MATHEMATICS

MATERIAL FOR GRADE 12

FUNCTIONS

M E M O R A N D A

QUESTION 1

1.1.1	$x = -2$ ✓ & $y = -3$✓	✓✓Answers (2)
1.1.2.1	$(2)\left(-\frac{5}{2}+3\right) = t$✓ $2\left(\frac{1}{2}\right) = t$ $t = 1$ ✓	✓Substituting $\left(0;\frac{-5}{2}\right)$ ✓$t = 1$ (2)
1.1.2.2	$p = 2$✓, $(2)^2 + q = 3$✓ $q = -1$ ✓	✓$p = 2$ ✓Substitution ✓$q = -1$ (3)
1.2	$(x+2)(y+3) = 1$✓ $y + 3 = \frac{1}{x+2}$✓ $y = \frac{1}{x+2} - 3$✓	✓Substituting 1 ✓Simplification ✓*Answer* (3)
1.3	$x - 1 = \frac{1}{x+2} - 3$ $x + 2 = \frac{1}{x+2}$ $(x+2)^2 = 1$ $x + 2 = \pm 1$ $x = -1$ *or* $x = -3$ $y = -2$ or $y = -4$ $(-1;-2)(-3;-4)$	✓Equating ✓Simplification ✓factors ✓x-values ✓✓ y-values (6)
1.4.1	$k < -1$	✓✓Answer (2)
1.4.2	$-1 < k < 3$	✓-1 & 3 ✓Notation (2)
1.5.1	$D: x \in \boldsymbol{R}$✓, $x \neq -2$ ✓ or $\boldsymbol{x} \in (-\infty;\infty)$, $x \neq -2$ or $x < -2$ *or* $x > 2$	✓ $x \in \boldsymbol{R}$ ✓ $x \neq -2$ (2)
1.5.2	$R: y \geq -1$✓ , $y \in \boldsymbol{R}$ or/*of* $\boldsymbol{y} \in [-1;\infty)$	✓Answer (1)
		[22]

QUESTION 2

2.1.1	$v(x)$ is not a function, ✓ there are two different $y-$values for each x. A vertical line test fails: Line cuts the graph at more than one point. ✓	✓Answer ✓Reason	(2)
2.1.2	$\boldsymbol{y} \in \{(-\infty;0]\mathbf{U}[0;\infty)\}$✓✓ **or** $y \geq 0$ or $y \leq 0$	✓✓ Ans	(2)
2.1.3.1	$y < 0$✓	✓Answer	(1)
2.1.3.2	$0 < x \leq \frac{49}{100}$✓	✓Notation ✓boundaries	(2)
2.1.4	$y = x^2$✓ $x\epsilon[0;\infty)$ ✓ $y = x^2$ $x\epsilon(-\infty;0]$ ✓	✓ $y = x^2$✓$x\epsilon[0;\infty)$ ✓ $y = x^2$ $x\epsilon(-\infty;0]$	(3)
2.1.5	$y \in (0;1)$ or $0 < y < 1$ $(-1;0)$ or $-1 < y < 0$	✓Answer	(1)
			[11]

QUESTION 3

3.1		At C ; $x = -\frac{b}{2a}$ $= -\frac{-2}{2(-1)} = -1$ $y = -(-1)^2 - 2(-1) + 3 = 4$ coordinated of C are $(-1;\ 4)$	✓ ✓ ✓	$x = -\frac{-2}{2(-1)}$ x value y value	(3)
3.2		$y = 0$ $-x^2 - 2x + 3 = 0$ $x^2 + 2x - 3 = 0$ $(x-1)(x-3) = 0$ $x = 1$ or $x = -3$ Coordinates are B(1; 0) and A$(-3\ ;0)$	✓ ✓ ✓	$y = 0$ $(x-1)(x-3) = 0$ $x-$ values	(3)
3.3		(0; 3)	✓	answer	(1)
3.4		equation of g $y - 0 = \frac{4-0}{-1-(-3)}\ (x-(-3))$ $y = 2(x+3)$ $y = 2x + 6$	✓ ✓ ✓	substitution value of m equation of g	

		E is (0; 6) C$(-1;\ 4)$ CE= $\sqrt{(0-(-1)^2+(6-4)^2}$ $=\sqrt{5}$	✓ ✓ ✓	E is (0; 6) substitution in the distance formula. answer	(6)
3.5		$x=2y+6$ $y=\frac{x}{2}-3=\frac{1}{2}x-3$ accept also $y=mx+c$, so inverse is $x=my+c$ $y=\frac{x}{m}-c$	✓ ✓	interchange x and y answer	(2)
					[15]

