education

Department:
Education
PROVINCE OF KWAZULU-NATAL

JUST IN TIME MATERIAL
GRADE 10

CURRICULUM GRADES 10 – 12 DIRECTORATE
TERM 2 – 2020

This document has been compiled by KZN Mathematics -FET advisors and Top teachers in the Province. It seeks to unpack the content and give more guidance to the teachers. Please note that this document is intended to supplement the Text book and not replace it!

TABLE OF CONTENTS

PROBABILITY

DATES	CURRICULUM STATEMENT	% COM-PLETED
04/08–18/08 (10 days)	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: • $P(A or B) = P(A) + P(B) - P(A and B)$ B); • A and B are mutually exclusive if $P(A and B) = 0$; • A and B are complementary if they are mutually exclusive and $P(A) + P(B) = 1$; Then: $P(B) = P(not A) = 1 - P(A)$.	83%

CAPS EXAM GUIDELINE WEIGHTING FOR FINAL EXAMINATION

PROBABILITY	15 ± 3 marks out of 100 marks in P1

1. SUB –TOPIC FROM ATP

- Probability of event
- Venn diagram
- Addition rule
- Mutual exclusive events
- Complementary events

2. METHODOLOGY
HOW WILL YOU TEACH PROBABILITY WITH SPECIFIC ATTENTION TO THE SUBTOPICS

Probability is an extent to which an event is likely to occur or not. It is measured by the ratio of favorable cases to the whole number of cases (sample space).

$$Probability = \frac{Total\ number\ of\ favorable\ outcomes}{Sample\ space}$$

$$P(E) = \frac{n(E)}{n(S)}$$

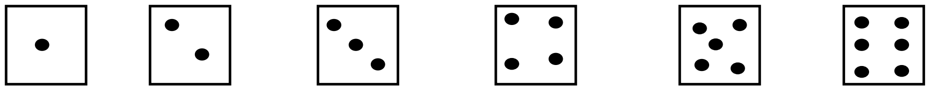
Sample space is a set of all possible outcomes in an experiment. e.g

1. In a coin there is a Head (H) and a Tail (T), which means there are only TWO possible outcomes. The probability of TOSSING a coin and land with a HEAD is $\frac{1}{2}$

$$P(H) = \frac{n(H)}{n(S)}$$

$$= \frac{1}{2}$$

2. In a die there is 1, 2, 3, 4, 5 and 6 that means there are ONLY SIX possible outcomes.



The probability of ROLLING a DIE and land with a 2 is $\frac{1}{6}$

$$P(2) = \frac{n(2)}{n(S)}$$

$$P(2) = \frac{1}{6}$$

Revise the following important concepts with learners:

A. Multiples
B. Even numbers
C. Odd numbers
D. Factors
E. Prime numbers
F. Perfect Square

The probability of ROLLING a DIE and land with:

A. Multiples of 3 $\{3 \text{ and } 6\}$ $= \frac{2}{6}$

B. Even number $\{2, 4 \text{ and } 6\}$ $= \frac{3}{6}$

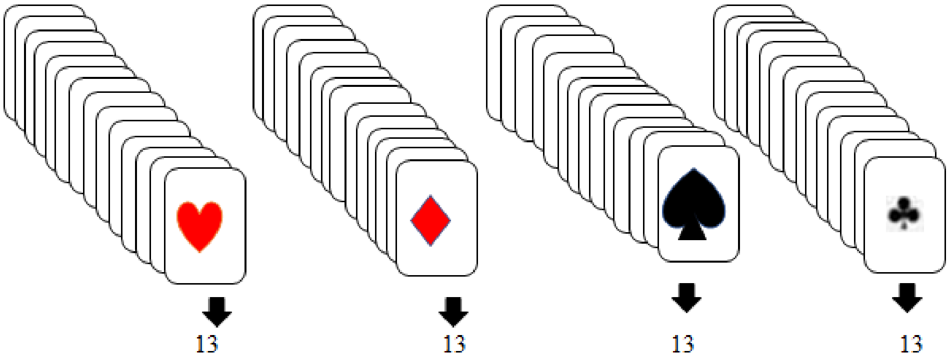
C. Odd number $\{1, 3 \text{ and } 5\}$ $= \frac{3}{6}$

D. Factor of 8 $\{1,2 \text{ and } 4\}$ $= \frac{3}{6}$

E. Prime numbers $\{2, 3 \text{ and } 5\}$ $= \frac{3}{6}$

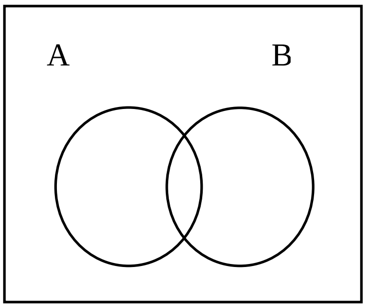
F. Perfect Square {1 and 4} $= \frac{2}{6}$

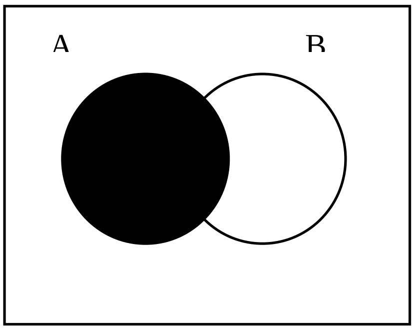
1. In a pack of cards there are 52 cards, therefore there are 52 possible outcomes



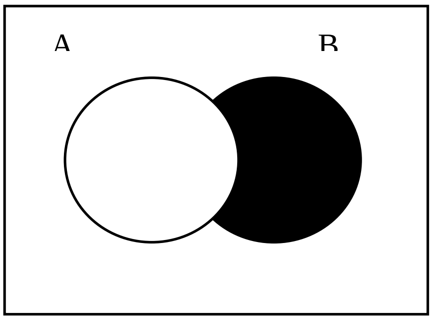
A learner must be able to write the sets of the above mentioned numbers in a given set of numbers.

Venn diagrams with TWO events

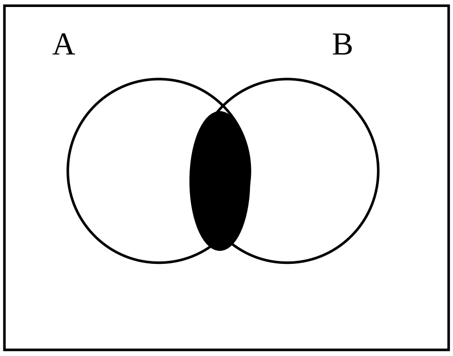




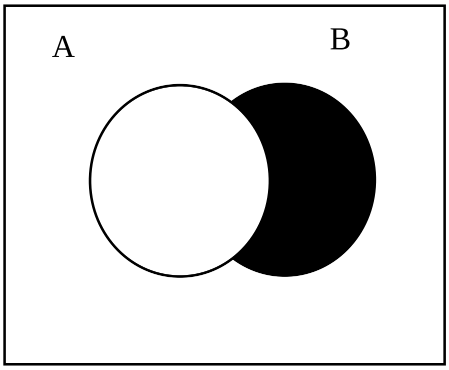
n(A)



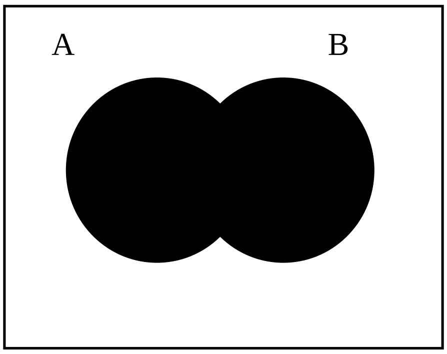
n(B)



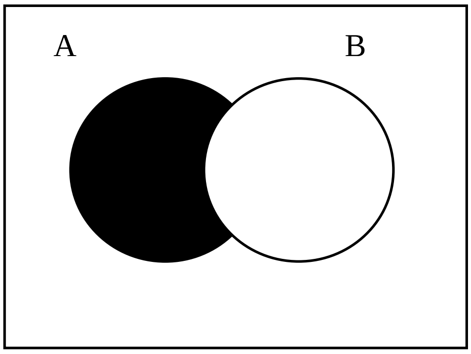
n(A and B)



