basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2016

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 28 pages.

INSTRUCTIONS AND INFORMATION

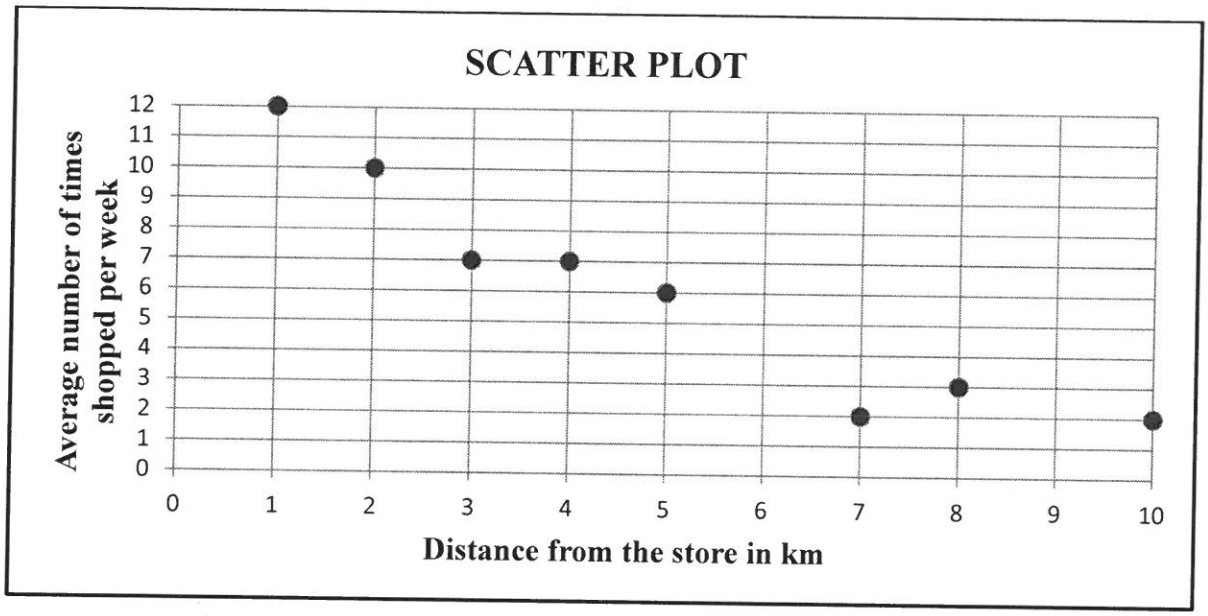
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



1.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

1.2 Calculate the correlation coefficient of the data. (1)

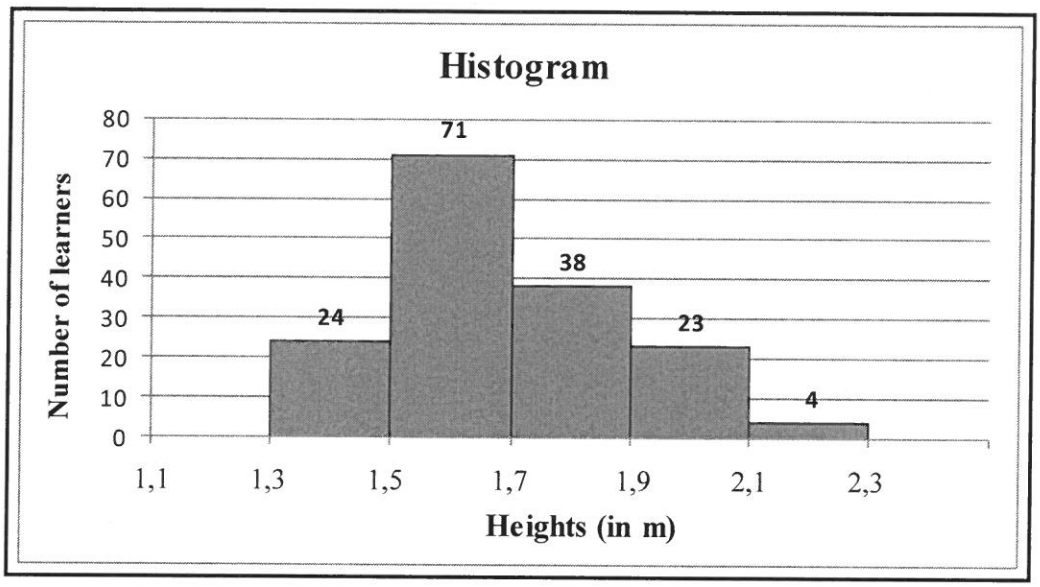
1.3 Calculate the equation of the least squares regression line of the data. (3)

1.4 Use your answer at QUESTION 1.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)

1.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)
[9]

QUESTION 2

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

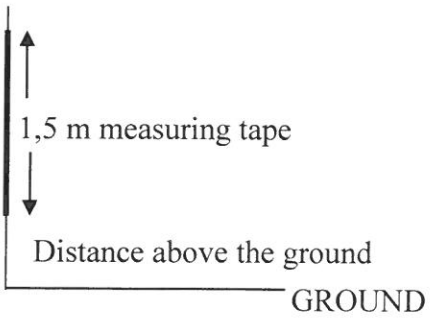
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

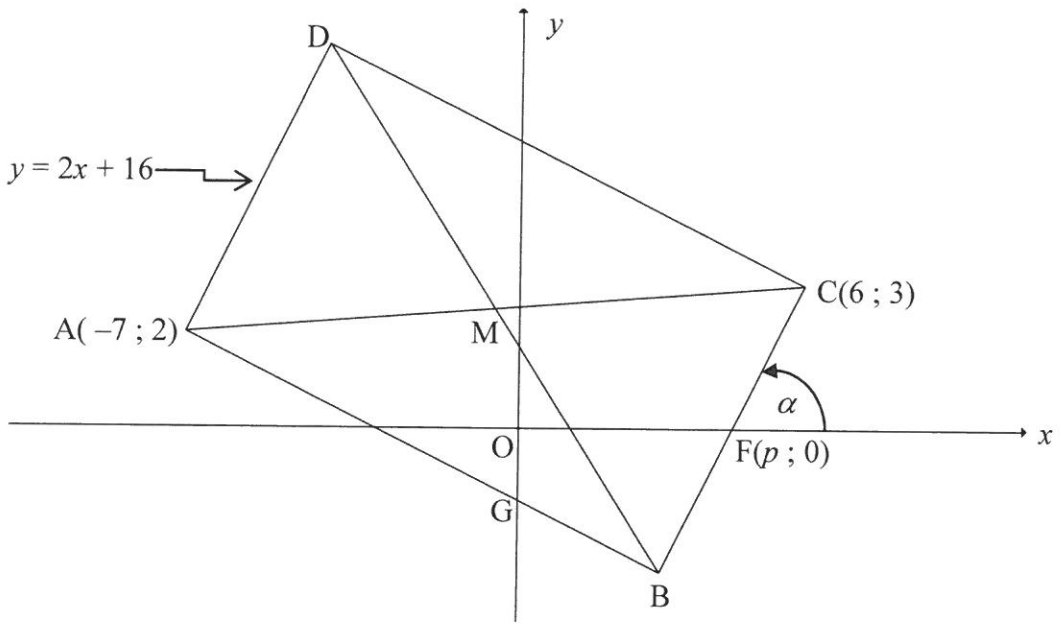
2.6.1 Mean of the data set (1)

2.6.2 Standard deviation of the data set (1)

[13]

QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α. (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

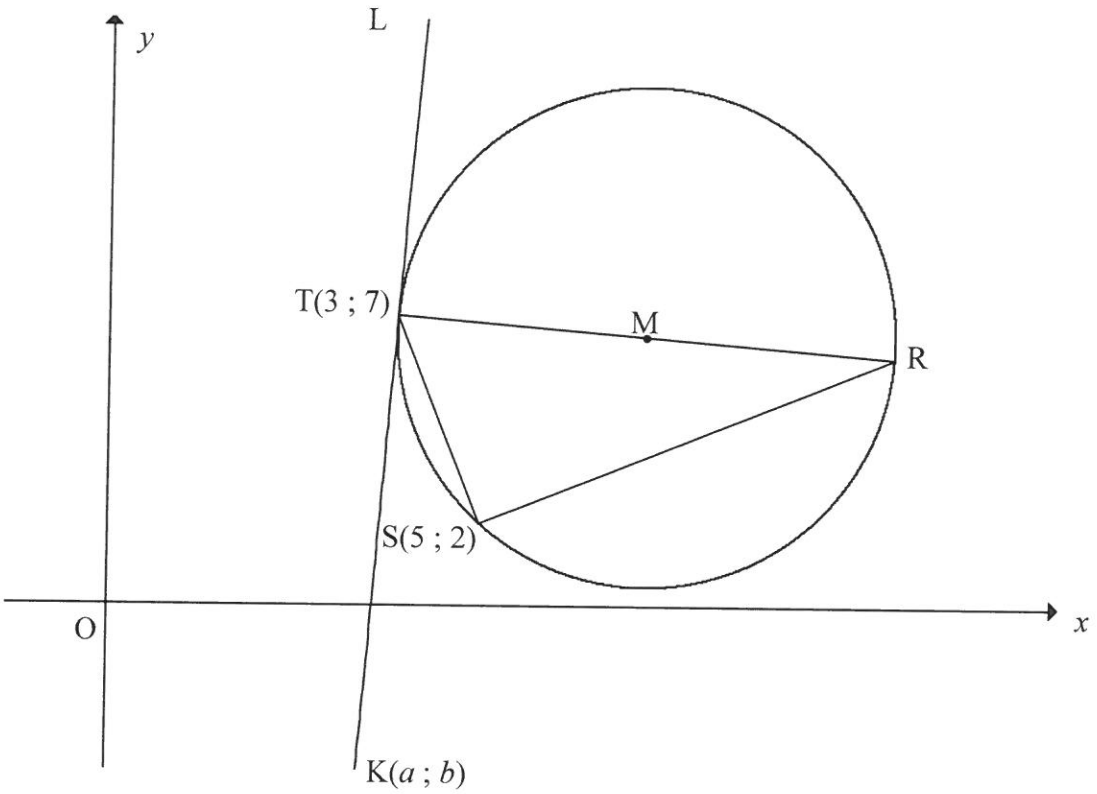
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2+(y-b)^2=r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)

[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

QUESTION 5

5.1 Given: $\sin 16° = p$

Determine the following in terms of p, **without using a calculator**.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

5.4.1 $\cos B$ (3)

