KZN DEPARTMENT OF EDUCATION

MATHEMATICS
JUST IN TIME MATERIAL
GRADE 10

TERM 1 – 2020

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give more guidance to teachers.

TABLE OF CONTENTS

EUCLIDEAN GEOMETRY

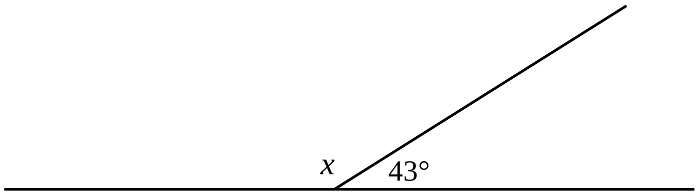
FROM GR. 10 Annual Teaching Plan 2020:

DATES	CURRICULUM STATEMENT
21/02 – 06/03 (11 days)	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures. 3. The following proofs of theorems are examinable: • The opposite sides and angles of a parallelogram are equal. • The diagonals of a parallelogram bisect each other. • If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram. • The diagonals of a rectangle are equal. • The diagonals of rhombus bisect each other at right angles and bisect the interior angles of the rhombus.
09/03 – 12/03 (4 days)	4. Investigate line segments joining the midpoints of two sides of a triangle

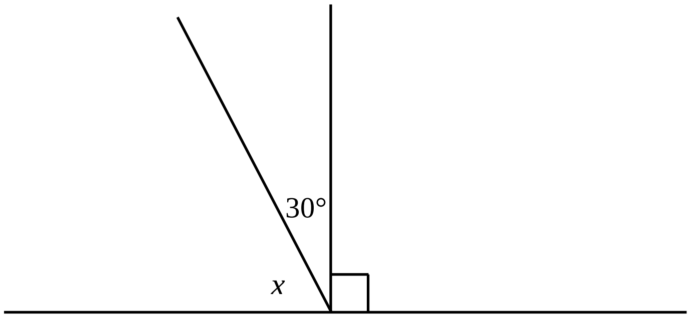
	Term 1
Week	
Topic	EUCLIDEAN GEOMETRY
Weighting	30±3 marks
Sub-topics/Clarification	Equation of a tangent, Concavity, Graph sketching and interpretation, factor and Remainder theorem
Related concepts/terms/vocabulary	• Straight line • Substitution
Prior-knowledge/ Background knowledge	• Derivative • Inequalities • Factorization
Resources	• Calculator. • Worksheets and Textbooks • Previous question papers
Activities	• See annexure A
Methodology	• Analyze the given information. • Revision on (factorization ,substitution, products and simultaneous equations)
Assessment	• Classwork. • Homework.
Related concepts/terms/vocabulary	• Straight line • Substitution • Factorization

BASELINE ASSESSMENT FOR GR. 10 EUCLIDEAN GEOMETRY:

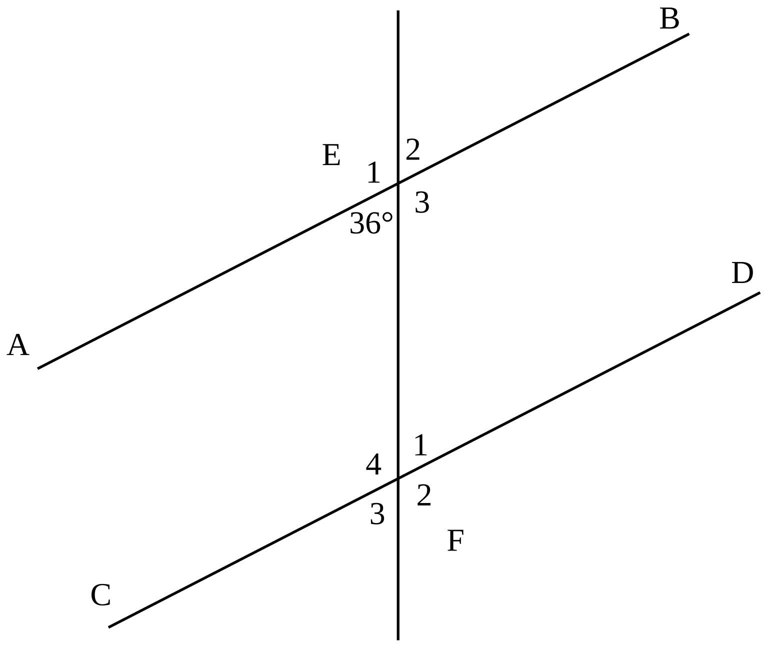
1. Calculate the value of x and give a reason for your answer (2)



2. Calculate the value of x and give a reason for your answer (2)



3. Calculate the sizes of the following angles and give reasons for your answers



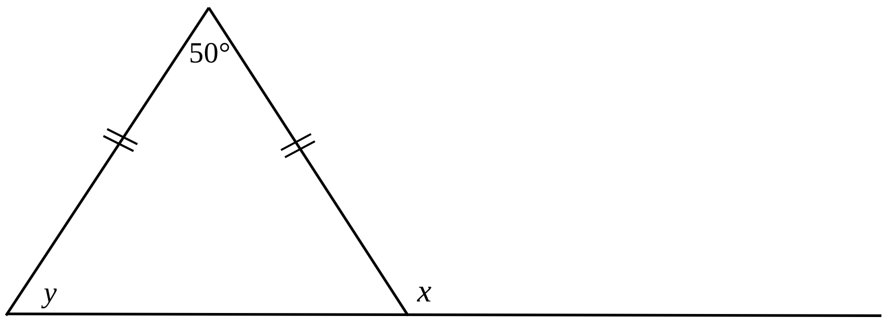
3.1 $\hat{F}_1$ (3)

3.2 $\hat{E}_2$ (3)

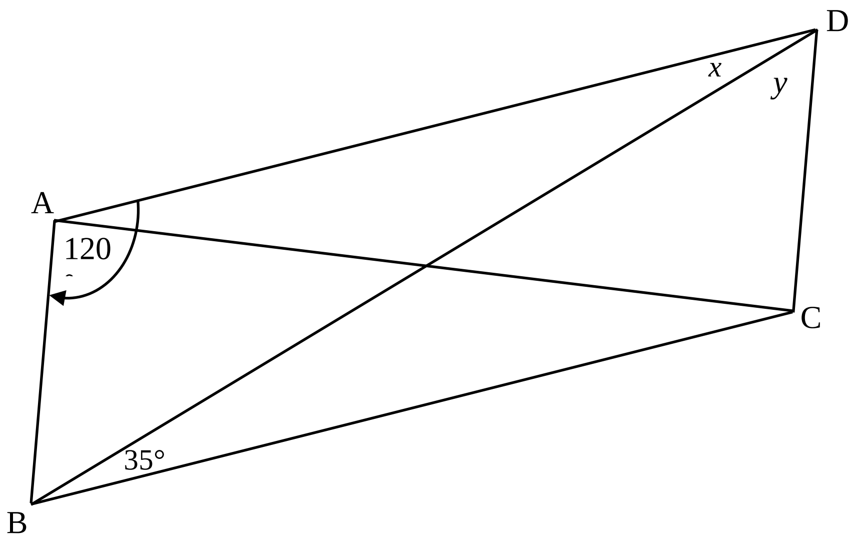
3.3 $\hat{F}_3$ (2)

3.4 $\hat{F}_4$ (2)

4. Calculate the values of x and y in the diagram below. Give reasons for your answers. (3)



5. Given that ABCD is a parallelogram



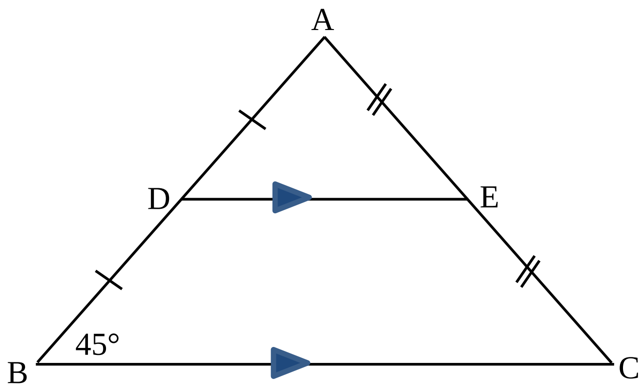
5.1 Calculate the values of x and y , give reasons for your answers (3)

5.2 Calculate the length of CD (1)

5.3 Is $\Delta ABC \equiv \Delta BDC$? (1)

5.4 State the condition of congruence in your answer to 5.3. (1)

6.



6.1 Calculate the size of $A\hat{D}E$ and give reason for your answer (2)

6.2 If the length of AB is 12cm find EB (1)

6.3 If AB=AC calculate the size of $C\hat{A}B$ and give a reason for your answer (4)

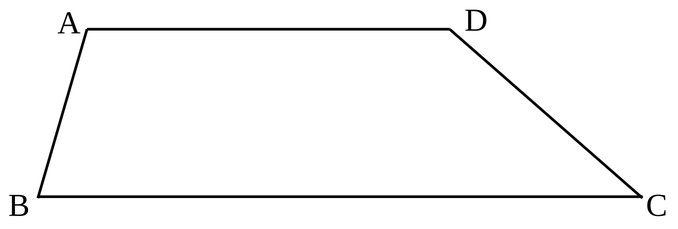
[30]

WORKSHEET: PROPERTIES OF SPECIAL QUADRILATERALS

For each of the following types of quadrilaterals, do the following:

- For each of the quadrilaterals draw in both diagonals, and call the point where they intersect E.
- List the properties of each of the quadrilaterals next to the quadrilateral.
- Refer to side lengths, sizes of angles, parallel lines and the properties of the diagonals.
- Also indicate the properties on the sketches.

TRAPEZIUM:



Properties:

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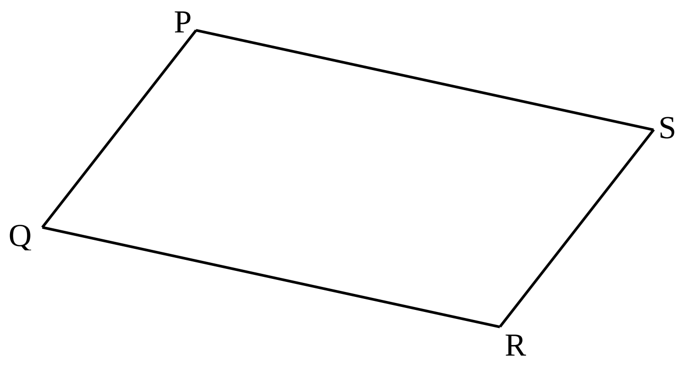
PARALLELOGRAM:



Properties:

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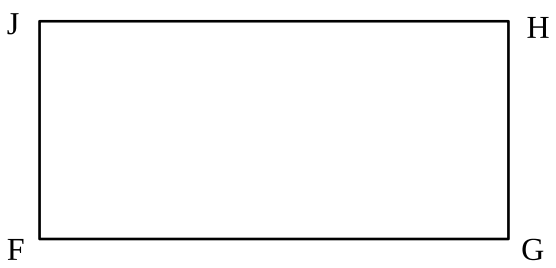
RECTANGLE:



Properties:

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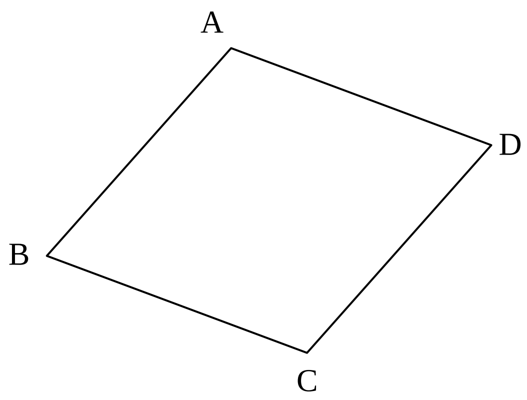
RHOMBUS:



Properties:

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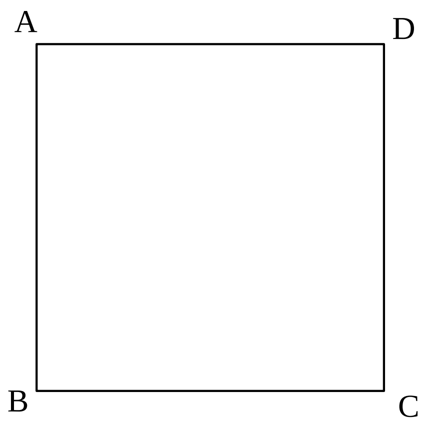
SQUARE:



Properties:

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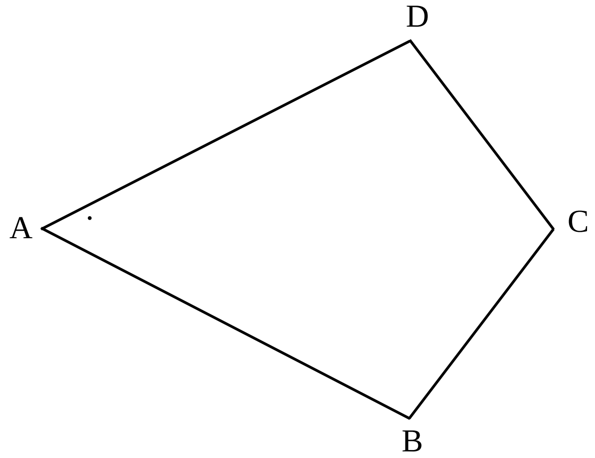
KITE:



Properties:

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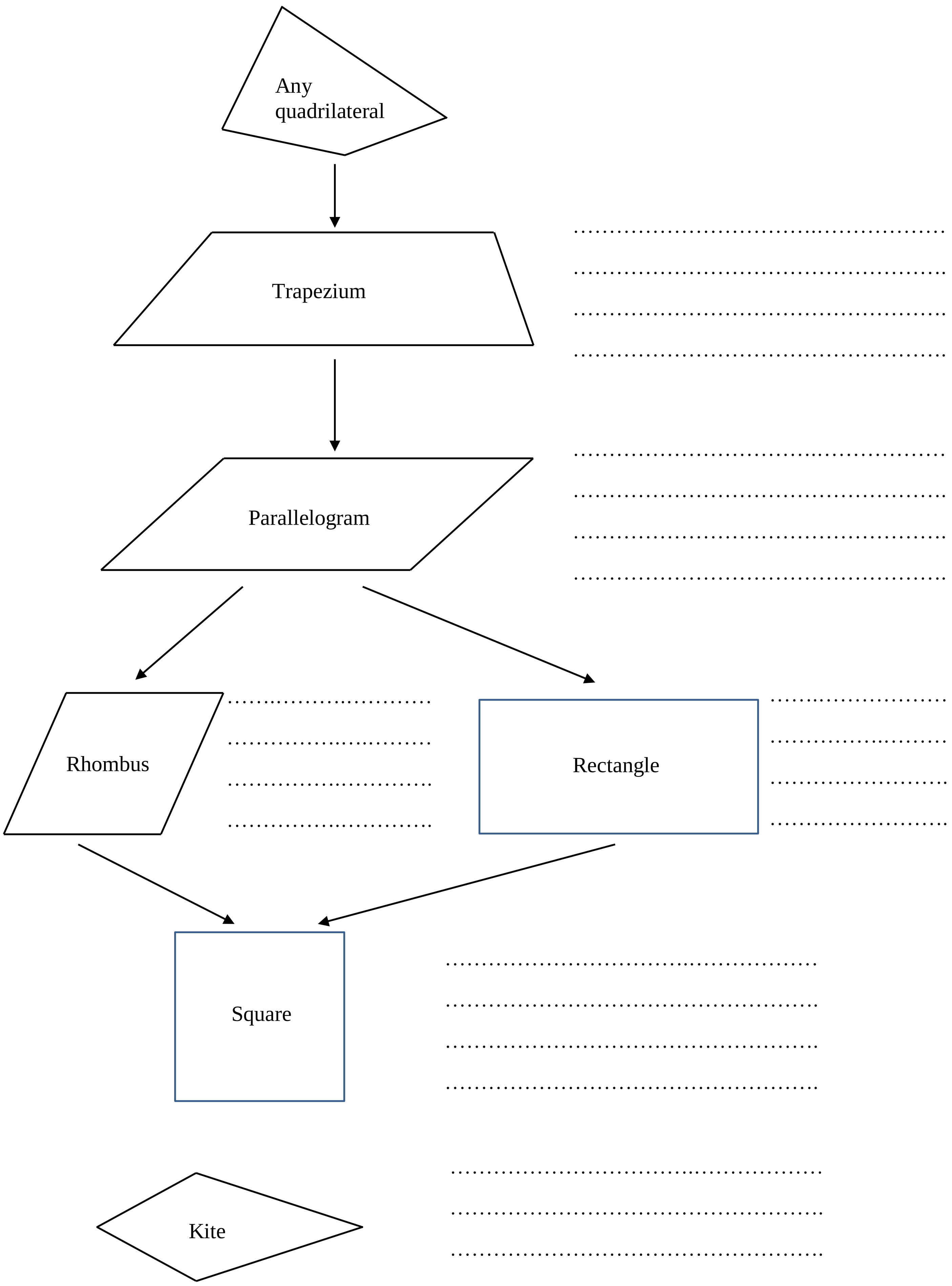
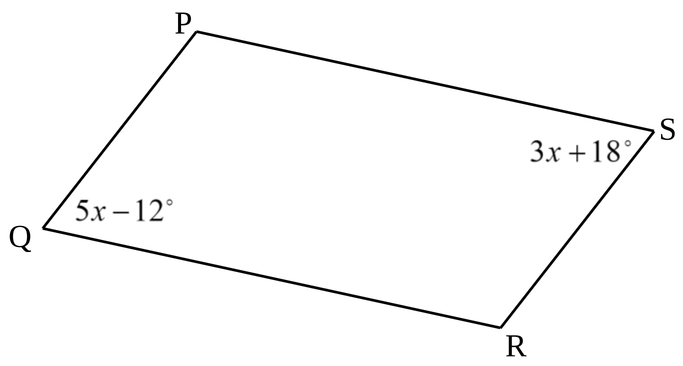
WORKSHEET: DEFINITIONS OF SPECIAL QUADRILATERALS

