education
Department:
Education
PROVINCE OF KWAZULU-NATAL

KZN DEPARTMENT OF EDUCATION

MATHEMATICS
JUST IN TIME MATERIAL
GRADE 12
ANSWERS / SOLUTIONS

TERM 1 – 2020

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EUCLIDEAN GEOMETRY

Proportionality Theorem Learner Activity Answers

QUESTION 1

$\frac{x}{2} = \frac{x+2}{3}$ (Prop theorem, $MS \parallel QR$)

$3x = 2x + 4$

$x = 4$

QUESTION 2

$x = \frac{30}{8}$ (Prop theorem, $DE \parallel BC$)

QUESTION 3

3.1 (a) $\frac{AG}{GH} = 4$ (b) $\frac{AD}{DE} = 4$

3.2 $DE = \frac{5}{4}$

QUESTION 4

4.1 PN = 12 units

4.2 MR = 24 units

QUESTION 5

5.1 AE = 15 units

5.2 AH = 8 units

QUESTION 6

6.1 CB:CA = 5:9

Let CB = 5k and CA = 9k then BA=4k

BA=8 units

6.2 $\frac{\text{Area } \Delta BDC}{\text{Area } \Delta BED} = \frac{CD}{DE}$ (shared height)

But $\frac{CD}{DE} = \frac{CB}{BA} = \frac{10}{8}$ (line parallel to one side of Δ)

$\therefore \frac{\text{Area } \Delta BDC}{\text{Area } \Delta BED} = \frac{5}{4}$

6.3 from above $\frac{CD}{DE} = \frac{5}{4}$ so let CD = 5p and DE = 4p so CE = 9p

Now $\frac{CE}{CF} = \frac{CB}{CA} = \frac{5}{9}$ (line parallel to one side of Δ)

QUESTION 7

7.1 $A\hat{D}L = 90^\circ$ (Angle in semi-circle)

$A\hat{C}B = 90^\circ$

∴ DL∥CB (Converse corresponding angles are equal)

7.2 LC = LA (radii)

SD=SL=SA (radii)

But LA=SA+SL

Therefore LC=2SD

7.3 AS = SL and AL = LB (radii)

$\frac{SL}{AB} = \frac{1}{4}$

7.4 LB = 15 units (radius)

$\frac{9}{16} = \frac{LM}{15}$ (line parallel to one side of Δ)

LM = 8,44 units

QUESTION 8

8.1 In Δ ABQ

$\frac{BR}{RA} = \frac{BT}{TQ}$ (Prop Theorem RT∥AQ)

$\frac{1}{2} = \frac{k}{TQ}$

∴ TQ = 2k

8.2.1 In Δ CRT

$\frac{CP}{PR} = \frac{5k}{2k}$ (Prop Theorem RT∥AQ)

$\frac{CP}{PR} = \frac{5}{2}$

8.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC} = \frac{\text{Area } \Delta RCT}{\text{Area } \Delta BRC} \times \frac{\text{Area } \Delta BRC}{\text{Area } \Delta ABC}$ (triangle with equal altitude)

$= \frac{7}{8} \times \frac{1}{3}$

$= \frac{7}{24}$

QUESTION 9:

9.1 In ΔAPQ and ΔACD

$\frac{AP}{PD} = \frac{AQ}{QC} = \frac{1}{3}$ (AP//DC)

$DB = \frac{5}{9} AB$

$AD = \frac{4}{9} AB$

$PD = 3AP$

$DB = \frac{3}{9} AB$ and $AP = \frac{1}{9} AB$

$\frac{\text{Area } \Delta QPA}{\text{Area } \Delta QPB} = \frac{AP}{PB} = \frac{\frac{1}{9} AB}{\frac{8}{9} AB} = \frac{1}{8}$ (Same height, PQ)

9.2 In ΔDBE and ΔPBQ

$\frac{BE}{EQ} = \frac{BD}{DP} = \frac{\frac{5}{9} AB}{\frac{3}{9} AB} = \frac{5}{3}$ (PQ//D

