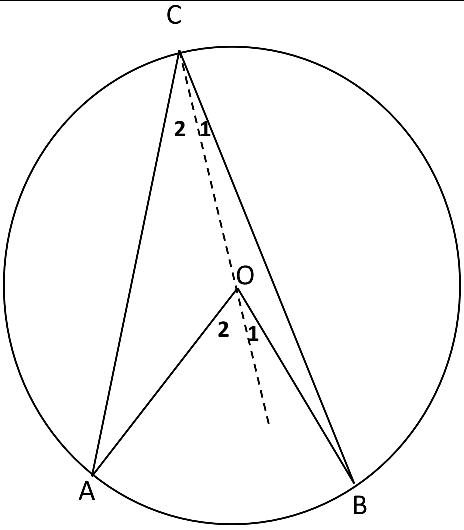
MATHEMATICS

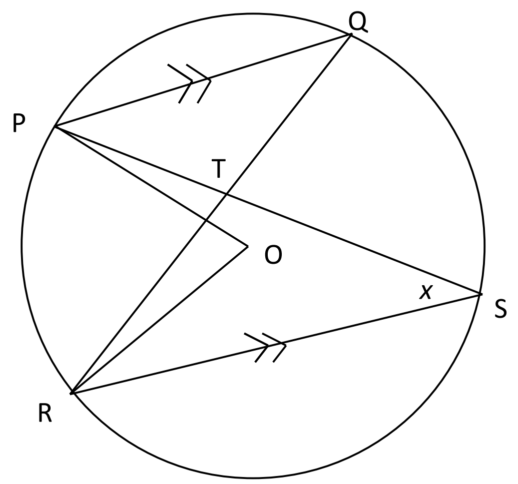
MATERIAL FOR GRADE 12

EUCLIDEAN GEOMETRY

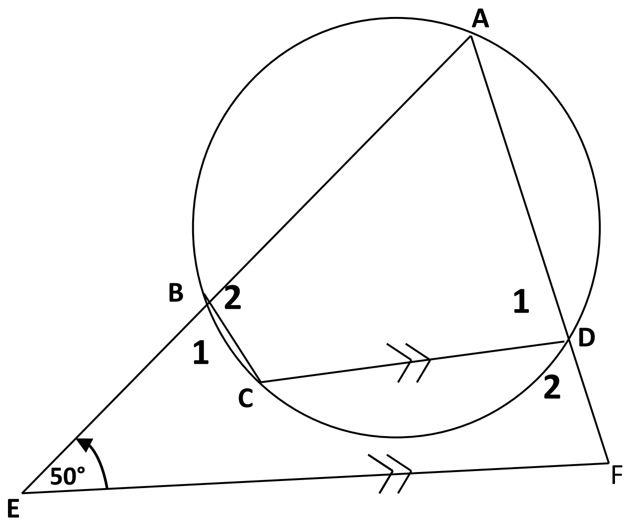
M E M O R A N D A

QUESTION 1

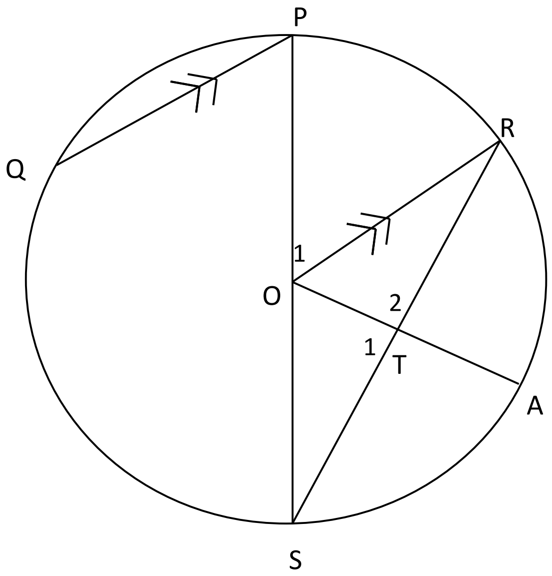
1.1	(a) Sum of two opposite interior angles	✓answer (1)
	(b) The angle on the alternate segment	✓answer (1)
	(c) Supplementary	✓answer (1)
		
1.2	**Construction:** Join CO and produce it. Proof: In ΔACO, $\hat{O}_2 = \hat{A} + \hat{C}_2$ (ext. Angle of a Δ) But $\hat{A} = \hat{C}_2$ (OA = OC = radii) $\therefore \hat{O}_2 = \hat{C}_2 + \hat{C}_2$ $\hat{O}_2 = 2\hat{C}_2$ Similarly we can prove that $\hat{O}_1 = 2\hat{C}_1$ $\hat{O}_2 + \hat{O}_1 = 2\hat{C}_2 + 2\hat{C}_1$ $\therefore A\hat{O}B = 2A\hat{C}B$	✓construction ✓ $\hat{O}_2 = \hat{A} + \hat{C}_2$ ✓ $\hat{A} = \hat{C}_2$ ✓ $\hat{O}_2 = 2\hat{C}_2$ ✓ $\hat{O}_1 = 2\hat{C}_1$ ✓ $\hat{O}_2 + \hat{O}_1 = 2\hat{C}_2 + 2\hat{C}_1$ (6)

		
1.3.1	$\hat{S} = \hat{Q} = x$ (sub.by same segment PR) $= Q\hat{R}S = Q = x$ (alt.angle PQ // RS) $= Q\hat{P}S$ (sub.by same segment QS or alt∠) $= x$	✓S ✓R ✓S ✓R ✓S ✓R (6)
1.3.2	$P\hat{T}R = x + x$ (ext. Angle of a ΔPQT) $= 2x$	✓S ✓2 x (2)
1.3.3	$P\hat{O}R = 2x$ (∠ at the centre =2 ∠ at the circumference) $= P\hat{T}R$ But they are sub. by same segment PR ∴ PTOR is cyclic	✓S ✓ R ✓They are sub. by same segment PR (3)
		[20]

QUESTION 2

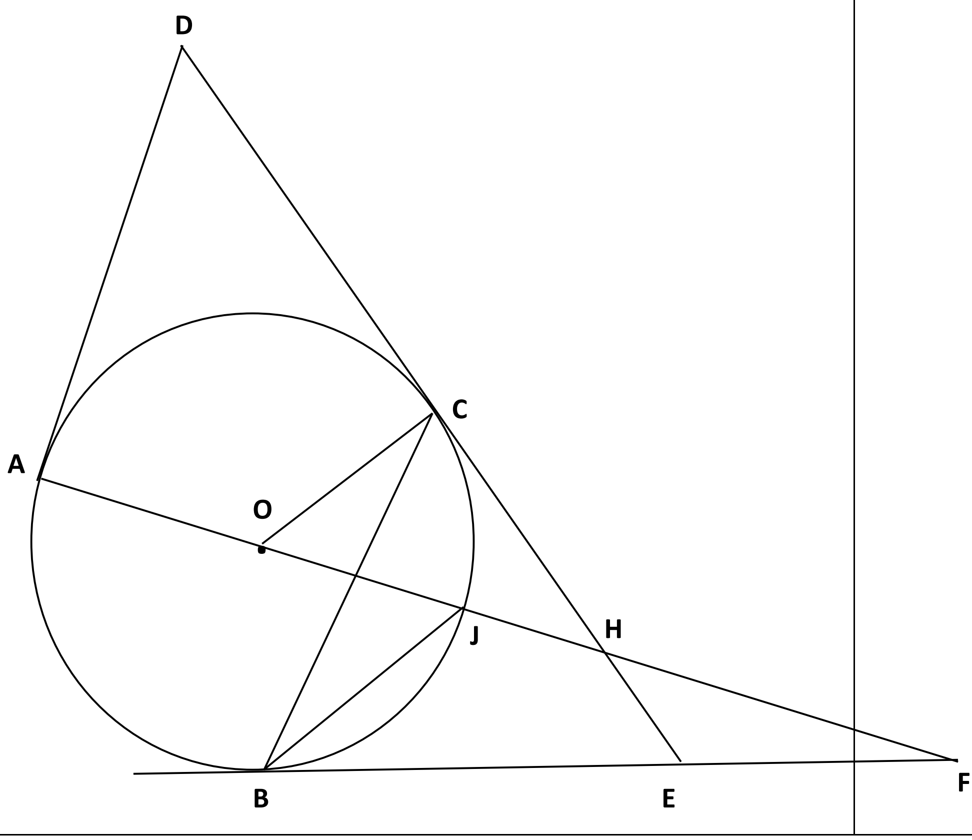
2.1	(a) Opposite angles of a cyclic quad are supplementary.	✓answer (1)
	(b) Angle at centre is twice angle at circumference.	✓answer (1)
	(c) $\hat{U} = 30°$	✓answer (1)
	(d) Tan chord theorem	✓answer (1)
	(e) Radius is perpendicular to tangent	✓answer (1)
		
2.2	$\hat{E} = \hat{F} = 50°$ (EA = AF) $\hat{F} = \hat{D}_1 = 50°$ (corr. angles EF // CD) $\hat{B}_2 + \hat{D}_1 = 180°$ (opp.angles of a cyclic) $\hat{B}_2 + 50° = 180°$ $\therefore \hat{B}_2 = 130°$ **OR** $\hat{E} = \hat{F} = 50°$ given $\hat{F} = \hat{D}_1 = 50°$ corr. angles EF // CD $\hat{D}_1 + \hat{D}_2 = 180°$ adjacent angles on a str line $\hat{D}_2 = 180° - 50°$ $= 50°$ But $\hat{D}_2 = \hat{B}_2$ ext angle of cyclic quad $\therefore \hat{B}_2 = 130°$	✓R ✓S ✓R ✓S ✓R ✓answer (6) ✓R ✓S ✓R ✓S ✓R ✓answer (6)
		[11]

QUESTION 3

		
3.1	$\hat{P} = \hat{O}_1$ (alternate angles) $\hat{O}_1 = 2\hat{S}$ (angle at the centre) but $\hat{S} = \hat{R}$ (OS = OR radii) $\hat{P} = 2\hat{R}$ **OR** $\hat{P} = \hat{O}_1$ (alternate angles) $Q_1 = \hat{S} + \hat{R}$ (ext.angle of a Δ) but $\hat{S} = \hat{R}$ (OS = OR radii) $\hat{P} = 2\hat{R}$	✓$\hat{P} = \hat{O}_1$ ✓$\hat{O}_1 = 2\hat{S}$ ✓angle at the centre ✓ $\hat{S} = \hat{R}$ ✓$\hat{P} = 2\hat{R}$ (5)

3.2.	$OS = OA = x + 2$ radii RT=ST= 4cm line from centre to mid-point $(OS)^2 = (ST)^2 + (OT)^2$ Pythagoras theorem $(x + 2)^2 = (4)^2 + (x)^2$ $x^2 + 4x + 4 = 16 + x^2$ $4x = 12$ $\therefore x = 3$ OA= $x + 2$ $= 3 + 2$ $OA = 5 \quad \therefore OS = 5$	✓OA $= x + 2$ ✓ RT = 4 ✓ Pythagoras ✓$x = 3$ ✓OA= 5 (5)
		[10]

