

Gr 9 Maths: Content Area 2

Patterns, Algebra & Graphs

QUESTIONS

Mainly past ANA exam content

- Patterns
- Algebraic Expressions
- Factorisation
- Algebraic Equations
- Graphs



PATTERNS

(Solutions on page A1)

The most common weakness that learners have when doing patterns is determining the general term.

What is 'the general term (or rule)'?

The general term (or rule) of a sequence gives us the value of any term if we know the position.

e.g. If the 'general term' of a sequence is $2n$, we are saying that: the n^{th} term is $2n$

So: the 1^{st} term is $2(1) = 2$ <

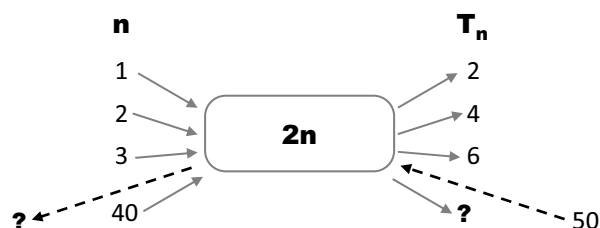
the 2^{nd} term is $2(2) = 4$ <

the 3^{rd} term is $2(3) = 6$ <

...

& the 40^{th} term is $2(40) = 80$ <

Note: n is the position of the term



As we see,
any term can
be 'generated'.

In REVERSE: If the term, $T_n = 50$, what will n be?
i.e. Which term has the value 50?

The 25^{th} term! So, $n = 25$.

In TABLE FORM:

n	1	2	3		?		40	
$2n$					50		?	

50 is the 25^{th} term <

The 40^{th} term is **80** <

The Questions

1.1 The next number in the sequence

1 ; 9 ; 25 ; ... is

A 33

B 36

C 49

D 50



(1)

1.2 The two missing numbers in the sequence below

18 ; 36 ; ____ ; 72 ; ____ ; 108 are

A 38 and 74

B 42 and 78

C 54 and 90

D 45 and 81

(1)

1.3 Which number is missing in the sequence?

1 ; $\frac{1}{2}$; $\frac{1}{4}$; ... ; $\frac{1}{16}$

A $\frac{1}{8}$

B $\frac{1}{10}$

C $\frac{1}{12}$

D $\frac{1}{14}$

(1)

1.4 Which number is missing in the number sequence?

$\frac{1}{3}$; ... ; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$

A $\frac{1}{6}$

B $\frac{1}{8}$

C $\frac{1}{9}$

D $\frac{1}{10}$

(1)

1.5 The next number in the sequence

3 ; 6 ; 11 ; 18 ; ... is

A 25

B 24

C 26

D 27

(1)



Get to know and understand the general term . . .

2.1 Write down the 1^{st} 3 terms of a sequence if the general term is:

a) $3n$

b) $5n$

c) $3n + 1$

d) $5n - 2$

e) n^2

f) n^3

(18)

2.2 Write down the 12^{th} term for each case in Question 2.1.

(6)

3. Use the table to answer the questions that follow:

x	1	2	3	4	a	10
y	3	6	9	12	21	b

3.1 Write down the relationship between x and y . (1)

3.2 Write down the values of a and b . (2)

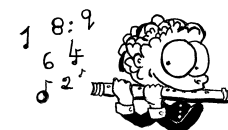
4. Study the given number sequence and answer the questions that follow:

3 ; 10 ; 17 ; 24 ; 31 ; ...

4.1 Determine the constant difference between the consecutive terms in the number sequence. (1)

4.2 Write down the next two terms in the sequence. (2)

4.3 Write down the general term of the sequence. (2)



5.1 Complete the table below:

Position in pattern	1	2	3	4	5
Term	1	8	27		

5.2 Write down the general term T_n of the above number pattern.

5.3 If $T_n = 512$, determine the value of n .

6.1 Write down the next TWO terms in the number sequence 7 ; 11 ; 15 ; ...

6.2 Write down the general term T_n of the above number sequence.

$T_n =$

6.3 Calculate the value of the 50th term.



7.1 Write down the next two terms in the given sequence:

3 ; 8 ; 13 ; ____ ; ____

7.2 Describe the pattern in Question 7.1 in your own words.

7.3 Write down the general term of the given sequence in the form

$T_n =$ _____.

7.4 Which term in the sequence is equal to 38?

8.

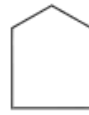


Figure 1

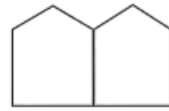


Figure 2

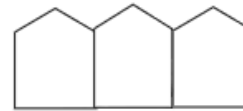


Figure 3

8.1 Study the above diagram pattern and complete the table.

Figure	1	2	3	4
Number of sides	5	9		

8.2 Describe the pattern in your own words.

8.3 Write down the general term of the pattern in the form, $T_n =$ _____

9. Matchsticks are arranged as shown in the following figures:

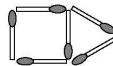


Figure 1

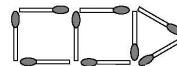


Figure 2

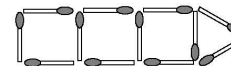


Figure 3

9.1 Determine the number of matchsticks in the next figure if the pattern is continued.

9.2 Write down the general term of the given sequence of the matchsticks in the form

$T_n =$ _____.

9.3 Determine the number of matchsticks in the 20th figure.

10. A tiler creates the following patterns with black and white tiles:

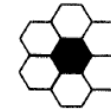


Figure 1

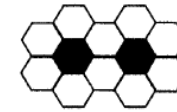


Figure 2

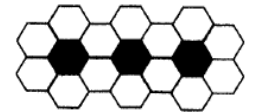


Figure 3

10.1 Study the above diagram pattern and complete the table.

Figure	1	2	3	4
Number of black tiles	1	2	3	4
Number of white tiles	6			

10.2 Write down the general term, T_n , of the number sequence created by the number of white tiles.

11. Natural numbers are arranged as shown below.

$$\begin{aligned} 1 + 2 &= 3 \\ 4 + 5 + 6 &= 7 + 8 \\ 9 + 10 + 11 + 12 &= 13 + 14 + 15 \end{aligned}$$

Find the first number in the 20th row if the pattern is continued another 17 times.

For further practice in this topic –
see The Answer Series
Gr 9 Mathematics 2 in 1 on p. 1.15



ALGEBRAIC EXPRESSIONS

(Solutions on page A3)

Terminology

1. Given the expression $2x - 7 - 8x^2$.
 - 1.1 Write down the coefficient of x^2 . (1)
 - 1.2 Write down the constant term. (1)
 - 1.3 Write the expression in descending powers of x . (1)
 - 1.4 Write down the exponent in the term $2x$. (1)
 - 1.5 Calculate the value of the expression $2x - 7 - 8x^2$ if $x = \frac{1}{2}$. (2)
2. Given the expression: $\frac{x-y}{3} + 4 - x^2$

Circle the letter of the **incorrect** statement.

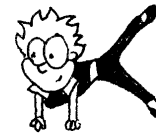
 - A The expression consists of 3 terms.
 - B The coefficient of x is 1.
 - C The coefficient of x^2 is -1 .
 - D The expression contains 2 variables. (1)

Substitution

- 3.1 Calculate the value of $2x^3 - 3x^2 + 9x + 2$ if $x = -2$. (4)
- 3.2 If $x = -1$, calculate the value of y if $y = 2x^2 - 3x + 5$. (2)
- 3.3 If $a = 2$, $b = -3$ and $c = \frac{1}{2}$, find the value of $\frac{5ac}{b}$. (4)
- 3.4 If $x = 2$ and $y = -3$, calculate the value of $3x^2 - 2xy - y^2$. (5)

Addition, Subtraction, Multiplication and Division

4. Answer the following questions.
 - 4.1 Add $2b - 3a - c$ and $a - 4b + 2c$. (3)
 - 4.2 Multiply $5x^2 - 3x$ by $-4x^2$. (2)
 - 4.3 Divide $8a + 16a^2 - 4a^3$ by $2a$. (3)
 - 4.4 Simplify $-3(x)(x) + 2x(-x)$. (3)
 - 4.5 Multiply $4m - 3mn^5 + 2n$ by $-3m^2n$. (3)
 - 4.6 Subtract $-2ab$ from $3ab$.
5. Simplify:
 - 5.1 $(3x)^3 + 2x^3$ (2)
 - 5.2 $(2x)^2 \times 3x^2$ (2)
 - 5.3 $(a^2b^3)^2 \cdot ab^2$ (2)
 - 5.4 $2^5 - 1^5$ (2)



Fractions (+, -, ×, ÷)

- 5.5 $\frac{x}{2} + \frac{x}{5}$ (3)
- 5.6 $\frac{5a}{8} - \frac{5a}{12}$ (3)
- 5.7 $\frac{a^2b^2}{ac^2} \times \frac{4a^2bc}{20b^3}$ (2)
- 5.8 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$ (3)
- 5.9 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$ (4)

- 5.10 $\frac{x-y}{y+x} \times \frac{(x+y)^2}{x-y}$ (2)
- 5.11 $\frac{x-2}{2x} - \frac{x-3}{3x}$ (5)
- 5.12 $\frac{4x^2}{2a^2} \div \frac{4x}{2a^2}$ (2)
- 5.13 $\frac{15x^2y^3 + 9x^2y^3}{8x^2y^3}$ (2)
- 5.14 $\frac{5a^2b}{3ab} \div \frac{20a^3b}{27}$ (5)
- 5.15 $\frac{5}{b} - \frac{4}{a} - \frac{a-b}{ab}$ (5)
- 5.16 $\frac{3a^{-2}b \times 24b^{-1}a^{-1}}{9a^{-4}b^{-3}}$ (3)
- 5.17 $\frac{x^2}{2} + \frac{2x^2}{3} - \frac{7x^2}{6}$ (3)
- 5.18 $\frac{6x^2}{7xy} \times \frac{3y^3}{2x}$ (2)

Square roots and cube roots

- 5.19 $\sqrt{225x^4} - \sqrt[3]{125x^6}$ (5)
- 5.20 $\sqrt{16x^{16} \times 25x^4}$ (3)
- 5.21 $\sqrt[3]{27x^{27}}$ (2)
- 5.22 $\sqrt{16a^2 + 9a^2}$ (2)



Note: $x^2 = x \times x$... x multiplied by itself!

STUDY THESE PRODUCTS OVER AND OVER AGAIN

So:

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y) \\ &= x^2 + xy + xy + y^2 \\ &= x^2 + 2xy + y^2\end{aligned}$$

So: $(x + y)^2$ **does not equal** $x^2 + y^2$

And:

$$\begin{aligned}(x - y)^2 &= (x - y)(x - y) \\ &= x^2 - xy - xy + y^2 \\ &= x^2 - 2xy + y^2\end{aligned}$$

So: $(x - y)^2$ **does not equal** $x^2 - y^2$

And, finally

$$\begin{aligned}(x + y)(x - y) &= x^2 - xy + xy - y^2 \\ &= x^2 - y^2 \quad \dots \text{the difference} \\ &\quad \text{of 2 squares!}\end{aligned}$$



6. Determine the following products and simplify if necessary.

6.1 $4ab(5a^2b^2 + 2ab - 3)$ (3)

6.2 $3a^2bc^2(3a^2 - 4b - c)$ (3)

6.3 $(x + 5)(x + 2)$ (3)

6.4 $(x - 2)(x - 3)$ (3)

6.5 $(x + 7)(x - 1)$ (3)



6.6 $(2x - 3)(x + 1)$ (3)

6.7 $x(x + 2) - (x - 1)(x - 3)$ (4)

6.8 $(x - 3)^2 - x(x + 4)$ (4)

6.9 $(2x - 1)^2 - (x + 1)(x - 1)$ (4)

6.10 $2(x + 2)^2 - (2x - 1)(x + 2)$ (4)

7. Complete the following products:

7.1 $(x + 5)^2 = (x + 5)(x + 5) = \dots\dots\dots$

7.2 $(p - 3)^2 = (p - 3)(p - 3) = \dots\dots\dots$

7.3 $(2a + 3)^2 = \dots\dots\dots$

7.4 $(4x - 1)^2 = \dots\dots\dots$

7.5 $(x + 5)(x - 5) = \dots\dots\dots = \dots\dots\dots$

7.6 $(p - 3)(p + 3) = \dots\dots\dots = \dots\dots\dots$

7.7 $(2a + 3)(2a - 3) = \dots\dots\dots = \dots\dots\dots$

7.8 $(4x - 1)(4x + 1) = \dots\dots\dots = \dots\dots\dots$

7.9 $(x + 3)(x + 4) = \dots\dots\dots$

7.10 $(x - 3)(x - 4) = \dots\dots\dots$

7.11 $(x + 3)(x - 4) = \dots\dots\dots$

7.12 $(x - 3)(x + 4) = \dots\dots\dots$

(24)

8.1 The value of $-x^2 - 2(2x - 1)$ when $x = -2$ is ...

- A 6 B 1
C -6 D -1 (2)

8.2 The LCM of $5a^3$ and $60a^2$ is ...

- A $60a^5$ B $30a^3$
C $60a^3$ D $300a^6$ (2)

8.3 $\frac{x}{y} - 1 =$

- A $\frac{y - x}{x}$ B $\frac{y - x}{y}$
C $x - y$ D $\frac{x - y}{y}$ (2)

8.4 $\left(\frac{x}{3} - 3y\right)\left(\frac{x}{3} + 3y\right) =$

- A $\frac{x^2}{9} + 3xy - 9y^2$ B $\frac{x^2}{9} + xy - 9y^2$
C $\frac{x^2}{9} + 9y^2$ D $\frac{x^2}{9} - 9y^2$ (2)

For further practice in this topic –
see The Answer Series
Gr 9 Mathematics 2 in 1 on pg 1.18 & 1.40



FACTORISATION

(Solutions on page A7)



STUDY THIS TOPIC VERY WELL!

