GR 11 MATHS P2

Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

COMMON TEST

JUNE 2017

NATIONAL SENIOR CERTIFICATE

GRADE 11

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages and 4 diagram sheets.

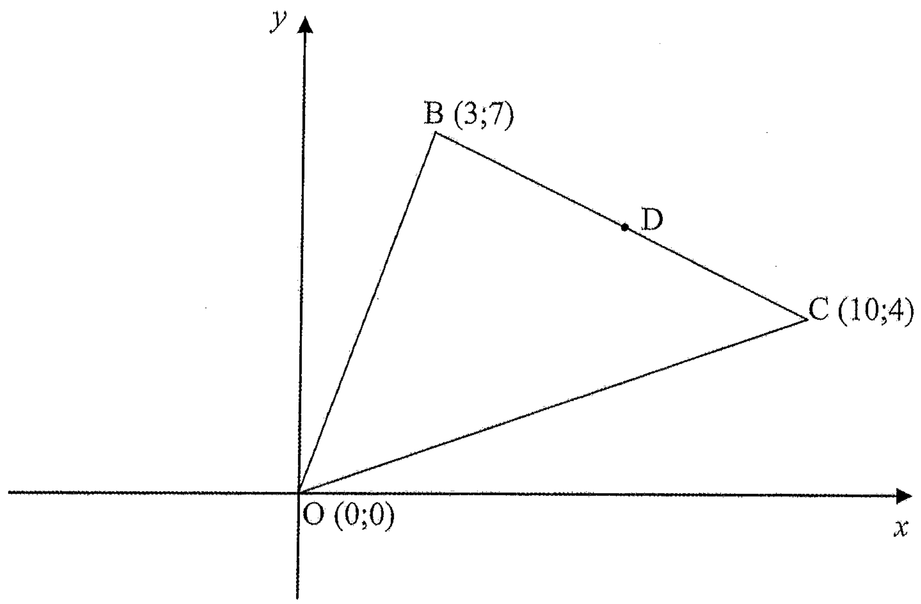
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. FOUR diagram sheets are attached at the end of this question paper. Write your name on these diagram sheets in the spaces provided and hand your diagram sheets in together with your ANSWER BOOK.
10. Write neatly and legibly.

QUESTION 1

In the diagram B (3;7), C(10;4) and O (0;0) are the vertices of $\triangle BCO$.
D is the midpoint of BC.



1.1 Calculate the lengths of BO and BC. Leave your answers in surd form. (4)

1.2 Determine the gradients of BO and BC. (4)

1.3 Prove that $O\hat{B}C = 90^\circ$. (2)

1.4 Calculate the area of ΔBCO. (3)

1.5 Calculate the coordinates of D. (2)

1.6 A straight line passes through the point (5 ; 2) and is parallel to BO.

1.6.1 Determine the equation of this line in the form $ax + by + c = 0$. (5)

1.6.2 Hence, show that D lies on this line. (2)

[22]

QUESTION 2

2.1 Given the points P $(-1; 4)$, S $(3; a)$ and W $\left(t; \frac{17}{2}\right)$. $a > 0$

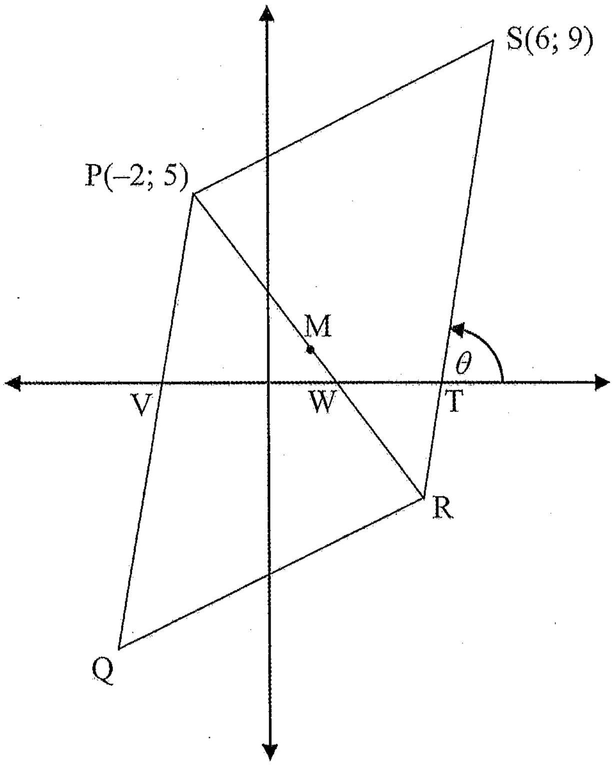
The length of PS is $2\sqrt{13}$.