QUESTION 4

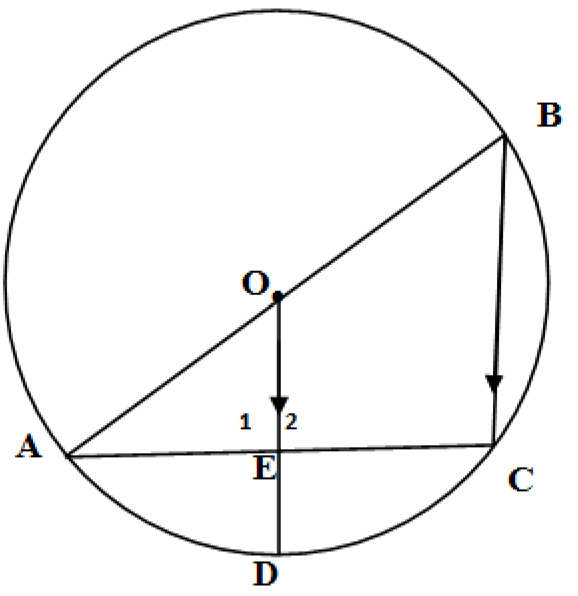


4.1	OA ⊥ AD rad ⊥ tan OC ⊥ DC rad ⊥ tan In ΔDAH and ΔOCH $\hat{H} = \hat{H}$ common $O\hat{C}H = H\hat{A}D$ both = 90° $A\hat{D}C = H\hat{O}C$ 3rd angle ∴ $\Delta DAH \mathbin{///} \Delta OCH$ (Đ ; Đ ; Đ)	✓R ✓ R ✓ R ✓ R (4)
4.2.	$\frac{DA}{OC} = \frac{AH}{CH} = \frac{DH}{OH}$ (ΔDAH ||| ΔOCH) $OH = \frac{OC \cdot DH}{DA}$ OC = OA (Radii) DC = DA (tangent from same point) $OH = \frac{OA \cdot DH}{DC}$	✓ S ✓ R ✓ $OH = \frac{OC \cdot DH}{DA}$ ✓R ✓S✓R (6)
		[10]

QUESTION 5

5.1	is perpendicular to the chord	✓ (1)
5.2	The line from the centre of the circle perpendicular to the chord, bisects the chord	✓The line from the centre of the circle perpendicular to the chord ✓ bisects the chord (2)

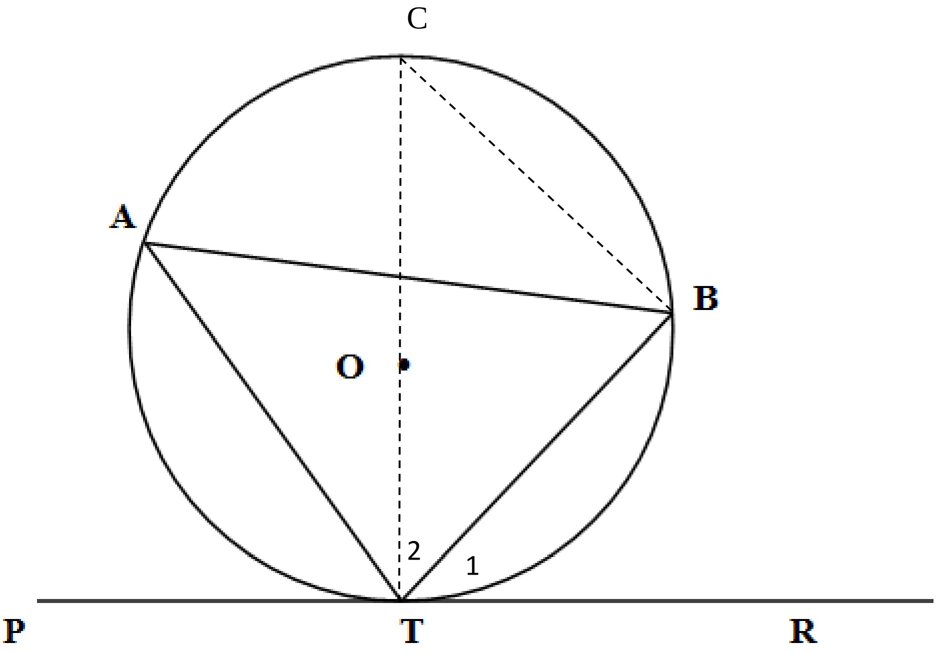
5.3



5.3.1	$\frac{AO}{OB}=\frac{AE}{EC}\cdots\cdots OE\,//\,BC$ AO=OB.....Radii $\Longrightarrow$ AE = EC	✓S✓R (2)
5.3.2	$\hat{C}=90°$ (angle in semi⨀) $\hat{E}_1=90°$ (corr. angles; OD ‖ BC)	✓S/R ✓R (2)
5.3.3	$OE^2=10^2-8^2$ (theorem of Pyth) $OE^2=100-64=36$ OE = 6 cm ∴ED = 4 cm	✓S ✓OE = 6 cm ✓answer (3)
		[10]

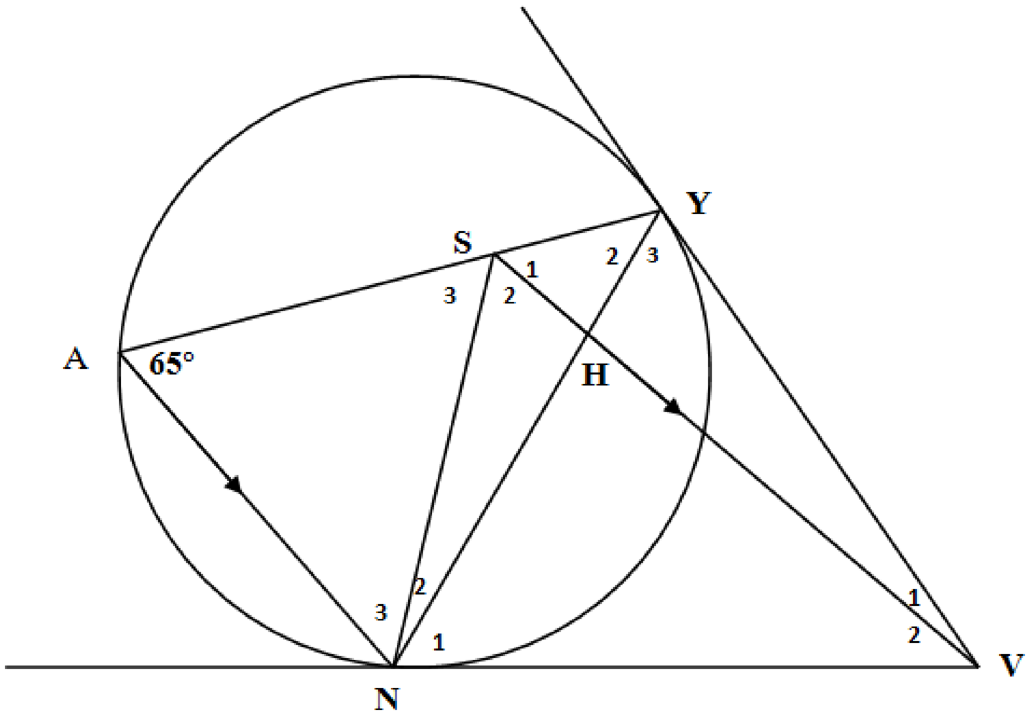
QUESTION 6

6.1



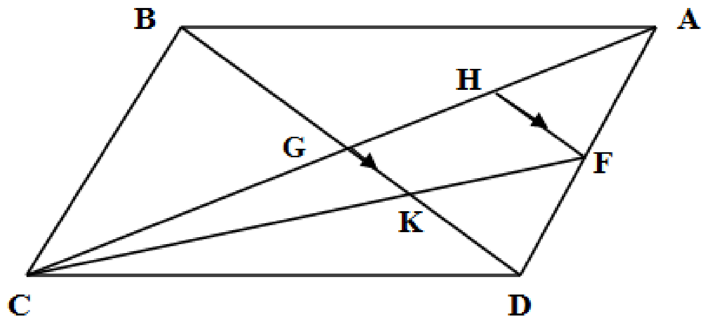
6.1	Construction: Draw diameter TC and join BC. $C\hat{B}T = 90°$ (∠ in semi ⊙) $\hat{C}+\hat{T}_2 = 90°$ ($\angle' s of\ \Delta$) $\hat{T}_1+\hat{T}_2 = 90°$ (tangent ⊥ r) $\therefore\ \hat{C}=\hat{T}_1$ But $\hat{C} = \hat{A}$ (∠'s in same segment) $\therefore \hat{T}_1 = \hat{A}$	✓construction ✓S / R ✓S ✓S/ R ✓S/ R ✓conclusion (6)

6.2



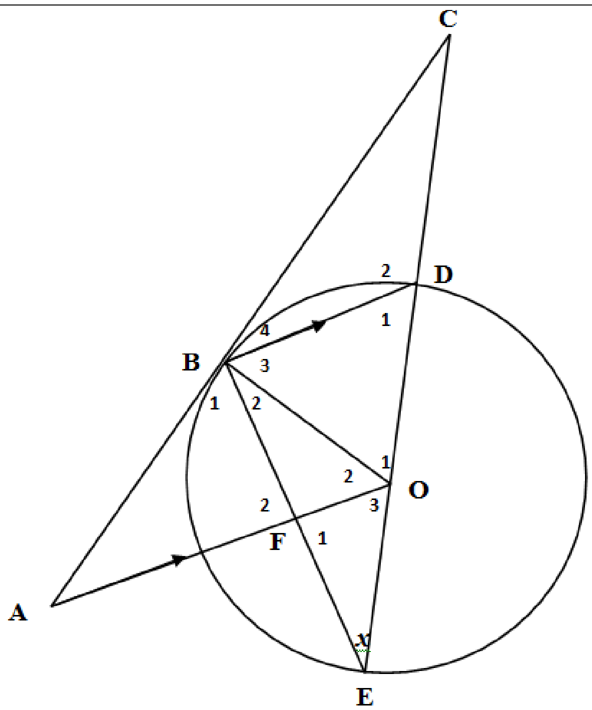
6.2.1	$\hat{S}_1 = 65°$ (corr ∠'s; AN || SV) $\hat{Y}_3 = 65°$ (tan-chord th) $\hat{N}_1 = 65°$ (tan-chord th)	✓S R ✓S R ✓S R (3)
6.2.2	$\hat{S}_1 = \hat{N}_1$ VYSN is a cyclic quad (YV subtends equal angles)	✓$\hat{S}_1 = \hat{N}_1$ ✓YV subtends equal angles (2)
6.2.3	$\hat{S}_2 = 65°$ (∠'s in same segment) $\hat{N}_3 = 65°$ (alt. ∠'s; AN || SV) $\therefore \hat{A} = \hat{N}_3$ AS = SN (sides opp equal angles)	✓S✓R ✓S✓R ✓R (5)
		[16]

