MATHEMATICS

MATERIAL FOR GRADE 12

Euclidean Geometry

QUESTIONS

QUESTION 1

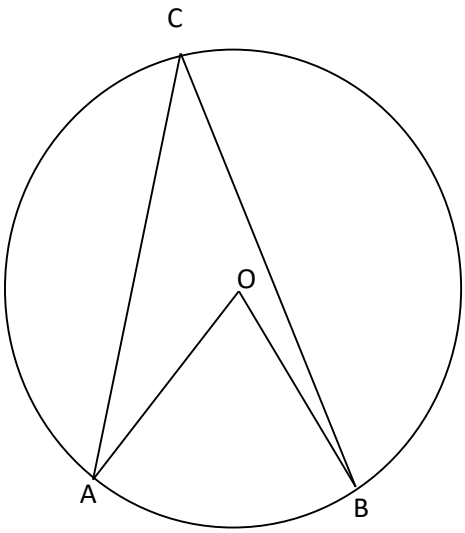
Complete such that the Euclidean Geometry statement is true:

1.1 (a) The exterior angle of a triangle is equal to … (1)

(b) The angle between a tangent and a chord is equal to the … (1)

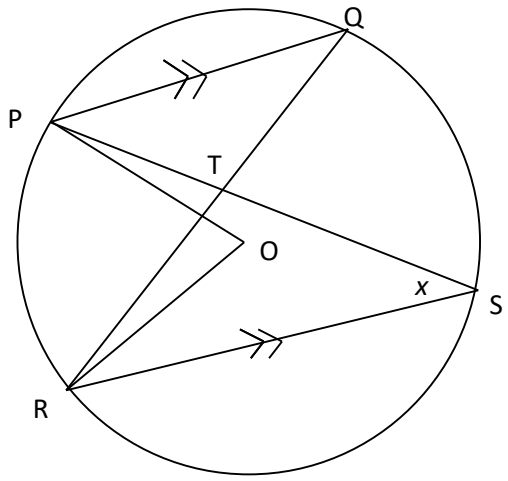
(c) Opposite angles of a cyclic quadrilateral are … (1)

1.2 In the diagram, O is the centre of the circle and A, B and C are points on the circumference of a circle. Use the diagram in the diagram sheet to prove the theorem which states that $A\hat{O}B = 2A\hat{C}B$ (6)



1.3 In the diagram, PQ and RS are parallel chords of a circle with centre O.

PS and RQ intersect at T. Let $\hat{S} = x$



1.3.1 Name with reasons, three other angles each equal to *x*. (6)

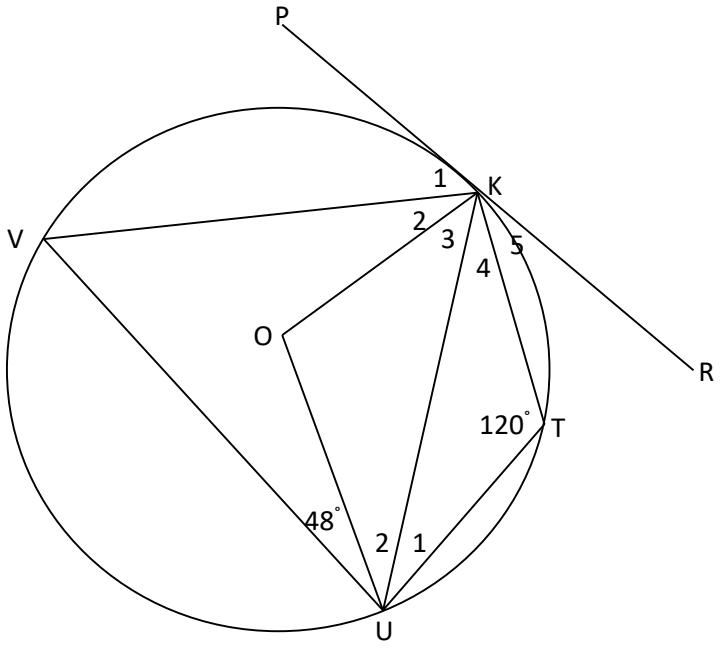
1.3.2 Calculate $P\hat{T}R$ in terms of x. (2)

1.3.3 Prove that PTOR is a cyclic quadrilateral. (3)

[20]

QUESTION 2

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K.
$O\hat{U}V = 48^\circ$ and $K\hat{T}U = 120^\circ$

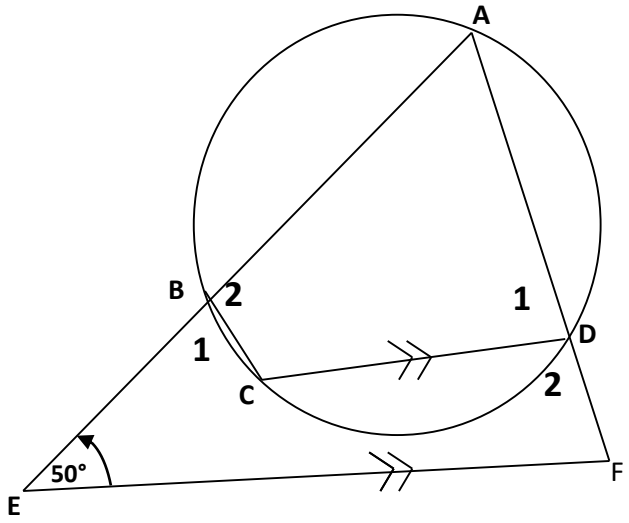


2.1 Use the above information to fill in the missing information for (a) to (e).

STATEMENT	REASON
$\hat{V} + 120° = 180°$ $\therefore \hat{V} = 60°$	(a)
$K\hat{O}U = 120°$	(b)
$\hat{U}_2 =$ (c)	$OU = OK$ radii
$\hat{K}_1 = 78°$	(d)
$\hat{K}_2 = 12°$	(e)

(5)

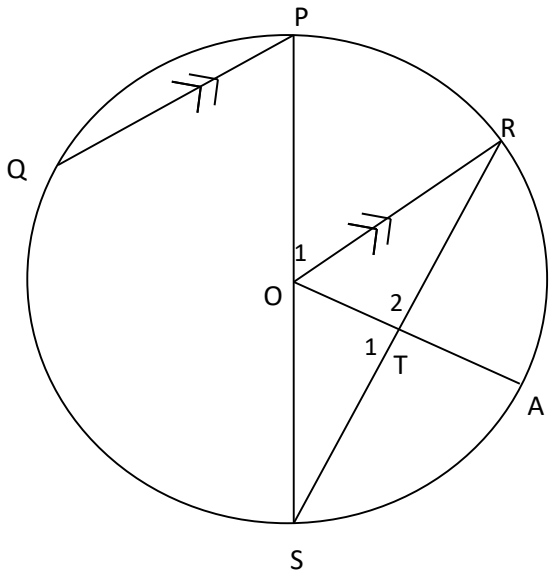
2.2 In the diagram below**,** ABCD is a cyclic quadrilateral with AD produced to F and AB produced to E. C D // EF. $\hat{E} = 50°$ and EA = AF. Calculate with reasons, the value of $\hat{B}C$. (6)



[11]

QUESTION 3

In the diagram, PS is a diameter of a circle with centre O. PQ and RS are two chords. OA is perpendicular to RS and QP//OR.

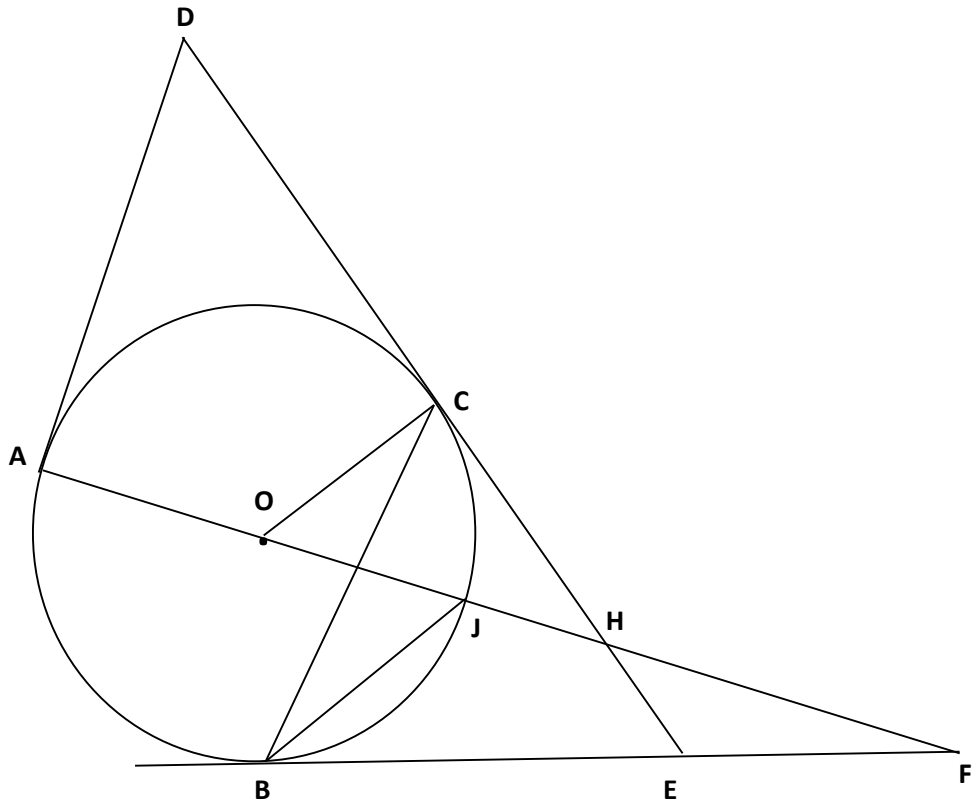


3.1 Prove that $\hat{P} = 2\hat{R}$ (5)

3.2 If it is given that RS = 8cm and TA $= 2cm$, determine the length of the radius(r). (5)

QUESTION 4

In the diagram below AD, DC and BE are tangents to the circle, centre O.
Diameter AJ and tangent BE are produced to meet at F.
Also tangent DC is produced to meet BF at E. H is the point of intersection of AF and DE.



4.1 Prove $\Delta DAH \,|||\, \Delta OCH$ (4)

4.2 Prove $OH = \dfrac{AO.DH}{DC}$

(6)

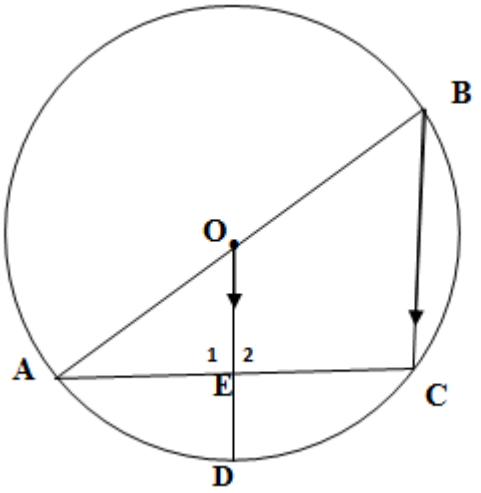
[10]

QUESTION 5

5.1 Complete the theorem that states: the line from the centre of the circle to the midpoint of the chord ..... (1)

5.2 Write down the converse of the theorem in 7.1. (2)

5.3 AB is a diameter of circle O. OD is drawn parallel to chord BC and intersects AC at E.



The radius is 10 cm and AC = 16 cm.