P, S and W are collinear points.

2.1.1 Calculate the value of a. (5)

2.1.2 If $a = 10$, calculate the value of t. (4)

2.2 In the diagram, PQRS is a parallelogram with vertices P$(-2; 5)$, Q, R and S$(6; 9)$.
M is the midpoint of diagonal PR. θ is the angle of inclination of SR.
PQ, PR and SR cut the x-axis at V, W and T respectively.



2.2.1 Show, by calculation, that the coordinates of R are $(4 ; -3)$. (2)

2.2.2 Calculate the size of angle θ. (3)

2.2.3 Calculate the size of $Q\hat{P}R$. (5)

2.2.4 Determine the coordinates of Q. (2)

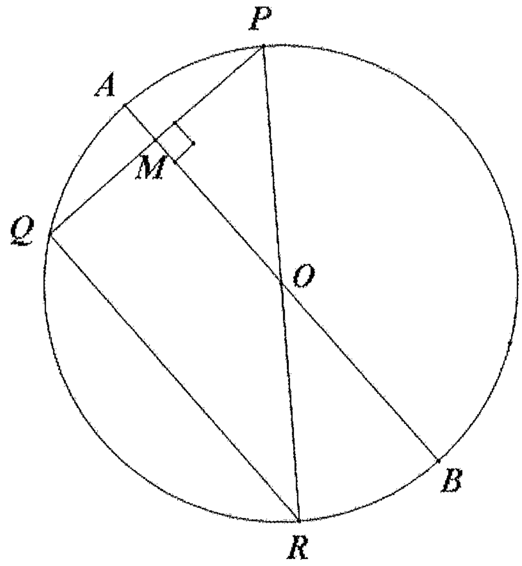
[21]

GIVE REASONS FOR YOUR STATEMENTS AND CALCULATIONS IN QUESTIONS 3 – 6.

QUESTION 3

3.1 Complete the statement so that it is valid:
The line drawn from the centre of a circle perpendicular to a chord .... (1)

3.2 In the diagram PQ and QR are chords of the circle with centre O.
Diameter AB intersects chord PQ perpendicularly at M.
AM = 2cm and MB = 32cm.



Calculate the length of the following, with reasons:

3.2.1 OP (2)

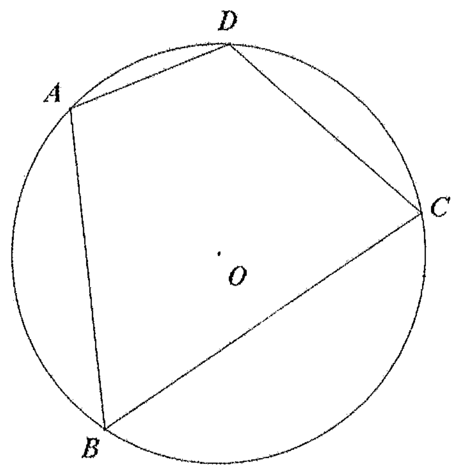
3.2.2 PQ (5)

3.2.3 QR (3)

[11]

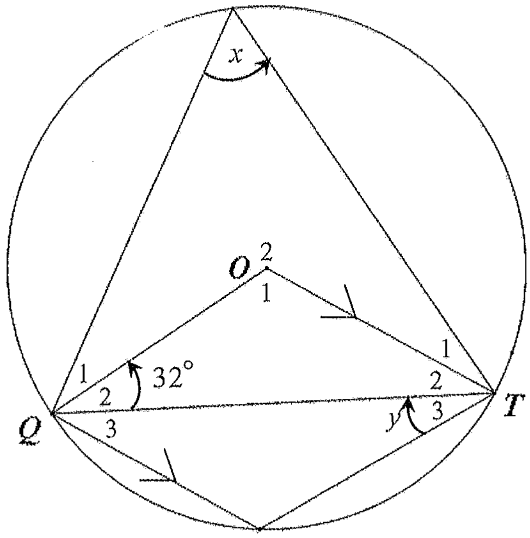
QUESTION 4

4.1 In the diagram O is the centre of the circle ABCD.



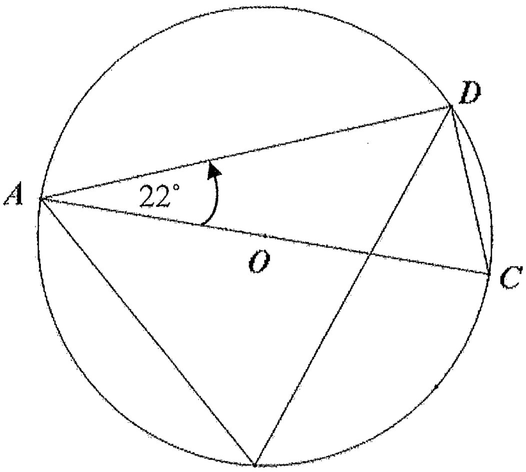
Prove the theorem which states that $\hat{B}+\hat{D}=180^\circ$. (6)