QUESTION 7



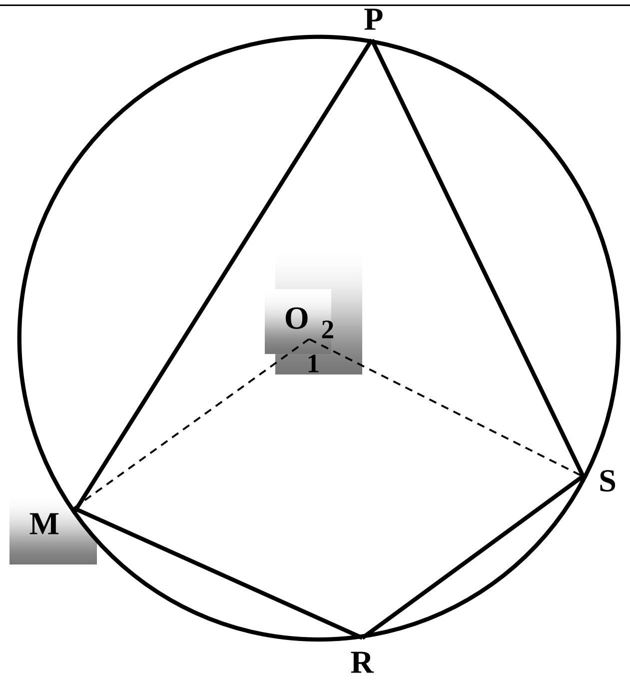
7.1	GA = 72 units (diagononals of parm) $\frac{AF}{FD} = \frac{40}{24} = \frac{5}{3}$ (line ||one side triangle) GH = $\frac{3}{8} \times 72$ = 27 units OR GA = 72 units (diagonals of parm) $\frac{GH}{72} = \frac{24}{64}$ (line ||one side triangle) GH = 27	✓ S R ✓S✓R ✓27 (4) ✓ S R ✓S✓R ✓27 (4)
7.2	$\frac{area\ \Delta\ AHF}{area\ \Delta\ ACD} = \frac{\frac{1}{2}AH.AF.sinA}{\frac{1}{2}.AC.AD.sinA}$ $= \frac{45.40}{144.64}$ $= \frac{25}{128}$ OR $\frac{area\ \Delta\ AHF}{area\ \Delta AGD} = \frac{\frac{1}{2}.45.40}{\frac{1}{2}.72.64} = \frac{25}{64}$ But $\Delta\ ACD = 2 \times \Delta AGD$ $\therefore \frac{area\ \Delta AHF}{area\ \Delta ACD} = \frac{25}{24 \times 6} = \frac{25}{128}$	✓area Δ AHF ✓area Δ ACD ✓correct substitution ✓answer (4) ✓area Δ AHF ✓area Δ ACD ✓$\Delta\ ACD = 2 \times \Delta AGD$ ✓answer (4)
		[8]

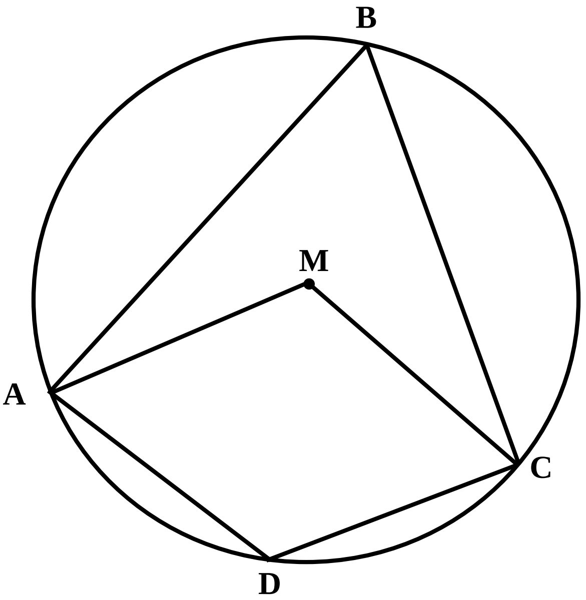
QUESTION 8

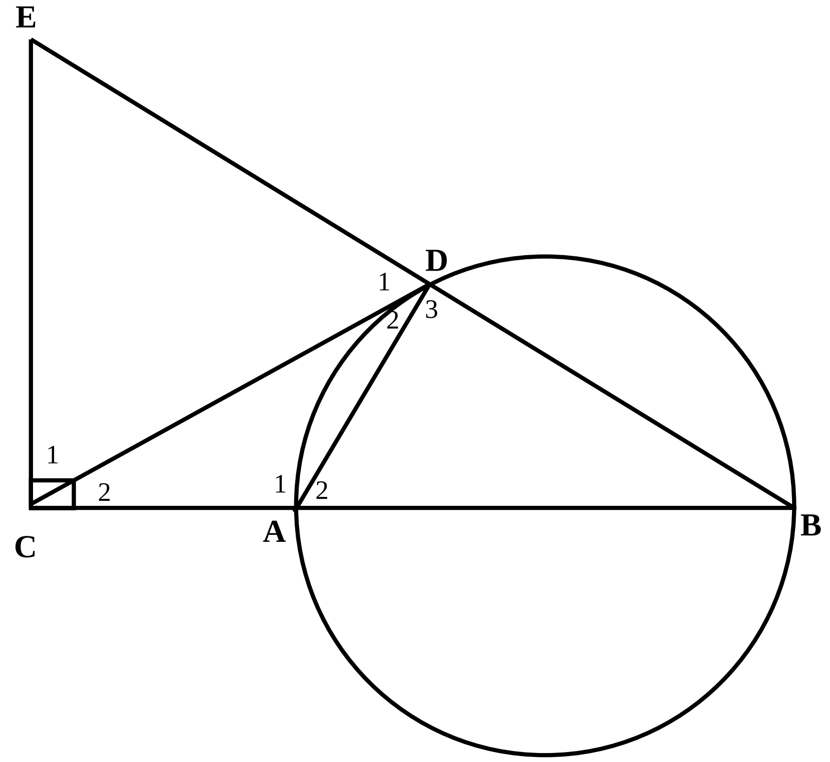


8.1	$\hat{B}_2 = x$ (radii =) $\hat{B}_4 = x$ (tan-chord th) $\hat{A} = x$ (corr ∠'s; BD ∥ AO)	✓S ✓SR ✓S (3)
8.2	$\hat{B}_2 + \hat{B}_3 = 90°$ (∠ in semi ⊙) C$\hat{B}$E = 90° + x	✓R ✓90° + x (2)
8.3.1	In Δ CBD and Δ CEB: $\hat{C} = \hat{C}$ $\hat{B}_4 = \hat{E} = x$ $\hat{D}_2$ = C$\hat{B}$E ∴ Δ CBD ∥∣ Δ CEB (∠∠∠)	✓S ✓S (2)
8.3.2	$\frac{CB}{CE} = \frac{BD}{EB}$ (∥∣ triangles) EB. CB = CE. BD $\hat{F}_1 = 90°$ (corr ∠'s; BD ∥ AO) BF = FE (line from centre to mdpt of chord) ∴ BE = 2EF ∴ 2EF.CB = CE.BD	✓S✓R ✓SR ✓SR ✓replacing BE (5)
8.3.3	$\frac{2EF}{CE} = \frac{BD}{BC}$ out of / uit 10.4 But/maar Δ BCD ∥∣ Δ ACO (∠∠∠) ∴ $\frac{BD}{AO} = \frac{BC}{AC}$ $\frac{BD}{BC} = \frac{AO}{AC}$ $\frac{2EF}{CE} = \frac{AO}{AC}$	✓S ✓SR ✓S ✓S (4)
		[16]

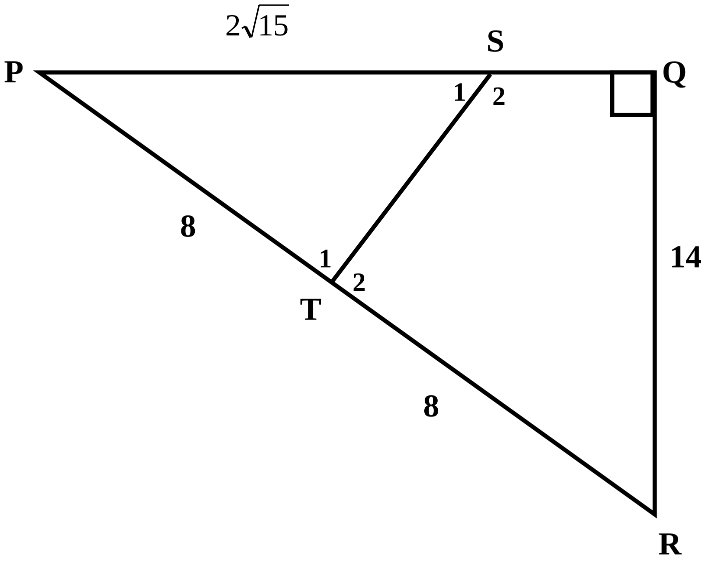
QUESTION 9

9.1		
9.1	Const: Draw radii OM and OS (also accepted on sketch) Proof: $\hat{O}_1 = 2\hat{P}$ ($\angle$ at centre $= 2\angle$ at the circumference) $\hat{O}_2 = 2\hat{R}$ ($\angle$ at centre $= 2\angle$ at the circumference) $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P} + \hat{R})$ but $\hat{O}_1 + \hat{O}_2 = 360°$ (revolution) $\therefore\ 2(\hat{P} + \hat{R}) = 360°$ $\hat{P} + \hat{R} = 180°$	✓construction ✓statement ✓reason ✓statement/reason ✓ $\hat{O}_1 + \hat{O}_2 = 360°$ ✓ $2(\hat{P} + \hat{R}) = 180°$ (6)

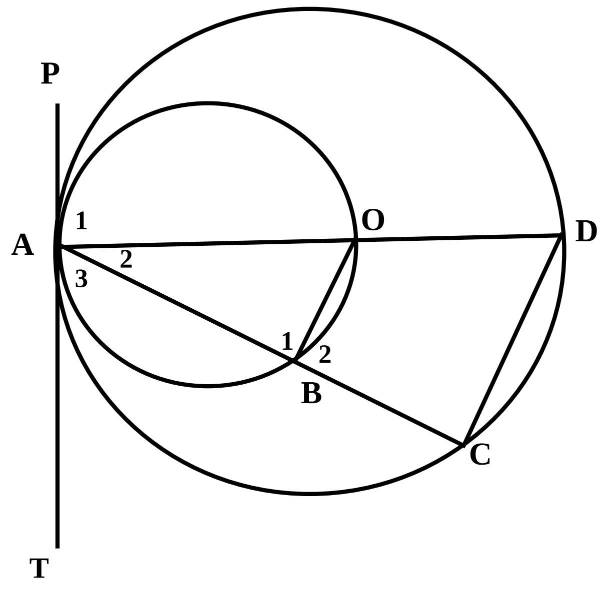
9.2	 	
9.2	$\hat{B} + \hat{D} = 180°$ (opp. $\angle s$ of a cyclic quad.) $\hat{B} = \frac{2}{5}.(180°)$ $= 72°$ $A\hat{M}C = 2\hat{B}$ ($\angle$ at the centre) $= 2(72°)$ $= 144°$ **OR** Let $\hat{B} = 2x$ $\therefore \hat{D} = 3x$ $2x + 3x = 180°$ (opp. $\angle s$ of a cyclic quad.) $5x = 180°$ $x = 36°$ $A\hat{M}C = 4x$ ($\angle$ at the centre) $= 4(36°)$ $= 144°$	✓ $\hat{B} + \hat{D} = 180°$ ✓ opp. $\angle s$ of a cyclic quad ✓ $\hat{B} = \frac{2}{5}.(180°)$ ✓ $A\hat{M}C = 2\hat{B}$ ✓ $\angle$ at the centre ✓144° (6) **OR** ✓ $\hat{B} = 2x$ and $\hat{D} = 3x$ ✓$2x + 3x = 180°$ ✓ opp. $\angle s$ of a cyclic quad ✓ $A\hat{M}C = 4x$ ✓ $\angle$ at the centre ✓answer