1. Common Factor

$$ab + ac = a(b + c)$$

BECAUSE: $a(b + c) = ab + ac$... reversed

Factorise:

1.1 $8p^3 + 4p^2$ (2)

1.2 $10t^2 - 5t$ (2)

1.3 $3x^2y - 9xy^2 + 12x^3y^3$ (2)

1.4 $2p^2 + 2$ (2)

1.5 $2(x + y) + a(x + y)$ (2)

1.6 $2(x + y) - t(x + y)$ (2)

* 1.7 $tx - ty - 2x + 2y$ (3)

* a challenging question

2. Difference between Squares

$$x^2 - y^2 = (x + y)(x - y)$$

BECAUSE: $(x + y)(x - y) = x^2 - y^2$... reversed

Factorise:

2.1 $4x^2 - y^2$ (2)

2.2 $4x^2 - 4y^2$ (2)

2.3 $81 - 100a^2$ (2)

2.4 $9p^2 - 36q^2$ (3)

2.5 $7x^2 - 28$ (3)

2 TERMS

Remember:

Always check for a
Common Factor first!

Always check
for this first!

NOTE

To factorise
is to reverse
a product!

3. Trinomials

3 TERMS

Complete the products:

► **Perfect Squares** ➔ **Perfect Square TRINOMIALS**



$$(x + 3)^2 = (x + 3)(x + 3) = x^2 + \dots + 9 = x^2 + \dots + 9$$

$$\& (x - 3)^2 = (x - 3)(x - 3) = x^2 + \dots + 9 = x^2 + \dots + 9$$

$$\therefore x^2 + 6x + 9 = \dots$$

$$\& x^2 - 6x + 9 = \dots$$

$$(a + b)^2 = (a + b)(a + b) = a^2 + \dots + b^2 = a^2 + \dots + b^2$$

$$\& (a - b)^2 = (a - b)(a - b) = a^2 + \dots + b^2 = a^2 + \dots + b^2$$

$$\therefore a^2 + 2ab + b^2 = \dots$$

$$\& a^2 - 2ab + b^2 = \dots$$



► **Other products**

TRINOMIALS



$$(x + 2)(x + 3) = \dots = \dots$$

$$(x - 2)(x - 3) = \dots = \dots$$

$$(x + 2)(x - 3) = \dots = \dots$$

$$(x - 2)(x + 3) = \dots = \dots$$

**Observe the results above to
understand factorising trinomials**



Factorise the following trinomials:

3.1 $a^2 + 8a + 16 = (a \dots)(a \dots) = (\dots)^2$

3.2 $p^2 - 10p + 25 = (p \dots)(p \dots) = (\dots)^2$

3.3 $x^2 + 5x + 6 = (x \dots)(x \dots)$

3.4 $x^2 - 5x + 6 = (x \dots)(x \dots)$

3.5 $x^2 + x - 6 = (x \dots)(x \dots)$

3.6 $x^2 - x - 6 = (x \dots)(x \dots)$

3.7 $x^2 - 11x + 18 = (x \dots)(x \dots)$

3.8 $x^2 + 11x + 18 = (x \dots)(x \dots)$

3.9 $x^2 - 7x - 18 = (x \dots)(x \dots)$

3.10 $x^2 + 7x - 18 = (x \dots)(x \dots)$

3.11 $x^2 + 9x + 18 = (x \dots)(x \dots)$

3.12 $x^2 - 9x + 18 = (x \dots)(x \dots)$

3.13 $x^2 + 3x - 18 = (x \dots)(x \dots)$

3.14 $x^2 - 3x - 18 = (x \dots)(x \dots)$

NOTE

To factorise
is to reverse
a product!



Mixed Factorisation

Factorise fully:


4.1 $3a^3 - 9a^2 - 6a$ (3)

4.2 $2a^2 - 18a + 36$ (3)

4.3 $4(a + b) - x^2(a + b)$ (3)

4.4 $6x^3(a - b) + x(b - a)$ (4)

4.5 $6a^3 - 12a^2 + 18a$ (3)

- 
- Always first check for a common factor; then,
 - make sure the factorisation is complete.

Use factorisation to simplify the following fractions

5.1 $\frac{x^2 - 1}{3x + 3}$ (3)

5.2 $\frac{x^2 - 4x}{x^2 - 2x - 8}$ (3)

5.3 $\frac{3a - 6b}{4b - 2a}$ (3)

5.4 $\frac{2x^2 - 8}{3x - 12} \times \frac{x^2 - 4x}{x - 2}$ (4)

5.5 $\frac{x^2 + 2x}{x^3 - 2x} \div \frac{x^2 - 4}{x - 2}$ (5)

For further practice in this topic –
see The Answer Series
Gr 9 Mathematics 2 in 1 on pg 1.40

**NOTES**

ALGEBRAIC EQUATIONS (Linear and Quadratic)

(Solutions on page A9)

- 1.1 If 3 is a root of the equation $x^2 + x + t = 0$, the value of t is ...

- A 12
B -12
C $\frac{1}{2}$
D $-\frac{1}{2}$

A 'root' of an equation is 'the solution' of the equation.



- 1.2 Calculate the value of p if $2p + 12 = 58$.

- A 22
B 12
C 18
D 23

- 1.3 If $(x-1)(x+2) = 0$ then $x =$

- A -1 or 0
B 1 or -2
C 1
D -2

- 1.4 If $\frac{3x}{2} = -6$ then $x =$

- A 9
B 4
C -9
D -4



- 1.5 The product of a number and 6 decreased by 4 is equal to 20. Which one of the following equations matches the statement?

- A $6x + 4 = 20$
B $6x - 4 = 20$
C $6(x + 4) = 20$
D $6 - 4x = 20$



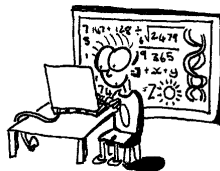
2. Solve for x in the following LINEAR equations (i.e. find the value of x which makes the equation true).

- 2.1 $x + 5 = 2$
2.2 $x - 3 = -4$
2.3 $2x = 12$
2.4 $\frac{x}{5} = 6$

These examples can be done by inspection.

3. Solve for x :

- 3.1 $3x - 1 = 5$
3.2 $2(x + 1) = 10$
3.3 $8x + 3 = 3x - 22$
3.4 $3(x + 6) = 12$
3.5 $2x - 5 = 5x + 16$
3.6 $x^3 + x^3 = 2$



Equations including fractions

4. Solve for x :

- 4.1 $\frac{x-2}{4} + \frac{2x+1}{3} = \frac{5}{3}$ (5)
4.2 $\frac{x+2}{3} - \frac{x-3}{4} = 0$ (3)
4.3 $\frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-1}{2}$ (4)
4.4 $x - \frac{x-1}{2} = 3$ (4)
4.5 $\frac{x+1}{3} - \frac{x-1}{6} = 1$ (3)

Quadratic Equations

5. Solve for x :

- 5.1 $(x-3)(x+4) = 0$ (2)
5.2 $x^2 - 5x - 6 = 0$ (3)
5.3 $x^2 - 1 = 0$ (3)
5.4 $x^2 - 2x = 0$ (3)

6. Solve for x :

- 6.1 $2(x-2)^2 = (2x-1)(x-3)$ (4)
6.2 $(x-2)^2 + 3x - 2 = (x+3)^2$ (4)
6.3 $(x-3)^2 = 16$ (6)

Other ...

7. Solve for x :

- 7.1 $\sqrt{\sqrt{x}} = 2$ 7.2 $\sqrt{\frac{1}{\sqrt{x}}} = 2$ (3)(3)

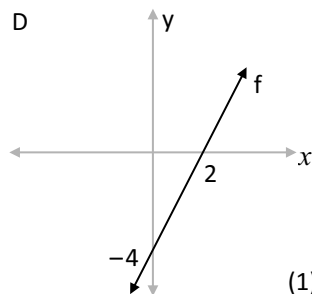
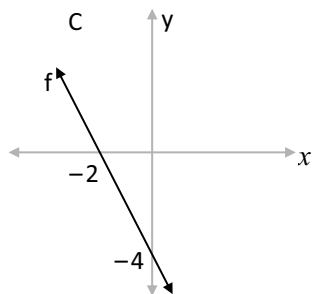
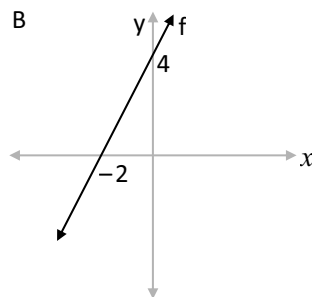
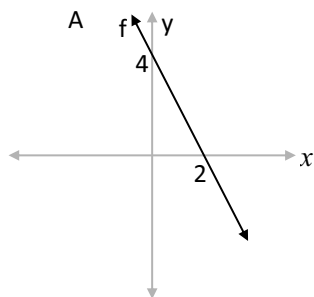
For further practice in this topic –
see The Answer Series
Gr 9 Mathematics 2 in 1 on pg 1.21 & 1.42



GRAPHS

(Solutions on page A13)

- 1.1 The graph of the straight line defined by $f(x) = 2x + 4$ is

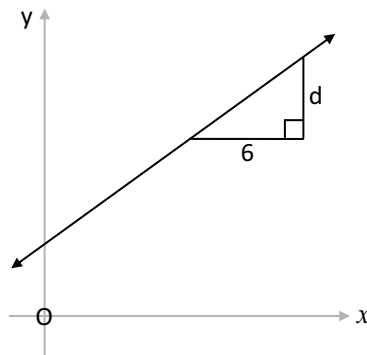


- 1.2 If T is a point on the line defined by $y = x$, the coordinates of T are ...

- A (5; -5)
- B (5; 0)
- C (-5; 5)
- D (-5; -5)

(1)

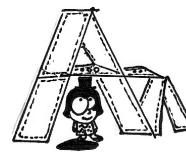
1.3



The gradient of the line shown above is $\frac{2}{3}$.

What is the value of d?

- A 3
- B 4
- C 6
- D 9



(1)

- 1.4 What is the y-intercept of the graph defined by $4x + 2y = 12$?

- A -4
- B -2
- C 6
- D 12

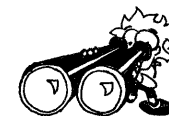
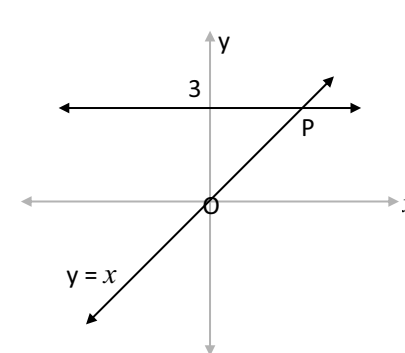
(1)

- 1.5 The straight line graph defined by $3y + 2x + 1 = 0$ will cut the x-axis at the point ...

- A (-2; 0)
- B $(-\frac{1}{2}; 0)$
- C (-3; 0)
- D $(-\frac{1}{3}; 0)$

(1)

2. Determine the co-ordinates of P in the graph below.



(1)

3. Use the given equation to complete each of the following tables.

3.1 $y = 3x - 5$

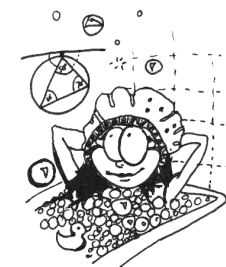
x	-2	-1	0	1
y				

(2)

3.2 $y = -\frac{2}{3}x - 1$

x	-3	-1	0	1
y				

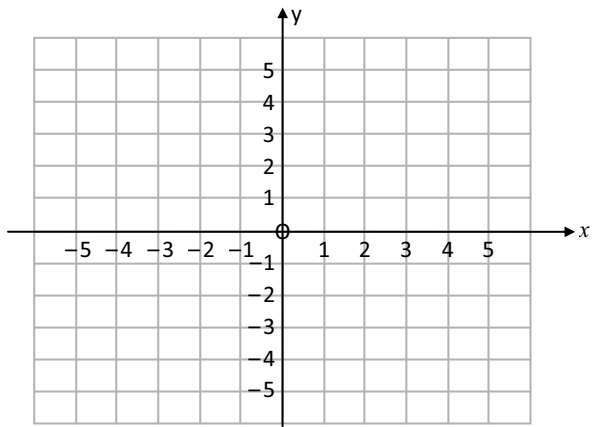
(2)



Questions: Graphs

- 4.1 On the given grid draw the graphs defined by $y = 3x - 2$ and $y = 3x + 1$ on the same set of axes.