4.2 In the diagram O is the centre of a circle which passes through P, Q, R and T. QT, OQ and OT are joined. OT is parallel to QR. $\hat{Q}_2 = 32°$, $\hat{P} = x$ and $\hat{T}_3 = y$.



Determine, with reasons, the size of x and y. (8)

4.3 In the diagram A, B,C and D are points on the circumference of the circle with centre O. AOC is a diameter. $D\hat{A}C = 22°$.

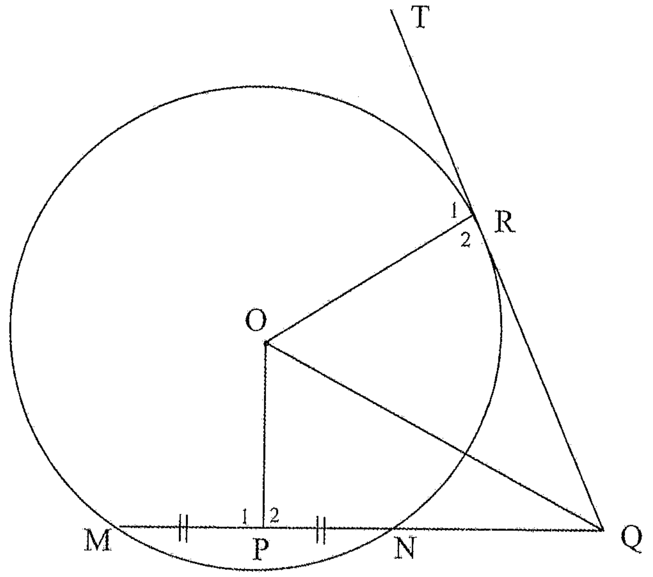


Calculate, with reasons, the size of $\hat{B}$. (5)

[19]

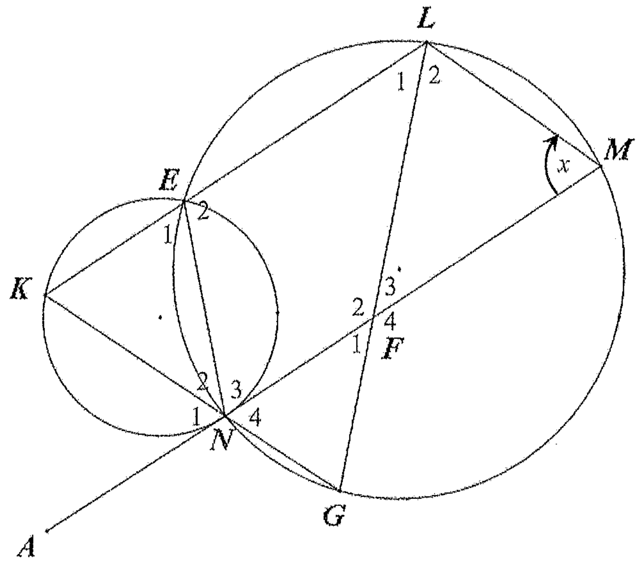
QUESTION 5

5.1 In the diagram, O is the centre of the circle. QRT is a tangent to the circle at R. MN is a chord of the circle and MNQ is a straight line. P is the midpoint of MN.



Prove that OPQR is a cyclic quadrilateral. (5)

5.2 In the diagram, two circles cut in E and N. K, L and M are points on the circles such that KLMN is a parallelogram and the chords MN and LG intersect at F. MN is produced to A. KNG and KEL are straight lines. Let $\hat{M} = x$.