5.3.1 Prove that AE = EC. (2)

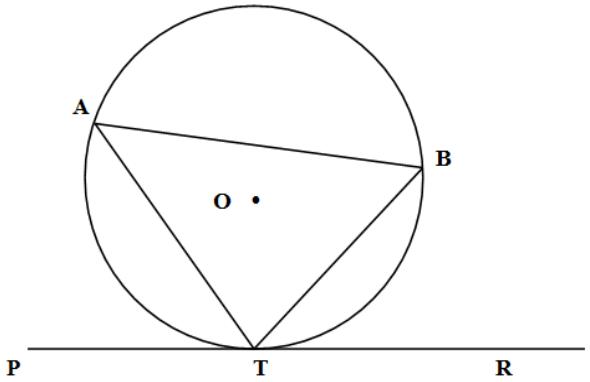
5.3.2 Prove $\hat{E}_1 = 90°$. (2)

5.3.3 Hence calculate the length of ED. (3)

[10]

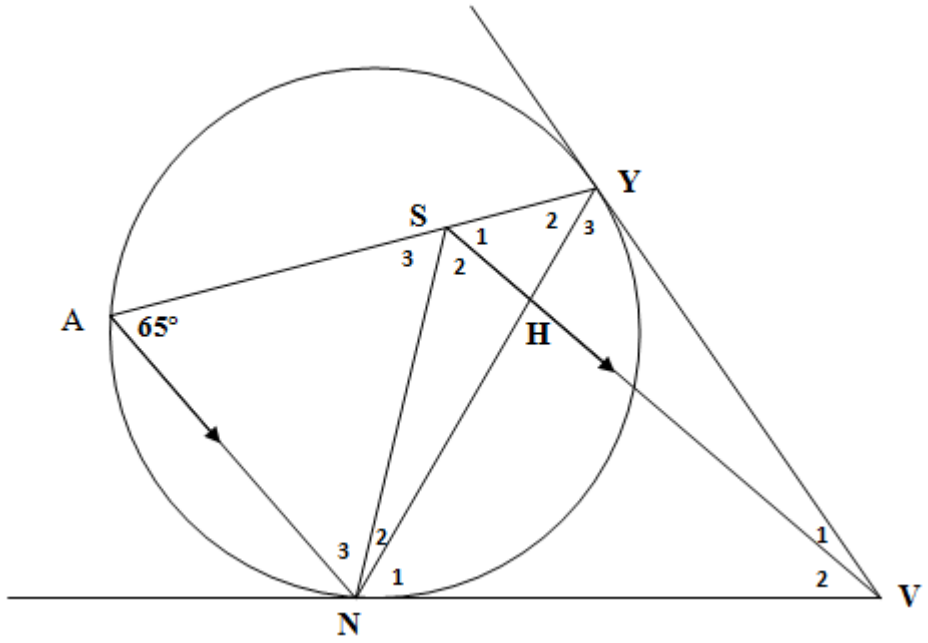
QUESTION 6

6.1 In the diagram the circle with centre O passes through points A, B and T.
PR is a tangent to the circle at T. AB, BT and AT are chords.



Prove that B$\hat{T}$R = $\hat{A}$. (6)

6.2 VN and VY are tangents to the circle at N and Y.
A is a point on the circle, and AN, AY and NY are chords so that $\hat{A}$ = 65°.
S is a point on AY so that AN || SV. S and N are joined.



6.2.1 Write down, with reasons, THREE other angles each equal to 65°. (3)

6.2.2 Prove that VYSN is a cyclic quadrilateral. (2)

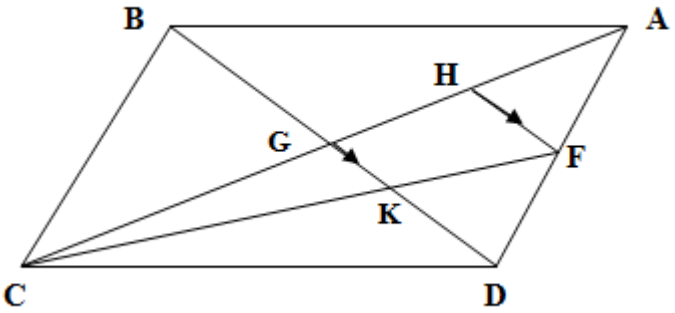
6.2.3 Prove that ΔASN is isosceles. (5)

[16]

QUESTION 7

ABCD is a parallelogram with diagonals BD and AC. $HF \parallel BD$.

$CG = 72$ units, $DF = 24$ units and $FA = 40$ units.



Determine with reasons:

7.1 the length of GH. (4)

7.2 the value of the $\frac{area\ \Delta\ AHF}{area\ \Delta\ ACD}$ (4)

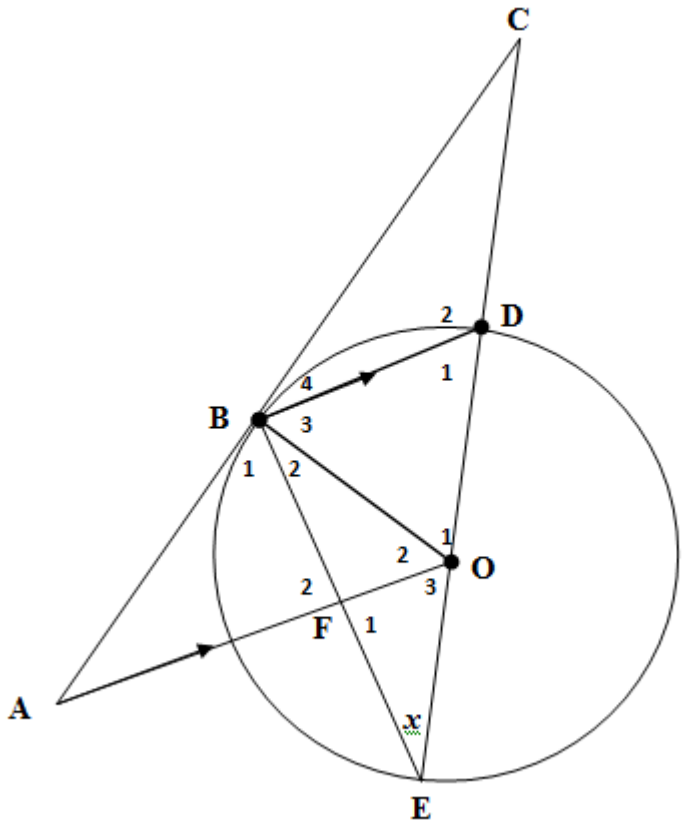
[8]

QUESTION 8

ED is a diameter of the circle with centre O. ED is extended to C.

CA is a tangent to the circle at B. AO intersects chord BE at F. BD || AO.

$\hat{E} = x$.



8.1 Give, with reasons, THREE other angles each equal to *x*. (3)

8.2 Express C$\hat{B}$E in terms of *x*. (2)

8.3 Prove that:

8.3.1 Δ CBD ||| Δ CEB (2)

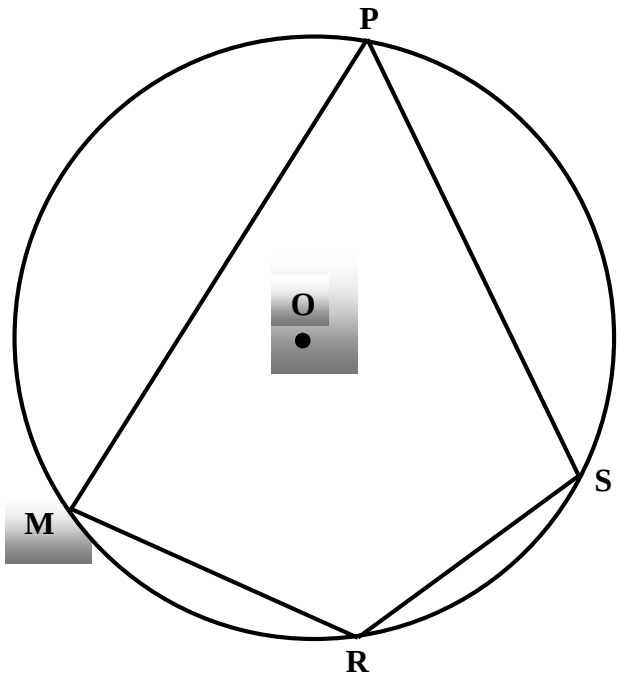
8.3.2 2EF. CB = CE.BD (5)

8.3.3 $\frac{2EF}{CE} = \frac{AO}{AC}$ (4)

[16]

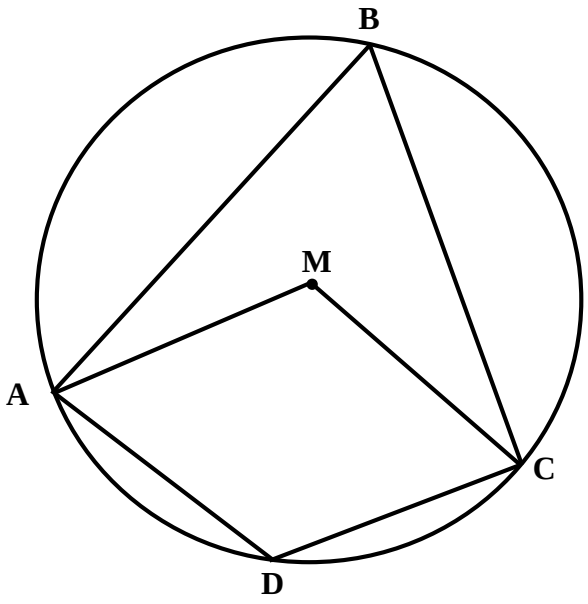
QUESTION 9

9.1 In the diagram below, O is the centre of the circle PMRS.



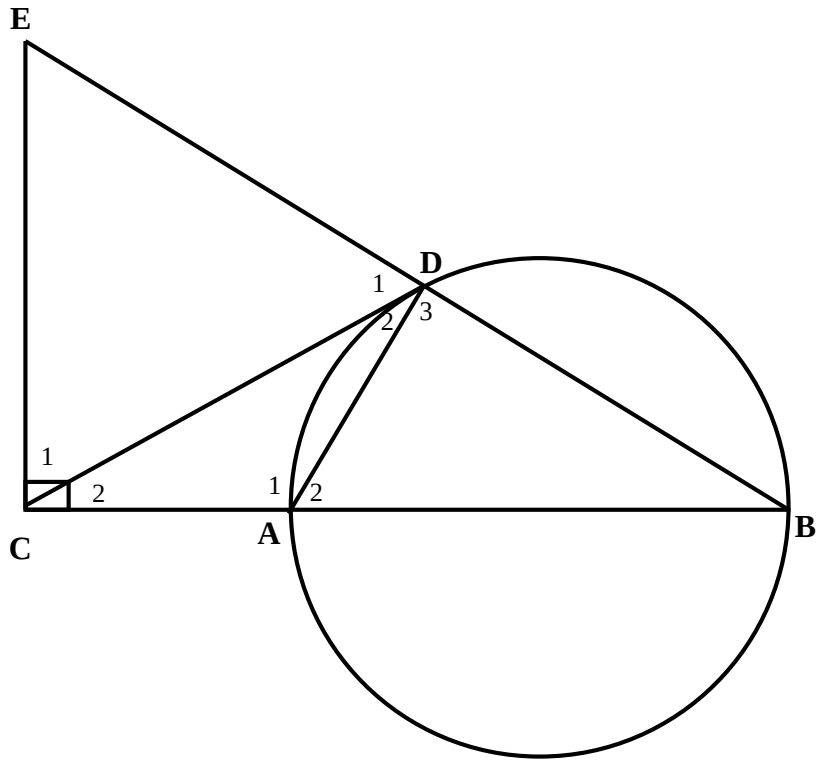
Use the diagram given to prove the theorem which states that: $\hat{P} + \hat{R} = 180°$. (6)

9.2 In the diagram below, ABCD is a cyclic quadrilateral of a circle with centre M.



If $\hat{B} : \hat{D} = 2:3$, calculate the size of $A\hat{M}C$. (6)

9.3 In the figure, AB is a diameter of the circle. CD is a tangent to the circle at D. EC $\perp$ BC. BAC and BDE are straight lines.