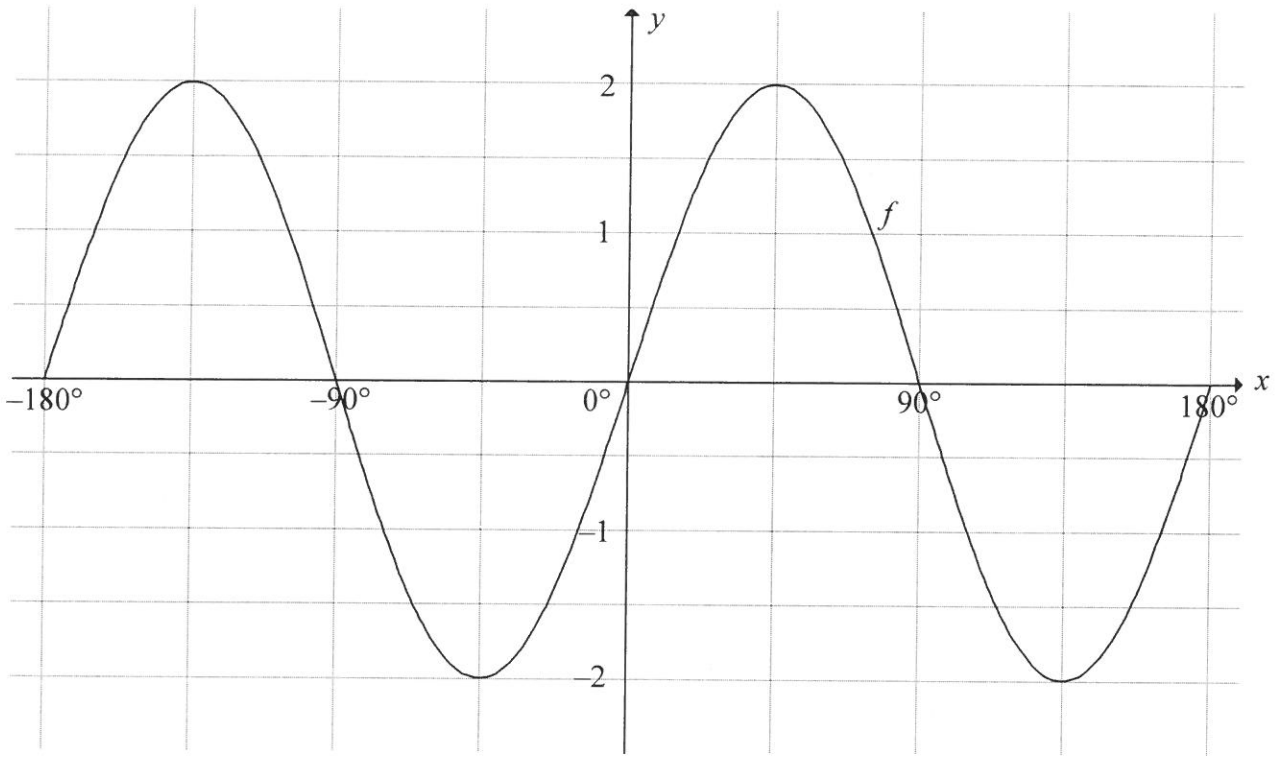
5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45°)$ (4)

[21]

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

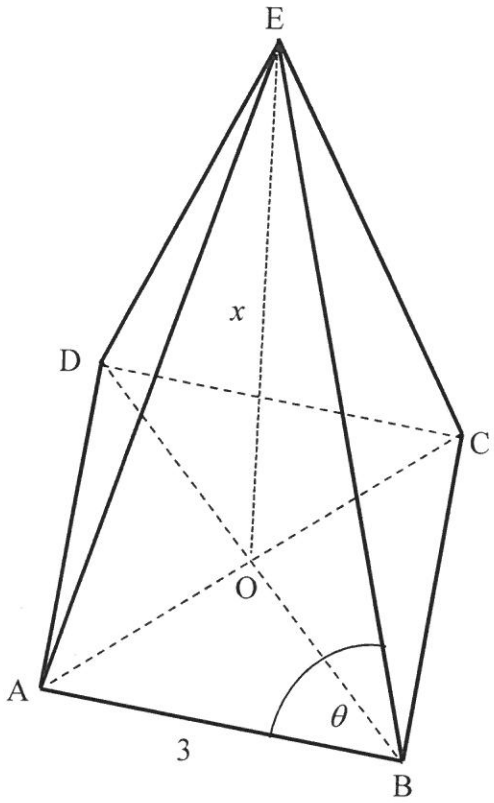
6.3 Determine the general solution of $f(x) = g(x)$. (4)

6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)
[12]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height)



7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

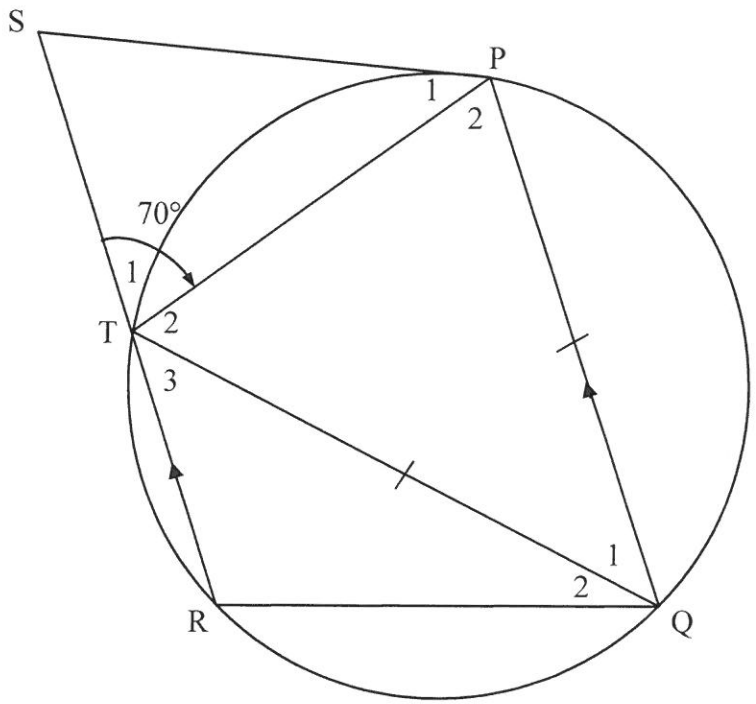
7.3 If the volume of the pyramid is 15 m^3, calculate the value of θ. (4)

[12]

Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below PQRT is a cyclic quadrilateral having RT || QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



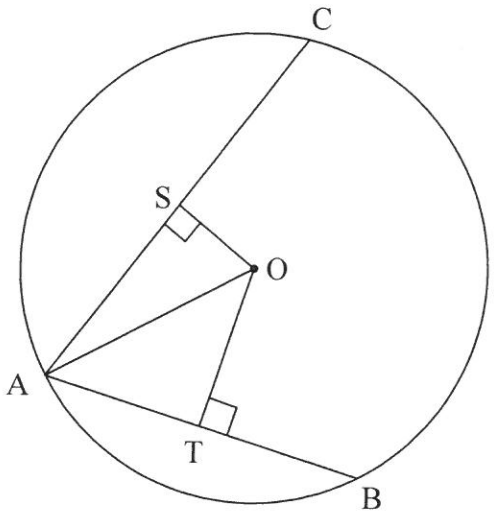
8.1.1 Give a reason why $\hat{P}_2 = 70°$. (1)

8.1.2 Calculate, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that OS ⊥ AC and OT ⊥ AB. AB = 40 and AC = 48.



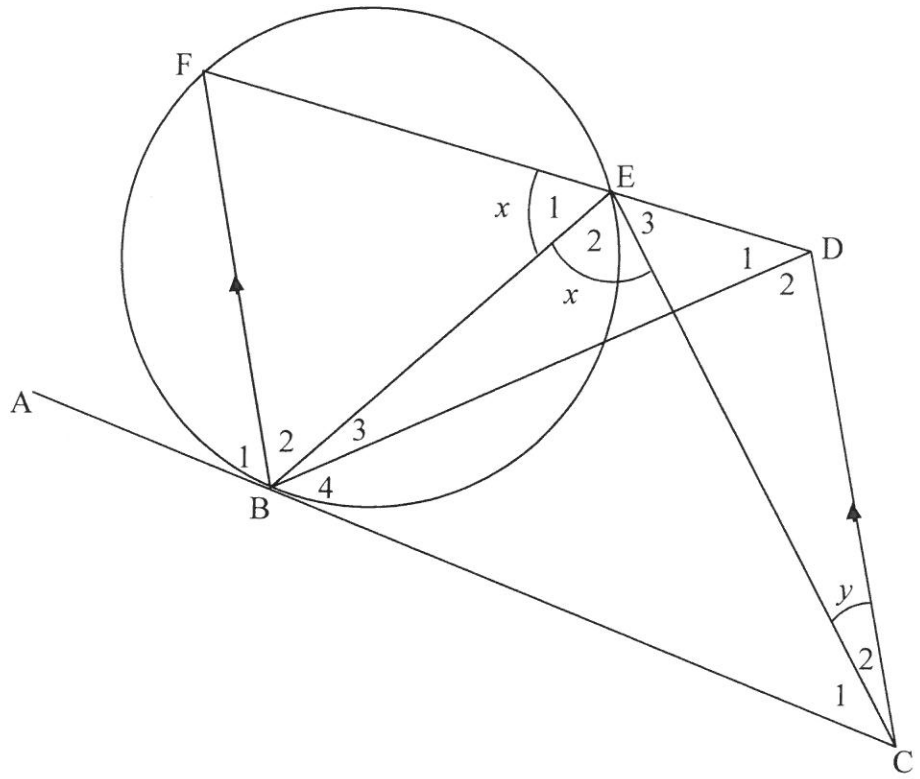
8.2.1 Calculate AT. (1)

8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

QUESTION 9

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1 Give a reason why EACH of the following is TRUE:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Prove that BCDE is a cyclic quadrilateral. (2)

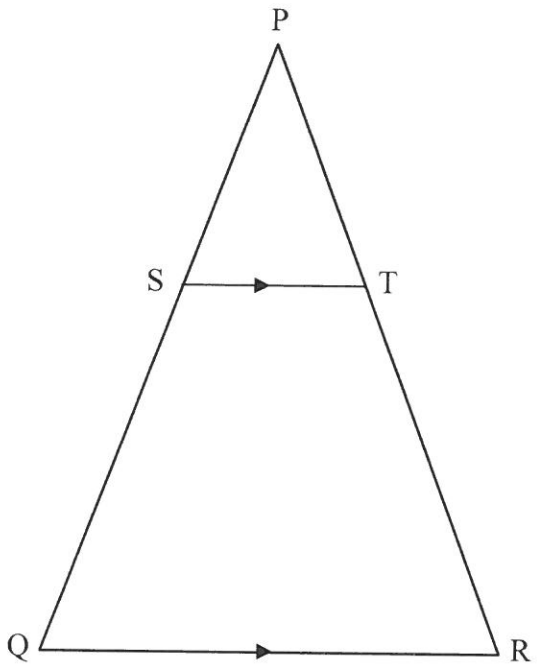
9.3 Which TWO other angles are each equal to x? (2)

9.4 Prove that $\hat{B}_2 = \hat{C}_1$. (3)

[9]

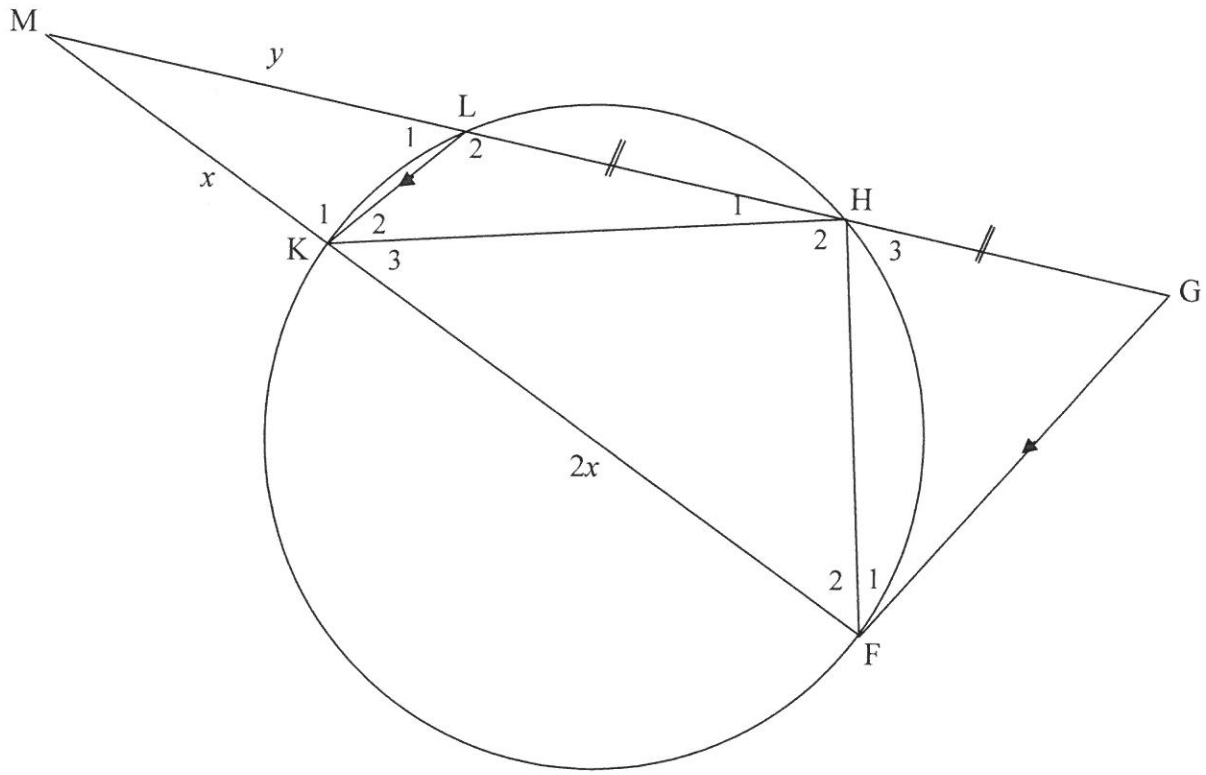
QUESTION 10

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that ST || QR.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. MK = x, KF = $2x$, ML = y and LH = HG.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Show that $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

