MATHEMATICS

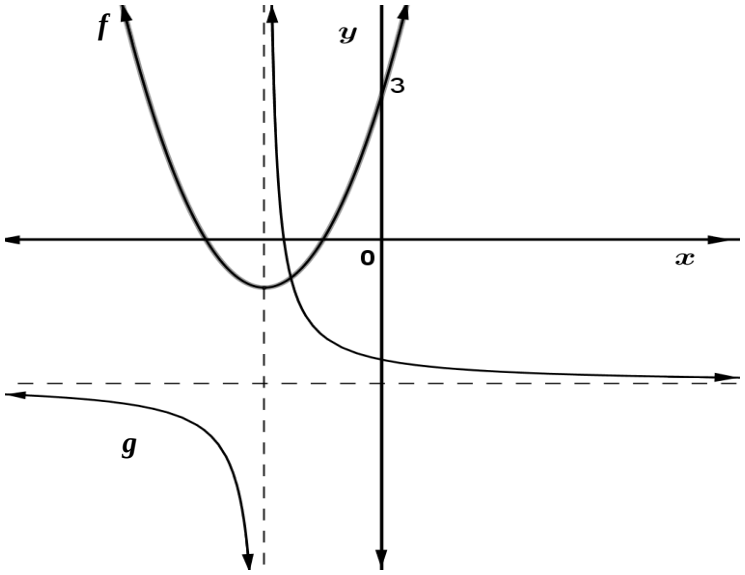
MATERIAL FOR GRADE 12

Functions

QUESTIONS

QUESTION 1

1.1 Sketched are the functions $f(x) = (x + p)^2 + q$, and $g(x)$: $(x + 2)(y + 3) = t$ If $g(0) = -\frac{5}{2}$ and g is a rectangular hyperbola with one of its asymptotes an axis of symmetry for f as shown. Answer the following:



1.1.1 Write down the equations of the asymptotes of g. (2)

1.1.2 Determine the values of:

1.1.2.1 t (2)

1.1.2.2 p and q (3)

1.2 Write g in the form $y = \frac{a}{x + p} + q$ (3)

1.3 If $h(x) = x - 1$ is the line of symmetry to g, determine the co-ordinates of the points of intersection h and g. (6)

1.4 If $k = x^2 + 4x + 3$. Determine the values of k if its roots are:

1.4.1 non-real (2)

1.4.2 negative and unequal (2)

1.5 Write down the:

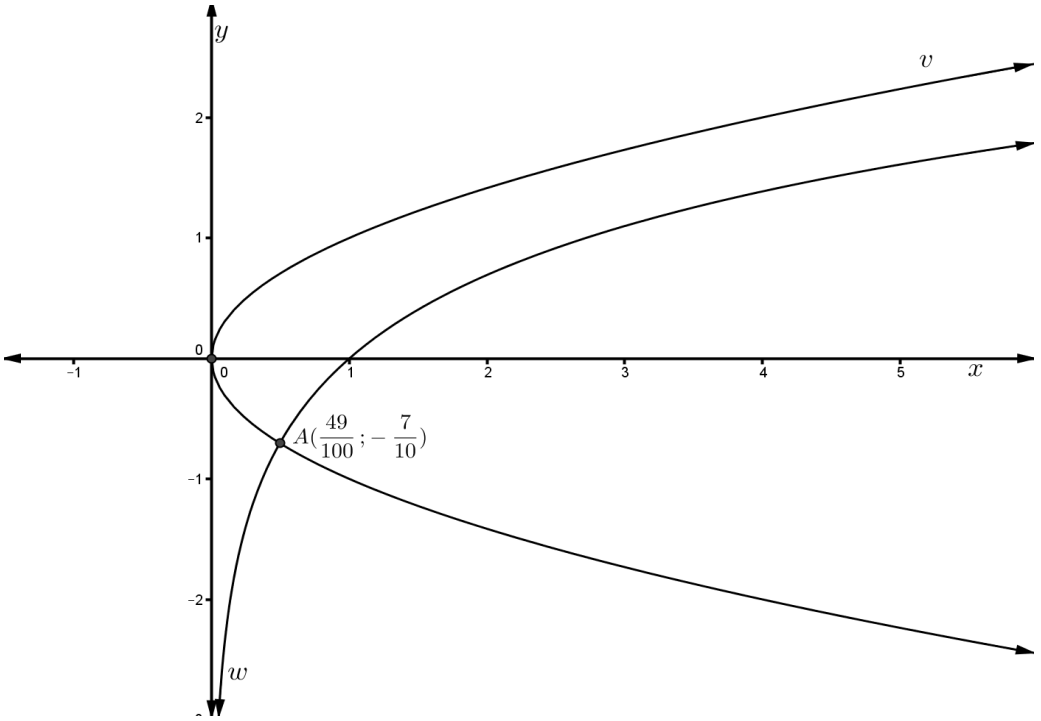
1.5.1 domain of g (2)

1.5.2 range of f (1)

[23]

QUESTION 2

2.1 Study the diagram which shows the sketch graphs of $v(x) = \pm\sqrt{x}$ and $w(x) = logx$ then answer the questions that follow:



2.1.1 State whether $v(x)$ is a function or not, motivate your answer. (2)

2.1.2 Write down conditions that will make v a function. (2)

2.1.3 Determine all values of:

2.1.3.1 y for which $w(x) < 0$ (1)

2.1.3.2 x for which $w(x) \leq -\frac{7}{10}$ (2)

2.1.4 If a function $v(x)$ is as determined in 5.1.2 write down the equation(s) of $v^{-1}(x)$. (3)

2.1.5 If $h(x) = w(x) - \sqrt{x}$ where the range of $\sqrt{x}$ is $(0;\infty)$, calculate the range of $h(1)$. (1)

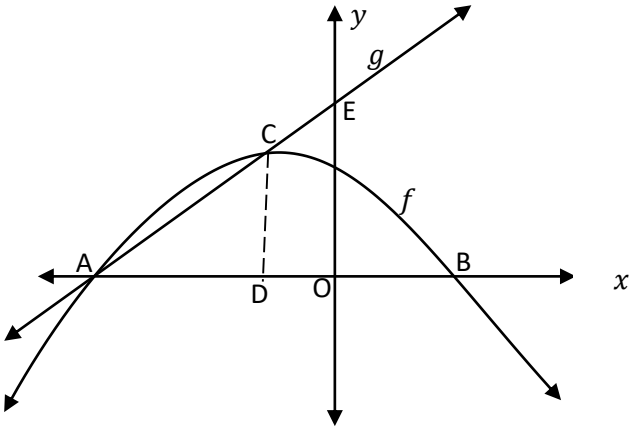
[11]

QUESTION 3

The functions $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + c$ are drawn below, with g passing through E, C and A.

A and B are the $x-$ intercepts of , and CD is the axis of symmetry of f.

E is the $y-$ intercept of g



3.1 Determine the coordinates of C, the turning point of the graph of f. (3)

3.2 Determine the coordinates of A and B. (3)

3.3 Write down the coordinates of the $y-$ intercept of the graph of f. (1)

3.4 Calculate the length of CE. (6)

3.5 Determine the equation of $g^{-1}(x)$ in the form y = …. (2)

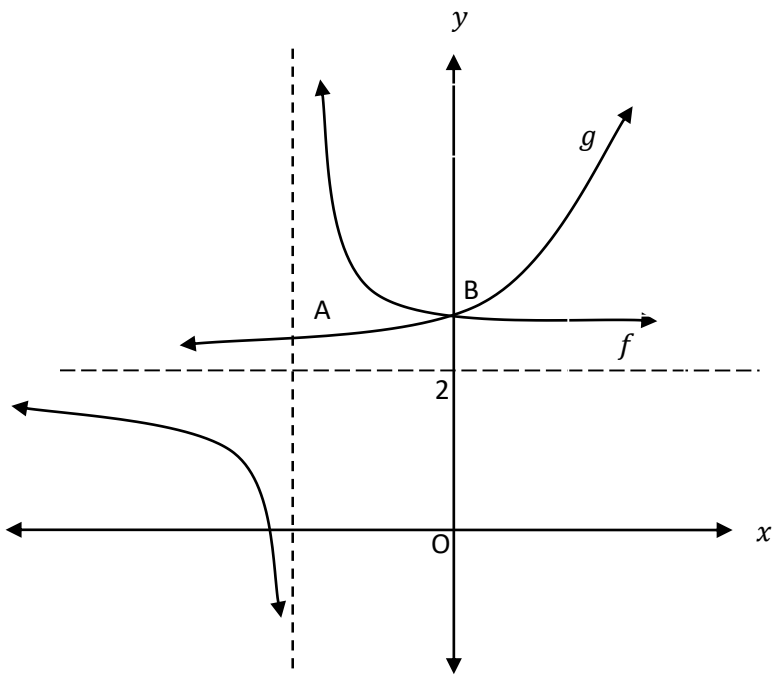
[15]

QUESTION 4

Below are graphs of the functions $f(x) = \frac{3}{x-p} + q$ and $g(x) = 2^x + e$.

Both f and g have the same horizontal asymptote and y −intercept B.

A is the point of intersection of g and the vertical asymptote of f.



