



**education**

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Department:

Education

**PROVINCE OF KWAZULU-NATAL**

**CURRICULUM GRADE 10 -12 DIRECTORATE**

**NCS (CAPS)**

**LEARNER SUPPORT DOCUMENT**

**GRADE 10**

**MATHEMATICS**

**STEP AHEAD PROGRAMME**

**2021**

## PREFACE

This support document serves to assist Mathematics learners on how to deal with curriculum gaps and learning losses as a result of the impact of COVID-19 in 2020. It also captures the challenging topics in the Grade 10 -12 work. Activities should serve as a guide on how various topics are assessed at different cognitive levels and also preparing learners for informal and formal tasks in Mathematics. It will cover the following topics:

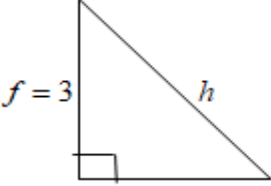
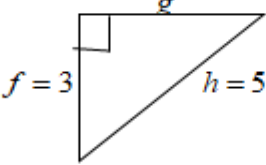
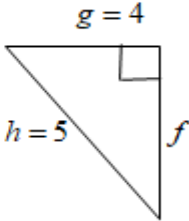
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1. PYTHAGORAS

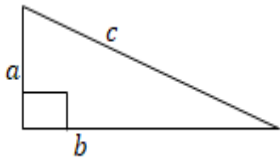
<b>TOPIC: PYTHAGORAS THEOREM(LESSON 1)</b>	Weighting	Grade: 9
<b>Sub-topics</b>	Determining the missing sides of right angled triangle	

<b>RESOURCES</b>
Platinum grade 9 and Oxford successful

<b>NOTES</b>
<p>Pythagorean theorem, the well-known geometric theorem that the sum of the squares on the legs of a right triangle is equal to the square on the hypotenuse (the side opposite the right angle) or, in familiar algebraic notation <math>a^2 + b^2 = c^2</math>.</p> <p><b>Examples:</b></p> <p>There are three possible unknowns, each case is outlined below:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p><math>f = 3</math></p> <p><math>h</math></p> <math display="block">h^2 = f^2 + g^2</math> <math display="block">h^2 = 3^2 + 4^2</math> <math display="block">h^2 = 25</math> <math display="block">h = \sqrt{25}</math> <math display="block">h = 5</math> </div> <div style="text-align: center;">  <p><math>f = 3</math></p> <p><math>g</math></p> <p><math>h = 5</math></p> <math display="block">h^2 = f^2 + g^2</math> <math display="block">5^2 = 3^2 + g^2</math> <math display="block">5^2 - 3^2 = g^2</math> <math display="block">16 = g^2</math> <math display="block">g = \sqrt{16}</math> <math display="block">g = 4</math> </div> <div style="text-align: center;">  <p><math>g = 4</math></p> <p><math>h = 5</math></p> <p><math>f</math></p> <math display="block">h^2 = f^2 + g^2</math> <math display="block">5^2 = f^2 + 4^2</math> <math display="block">5^2 - 4^2 = f^2</math> <math display="block">9 = f^2</math> <math display="block">g = \sqrt{9}</math> <math display="block">g = 3</math> </div> </div>

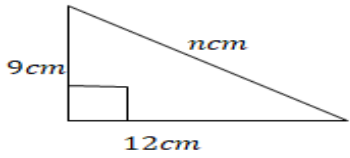
**ACTIVITIES/ASSESSMENT**

1.1.1 Equations represent the Pythagoras Theorem?

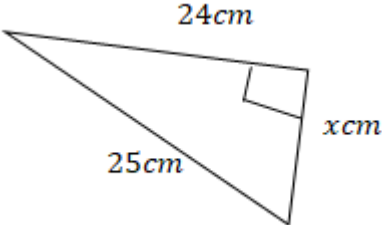


A:  $c^2 = a^2 + b^2$   
 B:  $a^2 + c^2 = b^2$   
 C:  $a^2 = b^2 + c^2$   
 D:  $(a + b)^2 = c^2$

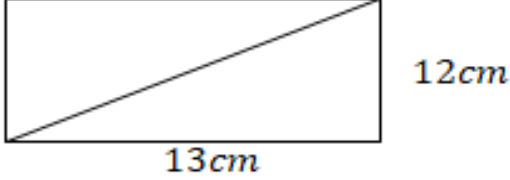
1.1.2 Determine the length of the hypotenuse in the triangle below.



