education

MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

MATHEMATICS GRADE 12
REVISION PACK

Paper 1

EASY TO SCORE QUESTIONS

Purpose:

To assist learners to obtain a minimum mark of 50 % in Mathematics in NSC Examinations.

Focus:

Knowledge and Routine Procedures cognitive levels questions.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (2)

1.1.2 $2x^2 - 11x + 7 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5x^2 + 4 > 21x$ (5)

1.1.4 $2^{2x} - 6.2^x = 16$ (4)

1.2 Solve for x and y simultaneously:

$y + 1 = 2x$

$x^2 - xy + y^2 = 7$ (6)

1.3 The roots of a quadratic equation are given by $x = \frac{-5 \pm \sqrt{20 + 8k}}{6}$, where $k \in \{-3; -2; -1; 0; 1; 2; 3\}$.

1.3.1 Write down TWO values of k for which the roots will be rational. (2)

1.3.2 Write down ONE value of k for which the roots will be non-real. (1)

1.4 Calculate a and b if $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a(7^b)$ and a is not a multiple of 7. (4)

[27]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 12 = 0$ (3)

1.1.2 $x(x+3) - 1 = 0$ (Leave your answer in simplest surd form.) (3)

1.1.3 $x(4 - x) < 0$ (3)

1.1.4 $x = \frac{a^2 + a - 2}{a - 1}$ if $a = 888\ 888\ 888\ 888$ (2)

1.2 Solve the following equations simultaneously:

$y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$ (6)

1.3 Determine the range of the function $y = x + \frac{1}{x}$, $x \neq 0$ and x is real. (6)

[23]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+1)=0$ (2)

1.1.2 $\sqrt{x^3}=512$ (3)

1.1.3 $x(x-4)<0$ (2)

1.2 Given: $f(x)=x^2-5x+2$

1.2.1 Solve for x if $f(x)=0$ (3)

1.2.2 For which values of c will $f(x)=c$ have no real roots? (4)

1.3 Solve for x and y:

$$x=2y+2$$
$$x^2-2xy+3y^2=4$$ (6)

1.4 Calculate the maximum value of S if $S=\dfrac{6}{x^2+2}$. (2)

[22]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2-6x-16=0$ (3)

1.1.2 $2x^2+7x-1=0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2-25<0$. (4)

1.3 Solve for x and y:

$-2y+x=-1$ and $x^2-7-y^2=-y$ (6)

1.4 Evaluate: $\dfrac{3^{2018}+3^{2016}}{3^{2017}}$ (2)

1.5 Given: $t(x)=\dfrac{\sqrt{3x-5}}{x-3}$

1.5.1 For which values of x will $\dfrac{\sqrt{3x-5}}{x-3}$ be real? (3)

1.5.2 Solve for x if $t(x)=1$. (4)

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-1)(x+4)=0$ (2)

1.1.2 $2x^2+9x-14=0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{3-26x}=3x$ (4)

1.1.4 $(x-1)(x-4)>x+11$ (5)

1.2 Simplify fully:

$$\frac{\sqrt{16x^7}-\sqrt{25x^7}}{\sqrt{x}}$$ (3)

1.3 Solve simultaneously for x and y:

$xy=9$ and $x-2y-3=0$ (5)

1.4 Prove that $x^2+2xy+2y^2$ cannot be negative for $x, y \in R$. (4)

[27]

QUESTION 2

2.1 Given the following quadratic sequence: $-2 ; 0 ; 3 ; 7 ; \ldots$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

[18]

QUESTION 2

Given the geometric sequence: $-\frac{1}{4} ; b ; -1 ; \ldots\ldots$

2.1 Calculate the possible values of b. (3)

2.2 If $b = \frac{1}{2}$, calculate the 19^{th} term (T_{19}) of the sequence. (3)

2.3 If $b = \frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)

[12]

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a+(n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100-3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

QUESTION 3

Consider the infinite geometric series: $45 + 40,5 + 36,45 + \dots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION 2