9.3.1 Prove that ACED is a cyclic quadrilateral. (5)

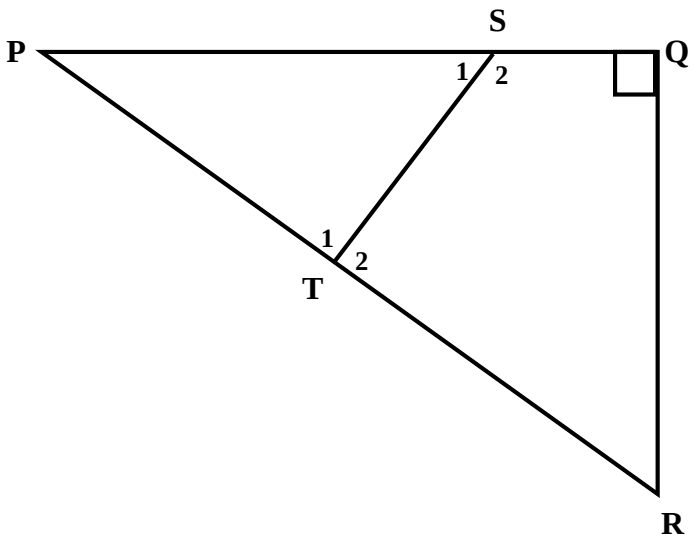
9.3.2 Prove that $\hat{A}_2 = \hat{D}_1$. (4)

9.3.3 Prove that ΔCDE is isosceles. (4)

[25]

QUESTION 10

In the accompanying diagram, ΔPQR is a right angled triangle with $\hat{Q} = 90°$.
ST is the perpendicular bisector of PR.



10.1 Prove that ΔPQR ||| ΔPTS. (3)

10.2 If PR = 16 cm and QR = 14 cm, calculate the value of:

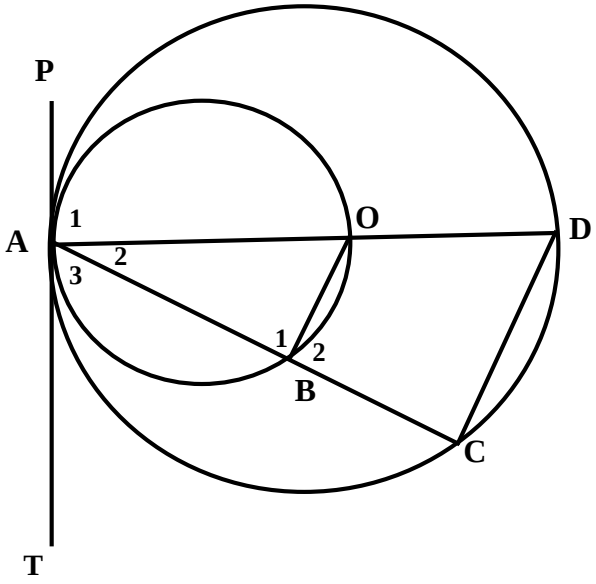
10.2.1 PS (5)

10.2.2 The perimeter of ΔPQR. (2)

[10]

QUESTION 11

In the diagram below, O is the centre of the bigger circle and the smaller circle passes through O. PAT is a common tangent to both circles at A. AC intersects the smaller circle at B. AO produced meets the bigger circle at D. DC and OB are drawn.



11.1 Prove that OB ‖ DC. (5)

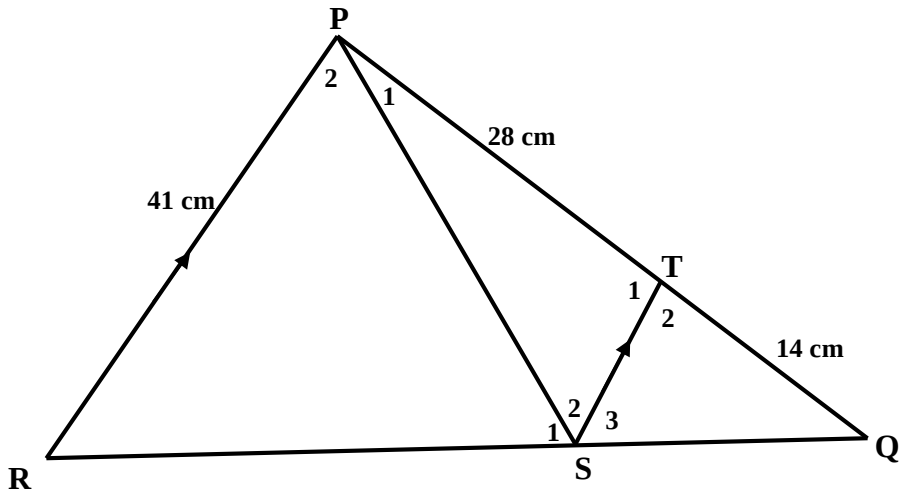
11.2 Write down the value of AB: AC. (2)

[7]

QUESTION 12

In the diagram below, PR = 41 cm, PT = 28 cm and TQ = 14 cm.

ST ∥ PR and $\hat{S}_1 = \hat{S}_2$.



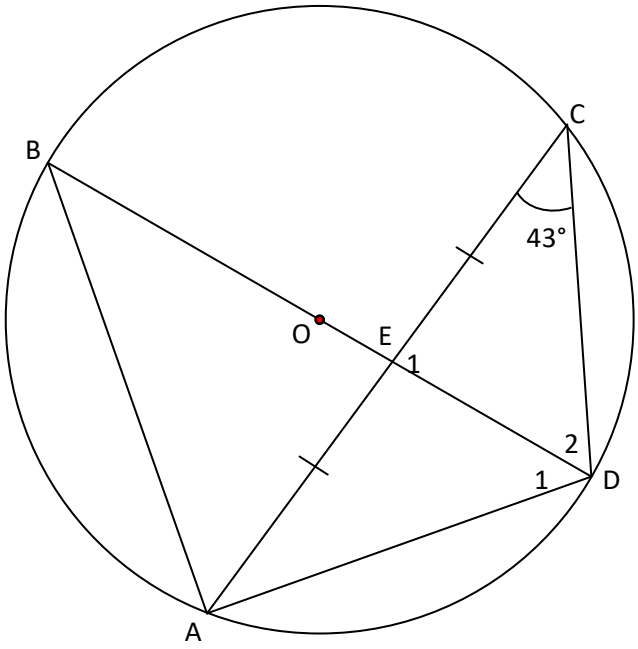
12.1 Determine the length of SQ. (5)

12.2 Write down the length of ST. (4)

[9]

QUESTION 13

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Diameter BD bisects chord AC at E. Chords AB, CD and AD are drawn. $\hat{C} = 43°$.



13.1 Give a reason for $DE \perp AC$. (1)

13.2 Calculate, giving reasons, the size of $\hat{B}$. (2)

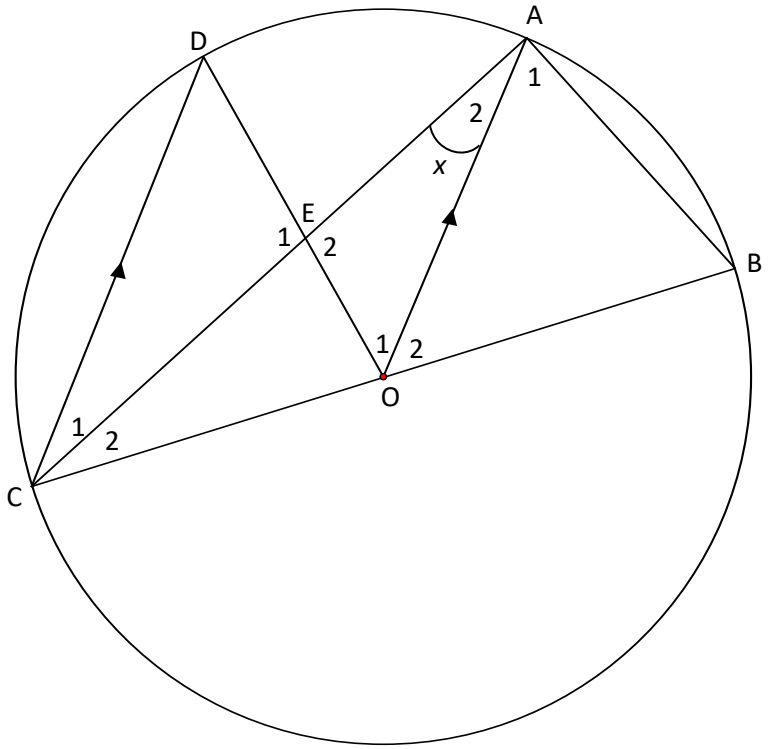
13.3 Prove that $\hat{E}_1 = B\hat{A}D$. (2)

13.4 The length of the diameter of the circle is 28 units. Calculate the length of AB. (2)

[7]

QUESTION 14

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersect radius OD at E. AB is drawn. CD | | OA and $\hat{A}_2 = x$.