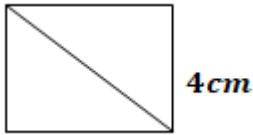
1.1.3 Determine the missing side.



1.1.4 Determine the length of the diagonal in the rectangle below.



1.1.5 Given the side of a square to be 4cm. Determine the length of the diagonal.



**TOPIC: PYTHAGORAS THEOREM(LESSON 2)**

Weighting

Grade:

**Sub-topics**

Calculating the missing sides in two triangles

**RESOURCES**

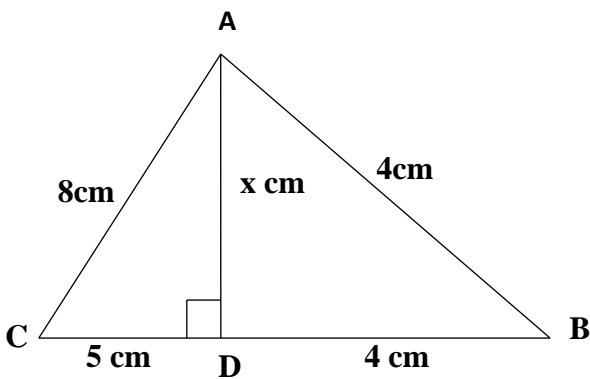
Platinum grade 9; Oxford successful; ANA question papers

**NOTES**

- Master all the geometric shapes involving more than one Right Angled triangle
- Move from one right angled triangle to another

**Example 1**

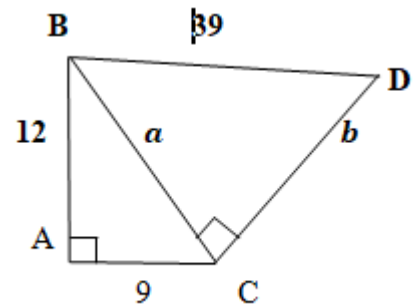
Calculate  $x$  and  $y$



$$\begin{aligned} 8^2 &= 5^2 + x^2 \\ 8^2 - 5^2 &= x^2 \\ 39 &= x^2 \\ x &= \sqrt{39} \text{ cm} \\ y^2 &= 4^2 + x^2 \\ y^2 &= 4^2 + (\sqrt{39})^2 \\ y^2 &= 55 \\ y &= \sqrt{55} \text{ cm} \end{aligned}$$

**Example 2**

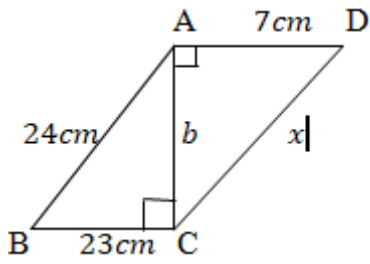
Calculate  $a$  and  $b$



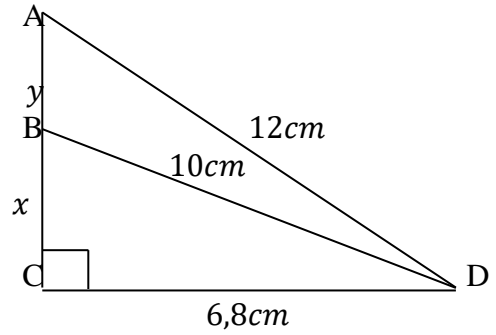
$$\begin{aligned} a^2 &= 9^2 + 12^2 \\ a^2 &= 225 \\ a &= \sqrt{225} \\ a &= 15 \\ 39^2 &= a^2 + b^2 \\ 39^2 &= 15^2 + b^2 \\ 39^2 - 15^2 &= b^2 \\ 1296 &= b^2 \\ b &= \sqrt{1296} \\ b &= 36 \end{aligned}$$