2.1 Given the following geometric sequence: $30\ ;\ 10\ ;\ \frac{10}{3}\ ;\ldots$

2.1.1 Determine n if the n^{th} term of the sequence is equal to $\frac{10}{729}$. (4)

2.1.2 Calculate: $30+10+\frac{10}{3}+\ldots$ (2)

2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d. (4)

[10]

QUESTION 2

2.1 Given the quadratic pattern: $5\ ;\ 10\ ;\ 17\ ;\ 26\ ;\ \ldots$

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the formula for the n^{th} term of the pattern. (4)

2.1.3 Which term of the pattern will have a value of 1 765? (4)

2.2 The first 24 terms of an arithmetic series are: $35 + 42 + 49 + \ldots + 196$.

Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)

[15]

QUESTION 3

3.1 $6\ ;\ 6\ ;\ 9\ ;\ 15\ ;\ \ldots$ are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term (T_5) of the pattern. (1)

3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

QUESTION 3

The first three terms of an arithmetic sequence are $-1\ ;\ 2$ and 5.

3.1 Determine the n^{th} term, T_n, of the sequence. (2)

3.2 Calculate T_{43}. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of n. (3)

QUESTION 4

Given: $g(x)=\dfrac{6}{x+2}-1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\dfrac{6}{x+2}-1\geq -x-3$. (2)

[13]

QUESTION 4

Given: $f(x)=2^{-x}+1$

4.1 Determine the coordinates of the y-intercept of f. (1)

4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of f between the points on the graph where $x=-2$ and $x=1$. (3)

4.4 If $h(x)=3f(x)$, write down an equation of the asymptote of h. (1)

[8]

QUESTION 5

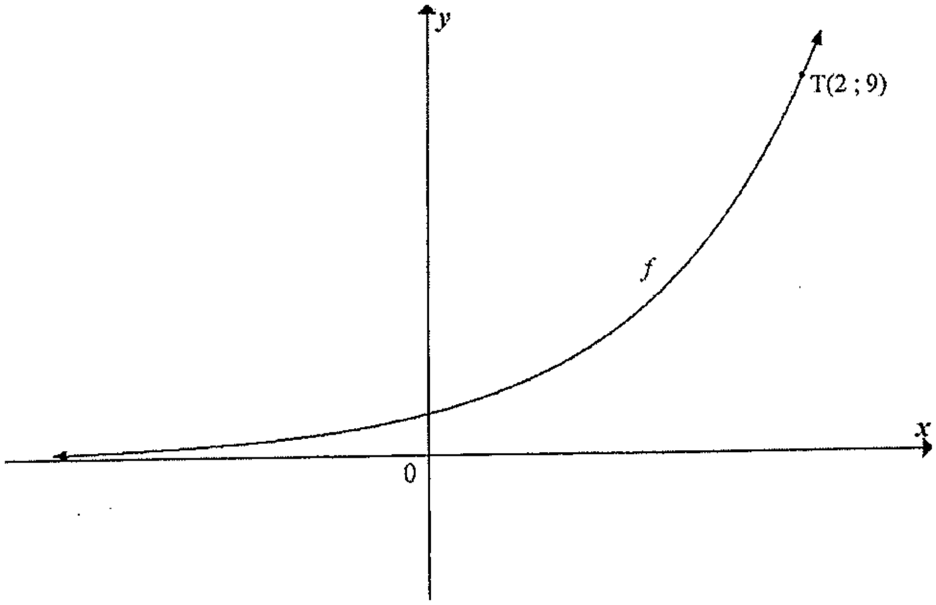
Given: $f(x)=x^2-5x-14$ and $g(x)=2x-14$

5.1 On the same set of axes, sketch the graphs of f and g. Clearly indicate all intercepts with the axes and turning points. (6)

5.2 Determine the equation of the tangent to f at $x=2\frac{1}{2}$. (2)

QUESTION 5

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on *f*.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

5.4 Is the inverse of f a function? Explain your answer. (2)
[7]

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = \ldots$ (3)