[20]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

basic education

Department:
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Candidate Barcode Label/*Stafieskodeplakker*

NSC Answer Book
NSS-antwoordeboek

National Senior Certificate/*Nasionale Senior Sertifikaat* (Grade 12/*Graad 12*)

Sequence no on mark sheet
Volgnommer op puntestaat

CENTRE NUMBER *SENTRUMNOMMER*	
EXAMINATION NUMBER *EKSAMENNOMMER*	
DATE *DATUM*	
BOOK NUMBER *BOEKNOMMER* / OF *VAN* / BOOKS *BOEKE*	
SUBJECT CODE *VAKKODE*	
PAPER NUMBER *VRAESTELNOMMER*	2
SUBJECT NAME *VAKNAAM*	MATHEMATICS/*WISKUNDE*

This answer book consists of 28 pages./*Hierdie antwoordeboek bestaan uit 28 bladsye.*

MARKER/*NASIENER*			MODERATOR'S INITIALS IN RELEVANT BLOCK *MODERATORPARAAF IN BETROKKE BLOKKIE*					
Question *Vraag*	Marks *Punte*	Marker's Code & Initials	Marks *Punte*	SM	DCM	CM	IM	EM
1								
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RE-MARK/RE-CHECK/ *HERNASIEN/HERKONTROLEER*		
Question *Vraag*	Marks *Punte*	Initials *Paraaf*
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		TOTAL *TOTAAL*			

TOTAL *TOTAAL*	
HASH TOTAL *KAFTOTAAL*	

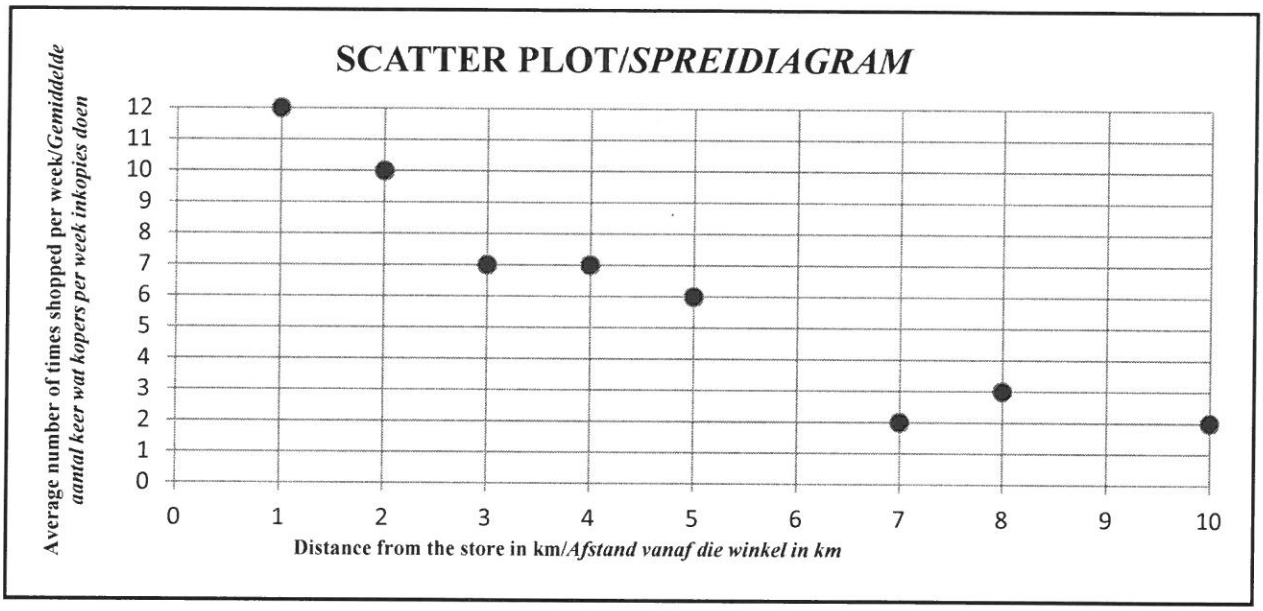
CONTROLLED AND CERTIFIED CORRECT (SURNAME AND INITIALS OF EA)	

READ INSTRUCTIONS ON THE NEXT PAGE.
LEES INSTRUKSIES OP VOLGENDE BLADSY.

FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG HIERDIE INSTRUKSIES NOUKEURIG
1. Write your examination number and centre number clearly in the spaces provided and attach your examination number sticker in the space provided.	1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes soos verskaf en plak jou eksamennommeretiket in die ruimte soos verskaf.
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. Onthou dat jou eie naam (of die naam van jou skool) nie op of in hierdie antwoordeboek mag voorkom nie.
3. Answer ALL questions in the spaces provided.	3. Beantwoord ALLE vrae in die ruimtes wat voorsien is.
4. No pages may be torn from this answer book.	4. Geen bladsye mag uit hierdie antwoordeboek geskeur word nie.
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. Lees die instruksies wat op jou eksamenrooster gedruk is sorgvuldig deur, asook enige ander instruksies wat in elke eksamenvraestel gegee word.
6. Candidates may not retain an answer book or remove it from the examination room. Answer books must be returned to the Invigilator at the end of the examination session.	6. Geen antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word nie. Antwoordeboeke moet aan die Toesighouer terugbesorg word aan die einde van die eksamensessie.
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Moenie in die kantlyn skryf nie.
8. Indicate the questions you have answered by drawing a circle around the relevant numbers on the front cover of the answer book where marks are to be recorded.	8. Dui die vrae wat jy beantwoord het aan op die voorblad van die antwoordeboek waar die punte aangebring word, deur 'n kringetjie te trek om die nommers van die vrae wat jy beantwoord het.
9. Rule off through any work/rough work that must not be marked.	9. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.
10 In the event that you use the additional space provided: 10.1 Write down the number of the question 10.2 Leave a line and rule off after your answer.	10 Ingeval jy die bykomende ruimte wat voorsien word, gebruik: 10.1 Skryf die nommer van die vraag neer 10.2 Laat 'n lyn oop en trek 'n lyn na jou antwoord.

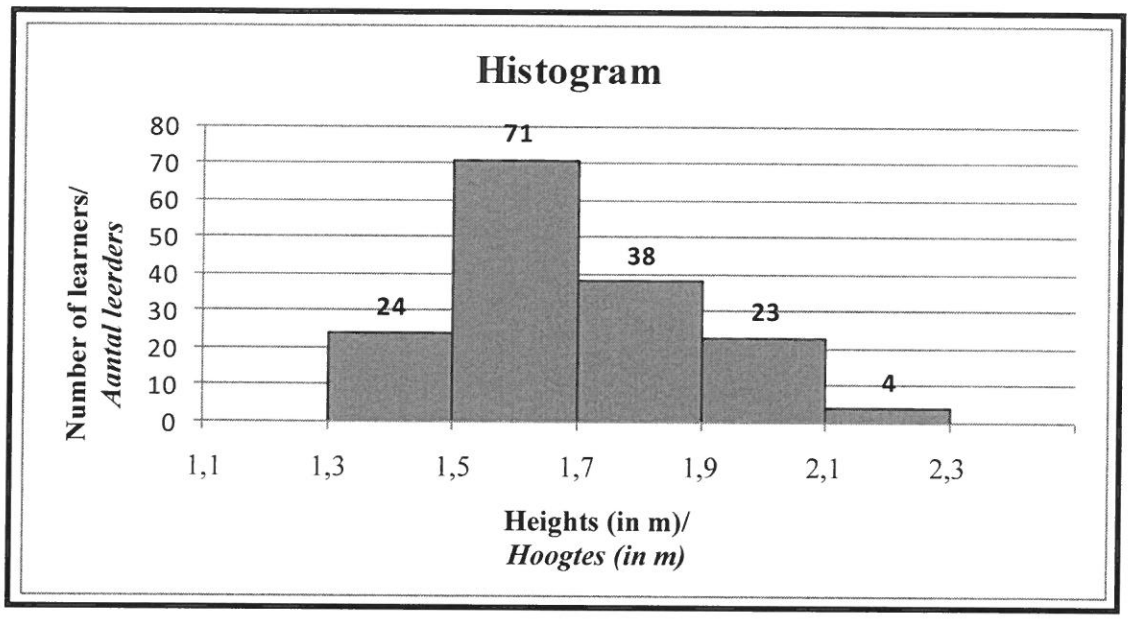
QUESTION/*VRAAG* 1

Distance from the store in km *Afstand vanaf die winkel in km*	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers per week inkopies doen*	12	10	7	7	6	2	3	2



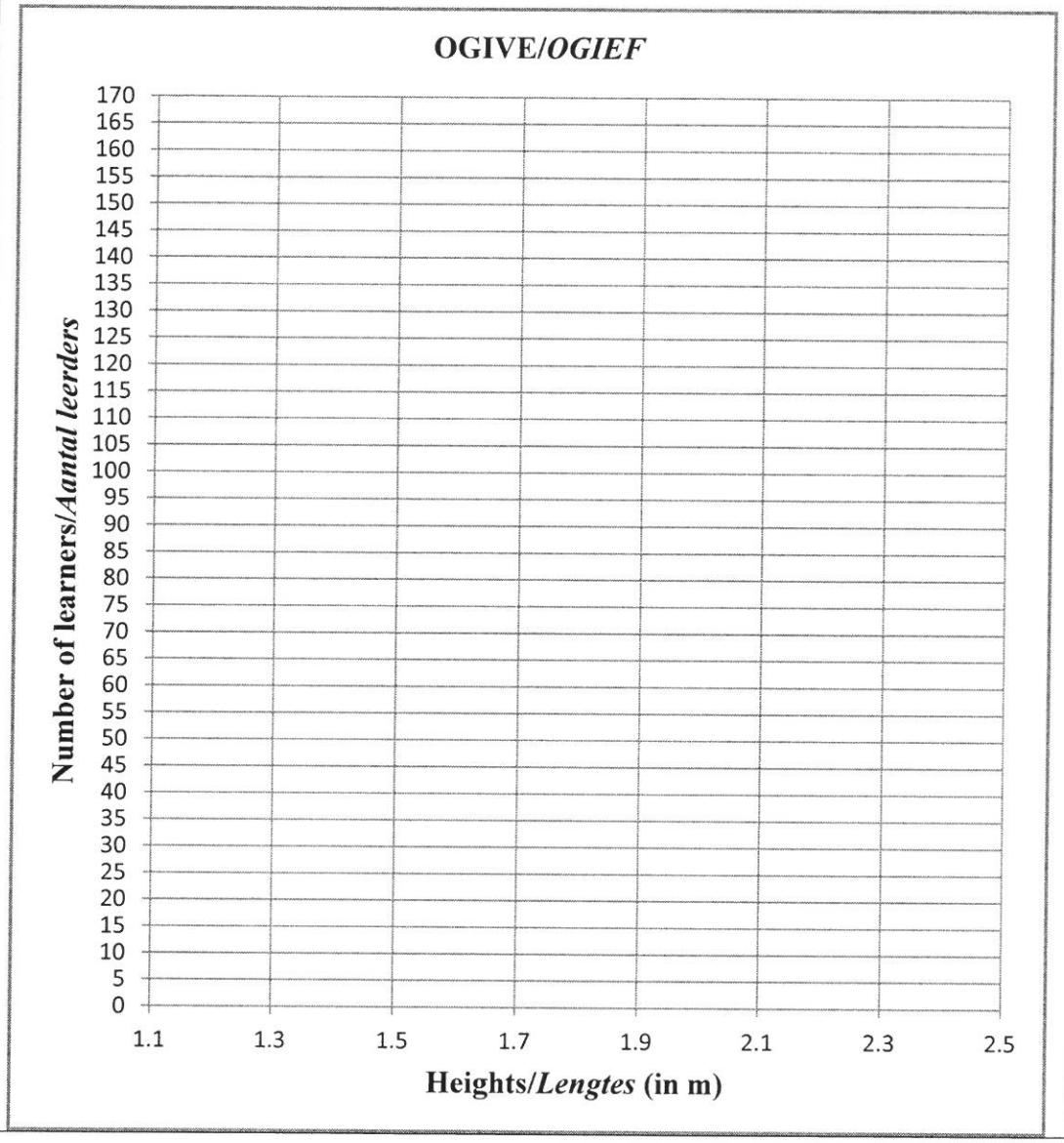
	Solution/*Oplossing*	**Marks** *Punte*
1.1		(1)
1.2		(1)
1.3		(3)
1.4		(2)
1.5	Draw on the scatter plot above./*Skets op die spreidiagram hierbo.*	(2)
		[9]