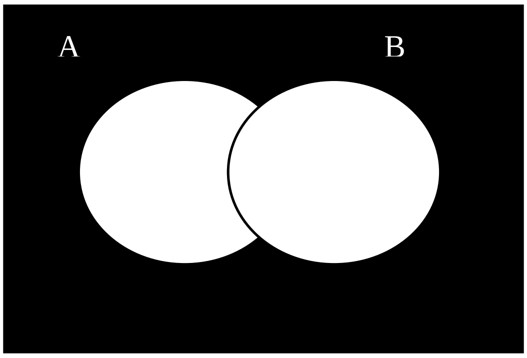
n(B only)



n(A or B)



n (A only)



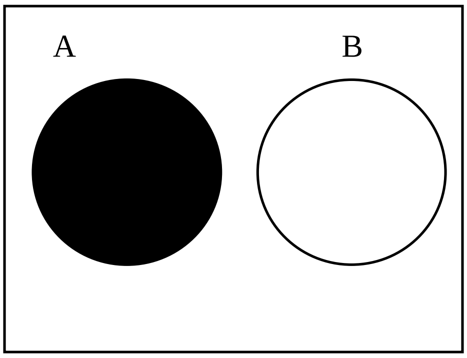
n (A or B)’

ADDITION RULE

$$\frac{n(A\ or\ B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\ and\ B)}{n(S)}$$

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

MUTUALLY EXCLUSIVE



- Events cannot occur at the same time
- There is no intersection
- $n(A\ and\ B) = 0$ Therefore $P(A\ and\ B) = 0$
- $\therefore P(A\ or\ B) = P(A) + P(B)$

COMPLEMENTARY EVENTS

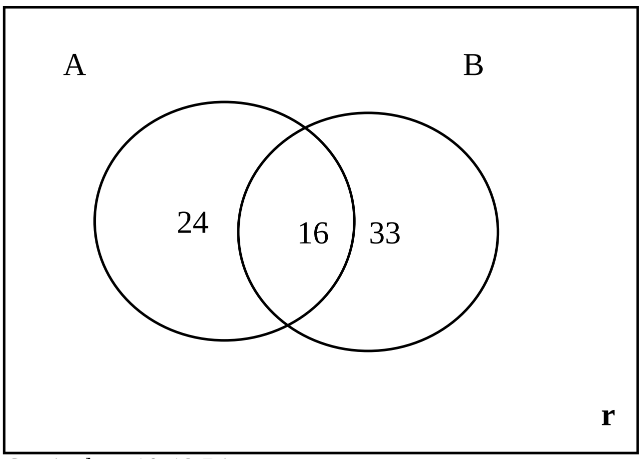
$P(A) + P(B) = 1$

$P(A) + P(not\ A) = 1$

$P(A) = 1 - P(not\ A)$

$P(notA) = 1 - P(A)$

Worked example
Events A and B are represented in the diagram below



N(S) = 80

1. Calculate the value of r.
2. Determine the following:

2.1 $P(A \text{ and } B)$
2.2 $P(A \text{ only})$
2.3 $P(A)$
2.4 $P(A \text{ or } B)$
2.5 $P(B \text{ only})$
2.6 $P(B)$
2.7 $P(A \text{ or } B)^{/}$
2.8 $P(\ B)^{/}$

3 Are the events A and B Mutually exclusive? Give reason for your answer?
4 Are the events A and B complementary? Give reason for your answer?

Solutions

1. $r + 16 + 24 + 33 = 80$ Therefore $r = 7$

2.1 $P(A \text{ and } B) = \frac{16}{80}$

2.2 $P(A \text{ only}) = \frac{24}{80}$

2.3 $P(A) = \frac{40}{80}$

2.4 $P(A \text{ or } B) = \frac{73}{80}$

2.5 $P(B \text{ only}) = \frac{33}{80}$

2.6 $P(B) = \frac{49}{80}$

2.7 $P(A \text{ or } B)^{/} = \frac{7}{80}$

2.8 $P(\ B)^{/} = \frac{31}{80}$

Activities

A group of 100 people have been surveyed to see who uses dental floss and who uses mouthwash. 83 people surveyed use mouthwash and 34 people use dental floss. All 100 people surveyed use either mouthwash, dental floss or both.

1.1 How many people use both? (2)

1.2 Draw a Venn diagram to represent the results of the survey. (3)

2. Your friend, Frank, has just started learning about probability theory. Help him to understand the principles by answering the following questions.

2.1 Frank has been watching the lotto draw on TV. He has noticed that certain numbers tend to come up more often than others do. He tells you that this means that he has a better chance of winning by picking numbers that have not come up yet. Do you agree? Give a reason for your answer. (2)

2.2 Frank has also read that the probability of being struck by lightning is smaller than 1 in a million. He has decided to take a dare and wear a lightning conductor on his head outside, during a thunderstorm. He things, because there is such a remote chance of being struck by lightning, he is basically safe. Do you agree with Frank's reasoning? Give a reason for your answer. (3)

3. A friend of yours is struggling to understand probability. She has asked you the following questions.

3.1 The probability of getting heads when I flip a coin is $\frac{1}{2}$. I decided to test this, so I flipped a coin three times. I got tails each time, so surely this means that the probability of getting heads is 0! Please explain what is going on. (3)

3.2 If I play the lotto, I could either win or lose. This means that the probability of winning is $\frac{1}{2}$. I have bought many tickets, so why haven I not won yet? (2)

4. You are looking for your friend, Marius, who lives in a block of 30 flats. You are not sure what number his flat is, so you do not know which number to ring at the front gate.

4.1 What is the probability of getting either Marius' flat or one of his next door neighbours, if you ring a number at random? (3)

4.2 You have remembered that his flat number has a 6 in it. What is the probability that you will be able to guess his flat number by considering this information? (2)

5. Answer the following questions about mutually exclusive events.

5.1 What are mutually exclusive events? Give a simple example of two events that are mutually exclusive. (2)

5.2 If A and B are mutually exclusive and complementary events, then what is P(A) +P(B)? (1)

5.3 If P(A) = P(B) = 0,6 and A and B are complementary, prove that A and B are not mutually exclusive. (2)

6. In many movies, we see people play "Russian Roulette", where a person puts one bullet in a gun (the gun has six chambers), spins the chambers, puts the gun to their head and pulls the trigger.

6.1 What is the probability that a person who plays Russian Roulette will shoot himself? (2)

6.2 How many times is it safe to play Russian Roulette? Give a reason for your answer. (3)

7. A friend of yours has asked for some help with probability. You are required to answer the following questions.

7.1 What are complementary events? (3)

7.2 What is the formula for probability? (2)

8. You have entered an SMS competition. You have to choose a letter of the alphabet and SMS it to the company.

8.1 What is the probability that you will choose the winning letter? (2)

8.2 What is the probability that the winning letter will be a letter from the word COMPETITION? (3)

9. Determine which of the following statements are **true** and which are **false**. If a statement is false, briefly explain why.

9.1 If you flip a coin and get heads, then you will definitely get tails the next time you flip the coin because the probability is $\frac{1}{2}$. (2)

9.2 If you throw an ordinary, six-sided die a very large number of times, you will find that on average, all of the numbers appear equally often. (1)

9.3 If you understand probability well enough, you can predict the future. (2)

10. Answer the following questions relating to probability.

10.1 A and B are events that can both occur together. If you know that P(A or B) = 0,8 and P(A) = P(B) = 0,7, find P(A and B). (2)

10.2 If one event is three times as likely as another and the two events are complementary, find the probability of the less-likely event. (3)

11. A teacher has found that some of his learner are not studying and are guessing on their multiple choice tests. He has designed a new question format to help solve this problem. The teacher thinks that if there is a smaller chance of guessing the correct answer, learners will be encouraged to study harder. In a normal multiple-choice question, you can pick any one of a, b, c or d as your answer. In the new question format, you can pick up to three letters (so your answer could be b and c, for example, or even a, b and d). Your friend, Bob, is planning to guess on his test. Help him figure out his chances of success by answering the following questions.