Label each graph and clearly mark the points where the graphs cut the axes.

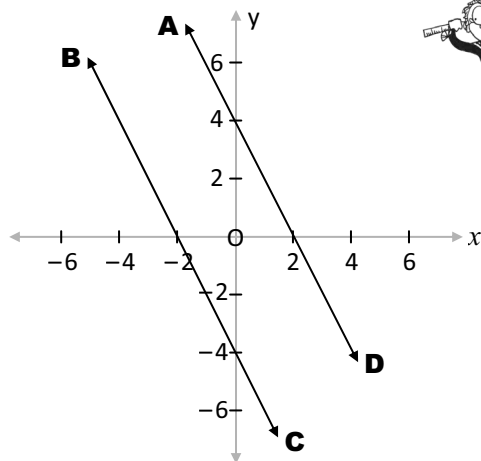


(6)

- 4.2 What is the relationship between the lines that you have drawn?

(1)

- 5.1 Write down the defining equation of each of the following straight line graphs.

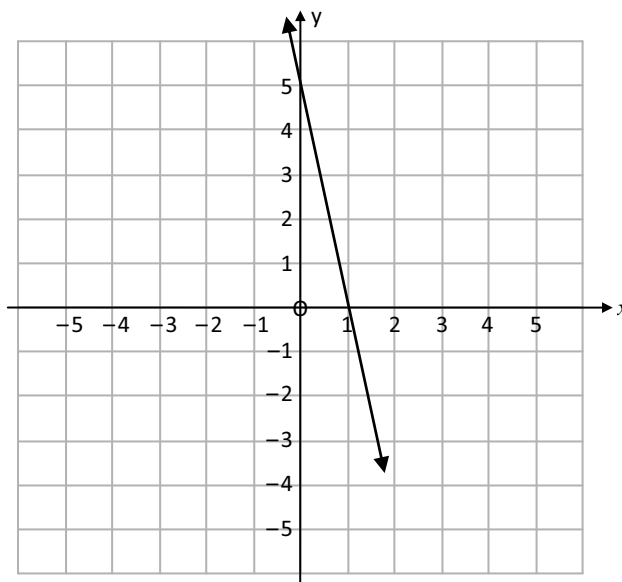


(4)

- 5.2 What can you deduce about lines **AD** and **BC**?
Give a reason for your answer.

(2)

6. Study the graph below.



- 6.1 Use the graph to calculate the gradient of the straight line.

(3)

- 6.2 Determine the equation of the straight line.

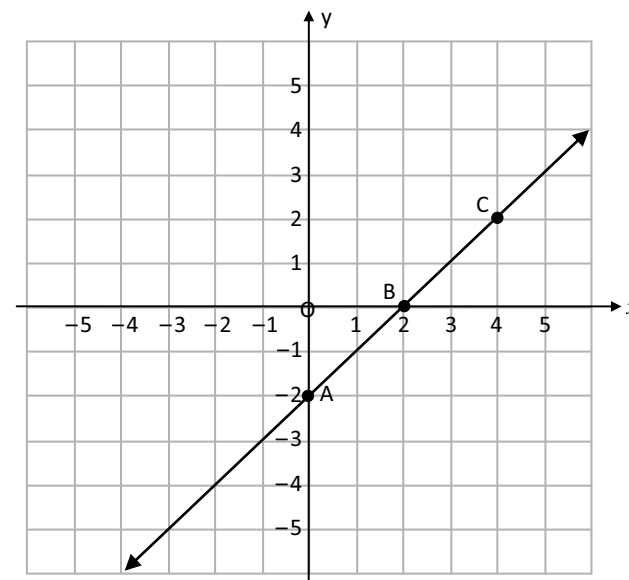
(2)

- 6.3 Write down the gradient of any other straight line which can be drawn parallel to the given line.

(1)



7. Use the graph below to answer the questions that follow.



- 7.1 Write down the coordinates of points A, B and C in the table.

	A	B	C
x-coordinate			
y-coordinate			

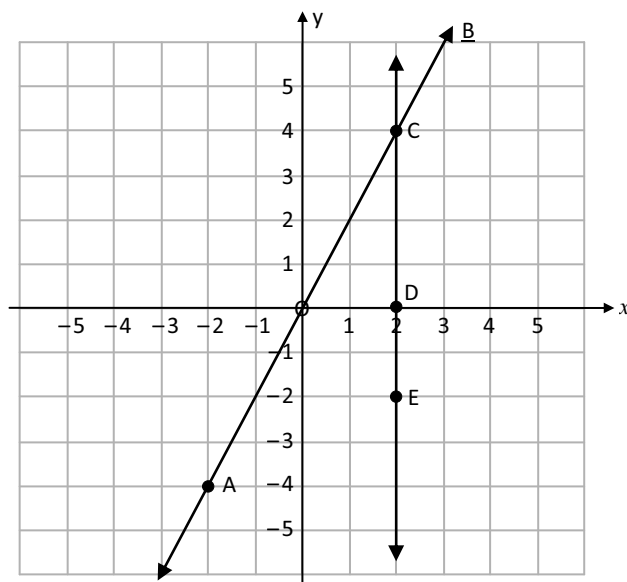
(3)

- 7.2 Use the table in Question 7.1 or any other method to determine the equation of line ABC.

(2)



8. Study the straight line graphs below and answer the questions that follow.



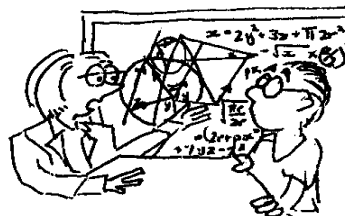
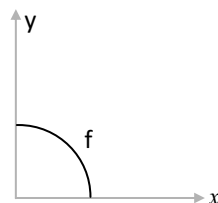
Complete:

- 8.1 The equation of the line CD is (1)
- 8.2 The equation of the line AB is (2)
- 8.3 If $DE = 2$, the co-ordinates of E are (2)
- 8.4 The length of CE is (1)



9. Underline the word, the number or the equation between brackets so that each of the following statements is correct.

- 9.1 The lines $x = 4$ and $x = -4$ are (parallel/perpendicular) to one another. (1)
- 9.2 The equation of the horizontal line through the point $P(3; -2)$ is ($x = 3$ / $y = -2$). (1)
- 9.3 The gradient of the line defined by $y - 4x + 5 = 0$ is equal to $(-4 / 4)$. (1)
- 9.4 This graph of f below represents a (linear / non-linear) function. (1)



- 10.1 On the given grid draw the graphs defined by

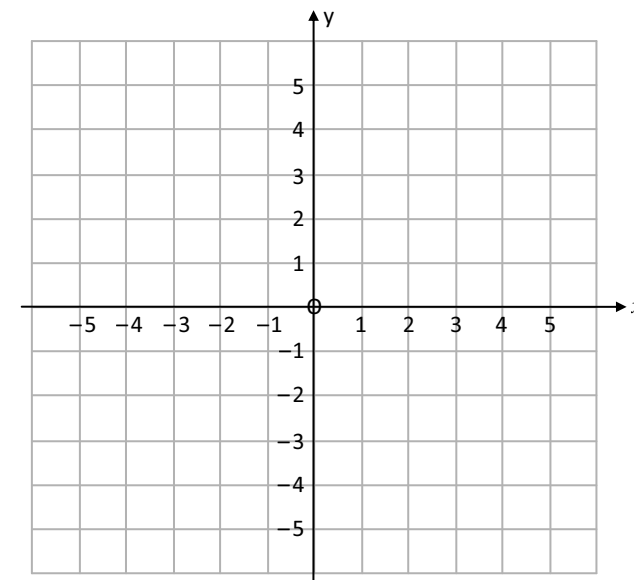
$$y = -\frac{2}{3}x + 1 \text{ and } y = \frac{3}{2}x - 1.$$

Label each graph and clearly mark the points where each graph cuts the x -axis and the y -axis. (6)

(1)

(1)

(1)



(1)

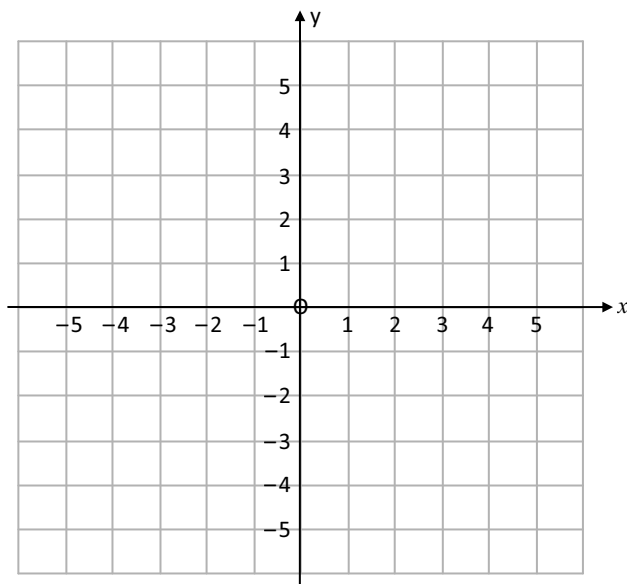
- 10.2 What is the relationship between the lines that you have drawn? (1)



Questions: Graphs

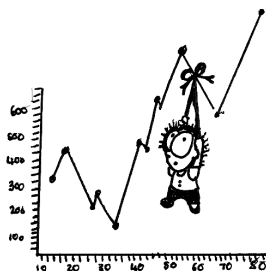
11. Use the grid below to answer the questions that follow.

- 11.1 Draw the graphs defined by $y = -2x + 4$ and $x = 1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes.



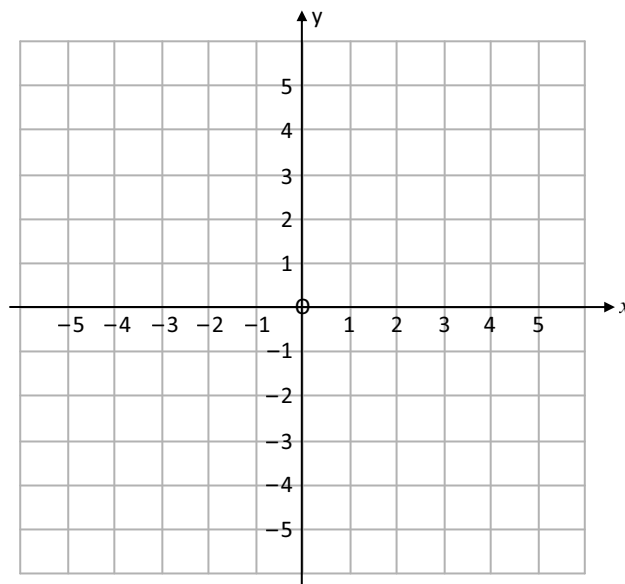
(6)

- 11.2 Write down the coordinates of the point where the two lines cut one another. (2)



12.1 On the same set of axes, draw and label the graphs defined by $y = -2x + 1$ and $y = x - 2$.

Use the given grid and clearly indicate the points where the lines cut the axes.



(8)

12.2 The lines intersect at T.

Show by calculation that the coordinates of T are $x = 1$ and $y = -1$ or $(1; -1)$.