9.3		
9.3.1	$\hat{D}_3 = 90°$ (AB is a diameter) $\hat{C}_1 + \hat{C}_2 = 90°$ (given) $\hat{D}_3 = \hat{C}_1 + \hat{C}_2$ (both = 90°) $\therefore$ ACED is a cyclic quadrilateral (ext. $\angle$ of a quad = interior opp $\angle$)	✓ $\hat{D}_3 = 90°$ ✓AB is a diameter ✓ $\hat{C}_1 + \hat{C}_2 = 90°$ ✓ $\hat{D}_3 = \hat{C}_1 + \hat{C}_2$ ✓ ext. $\angle$ of a quad = interior opp $\angle$ (5)
9.3.2	In ΔADB, $\hat{A}_2 + \hat{B} + \hat{D}_3 = 180°$ ($\angle$s in a Δ) $\hat{D}_1 + \hat{D}_2 + \hat{D}_3 = 180°$ ($\angle$s on a st. line) $\hat{A}_2 + \hat{B} + 90° = \hat{D}_1 + \hat{D}_2 + \hat{D}_3$ $\hat{D}_2 = \hat{B}$ ($\angle$between tan. and chord) $\therefore \hat{A}_2 = \hat{D}_1$	✓ $\hat{A}_2 + \hat{B} + \hat{D}_3 = 180°$ ✓ $\hat{D}_1 + \hat{D}_2 + \hat{D}_3 = 180°$ ✓ $\hat{D}_2 = \hat{B}$ ✓ $\angle$between tan and chord (4)
9.3.3	$\hat{A}_2 = \hat{E}$ (ext $\angle$ of a cyclic quad ACED) $\hat{A}_2 = \hat{D}_1$ (from above) $\hat{D}_1 = \hat{E}$ EC = DC (side opp of equal angles) $\therefore$ Δ CDE is isosceles	✓ $\hat{A}_2 = \hat{E}$ ✓ ext $\angle$ of a cyclic quad ACED ✓ $\hat{A}_2 = \hat{D}_1$ ✓ EC = DC (4) **[25]**

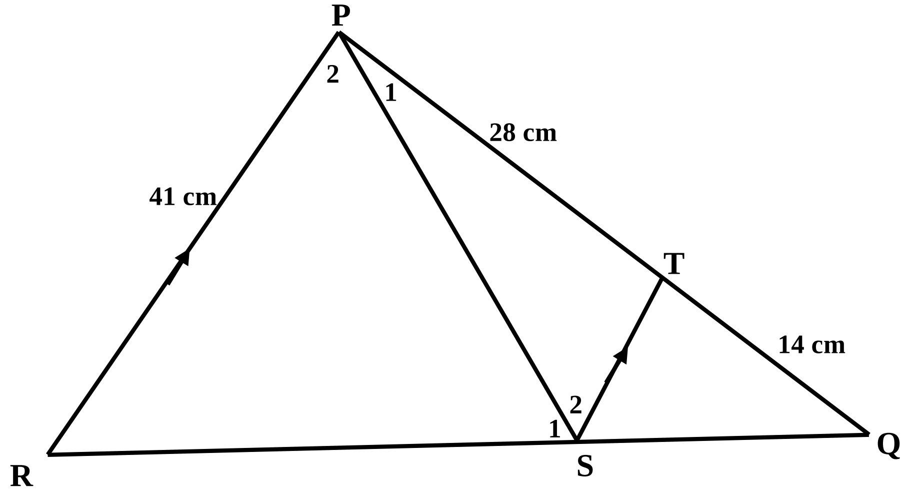
QUESTION 10

		
10.1	In ΔPQR and ΔPTS 1. $\hat{P}$ is common 2. $\hat{Q} = P\hat{T}S$ (both = 90°) 3. $\hat{R} = P\hat{S}T$ (rem. $\angle s$ of a Δ) ΔPQR ||| ΔPTS ($\angle\angle\angle$)	✓ $\hat{P}$ is common ✓ $\hat{Q} = P\hat{T}S$ (both= 90°) ✓ $\hat{R} = P\hat{S}T$ or ($\angle\angle\angle$) (3)
10.2.1	$\frac{PQ}{PT} = \frac{QR}{TS} = \frac{PR}{PS}$ (ΔPQR ||| ΔPTS) PQ = $\sqrt{16^2 - 14^2}$ (Pythagoras thm) = $\sqrt{60}$ or $2\sqrt{15}$ $\frac{PQ}{PT} = \frac{PR}{PS}$ $\frac{\sqrt{60}}{8} = \frac{16}{PS}$ PS = $\frac{16.8}{\sqrt{60}}$ = 16,52 cm	✓ $\frac{PQ}{PT} = \frac{QR}{TS} = \frac{PR}{PS}$ ✓ $\sqrt{16^2 - 14^2}$ ✓PQ = $2\sqrt{15}$ or $\sqrt{60}$ ✓ $\frac{\sqrt{60}}{8} = \frac{16}{PS}$ ✓answer (5)
10.2.2	Perimeter of ΔPQR = 16 + 14 + $\sqrt{60}$ = 37,75cm	✓16 + 14 + $\sqrt{60}$ ✓answer (2) **[10]**

QUESTION 11

<table>
<tr><td></td><td colspan="2">
</td></tr>
<tr><td>11.1</td><td>$\hat{A}_1 = \hat{B}_1$ (tan-chord thm)
$\hat{A}_1 = \hat{C}$ (tan-chord thm)
$\therefore \hat{B}_1 = \hat{C}$
But they are corresponding $\angle s$
$\therefore OB \parallel DC$</td><td>✓ $\hat{A}_1 = \hat{B}_1$
✓tan-chord thm
✓ $\hat{A}_1 = \hat{C}$
✓ $\hat{B}_1 = \hat{C}$
✓ But they are corresponding $\angle s$
(5)</td></tr>
<tr><td>11.2</td><td>$\frac{AB}{AC} = \frac{AO}{AD}$ (line drawn $\parallel$ to one side of a Δ)
$= \frac{1}{2}$ (OA = r ; AD = 2r)</td><td>✓ $\frac{AB}{AC} = \frac{AO}{AD}$
✓ $\frac{1}{2}$ (2)
[7]</td></tr>
</table>

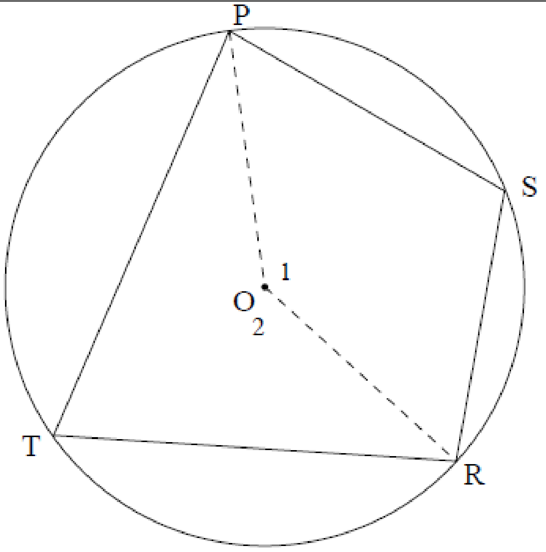
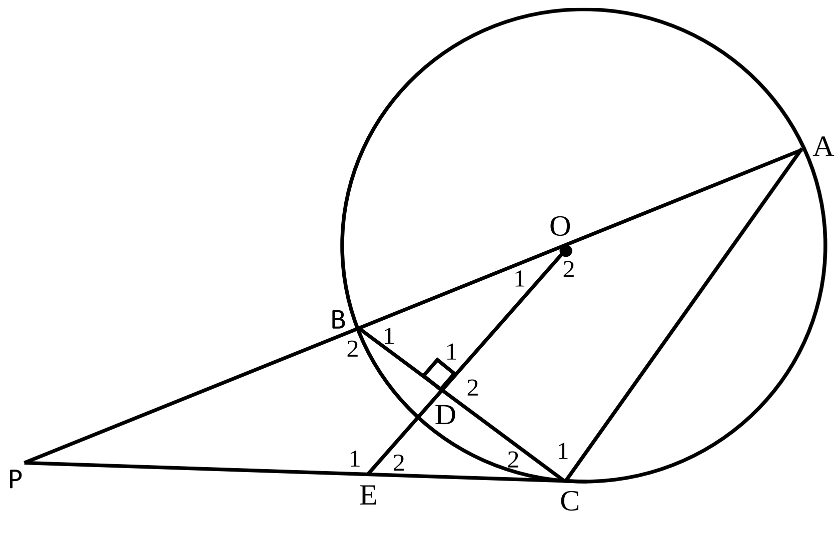
QUESTION 12

		
12.1	$\hat{P}_2 = \hat{S}_2$ (alt $\angle s$ PR ∥ TS) $= \hat{S}_1$ (given) ∴ RS = PR = 41 cm (side opp. to equal $\angle s$ **)** $\frac{SQ}{SR} = \frac{QT}{TP}$ (line drawn ∥ to one side of a Δ) $\frac{SQ}{41} = \frac{14}{28}$ ∴ $SQ = \frac{14}{28}(41) =$ 20,5 cm	✓ $\hat{P}_2 = \hat{S}_2$ ✓ RS = 41cm ✓ $\frac{SQ}{41} = \frac{14}{28}$ ✓ line drawn // to one side of a Δ ✓ answer (5)
12.2	In ΔPQR and ΔTQS $\hat{Q} = \hat{Q}$ (common) $R\hat{P}Q = S\hat{T}Q$ (corr. angles, PR ∥ TS) ΔPQR ||| ΔTQS ($\angle\angle\angle$) $\frac{ST}{41} = \frac{14}{42}$ (ΔPQR ||| ΔTQS) $ST = \frac{41,14}{42}$ $= 13,67$ cm	✓✓ ΔPQR ||| ΔTQS ✓ $\frac{ST}{41} = \frac{14}{42}$ ✓answer (4) **[9]**

QUESTION 17

17.1.1	Equal to the angle on the alternating segment.	✓ (1)
17.1.2	Supplementary	✓ (1)
17.2.1	$\hat{A}_2 = 40°$ [tan chord theorem] $\hat{A}_5 = 40°$ [vert. opp. Angles] $\hat{P}_2 = 40°$ [tan chord theorem]	✓✓S /R ✓✓S/R ✓✓S/R (6)
17.2.2	$\hat{P}_1 = \hat{Q}_1 = 40°$ but these are cor angl. (PN ∥ TQ) $\hat{P}_1 = \hat{A}_4$ given $\hat{A}_1 = \hat{P}_1$ but these are cor. angl. (PT ∥ NR) ∴ PNRT is a parallelogram pair of opp.sides ∥	✓✓S/R ✓✓S/R ✓S/R (5)
17.3.1	$B\hat{E}D = \hat{B}_1 = x$ [alt. angle, AB ∥ EC] $A\hat{D}E = \hat{B}_1 = x$ [angle in same segment] $\hat{A}_2 = B\hat{E}D = x$ [angle in same segment] $\hat{B}_2 = E\hat{A}B = x + y$ [tan. chord] $\hat{C} + \hat{B}_1 + \hat{B}_2 = 180°$ [co-int. angles, AB ∥ EC] $\therefore \hat{C} = 180° - 2x - y$	✓S/R ✓S/R ✓S/R ✓S/R ✓S/R ✓S (6)
17.3.2	$B\hat{F}D = 2x$ ext.angle of Δ FED $\hat{C} = 180° - 2x - y$ proven $B\hat{F}D + \hat{C} = 180° - 2x - y + 2x$ $= 180° - y$ $\therefore B\hat{F}D + \hat{C} \neq 180°$ opposite angles not supplementary ∴ Becky she is correct.	✓✓S/R ✓S ✓S ✓S (5)
		[24]