11.1 Write down the sample space for the new format test questions. (3)

11.2 What is the probability of guessing the correct answer? (2)

11.3 If a test contains 30 questions, how many would you expect Bob to get right if he guessed every answer? (2)

11.4 What advice would you give Bob? (1)

11.5 Bob knows that the answer to a question is two letters (e.g. a and b), but he does not know which two letters. What is the probability that he will get the question right by guessing? (2)

12. Sipho is an engineer. He has recently designed a new computer chip. At the moment, two types of fault can occur: from a power surge or from overheating. Sipho has calculated P(power surge) = 0,05 and P(overheating) = 0,12. He has also found that a quarter of the times that overheating happens, it is because there has also been a power surge.

12.1 What is the probability of overheating and a power surge occurring? (2)

12.2 Determine the probability of a fault occurring. (4)

12.3 What is the probability of no fault occurring with the chip? (2)

12.4 What is the probability of a power surge but no overheating occurring? (2)

13. For three complementary events, A, B and C, you are given the following information:

P(A and B) = 0,2

P(A and C) = 0

P(B and C) = 0,3

P(A) = 0,55

P(C) = 0,4

13.1 Draw a Venn diagram based on the information given above. (6)

Use your diagram to determine the following probabilities:

13.2 P(B only) (2)

13.3 P(A or B not C) (2)

14. The numbers from 1 to 20 are written on cards and placed in a box. You choose a number out of the box at random.

14.1 What is the probability that you choose a number greater than 20? (1)

14.2 What is the probability that you will choose a number larger than 15? (2)

14.3 What is the probability that you will choose a multiple of 3? (2)

You choose the number 8. You keep the card in your hand and choose another card out of the box.

14.4 What is the probability of choosing 3? (1)

14.5 What is the probability of choosing an even number? (2)

14.6 What is the probability of choosing a number that will result in the sum of your two numbers being greater than 15? (2)

15. You are on a TV game show and hope to win a prize. There are two boxes, each containing coloured balls. You have to choose a ball out of each box. If you get two balls of the same colour, you win a prize. If you get two red balls, you get a special bonus prize.

- One box contains a red ball, a blue ball and a green ball.
- The second box contains a red ball, a blue ball and a yellow ball.

15.1 Write down the sample space. (2)

15.2 What is the probability of winning the special bonus prize? (2)

15.3 What is the probability of winning any prize? (2)

15.4 What is the probability of not winning anything? (2)

15.5 If all the balls were put in one box instead of being in two separate boxes, would the game be any different? Briefly explain your answer. (2)

16. You have bought a box of chocolates as a gift for someone special. There are three types of chocolate: white, milk and dark, and three types of filling: almond, praline and plain. The box of chocolates contains one of each possible variety (i.e. combination of chocolate and filling).

16.1 Write down the sample space for the chocolates. (2)

16.2 What is the probability of picking a chocolate with praline filling from the box? (2)

16.3 What is the probability of picking a white chocolate from the box? (2)

16.4 What is the probability of picking a milk chocolate with either almond or praline filling? (2)

16.5 What is the probability of not picking a chocolate with almond filling? (2)

17. In a small town of 1 500 people, there are two main banks, Acorn Savings and Nest Egg Bank. Most of the people in the town bank with one or both of these banks. 600 people bank with Acorn Savings and 950 bank with Nest Egg Bank. 235 people in the town do not use either Acorn Savings or Nest Egg Bank.

17.1 Draw a Venn diagram to illustrate the information given above. (4)

17.2 What is the probability that a person from the town uses both banks? (2)

17.3 What is the probability that a person from the town does not use Nest Egg Bank? (2)

17.4 What is the probability that a person from the town uses Acorn Savings but not Nest Egg Bank? (2)

18. Determine which of the following statements are **true** and which are **false**. If the statement is false, explain why it is so.

18.1 Only 3 people in your class scored over 80 % for the last test. This means that, because there are 40 people in your class, the probability that you get an A for the next test is $\frac{3}{40}$. (2)

18.2 If P(A and B) = 0, then the events are mutually exclusive. (2)

18.3 Probability is useless because we can never be sure of what is going to happen. (2)

18.4 All events are equally likely, because either something will happen or it will not. (2)

18.5 One in every 50 jellybeans made by a certain company is purple. This means that if you buy a bag containing 60 jellybeans, you will definitely get a purple one. (2)

B.QUESTIONS FROM PAST EXAM PAPERS

Question 1

1.1 A bag contains 7 red marbles and 5 green marbles. One marble is drawn out of the bag at random.

Calculate the probability that it is:

1.1.1 a red marble (1)

1.1.2 an orange marble (1)

1.1.3 a red or a green marble (1)

1.1.4 not a red marble (1)

Question 2

2.2 There are 120 grade 10 learners at a school. 55 learners take Mathematics and 80 learners take Life Sciences. There are 25 learners who do not do Mathematics or Life Sciences and the number of learners who take both Mathematics and Life Sciences = x.

2.2.1 Represent this information in a Venn diagram. Let M = {learners who take Mathematics}, LS = {learners who take Life Sciences}. (3)

2.2.2 Calculate the value of x (2)

Use the Venn Diagram to calculate the probability that a randomly chosen learner:

2.2.3 takes Mathematics only (2)

2.2.4 takes Mathematics and Life Sciences (2)

2.2.5 takes Mathematics or Life Sciences (2)

[15]

Question 3

3.1 The letters of the word PARALLEL were put into a hat. Determine, in simplest form, the probability that:

3.1.1 the letter P is chosen. (1)

3.1.2 the letter A is chosen. (1)

3.2 Cards numbered from 1 to 10 were put into a container.

3.2.1 Write down the $n(S)$. (1)

3.2.2 List the set to represent the event A, of drawing a card which are the factors of 18. (1)

3.2.3 List the set to represent the event B, of drawing a card which is an even number. (1)

3.2.4 Draw a Venn diagram to illustrate the above information. (3)

3.2.5 Calculate, in simplest form, the $\mathrm{P}(\mathrm{A} \text{ and } \mathrm{B})$. (1)

3.2.6 Calculate, in simplest form, the $\mathrm{P}(\mathrm{A} \text{ or } \mathrm{B})$. (1)

3.2.7 Calculate, in simplest form, the $\mathrm{P}(\text{"not B" and A})$. (1)

3.3 Learners at the tuck shop were asked if they bought chips, pies or both. 100 learners were questioned and the results were as follows:

- 75 bought chips
- 48 bought pies
- 9 did not buy chips or pies.

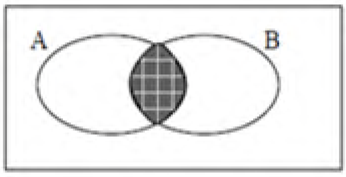
3.3.1 Draw a Venn diagram to illustrate the above information. (3)

3.3.2 Calculate how many learners bought both chips and pies. (2)
[16]

Question 4

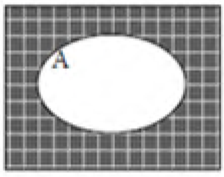
What expression BEST represents the shaded area of the following Venn diagrams?

4.1



(1)

4.2



(1)

4.3 State which of the following sets of events is mutually exclusive:

A Event 1: The learners in Grade 10 in the swimming team
Event 2: The learners in Grade 10 in the debating team

C Event 1: The learners who take Mathematics in Grade 10
Event 2: The learners who take Physical Sciences in Grade 10

4.4 In a class of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

4.5 How many learners in the class are right-handed and do NOT play soccer? (1)

4.6 Draw a Venn diagram to represent the above information. (4)

4.7 **Determine the probability that a learner is**

(a) Left-handed or plays soccer (2)

(b) Right-handed and plays soccer (2)

<u>Question 5</u>

Two events, A and B, are mutually exclusive.