QUESTION 10

10.1 $\frac{\text{Area } \Delta PRA}{\text{Area } \Delta QRA} = \frac{\frac{1}{2}.\ 3k\ .2a\ .\ \sin P}{\frac{1}{2}.\ 8k\ .2a\ .\ \sin P - \frac{1}{2}.\ 3k\ .\ 2a\ .\ \sin P}$

$= \frac{3\ .\ a\ .\ k\ .\ \sin P}{8\ .\ a.\ k\ .\ \sin P - 3\ .\ a\ .\ k\ .\ \sin P}$

$= \frac{3\ a\ k \sin P}{5\ a\ k \sin P}$

$= \frac{3}{5}$

10.2 $\frac{BD}{BQ} = \frac{AC}{CQ}$ (prop theorem CB ∥ AR)

$= \frac{1{,}5\ k}{5k + 1.5\ k}$

$= \frac{1{,}5\ k}{6{,}5\ k}$

$= \frac{3}{13}$

QUESTION 11:

11.1 $\frac{PQ}{RS} = \frac{PS}{QS} = \frac{QS}{QR} = \frac{2}{1}$

Δ PQS /// Δ SRQ

$P\hat{S}Q$ = alt $S\hat{Q}R$

PS // QR

11.2 $\frac{AREA\ \Delta QRS}{AREA\ PQRS} = \frac{\frac{1}{2} QS\ .QR\ .Sin\ S\hat{Q}R}{\frac{1}{2} QS.QR.Sin\ S\hat{Q}R + \frac{1}{2} PS.QS.Sin\ P\hat{S}Q}$

$= \frac{\frac{1}{2}(16)(8) Sin S\hat{Q}R}{\frac{1}{2}(16)(8+32) Sin S\hat{Q}R}$

$= \frac{8}{40}$

$= \frac{1}{5}$

11.3 In Δ POS and Δ ROQ

(i) $P\hat{S}O = O\hat{Q}R$ (PS//QR)

(ii) $P\hat{O}S = R\hat{O}Q$ (Vert. opp. ∠'s)

Similarity Theorem Learner Activity Answers

1.

(a) Similar (AAA)

(b) Similar (SSS)

2. $x = 5 \quad y = 9$

3.1 $\hat{C}_1 = \hat{A}_1$ (tan. chord)

$A_1 = A_2$ ($\angle$'s subt. by = chords)

$\hat{C}_1 = \hat{A}_2$

3.2 In Δ's BCS, DAC:

1. $\hat{C}_1 = \hat{A}_2$ (proven)
2. $\hat{B}_1 = \hat{D}$ (ext.$\angle$ cyclic quad.)
3. $\hat{S} = \hat{C}_3$ ($\angle$ sum Δ)

$\therefore \Delta BCS ||| \Delta DAC$ (AAA)

3.3 $\therefore \quad \frac{BC}{DA} = \frac{BS}{CD}$ (||| Δ's proven)

$\Rightarrow BC.CD = DA.BS$

but $\quad BC = CD$

$\therefore \quad BC^2 = DA.BS$

4.1 $\hat{A}_1$ (alt.$\angle$'s; BA//SP)

$\hat{R}_1$ (ext.$\angle$ cyclic quad.)

4.2 In Δ's PTS, PSR:

1. $\hat{S}_1 = \hat{R}_1 = x$ (proven)
2. $\hat{P}$ is common
3. $\hat{T}_2 = P\hat{S}R$ ($\angle$ sum Δ)

$\therefore \Delta PTS ||| \Delta PSR$ (AAA)

4.3.1 In Δ's PQT, PRQ:

1. $\hat{Q}_1 = \hat{R}_2$ (tan. chord)
2. $\hat{P}$ is common
3. $\hat{T}_1 = P\hat{Q}R$ ($\angle$ sum Δ)

$\therefore \Delta PQT ||| \Delta PRQ$ (AAA)