Write the definition of each of the special quadrilaterals next to its sketch:

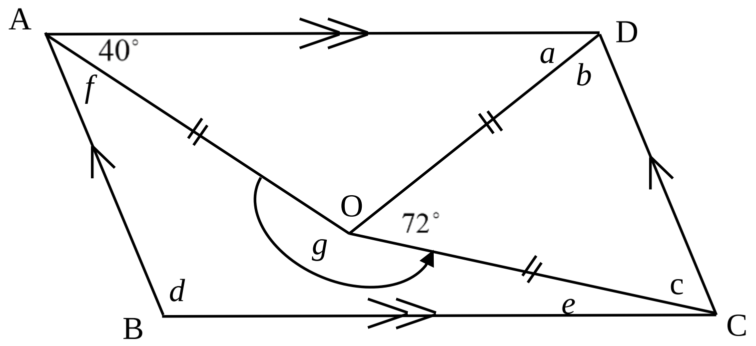


PRACTICE EXERCISES: SPECIAL QUADRILATERALS

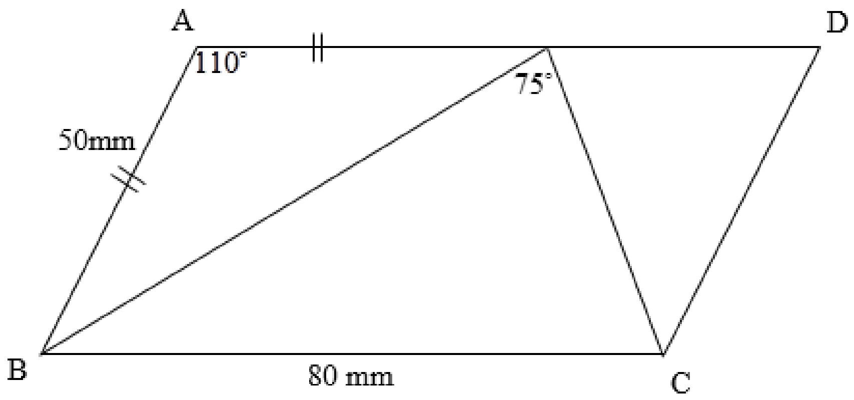
1. KLMN is a parallelogram. Calculate the sizes of its interior angles.



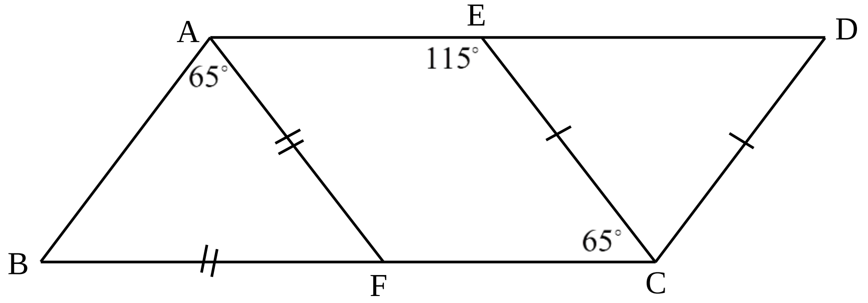
2. Find the sizes of the angles marked a to g. Give reasons for all statements.



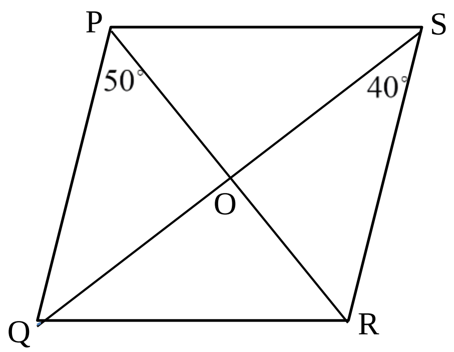
3. In parallelogram ABCD, AB = 50 mm, BC =80 mm and $B\hat{A}D = 110^\circ$. E is a point on AD such that AE = AB and $B\hat{E}C = 75^\circ$.
   Calculate the following:
   3.1 $C\hat{E}D$
   3.2 the lengths of the sides of ΔCED



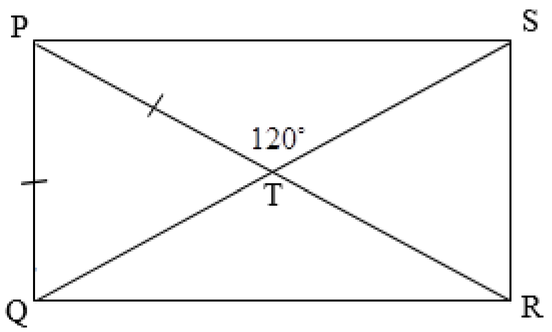
4. Prove that ABCD is a parallelogram.



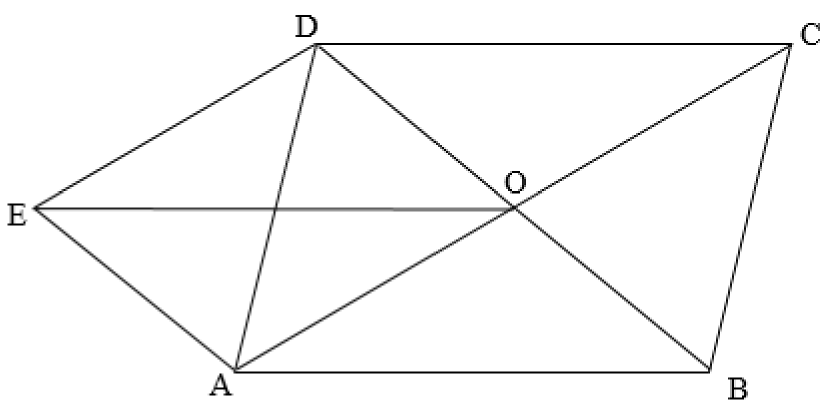
5. PQRS is a parallelogram with $Q\hat{P}R = 50^{\circ}$ and $Q\hat{S}R = 40^{\circ}$. Prove that PQRS is a rhombus.



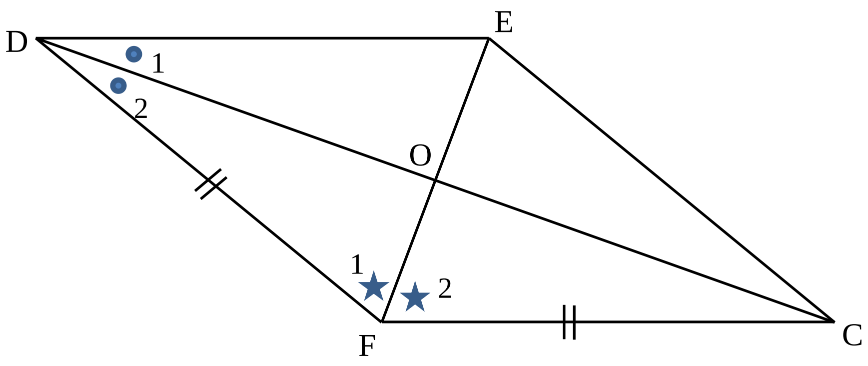
6. Diagonals PR and QS of parallelogram PQRS intersect at T. If PT = PQ and $P\hat{T}S = 120^{\circ}$, prove that PQRS is a rectangle.



7. ABCD and ABOE are parallelograms. Prove that EAOD is also a parallelogram.



8. In the diagram $\hat{D}_1 = \hat{D}_2$, $\hat{F}_1 = \hat{F}_2$ and DF = FC.



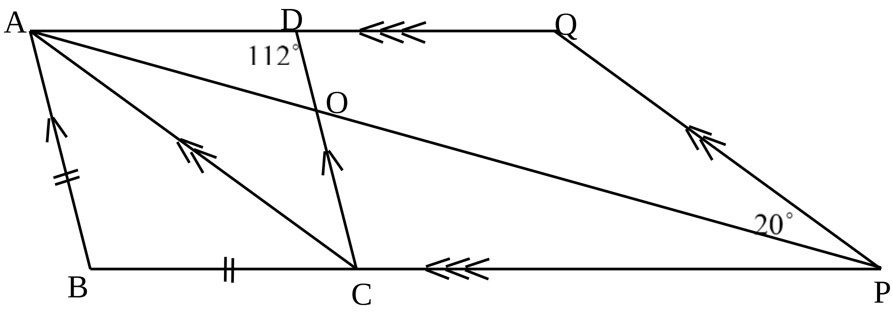
Prove that

8.1 DFCE is a rhombus.

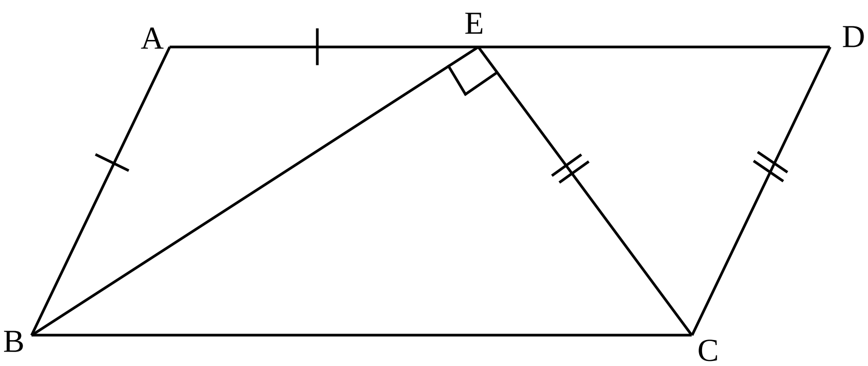
8.2 The circumference of DFCE is 68 cm. The length of DO exceeds the length of OF by 7 cm. Determine the lengths of DC and EF. (Hint: Let OF = *x*.)

8.3 Determine the area of rhombus DFCE.

9. In the diagram AQ || BP, AB || DC, QP || AC and AB = BC. $A\hat{D}C = 112^\circ$ and $Q\hat{P}A = 20^\circ$. Determine the magnitude of $A\hat{O}D$.



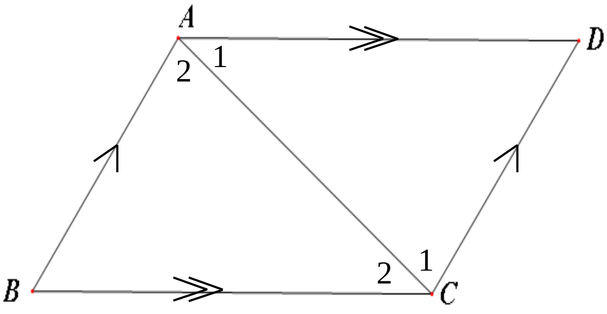
10. ABCD is a parallelogram. E is a point on AD such that AE = AB, and EC = CD. $B\hat{E}C = 90^\circ$. Calculate $E\hat{B}C$.



EXAMINABLE PROOFS OF THEOREMS : GEOMETRY GRADE 10

1. Prove that the opposite sides and angles of a parallelogram are equal:

Given: Parallelogram ABCD.



R.T.P.: AB = CD and AD = BC

$B\hat{A}D = B\hat{C}D$ and $\hat{B} = \hat{D}$

Construction: Draw AC and BD.

Proof: In ΔABC and ΔCDA:

1. $\hat{A}_2 = \hat{C}_1$ [alt. $\angle$'s; AB || CD]
2. $\hat{A}_1 = \hat{C}_2$ [alt. $\angle$'s; AD || BC]
3. $AC = AC$ [common]

$\therefore \Delta ABC \equiv \Delta CDA$ [$\angle;\angle;s$]

$\therefore AB = CD$ and $BC = AD$ and $\hat{B} = \hat{D}$ [$\equiv \Delta's$]

Also: $\hat{A}_1 + \hat{A}_2 = \hat{C}_1 + \hat{C}_2$ [$\hat{A}_2 = \hat{C}_1$; $\hat{A}_1 = \hat{C}_2$]

$B\hat{A}D = B\hat{C}D$

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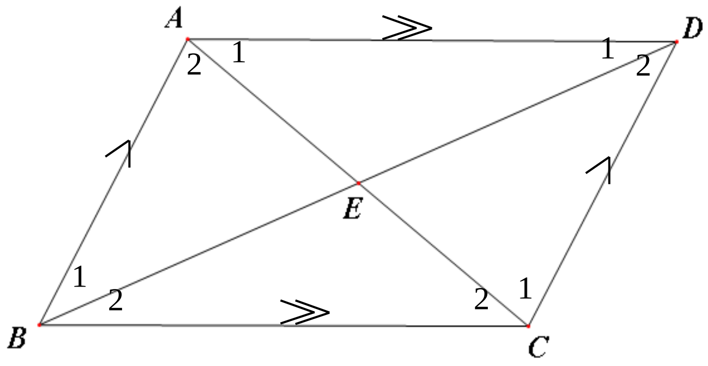
2. Prove that the diagonals of a parallelogram bisect each other:

Given: Parallelogram ABCD with diagonals AC and BC intersecting in E.



R.T.P.: AE = EC and BE = ED

Proof: In ΔABE and ΔCDE:

1. $\hat{A}_2 = \hat{C}_1$ [alt. $\angle$'s; AB || CD]
2. $\hat{B}_1 = \hat{D}_2$ [alt. $\angle$'s; AB || CD]
3. AB = CD [opp. sides of parm]

$\therefore \Delta ABE \equiv \Delta CDE$ [$\angle;\angle;s$]

$\therefore$ AE = EC and BE = ED [$\equiv \Delta's$]

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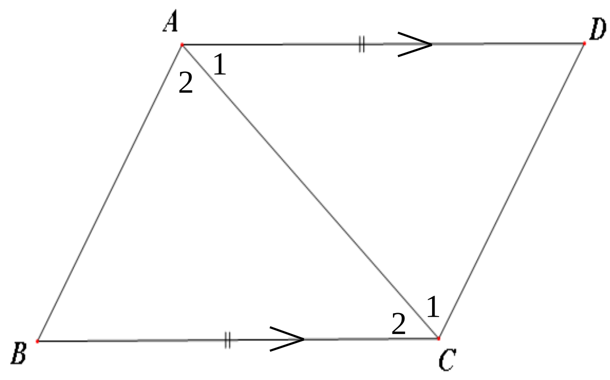
3. Prove that if one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

Given: Quadrilateral ABCD with AD || BC and AD=BC.



R.T.P.: ABCD is a parallelogram.

Construction: Draw diagonal AC.

Proof: In ΔABC and ΔCDA:

1. $\hat{C}_2 = \hat{A}_1$ [alt. $\angle$'s; AD||BC]
2. AC = AC [common]
3. BC = AD [given]

$\therefore \Delta ABC \equiv \Delta CDA$ [s; $\angle$; s]

$\therefore \hat{A}_2 = \hat{C}_1$ [$\equiv \Delta's$]

$\therefore$ AB || DC [alt. $\angle$'s =]

$\therefore$ ABCD is a parm. [both pairs of opp sides ||]

4. Prove that the diagonals of a rectangle are equal.

Given: Rectangle ABCD with diagonals AC and BD.

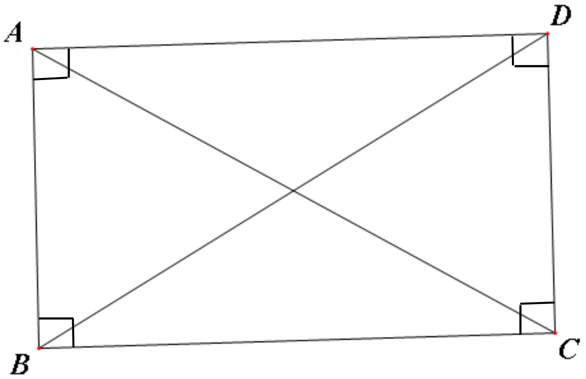
R.T.P.: AC = BD

Proof: In ΔABC and ΔBAD:

1. BC = AD [opp. sides of rectangle]
2. AB = AB [common]
3. $A\hat{B}C = B\hat{A}D$ [$= 90^\circ$; $\angle$'s of rectangle

$\therefore \Delta ABC \equiv \Delta BAD$ [s; $\angle$; s]

AC = BD [$\equiv \Delta's$]



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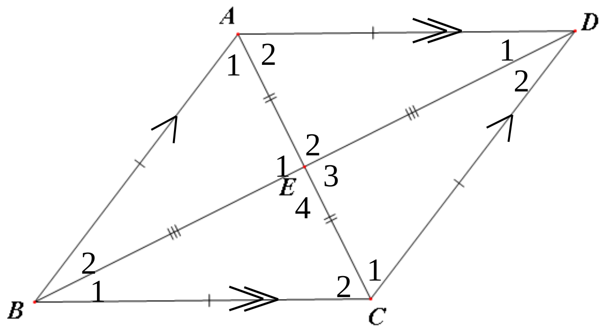
5. Prove that the diagonals of a rhombus bisect each other at right angles and bisect the interior angles of the rhombus.