4.1 Write down the values of e and q. (2)

4.2 Determine the value of p. (3)

4.3 Determine the coordinates of A. (3)

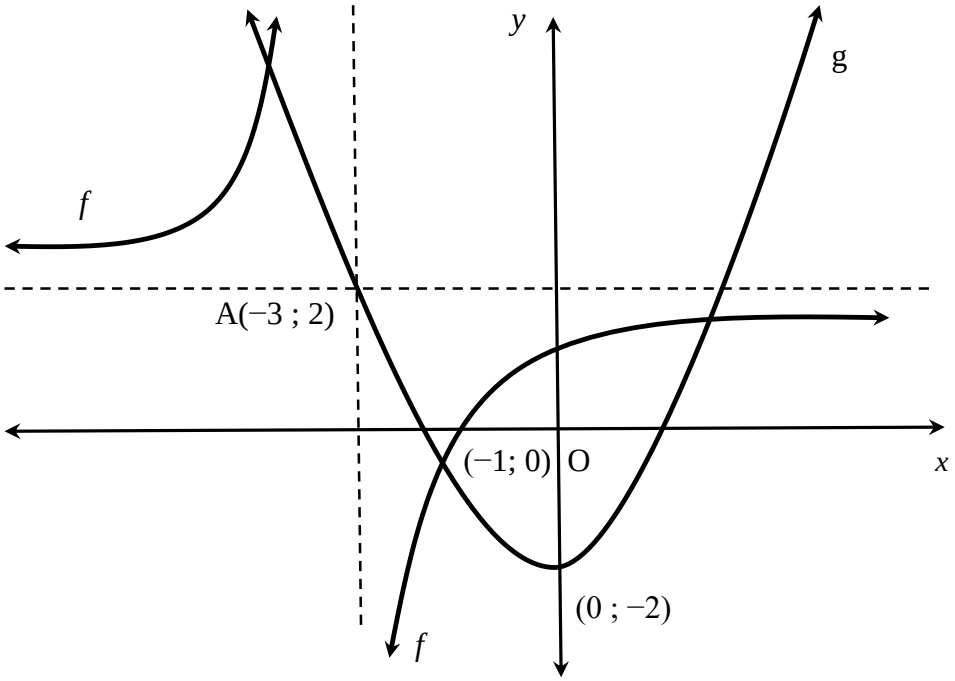
4.4 Determine x such that $f(x) - g(x) \geq 0$ (2)

4.5 If the graph of f is shifted two units to the right, determine the equation of the shifted graph of f. (2)

[12]

QUESTION 5

The sketch below represents the graphs of: $f(x) = \frac{a}{x + p} + q$ and $g(x) = bx^2 + c$.



The point A (−3; 2) is the point of intersection of the asymptotes of f. The graph of f intersects the x-axis at (−1; 0). The graph of g intersects the y-axis at (0; −2).

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Determine the equation of f. (3)

5.3 Write down the equation of the axes of symmetry of f in the form $y = mx + c$ if $m < 0$. (2)

5.4 Write down the domain of $5f(x - 1)$. (2)

5.5 Write down the equation of k, the reflection of f about the y-axis. Leave your answer in the form $y = \frac{a}{x + p} + q$. (2)

5.6 For which value(s) of x is $f(x).g'(x) \geq 0$? (2)

5.7 Determine the equation of g. (3)

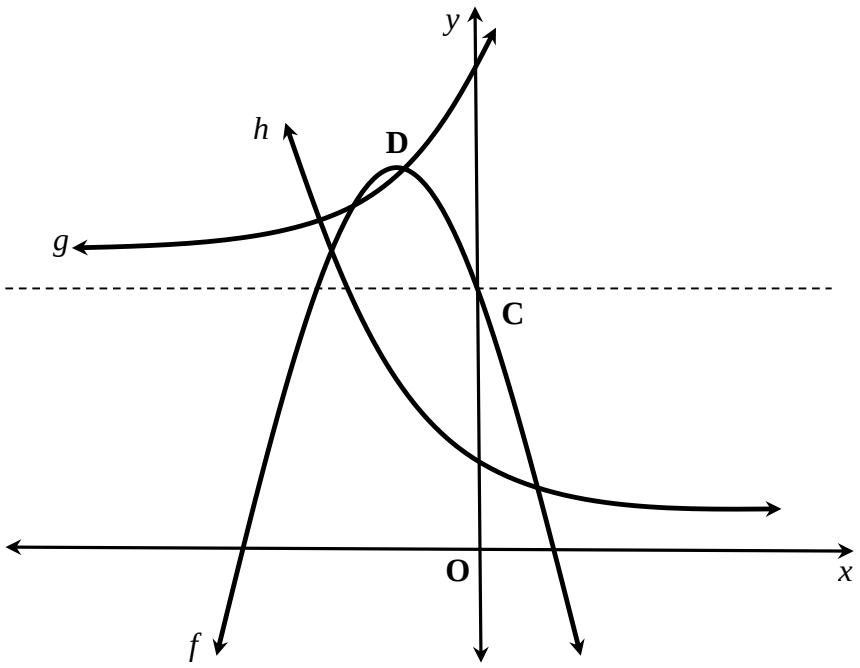
5.8 Determine the equation of $h^{-1}(x)$ if $h(x) = g(x) + 2$. Leave your answer in the form $y = ...$ (3)

5.9 The inverse of h is not a function. Restrict the domain of h such that h^{-1} is a function. Sketch the restricted graph of h and h^{-1} on the same system of axes. (2)

[21]

QUESTION 6

The graphs of $h(x) = 3^{-x}$, $f(x) = -(x+1)^2 + 9$ and $g(x) = a.2^x + q$ are represented in the sketch below. D, the turning point of *f*, is also a point of intersection of *g* and *f*. The asymptote of *g* passes through C, the *y*-intercept of *f*.