14.1 Give reasons why

14.1.1 $\hat{C}_1 = x$ (1)

14.1.2 $\hat{C}_2 = x$ (1)

14.2 Determine, giving reasons, the size of the following angles in terms of *x*.

14.2.1 $\hat{A}_1$ (3)

14.2.2 $\hat{O}_1$ (2)

14.2.3 $\hat{O}_2$ (2)

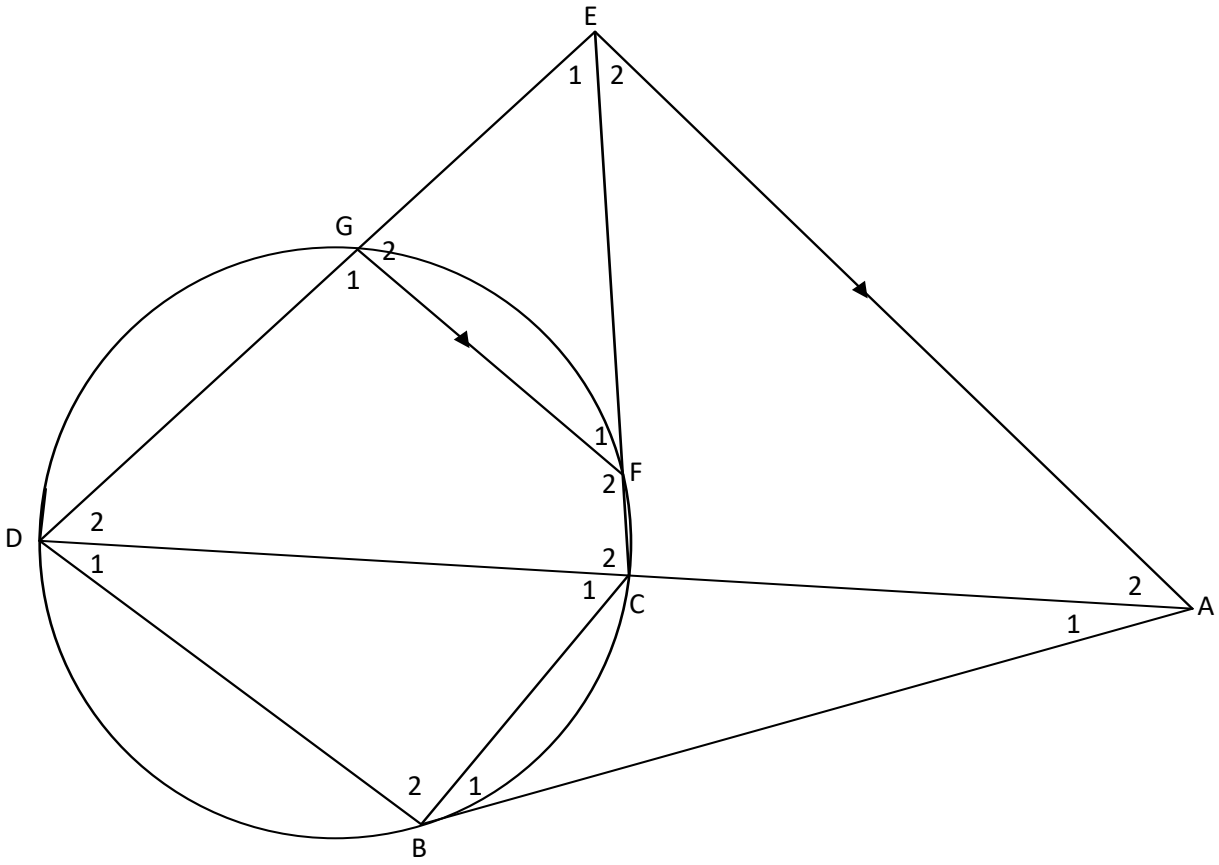
14.3 For which value of *x* will ABOE be a cyclic quadrilateral? (3)

[12]

QUESTION 15

15.1 Complete the following statement of the theorem in the ANSWER BOOK:
If two triangles are equiangular, then the corresponding sides are … (1)

15.2 In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG and CF produced meet at E and DC is produced to A. EA | | GF.



15.2.1 Give a reason why $\hat{B}_1 = \hat{D}_1$. (1)

15.2.2 Prove $\Delta ABC|||\Delta ADB$. (3)

15.2.3 Prove $\hat{E}_2 = \hat{D}_2$. (4)

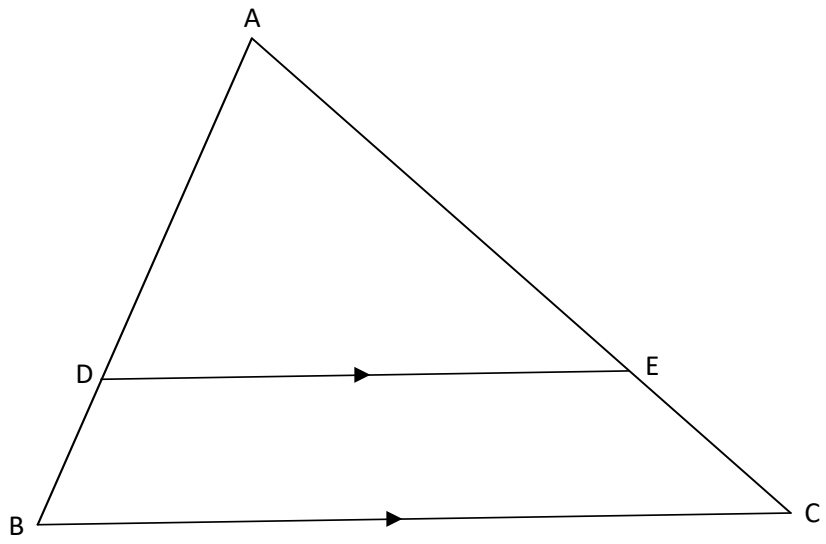
15.2.4 Prove $AE^2 = AD \times AC$. (4)

15.2.5 Hence, deduce that AE = AB. (3)

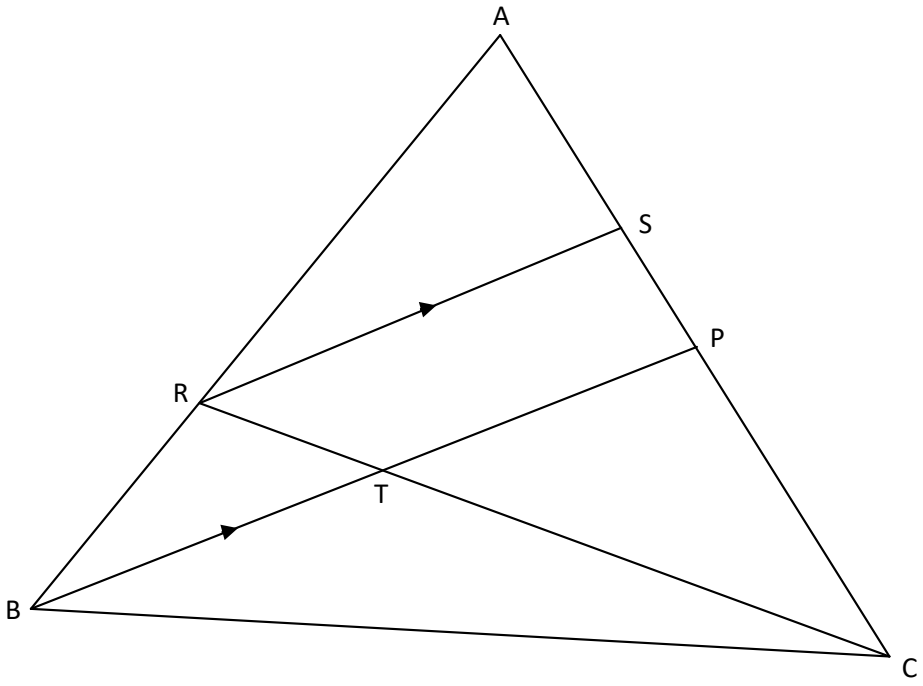
[16]

QUESTION 16

16.1 In ΔABC below, D and E are points on AB and AC respectively such that DE | | BC. Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



16.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS | | BP and $\frac{AR}{AB} = \frac{3}{5}$. RC cuts BP in T.



Determine, giving reasons, the following ratios:

16.2.1 $\frac{AS}{SC}$ (4)

16.2.2 $\frac{RT}{TC}$ (3)

16.2.3 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ (4)

[17]

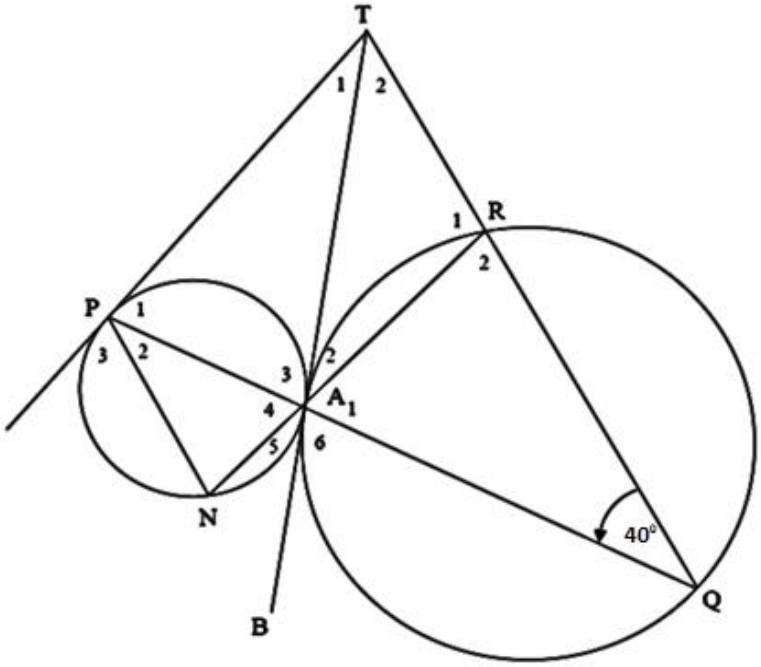
QUESTION 17

17.1 Complete the following statements so that they are true:

17.1.1 The angle between the tangent and chord is … (1)

17.1.2 Opposite angles of a cyclic quadrilateral are … (1)

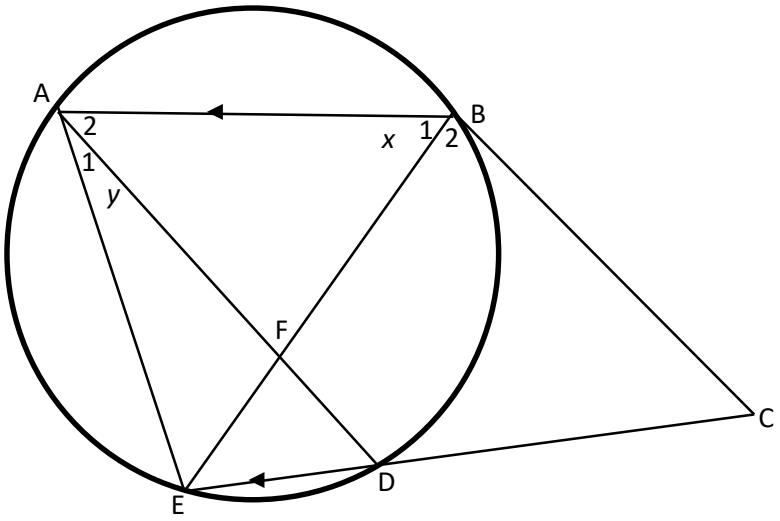
17.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines. Let $\widehat{Q} = 40°$.