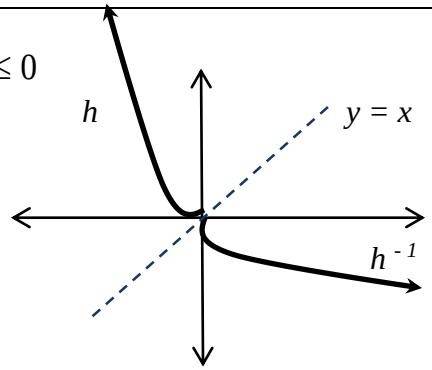
QUESTION 4

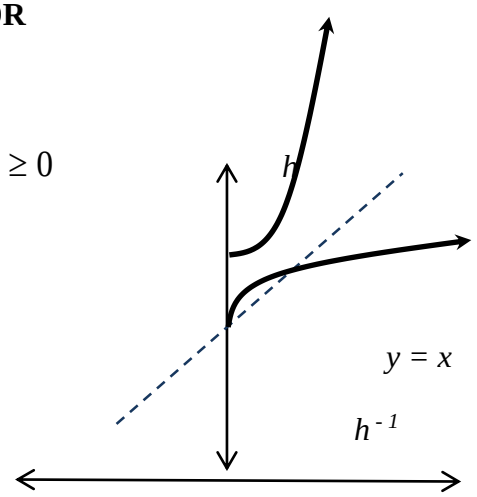
4.1		$e\ =\ 2$ $q\ =\ 2$	✓ ✓	$e\ =\ 2$ $q\ =\ 2$	(2)
4.2		$g(x)=\ 2^x+2$ $y-$int.; let $x\ =\ 0$ $g(0)=\ 2^0+2\ \ =3$ so both graphs pass at (0; 3) $3\ =\frac{3}{0-p}+2$ $\frac{3}{0-\ p}=1$ $-p=3\ \ \ \ \therefore p\ =\ -3$	✓ ✓ ✓	(0; 3) or $y-$ int. subst. (0;3) $p=\ -3$	(3)
4.3		at A: $x\ =\ -3$ $y\ =\ 2^{-3}+2$ $=2\ \frac{1}{8}$ A$(-3;\ 2\ \frac{1}{8})$ or A$(-3;\ 2,125)$	✓ ✓ ✓	at A : $x\ =\ -3$ subst. answer	(3)

4.4		$-3 < x \leq 0$	✓ ✓	$x > -3$ $x \leq 0$	(2)
4.5		$f(x) = \frac{3}{x+3} + 2$ after shifting $f(x-2) = \frac{3}{x+3-2} + 2$ $f(x-2) = \frac{3}{x+1} + 2$	✓ ✓	Subst. x by $x-2$ answer	(2)
					[12]

QUESTION 5

5.1	$x = -3$ $y = 2$	✓ $x = -3$ ✓ $y = 2$ (2)
5.2	$y = \frac{a}{x+p} + q$ $= \frac{a}{x+3} + 2$ $0 = \frac{a}{-1+3} + 2$ $-2 = \frac{a}{2}$ $-4 = a$ $\therefore f(x) = \frac{-4}{x+3} + 2$	✓ substitution of p and q ✓ substitution of $(-1; 0)$ ✓ $a = -4$ (3)
5.3	$y = -(x+3) + 2$ $= -x - 3 + 2$ $= -x - 1$ **OR** $y = -x + c$ $2 = -(-3) + c$ $c = -1$ $y = -x - 1$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
5.4	$x \in \mathrm{R}; x \neq -2$	✓ $x \in \mathrm{R}$ ✓ $x \neq -2$ (2)
5.5	$k(x) = \frac{-4}{-x+3} + 2$ $= \frac{-4}{-(x-3)} + 2$ $= \frac{4}{x-3} + 2$	✓✓ answer (2)
5.6	$-3 < x \leq -1$ or $x \geq 0$ **OR** $x \in (-3; -1]$ or $x \in [0; \infty)$	✓ $-3 < x \leq -1$ ✓ $x \geq 0$ (2) ✓ $x \in (-3; -1]$ ✓ $x \in [0; \infty)$ (2)

5.7	$g(x) = bx^2 - 2$ $2 = b(-3)^2 - 2$ $4 = b(9)$ $b = \frac{4}{9}$ $\therefore g(x) = \frac{4}{9}x^2 - 2$	✓ $c = -2$ ✓ substitution $(-3; 2)$ ✓ $b = \frac{4}{9}$ (3)
5.8	$h: \quad y = \frac{4}{9}x^2$ $h^{-1}: \quad x = \frac{4}{9}y^2$ $y^2 = \frac{9}{4}x$ $y = \pm\sqrt{\frac{9}{4}x};\ x \geq 0$ $y = \pm\frac{3}{2}\sqrt{x};\ \ x \geq 0$	✓ $h(x) = \frac{4}{9}x^2$ ✓ swop x and y ✓ answer with restriction (3)
5.9	$x \leq 0$  	✓ form of h ✓ form of h^{-1} (must fit form of h) (2) ✓ form of h ✓ form of h^{-1} (must fit form of h) (2)

	OR $x \geq 0$  	**[21]**

QUESTION 6

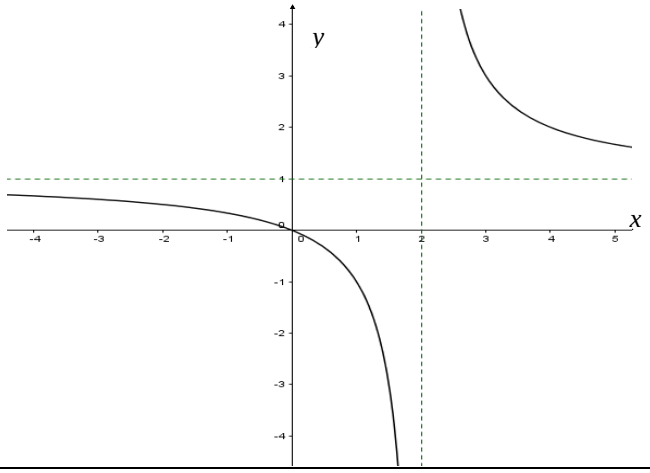
6.1	$x = 0$: $y = -(0+1)^2 + 9$ $= -1 + 9$ $= 8$ $\therefore C(0;8)$	✓ x-coordinate of C ✓ y-coordinate of C (2)
6.2	$q = 8$ (Horisontal asymptote) $y = a.2^x + 8$ Turning point: $D(-1;9)$ $9 = a.2^{-1} + 8$ $1 = \frac{a}{2}$ $2 = a$ $\therefore g(x) = 2.2^x + 8$ $= 2^{x+1} + 8$	✓ $q = 8$ ✓ substitution of $D(-1;9)$ ✓ $a = 2$ (3)
6.3	$y > 8$ **OR** $y \in (8;\infty)$	✓ notation