**ACTIVITIES/ASSESSMENT**

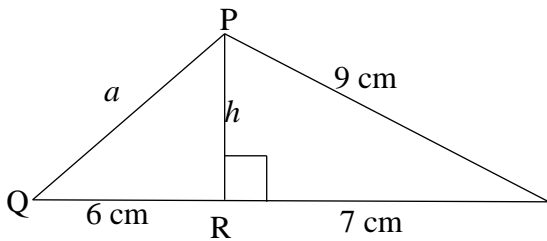
1.2.1 Calculate the unknown lengths  $b$  and  $x$



1.2.3 Calculate  $y$  and  $x$



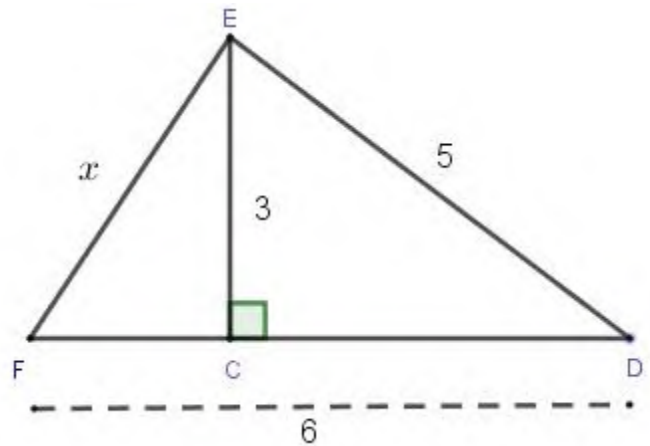
1.2.2 Calculate  $h$  and  $x$



1.2.4 the length of BC

1.2.5 the length of AB

1.2.5 Study the figure below and calculate the value of  $x$



**TOPIC: PYTHAGORAS THEOREM(LESSON 3)**

Weighting

Grade: 10

**Sub-topics**

Measurements

**RESOURCES**

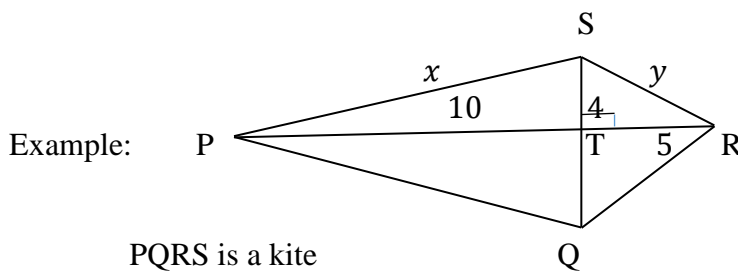
Platinum grade 9.

Oxford successful

Ana question papers

**NOTES**

- Application of theorem of Pythagoras
- Identify the geometric shapes and apply the properties of the figure.
- Measurements (refer to the formula sheet)



- 1.1 Solve for  $x$  and  $y$ .
- 1.2 State the length of PQ and QR.
- 1.3 Prove the;  $ST = TQ$