4.3.2 $\quad \frac{PQ}{PR} = \frac{PT}{PQ}$ (||| Δ's proven)

$\therefore PQ^2 = PR.PT$

4.4 $\quad \frac{PS}{PR} = \frac{PT}{PS}$ ($\Delta PTS ||| \Delta PSR$ proven)

$\Rightarrow \quad PS^2 = PR.PT$

but $PQ^2 = PR.PT$ (proven)

$\therefore \quad PQ^2 = PS^2$

$\Rightarrow \quad PQ = PS$

5.1 In Δ's DEC, BEA:

1. $\hat{D}_1 = \hat{B}_2$ (ext.∠ cyclic quad.)
2. $\hat{E}$ is common
3. $\hat{C}_3 = B\hat{A}E$ (∠ sum Δ)

$\therefore \Delta DEC ||| \Delta BEA$ (AAA)

5.2 In Δ's FAB, FCA:

1. $\hat{A}_1 = \hat{C}_1$ (ext.∠ cyclic quad.)
2. $\hat{F}$ is common
3. $\hat{B}_1 = F\hat{A}C$ (∠ sum Δ)

$\therefore \Delta FAB ||| \Delta FCA$ (AAA)

5.3 $\therefore \quad \frac{FA}{FC} = \frac{AB}{CA}$ (|||Δ's proven)

$\Rightarrow FA.CA = FC.AB$

6.1.1 $\hat{D}_2 = \hat{A}_2 + \hat{B}_1$ (ext.∠ Δ)

but $\hat{A}_3 = \hat{B}_1$ (tan. chord)

$\therefore \hat{D}_2 = \hat{A}_2 + \hat{A}_3$

6.1.2 $\hat{B}_2 = \hat{D}_1$ (tan. chord)

$\hat{D}_1 = \hat{A}_2$ (isos Δ)

$\therefore \hat{B}_2 = \hat{A}_2$

$\therefore$ DA // BC (alt.∠'s =)

6.2 $\frac{ED}{DB} = \frac{EA}{AC}$ (prop. int thm)

but $AB = DB$ (given)

$\therefore \quad \frac{ED}{AB} = \frac{EA}{AC}$

6.3 $EC : EA = 5 : 2 \Rightarrow EA : AC = 2 : 3$

$\therefore \frac{ED}{AB} = \frac{EA}{AC} \Rightarrow \frac{18}{AB} = \frac{2}{3}$

$\Rightarrow AB = 27$

6.4 In Δ's EDA, EAB

1. $\hat{A}_3 = \hat{B}_1$ (ext.∠ cyclic quad.)
2. $\hat{E}$ is common
3. $\hat{D}_2 = B\hat{A}E$ (∠ sum Δ)

$\therefore \Delta EDA ||| \Delta EAB$ (AAA)

6.5 $\therefore \frac{EA}{EB} = \frac{ED}{EA}$ (||| Δ's proven)

$\therefore EA^2 = ED.EB$

6.6 ΔEBC

7.1.1 $\hat{R}_3 = \hat{T}_2$ (∠'s in same segm.)

$\hat{T}_2 = \hat{T}_1$ (given)

$\hat{R}_4 = \hat{T}_1$ (tan. chord)

$\therefore \hat{R}_3 = \hat{R}_4$

7.1.2 In Δ's APR, MPT

1. $\hat{R}_3 = T_2$ (proven)
2. $\hat{P}_1 = \hat{P}_2$ (vert. opp.)
3. $\hat{A} = \hat{M}$ ($\angle$ sum Δ)

$\therefore \Delta APR \,|||\, \Delta MPT$ (AAA)

7.2 $\dfrac{AP}{MP} = \dfrac{PR}{PT}$

$AP.PT = PR.MP$

8.1 In Δ's JRS, JTK

1. $\hat{J}$ is common
2. $\hat{K}_1 = \hat{S}$ (ext.$\angle$ cyclic quad.)
3. $\hat{T}_1 = \hat{R}$ ($\angle$ sum Δ)