5.2.1 Prove that KN = EN. (4)

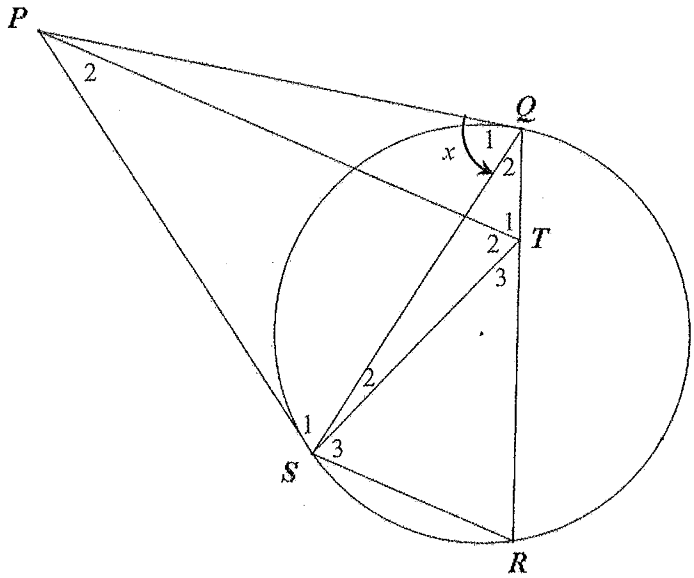
5.2.2 Prove that MNA is a tangent to circle KEN. (3)

5.2.3 Prove that KL = LG. (4)

[16]

QUESTION 6

In the diagram PQ and PS are tangents to the given circle, and R is a point on the circumference. T is a point on QR such that $\hat{T}_1 = \hat{Q}_1$. SQ, TS and SR are joined.
Let $\hat{Q}_1 = x$.



Prove that

6.1 PT ∥ SR (4)

6.2 TQPS is a cyclic quadrilateral. (4)

6.3 PT bisects $S\hat{T}Q$. (3)

[11]