6.1 Write down the coordinates of C. (2)

6.2 Calculate the values of *a* and *q*. (3)

6.3 Write down the range of *g*. (2)

6.4 Write down the coordinates of D', if D is reflected about the line $y = 8$. (1)

6.5 If $k(x) = (x+2)^2 + 9$, describe the transformation from f to k. (3)

6.6 Write down the equation of $h^{-1}(x)$ in the form $y = ...$ (1)

6.7 Determine the minimum value of $y = \left(\frac{1}{3}\right)^{f(x)-5}$. (2)

[14]

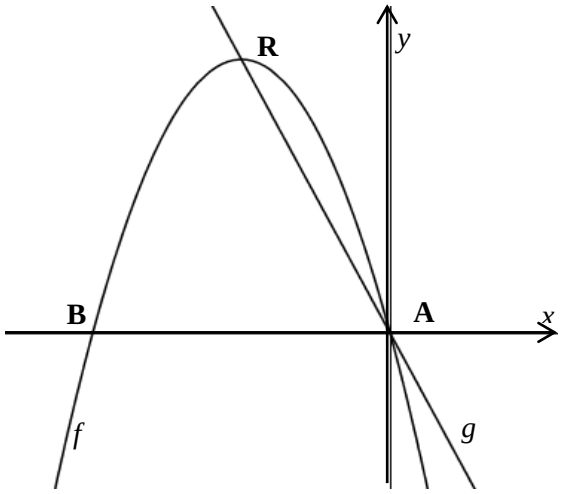
QUESTION 7

Sketched alongside are the graphs of

$f(x) = -(x+2)^2 + 4$ and

$g(x) = ax + q$

R is the turning point of f



7.1 Give the coordinates of R. (2)

7.2 Calculate the length of AB. (2)

7.3 Determine the equation of g. (2)

7.4 For which values of x is $g(x) > f(x)$? (2)

7.5 Write down the equation of the axis of symmetry of h if $h(x) = f(-x)$. (2)

7.6 Give the range of p if $p(x) = -f(x)$. (2)

[12]

QUESTION 8

Given: $h(x) = \frac{2}{x-2} + 1$

8.1 Give the equations of the asymptotes of h (2)

8.2 Determine the x- and y-intercepts of the graph of h. (3)

8.3 Sketch the graph of h using the grid on the DIAGRAM SHEET. (3)

8.4 Give the domain of h. (2)

8.5 Describe the transformation of h to f if :

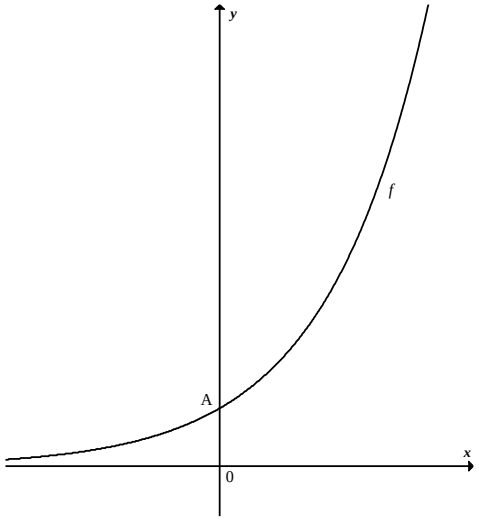
8.5.1 $f(x) = h(x+3)$ (2)

8.5.1 $f(x) = h(x) - 2$ (2)

[14]

QUESTION 9

9.1 The graph of $f(x) = 3^x$ is sketched alongside.



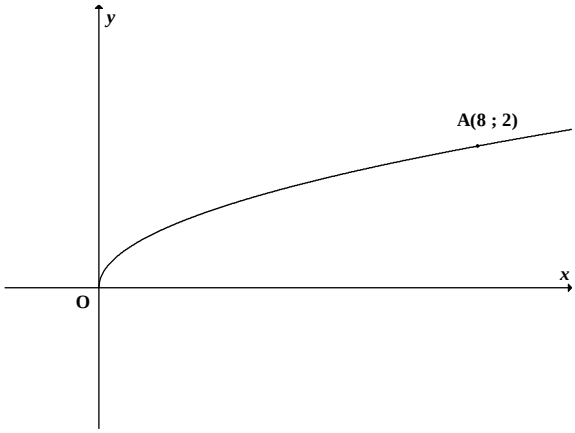
9.1.1 Give the coordinates of A. (1)

9.1.2 Write down the equation of f^{-1} in the form $y = \ldots$ (1)

9.1.3 For which value(s) of x will $f^{-1} \leq 0$? (2)

9.1.4 Write down the equation of the asymptote of $f(x-1)$ (1)

9.2 Sketched is the graph of f, the inverse of a restricted parabola. The point A (8; 2) lies on the graph of f.



9.2.1 Determine the equation of f in the form $y = \ldots$ (2)

9.2.2 Hence, write down the equation of f^{-1} in the form $y = \ldots$ (2)

9.2.3 Give the coordinates of the turning point of $g(x) = f^{-1}(x+3) - 1$. (1)

[10]

QUESTION 10

Given: $f(x) = \frac{3}{x-2} + 1$

10.1 Write down the equations of the asymptotes of f. (2)

10.2 Determine coordinates of B, the *x*-intercept of *f*. (2)

10.3 Write down the domain of *g* if $g(x) = f(x+1)$. (3)

10.4 One of the axes of symmetry of *f* is an increasing function. Write down the equation of this axis of symmetry. (2)

[9]

QUESTION 11

Given $h(x) = 3^{-x}$ and $k^{-1}(x) = 2x^2$ for $x \geq 0$.
A(0,57 ; 0,53) is a point of intersection between *h* and k.