QUESTION/*VRAAG* 2

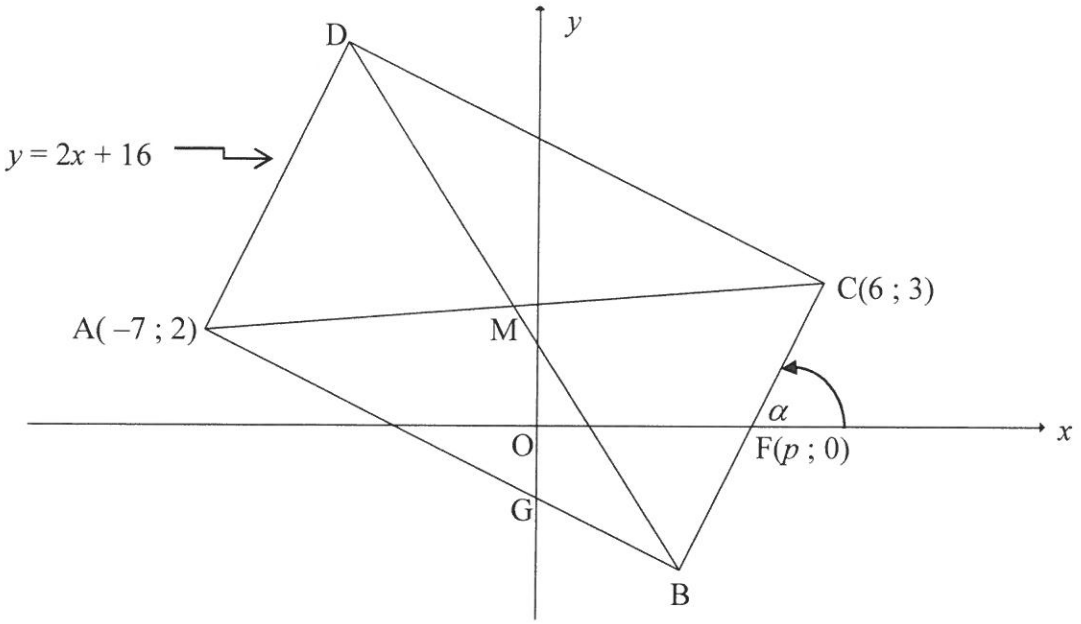


	Solution/*Oplossing*	Marks *Punte*
2.1		(1)
2.2		(2)
2.3		(2)

Intervals *Klasse*	Cumulative frequency *Kumulatiewe frekwensie*
$1,3 \leq x < 1,5$	
$1,5 \leq x < 1,7$	
$1,7 \leq x < 1,9$	
$1,9 \leq x < 2,1$	
$2,1 \leq x < 2,3$	

	Solution/*Oplossing*	Marks *Punte*
2.4		(4)
2.5		(2)
2.6.1		(1)
2.6.2		(1)
		[13]

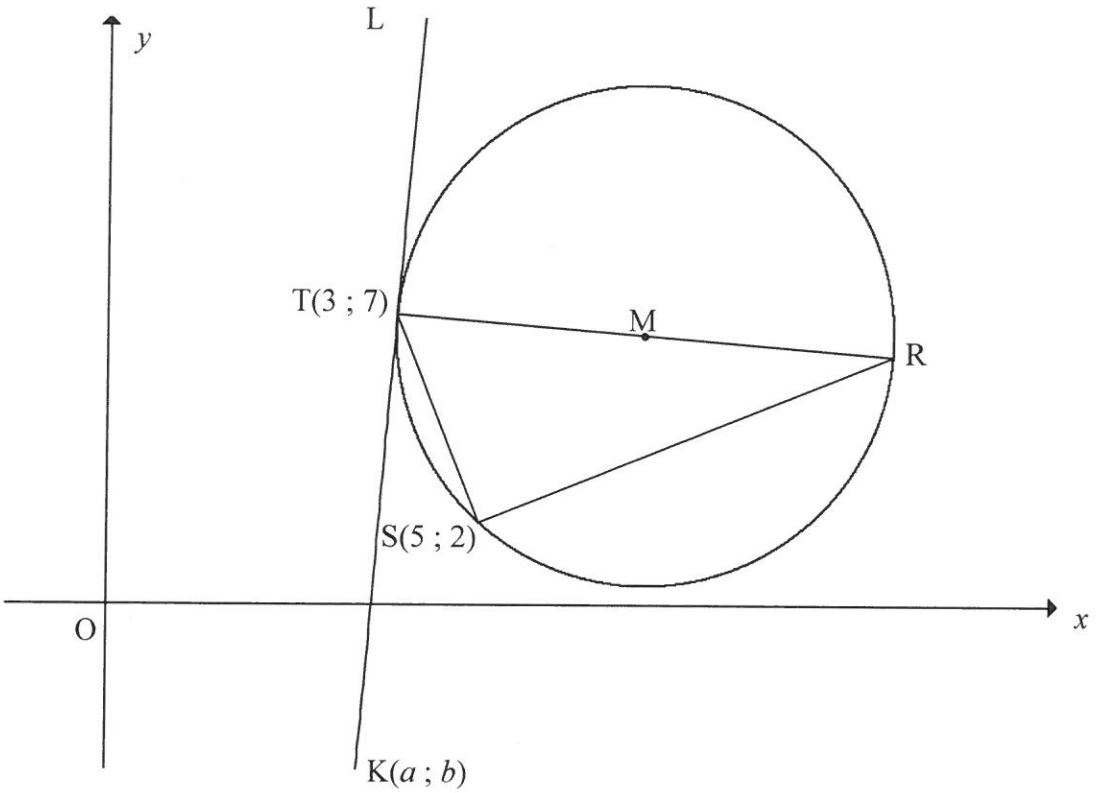
QUESTION/*VRAAG* 3



	Solution/*Oplossing*	**Marks** ***Punte***
3.1		(2)
3.2		(1)
3.3		(3)

	Solution/*Oplossing*	**Marks** ***Punte***
3.4		(3)
3.5		(2)
3.6		(3)
3.7		(3)
3.8		(2)
		[19]

QUESTION/*VRAAG* 4



	Solution/*Oplossing*	**Marks** ***Punte***
4.1		(1)
4.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
4.3		(3)
4.4.1		(2)
4.4.2		(3)
4.4.3		(3)

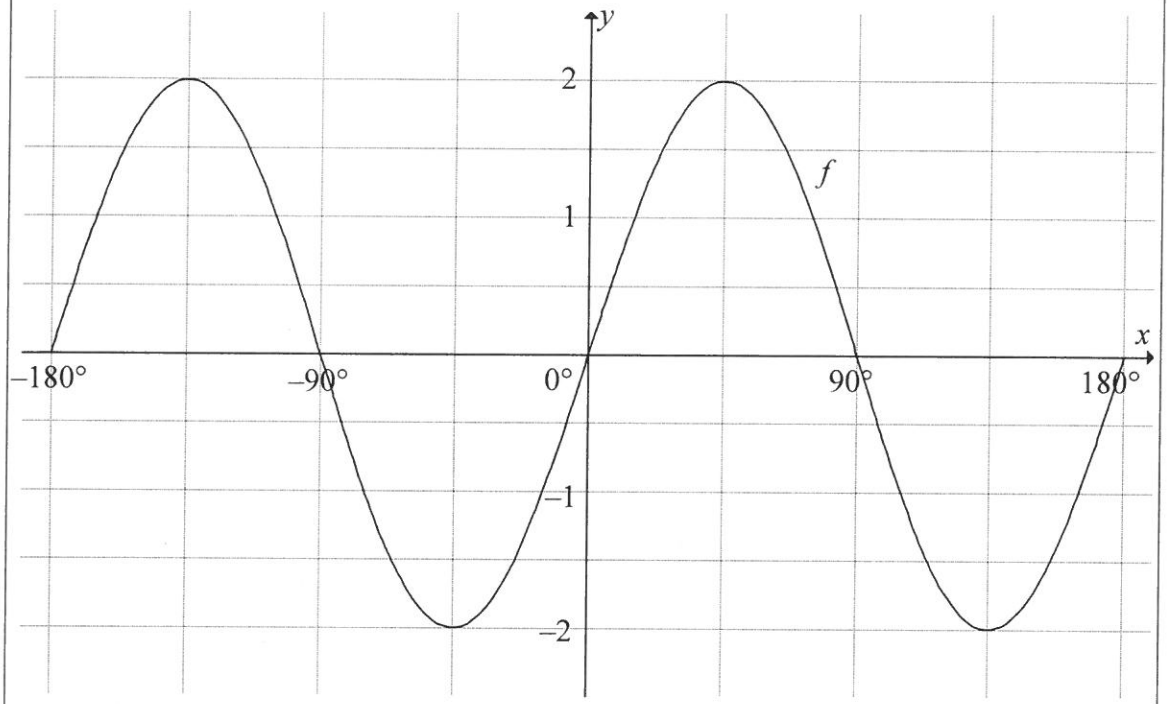
	Solution/*Oplossing*	**Marks** ***Punte***
4.4.4		(3)
4.4.5		(6)
		[23]

QUESTION/*VRAAG* 5

	Solution/*Oplossing*	**Marks** ***Punte***
5.1.1		(2)
5.1.2		(2)
5.2		(3)
5.3		(5)

	Solution/*Oplossing*	**Marks** ***Punte***
5.4.1		(3)
5.4.2		(2)
5.4.3		(4)
		[21]

