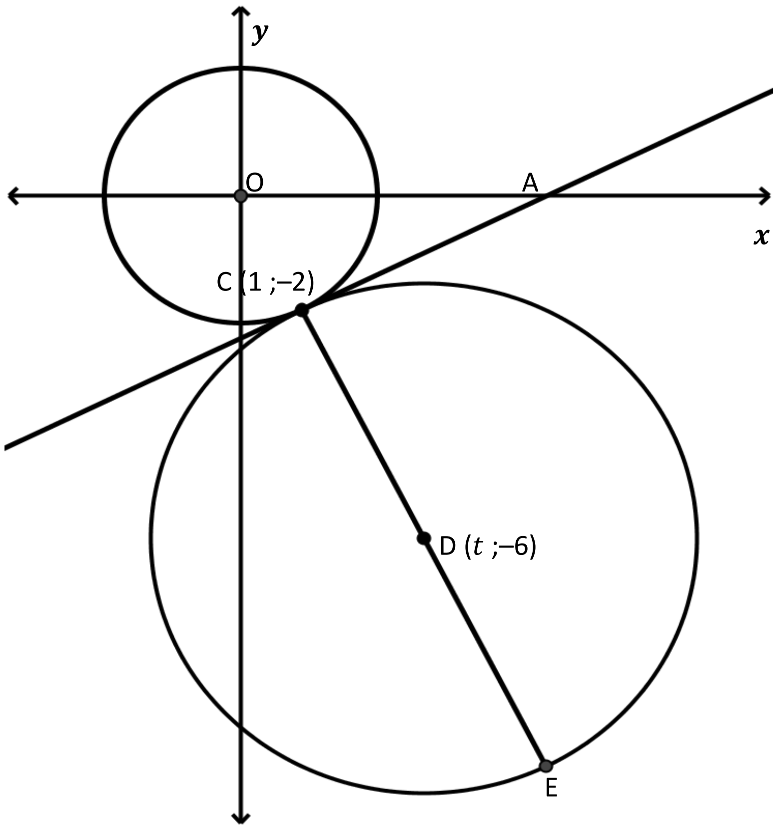


18.1	Calculate the length of AB.	(2)
18.2	Calculate the coordinates of C.	(2)
18.3	Calculate the coordinates of K, the midpoint of AC.	(2)
18.4	Determine the equation of AC in the form $y = mx + c$.	(3)
18.5	Calculate the size of θ.	(4)

18.6 Calculate the area of ΔABC.

QUESTION 19

The diagram below consists of two circles, which touch each other externally at C (1; –2). The smaller circle has its centre O at the origin. The other circle has centre D(t ; –6). CA is a common tangent which intersects the x-axis at A. CDE is the diameter of the larger circle.



19.1	Give a reason why the points O, C and D lie on a straight line.	(2)
19.2	Calculate the gradient of OC.	(2)
19.3	Hence, show that the value of $t = 3$.	(2)
19.4	Determine the equation of the tangent AC in the form $y = mx + c$.	(3)
19.5	Calculate the coordinates of E.	(2)

19.6 Determine the equation of a circle passing through the points A(5 ; 0), C and E in the form $(x-a)^2+(y-b)^2=r^2$. (6)

19.7 If a circle with centre D and equation $(x-t)^2+(y+6)^2=r^2$ has to cut the circle with centre O twice, give all possible values of r. (4)

[21]