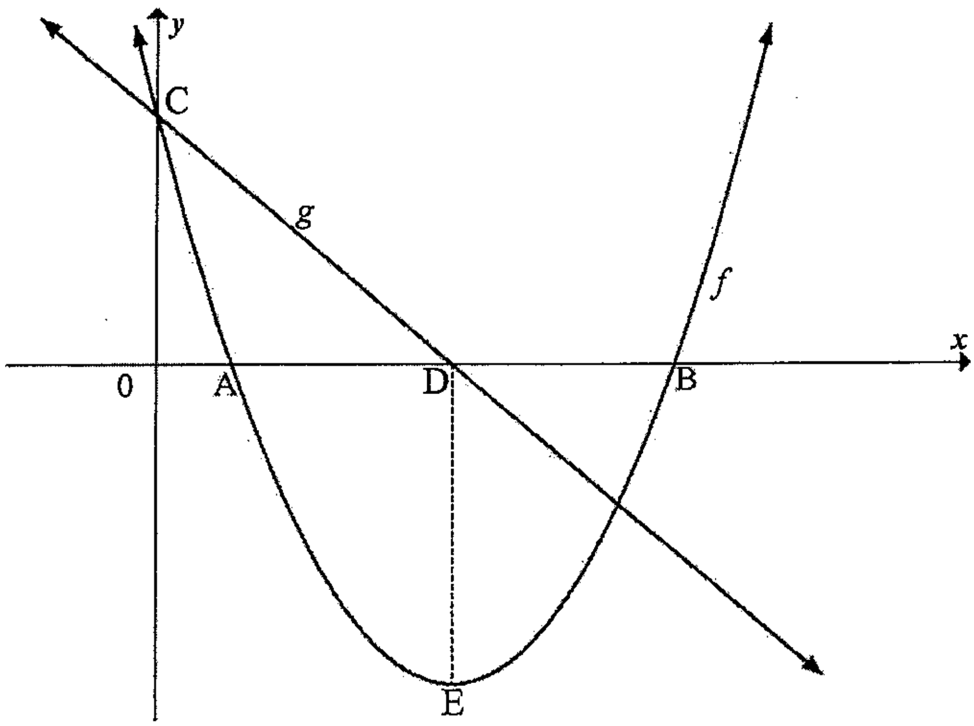
6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)
[8]

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.