17.2.1 Determine, with reasons, THREE other angles equal to 40°. (6)

17.2.2 If $\widehat{P}_1 = \widehat{A}_4$ prove that PTRN is a parallelogram. (5)

17.3 Tangent BC touches the circle ABDE at B. Chords AD and BE intersect at F. Chord ED is produced to C. AB || EC. It is further given that $\widehat{B}_1 = x$ and $\widehat{A}_1 = y$.



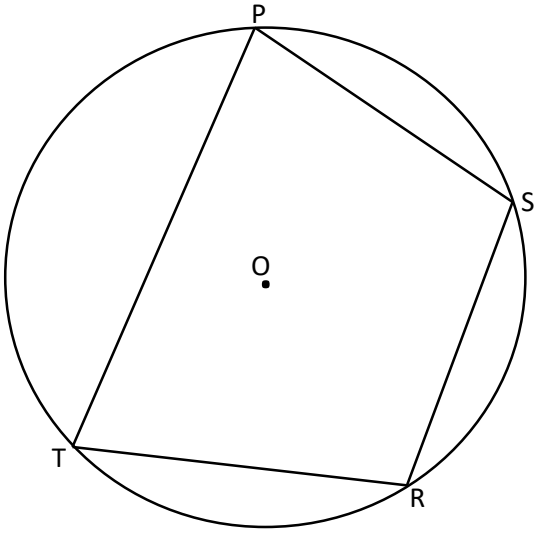
17.3.1 Determine the size of $\widehat{C}$ in terms of *x* and *y*. (6)

17.3.2 Becky says that BCDF is not a cyclic quadrilateral while Teboho insists that it is. Who is correct? Show all your working in determining your answer. (5)

[24]

QUESTION 18

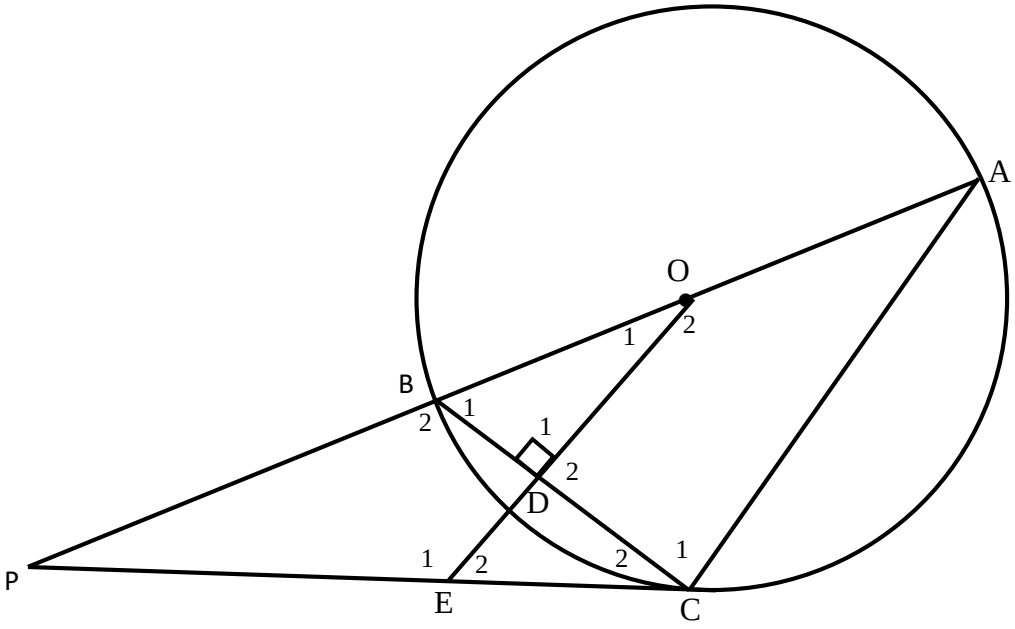
18.1 In the diagram below, O is the centre of the circle. PSRT is a cyclic quadrilateral. Prove the theorem that states: $P\hat{T}R + P\hat{S}R = 180°$.



(6)

18.2 In the diagram, AB is a diameter of circle, centre O. AB is produced to P.

PC is a tangent to the circle at C. OE intersects BC at D such that OE ⊥ BC.



18.2.1 Prove, with reasons, that EO ∥ CA. (4)

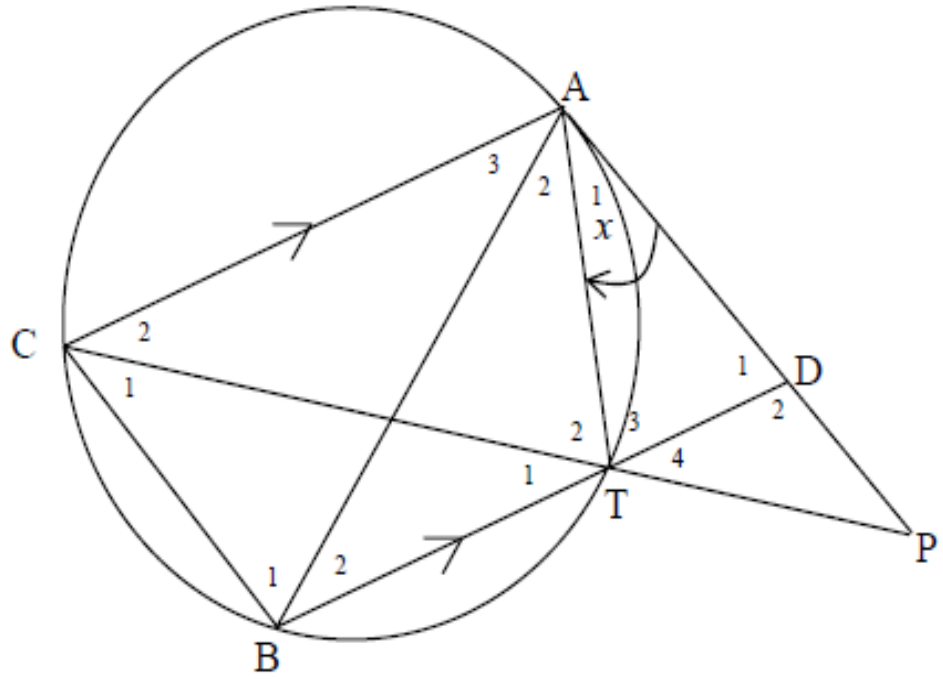
18.2.2 If $\hat{C}_2 = x$, name with reasons TWO other angles equal to *x*. (3)

18.2.3 Determine the size of $\hat{P}$ in terms of *x*. (3)

[16

QUESTION 19

In the diagram below DA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are drawn. AC is parallel to BD. Let $\hat{A}_1 = x$.



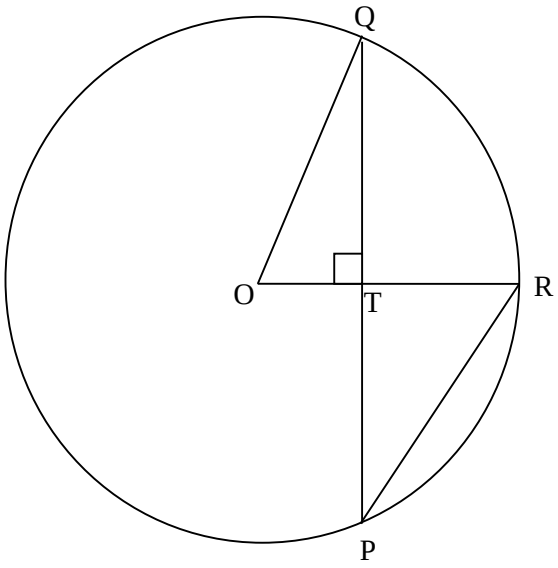
19.1 Prove that PT is a tangent to the circle ADT at T. (5)

19.2 Prove that $\Delta APT \,|||\, \Delta TPD$ (3)

19.3 If $AD = \frac{2}{3}AP$ show that $AP^2 = 3PT^2$ (4)

QUESTION 20

In the diagram below, PQ is the chord of circle O. OR is perpendicular to PQ and OR intersects PQ at T. If the radius of the circle is 13 cm and PT = 12 cm.



Calculate the length of:

20.1 PQ (2)

20.2 PR (4)

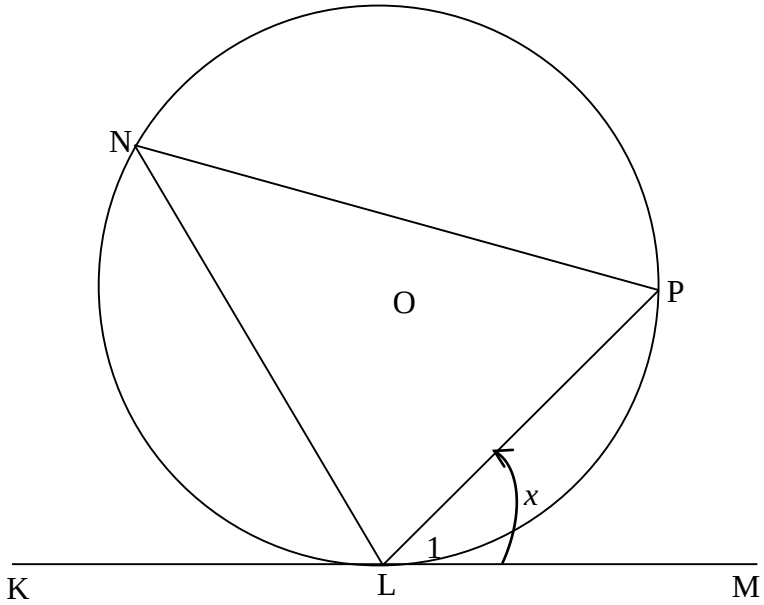
[6]

QUESTION 21

21.1 Complete the statement so that it is true:

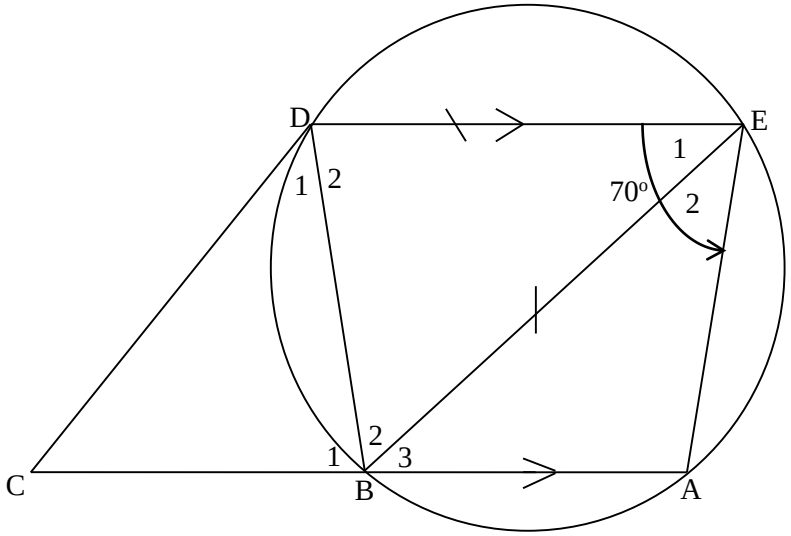
The exterior angle of a cyclic quadrilateral is equal to …
(1)

21.2 In the diagram below the circle with centre O passes through L, N and P. KLM is a tangent to the circle at L. NP, NL and LP are joined.