11.1 Write down the equation of *k* in the form *y* = … (2)

11.2 Sketch the graphs of *h* and *k* on the same set of axes, clearly indicating the intercepts with the axes. (5)

11.3 Write down the range of *h*. (1)

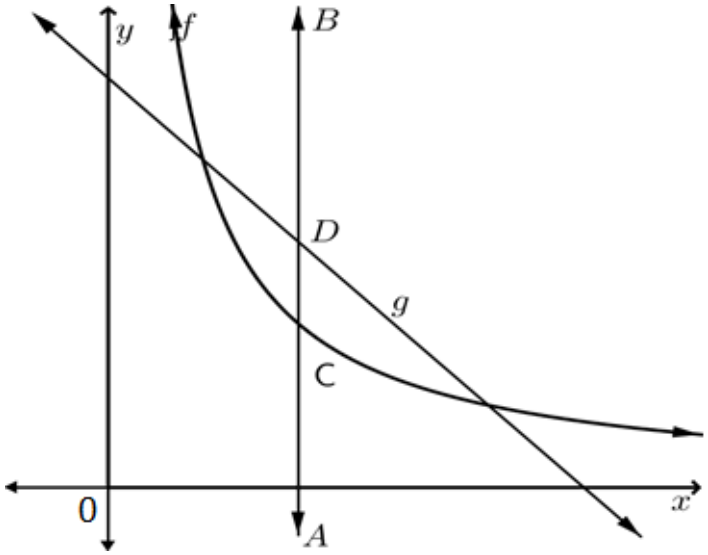
11.4 For which values of x is $k(x) \leq h(x)$? (2)

11.5 For which values of t will $k(x) + t = h(x)$ have no real roots? (2)

[12]

QUESTION 12

The diagram below shows the graphs of $f(x)=\frac{4}{x};\ x>0$ and $g(x)=6-x$. Line BA intersects f and g respectively at points C and D. BA is perpendicular to the *x*-axis.



12.1 Write down the *y*-intercept of *g*. (1)

12.2 Write down the equation of *h* if *h* is a translation of *f* one unit to the right. (1)

12.3 Calculate the values of *x* for which $h(x)=g(x)$. (3)

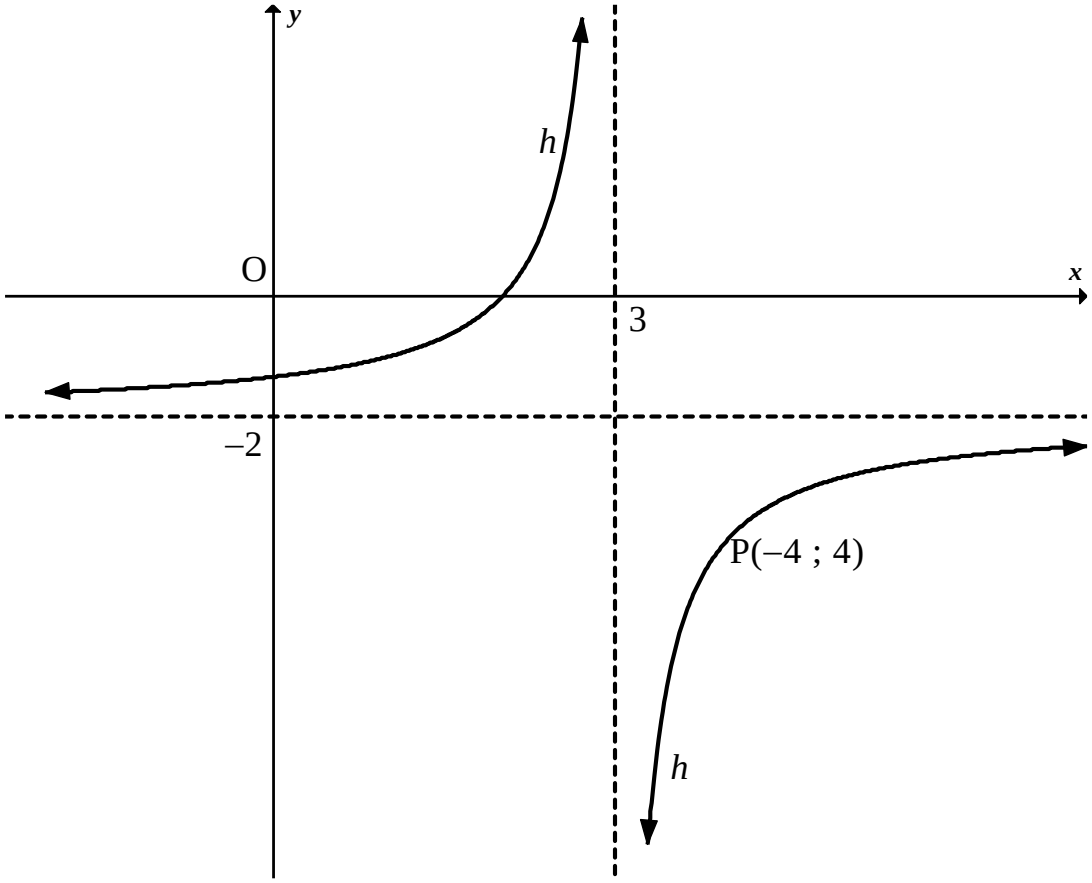
12.4 Write down the length of CD in terms of *x*. (2)

12.5 Determine the value of x for which CD has a maximum length. (4)

[11]

QUESTION 13

The diagram below shows the graph of $h(x) = \frac{a}{x+p} + q$. The lines $x = 3$ and $y = -2$ are asymptotes of h. $P(4;-4)$ is a point on h.