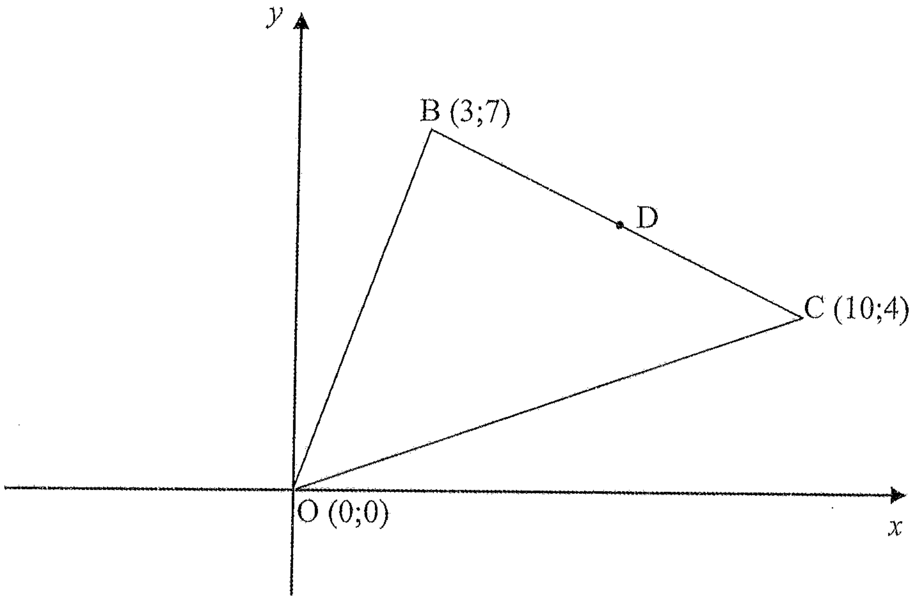
TOTAL 100

TEAR-OFF SHEET

NAME: ____________________

DIAGRAM SHEET 1

QUESTION 1



QUESTION 2.2

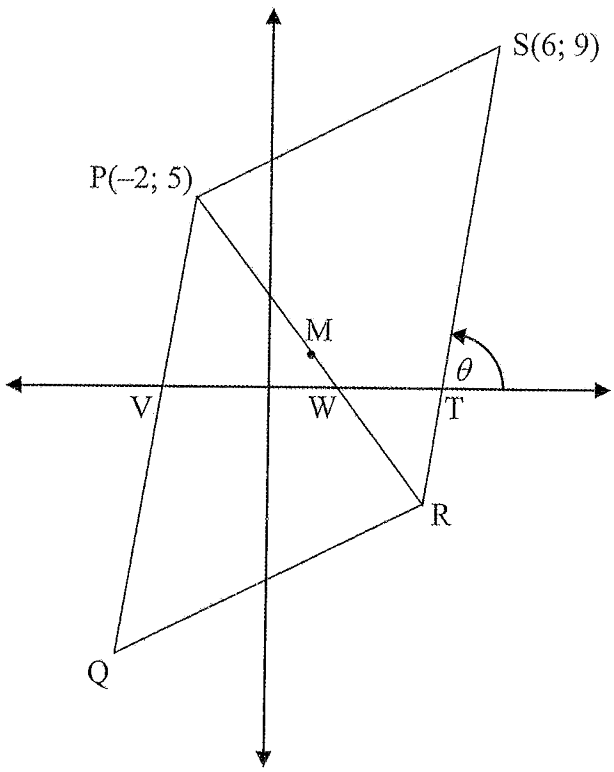


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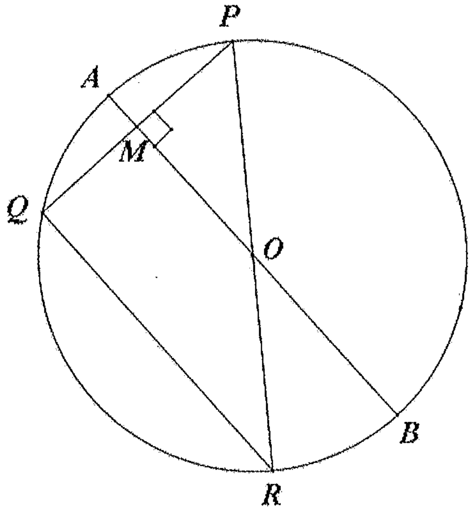
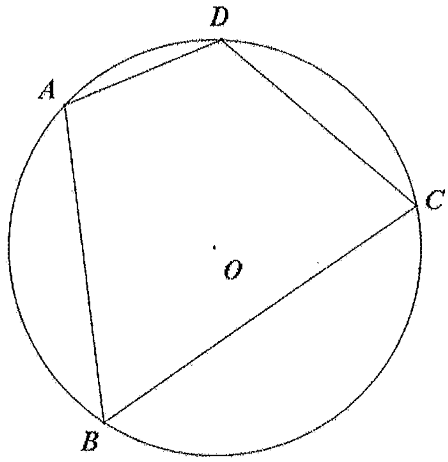
DIAGRAM SHEET 2