(2)

For further practice in this topic –
see The Answer Series
Gr 9 Mathematics 2 in 1 on pg 1.44



NOTES



Gr 9 Maths: Content Area 2

Patterns, Algebra & Graphs

ANSWERS

- Patterns
- Algebraic Expressions
- Factorisation
- Algebraic Equations
- Graphs



PATTERNS

1.1 C < ... 1^2 ; 3^2 ; 5^2 ; **7^2**

1.2 C < ... 18; 36; **54**; 72; **90**

1.3 A < ... $\frac{1}{2^0}$; $\frac{1}{2^1}$; $\frac{1}{2^2}$; **$\frac{1}{2^3}$** ; $\frac{1}{2^4}$

1.4 A < ... $\frac{1}{3}$; **$\frac{1}{6}$** ; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$

1.5 D < ... 1^2+2 ; 2^2+2 ; 3^2+2 ; 4^2+2 ; **5^2+2**



2.1 a) $T_n = 3n$:
3; 6; 9 < ... $3(1)$; $3(2)$; $3(3)$

b) $T_n = 5n$:
5; 10; 15 < ... $5(1)$; $5(2)$; $5(3)$

c) $T_n = 3n + 1$:
4; 7; 10 < ... $3(1) + 1$; $3(2) + 1$; $3(3) + 1$

d) $T_n = 5n - 2$:
3; 8; 13 < ... $5(1) - 2$; $5(2) - 2$; $5(3) - 2$

e) $T_n = n^2$:
1; 4; 9 < ... $(1)^2$; $(2)^2$; $(3)^2$

f) $T_n = n^3$:
1; 8; 27 < ... $(1)^3$; $(2)^3$; $(3)^3$

2.2 a) $T_{12} = 3(12) = 36$ <

b) $T_{12} = 5(12) = 60$ <

c) $T_{12} = 3(12) + 1 = 37$ <

d) $T_{12} = 5(12) - 2 = 58$ <

e) $T_{12} = (12)^2 = 144$ <

f) $T_{12} = (12)^3 = 1728$ <



3.1 $y = 3x$ <

3.2 $a = 7$ < ... $7 \times 3 = 21$

$b = 30$ < ... $10 \times 3 = 30$

4.1 The constant difference = **7** <

4.2 3; 10; 17; 24; 31; **38**; **45** <

4.3 The constant **difference** is **7** ... see Question 4.1

So, write down **the multiples of 7** ... where $T_n = 7n$:

7; 14; 21; 28; 35; ...

and compare the given sequence:

3; 10; 17; 24; 31; ...

Each term is **4 less** than the multiples of 7.

$\therefore T_n = 7n - 4$ <

Position in pattern	1	2	3	4	5
Term	1	8	27	64	125

5.2 $T_n = n^3$ <

5.3 If $T_n = 512$, then $n = 8$ <

$$\left(\begin{array}{l} \dots 512 = 2^9 \\ = 2^3 \times 2^3 \times 2^3 \\ = 8 \times 8 \times 8 \\ = 8^3 \end{array} \right. \left. \begin{array}{l} 2 \overline{)512} \\ 2 \overline{)256} \\ 2 \overline{)128} \\ 2 \overline{)64} \\ 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \overline{)2} \end{array} \right)$$

6.1 7; 11; 15; **19**; **23** <

6.2 The common **difference** is **4**

So, compare the **multiples of 4** ... where $T_n = 4n$:

4; 8; 12; 16; ...

to the given sequence:

7; 11; 15; 19; ...

Each term is **3 more** than the multiples of 4.

$$\therefore T_n = 4n + 3 \quad \dots 7; 11; 15; 19; 23$$

6.3 $T_{50} = 4(50) + 3$
 $= 203$ <

7.1 3 ; 8 ; 13 ; **18** ; **23** ◀

7.2 Each term is 5 more than the previous term ◀

7.3 Compare $T_n = 5n$: 5 ; 10 ; 15 ; ...
to : 3 ; 8 ; 13 ; ...

Each term is **2 less** than the multiples of 5

$$\therefore T_n = 5n - 2 \quad \blacktriangleleft$$



7.4 38 is 2 less than 40 ; $40 = 5 \times 8$

\therefore 38 is the **8th** term ◀

OR: Solve the equation:

$$5n - 2 = 38 \quad \dots \text{the } n^{\text{th}} \text{ term} = 38$$

$$\text{Add 2:} \quad \therefore 5n = 40$$

$$\text{Divide by 5:} \quad n = 8$$

\therefore The **8th** term ◀

8.1	Figure	1	2	3	4
	Number of sides	5	9	13	17

◀

8.2 Each figure has four more sides than the previous figure ◀

$$8.3 \quad T_n = 4n + 1 \quad \blacktriangleleft \quad \dots 5 ; 9 ; 13 ; 17$$

& each term is **1 more**
than the multiples of 4

9.	Figure	1	2	3
	Number of matchsticks	6	9	12

9.1 Number of matchsticks in Figure 4 = **15** ◀

$$9.2 \quad T_n = 3n + 3 \quad \blacktriangleleft \quad \dots 6 ; 9 ; 12$$

& each term is **3 more**
than the multiples of 3

$$9.3 \quad T_{20} = 3(20) + 3 \\ = 63 \text{ matchsticks} \quad \blacktriangleleft$$

10.1	Figure	1	2	3	4
	Number of black tiles	1	2	3	4
	Number of white tiles	6	10	14	18

◀

$$10.2 \quad T_n = 4n + 2 \quad \blacktriangleleft \quad \dots 6 ; 10 ; 14 ; 18$$

& each term is **2 more**
than the multiples of 4

11. Look at the pattern formed by the first numbers of each line:

$$\begin{array}{ccccccc} 1 & ; & 4 & ; & 9 & ; & \dots \text{ the squares!} \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{row 1} & & \text{row 2} & & \text{row 3} & & \\ (1^2) & & (2^2) & & (3^2) & & \end{array}$$

\therefore The first number in the **20th** row will be **$20^2 = 400$** ◀

NOTES



ALGEBRAIC EXPRESSIONS

Terminology

1.1 $-8 \blacktriangleleft \dots -8x^2$

1.2 $-7 \blacktriangleleft \dots$ the term with no variable

1.3 $-8x^2 + 2x - 7 \blacktriangleleft$

1.4 $1 \blacktriangleleft \dots 2x^1$

1.5 $2x - 7 - 8x^2$

$$\begin{aligned}
 x = \frac{1}{2}: \quad & 2\left(\frac{1}{2}\right) - 7 - 8\left(\frac{1}{2}\right)^2 \\
 &= \left(\frac{2}{1}\right)\left(\frac{1}{2}\right) - 7 - \left(\frac{8}{1}\right)\left(\frac{1}{4}\right) \\
 &= 1 - 7 - 2 \\
 &= -8 \blacktriangleleft
 \end{aligned}$$

2. **B** $\blacktriangleleft \dots$ x is part of the fraction $\frac{x-y}{3}$, which can also be written as $\frac{1}{3}(x-y) = \frac{1}{3}x - \frac{1}{3}y$.

\therefore The coefficient of x is $\frac{1}{3}$.

[The two variables are x and y .]

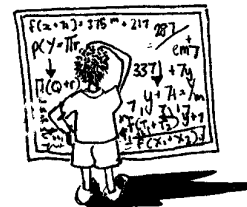
Substitution

$$\begin{aligned}
 3.1 \quad & 2x^3 - 3x^2 + 9x + 2 \\
 \underline{x = -2}: \quad & 2(-2)^3 - 3(-2)^2 + 9(-2) + 2 \\
 &= 2(-8) - 3(4) + 9(-2) + 2 \\
 &= -16 - 12 - 18 + 2 \\
 &= -44 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 3.2 \quad & y = 2x^2 - 3x + 5 \\
 \underline{x = -1}: \quad & y = 2(-1)^2 - 3(-1) + 5 \\
 &= 2 + 3 + 5 \\
 &= 10 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 3.3 \quad & \frac{5ac}{b} \\
 &= \frac{5\left(\frac{2}{1}\right)\left(\frac{1}{2}\right)}{(-3)} \quad \dots \text{it is useful to put whole numbers over 1 when fractions are involved} \\
 &= \frac{5(1)}{-3} \\
 &= -\frac{5}{3} \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 3.4 \quad & 3x^2 - 2xy - y^2 \\
 &= 3(2)^2 - 2(2)(-3) - (-3)^2 \\
 &= 3(4) - 2(-6) - (9) \\
 &= 12 + 12 - 9 \\
 &= 15 \blacktriangleleft
 \end{aligned}$$



Addition, Subtraction, Multiplication and Division

$$\begin{aligned}
 4.1 \quad & (2b - 3a - c) + (a - 4b + 2c) \\
 &= 2b - 3a - c + a - 4b + 2c \\
 &= -3a + a + 2b - 4b - c + 2c \\
 &= -2a - 2b + c \blacktriangleleft
 \end{aligned}$$

OR: $-3a + 2b - c$
Add $\underline{a - 4b + 2c}$
 $-2a - 2b + c \blacktriangleleft$

$$\begin{aligned}
 4.2 \quad & -4x^2(5x^2 - 3x) \\
 &= -20x^4 + 12x^3 \blacktriangleleft
 \end{aligned}$$



Distributive Property:

$$a(b + c) = ab + ac$$



$$\begin{aligned}
 4.3 \quad & \frac{8a + 16a^2 - 4a^3}{2a} \\
 &= \frac{8a}{2a} + \frac{16a^2}{2a} - \frac{4a^3}{2a} \quad \dots \text{Each term in the numerator must be divided by } 2a. \\
 &= 4 + 8a - 2a^2 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 4.4 \quad & -3(x)(x) + 2x(-x) \\
 &= -3x^2 - 2x^2 \\
 &= -5x^2 \blacktriangleleft \quad \dots \text{LIKE TERMS } \odot
 \end{aligned}$$

$$\begin{aligned}
 4.5 \quad & -3m^2n(4m - 3mn^5 + 2n) \\
 &= -12m^3n + 9m^3n^6 - 6m^2n^2 \blacktriangleleft \quad \dots \text{Distributive Property}
 \end{aligned}$$

$$\begin{aligned}
 4.6 \quad & 3ab - (-2ab) \\
 &= 3ab + 2ab \\
 &= 5ab \blacktriangleleft \quad \dots \text{LIKE TERMS } \odot
 \end{aligned}$$

$$\begin{aligned}
 5.1 \quad (3x)^3 + 2x^3 & \quad \dots (3x)^3 = 3^3 \cdot x^3 \\
 = 27x^3 + 2x^3 & \quad \dots \text{LIKE TERMS } \odot \\
 = 29x^3 <
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad (2x)^2 \times 3x^2 & \quad \dots (2x)^2 = 2x \times 2x \\
 = 4x^2 \times 3x^2 \\
 = 12x^4 <
 \end{aligned}$$

$$\begin{aligned}
 5.3 \quad (a^2b^3)^2 \cdot ab^2 & \\
 = a^4b^6 \cdot ab^2 \\
 = a^5b^8 <
 \end{aligned}$$



$$\begin{aligned}
 5.4 \quad 2^5 - 1^5 & \quad \dots 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \\
 = 32 - 1 & \quad 1^5 = 1 \times 1 \times 1 \times 1 \times 1 \\
 = 31 <
 \end{aligned}$$

Fractions (+, -, ×, ÷)

$$\begin{aligned}
 5.5 \quad \frac{x \times 5}{2 \times 5} + \frac{x \times 2}{5 \times 2} & \\
 = \frac{5x + 2x}{10} & \\
 = \frac{7x}{10} <
 \end{aligned}$$

When we add or subtract fractions, we must determine a common denominator

$$\begin{aligned}
 5.6 \quad \frac{5a \times 3}{8 \times 3} - \frac{5a \times 2}{12 \times 2} & \\
 = \frac{15a - 10a}{24} & \\
 = \frac{5a}{24} <
 \end{aligned}$$

NB: 'Keep' the denominator! Do not multiply by it.

$$\begin{aligned}
 5.7 \quad \frac{a^2b^2}{ac^2} \times \frac{4a^2bc}{20b^3} & \\
 = \frac{ab^2}{c^2} \times \frac{a^2c}{5b^2} & \\
 = \frac{a^3}{5c} &
 \end{aligned}$$

When we multiply fractions, we may cancel **FACTORS**.

NB: Understand the expression

$$\frac{a \times a \times b \times b}{a \times c \times c} \times \frac{4 \times a \times a \times b \times c}{20 \times b \times b \times b}$$

... and then cancel!

$$\begin{aligned}
 5.8 \quad \frac{6x^5}{x^4} - \frac{15x^3}{3x^2} & \\
 = 6x - 5x & \quad \dots \text{LIKE TERMS } \odot \\
 = x <
 \end{aligned}$$

NB: Understand the expression

$$\frac{6 \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}} - \frac{5 \times \cancel{15} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{3 \times \cancel{x} \times \cancel{x}}$$

$$\begin{aligned}
 5.9 \quad \frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4} & \\
 = \frac{2x+1 - 2(x+2) - 1}{4} & \\
 = \frac{2x+1 - 2x - 4 - 1}{4} & \\
 = \frac{-4}{4} & \\
 = -1 <
 \end{aligned}$$

This is an **expression**:

keep the value the same; do **not** multiply it!