**Answers:**

$$1.1 \quad x^2 = 10^2 + 4^2 \qquad y^2 = 4^2 + 5^2$$

$$x^2 = 116 \qquad y^2 = 41$$

$$x = \sqrt{116} \qquad y = \sqrt{41}$$

$$1.2 \quad PQ = x = \sqrt{116} \qquad QR = y = \sqrt{41}$$

$$1.3 \quad QR^2 = 5^2 + QT^2$$

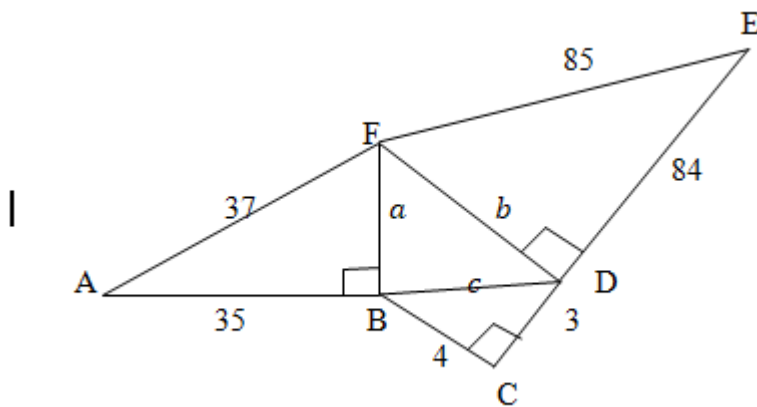
$$(\sqrt{41})^2 - 5^2 = QT^2$$

$$QT^2 = 16$$

$$QT = 4 = TQ$$

**ACTIVITIES/ASSESSMENT**

1.3.2 Study the figure and answer below



- (a) Determine the values of  $a, b$  and  $c$ .
- (b) Prove that ABD a straight line
- (c) Calculate the area of ABCDEF

**TOPIC: PYTHAGORUS THEOREM (LESSON 4)**

**Sub-topics** Area and perimeter

**RESOURCES**

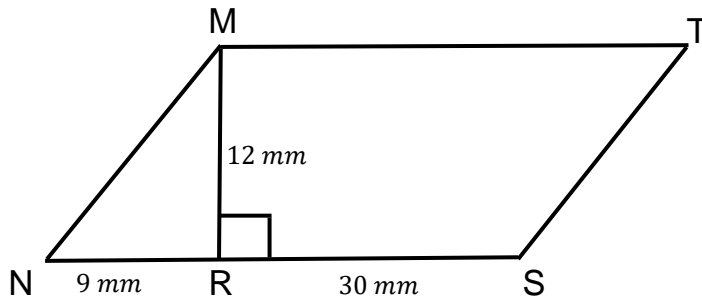
- Platinum grade 9.
- Oxford successful
- ANA question papers

**NOTES**

- Application of theorem of Pythagoras
- Identify the geometric shapes and apply the properties of the figure.
- Measurements (refer to the formula sheet)

**Example**

MNST is a parallelogram.  $NR = 9$  mm,  $RS = 30$  mm and  $MR = 12$  mm.



Calculate:

- 1.1 the area of  $\triangle MNR$
- 1.2 the perimeter of MNST

**ANSWERS**

1.1 Area =  $\frac{1}{2}bh$

$$= \frac{1}{2} \times 9 \text{ mm} \times 12 \text{ mm}$$

$$= 54 \text{ mm}^2$$

1.2  $MN = \sqrt{(9 \text{ mm})^2 + (12 \text{ mm})^2}$

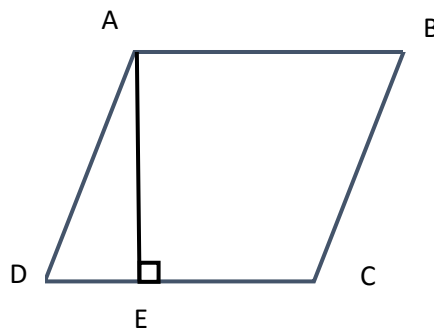
$$= \sqrt{225 \text{ mm}^2} = 15 \text{ mm}$$