		✓ answer (2)
6.4	D'(–1; 7)	✓ answer (1)
6.5	Reflection about the *x*-axis, and a translation of 1 unit left and 18 units up. **OR** Reflection about the line $y = 9$ and a translation of 1 unit left.	✓ Reflection *x*-axis ✓ 1-unit left ✓ 18 units up (3) ✓✓ Reflection $y = 9$ ✓ 1-unit left (3)
6.6	$y = \log_{\frac{1}{3}} x$ **OR** $y = -\log_3 x$ **OR** $y = \log_3 \frac{1}{x}$	✓ answer (1) ✓ answer (1) ✓ answer (1)
6.7	$y = \left(\frac{1}{3}\right)^x$ is a decreasing function $\therefore$ the bigger the $x-$value the smaller the $y-$value maximum value of $f = 9$ minimum value: $y = \left(\frac{1}{3}\right)^{9-5}$ $= \left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$ **OR** $y = 3^{-(f(x)-5)}$ $= 3^{-f(x)+5}$ $= 3^{(x+1)^2-9+5}$ $= 3^{(x+1)^2-4}$ $\therefore$ minimum $= 3^{-4}$ $= \frac{1}{81}$	✓ substitution of 9 ✓ answer $\left(\frac{1}{3}\right)^4$ or $\frac{1}{81}$ Accept 0,01 (2) ✓ substitution of $f(x)$ ✓ answer $\left(\frac{1}{3}\right)^4$ or $\frac{1}{81}$ Accept 0,01 (2) **[14]**

QUESTION 7

7.1	R(-2; 4)	✓ -2 ✓ 4	(2)
7.2	B (-4; 0) through symmetry ∴ AB = 4 units OR roots: $(x+2)^2 = 4$ OR $-x^2 - 4x = 0$ $\therefore x + 2 = \pm 2$ $x(x+4) = 0$ ∴ x = 0 or -4 ∴ AB = 4 units	✓ -4 ✓ 4 units	(2)
7.3	m = -2 eqn: $y = -2x$	✓ m = -2 ✓ eqn	(2)
7.4	$x < -2$ OR $x > 0$	✓ $x < -2$ ✓ $x > 0$	(2)
7.5	$h(x) = f(-x) = -(-x+2)^2 + 4$ sym- axis: x = 2	✓ h(x) ✓ x = 2 Answer Only = FULL marks	(2)
7.6	$p(x) = -f(x) = (x+2)^2 - 4$ range: $y \geq -4$; $y \in R$ OR $[-4; \infty)$	✓ p(x) ✓ $y \geq -4$ Answer Only = FULL marks	(2)
			[12]

QUESTION 8

8.1	$x = 2$; $y = 1$	✓ $x = 2$ ✓ $y = 1$	(2)
8.2	y-int: $y = 0$; x-int: $\frac{2}{x-2} = -1$ $\therefore x - 2 = -2$ $\therefore x = 0$	✓ $y = 0$ ✓ $\frac{2}{x-2} = -1$ ✓ $x = 0$	(3)
8.3		✓ asymptotes ✓ x/y intercept ✓ shape	(3)
8.4	$x \in R; x \neq 2$	✓ $x \in R$ ✓ $x \neq 2$	(2)
8.5.1	Graph shifts(translates) 3 units to the left	✓✓ 3 units to the left	(2)
8.5.2	Graph shifts(translates) 2 units down	✓✓ 2 units down	(2)
			[14]