Using the above diagram, prove the theorem that states that $P\hat{L}M = \hat{N}$. (5)

21.3 In the diagram below, BAED is a cyclic quadrilateral with BA || DE. BE = DE and $A\hat{E}D = 70^{o}$. The tangent to the circle at D meets AB produced at C.



Calculate, with reasons the sizes of the following angles.

21.3.1 $\hat{A}$ (2)

21.3.2 $\hat{B}_1$ (2)

21.3.3 $\hat{D}_2$ (2)

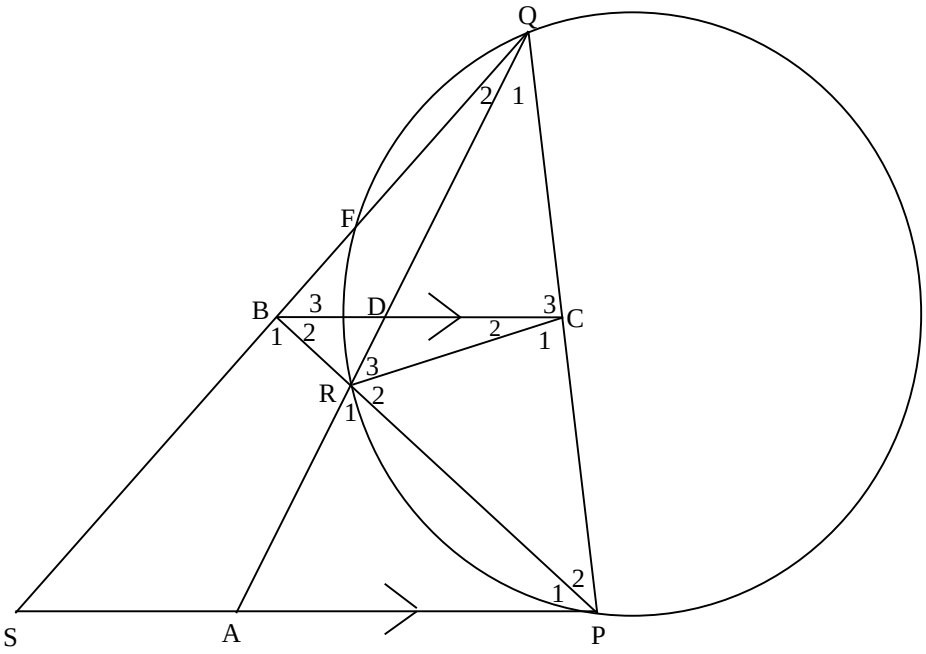
21.3.4 $\hat{B}_2$ (2)

21.3.5 $\hat{D}_1$ (3)

[17]

QUESTION 22

In the diagram below, SP is a tangent to the circle at P and PQ is a chord. Chord QF produced
meets SP at S and chord RP bisects $\hat{QPS}$. PR produced meets QS at B. BC || SP and cuts the chord QR at D. QR produced meets SP at A. Let $\hat{B}_2 = x$.



22.1 Name, with reasons, 3 angles equal to *x*. (4)

22.2 Prove that PC = BC (2)

22.3 Prove that RCQB is a cyclic quadrilateral. (2)

22.4 Prove that Δ PBS ||| Δ QCR. (5)

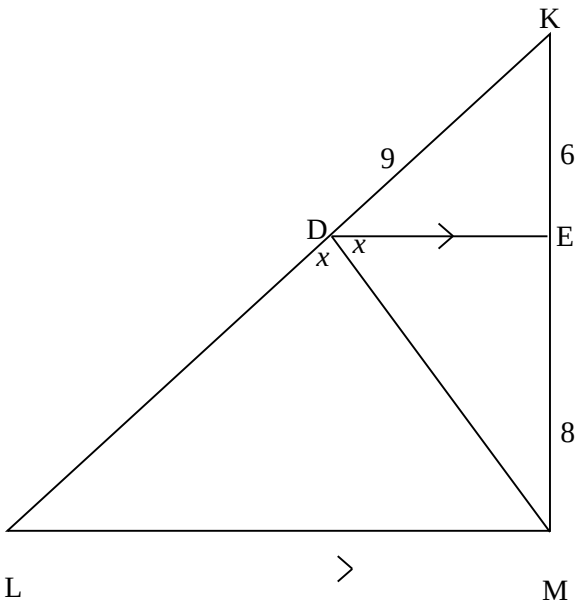
22.5 Show that PB.CR = QB. CP (4)

[17]

QUESTION 23

In the diagram alongside, Δ KLM,
DE || LM, $L\hat{D}M = M\hat{D}E = x$.
KD = 9, EM = 8 and EK = 6

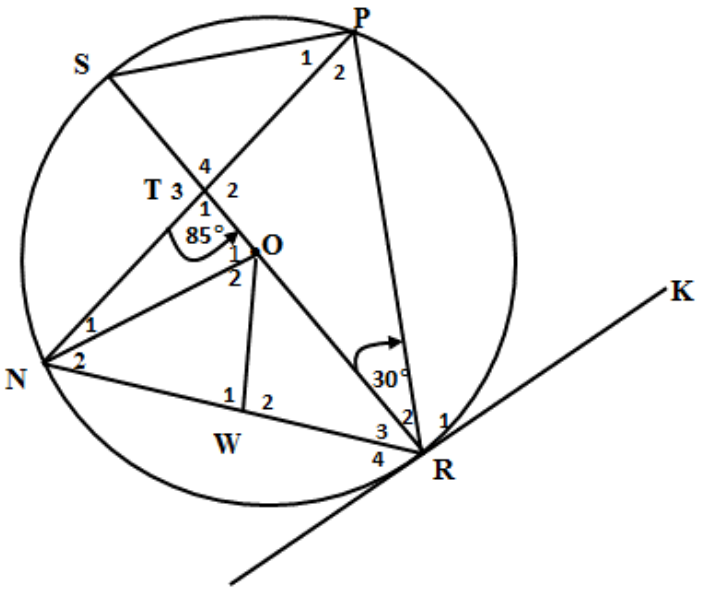
Calculate, with reasons LM.



(5)

QUESTION 24

The vertices of ΔPNR lie on the circumference of the circle with centre O. Diameter SR and chord NP intersect at T. $\hat{R}_2 = 30°$ and $\hat{T}_1 = 85°$. RK is a tangent to the circle at R.



24.1 Determine, stating reasons, the size of:

24.1.1 $\hat{S}$ (3)

24.1.2 $\hat{R}_3$ (4)

24.1.3 $\hat{N}_1$ (4)

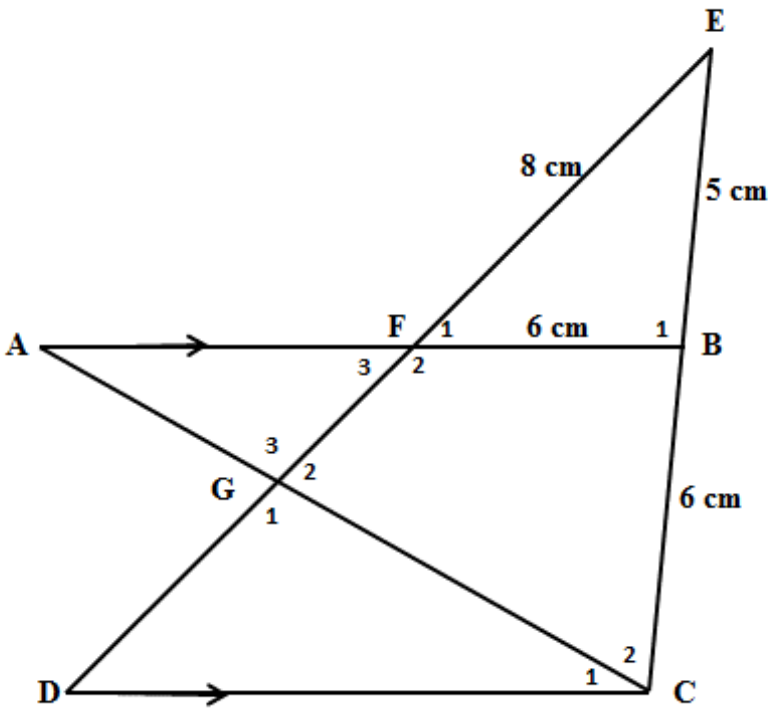
24.1.4 $\hat{R}_4$ (3)

24.2 Determine whether NT is a tangent to the circle passing through N, O and R. Justify your answer. (2)

[16]

QUESTION 25

In this diagram ΔABC and ΔCDE are drawn such that F is on AB and G is on AC. DGFE and CBE are straight lines. AFB ∥ DC. AB = 20 cm, BC = 6 cm, EF = 8 cm, EB = 5 cm and FB = 6 cm.



25.1 Determine, with reasons, the numerical value of $\frac{EF}{ED}$. (2)

25.2 Hence calculate the length of ED. (2)

25.3 Determine, with reasons, the length of DC. (6)

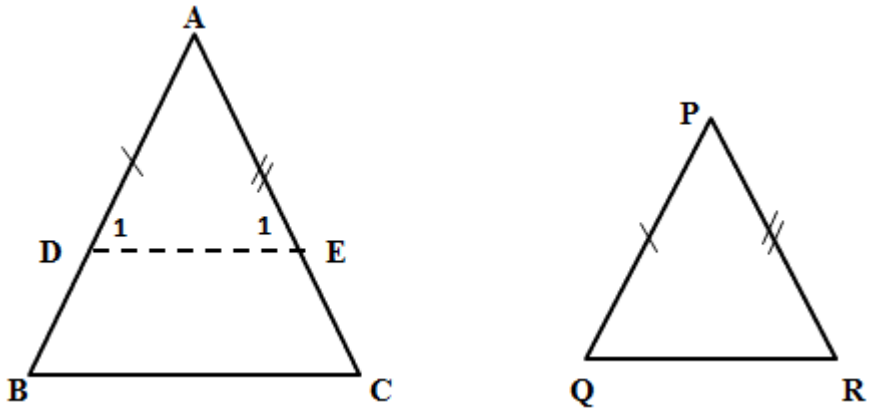
25.4 Determine, with reasons, $\frac{AG}{GC}$. (6)

25.5 Hence determine the length of GC, if AC = 18 cm. (2)

[18]

QUESTION 26

26.1 ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



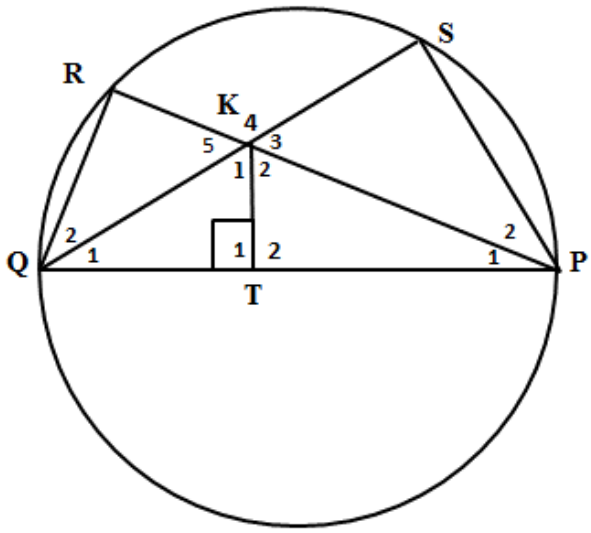
DE is drawn such that AD = PQ and AE = PR.