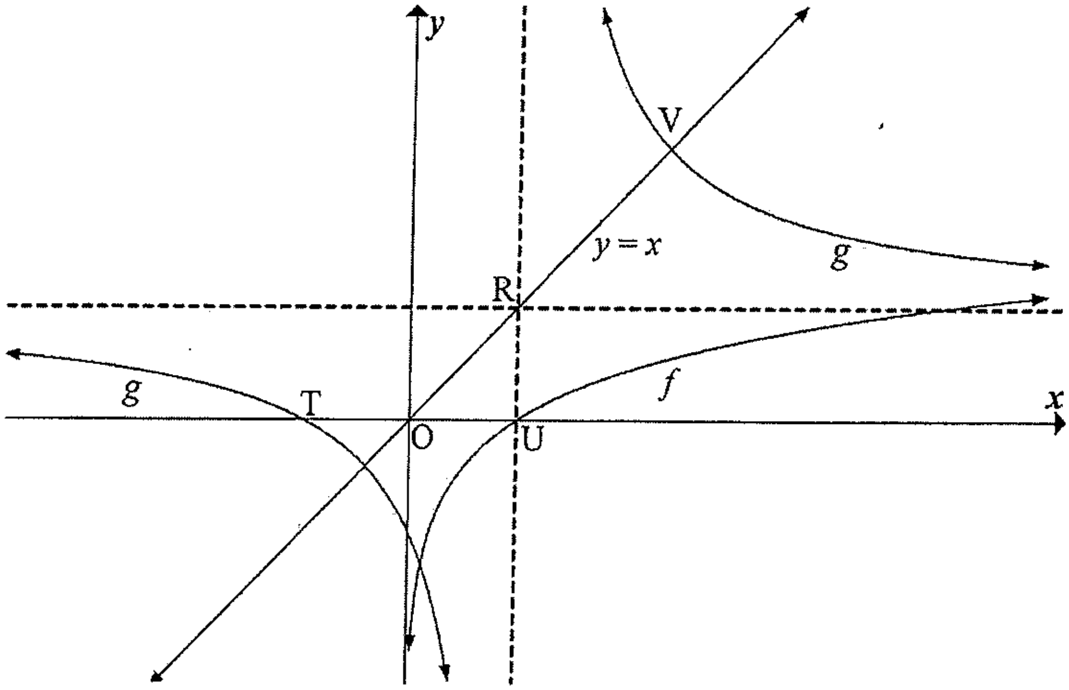


4.1	Write down the coordinates of E.	(2)
4.2	Calculate the coordinates of A.	(3)
4.3	M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M.	(3)
4.4	Determine the equation of g in the form $y = mx + c$.	(3)
4.5	Write down the equation of g^{-1} in the form $y = \ldots$	(3)

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



4.1 Write down the coordinates of U. (1)

4.2 Write down the equations of the asymptotes of g. (2)

4.3 Determine the coordinates of T. (2)

4.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = \ldots$ (2)

4.5 Write down the equation of the asymptote of $h(x-3)$. (1)

4.6 Calculate the coordinates of V. (4)

4.7 Determine the coordinates of T' the point which is symmetrical to T about the point R. (2)

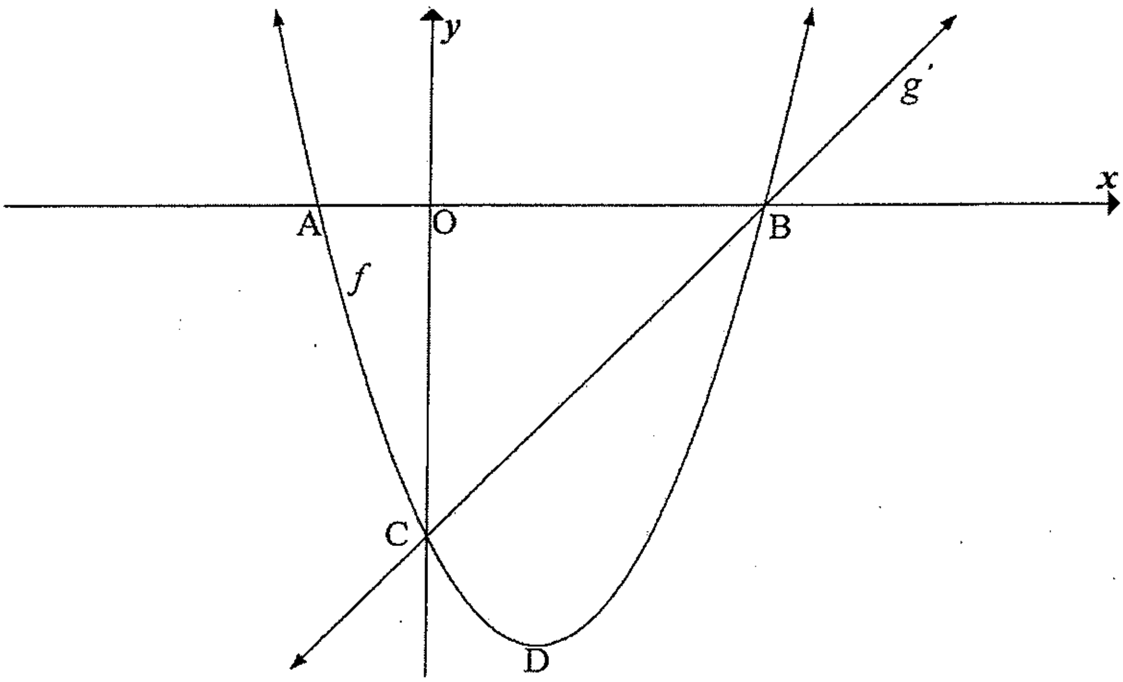
[14]