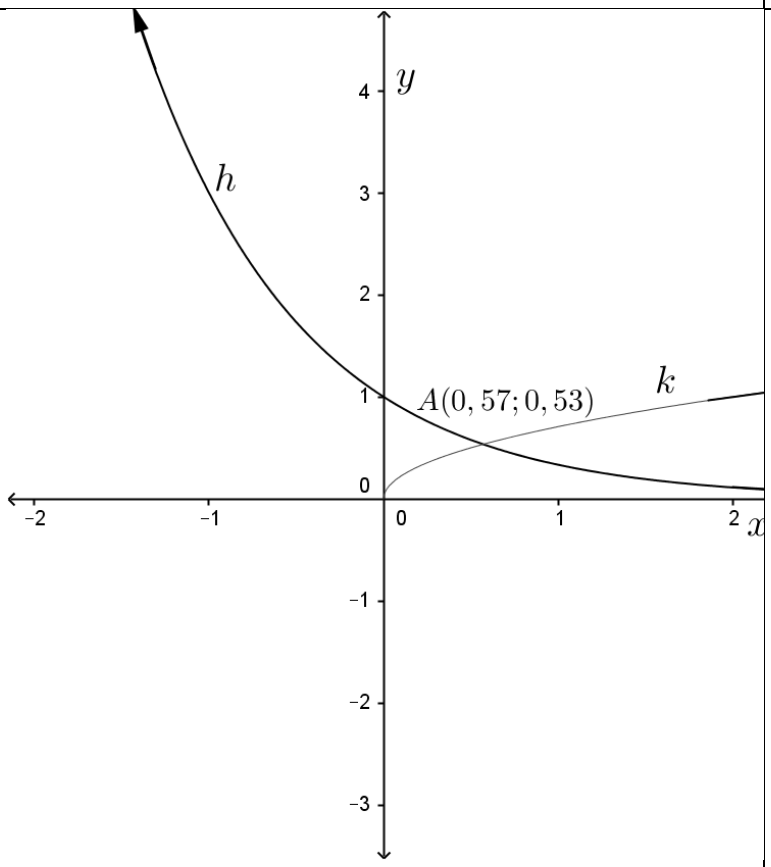
QUESTION 9

9.1.1	A(0; 1)	✓ Answer	(1)
9.1.2	f^{-1}: $y = \log_3 x$	✓ $y = \log_3 x$	(1)
9.1.3	$0 < x \leq 1$	✓ endpoints ✓ notation	(2)
9.1.4	$y = 0$	✓ $y = 0$	(1)
9.2.1	$f(x) = \sqrt{\frac{x}{a}}$ OR $f^{-1}(x) = ax^2$ (8; 2): $2 = \sqrt{\frac{8}{a}} \Rightarrow$ a = 2 (2; 8): $8 = a(2)^2$ $\Rightarrow$ a = 2 $\therefore f(x) = \sqrt{\frac{x}{2}}$ $\therefore$ $f^{-1}(x) = 2$ $f(x)$: $x = 2y^2$ $y = \sqrt{\frac{x}{2}}$	✓ a = 2 ✓ eqn	(2)
9.2.2	$f^{-1}(x) = ax^2$ (2; 8): $8 = a(2)^2 \Rightarrow$ a = 2 $\therefore f^{-1}(x) = 2x^2$	✓ eqn	(1)
9.2.3	$(-3; -1)$	✓ each value	(2)
			[10]

QUESTION 10

10.1	$x = 2$ and $y = 1$		✓ $x = 1$ ✓ $y = 1$ (2)
10.2	$\frac{3}{x-2} = -1$ $3 = -1(x-2)$ $3 = -x + 2$ $3 - 2 = -x$ $-1 = x$		✓ x = -1 ✓ y = 0 (2)
10.3	$g(x) = \frac{3}{x+1-2} + 1$ $g(x) = \frac{3}{x-1} + 1$ $x \in \mathrm{R}, x \neq 1$ **OR** $(-\infty; 1)$ or $(1; \infty)$		✓ $g(x) = \frac{3}{x-1} + 1$ ✓✓ $x \in \mathrm{R}, x \neq 1$ or ✓ $(-\infty; 1)$ or ✓ $(1; \infty)$ (3)
10.4	$y = x + p + q$ $= x - 2 + 1$ $= x - 1$	**OR** $y = x + c$ $1 = 2 + c$ $1 - 2 = c$ $y = x - 1$ (2;1)	✓x ✓-1 (2)
			[9]

QUESTION 11

11.1	$y = 2x^2$ $k: x = 2y^2$ $y = \sqrt{\frac{x}{2}} \quad ; y \geq 0$	✓interchange x and y ✓ $y = \sqrt{\frac{x}{2}}$ (2)
11.2		*h* ✓shape ✓ $y-$int. ✓asymptote. *k* ✓shape ✓*y*-intercept (5)
11.3	$(0;\infty)$ **OR** $y > 0$; $y \in$ IR	✓$(0;\infty)$ (1)
11.4	$0 < x \leq 0{,}57$ **OR** $x \in (0;0{,}57]$	✓$0 < x$ ✓$x \leq 0{,}57$ (2)
11.5	$t > 1$, OR $t \in (1;\infty)$	✓✓answer (2)
		[12]

QUESTION 12

12.1	$y = 6 - 0$ $y = 6$	✓$y = 6$ (1)
12.2	$h(x) = \frac{4}{x-1}$	✓ $h(x) = \frac{4}{x-1}$ (1)
12.3	$\frac{4}{x-1} = 6 - x$ $4 = (6 - x)(x - 1)$ $= 6x - x^2 - 6 + x$ $0 = -x^2 + 7x - 6 - 4$ $= -x^2 + 7x - 10$ $x^2 - 7x + 10 = 0$ $(x - 5)(x - 2) = 0$ $x = 5$ **or** $x = 2$	✓$\frac{4}{x+1} = 6 - x$ ✓standard form ✓both answers (3)
12.4	$CD = g(x) - f(x)$ $= 6 - x - \frac{4}{x}$	✓ $g(x) - f(x)$ ✓ $6 - x - \frac{4}{x}$ (2)
12.5	$\frac{dCD}{dx} = 0$ $6 - x - 4x^{-1}$ $-1 + \frac{4}{x^2} = 0$ $x^2 = 4$ $x = -2$ **or** $x = 2$ $\therefore x = 2$	✓=0 ✓ $-1 + \frac{4}{x^2}$ ✓ $x^2 = 4$ ✓ $x = 2$ (4)
		[11]