All terms need to be written over a common (the same) denominator.

$$\begin{aligned}
 5.10 \quad \frac{x-y}{y+x} \times \frac{(x+y)^2}{x-y} & \\
 = \frac{\cancel{(x-y)}}{\cancel{(x+y)}} \times \frac{(x+y)^{\cancel{2}}}{\cancel{(x-y)}} & \\
 = \frac{x+y}{1} & \\
 = x+y <
 \end{aligned}$$

Compare to: $\frac{a}{b} \times \frac{b^2}{a}$

$$= b <$$

$$\begin{aligned}
 5.11 \quad \frac{x-2}{2x} - \frac{x-3}{3x} & \quad \dots \text{Do not multiply!} \\
 = \frac{3(x-2) - 2(x-3)}{6x} & \quad \dots \text{NB: Brackets!} \\
 = \frac{3x - 6 - 2x + 6}{6x} & \quad \dots \text{Keep the denominator, } 6x! \\
 = \frac{x}{6x} & \\
 = \frac{1}{6} <
 \end{aligned}$$



NB:

It is only when working with fractions **IN EQUATIONS** (Page Q6, Question 4), that you **logically** multiply both sides of the equation!

$$\begin{aligned}
 5.12 \quad \frac{4x^2}{2a^2} \div \frac{4x}{2a^2} & \\
 = \frac{4x^2}{2a^2} \times \frac{2a^2}{4x} & \\
 = x <
 \end{aligned}$$

$$\begin{aligned}
 5.13 \quad \frac{15x^2y^3 + 9x^2y^3}{8x^2y^3} & \quad \dots \text{LIKE TERMS } \odot \\
 = \frac{24x^2y^3}{8x^2y^3} & \\
 = 3 <
 \end{aligned}$$

$$\begin{aligned}
 5.14 \quad \frac{5a^2b}{3ab} \div \frac{20a^3b}{27} & \\
 = \frac{5a}{3} \times \frac{27}{20a^3b} & \\
 = \frac{9}{4a^2b} <
 \end{aligned}$$



$$\begin{aligned}
 5.15 \quad & \frac{5}{b} - \frac{4}{a} - \frac{a-b}{ab} \quad \dots \text{Do not multiply!} \\
 &= \frac{5a - 4b - (a - b)}{ab} \quad \dots \text{NB: Brackets!} \\
 &= \frac{5a - 4b - a + b}{ab} \quad \dots \text{Keep the denominator, ab!} \\
 &= \frac{4a - 3b}{ab} \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 5.16 \quad & \frac{3a^{-2}b \times 24b^{-1}a^{-1}}{9a^{-4}b^{-3}} \\
 &= \frac{72a^{-3}}{9a^{-4}b^{-3}} \\
 &= 8ab^3 \blacktriangleleft \quad \dots \frac{a^{-3}}{a^{-4}} = a^{-3-(-4)} = a^1 \\
 &\quad \& \frac{1}{b^{-3}} = b^3
 \end{aligned}$$

$$\begin{aligned}
 5.17 \quad & \frac{x^2}{2} + \frac{2x^2}{3} - \frac{7x^2}{6} \\
 &= \frac{3x^2 + 2(2x^2) - 7x^2}{6} \quad \dots \text{Write all 3 terms over a common denominator, 6.} \\
 &= \frac{3x^2 + 4x^2 - 7x^2}{6} \quad \dots \text{Do not multiply (by 6)!} \\
 &= \frac{0}{6} \\
 &= 0 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 5.18 \quad & \frac{6x^2}{7xy} \times \frac{3y^3}{2x} \\
 &= \frac{6x}{7y} \times \frac{3y^3}{2x} \\
 &= \frac{9y^2}{7} \blacktriangleleft
 \end{aligned}$$



Square roots and cube roots

$$\begin{aligned}
 5.19 \quad & \sqrt{225x^4} - \sqrt[3]{125x^6} \\
 &= 15x^2 - 5x^2 \\
 &= 10x^2 \blacktriangleleft
 \end{aligned}$$

$$\begin{array}{r}
 3 \overline{)225} \\
 3 \overline{)75} \\
 5 \overline{)25} \\
 \hline
 5
 \end{array}$$

$$\therefore 225 = 3^2 \times 5^2$$

$$\therefore \sqrt{225} = 3 \times 5$$

$$\begin{array}{r}
 5 \overline{)125} \\
 5 \overline{)25} \\
 \hline
 5
 \end{array}$$

$$\therefore 125 = 5^3$$

$$\therefore \sqrt[3]{125} = 5$$

$$\begin{aligned}
 5.20 \quad & \sqrt{16x^{16} \times 25x^4} \quad \left[\text{OR: } = \sqrt{400x^{20}} \right] \\
 &= \sqrt{16x^{16}} \cdot \sqrt{25x^4} \\
 &= 4x^8 \cdot 5x^2 \\
 &= 20x^{10} \blacktriangleleft \\
 &\quad \left[= \sqrt{20x^{10}} \right]
 \end{aligned}$$

$$\begin{aligned}
 5.21 \quad & \sqrt[3]{27x^{27}} \quad \dots \left[\begin{array}{l} 3 \times 3 \times 3 = 27; \\ x^9 \times x^9 \times x^9 = x^{27} \end{array} \right] \\
 &= 3x^9 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 5.22 \quad & \sqrt{16a^2 + 9a^2} \quad \dots \text{1st add LIKE TERMS} \\
 &= \sqrt{25a^2} \\
 &= 5a \blacktriangleleft
 \end{aligned}$$

NB: $\sqrt{16a^2 + 9a^2} \neq 4a + 3a!$

$$\begin{aligned}
 6.1 \quad & 4ab(5a^2b^2 + 2ab - 3) \\
 &= 20a^3b^3 + 8a^2b^2 - 12ab \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 6.2 \quad & 3a^2bc^2(3a^2 - 4b - c) \\
 &= 9a^4bc^2 - 12a^2b^2c^2 - 3a^2bc^3 \blacktriangleleft
 \end{aligned}$$

Distributive Property:

$$\begin{aligned}
 a(b + c) \\
 &= ab + ac
 \end{aligned}$$

and

$$\begin{aligned}
 a(b - c) \\
 &= ab - ac
 \end{aligned}$$



$$\begin{aligned}
 6.3 \quad & (x+5)(x+2) = x^2 + 2x + 5x + 10 \\
 &= x^2 + 7x + 10
 \end{aligned}$$

$$\begin{aligned}
 6.4 \quad & (x-2)(x-3) = x^2 - 3x - 2x + 6 \\
 &= x^2 - 5x + 6
 \end{aligned}$$

$$\begin{aligned}
 6.5 \quad & (x+7)(x-1) = x^2 - x + 7x - 7 \\
 &= x^2 + 6x - 7
 \end{aligned}$$

$$\begin{aligned}
 6.6 \quad & (2x-3)(x+1) \\
 &= 2x^2 + 2x - 3x - 3 \\
 &= 2x^2 - x - 3 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 6.7 \quad & x(x+2) - (x-1)(x-3) \\
 &= x^2 + 2x - (x^2 - 3x - x + 3) \\
 &= x^2 + 2x - (x^2 - 4x + 3) \\
 &= x^2 + 2x - x^2 + 4x - 3 \\
 &= 6x - 3 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 6.8 \quad & (x-3)^2 - x(x+4) \\
 &= x^2 - 6x + 9 - x^2 - 4x \\
 &= -10x + 9 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 (x-3)^2 \\
 &= (x-3)(x-3) \\
 &= x^2 - 3x - 3x + 9 \\
 &= x^2 - 6x + 9
 \end{aligned}$$

Perfect Square Trinomial



$$\begin{aligned}
 6.9 \quad & (2x-1)^2 - (x+1)(x-1) \\
 &= (2x-1)(2x-1) - (x^2-1) \\
 &= 4x^2 - 4x + 1 - x^2 + 1 \\
 &= 3x^2 - 4x + 2 \quad \blacktriangleleft
 \end{aligned}$$

Perfect Square Trinomial

Difference between Squares

$$\begin{aligned}
 (2x-1)^2 &= (2x-1)(2x-1) \\
 &= 4x^2 - 2x - 2x + 1 \\
 &= 4x^2 - 4x + 1
 \end{aligned}$$

$$\begin{aligned}
 \& \quad (x+1)(x-1) &= x^2 - x + x - 1 \\
 &= x^2 - 1
 \end{aligned}$$

$$\begin{aligned}
 6.10 \quad & 2(x+2)^2 - (2x-1)(x+2) \\
 &= 2(x+2)(x+2) - (2x^2 + 4x - x - 2) \\
 &= 2(x^2 + 4x + 4) - (2x^2 + 3x - 2) \\
 &= 2x^2 + 8x + 8 - 2x^2 - 3x + 2 \\
 &= 5x + 10 \quad \blacktriangleleft
 \end{aligned}$$

Perfect Square Trinomial

$$\begin{aligned}
 (x+2)^2 &= (x+2)(x+2) \\
 &= x^2 + 2x + 2x + 4 \\
 &= x^2 + 4x + 4
 \end{aligned}$$



7.1 → 7.4:
Perfect Square Trinomials

$$\begin{aligned}
 7.1 \quad & (x+5)^2 = (x+5)(x+5) \\
 &= x^2 + 5x + 5x + 25 \\
 &= x^2 + 10x + 25 \quad \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 7.2 \quad & (p-3)^2 = (p-3)(p-3) \\
 &= p^2 - 3p - 3p + 9 \\
 &= p^2 - 6p + 9 \quad \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 7.3 \quad & (2a+3)^2 = (2a+3)(2a+3) \\
 &= 4a^2 + 6a + 6a + 9 \\
 &= 4a^2 + 12a + 9 \quad \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 7.4 \quad & (4x-1)^2 = (4x-1)(4x-1) \\
 &= 16x^2 - 4x - 4x + 1 \\
 &= 16x^2 - 8x + 1 \quad \blacktriangleleft
 \end{aligned}$$



7.5 → 7.8:
Difference between Squares

$$7.5 \quad (x+5)(x-5) = x^2 - 5x + 5x - 25 = x^2 - 25$$

$$7.6 \quad (p-3)(p+3) = p^2 + 3p - 3p - 9 = p^2 - 9$$

$$7.7 \quad (2a+3)(2a-3) = 4a^2 - 6a + 6a - 9 = 4a^2 - 9$$

$$7.8 \quad (4x-1)(4x+1) = 16x^2 + 4x - 4x - 1 = 16x^2 - 1$$



7.9 → 7.12:
Observe **how** the **trinomial** is obtained in each case.

$$7.9 \quad (x+3)(x+4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12 \quad \blacktriangleleft$$

$$7.10 \quad (x-3)(x-4) = x^2 - 4x - 3x + 12 = x^2 - 7x + 12 \quad \blacktriangleleft$$

$$7.11 \quad (x+3)(x-4) = x^2 - 4x + 3x - 12 = x^2 - x - 12 \quad \blacktriangleleft$$

$$7.12 \quad (x-3)(x+4) = x^2 + 4x - 3x - 12 = x^2 + x - 12 \quad \blacktriangleleft$$

$$\begin{aligned}
 8.1 \quad & \mathbf{B} \quad \blacktriangleleft \quad \dots -(-2)^2 - (2(-2) - 1) \\
 &= -(4) - (-4 - 1) \\
 &= -4 - (-5) \\
 &= -4 + 5 \\
 &= 1
 \end{aligned}$$

$$8.2 \quad \mathbf{C} \quad \blacktriangleleft$$

$$8.3 \quad \mathbf{D} \quad \blacktriangleleft \quad \dots \frac{x}{y} - \frac{1 \times y}{1 \times y} = \frac{x-y}{y} \quad \dots \text{Write the terms over the same denominator.}$$

$$\begin{aligned}
 8.4 \quad & \mathbf{D} \quad \blacktriangleleft \quad \dots \left(\frac{x}{3} - 3y\right)\left(\frac{x}{3} + 3y\right) \\
 &= \left(\frac{x}{3}\right)^2 - (3y)^2 \quad \dots \text{difference of squares} \\
 &= \frac{x^2}{9} - 9y^2
 \end{aligned}$$

FACTORISATION

1. Common Factor

Check each answer by
multiplying back (to the beginning)



$$1.1 \quad 8p^3 + 4p^2 = 4p^2(2p + 1) \quad \checkmark$$

$$1.2 \quad 10t^2 - 5t = 5t(2t - 1) \quad \checkmark$$

$$1.3 \quad 3x^2y - 9xy^2 + 12x^3y^3 = 3xy(x - 3y + 4x^2y^2) \quad \checkmark$$

$$1.4 \quad 2p^2 + 2 = 2(p^2 + 1) \quad \checkmark$$

$$1.5 \quad 2(x + y) + a(x + y) = (x + y)(2 + a) \quad \checkmark$$

$$1.6 \quad 2(x + y) - t(x + y) = (x + y)(2 - t) \quad \checkmark$$

$$1.7 \quad tx - ty - 2x + 2y = (tx - ty) - (2x - 2y) = t(x - y) - 2(x - y) = (x - y)(t - 2) \quad \checkmark$$



2. Difference between Squares

Check each answer by
multiplying back (to the beginning)



$$2.1 \quad 4x^2 - y^2 = (2x + y)(2x - y) \quad \checkmark$$

$$2.2 \quad 4x^2 - 4y^2 = 4(x^2 - y^2) = 4(x + y)(x - y) \quad \checkmark$$

Compare Question 2.1 and Question 2.2!