26.1.1 Prove that: ΔADE ≡ ΔPQR (2)

26.1.2 Prove that DE || BC (3)

26.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$ (2)

26.2 PQ is the diameter in semi- circle PSRQ. KT ⊥ PQ.



Prove, with reasons: 26.2.1 ΔQSP ||| ΔQTK (5)

26.2.2 $PS^2 = \frac{SP^2.QK^2}{TK^2} - SQ^2$ (4)

[16]

QUESTION 27

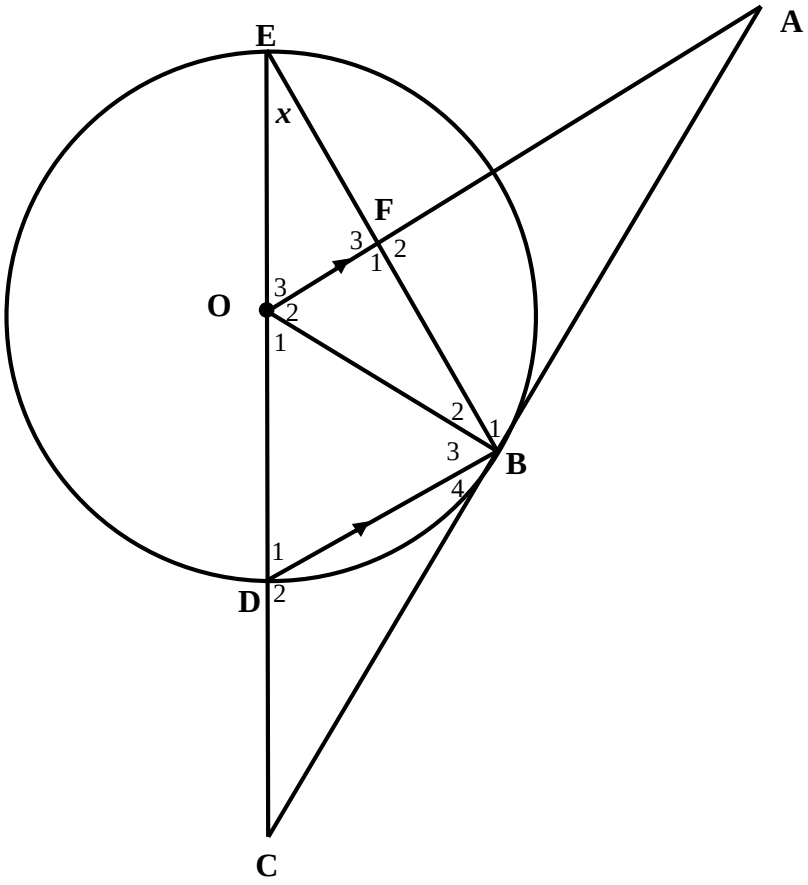
27.1 Complete the following statement:

27.1.1 The angle subtended at the circle by a diameter is ..... (1)

27.1.2 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to ..... (1)

27.2 In the diagram below, ED is a diameter of the circle with centre O.
ED is produced to C and CA is a tangent to the circle at B. AO intersects BE at F. BD ∥ AO.

Let $\hat{E} = x$



27.2.1 Write down, with reasons, THREE other angles each equal to x. (6)

27.2.2 Determine, with reasons, $C\hat{B}E$ in terms of x. (3)

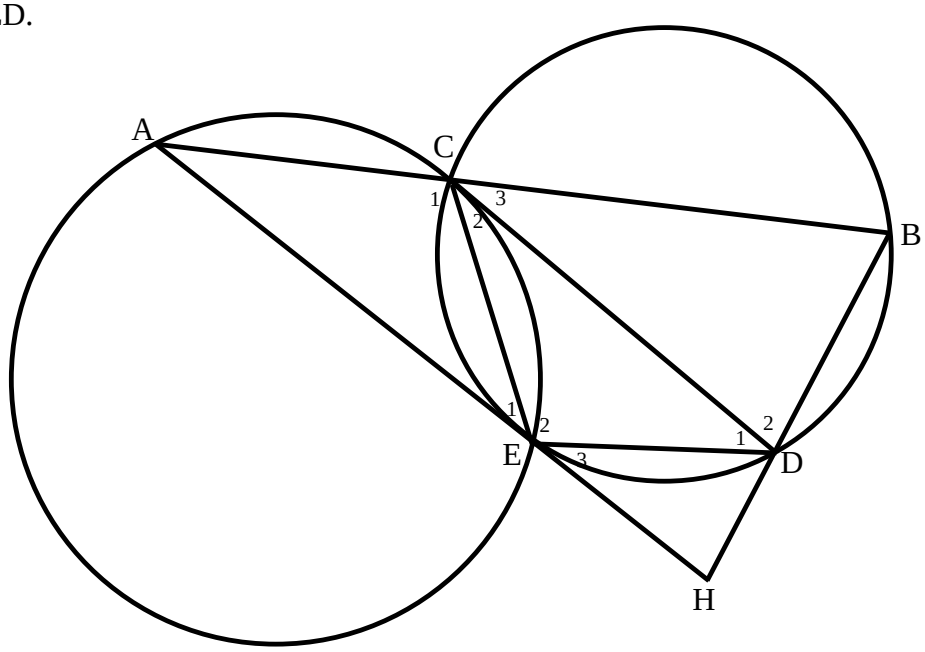
27.2.3 Prove that F is the midpoint of BE. (4)

27.2.4 Calculate the length of the diameter if it is further given that EB = 8 cm and OF = 3 cm. (4)

[19]

QUESTION 28

In the diagram below, CE is a common chord of the circles ACE and CBDE. CD is a tangent to circle ACE at C and AEH is a tangent to circle CBDE at E. ACB and BDH are straight lines. AC = ED.



28.1 Prove that AB ∥ ED. (5)

28.2 What type of quadrilateral is ACDE? Give reasons. (2)

28.3 Prove that $\frac{AC}{CB} = \frac{HE}{EA}$. (4)

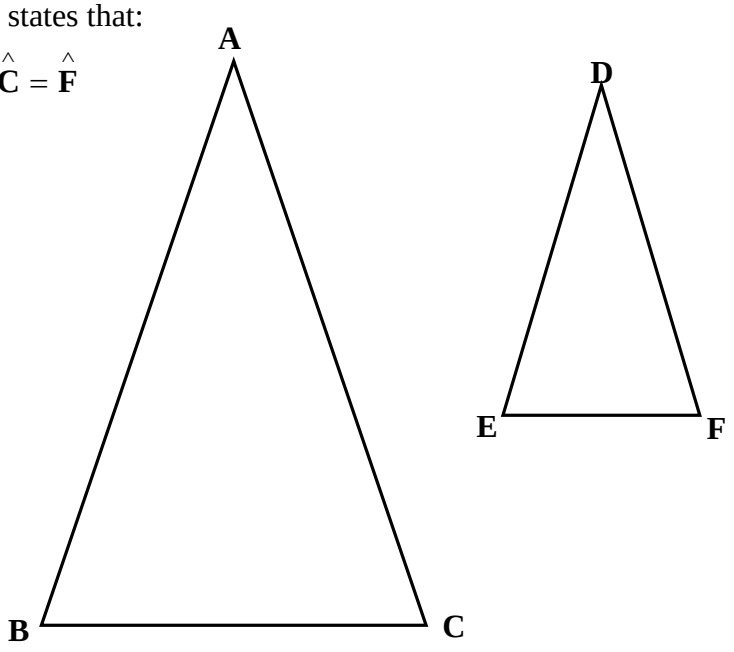
[11]

QUESTION 29

29.1 In the diagram alongside, ΔABC and ΔDEF are given.
Prove the theorem which states that:

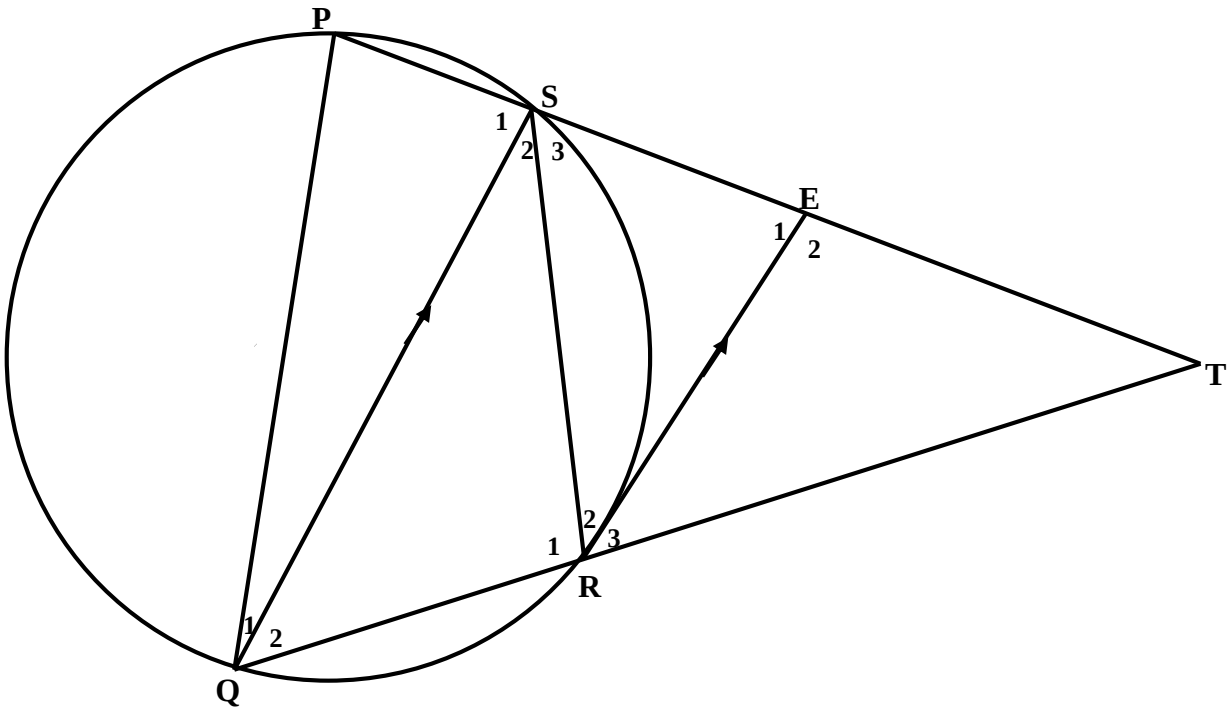
If $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ **and** $\hat{C} = \hat{F}$

then $\frac{AB}{DE} = \frac{AC}{DF}$



(7)