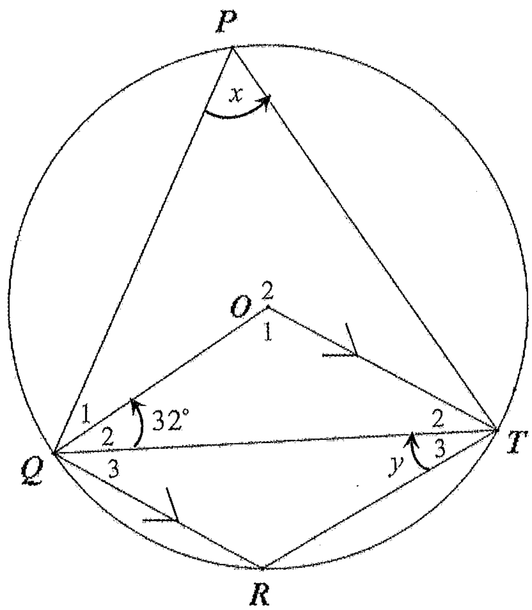
QUESTION 3.2



QUESTION 4.1



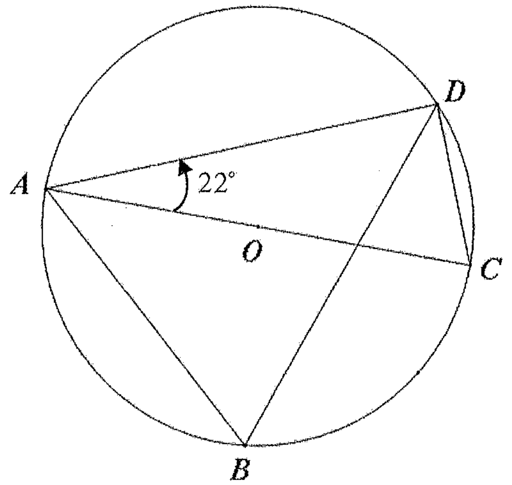
QUESTION 4.2



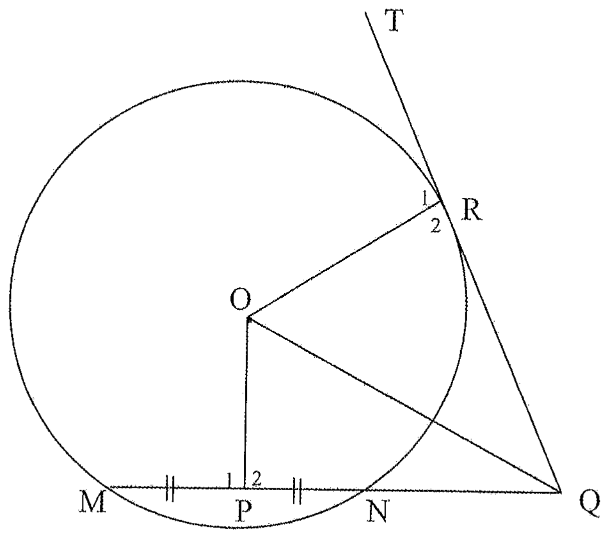
NAME: ______________________________

DIAGRAM SHEET 3

QUESTION 4.3



QUESTION 5.1

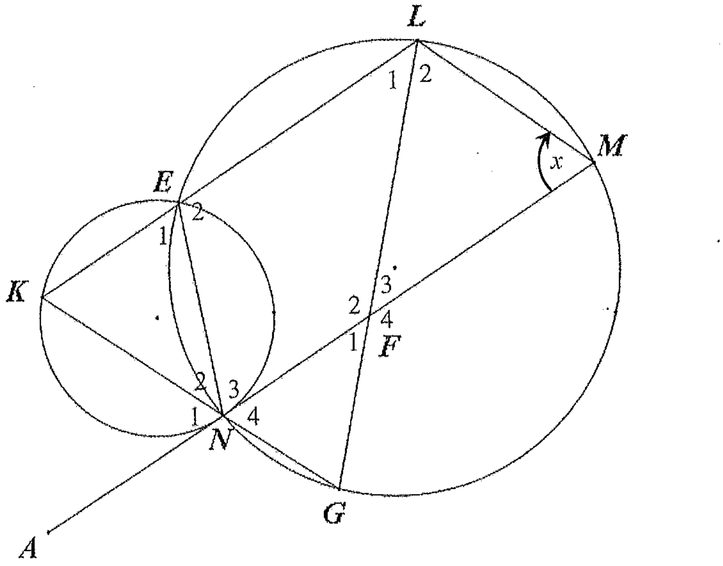


TEAR-OFF SHEET

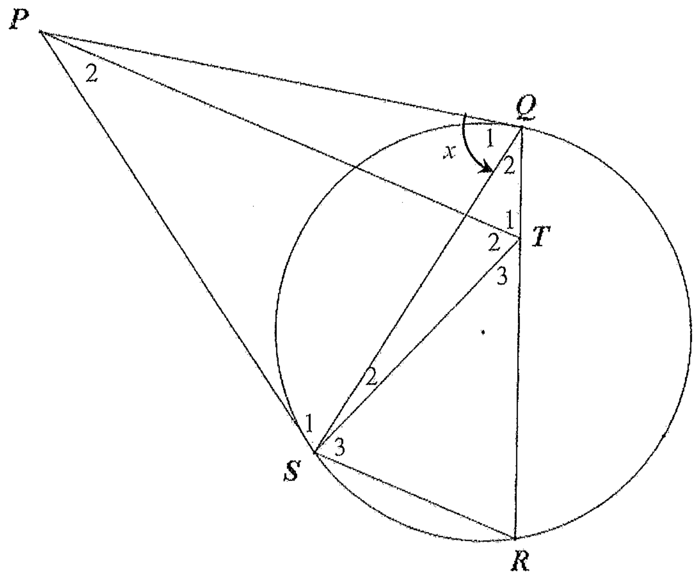
NAME: ______________________________

DIAGRAM SHEET 4

QUESTION 5.2



QUESTION 6



GR 11 MATHS + M.LIT P.2

Education
KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

MARKING GUIDELINE

COMMON TEST

JUNE 2017

NATIONAL
SENIOR CERTIFICATE

GRADE 11

MARKS: 100

N.B. This marking guideline consists of 8 pages.