- $P(B') = 0,4$
- $P(A \cup B) = 0,7$

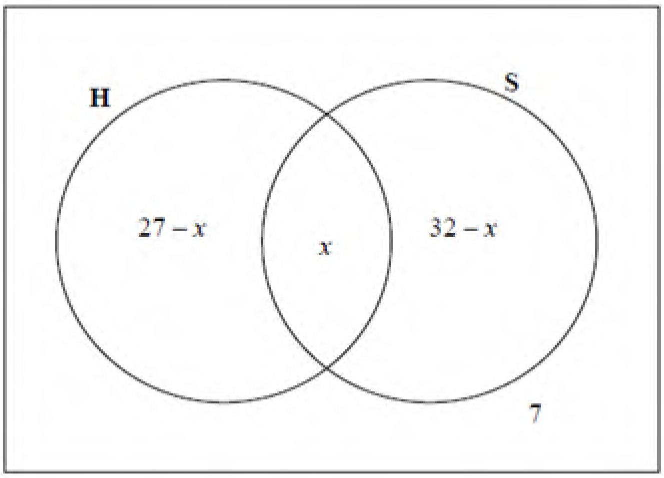
Calculate $P(A)$. (4)

Question 6

In a certain class of 42 boys:

- 27 play hockey (H)
- 32 play soccer (S)
- 7 do not play hockey or soccer
- An unknown number (x) play both hockey and soccer

The information is represented in the Venn diagram below.



6.1 Calculate the value of x. **(1)**

6.2 If a boy from the class is chosen at random, calculate the probability that he:

(a) Does not play hockey or soccer **(4)**

(b) Plays only soccer

6.3 A bag contains 3 blue balls and x yellow balls.

(a) Write down the total number of balls in the bag. **(1)**

(b) If a ball is drawn from the bag, write down the probability that it is blue. **(1)**

6.4 Complete the following statement:

If A and B are two mutually exclusive events, then
P (A and B) = … **(1)**

6.5 Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7.

Calculate P (A or B). **(3)**

Question 7

Customers at a supermarket were surveyed about their purchases of bread (B) and Milk (M).

- 80 customers were surveyed
- 54 bought bread
- 42 bought milk
- 73 bought bread or milk

Represent the given information as a Venn diagram
Show all relevant calculations and working out. (4)

Question 8

8.1 At a certain school there are 64 boys in Grade 10. Their sport preferences are indicated below:

- 24 boys play soccer
- 28 boys play rugby
- 10 boys play both soccer and rugby
- 22 boys do not play soccer or rugby

8.1.1 Represent the information above in a Venn diagram.

8.1.2 Calculate the probability that a Grade 10 boy at the school, selected at random, plays:

(a) Soccer and rugby

(b) Soccer or rugby

8.1.3 Are the events a Grade 10 boy plays soccer at the school and a Grade 10 boy plays rugby at the school, mutually exclusive? Justify your answer.

8.2 One morning Samuel conducted a survey in his residential area to establish how many passengers, excluding the driver, travel in a car. The results are shown in the table below:

Number of passengers, excluding the driver	0	1	2	3	4
Number of cars	7	11	6	5	1

Calculate the probability that, excluding the driver, there are more than two passengers in a car.

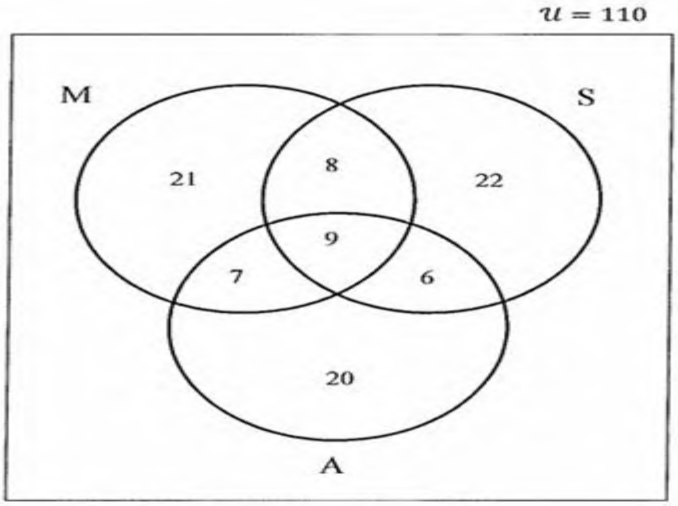
8.3 If you throw two dice at the same time, the probability that a six will be shown on one of the dice is $\frac{10}{36}$ and the probability that a six will be shown on both the dice, is $\frac{1}{36}$. What is the probability that a six will NOT show on either of the dice when you throw two dice at the same time?

Question 9 [Enrichment Only- not examinable in grade 10]

At a certain school, pupils doing the subjects

- Mathematics (M)
- Physical Science (S)
- Accounting (A)

were surveyed. The following Venn diagram represents the results of the survey :



9.1 Calculate the probability that a pupil chosen at random does:

9.1.1 none of the three subject
9.1.2 Only one of the three subjects
9.1.3 At least one of the three subjects

9.1.4 At least two of the three subjects
9.1.5 Mathematics and Accounting ,but not Physical Science
9.1.6 $M \cup S'$

Question 10

Leaners in grade 10 were surveyed about their subject choices

- 150 were surveyed
- 125 did life Sciences
- 85 did science
- 55 did life science , but not science
- 10 did neither

Draw a Venn diagram depicting the above information. Let A be life Science and B be Science. (4)

Question 11

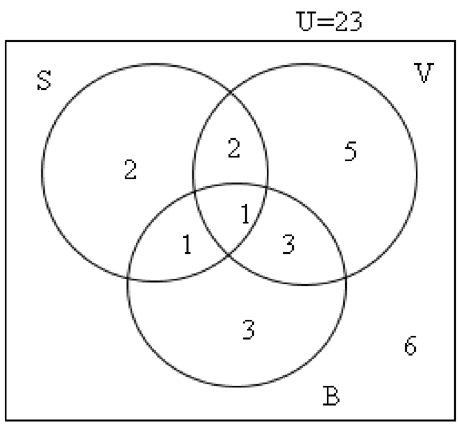
Two events A and B are mutually exclusive

- P(B') = 0.3
- P(A U B) = 0.8

Calculate P(A) (3)

Question 12 [For enrichment purposes]

The Venn diagram below represents the sports choices of 23 learners at a Sports Academy.Learners may choose to play soccer (S) , volleyball (V),baaseball (B) or any combination of three sports or neither.



Calculate the probability that a person chosen at random does:

12.1	only one sport	(1)
12.2	does all three sports	(1)
12.3	does any sport but baseball	(1)
12.4	does volleyball or soccer	(1)
12.5	does not play any sport	(1)

FUNCTIONS AND GRAPHS

DATES	CURRICULUM STATEMENT	% COM-PLETED
28/04 – 29/04 (2 days)	**1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value).** **Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations.** **Note that the graph defined by y = x should be known from Grade 9.**	**41%**
30/04 – 14/06 (10 days)	**2. Point by point plotting of basic graphs defined by** $y = x^2$, $y = \frac{1}{x}$ **and** $y = b^x$; $b > 0$ **and** $b \neq 1$ **to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable).** **3. Investigate the effect of** a **and** q **on the graphs defined by** $y = a.f(x) + q$, **where** $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ **and** $f(x) = b^x$, $b > 0$ **and** $b \neq 1$ **4. Sketch graphs, find the equations of given graphs and interpret graphs.** **Note: Sketching of the graphs must be based on the observation of number 3.**	**59%**
15/05 – 22/05 (6 days)	**1. Point by point plotting of basic graphs defined by** $y = \sin\theta$, $y = \cos\theta$ **and** $y = \tan\theta$ **for** $\theta \in [0^\circ; 360^\circ]$. **2. Study the effect of *a* and *q* on the graphs defined by** $y = a\sin\theta + q$; $y = a\cos\theta + q$ **and** $y = a\tan\theta + q$, **for** $\theta \in [0^\circ; 360^\circ]$. **3. Sketch graphs, find the equations of given graphs and interpret graphs.** **Note: Sketching of the graphs must be based on the observation of number 2 above.**	**63%**

JUNE COMMON TEST WEIGHTING

Functions and Graphs	30 ± 3 marks in P1
Trigonometry graphs	5± 3 marks in P2

1. SUB -TOPIC: LIST OF SUBTOPIC FROM ATP

- Linear function
- Quadratic function
- Exponential function
- Hyperbolic function
- Trigonometry functions

2. METHODOLOGY (HOW WILL YOU TEACH FUNCTIONS WITH SPECIFIC ATTENTION TO THE SUBTOPIC)

- Revise Grade 9 linear function $y = mx + c$, focus on the effect of m and c
- Also Grade 9 quadratic equation $y = ax^2$ and Discuss (range, domain, minimum & maximum turning point, symmetry line)
  Use table method to investigate the effect of q on the graphs, $y = x^2 + q$, by sketching these functions:
  - $f(x) = x^2$
  - $g(x) = x^2 - 1$
  - $h(x) = x^2 - 2$
  - $i(x) = x^2 + 1$
  - $j(x) = x^2 + 2$

  Write the conclusion about the effect of q on the above graphs.