Given: Rhombus ABCD with diagonals AC and BD bisecting each other at E.

R.T.P.: $\hat{E}_1 = \hat{E}_2 = \hat{E}_3 = \hat{E}_4 = 90^\circ$; and

$\hat{A}_1 = \hat{A}_2$; $\hat{B}_1 = \hat{B}_2$; $\hat{C}_1 = \hat{C}_2$; $\hat{D}_1 = \hat{D}_2$



Proof: In ΔABE and ΔCBE :

1. AE = EC [diagonals of parm. bisect each other]
2. BE = BE [common]
3. AB = BC [sides of rhombus]

$\therefore \Delta ABE \equiv \Delta CBE$ [s; s; s]

$\hat{E}_1 = \hat{E}_4$ [$\equiv \Delta's$]

But $\hat{E}_1 + \hat{E}_4 = 180^\circ$ [$\angle$'s on a straight line]

$\therefore \hat{E}_1 = 90^\circ = \hat{E}_4$

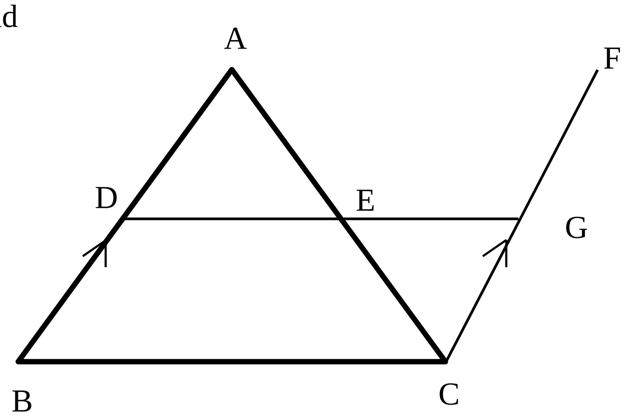
And: $\hat{E}_2 = 90^\circ = \hat{E}_3$ [vert. opp. $\angle$'s]

Also: $\hat{B}_2 = \hat{B}_1$ [$\equiv \Delta's$]

Similarly it can be proved that $\hat{A}_1 = \hat{A}_2$, $\hat{C}_1 = \hat{C}_2$ and $\hat{D}_1 = \hat{D}_2$.

PROOF OF THE MIDPOINT THEOREM
(NOT EXAMINABLE; FOR ENRICHMENT)

Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half the length of the third side.



Given: ΔABC with AD = DB (D is a midpoit of AB) and AE = EC (E is a midpoint of AC)

RTP: DE || BC and DE = ½ BC

Construction: Draw CF|| AB
Extend DE to G

Proof: In ΔADE and ΔCGE:

4. $D\hat{A}E = E\hat{C}G$ [alt. ∠'s; AB || CF]
5. $A\hat{E}D = G\hat{E}C$ [vert opp ∠'s;]
6. AE = EC [given]

$\therefore \Delta ADE \equiv \Delta CGE$ [∠;∠;s]

But AD = DB [given]
and AD = CG $\equiv\Delta$'s

$\therefore$ DB = BC

$\therefore$ DBCG is a parallelogram [one pair of opposite sides equal & parallel]

$\therefore$ DE || CG [opp sides of parm]

also: DE = EG [$\equiv\Delta$'s]

But DE + EG = BC [opp sides of parm]

$\therefore$ DE + DE = BC

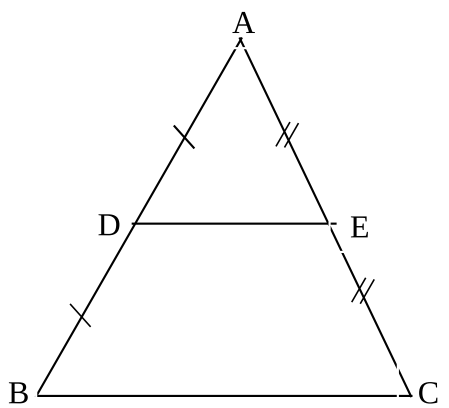
$\therefore$ 2DE = BC

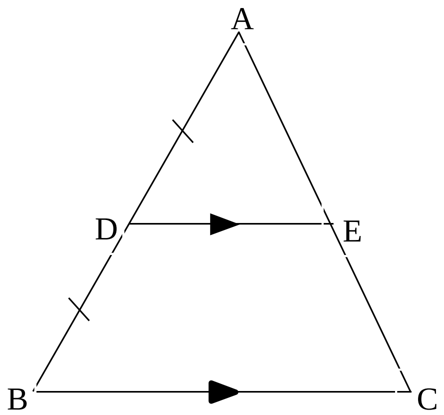
$\therefore$ DE = ½ BC

Ways in which the Midpoint Theorem can be stated:

1) If AD = DB and AE = EC,
then: DE|| BC and DE = ½ BC

2) If AD = DB and DE BC,
then: AE = EC and DE = ½ BC
(converse theorem)





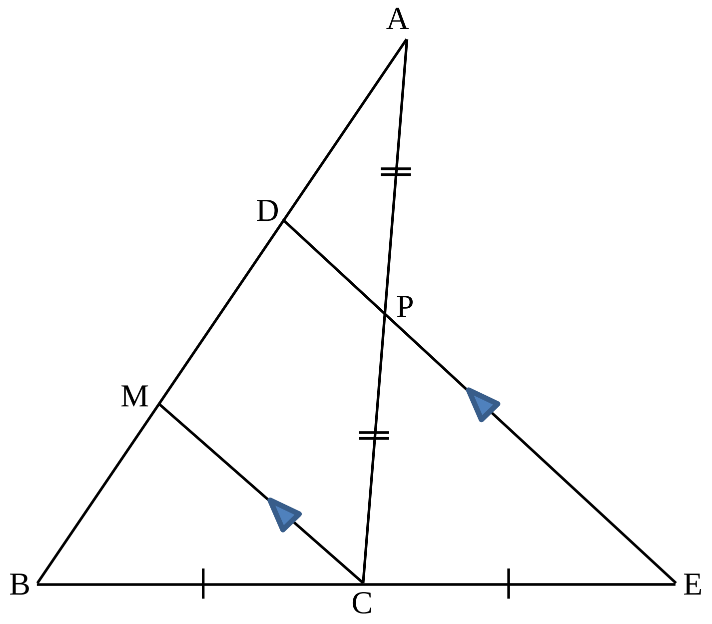
MIDPOINT THEOREM: PRACTICE EXERCISE

1. Given: AD = 5 cm and MC = 6 cm.
   Calculate, with reasons:

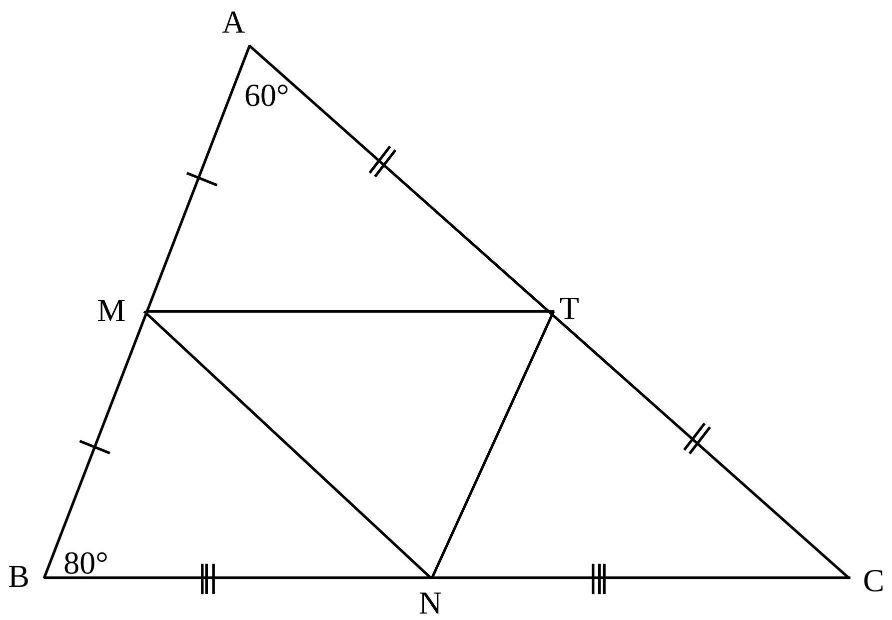
1.1 The length of BM

1.2 The length of DP

1.3 The length of DE

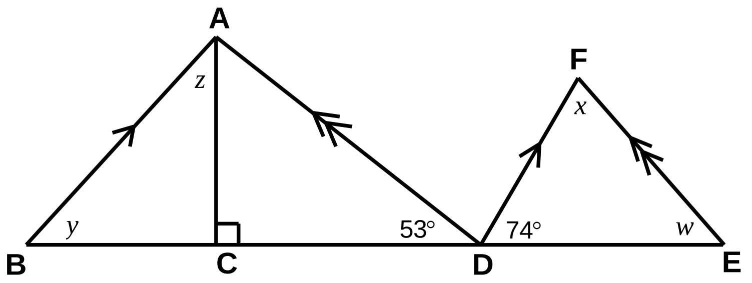


2. M, N and T are the midpoints of AB, BC and AC of ΔABC. $\hat{A} = 60^\circ$ and $\hat{B} = 80^\circ$.
   Calculate the interior angles of ΔMNT.



QUESTION 5 (KZN MARCH 2019)

Study the diagram below and calculate the unknown angles w , x , y and z.
Give reasons for your statements.



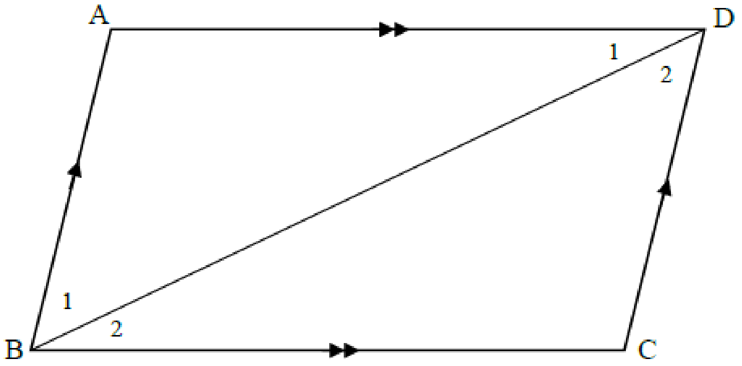
[4]

QUESTION 8 (DBE NOV 2016)

8.1 Complete the following statement:

If the opposite angles of a quadrilateral are equal, then the quadrilateral … (1)

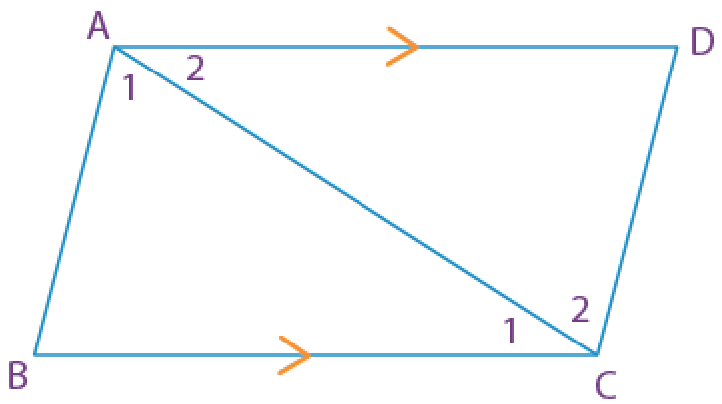
8.2 Use the sketch below to prove that the opposite sides of a parallelogram are equal.



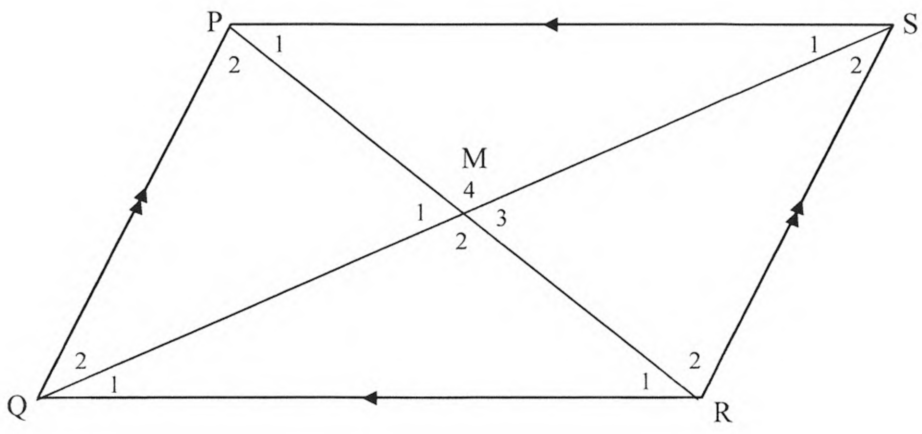
(6)

QUESTION 6 (GP JUNE 2016)

In quadrilateral ABCD, AD//BC and $\hat{B} = \hat{D}$. Prove that ABCD is a parallelogram.



8.2 Given parallelogram PQRS with diagonals PR and QS intersecting at M.

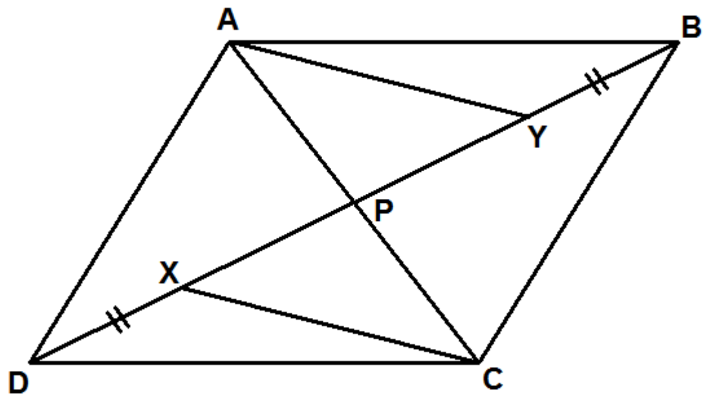


Prove that the diagonals bisect each other. (4)

QUESTION 7 (KZN MARCH 2019)

7.2 In the diagram ABCD is a parallelogram with diagonals intersecting at P.

AY and CX are drawn such that BY = DX.

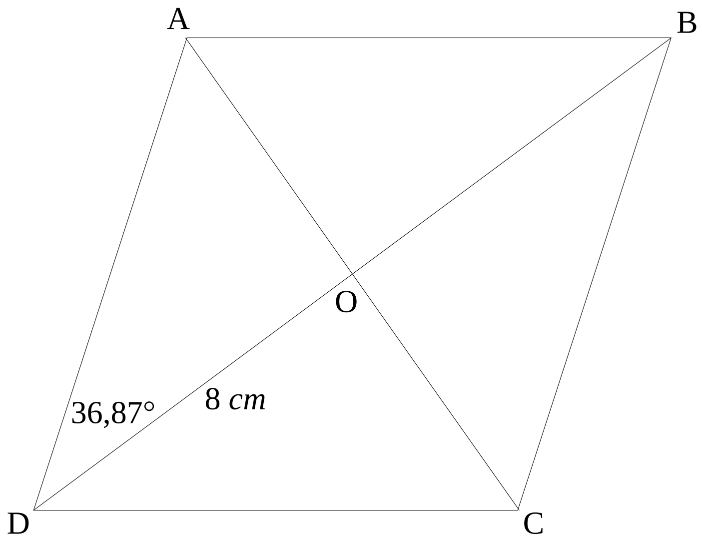


Prove that AYCX is a parallelogram. (3)

[8]

QUESTION 8 (DBE NOV 2015)

In the diagram, ABCD is a rhombus having diagonals AC and BD intersecting in O.
$A\hat{D}O = 36{,}87°$ and DO = 8 *cm*.