QUESTION 1

1.1	$BO = \sqrt{(3-0)^2+(7-0)^2}$ $= \sqrt{58}$ $BC = \sqrt{(3-10)^2+(7-4)^2}$ $= \sqrt{58}$	1A for substitution 1CA for answer 1A for substitution 1CA for answer (4)
1.2	Gradient of BO $= \frac{7-0}{3-0}$ $= \frac{7}{3}$ Gradient of BC $= \frac{7-4}{3-10}$ $= \frac{-3}{7}$	1A for substitution 1CA for answer 1A for substitution 1CA for answer (4)
1.3	$M_{BO} \times M_{BC} = \frac{7}{3} \times \frac{-3}{7}$ $= -1$ $\therefore O\hat{B}C = 90°$	1 A for product 1A for -1 (2)
1.4	Area of $\Delta BCO = \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times \sqrt{58} \times \sqrt{58}$ $= 29$ square units	1A for formula 1CA for substitution 1CA for answer (3)
1.5	$D = \left(\frac{3+10}{2}; \frac{7+4}{2}\right)$ $= \left(\frac{13}{2}; \frac{11}{2}\right)$	1A for $\frac{13}{2}$ 1A for $\frac{11}{2}$ (2)
1.6.1	Gradient of the line $= \frac{7}{3}$ $y = mx + c$ $2 = \frac{7}{3}(5) + c$ $c = \frac{-29}{3}$ $y = \frac{7}{3}x - \frac{29}{3}$ $7x - 3y - 29 = 0$ is the equation	1 CA for gradient 1CA for substitution of point 1CA for c - value 1CA for equation 1CA for $ax + by + c = 0$ form (5)
1.6.2	$7\left(\frac{13}{2}\right) - 3\left(\frac{11}{2}\right) - 29$ $= \frac{91}{2} - \frac{33}{2} - 29$ $= \frac{91-33-58}{2}$ $= 0$ $\therefore$ D lies on the line	1CA for substitution 1CA for simplification (2) [22]

QUESTION 2

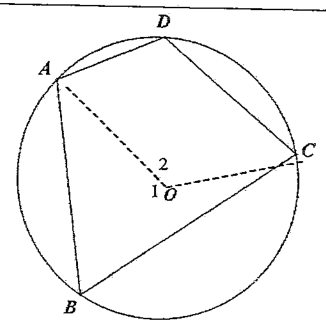
2.1.1	$PS=\sqrt{(3+1)^2+(a-4)^2}$ $\sqrt{16+a^2-8a+16}=2\sqrt{13}$ $32+a^2-8a=52$ $a^2-8a-20=0$ $(a-10)(a+2)=0$ $a=10$ *or* $a=-2$ $\therefore a=10$	1A for substitution in distance formula 1A for equating to $2\sqrt{13}$ 1CA for squaring both sides 1CA standard form 1CA factors	(5)
2.1.2	$m_{PS}=m_{SW}$ $\frac{10-4}{4}=\frac{\frac{17}{2}-10}{t-3}$ $\frac{6}{4}=\frac{\frac{-3}{2}}{t-3}$ $6t-18=6$ $t=2$ OR $m_{PS}=m_{PW}$ $\frac{10-4}{4}=\frac{\frac{17}{2}-4}{t+1}$ $\frac{6}{4}=\frac{\frac{9}{2}}{t+1}$ $6t+6=18$ $t=2$	1A for equating gradients 1A for substitution 1CA for simplification 1CA for answer OR 1A for equating gradients 1A for substitution 1CA for simplification 1CA for answer	(4)
2.2.1	$1=\frac{x+(-2)}{2}$ $x=4$ $1=\frac{y+5}{2}$ $y=-3$	1A for substitution 1A for substitution	(2)
2.2.2	$\tan\theta=m_{SR}$ $\tan\theta=\frac{9-(-3)}{6-4}$ $=6$ $\theta=80{,}54°$	1A for $\tan\theta=m_{SR}$ 1A for 6 1CA for answer	(3)

2.2.3	$P\hat{V}W$ = Angle of inclination of PQ = 80,54° [opp. sides of parm. PQRS are parallel.] $\tan P\hat{W}T=m_{PR}$ $=\frac{5-(-3)}{-2-4}$ $=-\frac{4}{3}$ $P\hat{W}T=180°-53{,}13°$ $=126{,}87°$ $Q\hat{P}R=P\hat{W}T-P\hat{V}W$ [ext.∠ of ΔPVW] $=126{,}87°-80{,}54°$ $=46{,}33°$	1A for $P\hat{V}W=80{,}54°$ 1A for $m_{PR}=-\frac{4}{3}$ 1CA for size of $P\hat{W}T$ 1CA for subtracting 1CA for answer	(5)
2.2.4	Q(−4;−7)	1A for −4 1A for −7	(2) [21]

QUESTION 3

3.1	bisects the chord	1A for answer (1)
3.2.1	AB = 34cm OP = 17cm	1A for length of diameter 1A for length of OP (2)
3.2.2	$PM^2 = OP^2 - OM^2$ [Theorem of Pythagoras] $= 17^2 - 15^2$ $= 64$ PM = 8cm $PQ = 2 \times PM$ [line from centre ⊥ to chord] $= 16cm$	1S for $PM^2 = OP^2 - OM^2$ or $PM^2 = 17^2 - 15^2$ 1R (for Theorem of Pythagoras) 1CA for length of PM 1CA for length of PQ 1R (for line from centre ⊥ to chord) (5)
3.2.3	$\hat{Q} = 90°$ [∠ in semicircle] $QR^2 = PR^2 - PQ^2$ [Theorem of Pythagoras] $= 34^2 - 16^2$ $= 900$ QR = 30cm OR $QR = 2 \times OM$ [midpoint theorem] $= 30cm$	1S/R 1CA for applying Theorem of Pythagoras 1CA for answer OR 1S 1R 1CA for answer (3) [11]