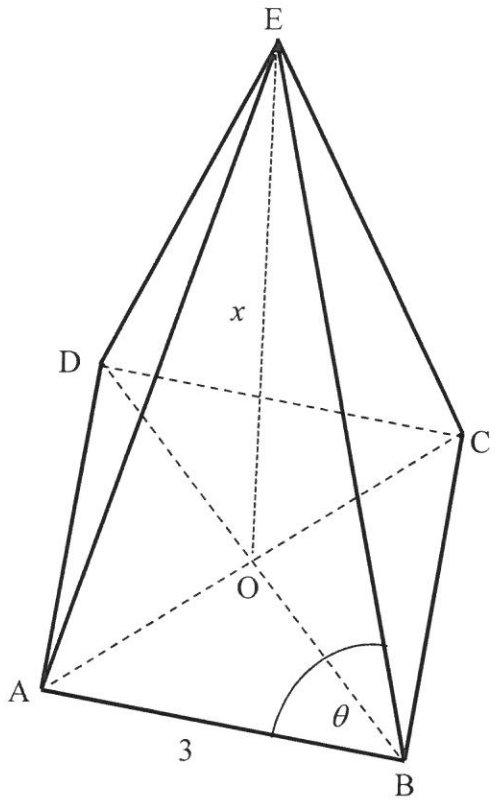
QUESTION/*VRAAG* 6

	Solution/*Oplossing*	**Marks** ***Punte***
6.1		(3)
6.2		(2)
6.3		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
6.4		(3)
		[12]

QUESTION/*VRAAG* 7

Volume of pyramid $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height)
Volume van piramide $= \frac{1}{3}$(oppervlakte van basis) $\times$ ($\perp$ hoogte)



	Solution/*Oplossing*	**Marks** ***Punte***
7.1		(3)

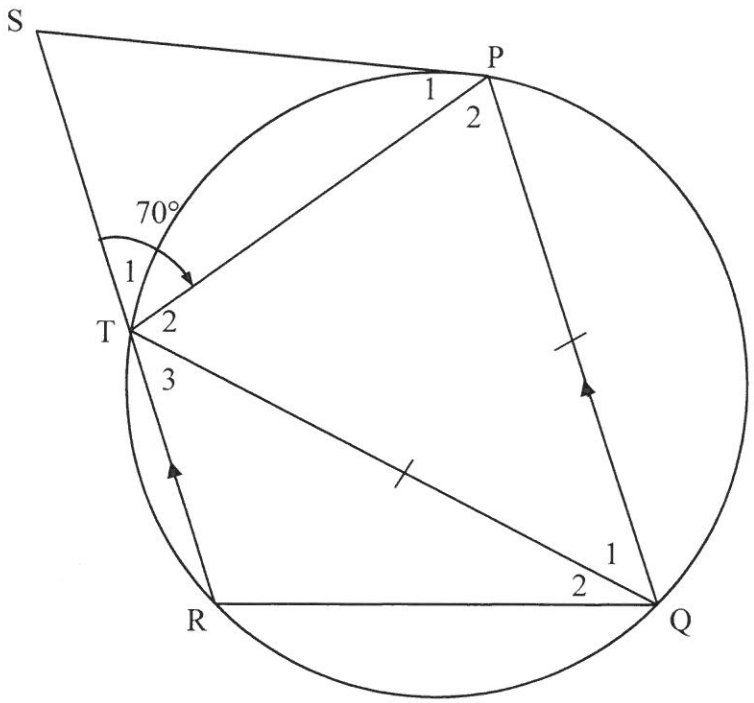
	Solution/*Oplossing*	**Marks** ***Punte***
7.2		(5)

7.3		(4)
		[12]

Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.
Gee redes vir ALLE bewerings en berekeninge in VRAAG 8, 9 en 10.

QUESTION/*VRAAG* 8

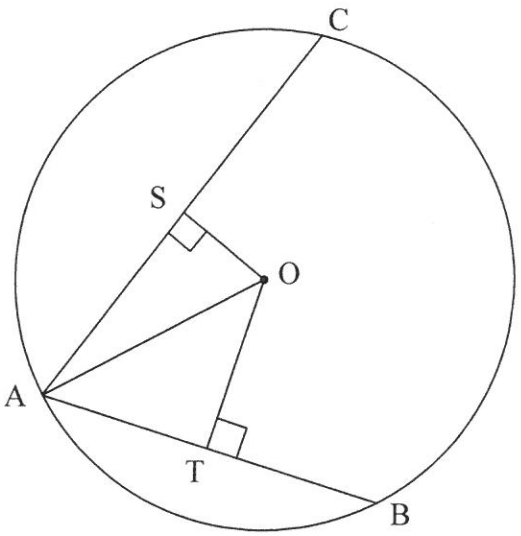
8.1



	Solution/*Oplossing*	Marks *Punte*
8.1.1		(1)

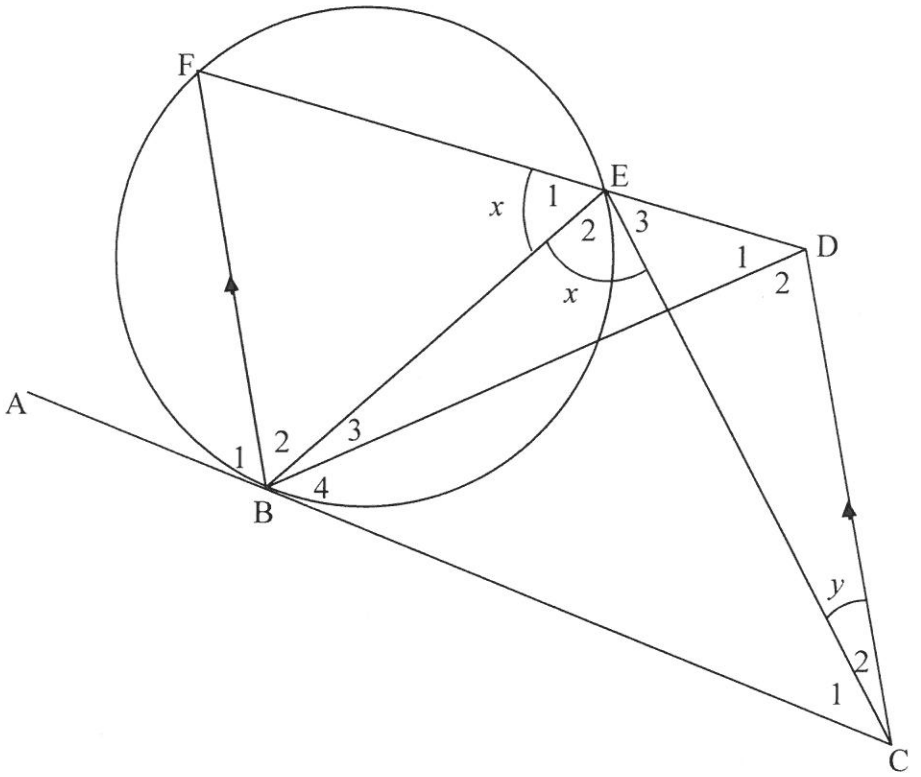
	Solution/*Oplossing*	**Marks** ***Punte***
8.1.2(a)		(3)
8.1.2(b)		(2)

8.2



	Solution/*Oplossing*	Marks *Punte*
8.2.1		(1)
8.2.2		(5)
		[12]

QUESTION/*VRAAG* 9

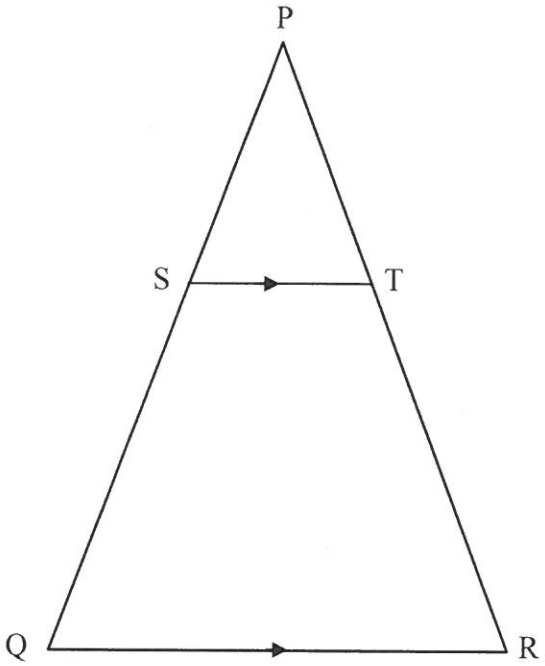


	Solution/*Oplossing*	**Marks** ***Punte***
9.1.1		(1)
9.1.2		(1)
9.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
9.3		(2)
9.4		(3)
		[9]

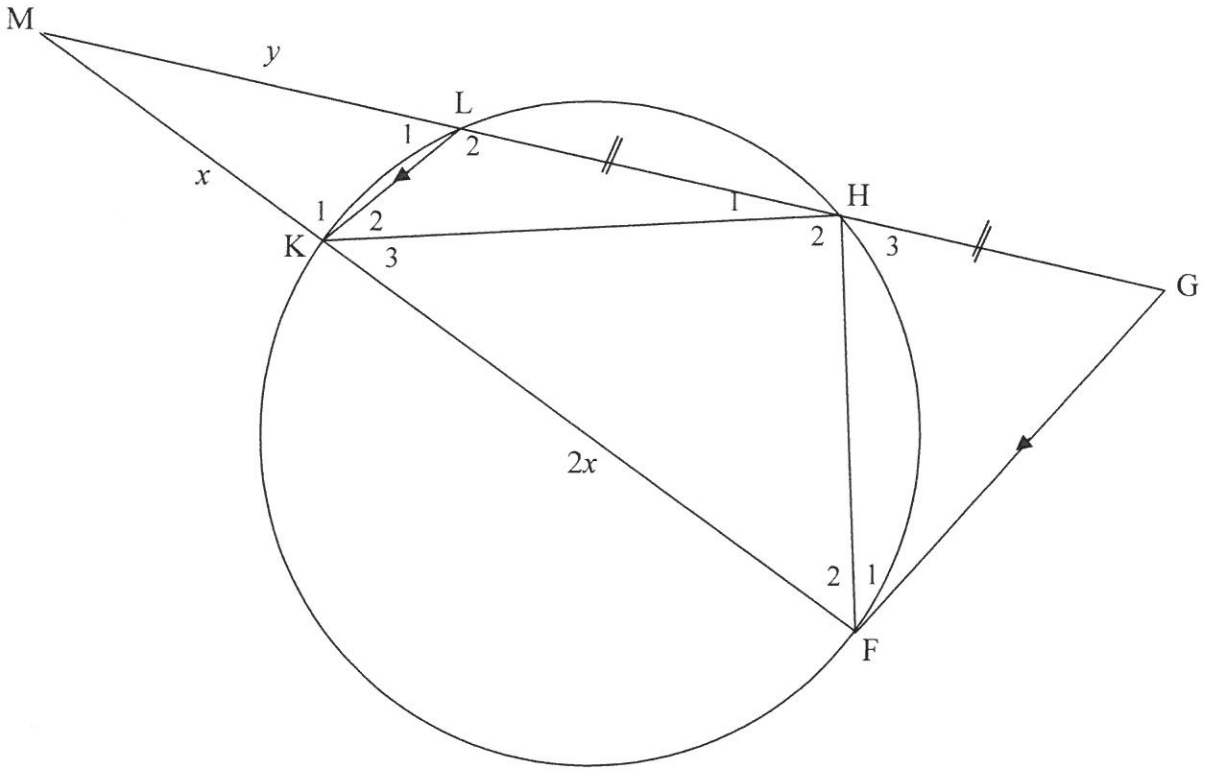
QUESTION/*VRAAG* 10

10.1



	Solution/*Oplossing*	**Marks** ***Punte***
		(6)

10.2



	Solution/*Oplossing*	**Marks** ***Punte***
10.2.1		(1)
10.2.2(a)		(3)

	Solution/*Oplossing*	**Marks** ***Punte***
10.2.2(b)		(5)
10.2.2(c)		(2)
10.2.3		(3)
		[20]

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks Punte**

	Additional space/*Bykomende ruimte*	Marks *Punte*

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE*
SENIOR
CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 26 pages.
Hierdie memorandum bestaan uit 26 bladsye.