$\therefore \Delta JRS \,|||\, \Delta JTK$ (AAA)

8.2 $\therefore \dfrac{JR}{JT} = \dfrac{RS}{TK}$ (||| Δ's proven)

$\Rightarrow JR.TK = RS.JT$

but $RS = JT$ (given)

$\therefore JR.TK = RS^2$

9.1 In Δ's CXY, XDY

1. $\hat{X}_1 = \hat{D}_1$ (tan. chord)
2. $\hat{C}_2 = \hat{X}_2$ (tan. chord)
3. $C\hat{Y}X = X\hat{D}Y$ ($\angle$ sum Δ)

$\therefore \Delta CXY \,|||\, \Delta XDY$ (AAA)

$\therefore \dfrac{XY}{DY} = \dfrac{YC}{XY}$ (||| Δ's proven)

$\Rightarrow XY^2 = DY.YC$

9.2 $\hat{A}_1 = 2\hat{X}_1$ ($\angle$ at centre)

$\hat{B}_1 = 2\hat{D}_2$ ($\angle$ at centre)

$\hat{X}_1 = 2\hat{D}_2$ (proven)

$\therefore A_1 = B_1$

9.3 $A\hat{C}Y = Y_4$ (isos Δ; radii)

$\therefore \hat{A}_1 = 180° - 2.\hat{Y}_4$ ($\angle$ sum Δ)

sim'ly $\hat{B}_1 = 180° - 2.\hat{Y}_2$

but $A_1 = B_1$ (proven)

$\therefore Y_4 = Y_2$

In Δ's CAY, YBX

1. $A_1 = B_1$ (proven)
2. $\hat{Y}_4 = \hat{Y}_2$ (proven)
3. $A\hat{C}Y = B\hat{X}Y$ ($\angle$ sum Δ)

$\therefore \Delta CAY \,|||\, \Delta YBX$ (AAA)

10.1 In Δ's GHK, EGK

1. $\hat{K}$ is common
2. $\hat{G}_3 = \hat{E}_1$ (tan. chord)
3. $\hat{H}_1 = E\hat{G}K$ ($\angle$ sum Δ)

$\therefore \Delta GHK ||| \Delta EGK$ (AAA)

$\therefore \frac{GK}{EK} = \frac{HK}{GK}$ (||| Δ's =)

$\therefore GK^2 = EK.HK$

10.2 In Δ's GHK, EFG

1. $\hat{H}_1 = \hat{F}$ (ext. $\angle$ cyclic quad.)
2. $\hat{G}_3 = \hat{E}_1$ (tan. chord)
   $\hat{E}_1 = \hat{E}_2$ (given)
   $\therefore \hat{G}_3 = \hat{E}_2$
3. $\hat{K} = \hat{G}_1$ ($\angle$ sum Δ)

$\therefore \Delta GHK ||| \Delta EFG$ (AAA)

$\therefore \frac{FG}{HK} = \frac{EF}{HG}$ (||| Δ's =)

$\therefore FG.HG = EF.HK$

but FG = GH (conv. $\angle$'s subt. by = chords)

$\therefore HG^2 = EF.HK$

$GK^2 = EK.HK$ (proven)

$\therefore \frac{GK^2}{HG^2} = \frac{EK.HK}{EF.HK} \Rightarrow \frac{GK^2}{HG^2} = \frac{EK}{EF}$

11.1.1 $PR = 3\sqrt{2} \Rightarrow QR = 2\sqrt{2}$

$PQ : QR = \sqrt{2} : 2\sqrt{2}$

$= 1 : 2 = PK : KT$

$\therefore TR \,//\, KQ$ (conv. prop. int thm)

11.1.2 $K\hat{T}S = 90°$ (rad. tan.)

$\hat{R}_1 = 90°$ ($\angle$ in semi-circle)

$\hat{Q}_1 = 90°$ (corres. $\angle$'s; TR // KQ)

$= K\hat{T}S$

$\therefore$ TKQS cyclic (conv. ext. $\angle$ cyclic quad.)