Hyperbolic function

- Use a calculator table mode to sketch the following functions & investigate the effect of a on the graphs of the form, $f(x) = \frac{a}{x}$:
  - $g(x) = \frac{1}{x}$
  - $h(x) = \frac{2}{x}$
  - $i(x) = \frac{-1}{x}$
  - $j(x) = \frac{-2}{x}$

  Write anything you have observed about the changes caused by the values of a.
- What do you notice about the values $f(1)$?

Exponential function

- Use a calculator table mode to sketch the following functions & investigate the shape of the graphs of the form, $y = b^x$:
  - $f(x) = 2^x$
  - $g(x) = 3^x$
  - $h(x) = 5^x$
  - $m\ x = -2^x$

  Write anything you have observed about the changes caused by the values of b.
  What do you notice about the values $f(0)$?

- Trig graphs refer to term 1 JIT document

3. Definition of Terms

- Range is the set of all possible y-values
- Domain is the set of all possible x-values
- Asymptotes is the line that the graph of the function approaches
- Axis of symmetry is the line that divides the graph into two congruent halves

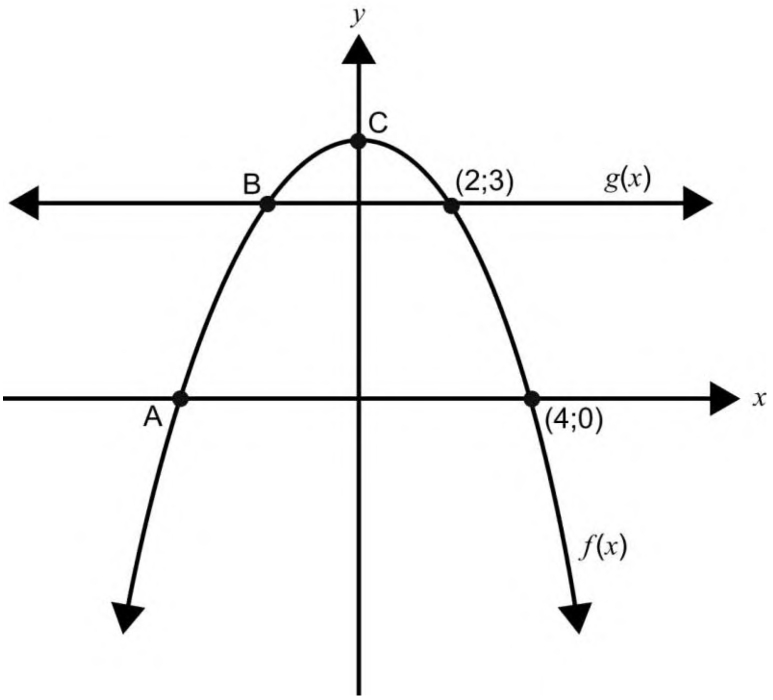
4. MISCONCEPTIONS

- Interchange a and b values.
- When simplifying, learners multiply values of a and b (exponential)
- Assuming lines are parallel when they are not told in the statement or shown in the diagram.

5. PRACTICE EXERCISE

Example 1

The graph below shows a parabola $f(x) = ax^2 + b$ and a straight line $g(x)$. $g(x)$ is parallel to the $x - axis$.



8.1 Write the coordinates of point:

a) A

b) B

8.2 Write down the equation of $g(x)$.

8.3 If point C is the reflection of point A about the line $y = -x$, find its coordinates.

8.4 If $f(x) - k$ has no x-intercepts, determine the value/s of k

8.5 Describe how $f(x)$ must be shifted so that it intersects $g(x)$ at only one x point.

8.6 Using the graph, determine for what value/s of x:

a) $f(x).g(x) < 0$

b) $f(x) - g(x) > 0$

Solutions:

8.1 a) A (-4; 0)

b) B (-2; 3)

8.2 $y = 3$

8.3 C (0; 4)

8. 4 k >4

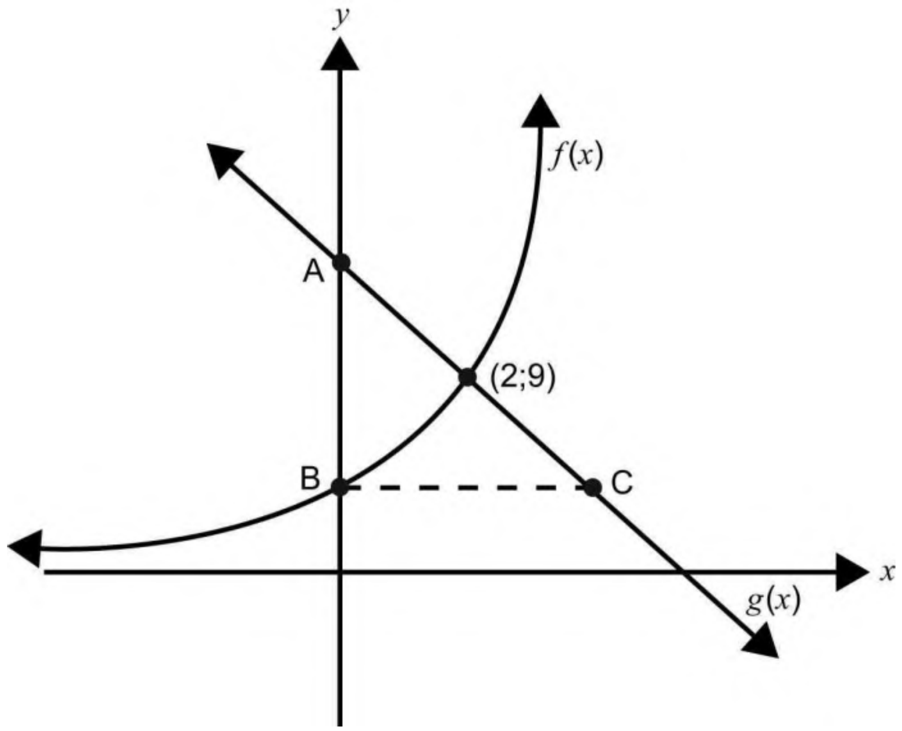
8.5 Shift graph of f 1 unit down

8.6 a) $x < -4$ or $x > 4$

b) $-2 < x < 2$

Example 2

Below are the graphs of $f(x) = a^x$ and $g(x) = -x + 11$. BC is perpendicular to the y-axis



2.1 Show that the value of a = 3.

2.2 Find the coordinates of A, B and C

2.3 What would the equation of $f(x)$ become if the graph was shifted so that it cut the y-axis at point A?

2.4 If $g(x)$ was reflected about the y-axis, what would its equation become?

2.5 Write down the range of $f(x)$.

2.6 Give the equation of the reflection of $f(x)$ about the x-axis.

Solutions

11.1 $f(x) = a^x$

$9 = a^2$

$3^2 = a^2$

$a = 3$

11.2 a) let $x = 0$

$g(0) = -(0) = 11$

$g(0) = 11$

$A\ (0; 11)$

b) let $x = 0$

$f(0) = 3^0$

$f(0) = 1$

$B\ (0; 1)$

c) $g(x) = -x + 11$

$1 = -x + 11$

$x = 10$

$C\ (10; 1)$

11.3 $f(x) = 3^x + 10$

11.4 $y = x + 11$

11.5 $y \in \mathbb{R}, y > 0$

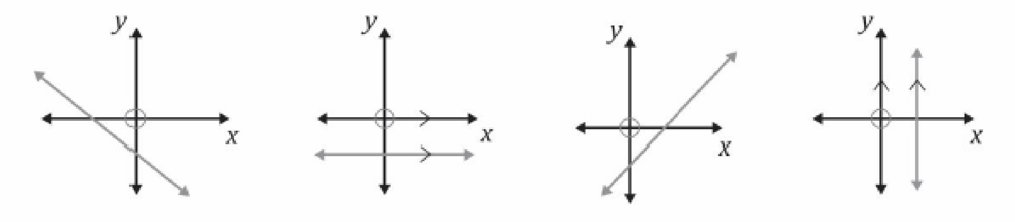
11.6 $y = -(3)^x$

A. STRAIGHT LINE

General representation or equation

$y = ax + q$ or $y = mx + x$. a or m is the gradient and q or c is the $y-$intercept