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1	Determine the coordinates of C.	(1)
5.1.2	Calculate the length of AB.	(4)
5.1.3	Determine the coordinates of D.	(2)
5.1.4	Calculate the average gradient of f between C and D.	(2)
5.1.5	Calculate the size of $O\hat{C}B$	(2)
5.1.6	Determine the values of k for which $f(x) = k$ will have two unequal positive real roots.	(3)
5.1.7	For which values of x will $f'(x) \cdot f''(x) > 0$?	(3)

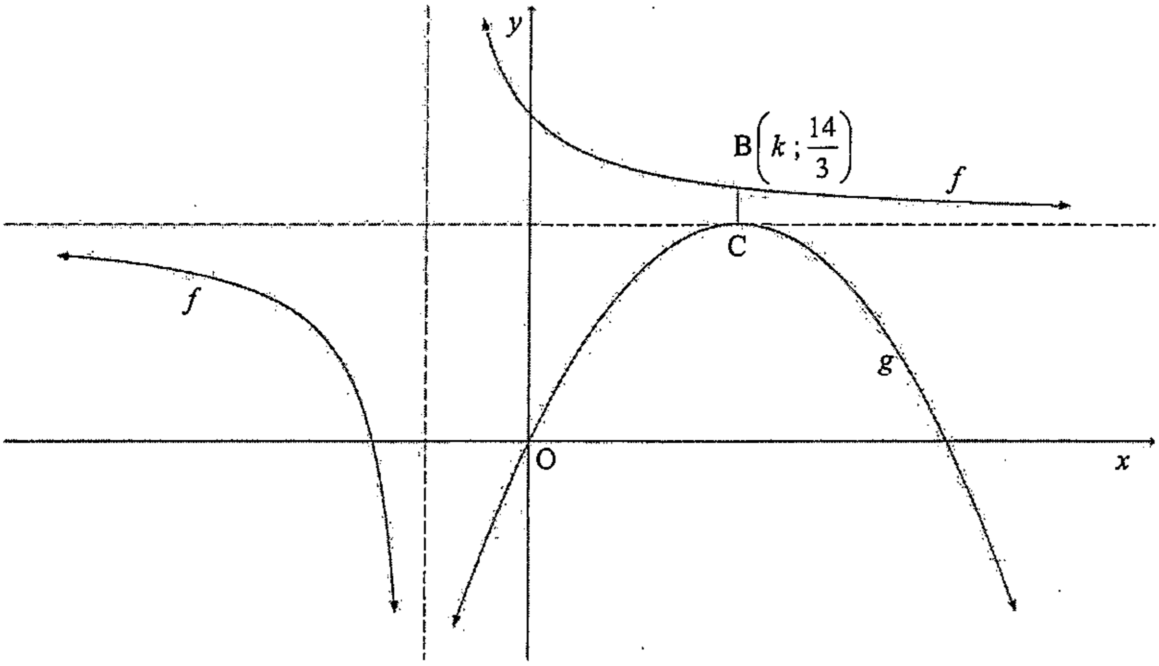
5.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

QUESTION 5

The graphs of $f(x)=\dfrac{2}{x+1}+4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptote of f.
- The graph of g passes through the origin.
- $B\left(k;\dfrac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.