$$P = 15 \text{ mm} + 39 \text{ mm} + 39 \text{ mm} + 15 \text{ mm}$$

$$= 108 \text{ mm}$$

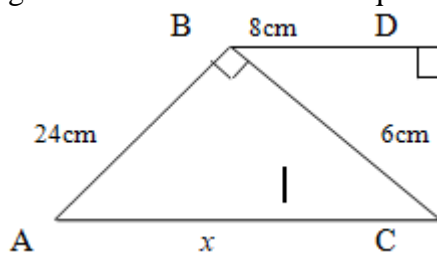
**ACTIVITIES/ASSESSMENT**

1.4.1 ABCD is a rhombus with  $AB = 4 \text{ cm}$  and  $AE = 3,5 \text{ cm}$ .

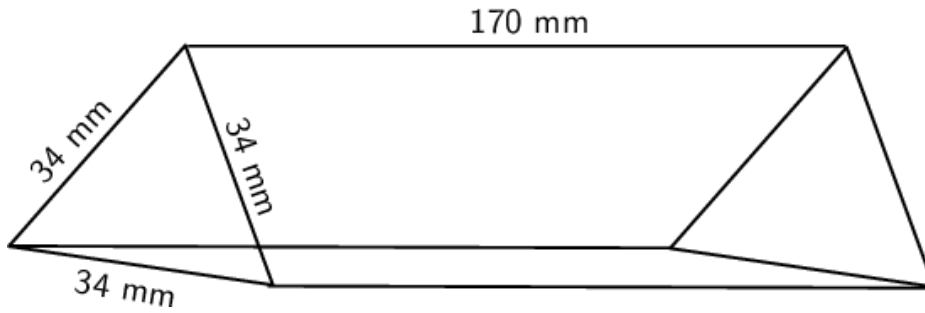


1.4.2 Determine the perimeter of ABCE correct to 1 decimal place.

1.4.3 Study the diagram below and answer the questions that follow.

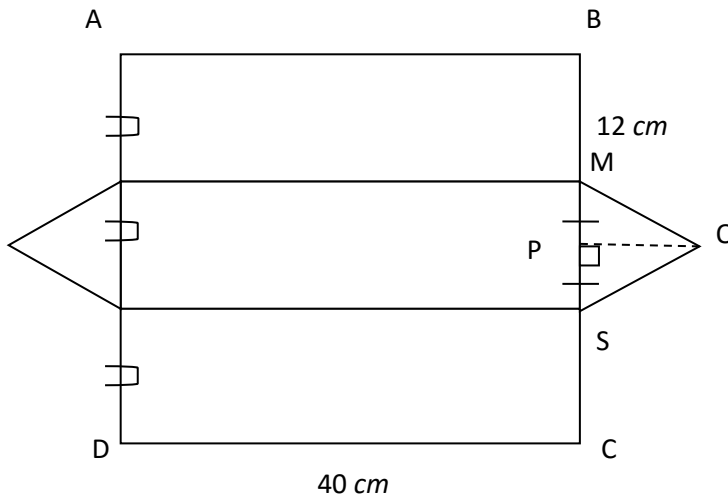


- 1.4.3.1 Calculate the length of BC.  
 1.4.3.2 Calculate the length marked  $x$  .  
 1.4.4 Given the diagram below, answer the following questions:



- 1.4.4.1 Identify the figure represented above.  
 1.4.4.2 Calculate the surface area of the above figure.

1.4.5



- 1.4.5.1 Identify the object represented in the above net.  
 1.4.5.2 Calculate the length of PQ. Leave your answer in surd form.  
 1.4.5.3 Calculate the volume of the above figure if  $PQ = 10,4$  cm.  
 1.4.5.4 Calculate the perimeter of  $\triangle MSQ$

**TOPIC: PYTHAGORUS THEOREM (LESSON 5)**

**Sub-topics** Practical problems

**RESOURCES**

- Platinum grade 9.
- Oxford successful
- Ana question papers

**NOTES**

- Analyse the statement and come up with the equation
- Identify the correct sides, substitutes and solve the unknown

**Example:**



1. Calculate the area of a rectangular carpet if the width is 8m and the diagonal measures 10m.

$$\begin{aligned}
 A &= (10\text{m})^2 - (8\text{m})^2 && \text{Pythagoras} \\
 &= 100\text{m}^2 - 64\text{m}^2 \\
 &= \sqrt{36\text{m}^2} \\
 &= 6 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 A &= l \times b \\
 &= 6 \times 8 \\
 &= 48 \text{ m}^2
 \end{aligned}$$