13.1 Write down the values of p and q. (2)

13.2 Calculate the value of a. (2)

13.3 Calculate the coordinates of the y – intercept of h. (2)

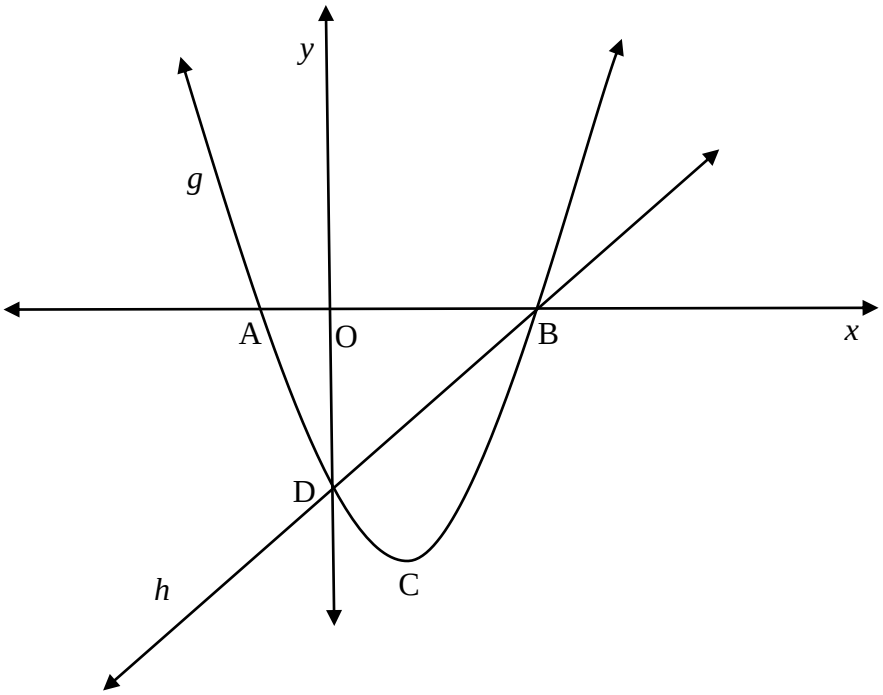
13.4 If $g(x) = h(x+2)$, write down the equation of the vertical asymptote of g. (2)

13.5 If the graph of h is symmetrical about the line $y = -x + c$, determine the value of c. (2)

[10]

QUESTION 14

The sketch below shows the graphs of $g(x) = x^2 - 3x - 10$ and $h(x) = ax + q$. The graphs intersect at B and D. The graph of *g* intersects the *x* – axis at A and B and has a turning point at C. The graph of *h* intersects the *y* – axis at D and the *x* – axis at B.



14.1 Write down the coordinates of D. (1)

14.2 Determine the coordinates of A and B. (4)

14.3 Write down the values of *a* and *q*. (2)

14.4 Calculate the coordinates of C, the turning point of *g*. (3)

14.5 Write down the turning point of *t*, if $t(x) = g(-x) + 3$. (2)

14.6 For which values of x will $g'(x).h'(x) \geq 0$? (2)

[14]

QUESTION 15

Given $p(x) = 3^x$.

15.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

15.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axes. Show clearly all the intercepts with the axes and at least one other point on each graph. (4)

15.3 Determine the values of x for which $p^{-1}(x) \leq 3$ (4)

[10]

QUESTION 16

Two functions are defined by $f(x) = (x - 4)(x + 2)$ and $g(x) = 2x - 12$.

A is a point on f such that g is a tangent to f at A.

16.1 Write down the gradient of g. (1)

16.2 Calculate the coordinates of A. (5)

16.3 Determine the equation of the graph h which is the reflection of f about the $y-$ axis. (2)

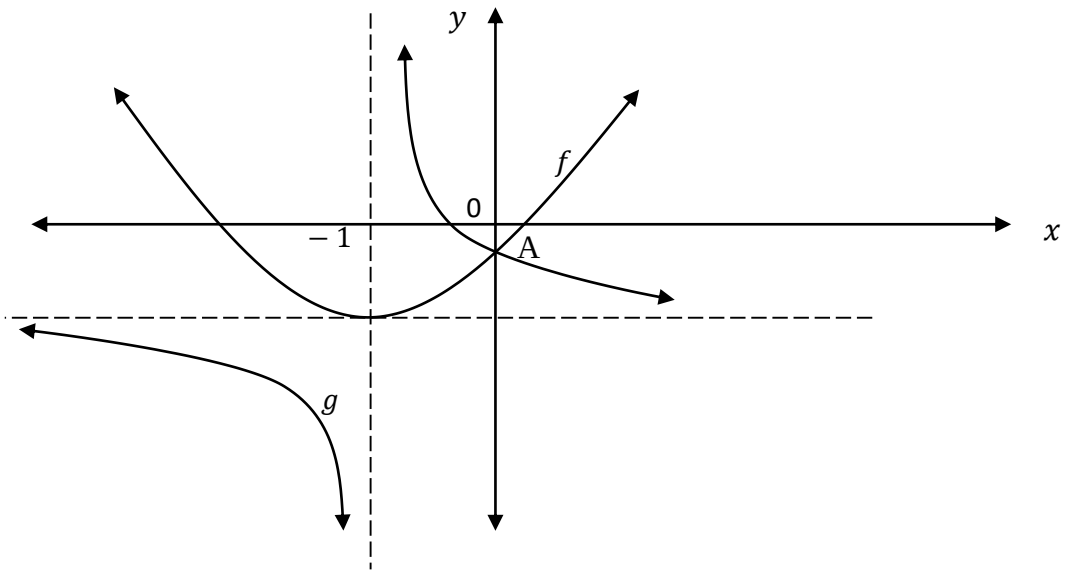
16.4 Determine value(s) of x for which $f(x).g(x) < 0$, given that $x > 0$. (2)