QUESTION 18

18.1	 Join RO and OP Let $\hat{O}_1 = 2x$ $\hat{O}_2 = 360° - 2x$ angles around a point $\hat{T} = x$ angle at centre = 2 angle at circ. $\hat{S} = 180° - x$ angle at centre = 2 angle at circum $\hat{S} + \hat{T} = x + 180° - x$ $\therefore \hat{S} + \hat{T} = 180°$	✓constr ✓$\hat{O}_1 = 2x$ ✓S ✓S ✓✓S/R (6)
18.2		

18.2.1	$\hat{C}_1 = 90°$ [angle in semi-circle] $\hat{D}_1 = 90°$ [OE⊥BC] $\therefore \hat{C}_1 = \hat{D}_1 = 90°$ $\therefore$ OE ∥ CA [corr. angles equal]	✓✓S/ R ✓S ✓R (4)
18.2.2	$\hat{C}_2 = \hat{A} = x$ [tan. chord] $\hat{A} = \hat{O}_1 = x$ [corr. Angles, OE ∥ CA]	✓✓S /R ✓ S/R (3)
18.2.3	$\hat{P} + \hat{A} + \hat{C}_1 + \hat{C}_2 = 180°$ [sum of the angles of triangle] $\hat{P} + x + 90° + x = 180°$ $\therefore \hat{P} = 90° - 2x$	✓✓S /R ✓answer (3)
		[16]

QUESTION 19

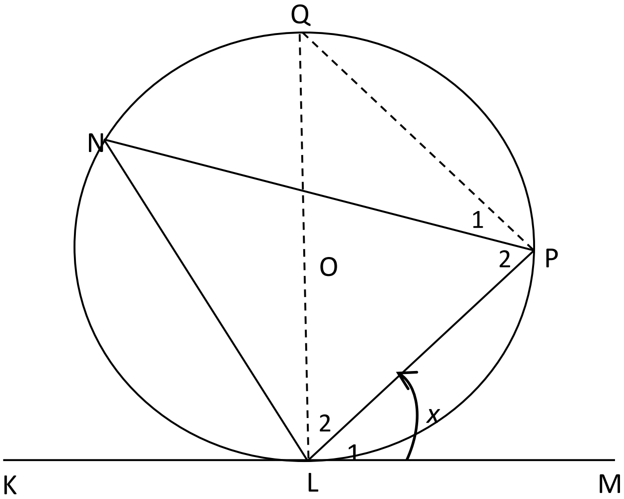
19.1	$\hat{A}_1 = \hat{B}_2 = x$ [tan chord] $\hat{A}_3 = \hat{B}_2 = x$ [alternating angles, AC // BT] $\hat{A}_3 = \hat{T}_1 = x$ [angles subt by same chord] But $\hat{T}_1 = \hat{T}_4 = x$ [vert opp. angles] $\therefore \hat{T}_4 = \hat{A}_1 = x$ $\therefore$ PT is a tangent to circle ADT. [Angle between line and chord]	✓S/R ✓S/R ✓S/R ✓ S/R ✓S (5)
19.2	In Δ APT and ΔTPD $\hat{P} = \hat{P}$ common $\hat{T}_4 = \hat{A}_1 = x$ proven $A\hat{T}P = \hat{D}_2$ 3rd angle of triangle $\therefore$ ΔAPT ||| ΔTPD angle, angle, angle	✓S/R ✓S/R ✓S/R (3)
19.3	$\frac{AP}{PT} = \frac{PT}{PD}$ [ΔAPT ||| ΔTPD] $AP.PD = PT.PT$ $AP.\frac{1}{3}AP = PT^2$ $AP^2 = 3PT^2$	✓✓S /R ✓DP= $\frac{1}{3}$ AP ✓S (4)
		[12]

QUESTION 20

20.1 PT = TQ = 12cm … (line from center perpendicular to chord PQ) $\therefore$ PQ = 12 cm + 12 cm = 24cm	A✓ R A✓ answer (2)
20.2 $OT^2 = OQ^2 - QT^2$ ….. pythagoras $= 13^2 - 12^2$ $= 169 - 144$ $= 25$ $\therefore$ OT = 5 $\therefore$ TR = OR– OT = 13cm – 5cm = 8cm In Δ PTR, $PR^2 = TR^2 + PT^2$ $= 8^2 + 12^2$ $= 64 + 144$ $= 208\ cm^2$ $\therefore PR = \sqrt{208}$ cm or $4\sqrt{13}$ cm or 14,42 cm	 A✓ OT = 5 CA✓ TR = 8cm CA✓ $PR^2 = 208$ CA✓ PR = $4\sqrt{13}$ or 14,42 (4) **[6]**

QUESTION 21

<table>
<tr><td>21.1 Interior opposite angle</td><td>A✓ S (1)</td></tr>
<tr><td>21.2



Construction : Draw diameter LOQ and join QP or

Join OL and OP

<table>
<tr><th>STATEMENT</th><th>REASON</th></tr>
<tr><td colspan="2">Let PL̂M = $\hat{L}_1 = x$</td></tr>
<tr><td>$\hat{P}_1 + \hat{P}_2 = 90^o$</td><td>angle subtended by the diameter</td></tr>
<tr><td>$\hat{L}_2 = 90^o - x$</td><td>LM $\perp$ OL, tan – radius</td></tr>
<tr><td>$\therefore$ Q̂ = x</td><td>Sum of the angles of a triangle</td></tr>
<tr><td>N̂ = x</td><td>Subtended by the same chord LP</td></tr>
<tr><td>PL̂M = N̂</td><td></td></tr>
</table>
</td><td>A✓construction

A✓S/R

A✓S/R

A✓S

A✓S/R (5)</td></tr>
</table>

21.3.1 $\hat{A} = 180^{\circ} - A\hat{E}D$ … co interior $\angle$'s, AB//ED $= 180^{\circ} - 70^{\circ}$ $= 110^{\circ}$	A✓ S/R A✓ 110° (2)
21.3.2 $\hat{B}_1 = 70^{\circ}$ ... ext $\angle$ cyclic quad ABDE	A✓ R A✓ 70° (2)
21.3.3 $\hat{D}_2 = \hat{B}_1 = 70^{\circ}$ ....(alt $\angle$s ; DE//CA)	CA✓ 70° A✓ S/R (2)
21.3.4 $\hat{B}_2 = \hat{D}_2 = 70^{\circ}$ ... ($\angle$s opp = sides)	CA✓ 70° A✓ S/R (2)
21.3.5 $\hat{E}_1 = 180^{\circ} - (\hat{B}_2 + \hat{D}_2)$...($\angle$sum of Δ) $= 180^{\circ} - 140^{\circ}$ $= 40^{\circ}$ $\therefore \hat{D}_1 = \hat{E}_1 = 40^{\circ}$ ... tan chord theorem	CA✓ $\hat{E}_1 = 40^{\circ}$ CA✓ $\hat{D}_1 = 40^{\circ}$ A✓ R (3) **[17]**

QUESTION 22

22.1 $\hat{P}_1 = \hat{B}_2 = x$ ... alt $\angle$s; SP//BC $\hat{P}_2 = \hat{P}_1 = x$ ... given $Q_1 = P_1 = x$ ... tan chord theorem	A✓ S A✓ R A✓ S/R A✓ S (4)
22.2 PC = BC … $\hat{P}_2 = \hat{B}_2 = x$ proved above (Δ PCB)	A✓ $\hat{P}_2 = \hat{B}_2 = C = x$ A✓reason (2)

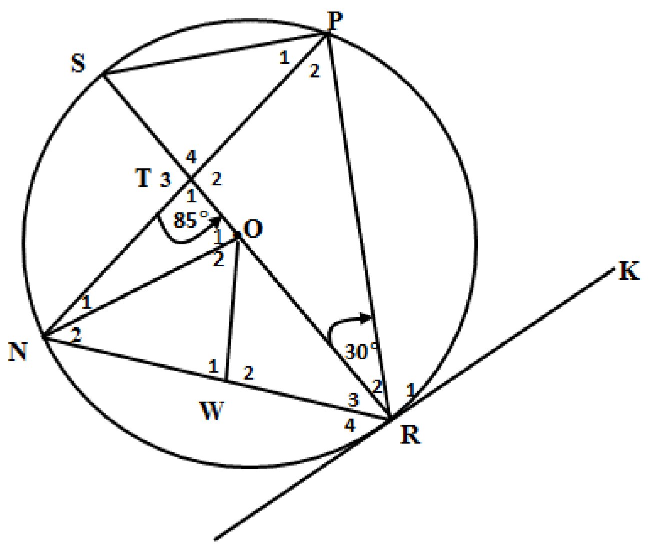
22.3 $\hat{Q}_1 = \hat{B}_2 = x$ ... proved	A✓ S
$\therefore$ RCQB is a cyclic quad	A✓ R (2)
… converse $\angle$'s in the same segment	
22.4 $\hat{S} = \hat{B}_3$ ... corresp $\angle$'s SP || BC	A✓ S/R
$= \hat{R}_3$ ...$\angle$'s in the same segment, cyclic quad RCQB	A✓ S/R
In Δ PBS and Δ QCR	
$\hat{P}_1 = \hat{Q}_1 = x$ ... proved	A✓ S/R
$\hat{S} = \hat{R}_3$ … proved	A✓ S/R
Remaining $\angle s$ equal	A✓ R (5)
$\therefore$ Δ PBS ||| Δ QCR	
22.5 In Δ PBQ and Δ PCR	A✓S
$\hat{P}_2$ is common	
$P\hat{Q}B = \hat{R}_2$ ... ext $\angle$ of cyclic quad RCQB	A✓ S/R
Δ PBQ ||| Δ PCR … (3^{rd} $\angle$ Δ)	A✓ S/R
$\therefore \frac{PB}{CP} = \frac{QB}{CR}$ (||| Δ s)	A✓ (4)
$\therefore$ PB. CR = QB. CP	**[17]**

QUESTION 23

In Δ KLM	
$\frac{LD}{9} = \frac{8}{6}$ … (LM//DE; proportionality theorem)	A✓S/R
$\therefore$ LD = 12	A✓ LD = 12

$D\hat{M}L = M\hat{D}E = x$ … alt $\angle s$, LM ∥ DE $\therefore$ LM = LD = 12 … (sides opp = $\angle s$)	A✓ S A✓answer A✓ R (5)
	[5]