QUESTION 13

13.1	$p = -3$ $q = -2$	A✓ p value A✓ q value	(2)
13.2	$h(x) = \frac{a}{x+p} + q$. P$(4;-4)$ is a point on h $-4 = \frac{a}{4-3} - 2$ $\therefore a = -2$	CA✓ subst. p, q and point P CA(negative)✓a value	(2)
13.3	$h(x) = \frac{-2}{x-3} - 2$ $h(0) = \frac{-2}{0-3} - 2$ $= -1\frac{1}{3}$ $\left(0;-1\frac{1}{3}\right)$	A✓substituting $x = 0$ CA(negative)✓answer	(2)
13.4	$x = 1$	CACA✓✓ answer	(2)
13.5	$y = -(x+p) + q$ $y = -(x-3) - 2$ $y = -x + 1$ $\therefore c = 1$	CA✓substitution of p and q values into equation of line of symmetry CA✓answer	(2)

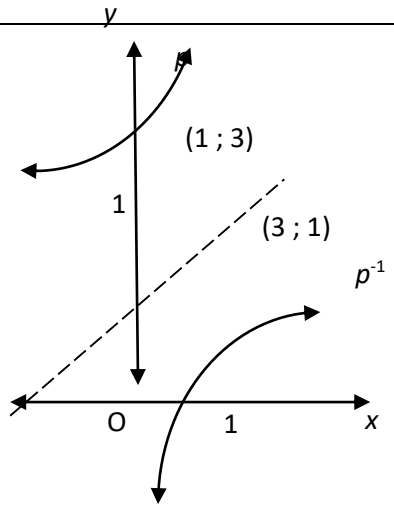
	OR $y=-x+c$ *Point of intersection of asymptotes* $(3\ ;\ -2)$ $-2=-3+c$ $c=1$	CA✓substitution of $(3\ ;\ -2)$ into equation of line of symmetry CA✓answer	(2)
			[10]

QUESTION 14

14.1	$D(0\ ;-10)$	A(must be in coordinate form)✓ answer	(1)
14.2	$x^2-3x-10=0$ $(x+2)(x-5)=0$ $x=-2 \quad or \quad x=5$ $A(-2;0)$ $B(5;0)$	A✓ $x^2-3x-10=0$ CA✓factors CACA(negative and positive)✓✓ each x – value $A(-2;0)$ $B(5;0)$	(4)
14.3	$a=2 \quad and \quad q=-10$	CA(positive)✓a – value A✓ q – value	(2)
14.4	$x=-\frac{b}{2a}=-\frac{(-3)}{2(1)}=\frac{3}{2} \quad or \quad x=\frac{-2+5}{2}=\frac{3}{2}$ $y=\left(\frac{3}{2}\right)^2-3\left(\frac{3}{2}\right)-10=-\frac{49}{4}/-12{,}25/-12\frac{1}{4}$ C(1,5 ; −12,25)	A✓ $x=-\frac{b}{2a}=-\frac{(-3)}{2(1)}=\frac{3}{2}$ or	

		CA✓ $x = \frac{-2+5}{2} = \frac{3}{2}$ CA✓substitution CA✓minimum value	(3)
14.5	$(-1{,}5\ ;\ -9{,}25)$	CA✓ x – value CA✓ y – value	(2)
14.6	$x \geq \frac{3}{2}$ **OR** $g'(x).h'(x) \geq 0$ $(2x-3).2 \geq 0$ $x \geq \frac{3}{2}$	CACA✓✓answer CA✓product of derivatives CA(positive)✓answer penalize 1 mark for incorrect notation	(2) (2)
			[14]

QUESTION 15

15.1	$y = \log_3 x$	AA✓✓ answer	(2)
15.2		A✓Shape of p and p^{-1} A✓y – intercept of p A✓ x – intercept of p^{-1} A✓ point on each graph	(4)
15.3	$\log_3 x = 3$ $x = 27$ $0 < x \leq 27$	M✓setting up equation CA✓$x = 27$ CACA✓✓for end points and inequality ANSWER ONLY full marks	(4)
			[10]

QUESTION 16

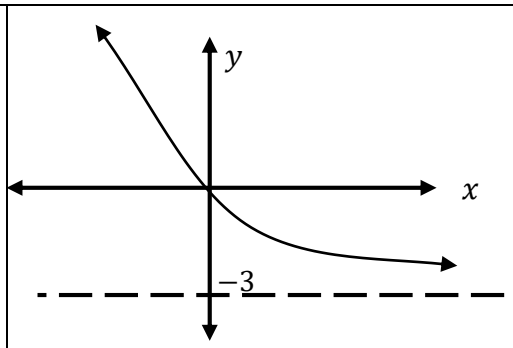
16.1		$y = 2x - 12$ $m = 2$	✓	$m = 2$	(1)
16.2		$(x - 4)(x + 2) = 2x - 12$ $x^2 - 2x - 8 - 2x + 12 = 0$ $x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$ $x = 2$ $y = 2(2) - 12$ $= -8$ A$(2; -8)$ OR $y = (x - 4)(x + 2)$ $= x^2 - 2x - 8$ $\frac{dy}{dx} = 2x - 2$ since g is a tangent passing through A $i.e. 2x - 2 = 2$ $x = 2$ $y = 2(2) - 12$ $= -8$ A$(2; -8)$	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	$(x - 4)(x + 2)$ $= 2x - 12$ standard form factors $x = 2$ $y = 8$ OR $y = x^2 - 2x - 8$ $\frac{dy}{dx} = 2x - 2$ $2x - 2 = 2$ $x = 2$ $y = -8$	(5)
16.3		$h(x) = f(-x)$ $= (-x - 4)(-x + 2)$ $= (x + 4)(x - 2)$ OR $h(x) = x^2 + 2x - 8$	✓ ✓	subst x by $-x$ answer	(2)
16.4		$(x - 4)(x + 2)(2x - 12) < 0.$ $4 < x < 6$	✓ ✓	 $x < 6$ $x > 4$	(2)
16.5		$x = 2y - 12$ $y = \frac{1}{2}x + 6$	✓ ✓	interchange x and y $y = \frac{1}{2}x + 6$	(2)
					[12]