16.5 Determine g^{-1}, the inverse of g, in the form $y = \ldots$. (2)

[12]

QUESTION 17

The graphs of $f(x) = x^2 + 2x - 3$ and $g(x) = \frac{a}{x+p} + q$ are drawn below. A is the $y-$ intercept of both f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f. The equation of the vertical asymptote of g is $x = -1$.



17.1 Determine the equations of the asymptotes of g. (3)

17.2 Write down the coordinates of A. (1)

17.3 Determine the equation of g. (4)

17.4 Determine points of intersection of g with its axis of symmetry that has a positive gradient. (4)

17.5 Write down the range of $-f(x)$. (2)

[14]

QUESTION 18

Given: $f(x) = 3^{-x+1} - 3$

18.1 Draw the graph of f showing all asymptotes and intercepts with the axes. (3)

18.2 Calculate the $x-$ value when $y = 5$. (3)

18.3 If $h(x) = 3^x$; explain what transformations f has undergone to become h. (3)

[9]

QUESTION 19

Given: $f(x) = \frac{-3}{x-2} + 1$

19.1 Calculate the coordinates of the *y*-intercept of *f*. (2)

19.2 Calculate the coordinates of the *x*-intercept of *f*. (2)

19.3 Sketch the graph of *f* in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

19.4 Write down the range of *f*. (2)

19.5 Another function *h*, is formed by translating *f* 3 units to the right and 4 units down. Write down the equation of *h*. (2)

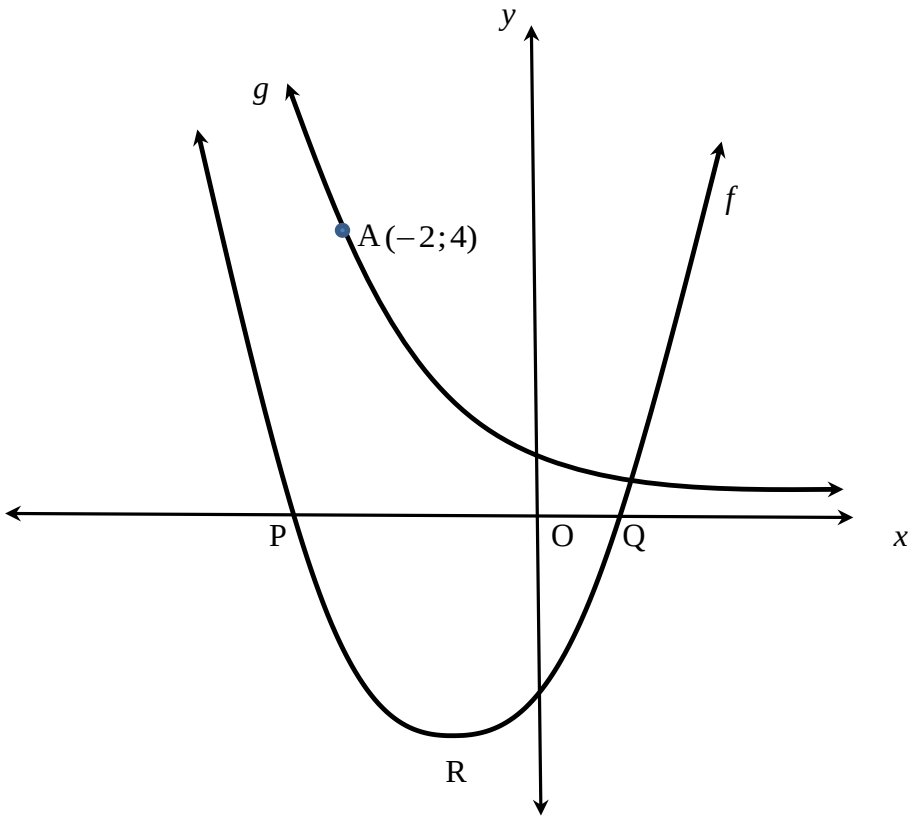
19.6 For which value(s) of *x* is $h(x) \leq -4$? (3)

19.7 Determine the equations of the asymptotes of $k(x) = \frac{3x-5}{x-1}$. (3)

[17]

QUESTION 20

The graphs of $f(x) = 2(x+1)^2 - 8$ and $g(x) = \left(\frac{1}{2}\right)^x$ are represented in the sketch below. P and Q are the *x*-intercepts of f and R is the turning point of f. The point $A(-2; 4)$ is a point on the graph of *g*.