11.1.3 In Δ's QRT, KTS

1. $R_1 = K\hat{T}S$ (proven)
2. $\hat{Q}_3 = \hat{K}_1$ ($\angle$'s in same segm.)
3. $\hat{T}_2 = \hat{S}_2$ ($\angle$ sum Δ)

$\therefore \Delta QRT ||| \Delta KTS$ (AAA)

11.1.4 $\Delta RTS ||| \Delta TPS$

In Δ's RTS, TPS

1. $\hat{S}$ is common
2. $\hat{T}_1 = \hat{P}$ (tan. chord)
3. $R_1 = K\hat{T}S$ (proven)

$\therefore \Delta RTS ||| \Delta TPS$ (AAA)

11.2.1 $\frac{ST}{PS} = \frac{RS}{ST}$ (||| Δ's proven)

$ST^2 = RS.PS$

$= \sqrt{2}.4\sqrt{2} = 8$

$ST = \sqrt{8} = 2\sqrt{2}$

11.2.2 $RT^2 = TS^2 - SR^2$ (Pythag. thm)

$= 8 - 2 = 6$

$RT = \sqrt{6}$

$\frac{KT}{QR} = \frac{ST}{RT}$ ($\Delta QRT ||| \Delta KTS$ proven)

$KT = \frac{ST.QR}{RT}$

$= \frac{2\sqrt{2}.2\sqrt{2}}{\sqrt{6}} = \frac{8}{\sqrt{6}}$

12.1 $\hat{C}_2 = 90°$ (∠ in semi-circle)

$= \hat{M}_2$ (given)

$\therefore NQ // CD$ (corres. ∠'s =)

12.2 $\hat{N} = \hat{C}_1$ (corres. ∠'s; NQ // CD)

$\hat{C}_1 = \hat{A}_2$ (tan. chord)

$\therefore \hat{N} = \hat{A}_2$

$\therefore$ ANCQ cyclic (conv. ∠'s in same segm.)

12.3.1 In Δ's PCD, PAC

1. $\hat{P}$ is common
2. $\hat{C}_1 = \hat{A}_2$ (tan. chord)
3. $\hat{D}_1 = A\hat{C}P$ (∠ sum Δ)

$\therefore \Delta PCD ||| \Delta PAC$ (AAA)

12.3.2 $\frac{PC}{PA} = \frac{PD}{PC}$ (||| Δ's proven)

$PC^2 = PD.PA$

12.4 In Δ's BCD, NBC

1. $B\hat{C}D = \hat{B}_1$ (alt. ∠'s; NQ // CD)
2. $\hat{D}_2 = \hat{C}_4$ (tan. chord)
3. $\hat{B}_2 = \hat{N}$ (∠ sum Δ)

$\therefore \Delta BCD ||| \Delta NBC$ (AAA)

$\therefore \frac{BC}{NB} = \frac{CD}{BC}$ (||| Δ's =)

$\Rightarrow BC^2 = CD.NB$

12.5 $PC^2 = PD.PA$ (proven)

$\therefore \frac{PC^2}{BC^2} = \frac{PD.PA}{CD.NB}$

but $PC = MC$ (given)

$\therefore \quad \frac{MC^2}{BC^2} = \frac{AP.DP}{CD.NB}$

$\Rightarrow \frac{BC^2 - BM^2}{BC^2} = \frac{AP.DP}{CD.NB}$ (Pythag. thm)

$\Rightarrow \quad 1 - \frac{BM^2}{BC^2} = \frac{AP.DP}{CD.NB}$

13.1 $\hat{P}_1 = x$ (tan. chord)

$\hat{S}_2 = 90°$ ($\angle$ in semi-circle)

$\hat{E}_2 = 90°$ (chord thm)

$\therefore RS // ON$ (alt. $\angle$'s =)

$\hat{N}_1 = \hat{P}_1 = x$ (corres. $\angle$'s; RS // ON)