QUESTION 17

17.1		$x = -1$ $y = (-1)^2 + 2(-1) - 3$ $= -4$	✓ ✓ ✓	$x = -1$ $y = (-1)^2 + 2(-1) - 3$ $y = -4$	(3)
17.2		$A(0; -3)$	✓	Answer	(1)
17.3		$y = \frac{a}{x+1} - 4$ $-3 = \frac{a}{0+1} - 4$ $1 = a$ $g(x) = \frac{1}{x+1} - 4$	✓ ✓ ✓ ✓	subst. of p subst. of q subst $(0; -3)$ answer	(4)
17.4		Before shifting, the points were at$(1\ ;\ 1)$and $(-1;\ -1)$ applying the shift on these points yields: $(1\ -1; 1-4)$ and $(-1-1; -1\ -4)$ so the points are: $(0;\ -3)$ and $(-2;\ -5)$ OR from the point of intersection of asymptotes (-1: - 4) we move a unit up and to the right or a unit down and to the left since $a = 1$ so the points are: $(-1 + 1; -4 + 1)$ and /or $(-1 - 1; -4 - 1)$ the points are $(0; -3)$ and $(-2; -5)$	✓ ✓ ✓ ✓ OR ✓ ✓ ✓ ✓	$(1\ -1; 1-4)$ $(-1-1; -1\ -4)$ $(0; -3)$ $(-2; -5)$ OR $(-1 + 1; -4 + 1)$ $(-1 - 1; -4 - 1)$ $(0; -3)$ $(-2; -5)$	(4)

		OR equation of the line of symmetry is $y = (x + 1) - 4$ $y = x - 3$ $x - 3 = \frac{1}{x+1} - 4$ $x + 1 = \frac{1}{x+1}$ $x^2 + 2x + 1 = 1$ $x^2 + 2x = 1$ $x = 0 \text{ or } x = -2$ $y = -3 \text{ or } y = -5$ The points are $(0; -3)$ and $(-2; -5)$	OR ✓ ✓ ✓ ✓	OR equation of the line of symmetry equating the two functions values of x values of y	
17.5		$y \leq 4; y \in \mathbb{R}$	✓✓	answer	(2)
					[14]

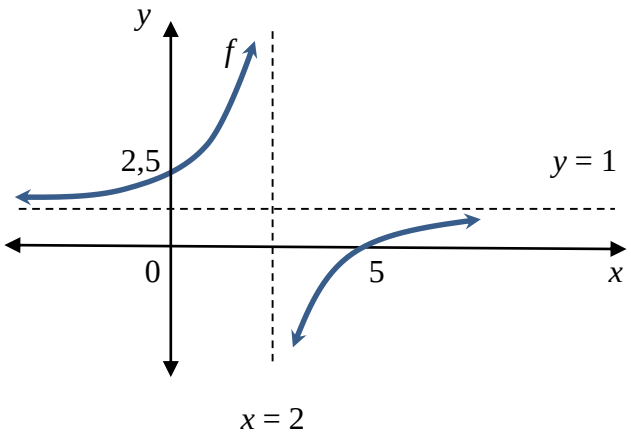
QUESTION 18

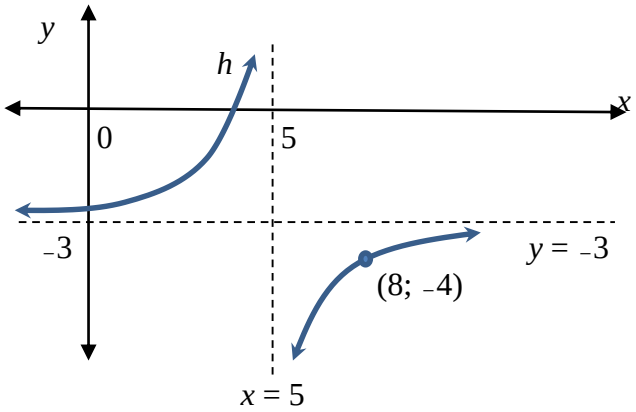
18.1			✓ ✓ ✓	shape graph passing though the origin asymptote	(3)
18.2		$5 = 3^{-x+1} - 3$ $8 = 3^{-x+1}$ $(-x + 1)\log 3 = \log 8$ $(-x + 1) = \frac{\log 8}{\log 3}$ $x = 1 - \frac{\log 8}{\log 3}$ $= -0,89$	✓ ✓ ✓	substitution applications of log answer	(3)
18.3		graph was reflected about the y −axis graph was shifted 3 units up graph was shifted one unit to left.	✓ ✓ ✓	reflect about y −axis shift 3 units up one unit to the left	(3)