NOTE:

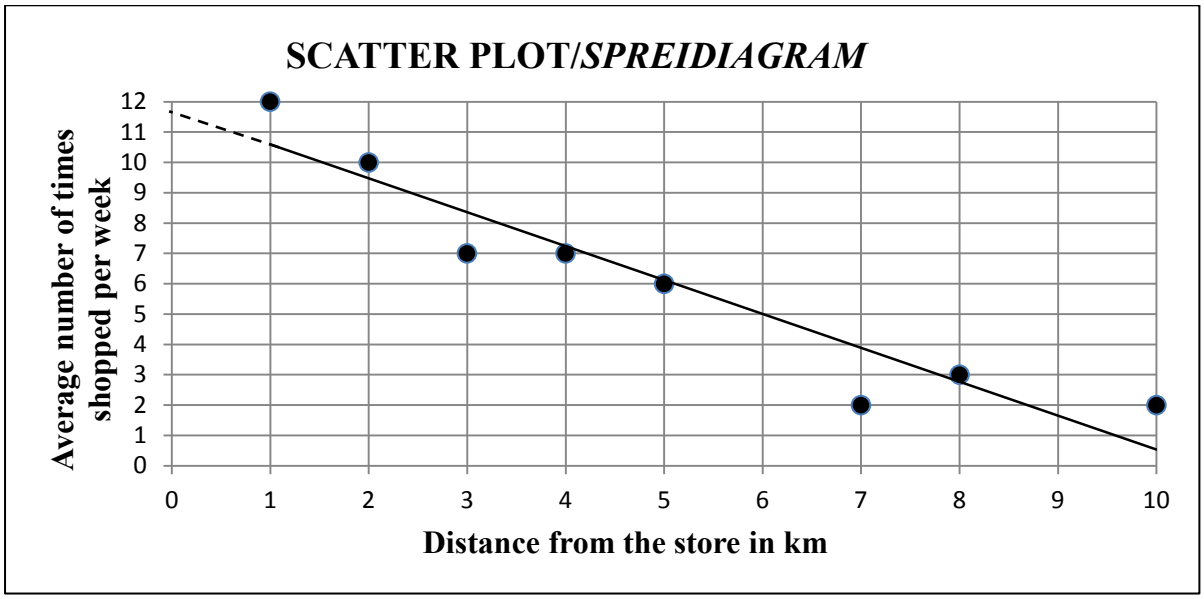
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

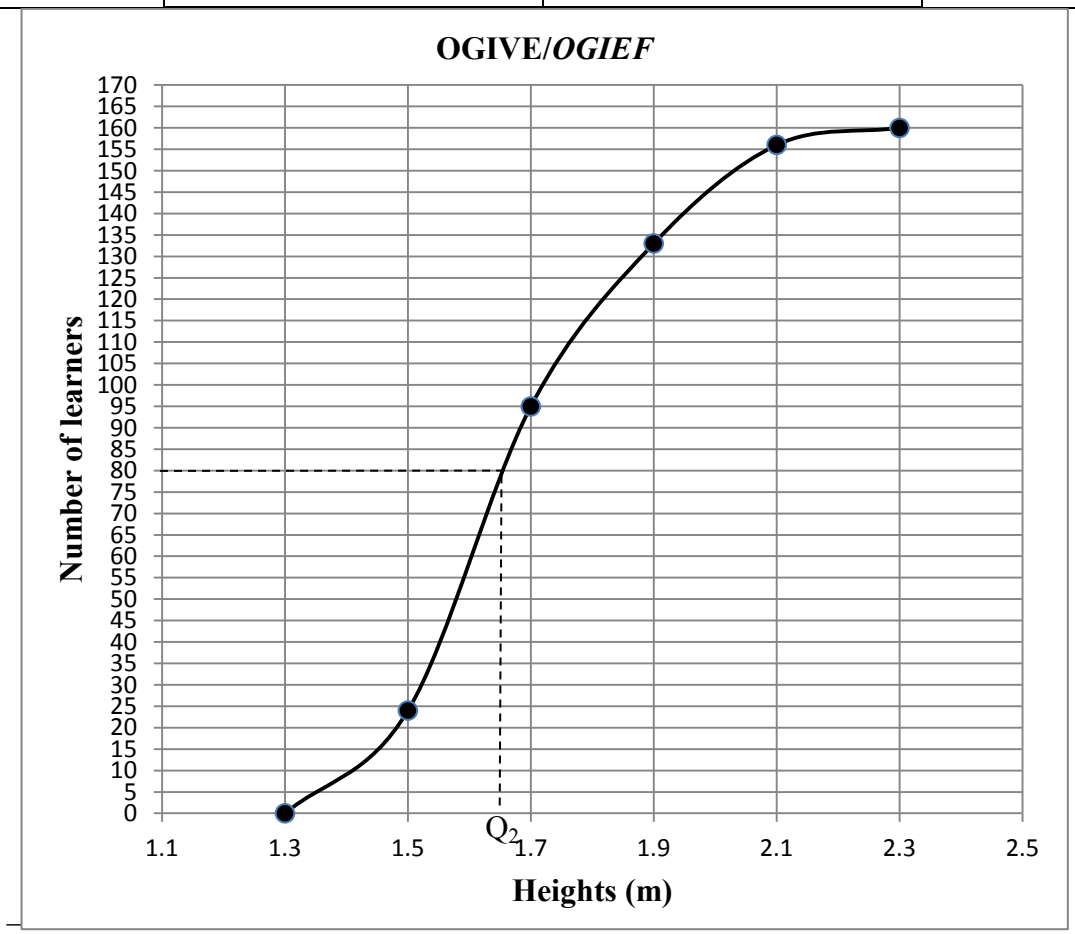
QUESTION/*VRAAG* 1

Distance from the store in km *Afstand vanaf die winkel in* **km**	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers die winkel per week besoek*	12	10	7	7	6	2	3	2

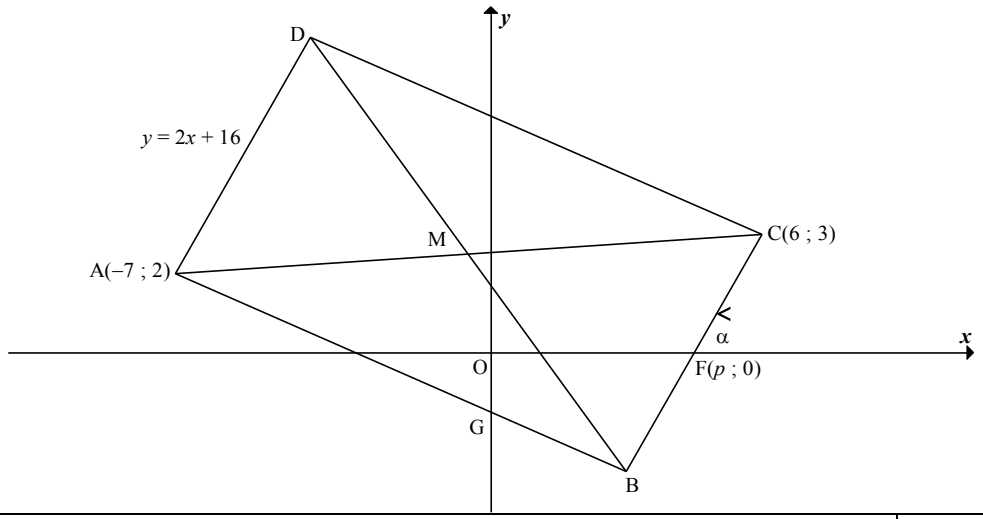


1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71$ (11,7132…) $b = -1{,}12$ (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of *a* ✓ value of *b* ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ = 5 times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

QUESTION/*VRAAG* 2

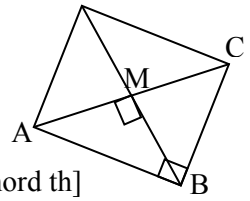
2.1	Positively skewed **OR** skewed to the right/*positief skeef* ***OF*** *skeef na regs*	✓ answer (1)
2.2	Range/*Omvang* = 2,21 – 1,39 = 0,82 m	✓ subtract values ✓answer (2)
2.3	**Intervals** / ***Klasse*** : **Cumulative frequency** / ***Kumulatiewe frekwensie*** $1{,}3 \le x < 1{,}5$: 24 $1{,}5 \le x < 1{,}7$: 95 $1{,}7 \le x < 1{,}9$: 133 $1{,}9 \le x < 2{,}1$: 156 $2{,}1 \le x < 2{,}3$: 160	✓95 , 133, 156 ✓160 (2)
2.4		✓ upper limits / *boonste limiete* ✓ cum *f*/ *kum f* ✓ shape/ *vorm* ✓ grounded *geanker* (4)
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)	✓ method ✓ answer (2)
2.6.1	The mean would change by 0,1 *m* *Die gemiddelde sal met 0,1 m verander*	✓ answer (1)
2.6.2	No influence/change as there is no difference in variation of data./*Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.*	✓ answer (1) **[13]**

QUESTION/*VRAAG 3*

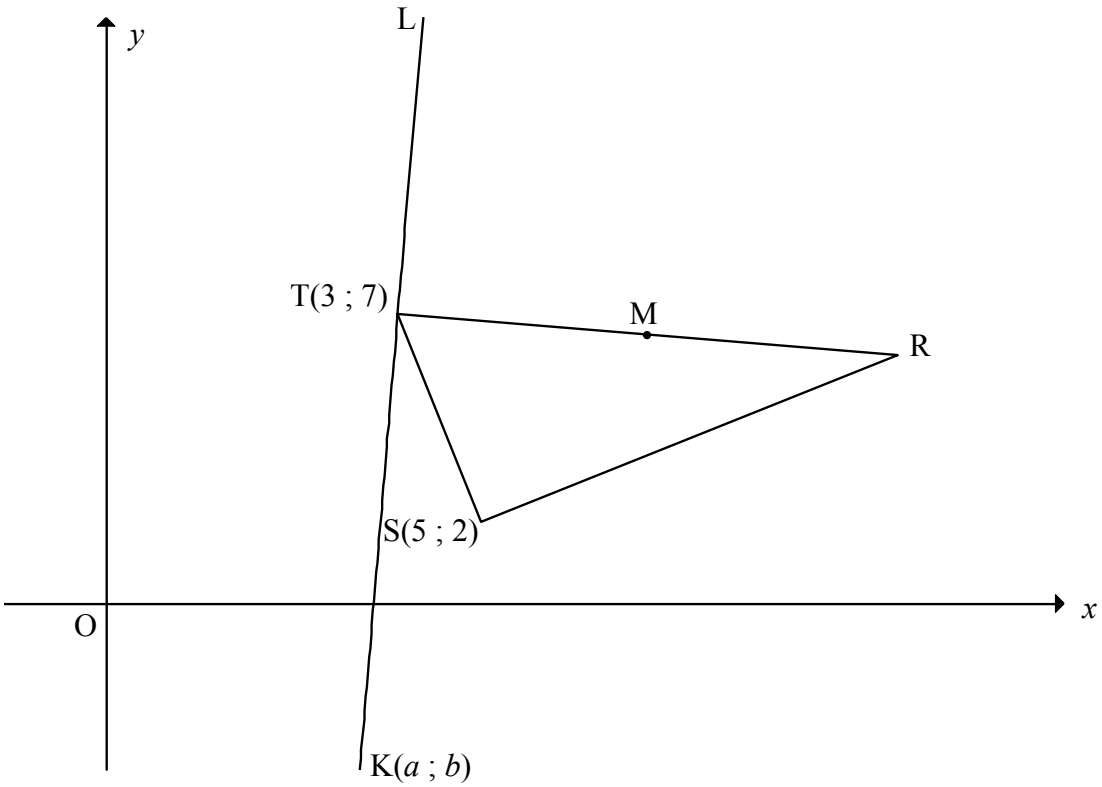


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= \text{M}\left(\frac{-7+6}{2}\ ;\ \frac{2+3}{2}\right)$ $= \text{M}\left(-\frac{1}{2}\ ;\ \frac{5}{2}\right)$	✓ *x*-value of M ✓ *y*-value of M (2)
3.2	$m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{AD} = m_{BC}$ [AD || BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6;3)$ $y - 3 = 2(x - 6)$ $\therefore y = 2x - 9$ *but* $y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{BC} = 2$ ✓ equating ✓answer (3) ✓ $m_{BC} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \quad \therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst ∠e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som ∠e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42{,}5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$C\hat{B}M = B\hat{A}M = 45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF*** $A\hat{M}B = 90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th] 	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION/*VRAAG 4*