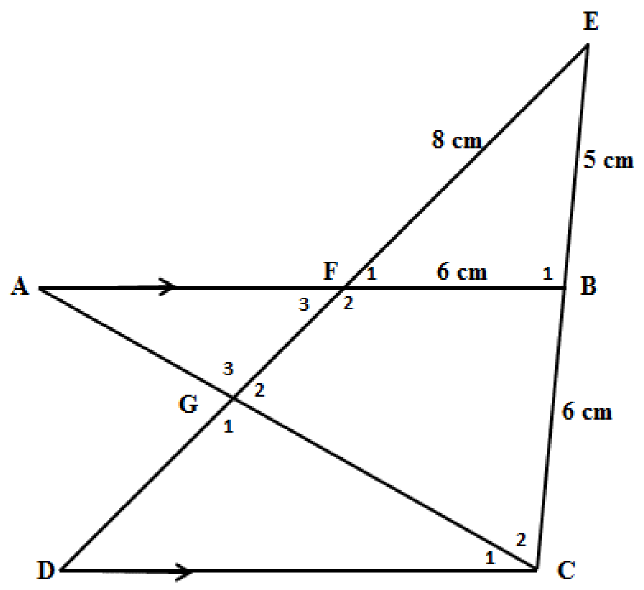
QUESTION 24



24.1.1	$\hat{P}_1+\hat{P}_2= 90°$ ($\angle$ *in semi* ⊙) $\hat{S} = 60°$ ($\angle s$ *of* Δ)	✓ S✓R ✓S (3)
24.1.2	$\hat{T}_4= 85°$ (vertically opposite $\angle s$ $\hat{P}_1 = 35°$ ($\angle s$ *of* Δ) $\hat{R}_3= 35°$ ($\angle s$ *in same segment*) OR $\hat{T}_4= 85°$ (vertically opposite $\angle s$) $\hat{P}_1 = 35°$ ($\angle s$ *of* Δ) $\hat{P}_2 = 55°$ $N\hat{O}R = 110°$ ($\angle$ *at centre*; $\angle$ *at circumf*) $\hat{R}_3= \frac{180°-110°}{2} =35°$ ($\angle s$ *of* Δ)	✓S ✓S ✓S✓R ✓S ✓S✓R ✓S (4)
24.1.3	$\hat{O}_1 = 70°$ ($\angle$*at centre* …) $\hat{N}_1= 25°$ ($\angle s$ *of* Δ) OR	✓S✓R ✓S✓R

	$\hat{P}_2 = 90° - 35° = 55°$ $\hat{N}_2 = 35°$ $(radii\ equal)$ $\hat{N}_1 = 180° - (35° + 35° + 30° + 55°) = 25°$ $(\angle s\ of\ \Delta)$	✓S ✓S ✓S ✓R (4)
24.1.4	$\hat{P}_2 = 55°$ $(\angle s\ of\ \Delta)$ $\hat{R}_4 = \hat{P}_2 = 55°$ (tan-chord theorem)	✓S ✓S✓R (3)
24.2	$\hat{N}_1 \neq \hat{R}_3$ ∴ NT is not a tangent to that circle	✓S (25° ≠ 35°) ✓Justification (2)
		[16]

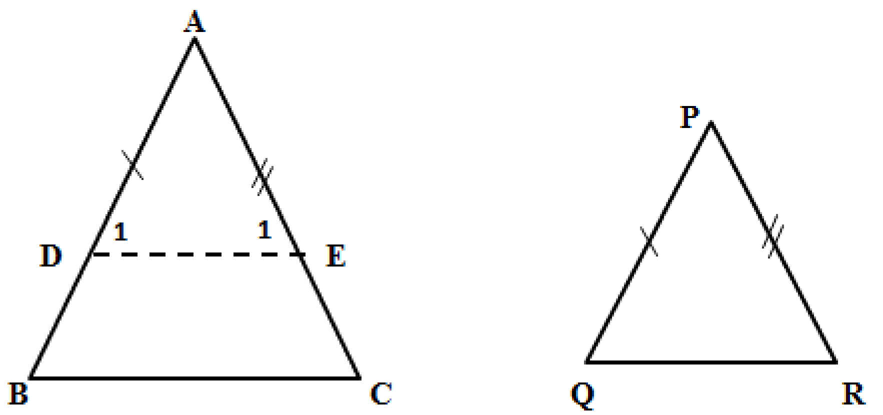
QUESTION 25



25.1	$\frac{EF}{ED} = \frac{5}{11}$ (Prop theorem – FB || DC)	✓S✓R (2)
25.2	$\frac{8}{ED} = \frac{5}{11}$ ED $= \frac{88}{5} = 17{,}6$	✓S ✓S (2)
25.3	In ΔEFB and ΔEDC : $\hat{E}$ is common $\hat{F}_1 = \hat{D}$ (corres ∠s; AB || DC); AB || DC $\hat{B}_1 = E\hat{C}D$ (∠s of Δ) ΔEFB |||ΔEDC (∠∠∠) $\frac{DC}{FB} = \frac{ED}{EF}$	 ✓ ✓S R ✓ΔEFB |||ΔEDC ✓R ✓S

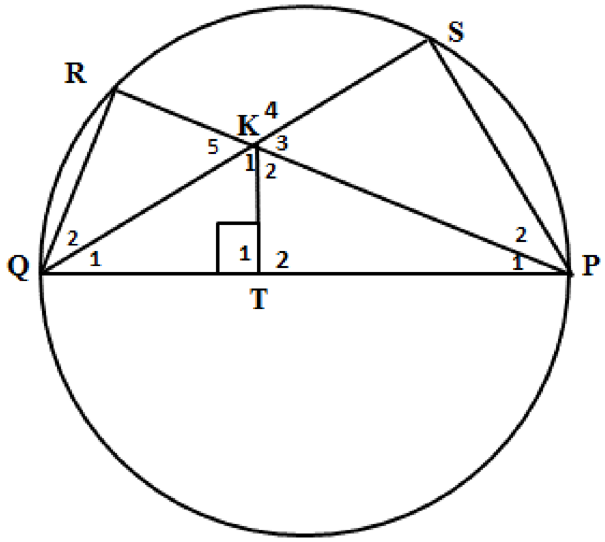
	$\frac{DC}{6} = \frac{11}{5}$ DC = 13,2	✓answer (6)
25.4	In ΔAGF and ΔCGD : $\hat{G}_1 = \hat{G}_3$ (vert opp $\angle s$) $\hat{A} = \hat{C}_1$ (alt $\angle s$ AB ∥ DC) $\hat{F}_3 = \hat{D}$ ($\angle s$ of Δ) ΔAGF ||| ΔCGD (∠∠∠) $\frac{AG}{GC} = \frac{AF}{DC}$ $\frac{AG}{GC} = \frac{14}{13,2} = \frac{35}{33}$	✓ ✓S R ✓ΔAGF ||| ΔCGD ✓R ✓S ✓S (6)
25.5	GC = $\frac{33}{68} \times 18 = 8,74$	✓$\frac{33}{68}$ ✓answer (2)
		[18]

QUESTION 26



26.1.1	In ΔADE *and* ΔPQR: AD = DQ AE = PR $\hat{A} = \hat{A}$ ∴ $\Delta ADE \equiv \Delta PQR$ ($S\angle S$)	✓all 3 statements ✓R (2)
26.1.2	$\hat{D}_1 = \hat{Q}$ (≡) $\hat{B} = \hat{Q}$ (given) $\hat{D}_1 = \hat{B}$ ∴ DE∥ BC (corresponding ∠s =)	✓$\hat{D}_1 = \hat{Q}$ ✓$\hat{D}_1 = \hat{B}$ ✓R (3)
26.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ (line ∥ to one side of Δ) But/ maar AD = PQ and AE = PR ∴ $\frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

26.2

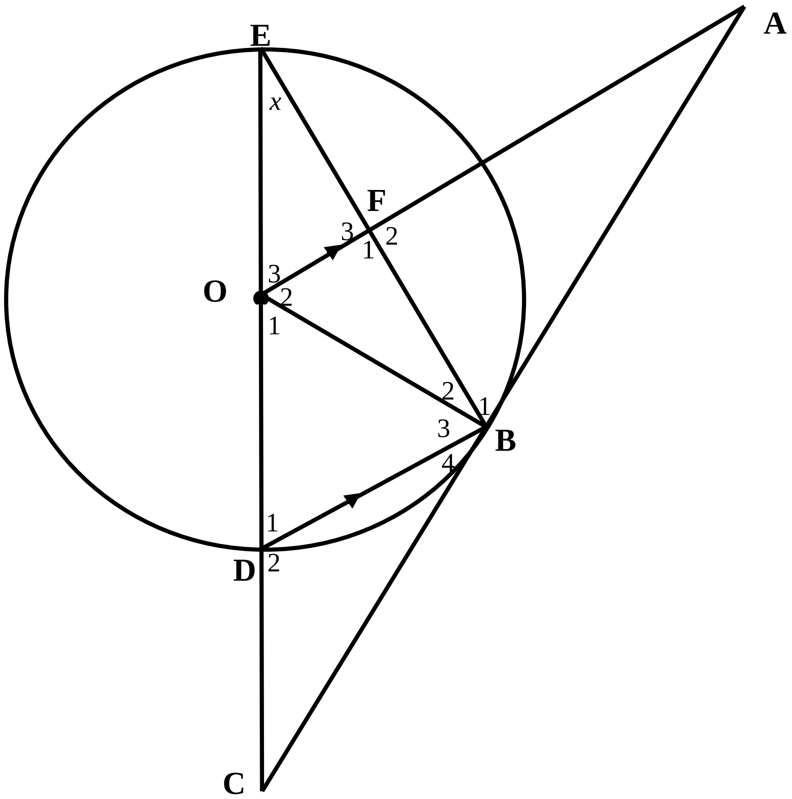


26.2.1	In ΔQSP and ΔQTK: $\hat{Q}_1$ is common $\hat{S}= 90°$ (∠ in semi ⊙) $\hat{S} = \hat{T}_1$ $\hat{P}_1+ \hat{P}_2= \hat{K}_1$ ($\angle s\ of \Delta$) $\Delta QSP \vert\vert\vert \Delta\ QTK$ (∠∠∠)	✓$\hat{Q}_1$ is common ✓$\hat{S}= 90°$ ✓R ✓$\hat{S} = \hat{T}_1$ ✓3rd angle or ∠∠∠ (5)
26.2.2	$PS^2 = PQ^2 - SQ^2$ (theorem of Pythagoras) But $\frac{PQ}{QK} = \frac{SP}{TK}$ ($\vert\vert\vert \Delta' s$) PQ = $\frac{SP.QK}{TK}$ $PS^2 = \dfrac{SP^2.QK^2}{TK^2} - SQ^2$	✓S ✓S✓R ✓PQ = $\frac{SP.QK}{TK}$ (4)
		[16]

QUESTION 27

27.1.1	90°	✓ ans (1)
27.1.2	Angle in the alternate segment	✓ ans (1)

27.2

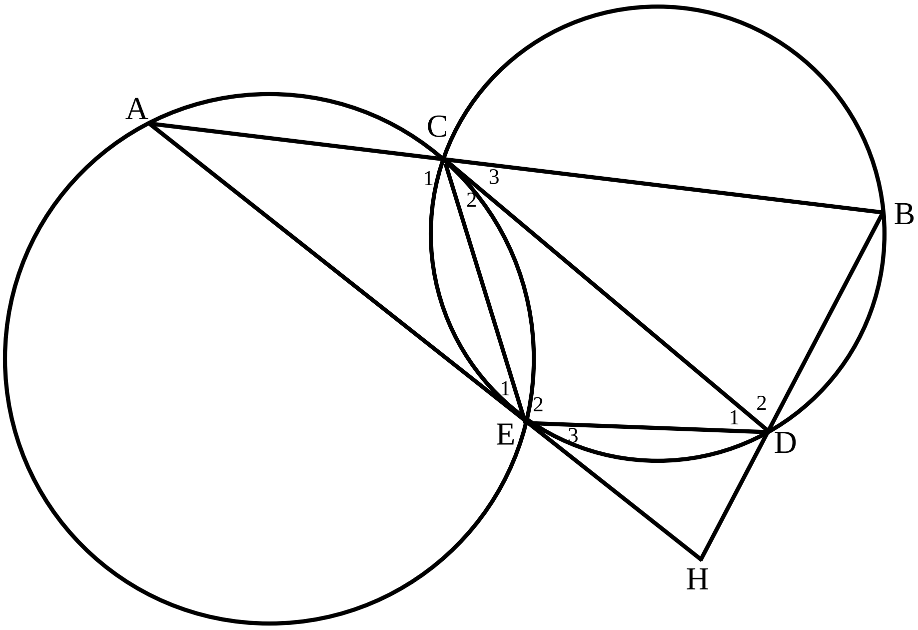


27.2.1	$\hat{B}_4 = \hat{E} = x$ (tan chord theorem) $\hat{B}_4 = \hat{A} = x$ (corresponding angles) $\hat{B}_2 = \hat{E} = x$ (radii OE = OB)	✓S ✓R ✓S ✓R ✓S ✓R (6)
27.2.2	$\hat{B}_2 + \hat{B}_3 = 90°$ (subtended by a diameter) $C\hat{B}E = 90° + x$	✓S ✓R ✓ ans (3)
27.2.3	In ΔDBE, $\frac{EO}{OD} = \frac{EF}{FB}$ (line ‖ to one side of a Δ) But/maar $\frac{EO}{OD} = 1$ (radii) $\frac{EF}{FB} = 1$ EF = FB F is the midpoint of EB **OR**	✓S ✓ R ✓ S ✓ EF = FB (4)