5.1 Write down the domain of f. (2)

5.2 Determine the x-intercept of f. (2)

5.3 Calculate the value of k. (3)

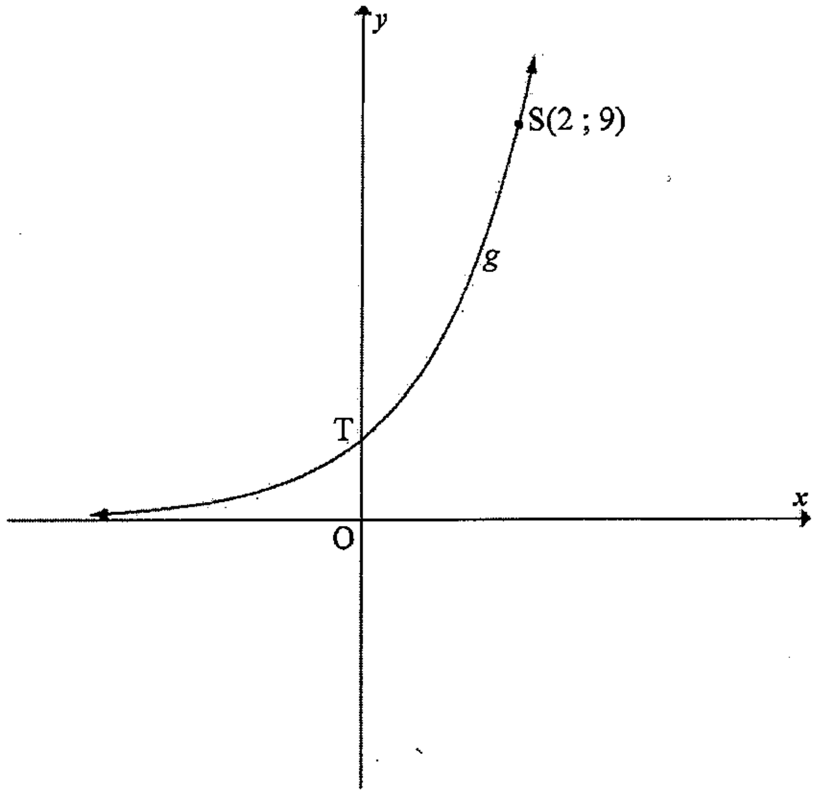
5.4 Write down the coordinates of C. (2)

5.5 Determine the equation of g in the form $y=a(x+p)^2+q$. (3)

5.6 For which value(s) of x will $\dfrac{f(x)}{g(x)}\leq 0$? (4)

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1 Write down the coordinates of T. (2)

5.2 Calculate the value of a. (2)

5.3 The graph h is obtained by reflecting g in the y-axis. Write down the equation of h. (2)

QUESTION 6

6.1 On the 2nd day of January 2015 a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

6.1.1 Calculate the book value of the printer on the 2nd day of January 2017. (3)

6.1.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)

QUESTION 6

6.1 Calculate how many years it will take for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method. (4)

QUESTION 7

7.1 A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method. (3)

QUESTION 7

7.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.

7.1.1 What is the quarterly interest rate for Diane's investment? (1)

7.1.2 Calculate the amount in Diane's savings account at the end of the 2 years. (3)

QUESTION 8

Given: $f(x)=2x^3-5x^2+4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

8.2 Prove that the equation $2x^3-5x^2+4x=0$ has only one real root. (3)

8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)

[14]

QUESTION 8

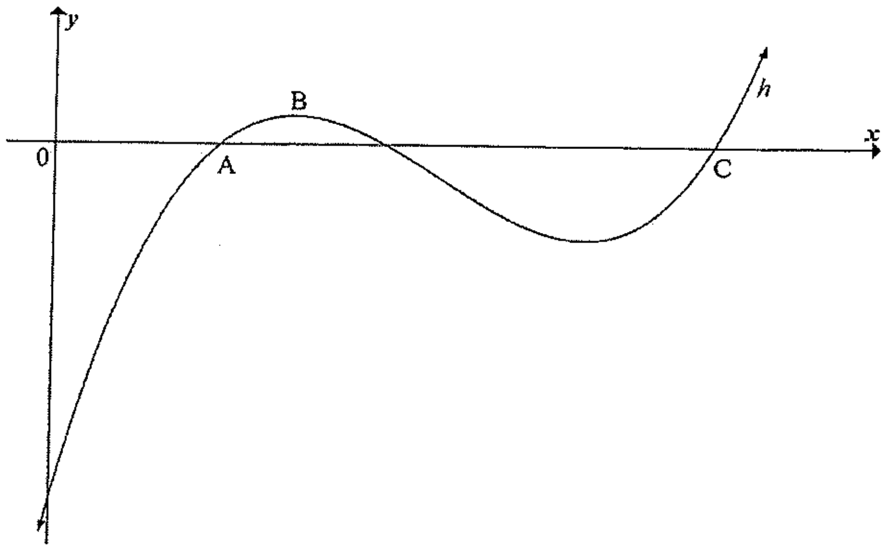
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