4.1	∠ in semi circle/ ∠ at centre = 2∠ on circle ∠ *in halfsirkel* /∠ by middelpt = 2∠ op sirkel	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution *m* and (5 ; 2) ✓ equation (3)

	$m_{TS} \times m_{RS} = -1 \quad [TS \perp SR]$ $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓ substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore$ R(15 ; 6) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)

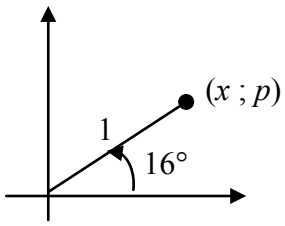
Answer only: full marks
Answer only: only 1 coordinate correct (1 mark)

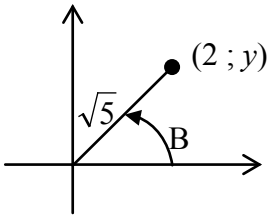
	$m_{TM} = \dfrac{9-3}{6\frac{1}{2}-7} = -\dfrac{1}{12}$ $TM: \quad 7 = -\dfrac{1}{12}(3) + c \quad y = -\dfrac{1}{12}x + \dfrac{29}{4}$ .....(1) $SR: \quad y = \dfrac{2}{5}x$ .....(2) $\dfrac{2}{5}x = -\dfrac{1}{12}x + \dfrac{29}{4}$ $\dfrac{29}{60}x = \dfrac{29}{4}$ $\therefore x = 15$ $\therefore y = \dfrac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$\text{ST} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\text{ST} = \sqrt{(5-3)^2 + (2-7)^2}$ $\text{ST} = \sqrt{4+25} = \sqrt{29}$ $\sin \text{R} = \dfrac{\text{TS}}{\text{TR}} = \dfrac{\sqrt{29}}{\sqrt{145}} or \dfrac{\sqrt{5}}{5} or \dfrac{1}{\sqrt{5}} or \ 0{,}45$ **OR/OF** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ $\text{area of } \Delta\text{TSR} = \dfrac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$ $29 = \dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \dfrac{\sqrt{5}}{5} or \dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{\text{TR}} = \dfrac{7 - 6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ **OR/*OF*** $m_{\text{TR}} = \dfrac{7-6}{3-15} = -\dfrac{1}{12}$ $m_{\text{TR}} \times m_{\text{KTL}} = -1 \quad [r \perp \text{tangent}]$ $m_{\text{KTL}} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x-3)$ $y = 12x - 29$ substitute $\text{K}(a;b)$: $b = 12a - 29$ **OR/*OF***	✓ $m_{\text{TR}} = -\dfrac{1}{12}$ ✓ $m_{\text{KTL}} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ $[r \perp \text{tangent}]$ $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a - 29$ **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓substitution (3 ; 7) & (a ; b) (3) ✓ subst into Pyth ✓multiplication ✓simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a - 29 - 7)^2 = 145$ $(a-3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore$ K(2 ; –5) **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	$TK = TR$ $\sqrt{(a-3)^2+(b-7)^2} = \sqrt{145}$ $(a-3)^2+(b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2 = 145$ $(a-3)^2+(12a-36)^2 = 145$ $(a-3)^2+144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ $a = 2$ or 4 $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5) **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2+(b-6)^2 = 145+145$ $(a-15)^2+(12a-29-6)^2 = 290$ $(a-15)^2+(12a-35)^2 = 290$ $a^2-30a+225+144a^2-840a+1225 = 290$ $145a^2-870a+1160 = 0$ $a^2-6a+8 = 0$ $\therefore (a-2)(a-4) = 0$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ K(2 ; –5)	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ $(a-3)^2 = 1$ ✓ ± 1 ✓ values of a ✓ value of b (6) ✓ substitution ✓ substitution of $b = 12a - 29$ ✓ standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

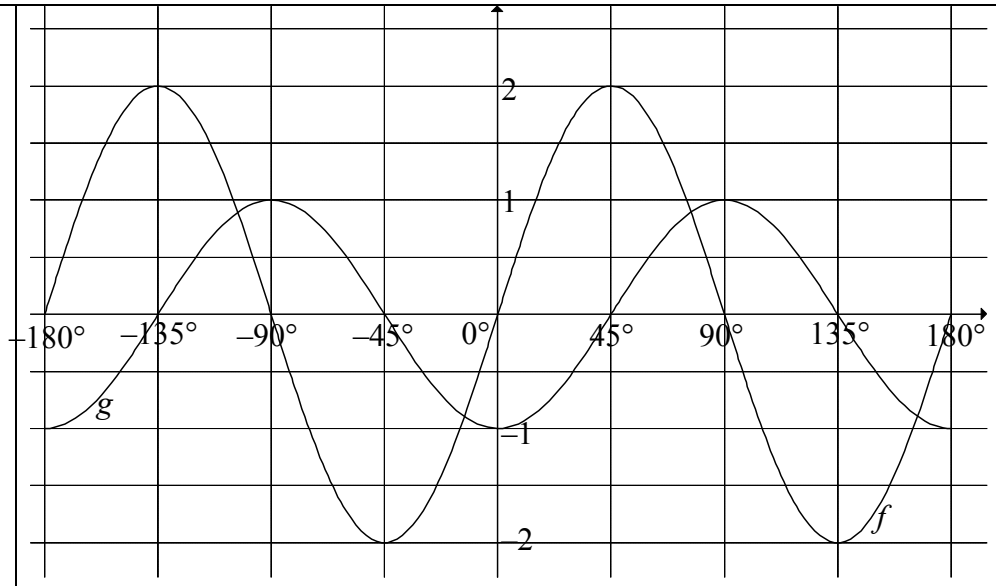
QUESTION/*VRAAG 5*

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF*** $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\frac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ $= \frac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \frac{\sin 2A}{\cos A.(-\sin A)}$ $= \frac{2\sin A\cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\frac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90° + A)} = \frac{\sqrt{1-(2\cos^2 A - 1)^2}}{\cos A.-\sin A}$ $= \frac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A.-\sin A}$ $= \frac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$ $= \frac{2\cos A.\sin A}{\cos A.-\sin A}$ $= -2$ **OR/OF**	✓$\sqrt{\sin^2 2A}$ ✓$\cos A$ ✓$-\sin A$ ✓$2\sin A\cos A$ ✓ answer (5) ✓$2\cos^2 A - 1$ ✓$\cos A$ ✓$-\sin A$ ✓identity ✓ answer (5)

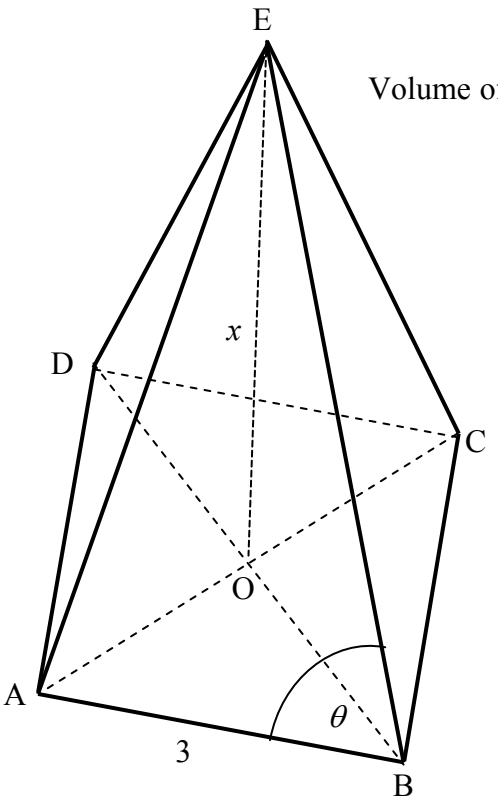
	$\dfrac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{1-(1-4\sin^2 A+4\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{2\sin A\sqrt{\cos^2 A}}{\cos A.-\sin A}$ $=-2$	✓ $1-2\sin^2 A$ ✓ cosA ✓ $-$ sinA ✓ identity ✓ answer (5)
5.4.1	$\cos 2B=\dfrac{3}{5}$ $2\cos^2 B-1=\dfrac{3}{5}$ $\cos^2 B=\dfrac{4}{5}$ $\therefore\ \cos B=\sqrt{\dfrac{4}{5}}\ or\ \dfrac{2}{\sqrt{5}}\ or\ \dfrac{2\sqrt{5}}{5}\quad [0°\le B\le 90°]$ **OR/*OF*** $\cos B=\dfrac{\sqrt{\cos 2B+1}}{2}$ $=\dfrac{\sqrt{\dfrac{3}{5}+1}}{2}$ $=\dfrac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2 B$ ✓ answer (3) ✓ $=\dfrac{\sqrt{\cos 2B+1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer (3)
5.4.2	$\sin^2 B=1-\cos^2 B$ $=1-\left(\dfrac{2}{\sqrt{5}}\right)^2$ $=\dfrac{1}{5}\qquad \therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2+y^2=(\sqrt{5})^2$ $4+y^2=5$ $y^2=1$ $y=1$ $\therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$  	✓ $\sin^2 B=\dfrac{1}{5}$ ✓ answer (2) ✓ $y=1$ ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1-2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B+45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ *or* $\frac{\sqrt{10}}{10}$ **OR/*OF*** $\cos(B+45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4) ✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

QUESTION/*VRAAG* 6

6.1		✓ *x*- intercepts/ *afsnitte* ✓ *y*- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ $\therefore$ maximum value = 2 – 3 = –1	✓ ✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z$ *or* $x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°$ *or* $333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°$ *or* $166{,}72°+k.180°;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or $-26{,}56^{o}$ ✓ 76,72° or – 13,28° ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & 333,43° ✓ 76,72° & 166,72° ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

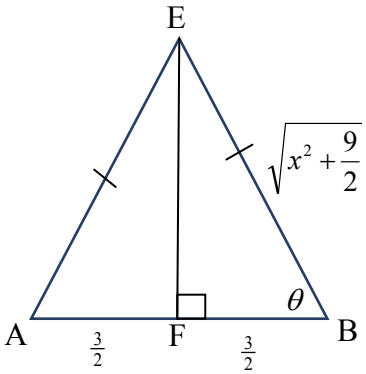
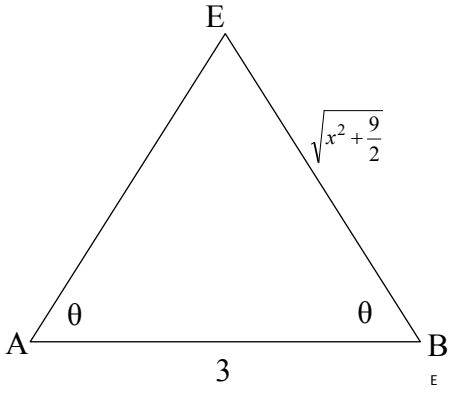
QUESTION/*VRAAG* 7