In Question 2.1: 4 is **not** a common factor.
In Question 2.2: 4 is a common factor.



$$2.3 \quad 81 - 100a^2 = (9 + 10a)(9 - 10a) \quad \checkmark$$

$$2.4 \quad 9p^2 - 36q^2 = 9(p^2 - 4q^2) = 9(p + 2q)(p - 2q) \quad \checkmark$$

$$2.5 \quad 7x^2 - 28 = 7(x^2 - 4) = 7(x + 2)(x - 2) \quad \checkmark$$



3. Trinomials

3 TERMS

Observe these PRODUCTS:

► Perfect Squares ➡ Perfect Square TRINOMIALS

$$\begin{aligned} \downarrow \\ (x+3)^2 &= (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9 \\ \& \ (x-3)^2 &= (x-3)(x-3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9 \\ \therefore x^2 + 6x + 9 &= (x+3)^2 \\ \& \ x^2 - 6x + 9 &= (x-3)^2 \end{aligned}$$

$$\begin{aligned} (a+b)^2 &= (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2 \\ \& \ (a-b)^2 &= (a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2 \\ \therefore a^2 + 2ab + b^2 &= (a+b)^2 \\ \& \ a^2 - 2ab + b^2 &= (a-b)^2 \end{aligned}$$



► Other products ➡ TRINOMIALS

$$\begin{aligned} \downarrow \\ (x+2)(x+3) &= x^2 + 2x + 3x + 6 = x^2 + 5x + 6 \\ (x-2)(x-3) &= x^2 - 2x - 3x + 6 = x^2 - 5x + 6 \\ (x+2)(x-3) &= x^2 + 2x - 3x - 6 = x^2 - x - 6 \\ (x-2)(x+3) &= x^2 - 2x + 3x - 6 = x^2 + x - 6 \end{aligned}$$

Observe the results above to
understand factorising trinomials



Check each answer by
multiplying back (to the beginning)



$$3.1 \quad a^2 + 8a + 16 = (a + 4)(a + 4) = (a + 4)^2 \quad \checkmark$$

$$3.2 \quad p^2 - 10p + 25 = (p - 5)(p - 5) = (p - 5)^2 \quad \checkmark$$

$$3.3 \quad x^2 + 5x + 6 = (x + 3)(x + 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & +3 \\ \times & +2 \\ \hline 1 & +5 \end{array}$$

$$3.4 \quad x^2 - 5x + 6 = (x - 3)(x - 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & -3 \\ \times & -2 \\ \hline 1 & -5 \end{array}$$

$$3.5 \quad x^2 + x - 6 = (x + 3)(x - 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & +3 \\ \times & -2 \\ \hline 1 & +1 \end{array}$$

$$3.6 \quad x^2 - x - 6 = (x - 3)(x + 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & -3 \\ \times & +2 \\ \hline 1 & -1 \end{array}$$

$$3.7 \quad x^2 - 11x + 18 = (x - 9)(x - 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & -9 \\ \times & -2 \\ \hline 1 & -11 \end{array}$$

$$3.8 \quad x^2 + 11x + 18 = (x + 9)(x + 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & +9 \\ \times & +2 \\ \hline 1 & +11 \end{array}$$

$$3.9 \quad x^2 - 7x - 18 = (x - 9)(x + 2) \quad \checkmark \quad \dots \quad \begin{array}{r|l} 1 & -9 \\ \times & +2 \\ \hline 1 & -7 \end{array}$$

$$3.10 \quad x^2 + 7x - 18 = (x + 9)(x - 2) \quad \leftarrow \quad \dots \quad \begin{array}{r} 1 \times 9 \\ 1 \times -2 \\ \hline +9 \\ -2 \\ \hline +7 \end{array}$$

$$3.11 \quad x^2 + 9x + 18 = (x + 6)(x + 3) \quad \leftarrow \quad \dots \quad \begin{array}{r} 1 \times 6 \\ 1 \times 3 \\ \hline +6 \\ +3 \\ \hline +9 \end{array}$$

$$3.12 \quad x^2 - 9x + 18 = (x - 6)(x - 3) \quad \leftarrow \quad \dots \quad \begin{array}{r} 1 \times -6 \\ 1 \times -3 \\ \hline -6 \\ -3 \\ \hline -9 \end{array}$$

$$3.13 \quad x^2 + 3x - 18 = (x + 6)(x - 3) \quad \leftarrow \quad \dots \quad \begin{array}{r} 1 \times 6 \\ 1 \times -3 \\ \hline +6 \\ -3 \\ \hline +3 \end{array}$$

$$3.14 \quad x^2 - 3x - 18 = (x - 6)(x + 3) \quad \leftarrow \quad \dots \quad \begin{array}{r} 1 \times -6 \\ 1 \times 3 \\ \hline -6 \\ +3 \\ \hline -3 \end{array}$$

FACTORISATION

There are **3 TYPES** of factorization:

- ❶ **Common Factor (CF):** Always try this first!
- ❷ **Difference between Squares (DbS):** 2 terms
- ❸ **Trinomials:** 3 terms



RECOGNISE THESE!

Mixed Factorisation

Check each answer by
multiplying back (to the beginning)



$$4.1 \quad 3a^3 - 9a^2 - 6a = 3a(a^2 - 3a - 2) \quad \leftarrow \quad \dots \text{ note: this does not factorise further}$$

$$4.2 \quad 2a^2 - 18a + 36 = 2(a^2 - 9a + 18) = 2(a - 6)(a - 3) \quad \leftarrow \quad \begin{array}{r} 1 \times 6 \\ 1 \times -3 \\ \hline -6 \\ -3 \\ \hline -9 \end{array}$$

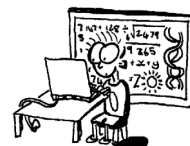
$$4.3 \quad 4(a + b) - x^2(a + b) = (a + b)(4 - x^2) = (a + b)(2 + x)(2 - x) \quad \leftarrow \quad \dots \text{ always check to see if you can factorise further}$$

$$4.4 \quad 6x^3(a - b) + x(b - a) = 6x^3(a - b) - x(a - b) = (a - b)(6x^3 - x) = (a - b) \cdot x(6x^2 - 1) = x(a - b)(6x^2 - 1) \quad \leftarrow \quad \dots \text{ switchround} \dots \text{ the 'new' factor can factorise further}$$

$$4.5 \quad 6a^3 - 12a^2 + 18a = 6a(a^2 - 2a + 3) \quad \leftarrow \quad \dots \text{ note: this does not factorise further}$$

Use factorisation to simplify the following fractions

$$5.1 \quad \frac{x^2 - 1}{3x + 3} \quad \dots \text{ DbS} \quad \dots \text{ CF} \\ = \frac{(x - 1)(x + 1)}{3(x + 1)} \\ = \frac{x - 1}{3} \quad \leftarrow$$



$$5.2 \quad \frac{x^2 - 4x}{x^2 - 2x - 8} \quad \dots \text{ CF} \quad \dots \text{ Trinomial} \quad \dots \quad \begin{array}{r} 1 \times -4 \\ 1 \times 2 \\ \hline -4 \\ +2 \\ \hline -2 \end{array} \\ = \frac{x(x - 4)}{(x - 4)(x + 2)} \\ = \frac{x}{x + 2} \quad \leftarrow$$



$$5.3 \quad \frac{3a - 6b}{4b - 2a} \quad \dots \text{ Common Factors} \\ = \frac{3(a - 2b)}{2(2b - a)} \\ = \frac{3(a - 2b)}{-2(a - 2b)} \quad \dots \quad 2b - a = -(a - 2b) \\ = -\frac{3}{2} \quad \leftarrow$$

$$5.4 \quad \frac{2x^2 - 8}{3x - 12} \times \frac{x^2 - 4x}{x - 2} \\ = \frac{2(x^2 - 4)}{3(x - 4)} \times \frac{x(x - 4)}{x - 2} \\ = \frac{2(x + 2)(x - 2)}{3(x - 2)} \\ = \frac{2(x + 2)}{3} \quad \leftarrow$$

Don't be frightened
by the look of these
fractions!

Just focus on
factorising where
possible; then
cancel the factors
where possible.



$$5.5 \quad \frac{x^2 + 2x}{x^3 - 2x} \div \frac{x^2 - 4}{x - 2} \\ = \frac{x^2 + 2x}{x^3 - 2x} \times \frac{x - 2}{x^2 - 4} \quad \dots \text{ note the 'flipped' fraction!} \\ = \frac{x(x + 2)}{x(x^2 - 2)} \times \frac{(x - 2)}{(x + 2)(x - 2)} \\ = \frac{1}{x^2 - 2} \quad \leftarrow$$

ALGEBRAIC EQUATIONS (Linear and Quadratic)

LOGIC IS KEY!


- 1.1 **B** ◀ ... If 3 is a root, then $x = 3$ will make the equation true

$$\begin{aligned} \text{i.e. } 3^2 + 3 + t &= 0 \\ \therefore 9 + 3 + t &= 0 \\ \therefore t &= -12 \end{aligned}$$

- 1.2 **D** ◀ ... $2p + 12 = 58$
 $\therefore 2p = 46$
 $\therefore p = 23$

$$\begin{aligned} \text{OR: } ? + 12 &= 58 \\ \text{Answer: } 46 \\ \text{So, } 2p &= 46 \\ \therefore 2 \times ? &= 46 \\ \therefore p &= 23 \end{aligned}$$

- 1.3 **B** ◀ ... **If** $(x-1)(x+2) = 0$,
then $x-1 = 0$ **or** $x+2 = 0$
 $\therefore x = 1$ $\therefore x = -2$

- 1.4 **D** ◀ ... $\frac{3x}{2} = -6$
 $\therefore 3x = -12$
 $x = -4$

$$\begin{aligned} \text{OR: } \frac{?}{2} &= -6 \\ \text{Answer: } -12 \\ \therefore 3x &= -12 \\ \text{So, } 3 \times ? &= -12 \\ \therefore x &= -4 \end{aligned}$$

- 1.5 **B** ◀ ... The product of a number, x , and 6 equals
 $x \times 6 = 6x$

2.1 $x + 5 = 2$... '5 more than a number is 2'
 $\therefore x = -3$ ◀

2.2 $x - 3 = -4$... '3 less than a number is -4'
 $\therefore x = -1$ ◀

2.3 $2x = 12$... 'double a number is 12'
 $\therefore x = 6$ ◀

2.4 $\frac{x}{5} = 6$... 'a fifth of a number is 6'
 $\therefore x = 30$ ◀

Check your answer by substituting in the given equation to see if it is 'true' for the value of x .

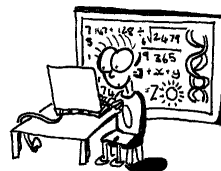
3.1 $3x - 1 = 5$
 $\therefore 3x = 6$
 $\therefore x = 2$ ◀



[Check: LHS = $3x - 1 = 3(2) - 1 = 5 = \text{RHS}$ ✓]

3.2 $2(x+1) = 10$
 $\therefore 2x + 2 = 10$... $\left[\text{OR: } x + 1 = 5 \right]$
 $\therefore 2x = 8$ $\therefore x = 4$ ◀
 $\therefore x = 4$ ◀

[Check: LHS = $2(4+1) = 2 \times 5 = 10 = \text{RHS}$ ✓]



3.3 $8x + 3 = 3x - 22$
 $\therefore 8x - 3x = -22 - 3$... Subtract $3x$ & Subtract 3
 $\therefore 5x = -25$
 $\therefore x = -5$ ◀ ... $\left[\div 5, \text{ OR: } 5 \times ? = -25 \right]$

Check: **LHS** = $8(-5) + 3 = -40 + 3 = -37$
& RHS = $3(-5) - 22 = -15 - 22 = -37$
 $\therefore \text{LHS} = \text{RHS}$ ✓ \therefore The answer is correct.
 i.e. The equation is 'true' for $x = -5$.
 We say that -5 is the **root** (or solution) of the equation.

3.4 $3(x+6) = 12$
 $\therefore 3x + 18 = 12$... $\left[\text{OR: } x + 6 = 4 \right]$
 $\therefore 3x = -6$ $\therefore x = -2$ ◀
 $\therefore x = -2$ ◀

Check your answer by substituting in the given equation to see if it is 'true' for the value of x .

3.5 $2x - 5 = 5x + 16$... Add 5 & Subtract $5x$
 $\therefore 2x - 5x = 16 + 5$
 $\therefore -3x = 21$
 $\therefore x = -7$ ◀ ... Divide by -3

Check your answer by substituting in the given equation to see if it is 'true' for the value of x .

3.6 $x^3 + x^3 = 2$
 $\therefore 2x^3 = 2$... **LIKE TERMS**
 $\therefore x^3 = 1$... divide by 2
 $\therefore x = 1$ ◀ ... take cube root

Check your answer by substituting in the given equation to see if it is 'true' for the value of x .

Equations including fractions

NOTE: In **EXPRESSIONS**, the previous section, we did **not** multiply. We kept the denominator!