Also note the shape of the following linear functions



$a < 0$	$a = 0$	$a > 0$	a is undefined
$q < 0$	$y = q$	$q < 0$	there is no q-value

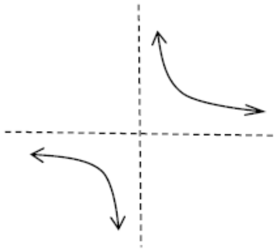
Domain and range is $x \in \Re$ and $y \in \Re$ respectively

B. HYPERBOLA

General representation or equation

$y=\frac{a}{x}$ or $xy=a$ $\qquad y=\frac{a}{x}+q$

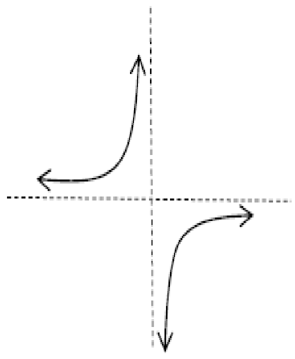
$a>0$



Dotted lines are asymptotes

- q is the vertical translation
- p is the horizontal translation

$a<0$



Dotted lines are asymptotes

- For $y=\frac{a}{x}$, $p=0$ and $q=0$. The **vertical** asymptote is $x=0$ and the **horizontal** asymptote is $y=0$. The **axis of symmetry** are $y=x$ (Positive) and $y=-x$ (Negative)

  Domain is $x\neq 0, x\in\Re$ and **Range** is $y\neq 0,\ y\in\Re$

- For $y=\frac{a}{x}+q$, $p=0$. The **vertical** asymptote is $x=0$ and the **horizontal** asymptote is $y=q$. The **axis of symmetry** are $y=x+q$ (Positive) and $y=-x+q$ (Negative).

  Domain is $x\neq 0,\ x\in\Re$ and **Range**, $y\neq q,\ y\in\Re$

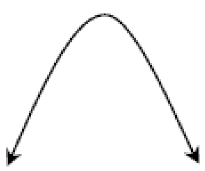
C. PARABOLA

General representation or Equation

$y=ax^2$ or $y=ax^2+q$

Important Deductions

for $a<0$



for $a>0$

- For $y = ax^2$, $p = 0$ and $q = 0$, the **turning point** is $(0;0)$ and **y-intercept** is $y = 0$

  The **domain** is $x \in \mathbb{R}$ and the **range** is $y \geq 0; y \in \mathbb{R}$ if $a > 0$ or $y \leq 0; y \in \mathbb{R}$ if $a < 0$

- For $y = ax^2 + q$, $p = 0$, the **turning point** is $(0;q)$ and **y-intercept** is $y = q$

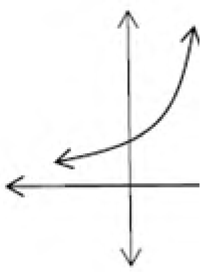
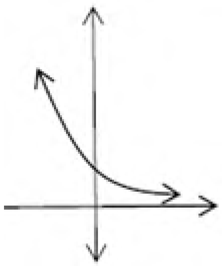
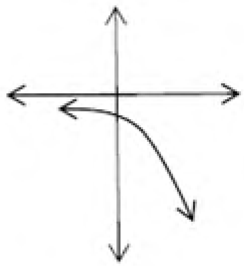
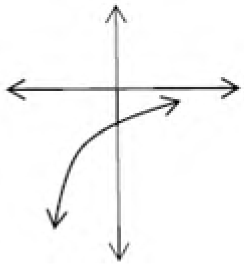
  The **domain** is $x \in \mathbb{R}$ and the **range** is $y \geq q; y \in \mathbb{R}$ if $a > 0$ or $y \leq 0; y \in \mathbb{R}$ if $a < 0$

D. EXPONENTIAL

General representation or Equation:

$y = ab^x$ or $y = ab^x + q$

The restriction is $b > 0;\ b \neq 1$

$a > 0$ *and* $b > 1$	$a > 0$ *and* $0 < b < 1$	$a < 0$ *and* $b > 1$	$a < 0$ *and* $0 < b < 1$
			
• For $y = ab^x$, the **asymptote** is $y = 0$ and the **y-intercept** is $y = a$			
• For $y = ab^x + q$, the **asymptote** is $y = q$ and **y-intercept** is $y = a + q$			

E. Basic Function concepts

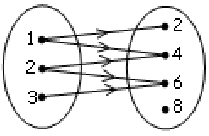
Relations and functions

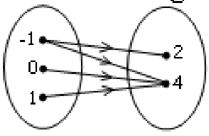
1 For each of the following relations, state the domain and the range:

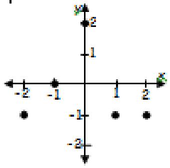
1.1.1 $\{(10;9);(8;7);(7;6);(6;5);(5;6)\}$

1.1.2 $\{(0;2);(0;3);(0;4)\}$

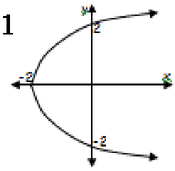
1.2 List the relation and state the domain and range:

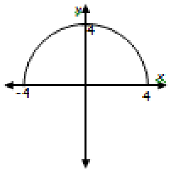
1.2.1 

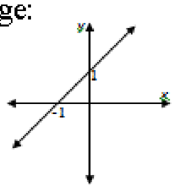
1.2.2 

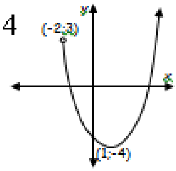
1.2.3 

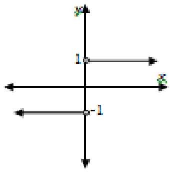
1.3 In each of the following cases determine the domain and range:

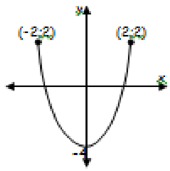
1.3.1 

1.3.2 

1.3.3 

1.3.4 

1.3.5 

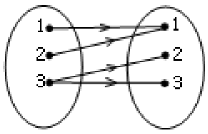
1.3.6 

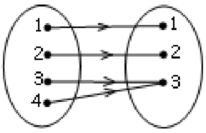
2 Determine whether each of the following is a function or not:

2.1 $\{(2;1);(2;2);(2;3)\}$

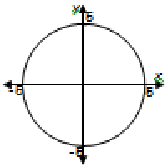
2.2 $\{(1;2);(2;2);(3;2)\}$

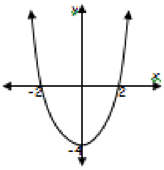
2.3 $\{(1;1);(2;2);(3;1);(3;2)\}$

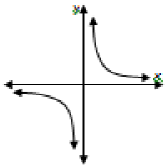
2.4 

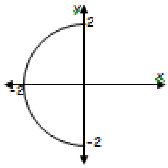
2.5 

2.6 $\{(x;y): y = x+1\}$

2.7 

2.8 

2.9 

2.10 

3 3.1 If $f(x) = 3x+3$, determine: 3.1.1 $f(-2)$ 3.1.2 $f(a)$ 3.1.3 m if $f(m) = 0$