		OR graph was shifted 3 units up graph was reflected about the y axis graph was shifted to one the left. NB reflection should not be mentioned after horizontal shift	OR ✓ ✓ ✓	OR shift 3 units up reflect about y −axis one unit to the left	
					[9]

QUESTION 19

19.1	$y = \frac{-3}{0-2} + 1$ $= \frac{3}{2} + 1$ $= 2,5$ or $y = \frac{5}{2}$ $\therefore (0; 2,5)$ or $\left(0; \frac{5}{2}\right)$	✓ $x = 0$ ✓ $y = 2,5$ or $y = \frac{5}{2}$ (2)
19.2	$0 = \frac{-3}{x-2} + 1$ $-1 = \frac{-3}{x-2}$ $x - 2 = 3$ $x = 5$ $\therefore (5; 0)$	✓ $y = 0$ ✓ $x = 5$ (2)

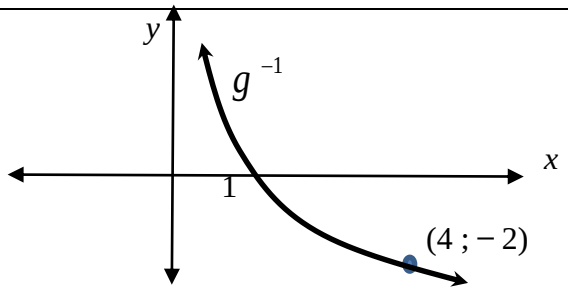
19.3		✓ shape ✓ both intercepts ✓ both asymptotes (3)

19.4	$y \in \mathbb{R};\ y \neq 1$ **OR** $y < 1 \quad \text{or} \quad y > 1$ **OR** $y \in (-\infty; 1) \cup y \in (1; \infty)$	✓ $y \in \mathbb{R}$ ✓ $y \neq 1$ (2) ✓ $y < 1$; ✓ $y > 1$ (2) ✓ $y \in (-\infty; 1)$ ✓ $y \in (1; \infty)$ (2)
19.5	$h(x) = \dfrac{-3}{x-5} - 3$	✓ $\dfrac{-3}{x-5}$ ✓ -3 (2)
19.6	From the graph of h:  $5 < x \leq 8$ or $x \in (5; 8]$ **OR** From translations: $h(x) \leq -4 \ \therefore\ f(x) \leq 0$ (4 units up) If $f(x) \leq 0$, then $2 < x \leq 5$ $\therefore$ for $h(x)$: $5 < x \leq 8$ (3 units to the right)	✓ (8; -4) ✓ $5 < x$ ✓ $x \leq 8$ (3) ✓ $f(x) \leq 0$ ✓ $f(x)$: $2 < x \leq 5$ ✓ $h(x)$: $5 < x \leq 8$ (3)
19.7	$k(x) = \dfrac{3x-5}{x-1}$ By dividing $x-1$ into $3x-5$: $k(x) = \dfrac{-2}{x-1} + 3$ $\therefore$ The asymptotes are: $x = 1$ and $y = 3$	✓ $k(x) = \dfrac{-2}{x-1} + 3$ ✓ $x = 1$ ✓ $y = 3$ (3)

	OR $k(x) = \dfrac{3x-5}{x-1}$ $k(x) = \dfrac{3(x-1)-2}{x-1}$ $k(x) = 3 - \dfrac{2}{x-1}$ $\therefore$ The asymptotes are: $x = 1$ and $y = 3$	✓ $k(x) = \dfrac{-2}{x-1} + 3$ ✓ $x = 1$ ✓ $y = 3$ (3) **[17]**

QUESTION 20

20.1	$x = -1$	✓ $x = -1$ (1)
20.2	$(-1; -8)$	✓ answer (1)
20.3	$2(x+1)^2 - 8 = 0$ $2(x+1)^2 = 8$ $(x+1)^2 = 4$ $x + 1 = \pm 2$ $x = 1$ of $x = -3$ $\therefore$ PQ = 1 + 3 = 4 units **OR** $2(x^2 + 2x + 1) - 8 = 0$ $2x^2 + 4x + 2 - 8 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = 1$ of $x = -3$ $\therefore$ PQ = 1 + 3 = 4 units	✓ $y = 0$ ✓ $(x+1)^2 = 4$ ✓ $x + 1 = \pm 2$ ✓ PQ = 4 units (4) ✓ $y = 0$ ✓ standard form ✓ factors ✓ PQ = 4 units (4)
20.4	$k(x) = 2(-x+1)^2 - 8$ $= 2(x^2 - 2x + 1) - 8$ $= 2x^2 - 4x + 2 - 8$ $= 2x^2 - 4x - 6$	✓ substituting x by $-x$ ✓ simplification $(x^2 - 2x + 1)$ ✓ answer $(2x^2 - 4x - 6)$ (3)