QUESTION 4

4.1	 Construct AO and OC $\hat{O}_1 = 2\hat{D}$ [∠ at centre = 2 x ∠ at circumference] $\hat{O}_2 = 2\hat{B}$ [∠ at centre = 2 x ∠ at circumference] but $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s around a pt] $2\hat{D} + 2\hat{B} = 360°$ $\hat{B} + \hat{D} = 180°$	1A construction 1S 1R 1S/R 1S/R 1A for substitution (6)
4.2	$\hat{T}_2 = 32°$ [∠s opp equal sides] $\hat{O}_1 = 118° - 2(32°)$ [sum of ∠'s of a triangle] $= 116°$ $\hat{P} = \frac{1}{2}(116°)$ [∠ at centre = 2×∠ at circumference] $= 58°$ $= x$ $\hat{R} = 180° - 58°$ [opp ∠s of cyclic quad] $= 122°$ $\hat{Q}_3 = \hat{T}_2$ [alt ∠'s; OT ∥ QR] $= 32°$ $y = 180° - (122° + 32°)$ [sum of ∠'s of a triangle] $= 26°$	1S/R 1S 1 S 1R 1S 1R 1S/R 1A answer (8)
4.3	$A\hat{D}C = 90°$ [∠ in a semicircle] $\hat{C} = 180° - (90° + 22°)$ [sum of ∠s of Δ] $= 68°$ $\hat{B} = \hat{C}$ [∠s in same segment] $= 68°$	1S 1R 1S/R 1S 1R (5) [19]

QUESTION 5

5.1	$\hat{P}_2 = 90°$ [line from centre to midpoint of chord] $\hat{R}_2 = 90°$ [radius $\perp$ tangent] $\therefore$ OPQR is a cyclic quadrilateral [converse: opp $\angle$s of cyclic quad are supplementary] OR $\hat{P}_1 = 90°$ [line from centre to midpoint of chord] $\hat{R}_2 = 90°$ [radius $\perp$ tangent] $\therefore$ QPQR is a cyclic quadliteral [converse: ext. $\angle$ of a cyclic quad = opp. interior angle]	1S 1R 1S 1R 1R OR 1S 1R 1S 1R 1R	(5)
5.2.1	$\hat{K} = \hat{M}$ [opp. $\angle$'s of parm.] $= x$ $\hat{E}_1 = \hat{M}$ [ext. $\angle$ of cyclic quad] $\hat{K} = \hat{E}_1$ [both $= \hat{M}$] KN = NE [sides opp. = angles]	1S/R 1S 1R 1S/R	(4)
5.2.2	$\hat{N}_3 = \hat{E}_1$ [alt. $\angle$'s; KL $\parallel$ MN] MFN is a tangent to circle KEN [converse: tan - chord - theorem]	1S 1R 1R	(3)
5.2.3	$\hat{G} = \hat{M}$ [$\angle$'s in the same segment] $= x$ $= \hat{K}$ [proved] KL = LG [sides opp. to = angles]	1 S 1R 1 S 1 S/R	(4) [16]

QUESTION 6

6.1	$\hat{Q}_1 = \hat{R}$ [tan - chord - theorem] $= x$ $= \hat{T}_1$ [given] PT $\parallel$ SR [correcsponding $\angle$'s are equal]	1S 1R 1S 1 S/R	(4)
6.2	PQ = PS [2 tangents from same point] $\hat{S}_1 = \hat{Q}_1$ [$\angle$'s opp to = sides] $= x$ $\hat{S}_1 = \hat{T}_1$ [both $= x$] TQPS is a cyclic quadlateral [converse: $\angle$'s in same segment]	1S/R 1 S/R 1S 1S/R	(4)
6.3	$\hat{T}_2 = \hat{Q}_1$ [$\angle$'s in same segment] $= x$ $\hat{T}_1 = \hat{Q}_1$ [given] $\hat{T}_1 = \hat{T}_2$ $\therefore$ PT bisects $S\hat{T}Q$	1S 1R 1 S	(3) [11]

TOTAL: 100