$\hat{N}_2 = \hat{N}_1 = x$ ($\Delta SNE \equiv \Delta PNE$)

13.2.1 In Δ's SEN, OPN

1. $\hat{N}_1 = \hat{N}_2$ (proven)
2. $\hat{P} = 90°$ (rad. tan)

   $= \hat{E}_1$ (proven)
3. $\hat{S}_3 = \hat{O}_1$ ($\angle$ sum Δ)

$\therefore \Delta SEN ||| \Delta OPN$ (AAA)

13.2.2 $\therefore \frac{SN}{ON} = \frac{EN}{PN}$ (||| Δ's proven)

$\Rightarrow SN.PN = EN.ON$

but SN = PN (tans from common pt)

$\Rightarrow PN^2 = EN.ON$

13.3 In Δ's PEN, OEP

1. $\hat{E}_4 = \hat{E}_1 = 90°$ (proven)
2. $\hat{P}_2 = \hat{S}_3$ (isos Δ)

   $= \hat{O}_1$ (proven)
3. $\hat{N}_2 = \hat{P}_1$ ($\angle$ sum Δ)

$\therefore \Delta PEN ||| \Delta OEP$ (AAA)

$\therefore \quad \frac{PE}{OE} = \frac{EN}{PE}$ (||| Δ's proven)

$\Rightarrow \quad PE^2 = OE.EN$

13.4 $\frac{OE}{RS} = \frac{PE}{PS} = \frac{1}{2}$

13.5 $\frac{PN^2}{PE^2} = \frac{EN.ON}{OE.EN} = \frac{ON}{OE} = \frac{ON}{\frac{1}{2}RS} = \frac{2ON}{RS}$

TRIGONOMETRY

MIXED EXERCISES SOLUTIONS

1.1 OP = $\sqrt{61}$ units

1.2 $\sin\theta = \frac{5}{\sqrt{61}}$

1.3 $\frac{6}{5}$

1.4 $-\frac{6}{5}$

1.5 $\frac{2\cos^2\theta - 1}{2\sin\theta\cos\theta}$

2.1 $\cos\alpha = -\frac{5}{35}$

2.2 $\frac{25}{169}$

2.3 $-\frac{120}{119}$

3. -1

4.1 $\frac{24}{25}$

4.2 $\frac{7}{5}$

5. $-\frac{171}{221}$

6.1 1
6.2 0

7.1 $\sin 52^\circ = \sqrt{1-m^2}$
7.2 $\sin 38^\circ = m$
7.3 $\cos(-128^\circ) = -m$
7.4 $\cos 8^\circ = \frac{m + \sqrt{3-3m^2}}{2}$
7.5 $\cos 104^\circ = 2m^2 - 1$
7.6 $\frac{m-1}{2}$

8.1 $1-2k^2$

8.2 $2k\sqrt{1-k^2}$

8.3 $\sqrt{1-k^2}$

8.4 $\dfrac{\sqrt{1-k^2}+\sqrt{3}k}{2}$

8.5 $\dfrac{k}{\sqrt{1-k^2}}$

9. $-\dfrac{t}{8\sqrt{1-t^2}}$

10.1 m
10.2 m

11.1 $tanx$
11.2 $sinx$

12.1 1

12.2 $-\dfrac{\sqrt{3}}{2\cos x}$

14.1. $\alpha = 30° + k.360°$ or $\alpha = 150° + k.360°$ or $\alpha = 90° + k.360°$ or $\alpha = 270° + k.360° k \in z$