8.3.3 Calculate the coordinates of C. (4)

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

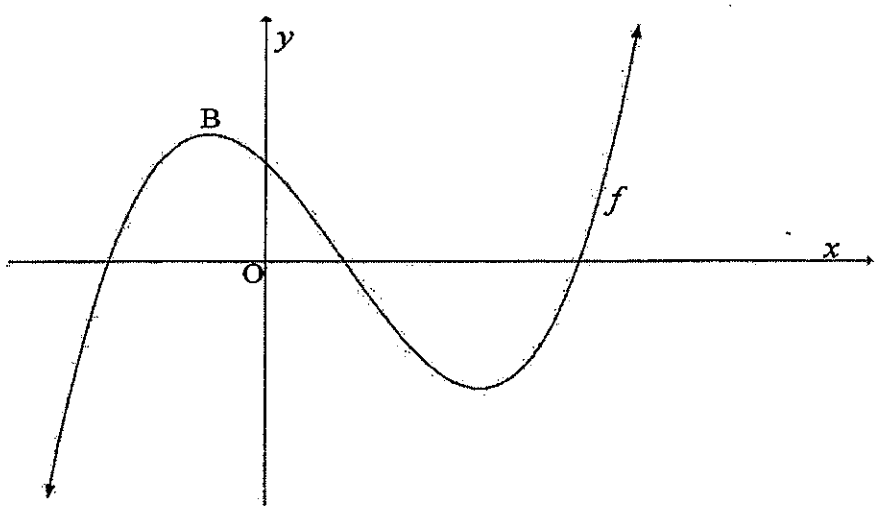
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

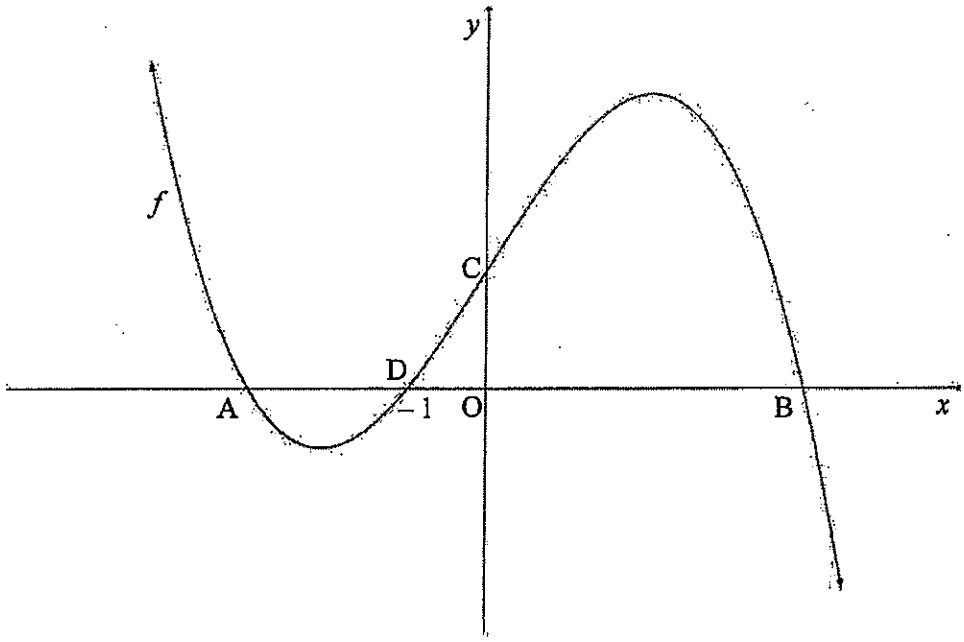
9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.
A, B and D(–1 ; 0) are the x-intercepts of f.
C is the y-intercept of f.