8.1 Write down the size of the following angles:

8.1.1 $C\hat{D}O$ (1)

8.1.2 $A\hat{O}D$ (1)

8.2 Calculate the length of AO. (2)

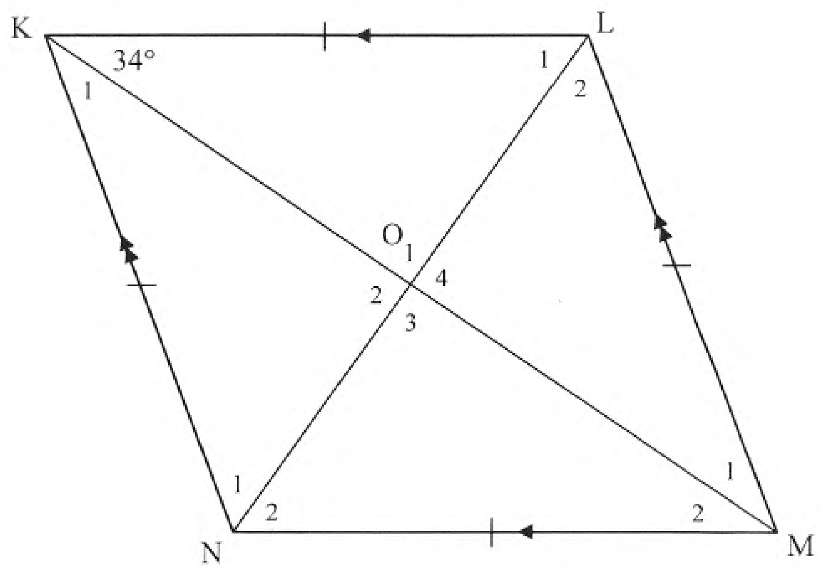
8.3 If E is a point on AB such that OE | | AD, calculate the length of OE. (4)

[8]

DBE NOV 2017

QUESTION 8:

8.1 KLMN is a rhombus with diagonals intersecting at O. $L\hat{K}M = 34°$.



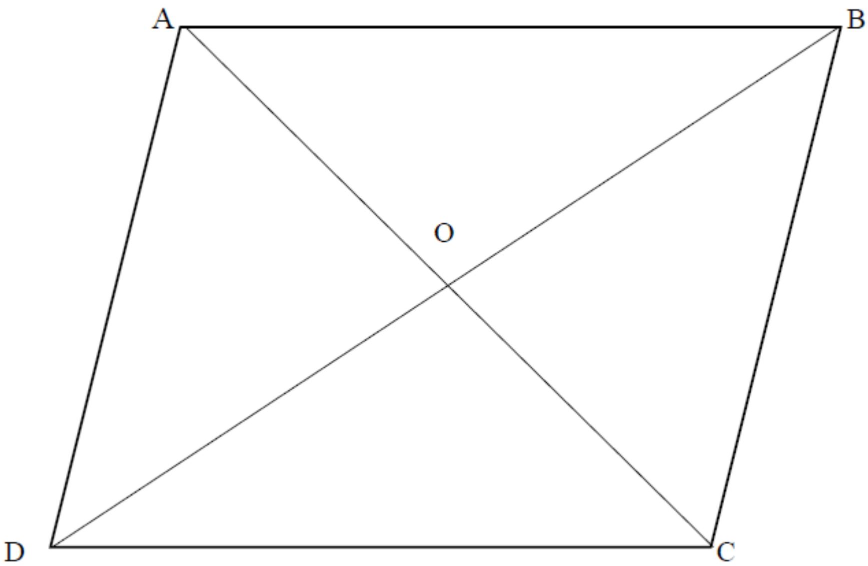
8.1.1 Write down the size of $\hat{O}_1$. (1)

8.1.2 Calculate the size of $\hat{L}_1$. (2)

8.1.3 Calculate the size of $K\hat{N}M$. (2)

GP JUNE 2016

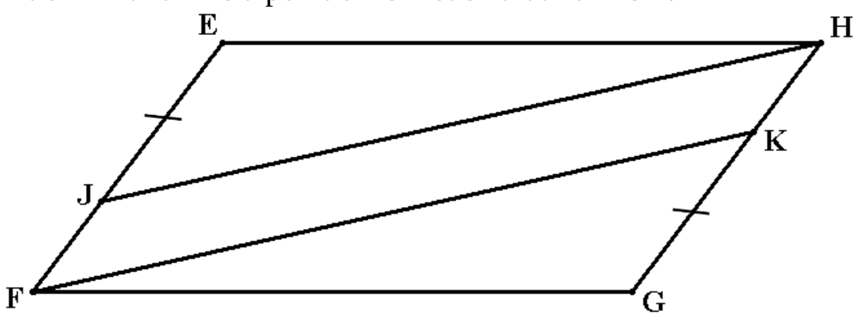
7.1 In the quadrilateral, diagonals, AC and BD bisect at O. If $AC = 4xy$; $BC = x^2 + y^2$ and $BD = 2x^2 - 2y^2$, prove that ABCD is a rhombus.



(5)

QUESTION 9 (KZN JUNE 2016)

9.2 In the sketch below EFGH is a parallelogram.
J is a point on EF and K is a point on GH such that EJ = GK.



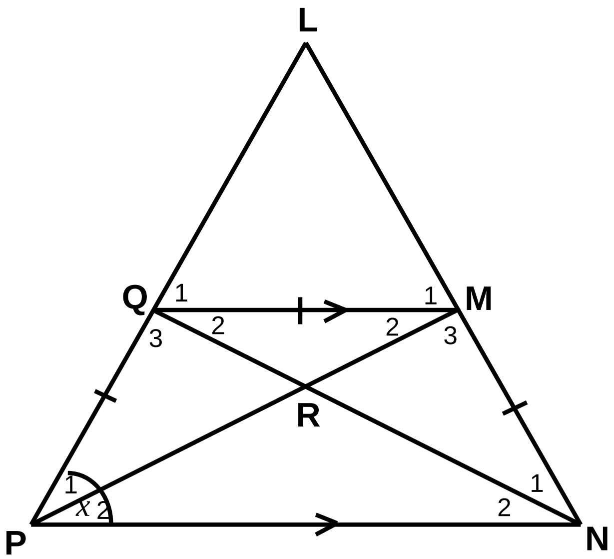
9.2.1 Prove that $\Delta EJH \equiv \Delta GKF$ (3)

9.2.2 Prove that JFKH is a parallelogram. (3)

[10]

8.2 **(KZN MARCH 2019)**

In the diagram below $PQ = QM = MN$, $QM \parallel PN$ and $L\hat{P}N = L\hat{N}P$.



8.2.1 Show that MP bisects $\hat{P}$. (Hint: let $\hat{P}_1 = x$) (3)

8.2.2 Prove that $\Delta PRN \,|||\, \Delta QRM$. (3)

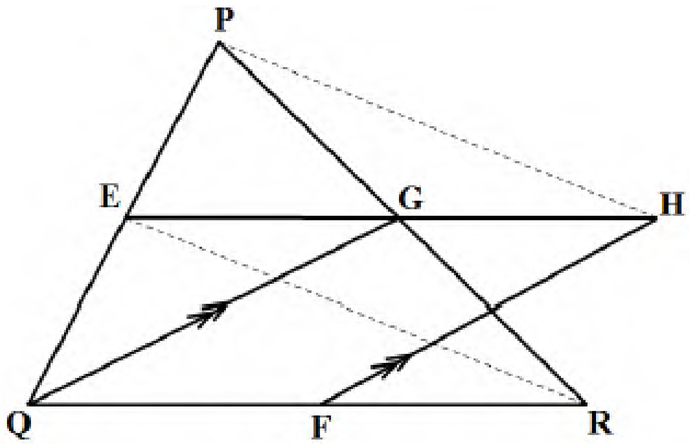
KZN JUNE 2019

QUESTION 3

3.1 Complete the following:

The line joining the mid-points of two sides of a triangle is ……. to the third side and equal to ……. the length of the third side. (2)

3.2 In the diagram below, ΔPQR has E, F and G the midpoints of PQ, QR and PR respectively. $QG \,//\, FH$.



Prove:

3.2.1 QGHF is a parallelogram (3)

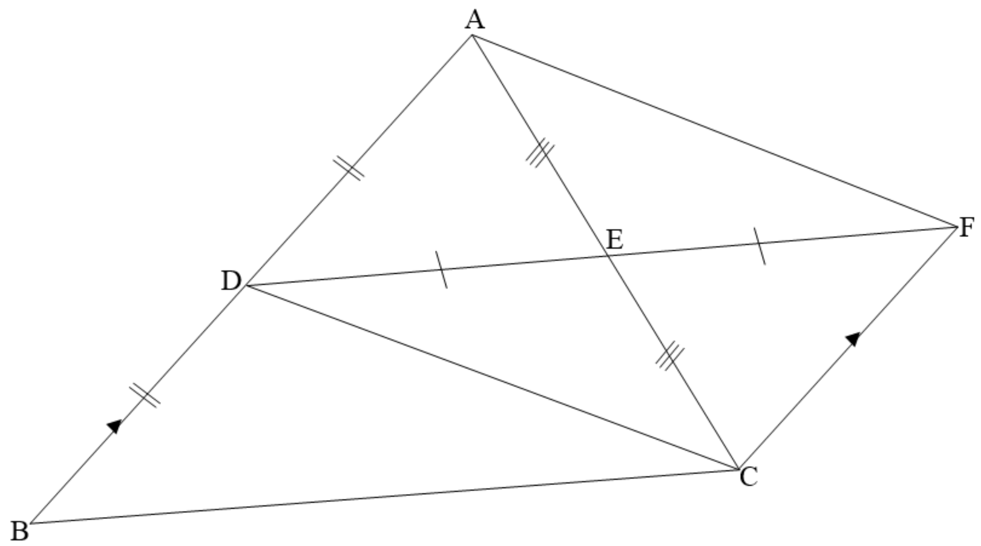
3.2.2 EG = GH (3)

3.2.3 EF || PH (3)

[11]

QUESTION 9 (DBE NOV 2015)

9.1 In the diagram below, D is the midpoint of side AB of ΔABC. E is the midpoint of AC. DE is produced to F such that DE = EF. CF | | BA.



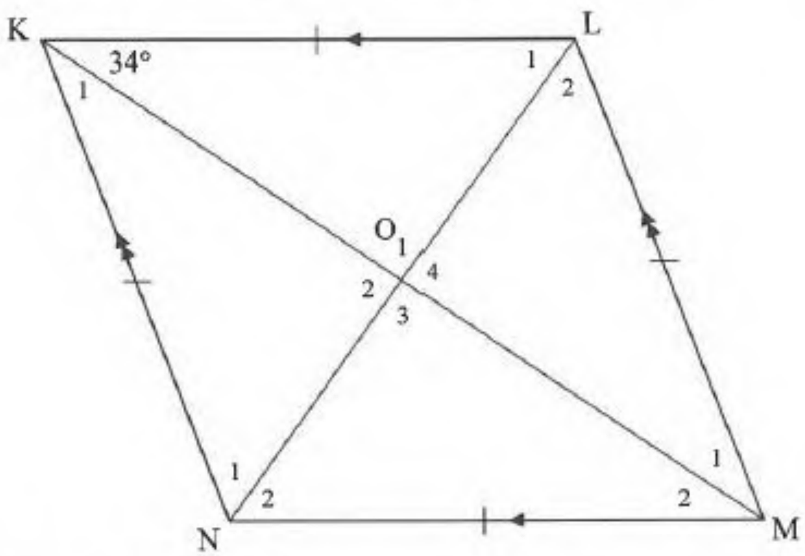
9.1.1 Write down a reason why ΔADE ≡ ΔCFE. (1)

9.1.2 Write down a reason why DBCF is a parallelogram. (1)

9.1.3 Hence, prove the theorem which states that $DE = \frac{1}{2}BC$. (2)

QUESTION (DBE NOV 2017)

8.1 KLMN is a rhombus with diagonals intersecting at O. $L\hat{K}M = 34°$.



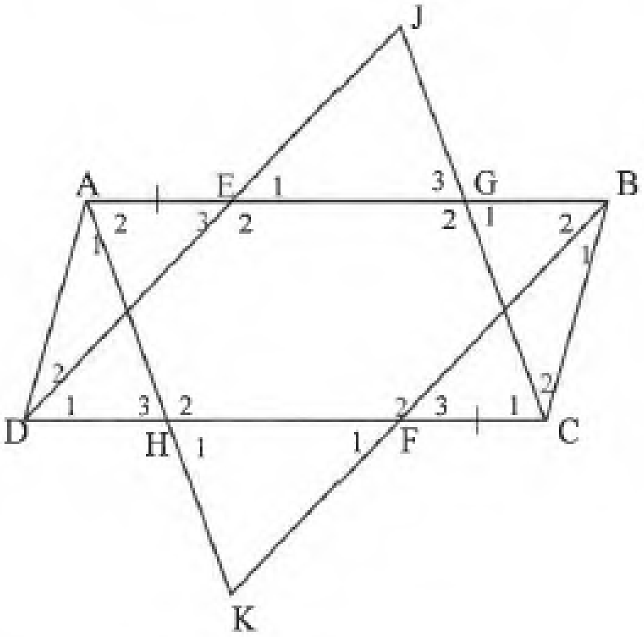
8.1.1 Write down the size of $\hat{O}_1$. (1)

8.1.2 Calculate the size of $\hat{L}_1$. (2)

8.1.3 Calculate the size of $K\hat{N}M$. (2)

QUESTION 8 (DBE NOV 2018)

8.1 ABCD is a parallelogram. E and F are points on AB and DC respectively such that AE = CF. DE is produced to J and CJ is drawn. BF is produced to K and AK is drawn.



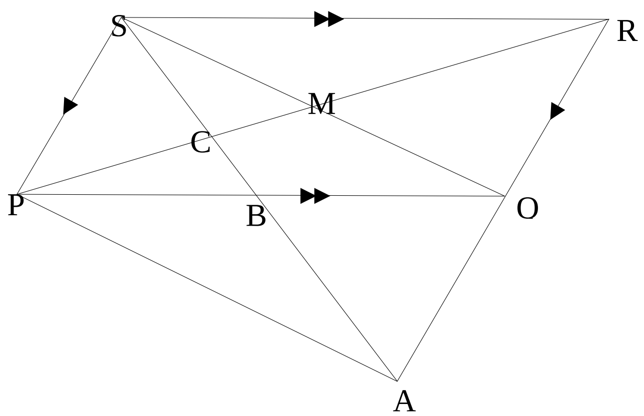
Prove that:

8.1.1 DJ || BK (5)

8.1.2 $\hat{E}_1 = \hat{F}_1$ (4)

DBE NOV 2015 GRADE 10

9.2 In the diagram below, PQRS is a parallelogram having diagonals PR and QS intersecting in M. B is a point on PQ such that SBA and RQA are straight lines and SB = BA. SA cuts PR in C and PA is drawn.



9.2.1 Prove that SP = QA. (4)

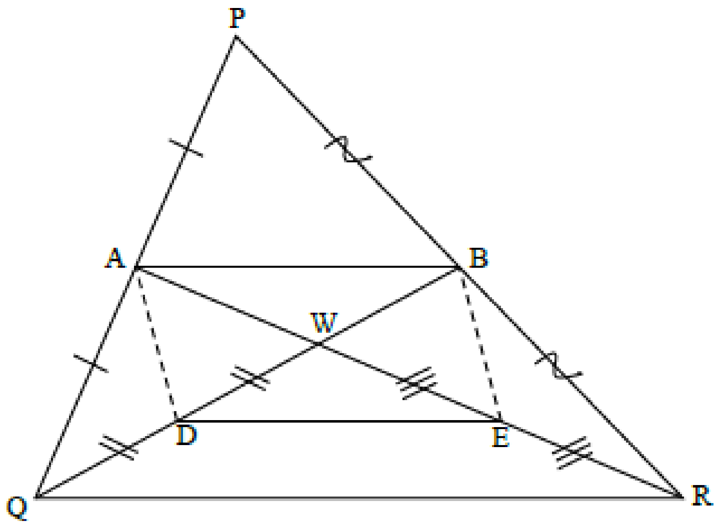
9.2.2 Prove that SPAQ is a parallelogram (2)

9.2.3 Prove that AR = 4MB. (4)

[14]

DBE NOV 2016 GRADE 10 P2

9.2 In ΔPQR, A and B are the midpoints of sides PQ and PR respectively. AR and BQ intersect at W. D and E are points on WQ and WR respectively such that WD = DQ and WE = ER.

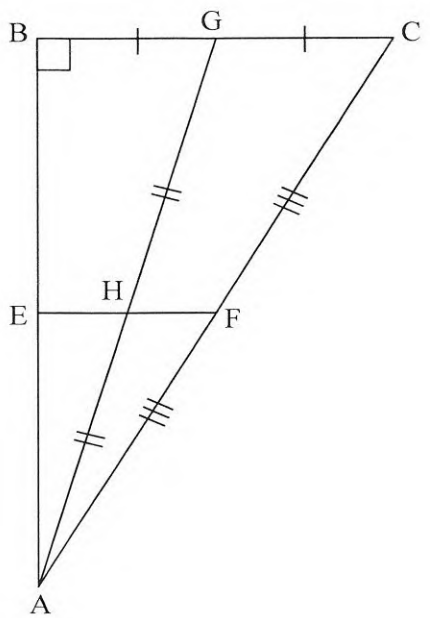


Prove that ADEB is a parallelogram. (5)
[6]

DBE NOV 2017 GRADE 10 P2

QUESTION 9

ΔABC is right-angled at B. F and G are the midpoints of AC and BC respectively. H is the midpoint of AG. E lies on AB such that FHE is a straight line.