20.1 Write down the equation of the axis of symmetry of *f*. (1)

20.2 Write down the coordinates of the turning point of *f*. (1)

20.3 Determine the length of PQ. (4)

20.4 Write down the equation of *k*, if *k* is the reflection of f in the *y*-axis. Give your answer in the form $y = ax^2 + bx + c$. (3)

20.5 Write down the equation of g^{-1}, the inverse of *g*, in the form $y = \ldots$ (1)

20.6 Sketch the graph of g^{-1} in your ANSWER BOOK, clearly showing the intercept with the axis as well as ONE other point on the graph of g^{-1}. (3)

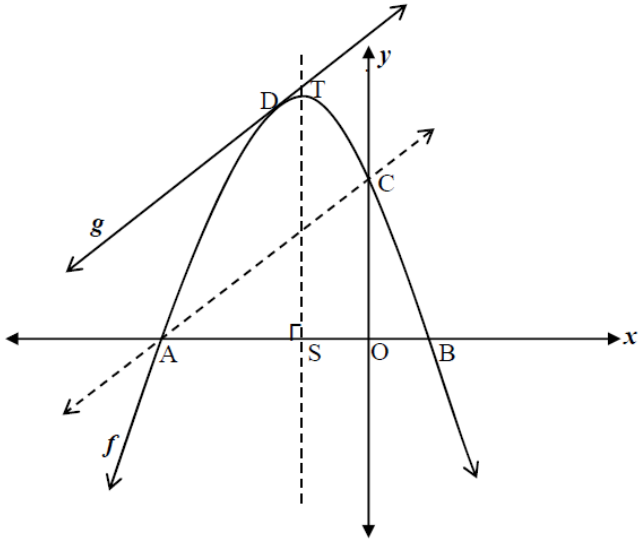
20.7 For which value(s) of *x* will:

20.7.1 $g^{-1}(x) \geq -2$ (2)

20.7.2 $x.f(x) < 0$ (4)

QUESTION 21

The diagram shows the graphs of $f(x) = -2x^2 - 4x + 6$ and $g(x) = mx + c$. A, B and C are the intercepts of f with the axes. T is the turning point of the graph of f. The graph of g is a straight line parallel to AC, and is a tangent to the graph of f at D.



21.1 Determine the lengths of OC and AB. (5)

21.2 Determine the equation of the axis of symmetry of the graph of f. (2)

21.3 Show that the length of ST = 8 units. (2)

21.4 Calculate the gradient of AC. (2)

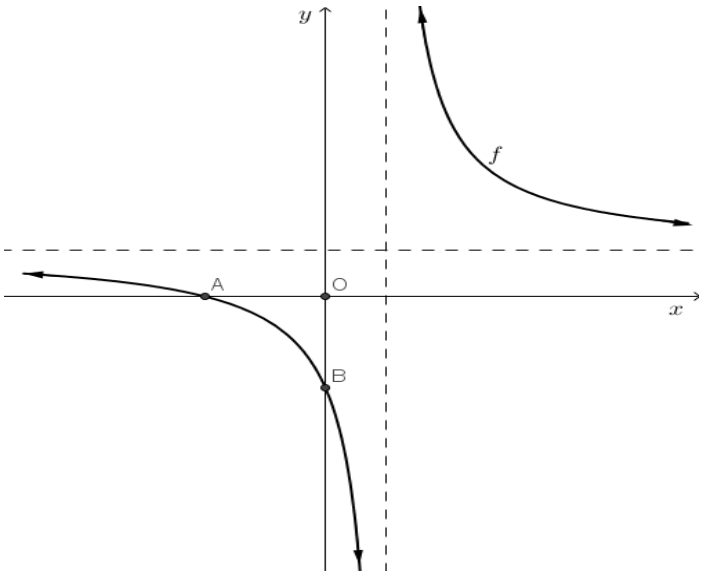
21.5 Hence, or otherwise, calculate the coordinates of D. (5)

21.6 For which value(s) of a will $f(a + t) = f(a - t)$ for all values of t? (2)

[18]

QUESTION 22

The sketch of $f(x) = \frac{2+x}{x-1}$ is drawn below.



22.1 Write down the equation of the vertical asymtote of *f*. (1)

22.2 Determine the coordinates of A, the *x*-intercept of the hyperbola. (2)

22.3 Calculate the area of ΔAOB. (3)

22.4 Show that *f* (*x*) can be rewritten as $f(x) = \frac{3}{x-1} + 1$ (2)

22.5 The graph of *f* is shifted such that point A lies on the origin. What are the coordinates of the point of intersection of the asymptotes of the new graph? (2)

[10]

QUESTION 23

23.1 Given: $f(x) = 2.2^x - 1$

23.1.1 Write down the range of f. (2)

23.1.2 $g(x) = f(x-1) + 1$. Write down the equation of g^{-1}, the inverse of g in the form y =... (2)

23.2 Given: $h(x) = -\sqrt{\frac{x}{3}}$; $x \geq 0$

23.2.1 If $k(x)$ is the inverse of h, give the equation of $k(x)$ (2)

23.2.2 Give the coordinates of the point of intersection of $h(x)$ and $k(x)$ (2)

[8]