	OR $k(x) = 2(-x)^2 + 4(-x) - 6$ $= 2x^2 - 4x - 6$ **OR** $k(x) = 2(x-1)^2 - 8$ $= 2(x^2 - 2x + 1) - 8$ $= 2x^2 - 4x + 2 - 8$ $= 2x^2 - 4x - 6$	✓ substituting x by $-x$ ✓✓ answer (3) ✓ substituting $(x+1)$ by $(x-1)$ ✓ simplification $(x^2 - 2x + 1)$ ✓ answer $(2x^2 - 4x - 6)$ (3)
20.5	$x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ **OR** $y = -\log_2 x$ **OR** $y = \log_2 \frac{1}{x}$	✓ answer (1) ✓ answer (1) ✓ answer (1)
20.6		✓ shape ✓ x-intercept ✓ point (4;−2) or any other point (3)
20.7.1	$0 < x \leq 4$ **OR** $x \in (0; 4]$	✓ $0 < x$ ✓ $x \leq 4$ (2) ✓✓ answer (2)
20.7.2	If x < 0 and f (x) > 0: $\therefore x < -3$ or if $x > 0$ and f (x) < 0: $\therefore 0 < x < 1$ **OR** $x \in (0; 1) \cup (-\infty; -3)$	✓✓ $x < -3$ ✓✓ $0 < x < 1$ (4) ✓✓ $(0; 1)$ ✓✓ $(-\infty; -3)$ (4) **[19]**

QUESTION 21

#	SUGGESTED ANSWER	DESCRIPTORS	Ma k
21.1	OC = 6 units A; B: *x*-intercepts: Let $y = 0$ $\therefore -2x^2 - 4x + 6 = 0$ $\therefore x^2 + 2x - 3 = 0$ $\therefore (x + 3)(x - 1) = 0$ $\therefore x = -3$ OR $x = 1$ A$(-3;\ 0)$ and B$(1;\ 0)$ $\Rightarrow$AB = 4 units	✓ OC = 6 ✓ Let $y = 0$ ✓ Factors ✓ Both *x*-values ✓ answer	(5)
21.2	$x = -\frac{b}{2a} = -\left[\frac{-4}{2(-2)}\right] = -1$ OR $f'(x) = -4x - 4 = 0 \Rightarrow x = -1$	✓ substitution ✓ $x = -1$ OR ✓ derivative ✓ $x = -1$ **Answer only = FULL MARKS**	(2)
21.3	Subst. $x = -1$ in *f*(*x*) $\therefore$ ST $= -2(-1)^2 - 4(-1) + 6$ $= 8$ units	✓ Subst. x = -1 ✓ answer	(2)
21.4	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0-6}{-3-0} = 2$	✓ Subst. in *m* ✓ answer	(2)

21.5	$m_g = 2$; // lines
	but $m_g = f'(x) = -4x - 4 = 2$
	$\therefore x = -\frac{3}{2}$
	$\therefore y = -2\left(-\frac{3}{2}\right)^2 - 4\left(-\frac{3}{2}\right) + 6 = 7\frac{1}{2}$
	$\Rightarrow$D$\left(-\frac{3}{2};\ 7\frac{1}{2}\right)$
		✓ $m_g = 2$
		✓ $m_g = f'(x)$
		✓ $-4x - 4 = 2$
		✓ $x = -\frac{3}{2}$
		✓ $y = 7\frac{1}{2}$
			(5)

21.6	$a = -1$; the axis of symmetry OR $f(a + t) = f(a - t)$ $\therefore -2(a + t)^2 - 4(a + t) + 6$ $= -2(a - t)^2 - 4(a - t) + 6$ $\therefore -2a^2 - 4at - 2t^2 - 4a - 4t + 6$ $= -2a^2 + 4at - 2t^2 - 4a + 4t + 6$ $\therefore 8at + 8t = 0$ $\therefore 8t(a + 1) = 0$ $\therefore t = o$ or $\boldsymbol{a = -1}$	✓✓ $a = -1$ ✓ Subst. ✓ $a = -1$	(2)
			[18

QUESTION 22

#	SUGGESTED ANSWER	DESCRIPTORS	Mark
22.1	$x = 1$	✓ answer	(1)
22.2	x -int: Let $y = 0$ $\frac{2+x}{x-1} = 0 \Rightarrow x = -2$ $\therefore A(-2;\ 0)$	✓ $y = 0$ ✓ $x = -2$	(2)
22.3	y -int: Let $x = 0$ $\frac{2+0}{0-1} = y \Rightarrow y = -2$ $\therefore B(0;\ -2)$ Area $\Delta AOB = \frac{1}{2} AO \times OB$ $= \frac{1}{2}(2)(2) = 2$ units^2	✓ $B(0;\ -2)$ ✓ Subst. in Area formula ✓ answer	(3)
22.4	$f(x) = \frac{2+x}{x-1} = \frac{x-1+3}{x-1}$ $= \frac{x-1}{x-1} + \frac{3}{x-1}$ $= \frac{3}{x-1} + 1$	✓ $\frac{2+x}{x-1} = \frac{x-1+3}{x-1}$ ✓ Simplify to: $\frac{x-1}{x-1} + \frac{3}{x-1}$	(2)
22.5	$(3;\ 1)$	✓ 3 (CA from 5.2 - shift 2 units to the right) ✓ 1	(2)
			[10]

QUESTION 23

#	SUGGESTED ANSWER	DESCRIPTORS	Mark
23.1.1	$y > -1; y \in R$	✓✓ $y > 0; y \in R$	(2)
23.1.2	$g(x) = 2^x$ $\therefore g^{-1}: y = \log_2 x$	✓ $g(x) = 2^x$ ✓ $y = \log_2 x$	(2)
23.2.1	$k(x) = 3x^2\ ;\ x \leq 0$	✓ $k(x) = 3x^2$ ✓ $x \leq 0$	(2)
23.2.2	$(0;0)$ OR origin	✓✓ Answer	(2)
			[8]