9.1 Prove that E is the midpoint of AB. (3)

9.2 If EH = 3,5 cm and the area of $\Delta AEH = 9{,}5\text{cm}^2$, calculate the length of AB. (3)

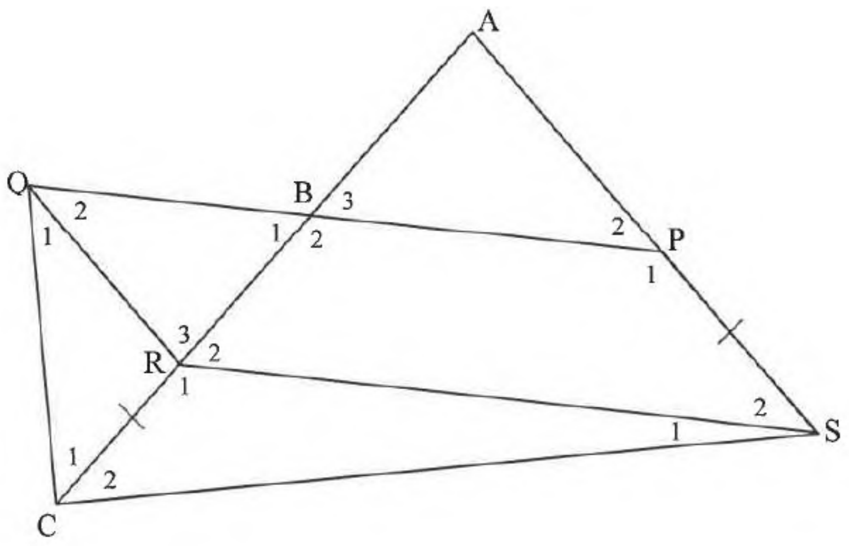
9.3 Hence, calculate the area of ΔABC. (3)
[9]

QUESTION 7

7.1 Complete the statement so that it is TRUE:

The line drawn from the midpoint of the one side of a triangle, parallel to the second side, … (1)

7.2 ACS is a triangle. P is a point on AS and R is a point on AC such that PSRQ is a parallelogram. PQ intersects AC at B such that B is the midpoint of AR. QC is joined. Also, CR = PS, $\hat{C}_1 = 50°$ and BP = 60 mm.



7.2.1 Calculate the size of $\hat{A}$. (5)

7.2.2 Determine the length of QP. (3)

[9]

TRIGONOMETRY

FROM GR. 10 Annual Teaching Plan 2020:

DATES	CURRICULUM STATEMENT
13/03 – 20/03 (6 days) TERM 1	1. Define the trigonometric ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$ using right-angled triangles. 2. Define the reciprocals of the trigonometric ratios $\text{cosec}\,\theta$, $\sec\theta$ and $\cot\theta$ using right-angled triangles. (These three reciprocals should be examined in grade 10 only.) 3. Derive values of the trigonometric ratios for the special cases (without using a calculator), $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$.
31/03 – 03/04 (4 days) TERM 2	4. Solve simple trigonometric equations for angles between 0^0 and 90^0. 5. Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for $0^\circ \leq \theta \leq 360^\circ$. 6. Use diagrams to determine the numerical values of ratios for angles from 0° to 360°.
15/05 – 22/05 (6 days) TERM 2	1. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0^\circ; 360^\circ]$. 2. Study the effect of *a* and *q* on the graphs defined by $y = a\sin\theta + q$, $y = a\cos\theta + q$ and $y = a\tan\theta + q$, for $\theta \in [0^\circ; 360^\circ]$. 3. Sketch graphs, find the equations of given graphs and interpret graphs. **Note**: Sketching of the graphs must be based on the observation of number 2 above.
07/07– 20/07 (10 days) TERM 3	Solve two-dimensional problems involving right-angled triangles.

	Term 1
Week	
Topic	EUCLIDEAN GEOMETRY
Weighting	30±3 marks
Sub-topics/Clarification	Equation of a tangent, Concavity, Graph sketching and interpretation, factor and Remainder theorem
Related concepts/terms/vocabulary	• Straight line • Substitution
Prior-knowledge/ Background knowledge	• Derivative • Inequalities
Resources	• Calculator. • Worksheets and Textbooks
Activities	• See annexure A
Methodology	• Analyze the given information. • Using long division method, inspection and synthetic method
Assessment	• Classwork. • Homework.
Related concepts/terms/vocabulary	• Straight line

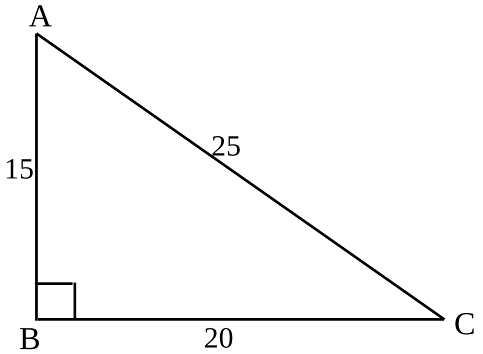
BASELINE ASSESSMENT:

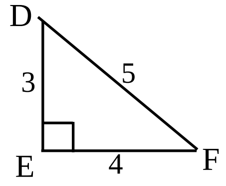
Ratios and Theorem of Pythagoras

Learner Activities

Question 1

Given ΔABC///ΔDEF (A, A, A)





Complete the following ratios and write them in simplest form.

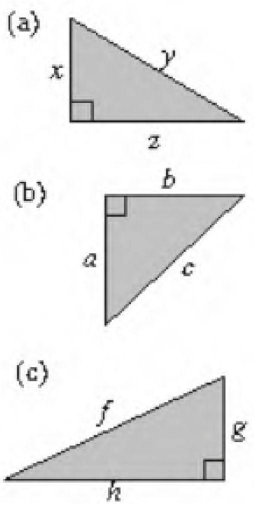
1.1 $\frac{AB}{DE} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

1.2 $\frac{BC}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

1.3 $\frac{AC}{DF} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

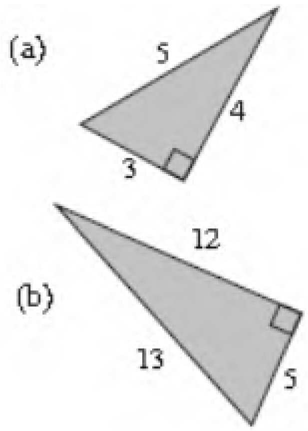
Question 2

State which side of each of the following triangles is the hypotenuse?



Question 3

What is the length of the hypotenuse in the following triangle?

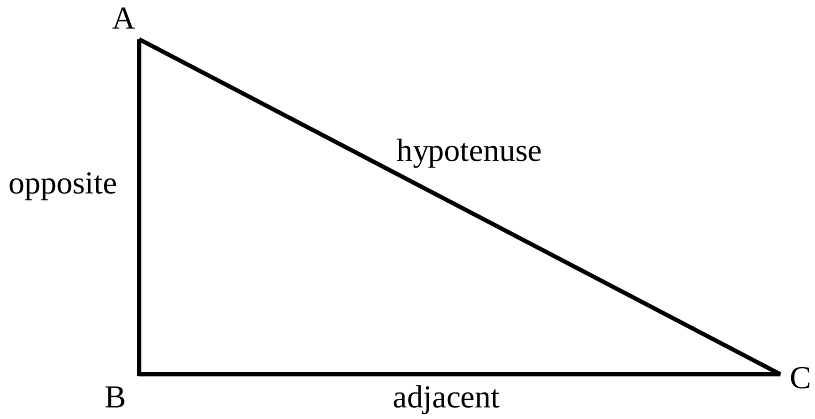


Question 4

The three side lengths of two right-angled triangles are listed below. For each triangle state the length of the hypotenuse.

(a) 35, 12, 37 (b) 60, 61, 11

DEFINITIONS of sin ϴ, cos ϴ and tan ϴ:



Hypotenuse – The side opposite the 90° angle (longest side)

Opposite – The side opposite the angle C

Adjacent – The remaining side next to C

SINOH **COSAH** **TANOA**

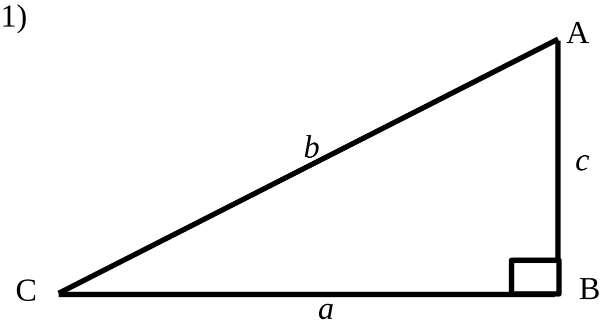
The ratio $\frac{\text{Opp}}{\text{Hyp}}$ is called sine of angle C. $\sin C = \frac{O}{H}$

The ratio $\frac{\text{Adj}}{\text{Hyp}}$ is called cosine of angle C. $\cos C = \frac{A}{H}$

The ratio $\frac{\text{Opp}}{\text{Adj}}$ is called tangent of angle C. $\tan C = \frac{O}{A}$

Define the trigonometric ratios sinθ, cosθ and tanθ using a right-angled triangle:

1)

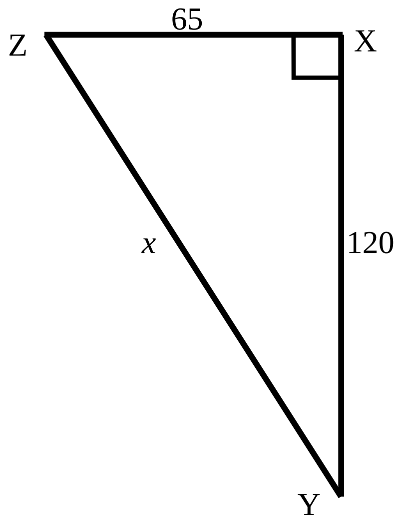


cos A =

sin C =

tan A =

2)



Determine the value of *x*.

sin Z =

cos Z =

tan Z =

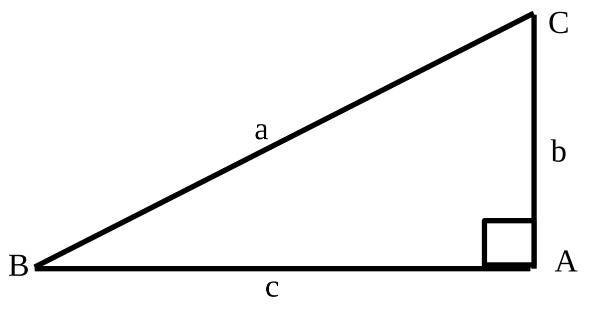
sin Y =

cos Y =

tan Y =

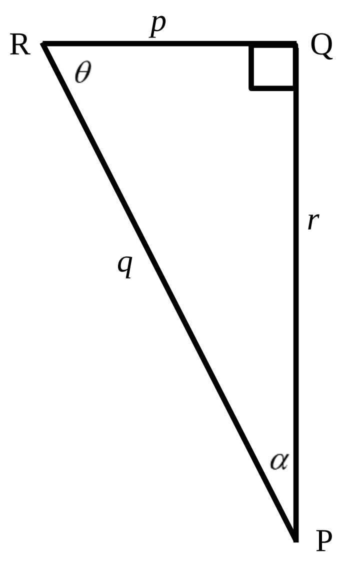
Learner Activity:

1. State the following:



a) sin C
b) cos C
c) tan C
d) sin B
e) cos B
f) tan B

2. State the following:



a) sin α
b) cos α
c) tan α
d) sin θ
e) cos θ
f) tan θ

Extended opportunity: Encourage learners to identify the relationship between the ratios from the solutions

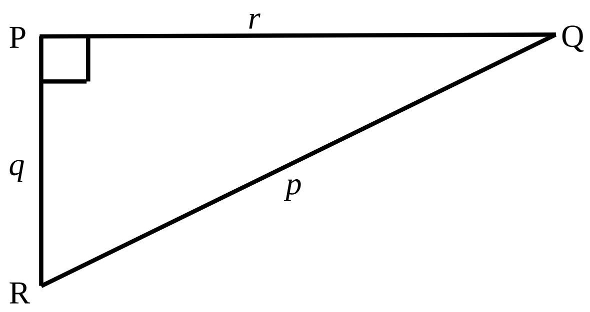
DEFINITION OF RECIPROCAL FUNCTIONS: cosec θ, sec θ and cot θ

Example: $\frac{1}{2}$ has a reciprocal of $\frac{2}{1}$ and $\frac{2}{3}$ has a reciprocal of $\frac{3}{2}$

cosec $\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$

sec $\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$

cot $\theta = \frac{1}{\tan\theta} = \frac{\text{Opposite}}{\text{Adjacent}}$



$\sin Q = \frac{q}{p}$ $\qquad$ $\text{cosec} Q = \frac{p}{q}$

$\cos Q = \frac{r}{p}$ $\qquad$ $\sec Q = \frac{p}{r}$

$\tan Q = \frac{q}{r}$ $\qquad$ $\cot Q = \frac{r}{q}$

Learners to complete:

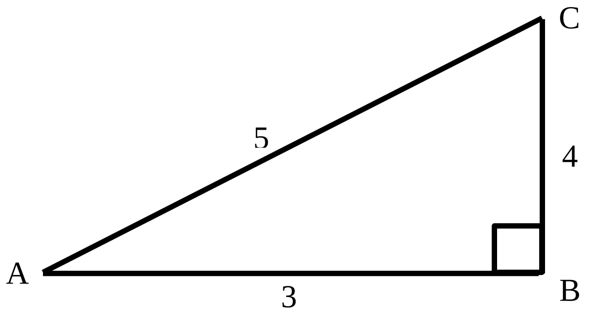
sin R = $\qquad$ cosec R = $\qquad$ cos R = $\qquad$ sec R = $\qquad$ tan R = $\qquad$ cot R =

Learner Activity:

1) Refer to the diagram alongside to answer the following question

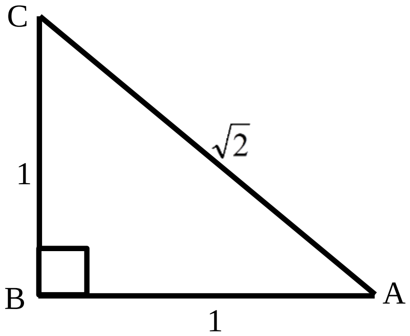
State the following:

a) sec A
b) cot A
c) cosec A
d) cot C
e) cosec C
f) sec C



Special Angles

Begin by constructing an isosceles right-angled triangle: (angles of 90°; 45°; 45°)

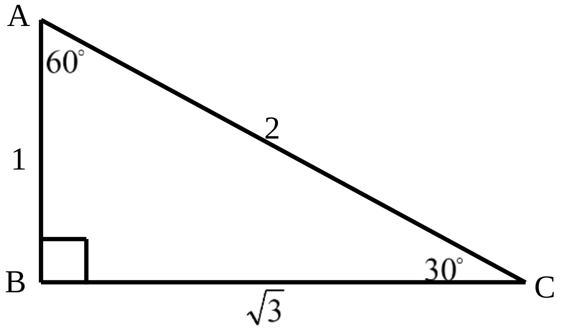


$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = 1$

Also construct a right-angled triangle including angles of 30° and 60°:



$\sin 30^\circ =$ $\qquad$ $\sin 60^\circ =$

$\cos 30^\circ =$ $\qquad$ $\cos 60^\circ =$

$\tan 30^\circ =$ $\qquad$ $\tan 60^\circ =$

Extend the activity to include the reciprocal trig functions for the special angles.

Learner Activity:

Determine the values of the following without using a calculator.

1) $\sin^2 30^\circ + \cos^2 30^\circ$
2) $\sin^2 30^\circ + \cos^2 60^\circ$
3) $\sin 30^\circ . \tan 45^\circ \cos 45^\circ$
4) $\frac{\sin 45^0}{\cos 45^0}$
5) $\cos 30^\circ . \tan 60^\circ + \text{cosec}^2 45^\circ . \sin^2 60^\circ$
6) $\frac{\sin 30^\circ . \sec 45^\circ}{\frac{1}{\sin^2 60^\circ}}$

Extended opportunity: Learners to identify the relationship between ratios to form identities.

$\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Evaluate the value of the following trigonometric ratios with the use of calculators. Round off all answers to TWO decimal places.

1) $\cos 10^\circ =$
2) $\sin 312^\circ =$
3) $\sin 35^\circ + \sin 75^\circ =$
4) $\sin^2 43^\circ + \cos^2 43^\circ =$
5) $\frac{\cos 24^\circ}{24}$

Trigonometric Equations:

Solving of simple trigonometric equations for angles between 0^0 and 90^0.

Subtopic: calculating the size of an angle by manipulation of equations.

Examples:

Consider cos β = 0.5

Methodology – use a calculator and making use of the button cos^{-1} on the calculator.

Use the shift button and enter cos.

$\beta = cos^{-1}(0,5)$

$= 60^0$

Example 2

$2 \sin \alpha - 1 = 0$

$\sin \alpha = \frac{1}{2}$

$\alpha = 30^0$

$2 \sin \alpha - 3 = 0$

$\sin \alpha = \frac{3}{2}$

$\alpha = sin^{-1}(\frac{3}{2})$

Calculator shows math error. This means that the equation has no solution. Therefore the ratios of cos and sin cannot be solved for values greater than 1.