	In Δ EOF and ΔBOF $\hat{E} = \hat{B}_2$ (Proven above) EO = OB (radii) $\hat{D}_1 = \hat{B}_3$ (∠s opp = sides) $\hat{D}_1 = \hat{O}_3$ (corresp ∠ s) $\therefore \hat{B}_3 = \hat{O}_3$ $\therefore \hat{B}_3 = \hat{O}_2$ (alt ∠ s) $\therefore \hat{O}_3 = \hat{O}_2$ Δ EOF ≡ ΔBOF (AAS) EF = FB	✓$\hat{E} = \hat{B}_2$ ✓$\hat{D}_1 = \hat{B}_3$ ✓$\hat{D}_1 = \hat{O}_3$ ✓Δ EOF ≡ ΔBOF (AAS)
27.2.4	OF ⊥ EB (line from centre to a midpoint) EF = 4 (F is the midpoint) $OE^2 = OF^2 + EF^2$ $= 32 + 4^2$ $= 25$ OE = 5 ED = 10 cm **OR** $\hat{F}_3 = 90°$ (corresponding angles) EF = 4 (F is the mid pint) $OE^2 = OF^2 + EF^2$ $= 3^2 + 4^2$ $= 25$ OE = 5 ED = 10 cm	✓ S/R ✓ EF = 4 ✓ OE = 5 ✓ans **OR** ✓ S/R ✓ EF = 4 ✓ OE = 5 ✓ans (4)

	OR OF = $\frac{1}{2}$DB (midpoint theorem) DB = 6 cm In ΔEDB, $ED^2 = 6^2 + 8^2$ (Pythagoras thm) = 100 ED = 10	**OR** ✓ OF = $\frac{1}{2}$DB ✓ DB = 6 ✓Application of Pythagoras thm ✓ ans (4) **[19]**

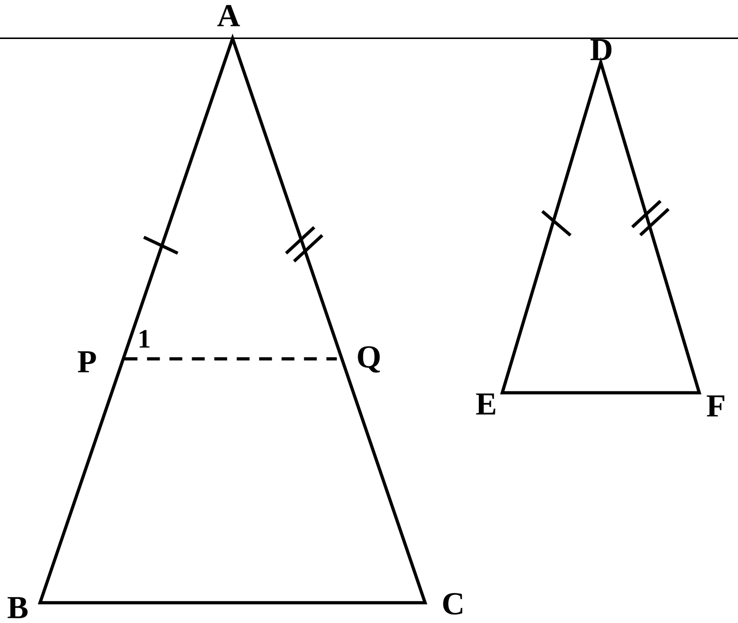
QUESTION 28

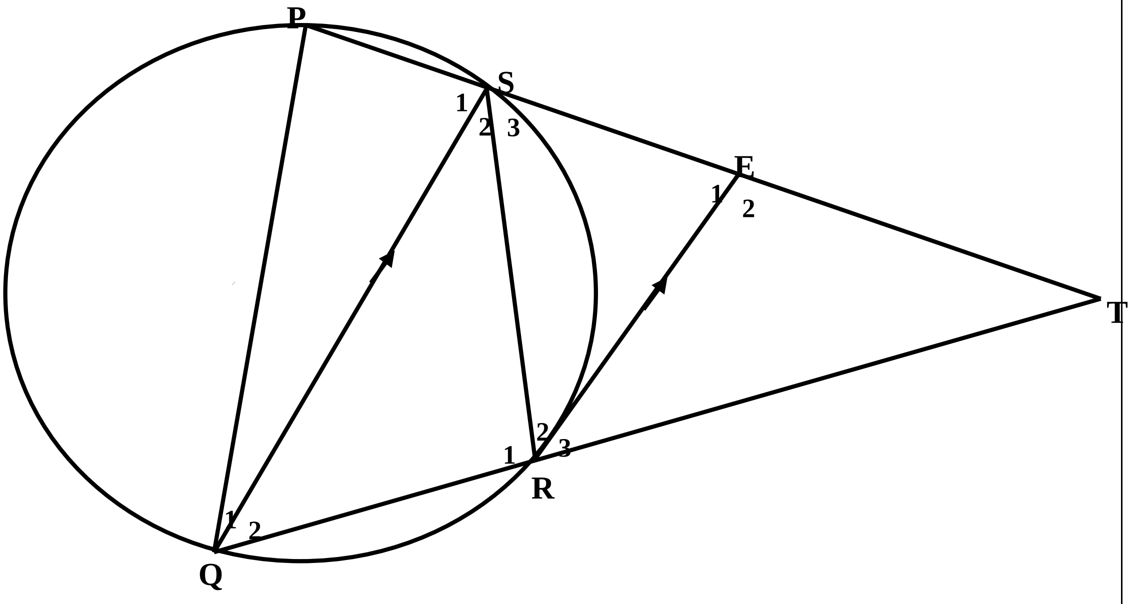


28.1	$\hat{A} = \hat{C}_2$ (tan CD and chord C) $= \hat{E}_3$ (tan AEH and chord ED) But they are corresponding angles AB ∥ ED	✓ $\hat{A} = \hat{C}_2$ ✓reason ✓ $\hat{C}_2 = \hat{E}_3$ ✓ reason ✓corresponding angles

	OR $\hat{A}+\hat{C}_1+\hat{E}_1=180^\circ$ (sum of angles in a Δ) $\hat{C}_2+\hat{E}_2+\hat{D}_1=180^\circ$ (Sum of angles in a Δ) But $\hat{A}=\hat{C}_2$ (tan CD and chord CE) $\hat{E}_1=\hat{D}_1$ (tan AEH and chord CE) $\therefore\ \hat{C}_1=\hat{E}_2$ but they are alt.angles) $AB \parallel ED$	**OR** ✓ $\hat{A}+\hat{C}_1+\hat{E}_1=180^\circ$ and $\hat{C}_2+\hat{E}_2+\hat{D}_1=180^\circ$ ✓ $\hat{A}=\hat{C}_2$ ✓reason ✓ $\hat{E}_1=\hat{D}_1$ ✓corresponding angles (5)
28.2	ACDE is a parallogram because one pair of opposite sides (AC and ED) are equal and parallel	✓answer ✓reason (2)
28.3	In ΔABH, $\frac{AC}{CB}=\frac{HD}{DB}$ (proportionality thm or $AH \parallel CD$) $\frac{HE}{EA}=\frac{HD}{DB}$ (proportionality thm or $AB \parallel ED$) $\frac{AC}{CB}=\frac{HE}{EA}$	✓ $\frac{AC}{CB}=\frac{HD}{DB}$ ✓reason ✓ $\frac{HE}{EA}=\frac{HD}{DB}$ ✓reason (4) **[11]**

QUESTION 29

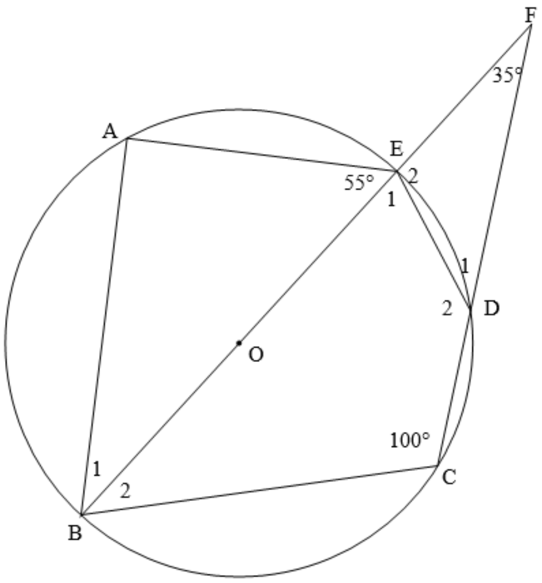
		
29.1	Const: On AB ,mark off AP = DE and on AC, mark off AQ = DF.	✓Construction

	Proof: In ΔAPQ and ΔDEF: $AP = DE$ (const) $AQ = DF$ (const) $\hat{A} = \hat{D}$ (given / gegee) $\Delta APQ \equiv \Delta DEF$ (SAS / SHS) $\hat{P}_1 = \hat{E}$ $\hat{P}_1 = \hat{B}$ $(\hat{E} = \hat{B})$ $PQ \parallel BC$ (corresp. angles =) $\frac{AB}{AP} = \frac{AC}{AQ}$ (line // one side of a Δ / lyn // aan een sy van Δ) $\frac{AB}{DE} = \frac{AC}{DF}$ (AP = DE and AQ = DF)	✓ $\Delta APQ \equiv \Delta DEF$ (SAS) ✓ $\hat{P}_1 = \hat{E}$ ✓ $\hat{P}_1 = \hat{B}$ ✓ $PQ \parallel BC$ ✓ $\frac{AB}{AP} = \frac{AC}{AQ}$ ✓line // to one side of a triangle (7)
	 	
29.2.1	$\hat{Q}_2 = \hat{R}_2$ (tan-chord theorem) $= \hat{S}_2$ (alt angles QS//RE)	✓S✓R ✓S