Volume of pyramid $= \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$

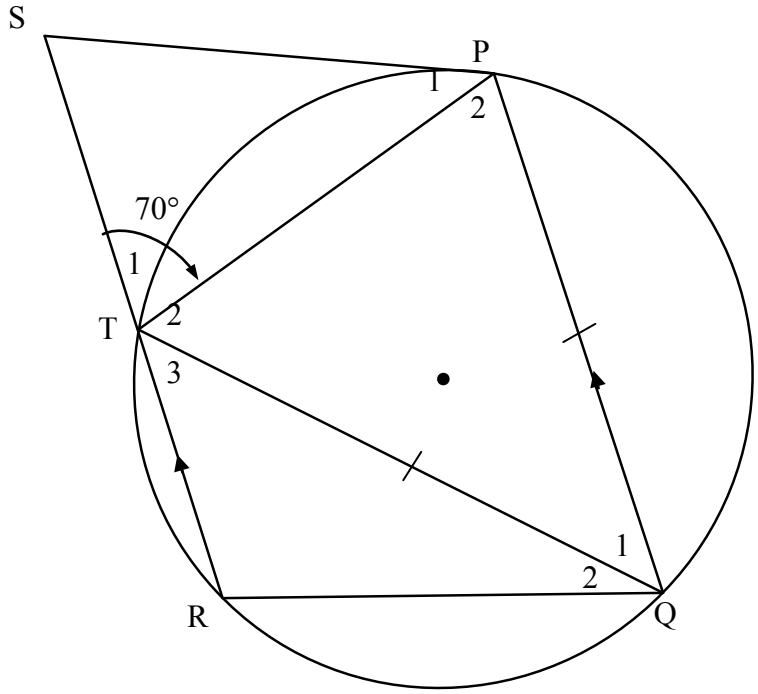
7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12 **OR/*OF*** $\sin 45° = \frac{OB}{3}$ $OB = 3\sin 45°$ $OB = \frac{3\sqrt{2}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* 2,12 **OF/OR** $\cos 45° = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)

	OR/OF $A\hat{O}B = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ cos θ as subject ✓ simplification (5)
	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right).\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	s ✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ substituting ✓ cos θ as subject (5)

	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  **OR/OF** $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\sin\theta\cos\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\cos\theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5) ✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	$\text{Volume} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$ $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

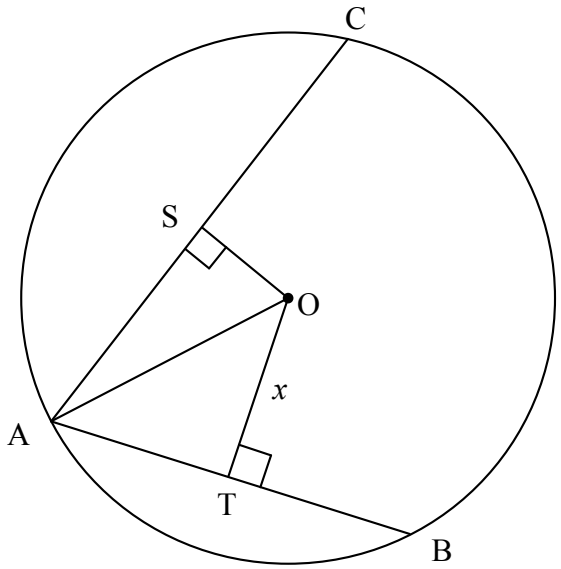
QUESTION/*VRAAG 8*

8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ‖ SR		✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ $\therefore \hat{Q}_1 = 180° - 2(70°)$ $= 40°$	[∠s opp = sides/*∠e teenoor = sye*] [∠s/*e* Δ = 180°]	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$	[tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

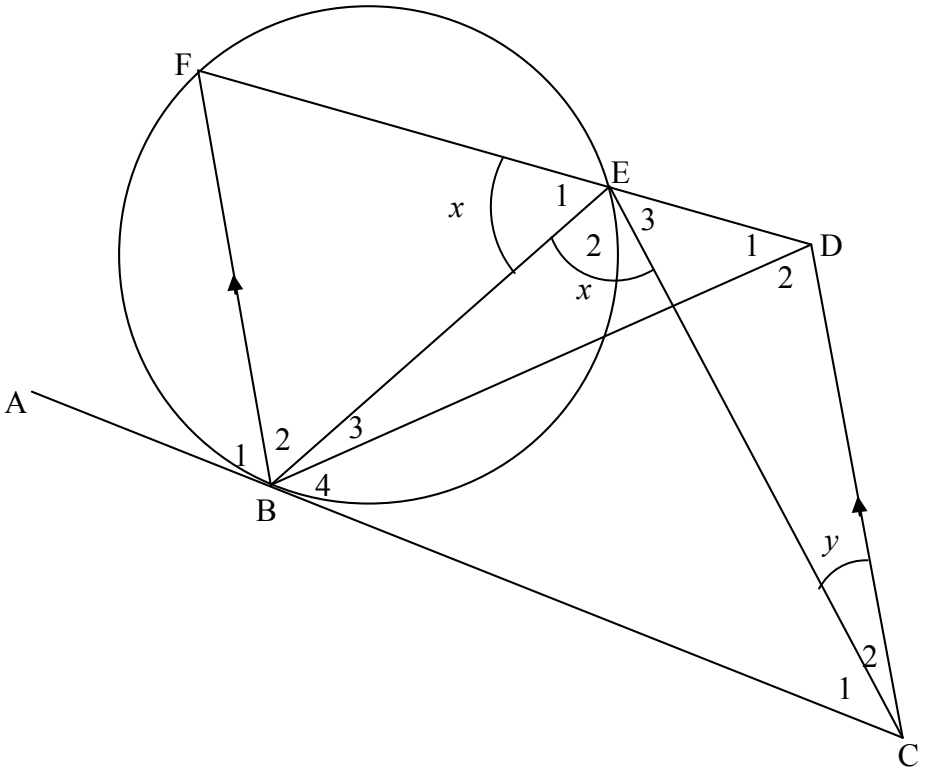
8.2



8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore$ OA = 25 **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] Let OT = 15*x*. Then OS = 7*x* But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ equating ✓ AS = 24 ✓ substitution ✓ *x* = 1 ✓ radius (5) ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth ✓ radius (5) **[12]**

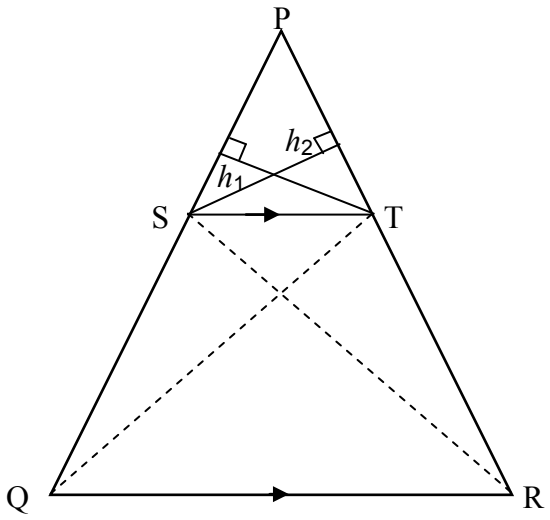
QUESTION/*VRAAG 9*



9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige* ∠s/*e*; FB ∥ DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ $\therefore$ BCDE = cyclic quad [converse ext ∠ cyc quad/*omgek: buite∠kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [∠*s* in the same segment/∠*e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt ∠*s*, BF ∥CD/*verwiss* ∠*e,BF*∥*CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [∠s in the same segment/∠*e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [$= x$] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \Delta BFE /// \Delta CBE$ [∠,∠,∠] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

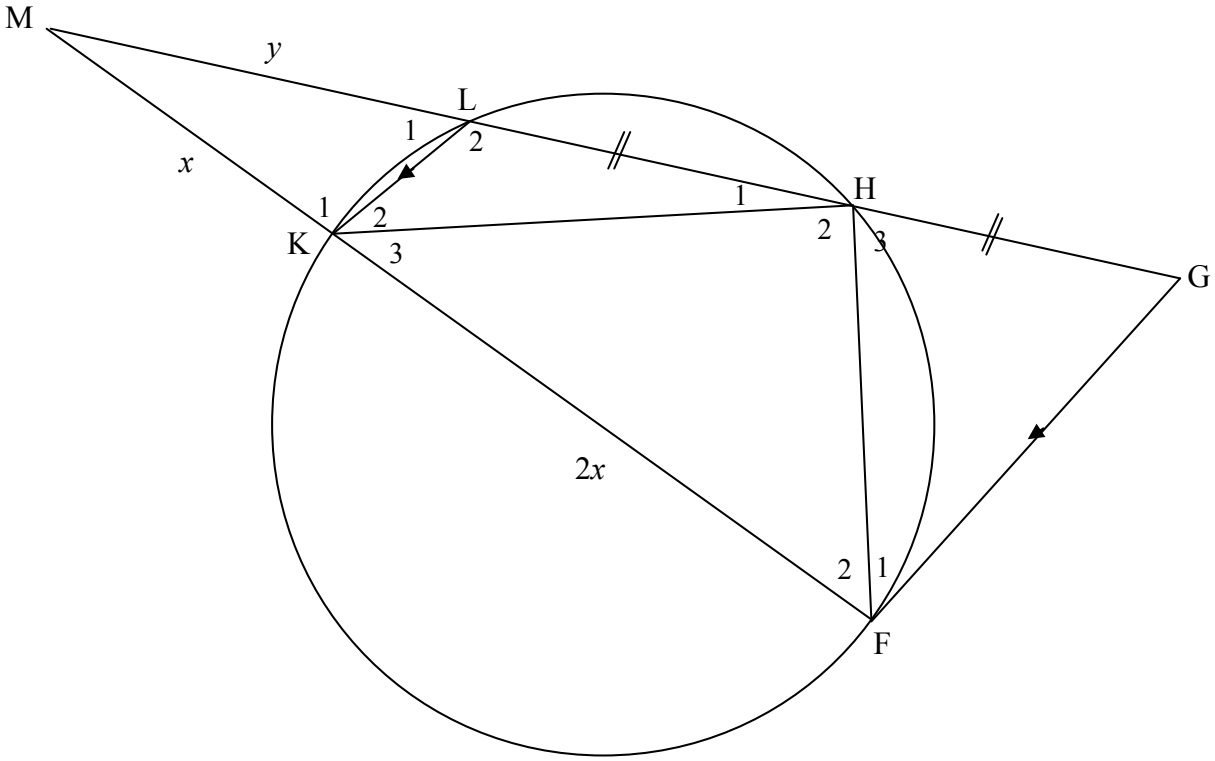
QUESTION/*VRAAG* 10

10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ *Verbind SR en TQ en trek* h_1 *van S* $\perp$ *PT en* h_2 *van T* $\perp$ *PS*] Proof : $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$ equal altitudes $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes area ΔPST = area ΔPST [common] But area ΔQST = area ΔSTR [same base, height; ST || QR] $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ constr/*konstruksie* ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST}$ $= \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2}$ ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$ ✓ S ✓ R ✓ S (6)

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF || LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM}=\frac{FK}{KM}$ OR $\frac{GL}{y}=\frac{2x}{x}$ [prop theorem/*eweredighst*; GF || LK] $\frac{2GH}{y}=\frac{2x}{x}$ [LH = HG] $\therefore GH = y$	✓S ✓ R ✓ GL = 2GH (3)

10.2.2(b)	$\hat{K}_1 = G\hat{F}M$ [corresponding/*ooreenkomst* ∠ s; GF ‖ LK] $L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$ [ext ∠ cyclic quad/*buite∠koordevh*] $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $M\hat{H}F = G\hat{F}M$ [proven/*bewys*] ∴ ΔMFH ||| ΔMGF [∠∠∠] **OR/*OR*** $\hat{K}_1 = G\hat{F}M$ [corresponding/*ooreenkomst* ∠ s; GF ‖ LK] $L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$ [ext ∠ cyclic quad/*buite∠koordevh*] $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $M\hat{H}F = G\hat{F}M$ [proven/*bewys*] $\hat{F}_2 = \hat{G}$ [∠s of Δ = 180°] ∴ ΔMFH ||| ΔMGF	✓S ✓ R ✓ S ✓ S ✓ R (5) ✓S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$ [‖| Δs] $= \frac{3x}{2y}$	✓S ✓R (2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$ [‖| Δs] $\frac{3x}{2y} = \frac{3y}{3x}$ [from 10.2.2(*c*)] $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	✓ S ✓substitution ✓ simplification (3) **[20]**
	TOTAL MARKS	**150**