Extended opportunity: Learner can be given an investigation to find the maximum and minimum values for sin, cos and tan.

$\sin 3\theta = 0{,}157$

$3\theta = sin^{-1}(0{,}157)$

$3\theta = 9{,}0328 \ldots..$

$\theta = 3{,}01$ Explain rounding off to 2 decimal places

$cos\,(x + 60^0) = 0{,}5$

$(x + 60^0) = cos^{-1}(0{,}5)$

$x + 60^0 = 60^0$

$x = 60^0 - 60^0$

$x = 0^0$

Learner Activity:

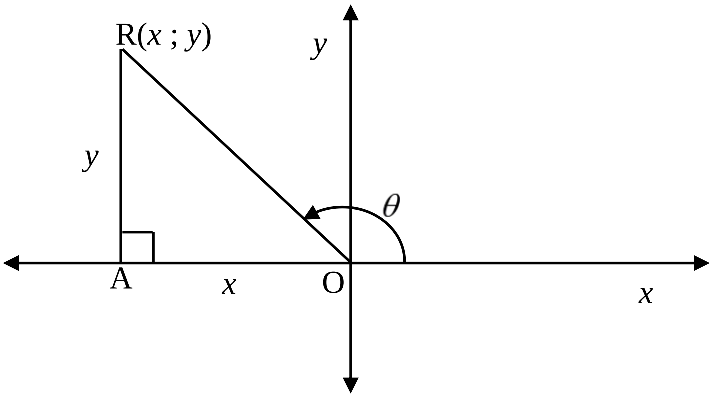
Solve the following equations and round off the answers to 2 decimal places.

1. $\tan\theta = 0{,}357$
2. $2\cos\alpha = \sqrt{3}$
3. $2\sin x + 1 = 2$
4. $2\tan(\beta + 10^\circ) + 3 = 5$
5. $\frac{1}{3}\cos 3x = 0{,}12$
6. $3\cos(2\theta - 12) - 2 = 1$
7. $\frac{3}{2}\sin x = \cos 33^\circ$
8. $\sec(x + 10^\circ) = 5{,}648$
9. $\frac{\tan\theta}{0{,}3} - 1 = 2{,}32$
10. $4\operatorname{cosec}\beta - 2 = 1$

Trigonometric functions in a Cartesian plane:

The definition of trigonometric ratio can be extended to include any angle from $0^\circ - 360^\circ$.

Let R (x, y) be any point in the Cartesian plane. Let θ be the angle measured in an anti – clockwise direction from the positive x – axis to OR. We say OR is the standard position. Let OR = r (radius). Draw RA perpendicular to the x – axis.

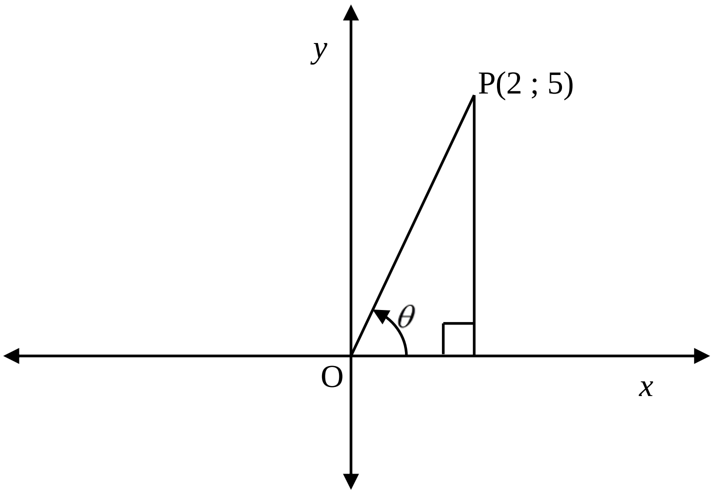


We define the trig ratios of any angle θ in terms of x, y and r by:

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $(x \neq 0)$

Learner Activity:

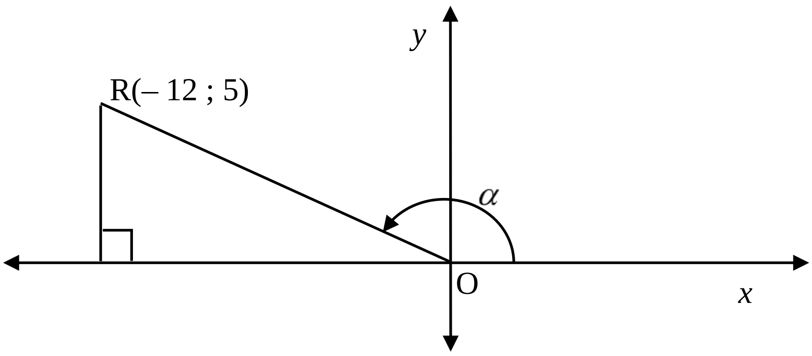
1. P(2 ; 5) is a point in the Cartesian plane. OP makes an angle of θ with positive x – axis.



Determine the following, leaving your answers in surd form if necessary:

1.1 OP 1.2 $\sin\theta$ 1.3 $\cos\theta$ 1.4 $\tan\theta$

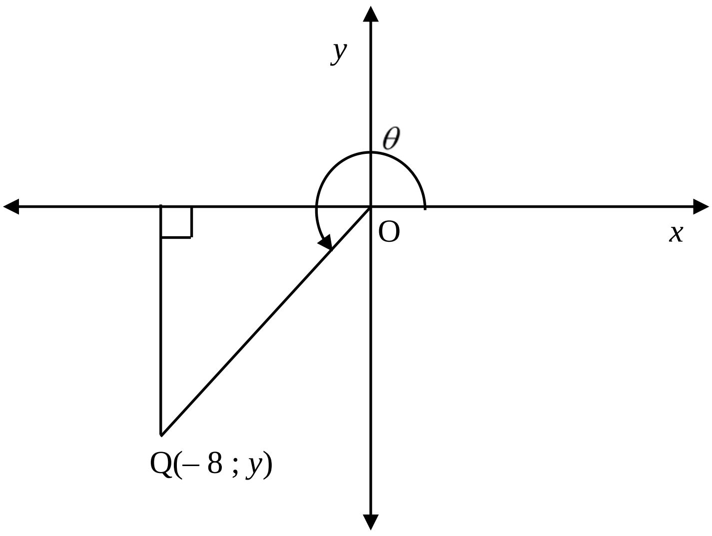
2. R is the point (– 5 ; 12) and OR makes an angle α in the anti – clockwise direction with the positive x – axis.



2.1 Calculate the length of OR.

2.2 State the values of the six trigonometric ratios of α. (sin, cos, tan, cosec, sec and cot)

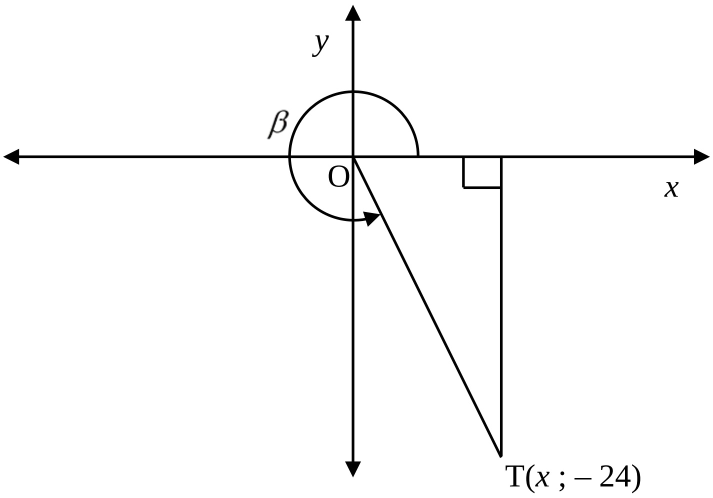
3. OQ = 10 cm and Q(– 8 ; y):



Determine

3.1 y 3.2 $\sin\theta$ 3.3 $\cos\theta$ 3.4 $\tan\theta$

4. OT = 25 cm and T(x ; – 24)



Determine

4.1 sin β 4.2 cosec β 4.3 tan β 4.4 (sin β) (sin β) + (cos β) (cos β)

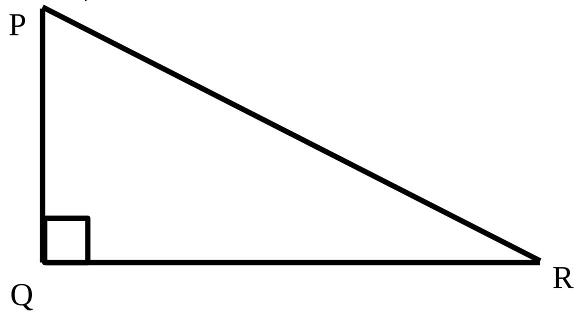
Solving Right-Angled Triangles:

In a right-angled triangle, we can calculate all the remaining side lengths and angle sizes if we know:

The lengths of two sides *or* the size of one angle and the length of one side.

Learner Activity

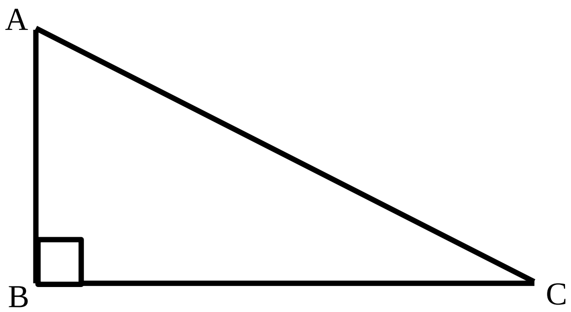
1. In ΔPQR, $\hat{Q} = 90^\circ$, $\hat{R} = 42^\circ$ and $PR = 22\ units$



Calculate, correct to two decimal places:

1.1 the length of QR; 1.2 the length PQ.

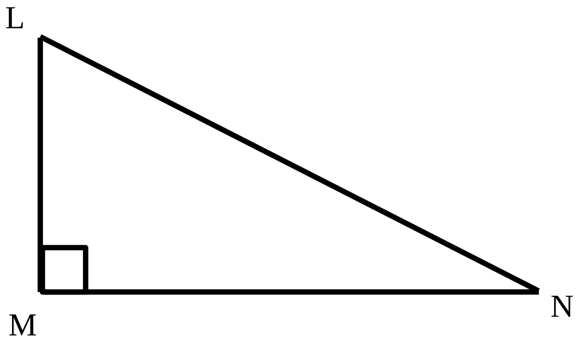
2. Given ΔABC with $\hat{C} = 25^\circ$ and AB = 40 units.



Calculate, correct to two decimal places:

2.1 the length of AC; 2.2 the length BC.

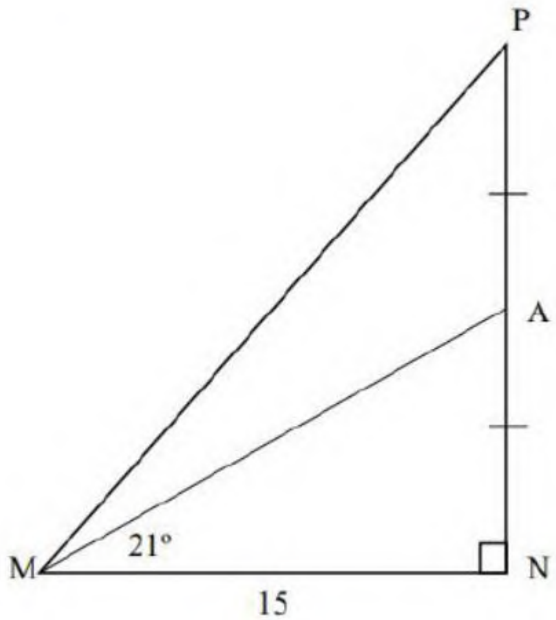
3. In the right-angled ΔLMN, $\widehat{M} = 90^\circ$, LM = 12 cm and MN = 36 cm.



Calculate, correct to two decimal places:

3.1 the size of $\hat{L}$; 3.2 the size of $\widehat{N}$.

4. In the sketch below, ΔMNP is drawn having a right angle at N and MN = 15 units. A is the midpoint of PN and $A\hat{M}N = 21^\circ$.



Calculate, correct to two decimal places:

4.1 the length of AN; 4.2 the size of $P\widehat{M}N$; 4.3 the length of MP.

Point by point plotting of graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0^\circ; 360^\circ]$:

Explanation of terminology:

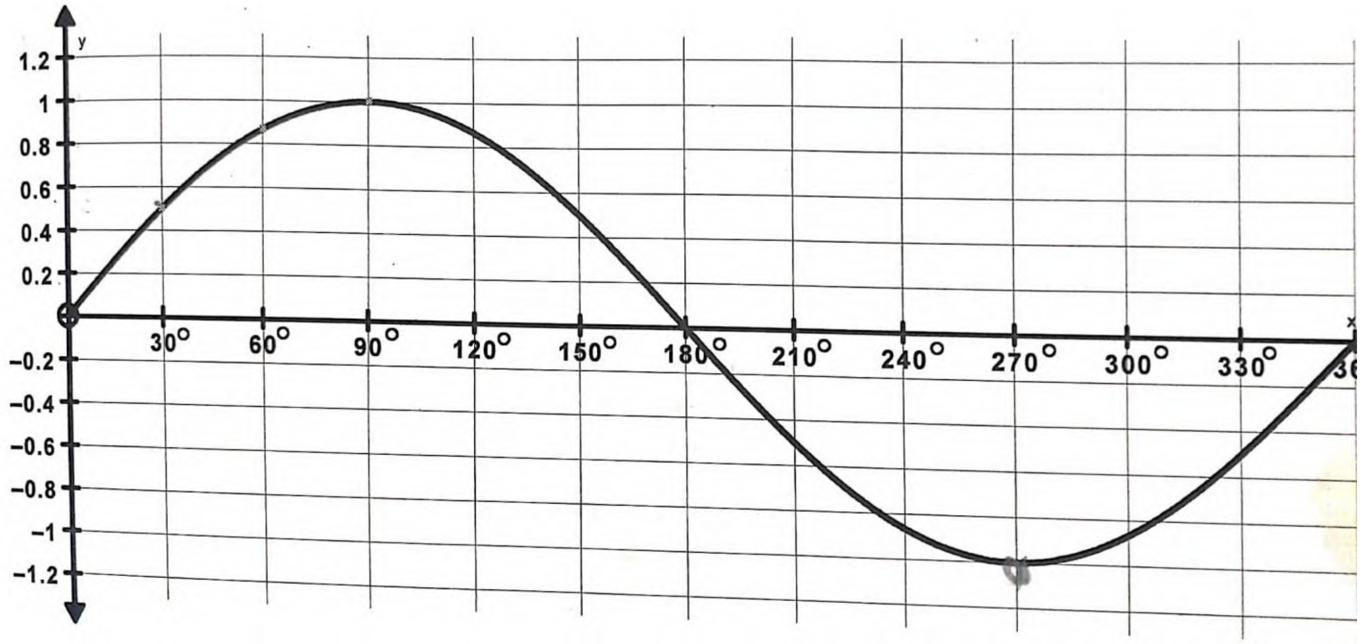
- Domain: the x values (input) that are represented on the x – axis.
- Range: the y values (output) that are represented on the y- axis (definitions of these terms would have been covered in the previous topic of functions).
- The interval notation and the meaning thereof:

  Square brackets mean that the end values are included and round brackets mean that the end values are not included.
- From algebra the concept of substitution and calculator skills.

Using the table method to sketch $y = \sin x$: all values have to be rounded off to 2 decimal places. Learners should use decimal values, because those are easier to plot than fractions.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin x$	0	0,5	0,87	1	0,87	0,5	0	– 0,5	– 0,87	– 1	0,87	– 0,5	0

Sketching the graph onto the Cartesian plane by drawing the axes and plotting point by point.