3.2 Given that $g(x) = 4 - x^2$, determine: 3.2.1 $g(2)$ 3.2.2 $g(0)$ 3.2.3 k if $g(k) = 0$

3.3 If $p = \{(-1;2);(2;-1);(3;2);(-3;1);(5;6)\}$ determine:

3.3.1 $p(3)$ 3.3.2 $p(-1)$ 3.3.3 m if $p(m) = -1$ 3.3.4 m if $p(m) = 6$

3.4 If $j(x) = 2+x$, determine:

3.4.1 $j(2)$ 3.4.2 $j(-4)$ 3.4.3 $2j(2) - j(-4)$

3.5 If $k(x) = x^2$, determine:

3.5.1 $k(-1)$ 3.5.2 $k(3)$ 3.5.3 $k(a+1)$ 3.5.4 p if $k(p) = 0$

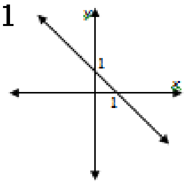
3.6 If $f(x) = 3x+4$, determine:

3.6.1 $f(\frac{1}{2})$ 3.6.2 $f(a)$ 3.6.3 $f(a+1)$ 3.6.4 x if $f(x) = -2$

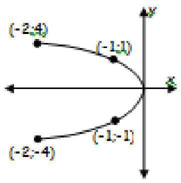
4 4.1 For each of the following determine:

- whether the relation is a function or not
- the domain
- the range

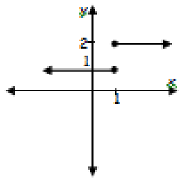
4.1.1



4.4.2



4.4.3



G. QUESTIONS FROM PAST EXAMINATION PAPERS

QUESTION 1

Consider the following functions: $f(x) = -\frac{3}{x} + 1$ and $g(x) = 2^x - 1$.

1.1 Sketch the graphs of *f* and g. Show all intercepts with the axes and asymptotes where applicable. **(6)**

1.2 Determine $f(-2)$. **(1)**

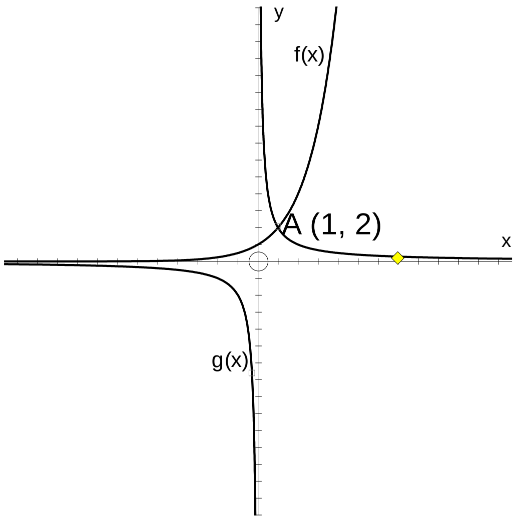
1.3 Solve for x if $g(x) = 7$. **(3)**

[10]

QUESTION 2

2.1 The graph of $f(x) = 2^x$ and $g(x) = \frac{k}{x}$ are represented alongside.

The graphs intersect at A.



Use the graph to answer the following questions

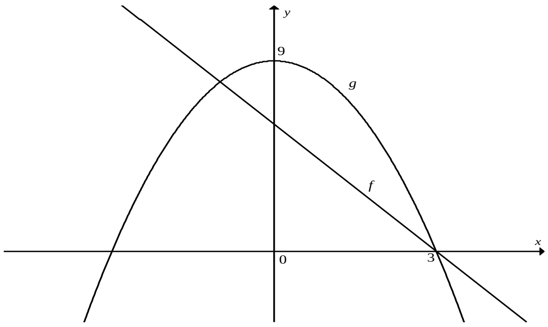
2.1.1 Determine the domain of f ? (1)

2.1.2 Give the equation of the asymptote of the graph of $y = 2^x - 2$. (1)

2.1.3 Determine the value of k ? (2)

QUESTION 3

Sketched below are the graphs of $f(x) = -2x + 6$ and $g(x) = ax^2 + q$.



3.1 Determine the values of a and q. (3)

3.2 Calculate the values of x for which $f(x) = g(x)$. (4)

3.3 Hence or otherwise, write down the values of x for which $g(x) > f(x)$. (2)

3.4 Write down the coordinates of the turning point of h if $h(x) = g(x) - 4$. (2)
[11]

QUESTION 4

4.1 Given the functions: $f(x) = -x^2 + 4$ and $g(x) = 2x + 4$

4.1.1 Draw f and g on the same system of axes. (6)

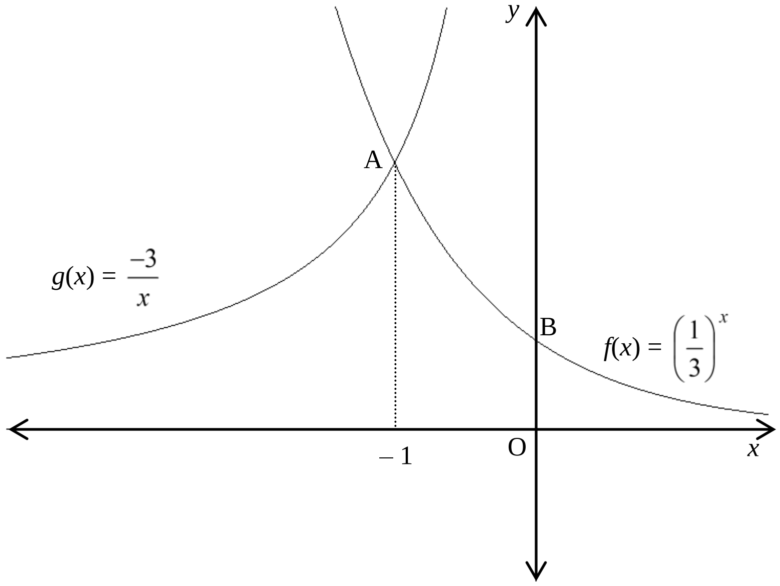
4.1.2 Use your graphs to solve for x if:

$f(x) \leq g(x)$ (3)

4.1.3 Give the equation of p, the reflection of f in the x-axis. (2)

QUESTION 5

The graphs of $f(x) = \left(\frac{1}{3}\right)^x$ and $g(x) = \frac{-3}{x}$ where $x < 0$ are represented below:



Answer the following questions by using the graphs:

5.1 Write down the co-ordinates of A and B. (2)

5.2 Write down the domain of $f(x)$. (1)

5.3 Write down the equation of the reflection of f in the y-axis. (2)

5.4 Give the equations of the asymptote of the graph of $y = g(x) + 2$. (2)
[18]

QUESTION 6

Given functions $f(x) = -x^2 + 9$ and $g(x) = 3x + 9$

6.1 Draw f and g on the same system of axes. (6)

6.2 Use your graphs to solve for x if:

6.2.1 $f(x) = g(x)$ (2)

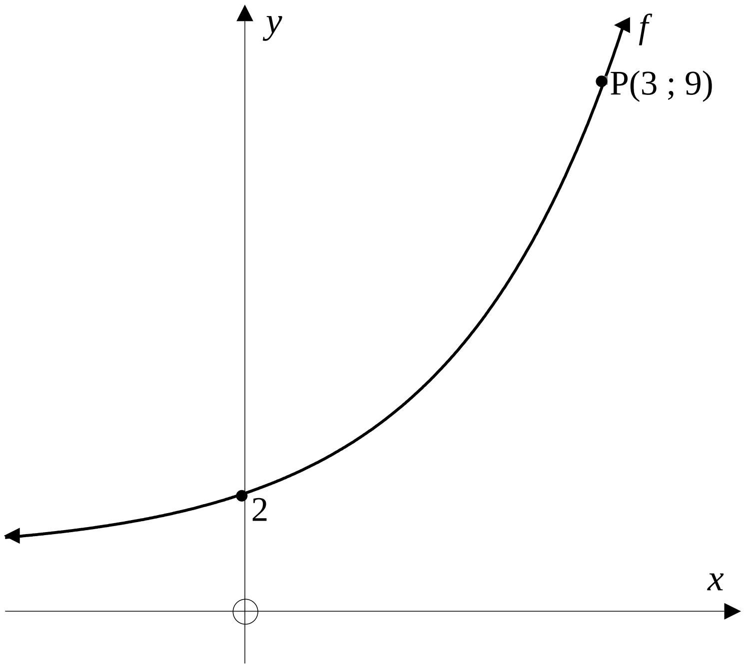
6.2.2 $f(x) > 0$ (2)

6.3 How does the graph of $h(x) = -x^2 - 9$ compare with the graph of $f(x)$? (2)

6.4 Give the equation of the reflection of f in the x–axis. (2)
[14]

QUESTION 7

In the sketch below, f is the graph of the function $y = a^x + q$.