Now, in **EQUATIONS**, we do multiply, applying logic !!! – 'what we do to the left hand side (**LHS**) of an equation, we also do to the right hand side (**RHS**)'.

$$4.1 \quad \frac{x-2}{4} + \frac{2x+1}{3} = \frac{5}{3}$$

NB:
Brackets!

$$\text{x12} \quad \therefore 3(x-2) + 4(2x+1) = 4(5)$$

$$\therefore 3x - 6 + 8x + 4 = 20$$

$$\therefore 11x - 2 = 20$$

$$\therefore 11x = 22$$

$$\therefore x = 2 \quad \blacktriangleleft$$

Check your answer!



The LCM of the denominators is 12.

The logic: $12 \times \text{the LHS} = 12 \times \text{the RHS}$

$$4.2 \quad \frac{x+2}{3} - \frac{x-3}{4} = 0$$

NB: Brackets!

$$\text{x12} \quad \therefore 4(x+2) - 3(x-3) = 0$$

$$\therefore 4x + 8 - 3x + 9 = 0$$

$$\therefore x + 17 = 0$$

$$\therefore x = -17 \quad \blacktriangleleft$$

Check your answer!



$$4.3 \quad \frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-1}{2}$$

NB:
Brackets!

$$\text{x6} \quad 3(2x-3) - 2(x+1) = 3(3x-1)$$

$$\therefore 6x - 9 - 2x - 2 = 9x - 3$$

$$\therefore 4x - 11 = 9x - 3$$

$$\therefore 4x - 9x = -3 + 11$$

$$\therefore -5x = 8$$

$$\therefore x = -\frac{8}{5} \quad \blacktriangleleft$$

Check your answer!



$$4.4 \quad \frac{x}{1} - \frac{x-1}{2} = \frac{3}{1}$$

$$\text{x2} \quad \therefore 2x - (x-1) = 2(3)$$

NB: Brackets!

$$\therefore 2x - x + 1 = 6$$

$$\therefore x + 1 = 6$$

$$\therefore x = 5 \quad \blacktriangleleft$$

Check your answer!



$$4.5 \quad \frac{x+1}{3} - \frac{x-1}{6} = \frac{1}{1}$$

$$\text{x6} \quad \therefore 2(x+1) - (x-1) = 6(1)$$

NB: Brackets!

$$\therefore 2x + 2 - x + 1 = 6$$

$$\therefore x + 3 = 6$$

$$\therefore x = 3 \quad \blacktriangleleft$$

Check your answer!



Compare the position of the = signs

In Algebraic Expressions (in the previous section):

The = signs are down the left

In Algebraic Equations (Q4.1 to 4.5 above):

The = signs are in the middle and the \therefore signs are on the left

Quadratic Equations

The LOGIC:

If the product of 2 numbers = 0, then either one or the other number must = 0.

$$5.1 \quad (x-3)(x+4) = 0 \quad \dots \text{the product of } x-3 \text{ and } x+4 \text{ equals } 0, \text{ so:}$$

$$\text{Either } x-3 = 0 \quad \text{or} \quad x+4 = 0$$

$$\therefore x = 3 \quad \blacktriangleleft \quad \therefore x = -4 \quad \blacktriangleleft$$

Check: If $x = 3$:

$$\text{LHS} = (3-3)(3+4) = 0(7) = 0 = \text{RHS} \quad \checkmark$$

If $x = -4$:

$$\text{LHS} = (-4-3)(-4+4) = (-7) \times 0 = 0 = \text{RHS} \quad \checkmark$$

\therefore Both answers are correct

$$5.2 \quad x^2 - 5x - 6 = 0 \quad \dots \text{Factorise the trinomial so that you have a product}$$

$$\therefore (x-6)(x+1) = 0$$

$$\therefore \text{Either } x-6 = 0 \quad \text{or} \quad x+1 = 0$$

$$\therefore x = 6 \quad \blacktriangleleft$$

$$x = -1 \quad \blacktriangleleft$$

$$\begin{array}{r|l} 1 & -6 \\ \times & +1 \\ \hline 1 & -5 \end{array}$$

Check your answers!



$$5.3 \quad x^2 - 1 = 0 \quad \dots \text{Factorise!}$$

(Difference between squares)

$$\therefore (x+1)(x-1) = 0$$

$$\therefore \text{Either } x+1 = 0 \quad \text{or} \quad x-1 = 0$$

$$\therefore x = -1 \quad \blacktriangleleft$$

$$x = 1 \quad \blacktriangleleft$$


Check your answers!



Solutions: Algebraic Equations

5.4 $x^2 - 2x = 0$... Factorise! (**Common Factor**)
 $\therefore x(x-2) = 0$

\therefore Either $x = 0$ < or $x - 2 = 0$
 $\therefore x = 2$ <

Check your answers! 

In Questions 6.1 and 6.2:

$$\begin{aligned}(x-2)^2 &= (x-2)(x-2) \\ &= x^2 - 2x - 2x + 4 \\ &= x^2 - 4x + 4\end{aligned}$$



In Question 6.3:

$$\begin{aligned}(x+3)^2 &= (x+3)(x+3) \\ &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9\end{aligned}$$

Now, the equations ...

6.1 $2(x-2)^2 = (2x-1)(x-3)$

$\therefore 2(x-2)(x-2) = 2x^2 - 6x - x + 3$

$\therefore 2(x^2 - 4x + 4) = 2x^2 - 7x + 3$

$\therefore 2x^2 - 8x + 8 = 2x^2 - 7x + 3$

$\therefore -8x + 7x = 3 - 8$

$\therefore -x = -5$

$\therefore x = 5$ <

Check your answer! 

the two $2x^2$ terms cancel each other and the equation becomes **linear** (no longer **quadratic**).

6.2 $(x-2)^2 + 3x - 2 = (x+3)^2$
 $\therefore (x-2)(x-2) + 3x - 2 = (x+3)(x+3)$


$\therefore x^2 - 4x + 4 + 3x - 2 = x^2 + 6x + 9$

$\therefore -x + 2 = 6x + 9$...

$\therefore -x - 6x = 9 - 2$

$\therefore -7x = 7$

$\therefore x = -1$ <

Check your answer! 

the two $2x^2$ terms cancel each other and the equation becomes **linear**.

6.3 $(x-3)^2 = 16$

$(x-3)(x-3) = 16$

$\therefore x^2 - 6x + 9 - 16 = 0$

$\therefore x^2 - 6x - 7 = 0$ < Remember the logic? The product must = 0!

$\therefore (x-7)(x+1) = 0$ < The **trinomial** is factorised


The logic:

$(x-7)$ times $(x+1)$ equals 0, so ...

Either $x-7$ equals 0 or $x+1$ equals 0

i.e. Either $x-7 = 0$ or $x+1 = 0$

$\therefore x = 7$ < or $x = -1$ <

Check your answers! 



Other ...

Note:
Solving equations requires reversing operations



+ \leftrightarrow -

x \leftrightarrow \div

powers \leftrightarrow roots

$x + 3 = 8$

$\therefore x + 3 - 3 = 8 - 3$
 $\therefore x = 5$ <

$3x = 12$

$\therefore \frac{3x}{3} = \frac{12}{3}$
 $\therefore x = 4$ <

$x^3 = 8$

Take $\sqrt[3]{}$ on both sides
 $\therefore x = 2$ <

Note: $x^2 = 9$
 $\therefore x = \pm 3$
 If the power is even, there are 2 roots!

$x - 3 = 8$

$\therefore x - 3 + 3 = 8 + 3$
 $\therefore x = 11$ <

$\frac{x}{3} = 12$

$\therefore \frac{x}{3} \times 3 = 12 \times 3$
 $\therefore x = 36$ <

$\sqrt{x} = 5$

Square both sides
 $(\sqrt{x})^2 = 5^2$
 $\therefore x = 25$ <



ALWAYS CHECK YOUR ANSWER!

So, solve for x : (a) $\sqrt{x} = 2$ (b) $\sqrt{\sqrt{x}} = 2$

Square both sides

$\therefore x = 4$ <

Square both sides

$\therefore \sqrt{x} = 4$

Square both sides again

$\therefore x = 16$ <

Now see Q7.1

7.1

$$\sqrt{\sqrt{\sqrt{x}}} = 2$$

Square both sides

$$\therefore (\sqrt{\sqrt{\sqrt{x}}})^2 = (2)^2$$

$$\therefore \sqrt{\sqrt{x}} = 4$$

Square both sides again

$$\therefore (\sqrt{\sqrt{x}})^2 = (4)^2$$

$$\therefore \sqrt{x} = 16$$

Square both sides again!

$$\therefore (\sqrt{x})^2 = (16)^2$$

$$\therefore x = 256 \quad \blacktriangleleft$$

Check your answer!



7.2

$$\sqrt{\frac{1}{\sqrt{x}}} = 2$$

$$\therefore \left(\sqrt{\frac{1}{\sqrt{x}}}\right)^2 = (2)^2$$

$$\therefore \frac{1}{\sqrt{x}} = 4$$

$$\therefore \left(\frac{1}{\sqrt{x}}\right)^2 = (4)^2$$

$$\therefore \frac{1}{x} = \frac{16}{1}$$

$$\therefore x = \frac{1}{16} \quad \blacktriangleleft$$

Check your answer!

**NOTES**

GRAPHS

1.1 B ◀ ... $f(x) = 2x + 4$:

- positive gradient of $\frac{2}{1}$... $2x$,
- y-int of 4 ... $y = 4$ when $x = 0$

If a point lies on a line, then the **equation** of the graph will be **true** for its coordinates. (See Question 1.2)

1.2 D ◀ ... The equation is $y = x$, so x and y will have to be equal (i.e. the coordinates must have the same value)

1.3 B ◀ ... $\frac{d}{6} = \frac{2}{3}$ $\therefore d = 4$... equivalent fractions

Very important to know:

On the **Y-axis**, the **x**-coordinate is (always) **0** (See Question 1.4)

On the **X-axis**, the **y**-coordinate is (always) **0** (See Question 1.5)

1.4 C ◀ ... Substitute $x = 0$; then

$$y\text{-intercept: } 4(0) + 2y = 12$$

$$\therefore 2y = 12$$

$$\therefore y = 6$$

[So, the point on the y-axis is (0; 6)]

1.5 B ◀ ... Substitute $y = 0$; then

$$x\text{-intercept: } 3(0) + 2x + 1 = 0$$

$$\therefore 2x = -1$$

$$\therefore x = -\frac{1}{2}$$

$\therefore \left(-\frac{1}{2}; 0\right)$... the coordinates of the x-intercept

2. P is the intersection of the lines $y = x$ and $y = 3$ and so at point P, both these equations must 'be true'.

So, y must equal x and y must equal 3. $\therefore y = x = 3$

$\therefore P(3; 3)$ ◀

3.1 $y = 3x - 5$

x	-2	-1	0	1
y	-11	-8	-5	-2

$$y = 3(-2) - 5 = -11$$

$$y = 3(-1) - 5 = -8$$

$$y = 3(0) - 5 = -5$$

$$y = 3(1) - 5 = -2$$

We substitute the values of x into the equation to find y .

3.2 $y = -\frac{2}{3}x - 1$

x	-3	-1	0	1
y	1	$-\frac{1}{3}$	-1	$-\frac{5}{3}$

$$y = -\frac{2}{3}\left(-\frac{3}{1}\right) - 1 = 2 - 1 = 1$$

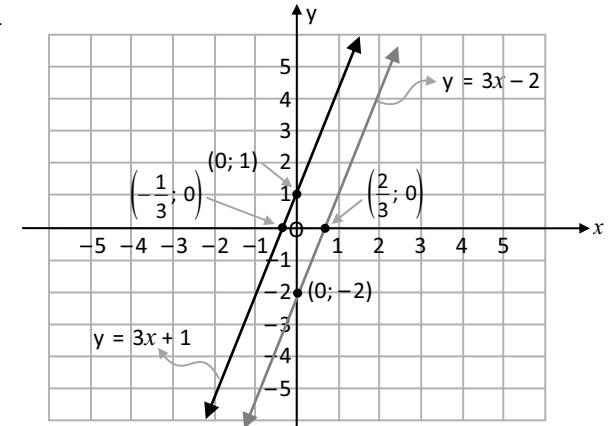
$$y = -\frac{2}{3}(-1) - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$y = -\frac{2}{3}(0) - 1 = -1$$

$$y = -\frac{2}{3}(1) - 1 = -\frac{2}{3} - 1 = -\frac{5}{3}$$



4.1



To find the points where the graphs cut the axes:

$y = 3x - 2$:

$y = 3x + 1$:

For the **Y-intercept**, substitute $x = 0$

$$\therefore y = 3(0) - 2 = -2$$

$$\therefore y = 3(0) + 1 = 1$$

\therefore The graph cuts the **y-axis** at -2.