Explanation of the characteristics of the graph sketched:

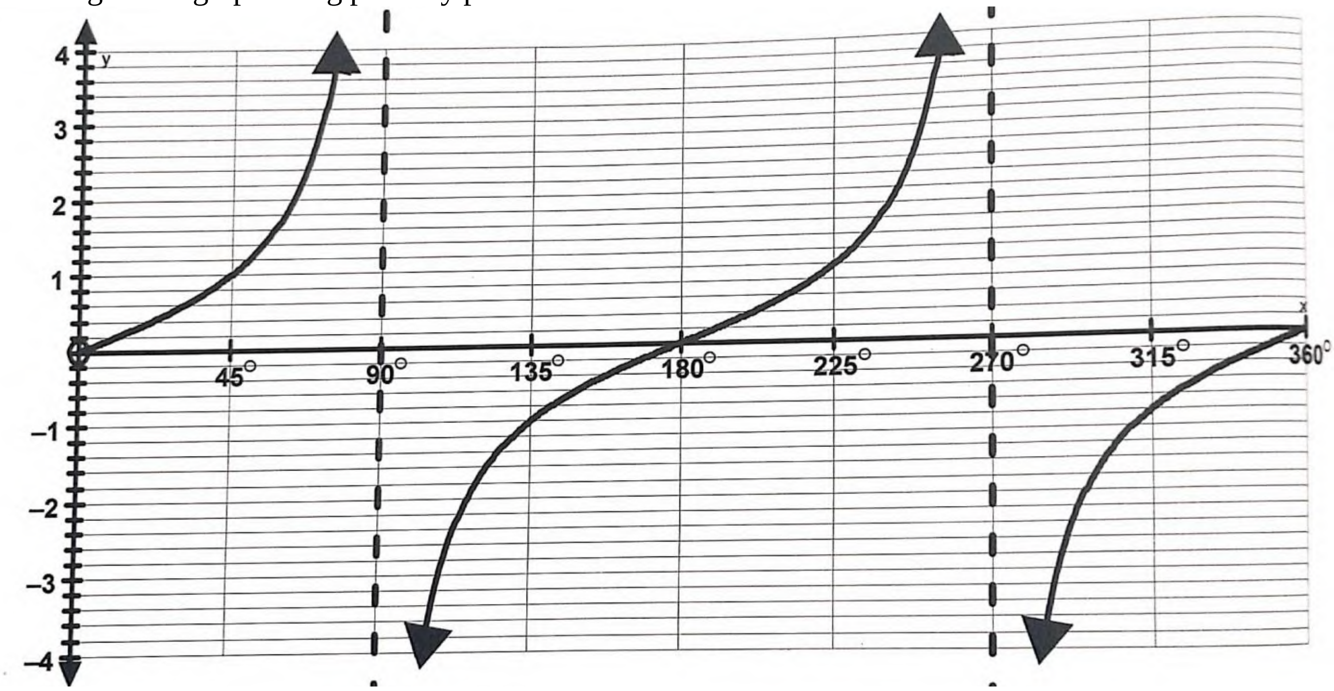
- Period: the number of degrees taken for the graph to complete one cycle (cycle being one complete shape of the graph.
- Learners identify highest and lowest points on the graph. Link highest to the term maximum and lowest to the term minimum.
- Amplitude: half the distance between the maximum and minimum value (must always be positive): $\frac{max\ value - \min value}{2}$
- Range: [min value; max value]

Teacher to draw the graph of $y = \tan x$, due to the differences in characteristics between the graph of tan x and those of sin x and cos x. The term asymptotes needs to be reintroduced.

$y = \tan x$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y=tan x	0	0,58	1,73	Undef.	– 1,73	– 0,58	0	0,58	1,73	Undef.	– 1,73	– 0,58	0

Sketching of the graph using point by point method:



- Teacher to explain to learners that the $y = \tan x$ graph has no maximum and no minimum values, therefore resulting in the amplitude not existing, and also the range then becomes $y \in R$ or $y \in (-\infty;\infty)$.
- The function has a period of 180°.
- The values from the table are then represented on the graph as broken lines to show the asymptotes, i.e. $x = 90^\circ$; 270°.

Learner activity:

1.1. Sketch the graph of $y = \cos x$; $x \in [0^\circ; 360^\circ]$

1.2. What is the maximum value?

1.3. What is the minimum value?

1.4. Determine the amplitude.

1.5. State the range.

1.6. What is the period of the function?

2.1 Complete the following table by using a calculator, and rounding off answers to 2 decimal places.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin x$	0	0,5	0,87	1	0,87	0,5	0	– 0,5	– 0,87	– 1	0,87	– 0,5	0
$y = 2 \sin x$													
$y = 3 \sin x$													
$y = \frac{1}{2} \sin x$													

2.2 On the graph paper provided, sketch the graphs on the same set of axes. (it is advisable to use different coloured pencils to highlight the differences between the graphs.)

2.3 Using the graphs sketched, explain what the effect is of the 2 in the graph of $y = 2 \sin x$ (the a-value of 2). Similarly explain the effects of $a = 3$ and $a = \frac{1}{2}$.

3.1 Complete the following table by use of a calculator, rounding off answers to 2 decimal places:

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$													
$y = \cos x + 2$													
$y = \cos x - 2$													
$y = 2\cos x + 1$													

3.2. Sketch the graph of $y = \cos x$, (using this as the 'mother graph, once again use different coloured pencils for each graph to highlight the differences and similarities.

3.3. Explain the effect of the 2 in the graph of $y = \cos x + 2$ (q-value of 2).

3.4. Similarly explain the effect of $q = -2$ on the 'mother graph' of $y = \cos x$.

3.5. Comparing your answers to questions 2 and 3, explain the effects of the values of a and q on the trigonometric functions. (Hint: use the words, stretch and shift to differentiate between the effects.)

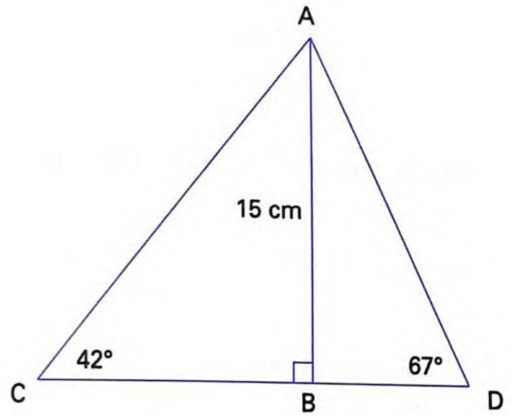
Extended opportunities:

- Learners to investigate the effects of a and q on the $y = \tan x$ function.
- Learners to use the graphs of all three functions and on the same system of axes than the graph of $y = \sin x$ also sketch $y = -\sin x$. On the same system of axes as the graph of $y = \cos x$ to sketch $y = -\cos x$ and similarly on the same system of axes than $y = \tan x$ to also sketch $y = -\tan x$.
- Learners to compare the similarities and differences between these graphs, i.e. to list the maximum and minimum values, the amplitude, the range and the period.
- Learners to reach a conclusion on what the effect of a negative value of $\boldsymbol{a}$ in on the trigonometric functions, (i.e. does a negative value of a result in a stretch or in a reflection?)

<u>Solving two – dimensional problems involving right-angled triangles</u>:

Learner activity: Revision of solving triangles using trigonometry

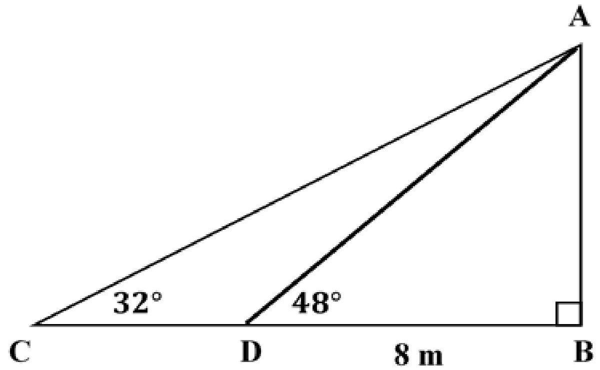
1. AB is perpendicular to CD, $A\hat{C}B = 42^\circ$, $A\hat{D}B = 67^\circ$ and AB = 15 cm.



1.1 Calculate the length of CB

1.2 Calculate the length of CD

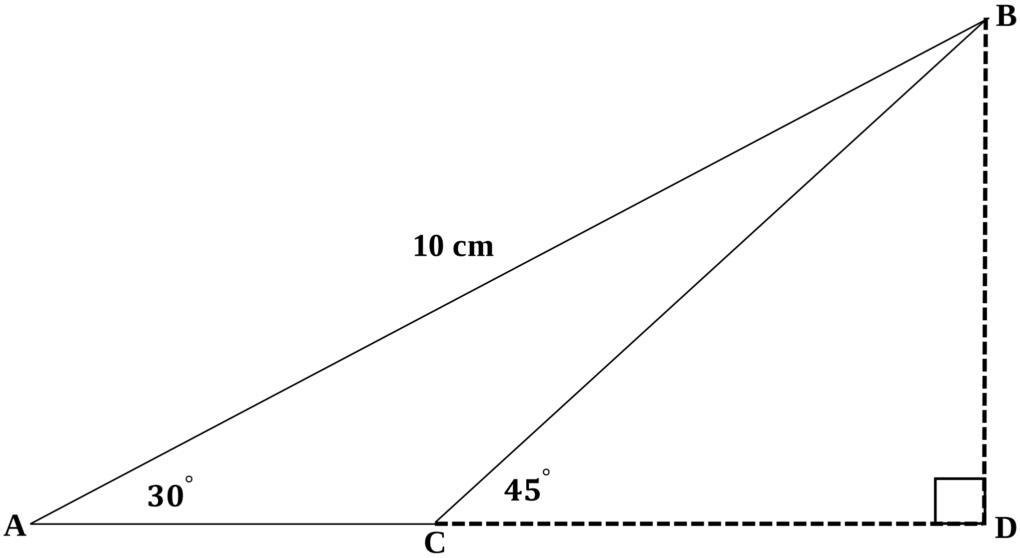
2. ΔABC is right angled at B; $\hat{C} = 32^\circ$, $A\hat{D}B = 48^\circ$ and DB = 8 m.



2.1 Calculate the length of AB.

2.2 Calculate the length of CD.

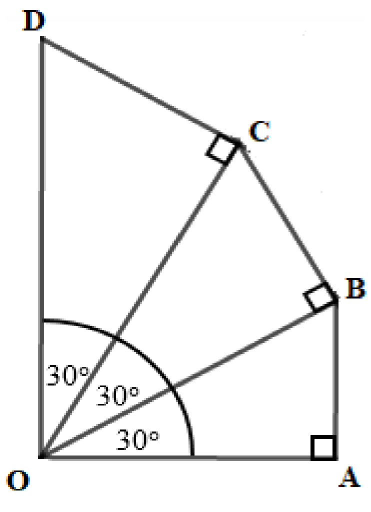
3. In the diagram, AB = 10 cm, $B\hat{A}C = 30^\circ$, $B\hat{C}D = 45^\circ$ and $B\hat{D}C = 90^\circ$.



3.1 Find the length of BD.

3.2 Show that AC = $5(\sqrt{3} - 1)$ cm.

4. In the diagram OA = 1 cm, $A\hat{O}B = B\hat{O}C = C\hat{O}D = 30^\circ$ $O\hat{A}B = O\hat{B}C = O\hat{C}D = 90^\circ$.

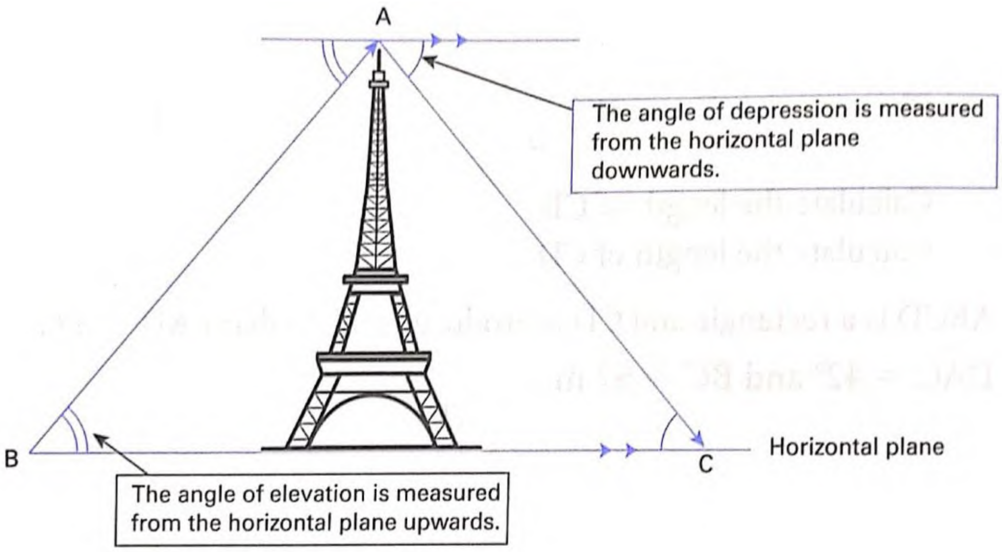


4.1 Find the length of OD giving your answers in the form $a\sqrt{3}$.

4.2 Show that the perimeter of OABCD is $\frac{5}{3}(1 + \sqrt{3})$ cm.

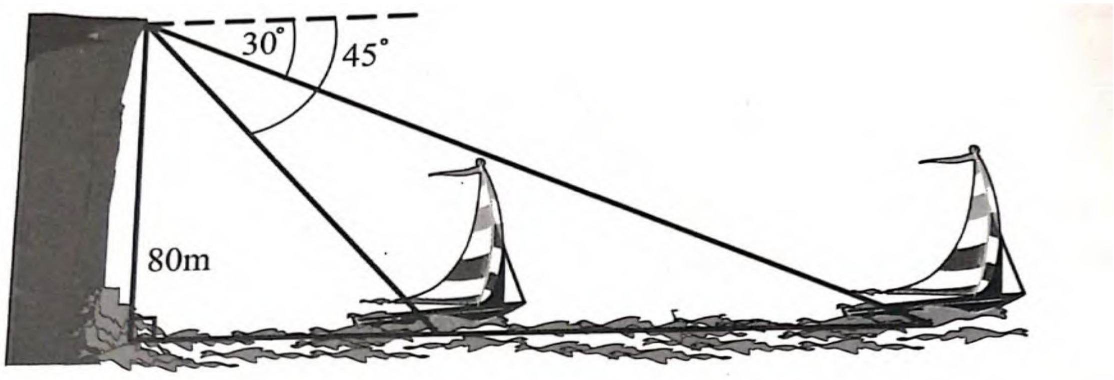
Angles of Elevation and Depression:

- The angle of **elevation** is the angle between the horizontal and a direction **above** the horizontal.
- The angle of **depression** is the angle between the horizontal and a direction **below** the horizontal.

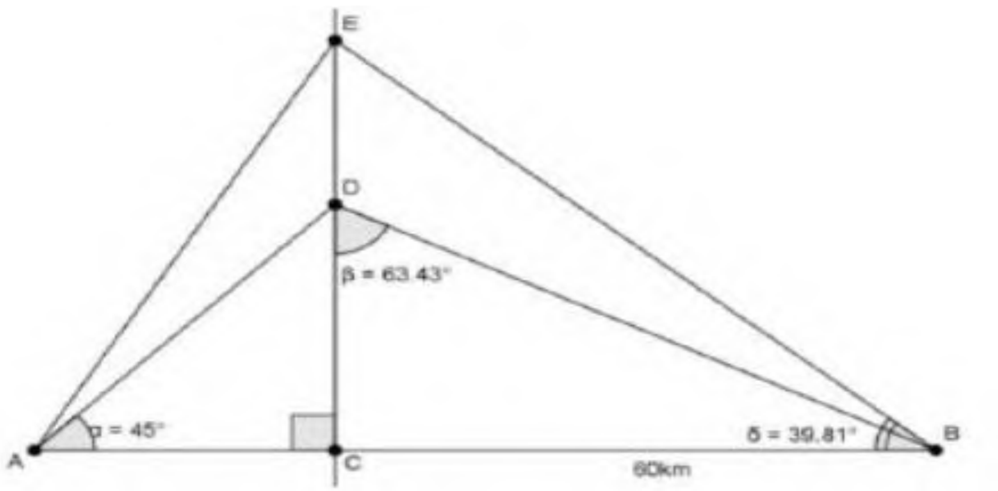


Learner activity: Revision of solving triangles using trigonometry

1. A man watching a boat from a cliff notices that the angle of depression from the cliff to the boat changes from 45^0 to 30^0. What distance did the boat cover while the man was watching it?



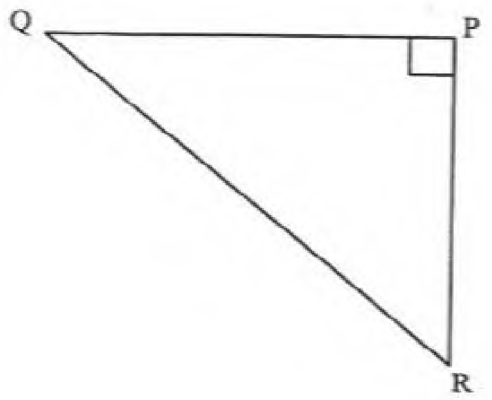
2. Two friends are arguing about a route they should follow. Themba says that travelling from E to B is the shortest route, while Sipho says that travelling from E to D and then to B is the shortest route. Who is right? (Show all your workings, to the nearest km.)



QUESTIONS FROM PAST EXAM PAPERS: TRIGONOMETRY

Question 1

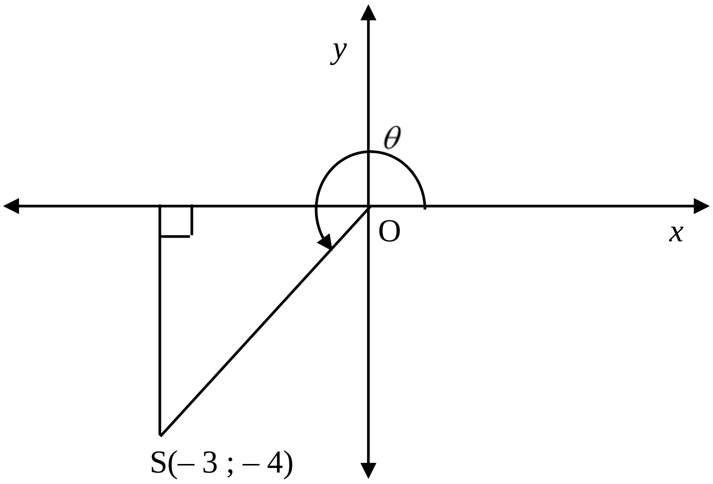
1.1 In the diagram below, ΔPQR is a right angled triangle with $P\hat{Q}R = 90°$



1.1.1 Use the sketch to determine the ratio of $\tan(90° - R)$ (1)

1.1.2 Write down the trigonometric ratio that is equal to $\frac{QR}{QP}$ (1)

1.2 $S(-3;\ -4)$ is a point on the Cartesian plane such that OS makes an angle θ with the positive x-axis.