7.1 Calculate the value of a and q if the point P(3; 9) lies on the graph. (5)

7.2 Write down the equation of the asymptote of f. (1)

7.3 Write down the equation of $h(x)$ if:

7.3.1 h is the reflection of f in the y-axis. (2)

7.3.2 h is the reflection of f in the x-axis. (2)

7.4 For which value(s) of x will $f(x) = 1\frac{1}{27}$? (3)

QUESTION 8

Given: $p(x) = \frac{4}{x} - 3$

8.1 Write down the equations of the asymptotes of p. (2)

8.2 Write down the domain of p. (2)

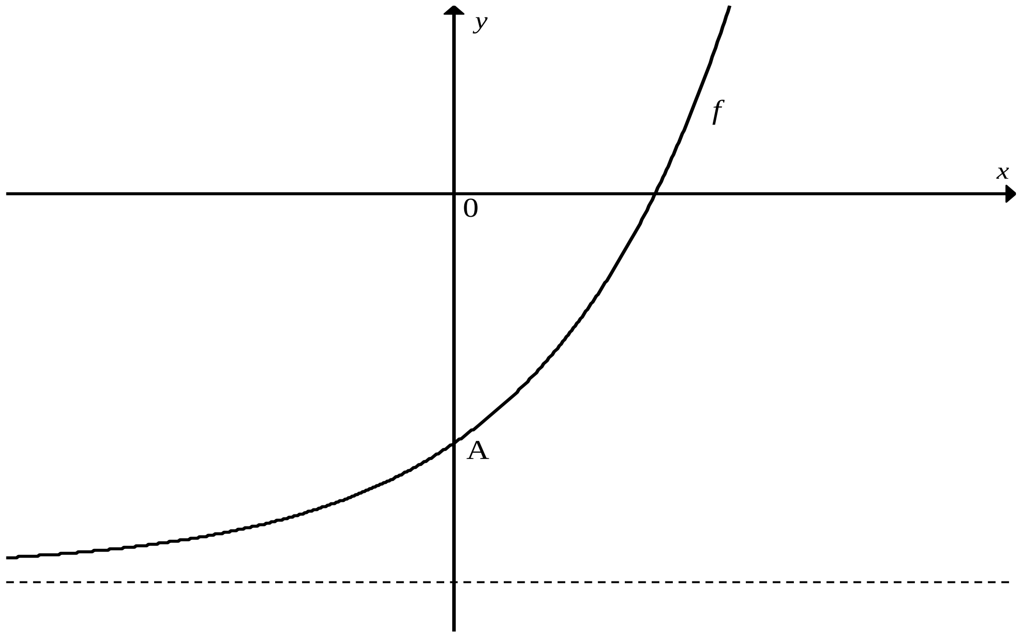
8.3 Calculate the x-intercepts of the graph p. (3)

8.4 Draw a neat sketch of p, showing clearly all intercepts with the axes and asymptotes. (4)

[11]

QUESTION 9

The graph of $f(x) = 3^x - 3$ is sketched below.



9.1 Write down the coordinates of A. (2)

9.2 Write down the range of *h* if $h(x) = -f(x)$. (2)

9.3 Write down the asymptote of *g* if $g(x) = f(-x)$. (1)
[5]

Question 10

The following functions are given: $f(x) = \left(\frac{1}{3}\right)^x$ where $x \in R$ and $g(x) = -\frac{3}{x}$ where $x < 0$

10.1 Sketch the graphs of f and g on the same set of axes.
Show all intercepts with the axes and asymptotes where applicable. (5)

10.2 Write down the range of k if $k(x) = f(x) - 2$. (2)

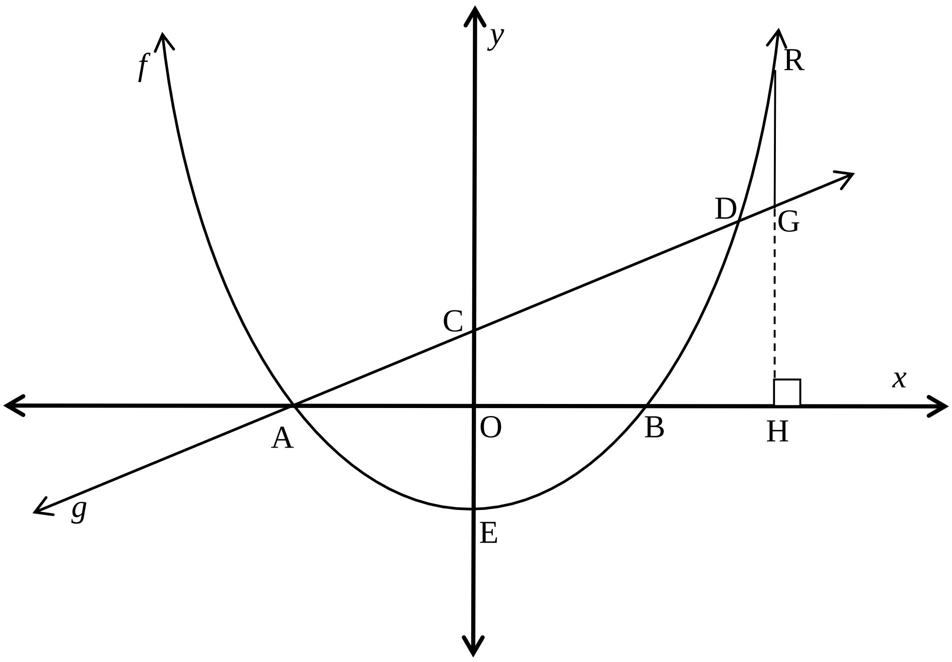
10.3 Write down the equations of the asymptotes of h if $h(x) = g(x) + 3$. (2)

10.4 Write down the values of x for which $f(x) \leq g(x)$. (2)
[11]

QUESTION 11

Sketched below are the graphs of $f(x) = x^2 - 1$ and $g(x) = x + 1$

The graph of f intersects the x-axis at A and B and the y-axis at E. The graph of g intersects the x-axis at A and the y-axis at C. f and g intersect at D.



Use the graphs and the information above to determine the following:

11.1 The coordinates of A and B. (3)

11.2 The coordinates of C. (1)

11.3 The coordinates of D. (5)

11.4 The range of f. (2)

11.5 The length of RG if OH is 6 units and F lies on f and G lies on g. (3)

11.6 The value(s) of x for which $g(x) > f(x)$. (2)

[16]

QUESTION 12

12.1 Draw a sketch graph of $y = \frac{4}{x} + 2$. Clearly indicate the asymptote(s) and the intercept(s) with the axes. (3)

12.2 Given the two functions $f(x) = -x^2 + 9$ and $g(x) = -x + 3$:

12.2.1 Sketch the graph of f and g on the same axes , showing the Co-ordinates of all the intercepts with the axes. (6)

12.2.2 Calculate the co-ordinates of the points at which $f(x) = g(x)$ (4)

12.2.3 Use your graph to write down the values of x for which $f(x) > 0$

12.2.4 Draw a dotted line on your graph, showing the graph of $y = \frac{1}{2} f(x)$. The intercepts on the axes must be shown. (3)

12.3 Determine the equation of a linear function $f(x) = mx + c$, if $f(0) = -7$ and $f(2) = 0$. (3)

REFERENCES

1. **Department of Basic Education –KZN Provincial grade 11 examination papers.**
2. **KZN -June 2011, Freestate- June 2014 ,KZN June 2008, KZN- June 2014,KZN-June 2013.**
3. **Mindset Learn www.mindset.co.za**
4. **KZN Revision document -2014**