29.2 In the diagram below, PQRS is a cyclic quadrilateral. PS and QR are produced to meet at T. RE is a tangent to the circle at R, with E on PT and RE ∥ QS



Prove that:

29.2.1 QR = RS (4)

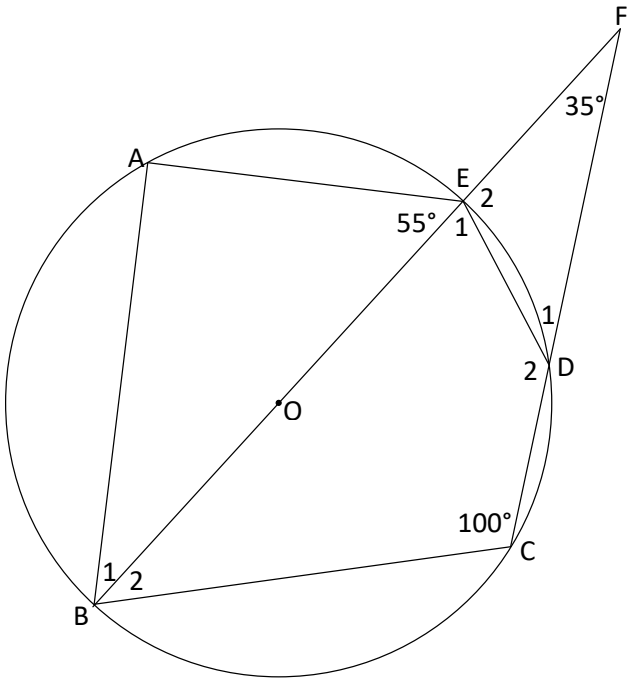
29.2.2 ΔRST ||| Δ PQT (4)

29.2.3 $\frac{PQ}{PT} = \frac{SE}{ET}$ (5)

[20]

QUESTION 30

In the diagram, O is the centre of the circle. A, B, C, D and E are points on the circumference of the circle. Chords BE and CD produced meet at F. $\hat{C} = 100°$, $\hat{F} = 35°$ and $A\hat{E}B = 55°$



30.1 Calculate, giving reasons, the size of each of the following angles:

30.1.1 $\hat{A}$ (2)

30.1.2 $\hat{E}_1$ (3)

30.1.3 $\hat{D}_1$ (2)

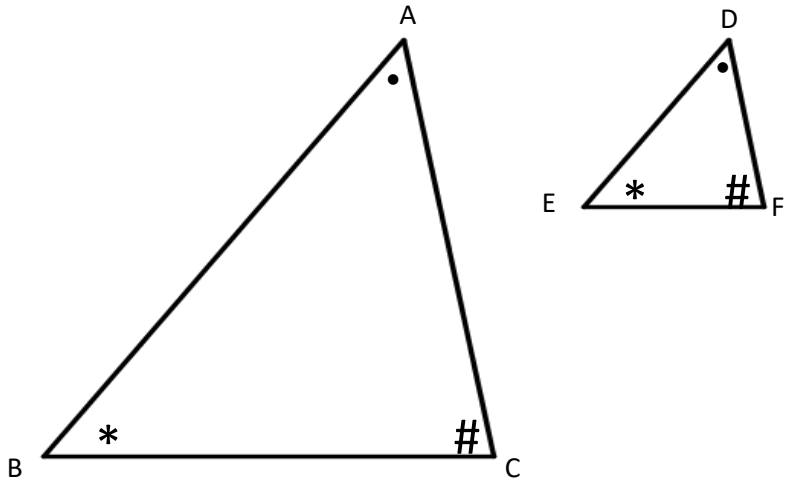
30.2 Prove, giving reasons, that AB || CF. (3)

[10]

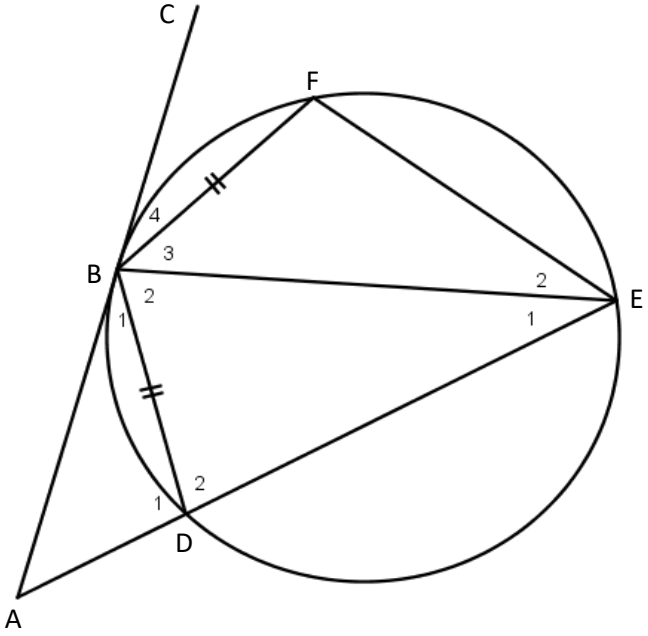
QUESTION 31

31.1 In the diagram below, ΔABC and ΔDEF are given with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Use the diagram to prove the theorem that states that $\frac{DE}{AB} = \frac{DF}{AC}$.

(7)



31.2 In the diagram, ABC is a tangent to the circle at B. BDEF is a cyclic quadrilateral with DB = BF. BE is drawn and ED produced meets the tangent at A.



Prove that:

31.2.1 $\hat{B}_1 = \hat{E}_2$ (3)

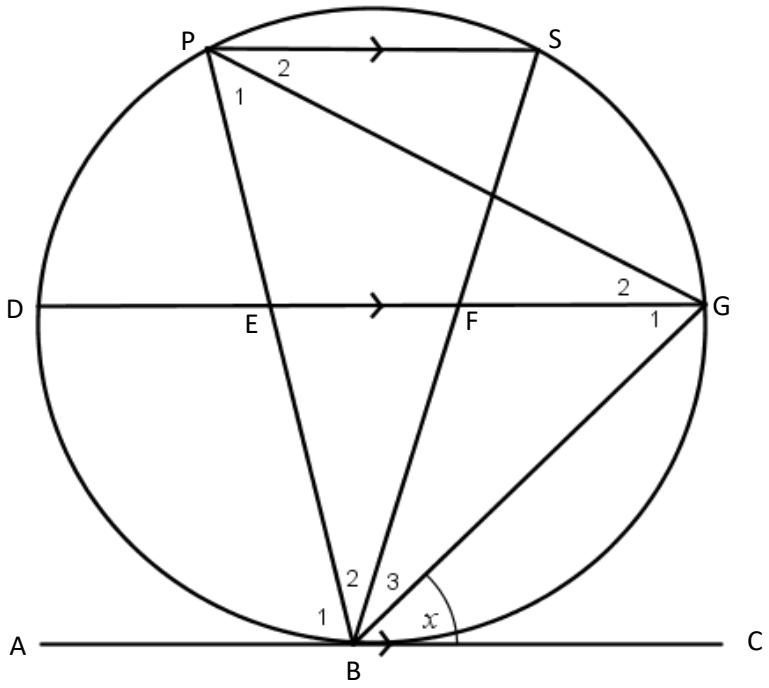
31.2.2 $\Delta BDA \,|||\, \Delta EFB$ (4)

31.2.3 $BD^2 = AD.EF$ (2)

[16]

QUESTION 32

In the diagram, P, S, G, B and D are points on the circumference of the circle such that PS || DG || AC. ABC is a tangent to the circle at B. $G\widehat{B}C = x$.



32.1 Give a reason why $\hat{G}_1 = x$. (1) (1)

32.2 Prove that:

32.2.1 $BE = \frac{BP.BF}{BS}$ (2) (2)

32.2.2 $\Delta BGP \,|||\, \Delta BEG$(4) (4)

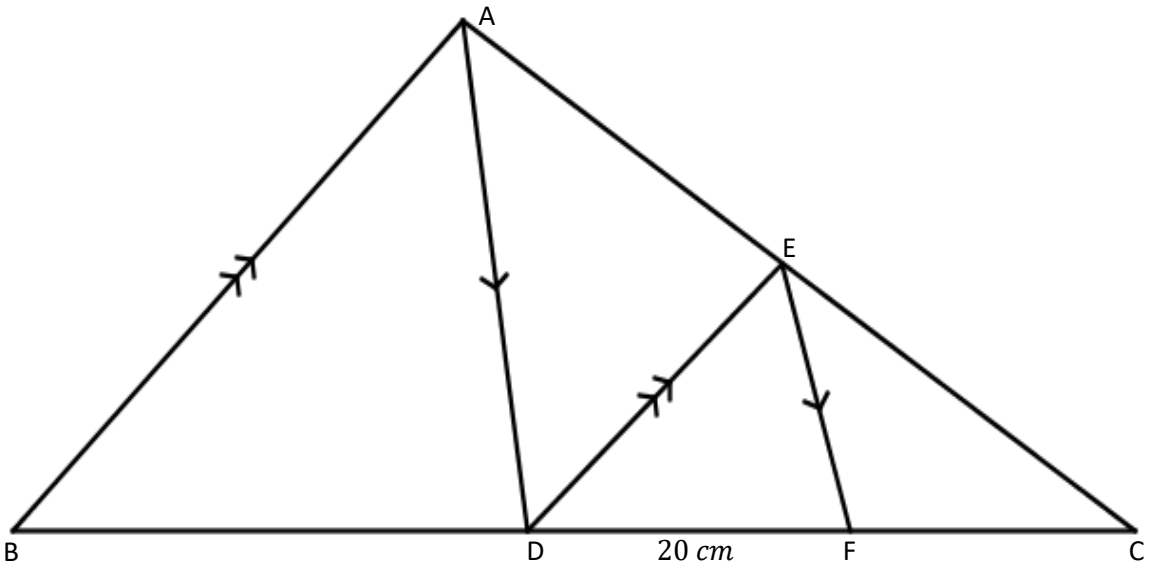
32.2.3 $\frac{BG^2}{BP^2} = \frac{BF}{BS}$ (3) (3)

[10]

[10]

QUESTION 33

In the diagram, ΔABC with points D and F on BC and E a point on AC such that EF || AD and DE || BA. Further it is given that $\frac{AE}{EC} = \frac{5}{4}$ and DF = 20 cm.



33.1 Calculate, giving reasons, the length of

33.1.1 FC (3)

33.1.2 BD (4)

33.2 Determine the following ratio:

$$\frac{\text{Area } \Delta \text{ ECF}}{\text{Area } \Delta \text{ ABC}}$$ (4)

[11]