1.2.1 The length of OS (2)

1.2.2 The value of $\sec\theta + \sin^2\theta$ (3)

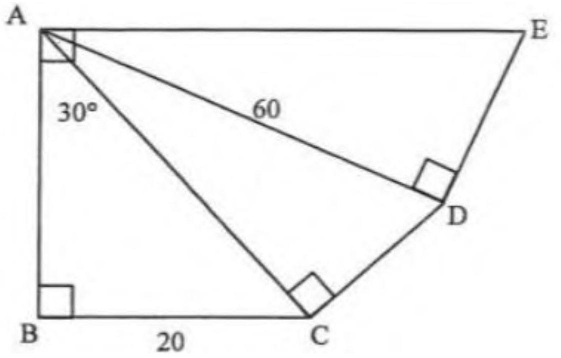
1.3 Determine the value of the following WITHOUT using a calculator:

$$\frac{\text{cosec } 45°}{\sin 90°.\tan 60°}$$ (4)

[11]

Question 2

2.1 In the diagram below, ABC, ACD and ADE are right-angled triangles.
$B\hat{A}E = 90°$ and $B\hat{A}C = 30°$. BC = 20 units and AD = 60 units.



Calculate the:

2.1.1 Length of AC (2)

2.1.2 Size of $C\hat{A}D$ (2)

2.1.3 Length of DE (3)

2.2 Solve for x, correct to ONE decimal place, where $0° \leq x \leq 90°$.

2.2.1 $\tan x = 2{,}01$ (2)

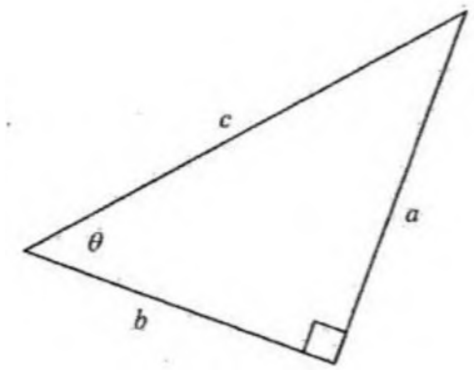
2.2.2 $5\cos x + 2 = 4$ (3)

2.2.3 $\dfrac{\operatorname{cosec} x}{2} = 3$ (3)

[15]

Question 3

3.1 A right-angled triangle has sides a, b and c and the angle θ as shown below.



3.1.1 Write the following in terms of a, b and c:

(a) $\cos\theta$ (1)

(b) $\tan\theta$ (1)

(c) $\sin(90° - \theta)$ (2)

3.1.2 If it is given that $a = 5$ and $\theta = 50°$, calculate the numerical value of b. (2)

3.2 Given that $\hat{A} = 38{,}2°$ and $B = 146{,}4°$.
Calculate the value of $2\operatorname{cosec} A + \cos 3B$ (3)

3.3 Simplify fully, WITHOUT the use of a calculator

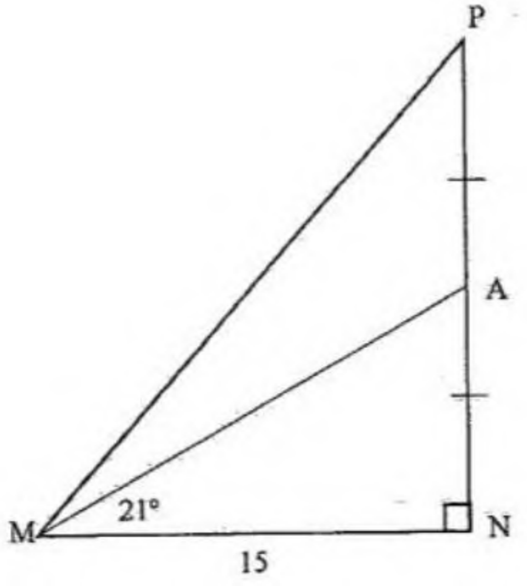
$$\frac{\sin 45°.\tan^2 60°}{\cos 45°}$$ (4)

3.4 Given that $5\cos\beta - 3 = 0$ and $0° \leq \beta \leq 90°$
If $\alpha + \beta = 90°$ and $0° \leq \alpha \leq 90°$, calculate the value of $\cot\alpha$ (4)

[17]

Question 4

4.1 In the sketch below, ΔMNP is drawn having a right angle at N, and MN = 15 units. A is the midpoint of PN and $A\hat{M}N = 21°$.



Calculate:

4.1.1 AN (3)

4.1.2 $P\hat{M}N$ (3)

4.1.3 MP (3)

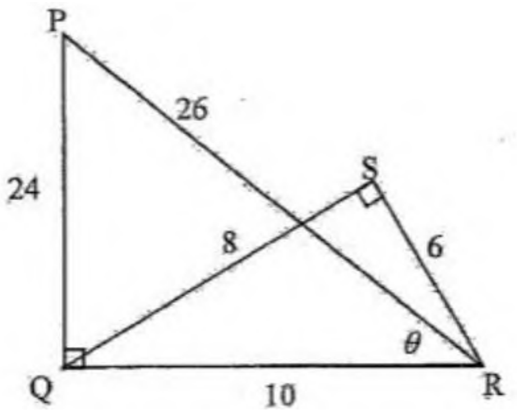
4.2 Calculate θ if $2\sin(\theta + 15°) = 1{,}462$ and $0° \leq \theta \leq 90°$ (3)

[12]

Question 5

ΔPQR and ΔSQR are right-angled triangles as shown in the diagram below.

PR = 26, PQ = 24, QS = 8, SR = 6, QR = 10 and $P\hat{Q}R = \theta$.



5.1 Refer to the diagram above and, WITHOUT using a calculator write down the value of

5.1.1 $\tan \hat{P}$ (1)

5.1.2 $\sin S\hat{Q}R$ (1)

5.1.3 $\cos \theta$ (1)

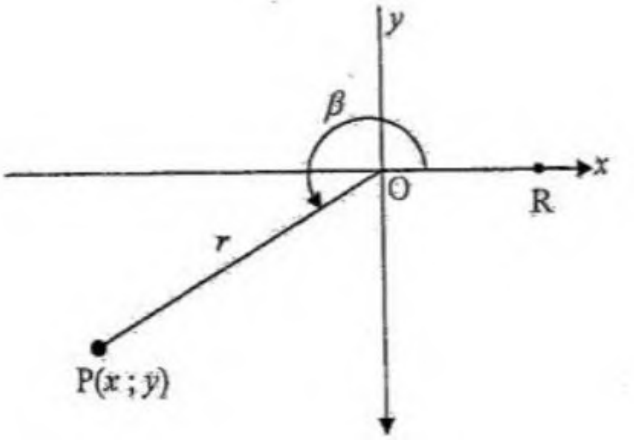
5.1.4 $\dfrac{1}{\cos S\hat{R}Q}$ (1)

5.2 WITHOUT using a calculator, determine the value of $\frac{\sin Q\hat{R}S}{\tan \theta}$ (3)

[7]

Question 6

6.1 In the diagram below, P($x; y$) is a point in the third quadrant. $R\hat{O}P = \beta$ and $17\cos\beta + 15 = 0$.



6.1.1 Write down the values of x, y, and r. (4)

6.1.2 WITHOUT using a calculator, determine the value of

(a) $\sin\beta$ (1)

(b) $\cos^2 30° . \tan\beta$ (3)

6.1.3 Calculate the size of $R\hat{O}P$ correct to TWO decimal places. (2)

6.2 In each of the following equations, solve for x, where $0° \leq x \leq 90°$. Give your answer correct to TWO decimal places

6.2.1 $\tan x = 2{,}22$ (2)

6.2.2 $\cos(x + 10°) = 0{,}179$ (3)

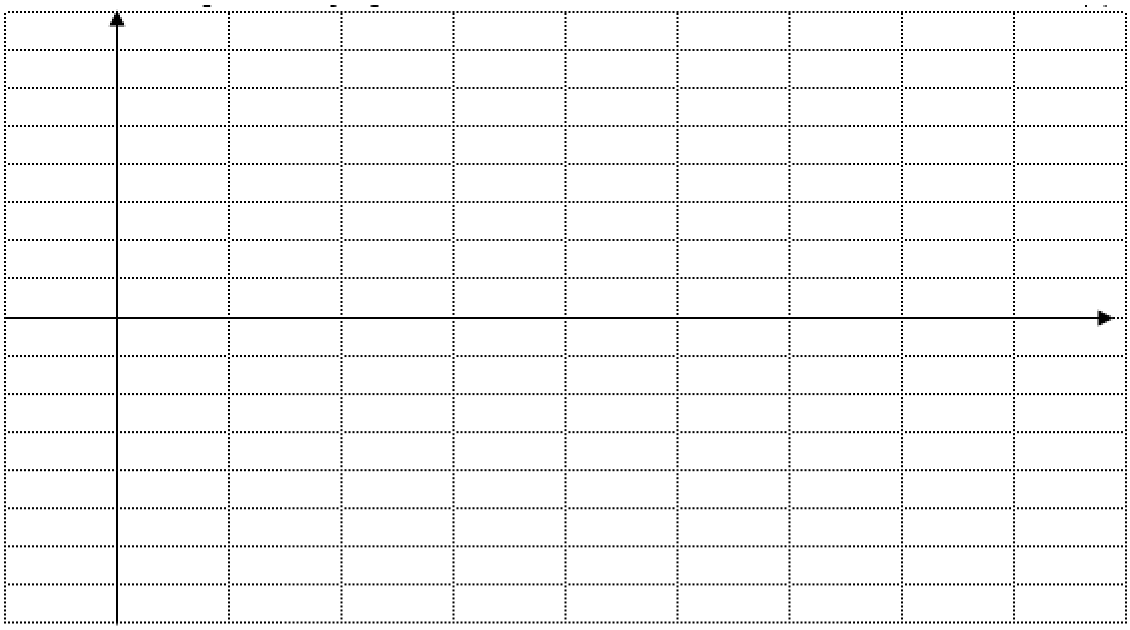
6.2.3 $\frac{\sin x}{0{,}2} - 2 = 1{,}24$ (3)

[18]

Question 7

7.1 Consider the function $f(x) = -3\tan x$.

7.1.1 Sketch, on the grid below, the graph of f for $0° \leq x \leq 360°$. Clearly show all intercepts and asymptotes. (3)

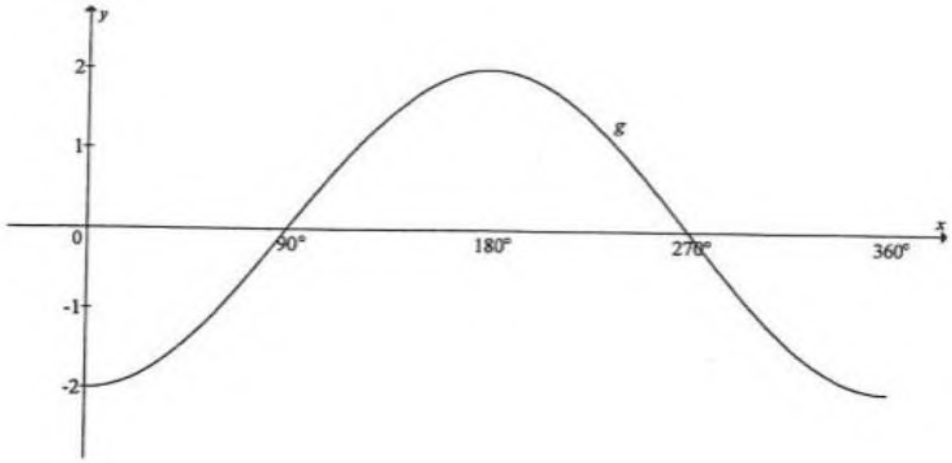


7.1.2 Hence, or otherwise, write down the:

(a) Period of f. (1)

(b) Equation of h, if h is the reflection of f about the x-axis. (2)

7.2 Sketched below is the graph of $g(x) = a\sin bx$.



7.2.1 Write down the values of a and b. (2)

7.2.2 Using the graph, determine the values of x for which $g(x) > 0$. (1)

7.2.3 Determine the range of h if h is the image of g if g is shifted down TWO units. (2)

7.2.4 Determine, using the graph, the value of:

$-2(\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 358° + \cos 359° + \cos 360°)$ (2)

[13]

Question 8

8.1 Use the calculator to evaluate the following expressions correct to TWO decimal places.

8.1.1 $\dfrac{\tan 70°}{3} + \sqrt{\cos^2 85°}$ (1)

8.1.2 $5.\text{cosec}\, x$ if $x = 99°$ (1)

8.2 Simplify the following WITHOUT the use of a calculator

$$\frac{\tan^2 30°.\sec 45°}{\frac{1}{\sin^2 60°}}$$ (4)

8.3 If $\tan\theta = \dfrac{8}{6}$, $\theta \in [180°;\ 360°]$, use a diagram to calculate the following:

$$\sin\theta - \cos\theta$$ (4)

8.4 If $\sin\alpha = p$, where $0° \leq p \leq 90°$, write the following in terms of p.

8.4.1 $\cos^2\alpha$ (2)

8.4.2 $\tan\alpha$ (1)

8.5 Solve for x, correct to 2 decimal places, for $0° \leq x \leq 90°$:

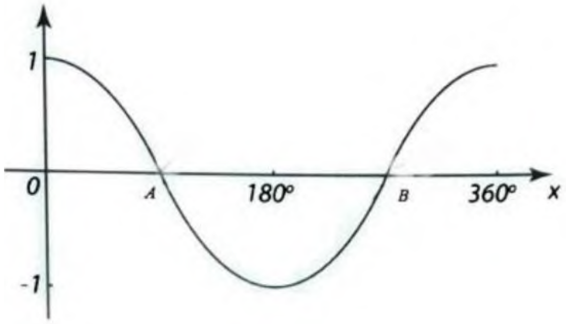
8.5.1 $\sin 2x = 0{,}682$ (2)

8.5.2 $\sin(x - 40°) = 0{,}58$ (2)

[17]

Question 9

9.1 The graph of f is drawn below:



9.1.1 Determine the equation of f. (1)

9.1.2 Write down the equation of A and B. (2)

9.1.3 State the domain and the range of f. (2)

9.1.4 Write down the amplitude and period of f. (2)

9.1.5 Write down the equation of $g(x)$ if $g(x)$ if the graph of f reflected across the x-axis shifted 2 units down. (2)

9.2 On the same system of axis sketch the graphs of:

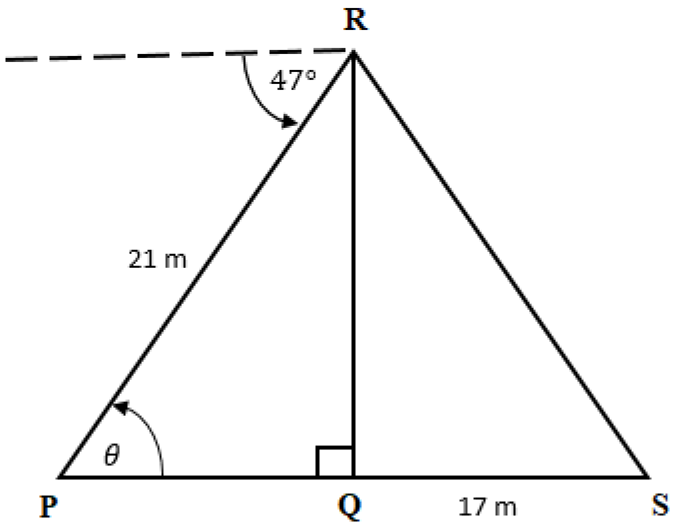
$$f(x) = \sin x - 1 \text{ and } g(x) = 2\cos x, \text{ for } x \in [0°; 360°].$$

Clearly indicate the x and y intercepts. (6)

[15]

Question 10

RQ is a vertical pole. The foot of the pole, Q, is on the same horizontal plane as P and S. The pole is anchored by wire cables RS and RP. The angle of depression from the top of the pole to the point P is 47°. PR is 21 m and QS is 17 m. $R\hat{P}Q = \theta$.



10.1 Write down the size of θ. (1)

10.2 Calculate the length of RQ. (3)

10.3 Hence. Calculate the size of $\hat{S}$. (2)

10.4 If P, Q and S lie in a straight line, how far apart are the anchors of the cables? (4)

[10]

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4. Examination Aid Mathematics Gr. 10 Papers with Solutions
5. Classroom Mathematics Gr. 10
6. Mathematics Handbook and Study Guide

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7\. DBE Gr. 10 Question Papers

8\. Maths for Africa Study Guide by Katie Müller