8.1 Write down the coordinates of C. (1)

8.2 Calculate the coordinates of A and B. (5)

8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)

8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)

[14]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

9.5 Write down the values of x for which $f'(x) < 0$. (2)

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=-x^2+4$. (5)

8.2 Determine the derivative of:

8.2.1 $y=3x^2+10x$ (2)

8.2.2 $f(x)=\left(x-\frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x)=2x^3-23x^2+80x-84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

QUESTION 10

10.1 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	b	c	80
Fruit juice	d	e	20
TOTAL	60	100	a

10.1.1 Write down the value of a. (1)

10.1.2 What is the probability that a randomly selected passenger is male? (2)

10.1.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of b. (4)

QUESTION 10

10.1 The events S and T are independent.

- $P(S \text{ and } T)=\frac{1}{6}$
- $P(S)=\frac{1}{4}$

10.1.1 Calculate $P(T)$. (2)

10.1.2 Hence, calculate $P(S \text{ or } T)$. (2)

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

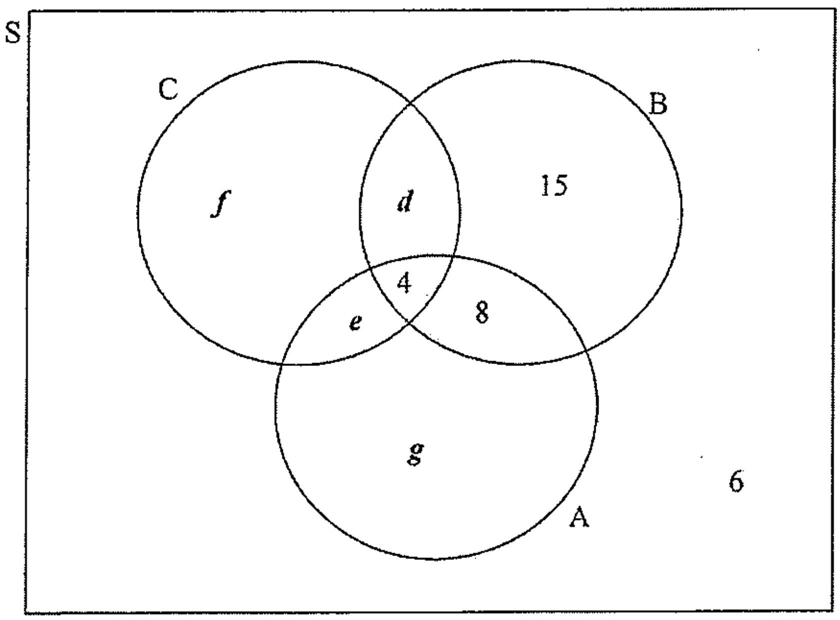
10.2.2 Repetition of digits IS allowed in the code (1)

QUESTION 10

10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



10.1.1 Use the given information and the Venn diagram to determine the values of d, e, f and g. (4)

10.1.2 For a randomly selected country, calculate:

(a) P(A and B and C) (1)

(b) P(A or B or C) (1)

(c) P(only C) (1)

(d) P(that a country uses exactly two methods) (1)

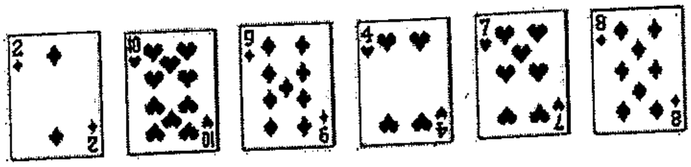
QUESTION 11

11.1 The letters of the word EQUATION are randomly used to form a new word consisting of five letters. How many of these words are possible if letters may not be repeated? (2)

11.2 It is given that two events, A and B, are independent. $P(A)=\frac{2}{5}$ and $P(B)=0{,}35$.

Calculate P(A or B). (4)

11.2 The cards below are placed from left to right in a row.



11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$