	QR = RS (side opp. of equal angles)	✓R (4)
29.2.2	In ΔRST and ΔPQT $\hat{T} = \hat{T}$ (common) $\hat{R}_2 + \hat{R}_3 = \hat{P}$ (ext. angle of a c.q PQRS) $\hat{S}_3 = \hat{Q}_1 + \hat{Q}_3$ (ext. angle of c.q **or** 3rd angle inΔ) ΔRST ||| Δ PQT (AAA)	✓$\hat{T} = \hat{T}$ ✓ $\hat{R}_2 + \hat{R}_3 = \hat{P}$ ✓R ✓3rd angle or Reason (4)
29.2.3	$\frac{RS}{PQ} = \frac{ST}{QT} = \frac{RT}{PT}$ (ΔRST ||| Δ PQT) $\frac{RS}{PQ} = \frac{RT}{PT}$ ....(1) In ΔQST, QS ∥ RE $\therefore \frac{SE}{ET} = \frac{QR}{RT}$ (line drawn parallel to one side of a Δ) $\therefore \frac{SE}{ET} = \frac{RS}{RT}$ (QR = RS proved above) $\therefore \frac{SE}{ET} = \frac{RS}{RT}$ (from equation (1)) $= \frac{PQ}{PT}$	✓ R (ΔRST ||| Δ PQT) ✓ $\frac{RS}{PQ} = \frac{RT}{PT}$ ✓ $\therefore \frac{SE}{ET} = \frac{QR}{RT}$ ✓ Reason ✓$\therefore \frac{SE}{ET} = \frac{RS}{RT}$ (5) **[20]**

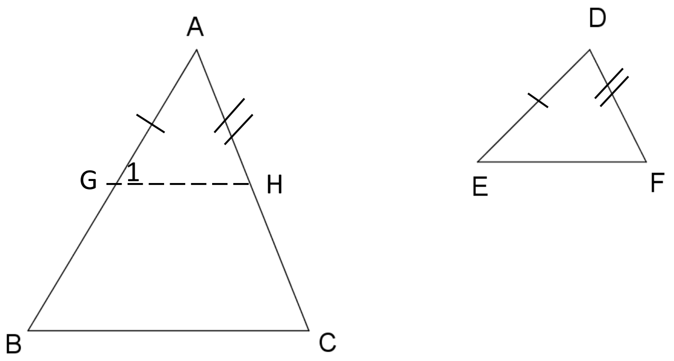
QUESTION 30

30.1

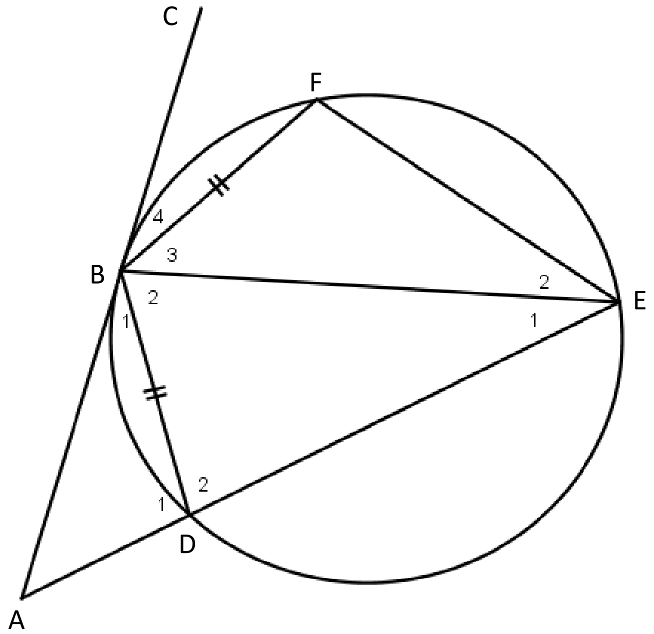


Q30	SUGGESTED ANSWER	DESCRIPTORS	MARK
30.1.1	BÂE = 90° ∠ in semi-circle	✓ S ✓R	(2)
30.1.2	$\hat{E}_1$ = 80 ° opp angles of cyclic quad	✓ S ✓R	(2)
30.1.3	D_1 =45° ext∠ of Δ FED	✓ S ✓R	(2)
30.2	$\hat{B}_1$ = 35° Interior ∠ of Δ $\hat{F}$ = 35° given ∴ $AB \parallel CF$ Alternate angles =	✓ S✓R ✓ S ✓R	(4)
			[10]

QUESTION 31

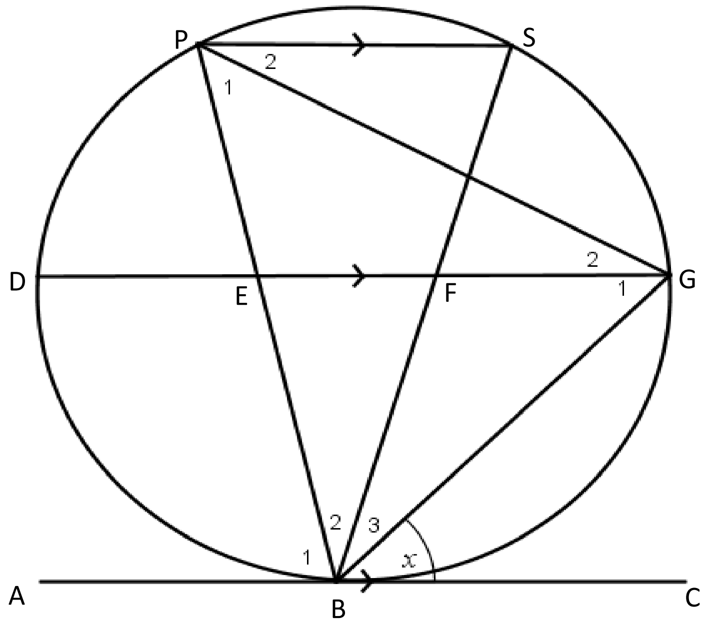
Q31	SUGGESTED ANSWER	DESCRIPTORS	MARK
31.1	 Constr.: Measure AG = DE on AB and AH = DF on AC . Draw GH Proof: $\hat{A} = \hat{D}$…given $AG = DE$ ...Constr. $AH = DF$ ...Constr $\therefore \Delta GAH \equiv \Delta EDF$ $(s; \angle; S)$ $\therefore \widehat{G_1} = \hat{E}$ But $\hat{B} = \hat{E}$...given $\therefore \widehat{G_1} = \hat{B}$ ∴ GH‖BC … corresp ∠s = $\therefore \frac{AG}{AB} = \frac{AH}{AC}$ $\therefore \frac{DE}{AB} = \frac{DF}{AC}$ … AG = DE ; AH = DF	**Consider other proofs as well** ✓constr. ✓S✓R ✓$\widehat{G_1} = \hat{B}$ ✓**S&**R ✓**S** ✓**S&R**	(7)

31.2



31.2.1	1) $\widehat{B_1} = \widehat{E_1}$ …tan-chord thm $\widehat{E_2} = \widehat{E_1}$…equal chords subtend equal ∠'s $\therefore\ \widehat{E_2} = \widehat{B_1}$	✓S&R ✓S	(3)
31.2.2	In ΔBDA and ΔEFB : $B\hat{D}A = \hat{F}$…ext∠ of cyclic quad $\therefore\ \widehat{E_2} = \widehat{B_1}$ Proven $\therefore\ \Delta BDA ||| \Delta EFB$ (∠;∠;∠)	✓✓S&R ✓S ✓S&R	(4)
	OR		
	In ΔBDA and ΔEFB : 1) $B\hat{D}A = \hat{F}$ … ext∠ of cyclic quad 2) $\widehat{B_1} = \widehat{E_1}$… tan-chord thm $\widehat{E_2} = \widehat{E_1}$ … equal chords opposite equal ∠'s $\therefore\ \widehat{E_2} = \widehat{B_1}$ $\hat{A} = \widehat{B_3}$ … sum of ∠'s in Δ $\therefore\ \Delta BDA ||| \Delta EFB$ (∠;∠;∠)	✓S&R ✓S&R ✓S &R ✓S &R	
31.2.2	$\frac{BD}{EF} = \frac{DA}{FB}$ $\therefore BD.FB = EF.DA$ $\therefore$ BD² = DA. EF … BD = FB	✓S ✓S&R	(2)
			[16]

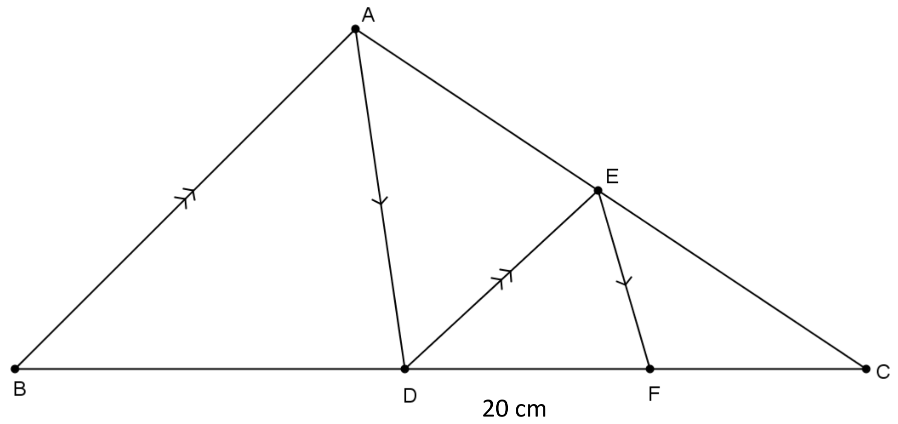
QUESTION 32



Q32	SUGGESTED ANSWER	DESCRIPTORS	MARK
32.1.	alternate ∠s ; AC ∥ DG	✓R	(1)
32.2.1	$\frac{BP}{BE}=\frac{BS}{BF}$… Prop. Thm; EF ll PS $BE=\frac{BP.BF}{BS}$	✓S✓R	(2)
32.2.2	In ΔBGP and ΔBEG: 1) $\hat{G}_1=\hat{P}_1$… Tan chord thm 2) $\hat{B}=\hat{B}$… common ∠ ∴ $\Delta BGP ||| \Delta BEG$ (∠;∠;∠)	✓✓ S&R ✓S &R ✓S &R	(4)
	OR		
	In ΔBGP and ΔBEG 1) $\hat{G}_1=\hat{P}_1$… Tan chord thm 2) $\hat{B}=\hat{B}$… common ∠ 3) $B\hat{G}P=B\hat{E}G$ … sum of ∠'s in Δ ∴ $\Delta BGP ||| \Delta BE$	✓✓ S&R ✓S &R ✓S	
32.2.3	$\frac{BG}{BE}=\frac{BP}{BG}$ …$\Delta BGP ||| \Delta BEG$ ∴ $BG^2 = BP.BE$ $BG^2=BP.\frac{BP.BF}{BS}$	✓S ✓S ✓ Subst	(3)

	$BG^2 = \frac{BP^2.BF}{BS}$ $\therefore \frac{BG^2}{BP^2} = \frac{BF}{BS}$		
			[10]

QUESTION 33



Q33	**SUGGESTED ANSWER**	**DESCRIPTORS**	**MARK**
33.1.1	$\frac{FC}{20} = \frac{4}{5}$ … EF ll AD, Prop. Thm $\therefore$ FC=16	✓✓S&R ✓answer	(3)
33.1.2	$\frac{36}{DB} = \frac{4}{5}$ … DE ll AB, Prop. Thm $\therefore$ DB= 45	✓DC = 36 ✓✓S&R ✓answer	(4)
33.2	$\frac{Area\ \Delta\ ECF}{Area\ \Delta ABC} = \frac{\frac{1}{2}.4k.16.\sin C}{\frac{1}{2}.9k.81.\sin C}$ $\frac{Area\ \Delta\ ECF}{Area\ \Delta ABC} = \frac{64}{729}$	✓½. $4k.16.sinC$ ✓½. $9k.81.sinC$ ✓ ✓ answer	(4)
			[11]