\therefore The graph cuts the **y-axis** at 1.

The **point** is (0; -2)

The **point** is (0; 1)

For the **X-intercept**, substitute $y = 0$

$$\therefore 0 = 3x - 2$$

$$\therefore 3x = 2$$

$$\therefore x = \frac{2}{3}$$

\therefore The graph cuts the **x-axis** at $\frac{2}{3}$.

The **point** is $\left(\frac{2}{3}; 0\right)$

$$\therefore 0 = 3x + 1$$

$$\therefore 3x = -1$$

$$\therefore x = -\frac{1}{3}$$

\therefore The graph cuts the **x-axis** at $-\frac{1}{3}$.

The **point** is $\left(-\frac{1}{3}; 0\right)$

4.2 They are parallel ◀ ... they have equal gradients

5.1 **AD:** The gradient = $-\frac{4}{2} = -2$... $\therefore m = -2$
& the y-intercept is 4 ... $\therefore c = 4$

\therefore The equation is $y = -2x + 4$ ◀ ... $m = -2$ & $c = 4$
in $y = mx + c$

BC: The gradient = $-\frac{4}{2} = -2$... $\therefore m = -2$
& the y-intercept is -4 ... $\therefore c = -4$

\therefore The equation is $y = -2x - 4$ ◀ ... $m = -2$ & $c = -4$
in $y = mx + c$

The standard form of the equation of a straight line is **$y = mx + c$** , where
m = the gradient and **c** = the y-intercept.



5.2 They are parallel.
They both have gradients of -2.

Both gradients are negative and are measured as $\frac{\text{number of units down}}{\text{number of units across}}$
i.e. $\frac{\text{vertical change}}{\text{horizontal change}}$

6.1 The gradient = $-\frac{5}{1} = -5$ ◀

By inspection

- negative gradient
- $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{vertical change}}{\text{horizontal change}}$

So, — and **5 units down**
1 unit across

The use of a formula for the gradient is not ideal for grade 9 learners.

6.2 $y = -5x + 5$ ◀ ... gradient, $m = -5$ & y-intercept, $c = 5$

6.3 The gradient of any other straight line drawn parallel to this line is -5. ◀ ... parallel lines have the same gradient

7.1

	A	B	C
x-coordinate	0	2	4
y-coordinate	-2	0	2



Note: $x = 0$ on the **y-axis** (at A)
& $y = 0$ on the **x-axis** (at B)

7.2 $y = x - 2$ ◀ ... By inspection:
The y-coordinates are all 2 less than the x-coordinates.

or: Gradient = $+\frac{2}{2} = 1$
& y-intercept, $c = -2$

8.1 The equation of CD: $x = 2$ ◀

... because every point on (vertical) line CD has an x-coordinate equal to 2

$\therefore x = 2$ is the **equation** of CD



The equation of a line is a 'rule' which is true for all points on the line.

8.2 The equation of AB: $y = 2x$ ◀

Method 1:

Observe various points on the graph:
e.g. (-2; -4); (-1; -2); (1; 2)
and notice that y always equals twice x

Method 2:

m, the gradient = $+\frac{2}{1} = 2$
& c, the y-intercept, is 0

8.3 **E(2; -2)** ◀ ... $x = 2$ and $y = -2$ at point E

8.4 **CE = 6 units** ◀ ... $CE = CD + DE = 4 + 2 = 6$ units

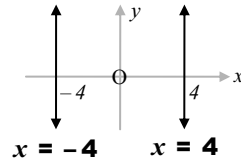
OR $CE = Y_C - Y_E$... the difference of the y-coordinates of C and E
= $4 - (-2)$
= 6



Solutions: Graphs

- 9.1 The lines $x = 4$ and $x = -4$ are **parallel** to one another. ◀

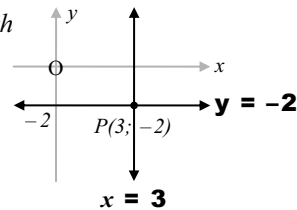
... The lines $x = 4$ and $x = -4$:
are both parallel to the y -axis



- 9.2 The equation of the horizontal line through the point $P(3; -2)$ is $y = -2$. ◀

... The horizontal line through
 $P(3; -2)$ is $y = -2$;

The vertical line through
 $P(3; -2)$ is $x = 3$;



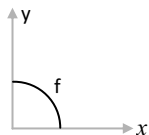
- 9.3 The gradient of the line defined by $y - 4x + 5 = 0$ is equal to **4**. ◀

$$\dots y - 4x + 5 = 0$$

$$\therefore y = 4x - 5 \quad \dots y = mx + c$$

\therefore The gradient, which is the coefficient of x , is **4**

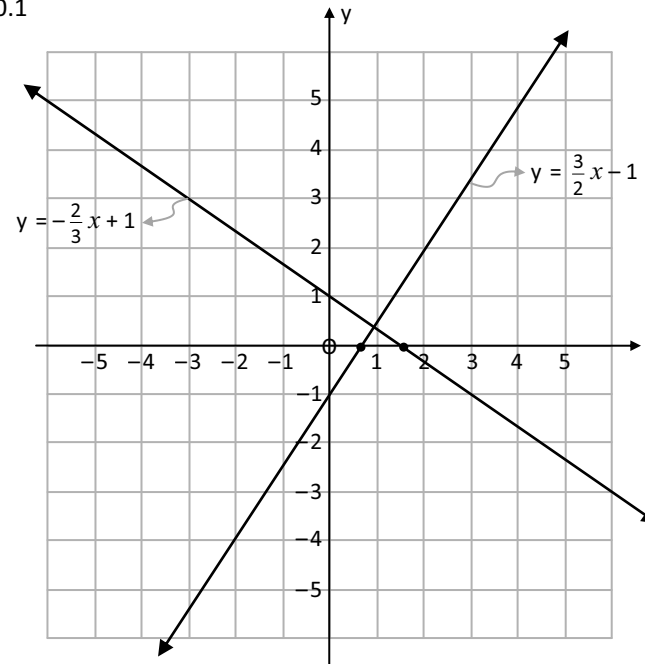
- 9.4 This graph of f below represents a **non-linear** function. ◀



... A linear function is
a straight line,
not a curve.



10.1



To find the points where the graphs cut the axes:

$$y = -\frac{2}{3}x + 1:$$

$$y = \frac{3}{2}x - 1:$$

For the **Y-intercept**, substitute $x = 0$

$$\begin{aligned} \therefore y &= -\frac{2}{3}(0) + 1 \\ &= 1 \end{aligned}$$

\therefore The graph cuts the
y-axis at 1.

The **point** is **(0; 1)**

$$\begin{aligned} \therefore y &= \frac{3}{2}(0) - 1 \\ &= -1 \end{aligned}$$

\therefore The graph cuts the
y-axis at -1.

The **point** is **(0; -1)**

For the **X-intercept**, substitute $y = 0$

$$\therefore 0 = -\frac{2}{3}x + 1$$

$$\therefore \frac{2}{3}x = 1$$

$$\therefore 2x = 3 \quad \dots \times 3$$

$$\therefore x = \frac{3}{2} \quad \dots \div 2$$

\therefore The graph cuts the
x-axis at $\frac{3}{2}$.

The **point** is **($\frac{3}{2}$; 0)**

$$\therefore 0 = \frac{3}{2}x - 1$$

$$\therefore \frac{3}{2}x = 1$$

$$\therefore 3x = 2 \quad \dots \times 2$$

$$\therefore x = \frac{2}{3} \quad \dots \div 3$$

\therefore The graph cuts the
x-axis at $\frac{2}{3}$.

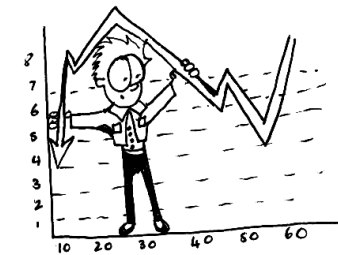
The **point** is **($\frac{2}{3}$; 0)**

- 10.2 They are perpendicular.

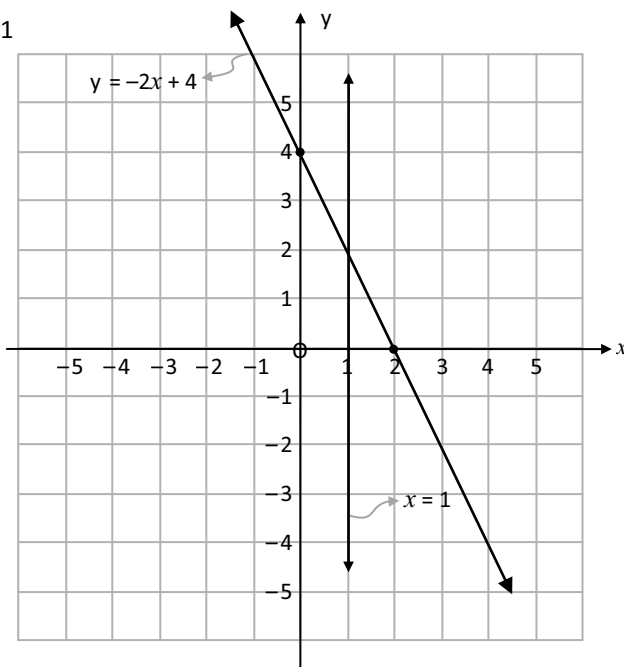
Out of interest:

Compare the gradients, $-\frac{2}{3}$ and $\frac{3}{2}$.

They are negative inverses of one another.



11.1



To find the points where the graphs cut the axes:

$$y = -2x + 4:$$

$$\text{y-intercept (substitute } x = 0): y = -2(0) + 4 = 4$$

$$\text{x-intercept (substitute } y = 0): 0 = -2x + 4 \\ \therefore 2x = 4 \\ \therefore x = 2$$

$x = 1$: This graph is a **vertical** line through $x = 1$.
Every point on the graph has an x -coordinate equal to 1.

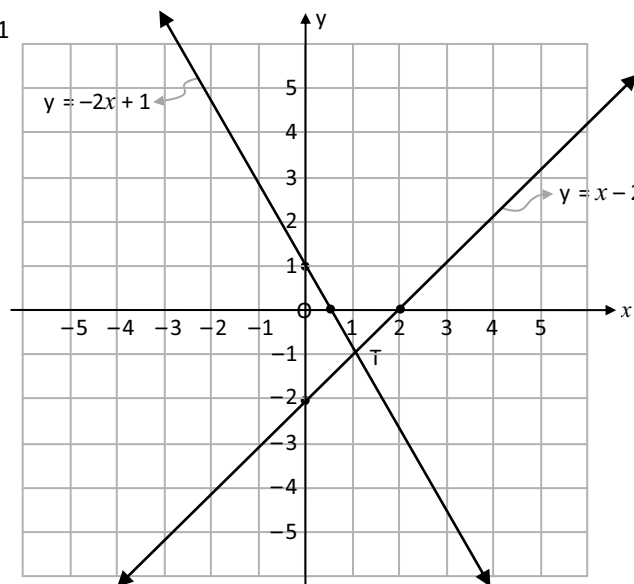
11.2 The point of intersection is $(1; 2)$ ◀

... At this point, $x = 1$ and $y = -2x + 4$
(i.e. both equations are true)

$$\therefore y = -2(1) + 4 = 2$$

\therefore The point is $(1; 2)$

12.1



To find the points where the graphs cut the axes:

$$y = -2x + 1:$$

$$\text{For the Y-intercept, substitute } x = 0 \\ \therefore y = -2(0) + 1 = 1$$

\therefore The graph cuts the **y-axis** at 1.

The **point** is $(0; 1)$

$$\text{For the X-intercept, substitute } y = 0$$

$$\therefore 0 = -2x + 1 \\ \therefore 2x = 1 \\ \therefore x = \frac{1}{2}$$

\therefore The graph cuts the **x-axis** at $\frac{1}{2}$.

The **point** is $(\frac{1}{2}; 0)$

$$y = x - 2:$$

$$\therefore y = (0) - 2 = -2$$

\therefore The graph cuts the **y-axis** at -2.

The **point** is $(0; -2)$

$$\therefore 0 = x - 2 \\ \therefore x = 2$$

\therefore The graph cuts the **x-axis** at 2.

The **point** is $(2; 0)$ 12.2 $y = -2x + 1 \dots \textcircled{1}$ & $y = x - 2 \dots \textcircled{2}$

$$\begin{aligned} \textcircled{1} = \textcircled{2}: -2x + 1 &= x - 2 \\ \therefore -2x - x &= -2 - 1 \\ \therefore -3x &= -3 \\ \therefore x &= 1 \end{aligned}$$

Both equations must be true at T, the point of intersection.



$$\text{Substitute } x = 1 \text{ into } \textcircled{2}: y = (1) - 2 \quad [\text{OR into } \textcircled{1}] \\ = -1$$

$$\therefore T(1; -1) \blacktriangleleft$$

NOTES