14.2 $\theta = \pm 55° + k.90°; k \in z$

14.3 $y = 71{,}57° + k.360°$ or $y = 135° + k.360°; k \in z$

14.4 $\alpha = \pm 60° + k.360°$ or $\alpha = \pm 180° + k.360°; k \in z$

14.5 $x = 45° - k.360°; k \in z$ or $x = 225° - k.360°; k \in z$

14.6 $x = 21° + k.360°; k \in z$ or $x = 153° + k.180°; k \in z$

14.7 $x = 0° + k.180°; k \in z$ or $x = 120° + k.180°; k \in z$

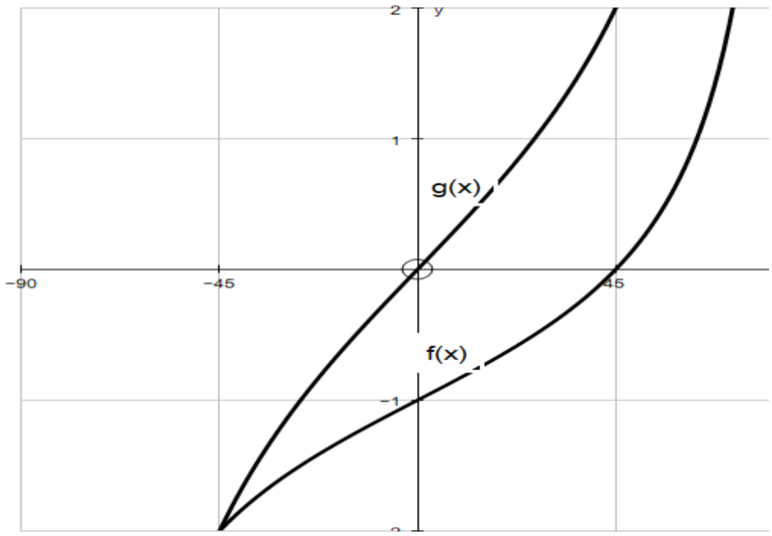
14.8 $x = -18{,}43°; \pm 161{,}57°$ or $x = \pm 45°; -135°; k \in z$

14.9 $x = 0° + k.360°$ or $x = 180° + k.360°$ or $x = 30° + k.360°$ or $x = 150° + k.360°; k \in z$

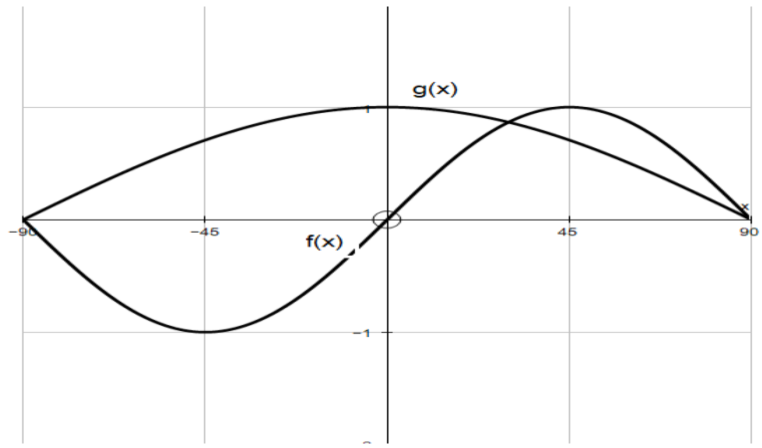
14.10 $x = 30° + k.180°; k \in z$

14.11 $x = [30°; -30°]$

15.



16.1



16.2 $x = 30^\circ$

16.3 $90^\circ \angle x \angle 30^\circ$

16.4 $-45^\circ \angle x \angle 45^\circ$

17.1

17.2 (i) $-150<x<90$

(ii) $0 \angle x \angle 180^\circ$

(iii) $-45^\circ \angle x \angle 45^\circ$

18.1 (i) b = -1

(ii) 1

(iii) k = 2

(iv) $\left(-45°;0°\right)$ and $\left(45°;0\right)$

18.2 (i) $-45° \angle x \angle 45°$

-45

19. $PQ = \dfrac{2A\tan x}{a\sin y}$

20. $AP = \dfrac{10\tan\alpha}{\cos\beta}$

21. $h = \tan\beta\left(x\sqrt{2(1+\cos 2x}\right)$

22. $HG = \dfrac{P\cos\beta\sin(\theta + x)\tan\alpha}{\sin\theta}$