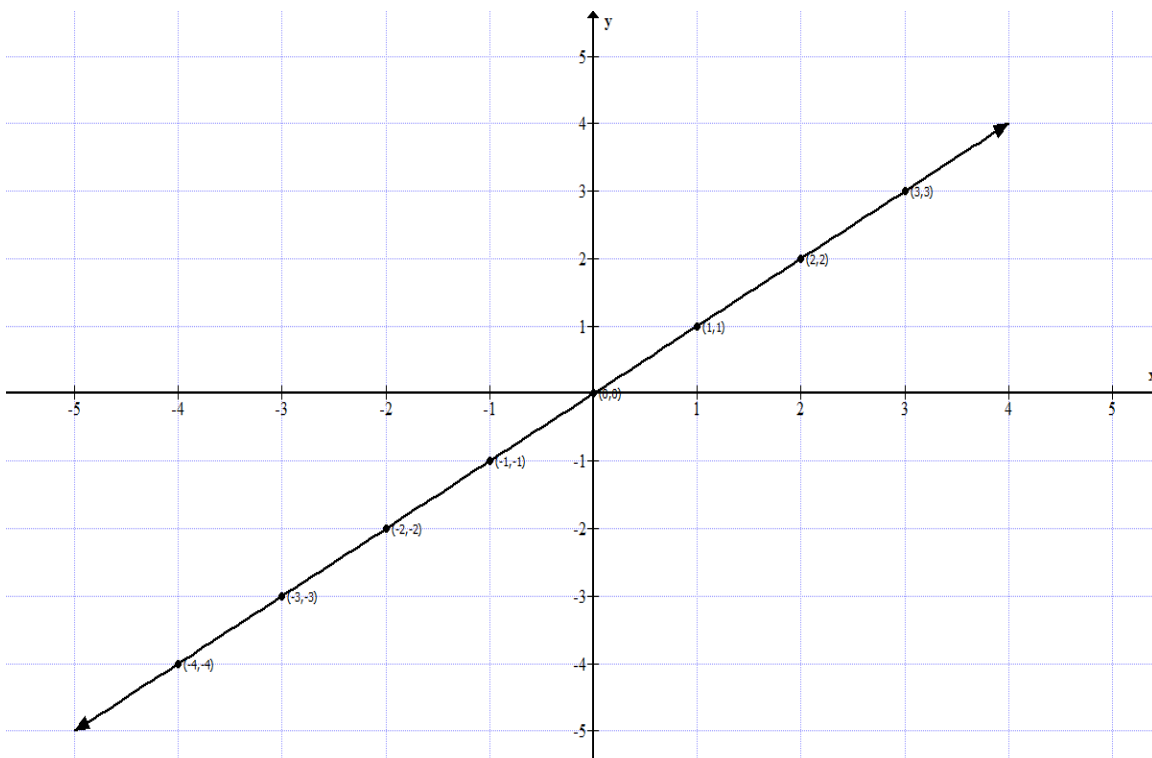
**ACTIVITIES/ASSESSMENT**

- 1.5.1 The bottom of a ladder must be placed 3 units from a wall. The ladder is 12 units long. How far above the ground does the ladder touch the wall?
- 1.5.2 A soccer field is a rectangle 90 meters wide and 120 meters long. The coach asks players to run from one corner to the corner diagonally across the field. How far do the players run?
- 1.5.3 How far from the base of the house do you need to place a 15m ladder so that it exactly reaches the top of a 12m wall?
- 1.5.4 What is the length of the diagonal of a 10 cm by 15 cm rectangle?
- 1.5.5 The diagonal of a rectangle is 25m. The width is 15m in. What is the area of the rectangle?
- 1.5.6 Two sides of a right triangle are 8” and 12”.
- 1.5.6.1 Determine the area of the triangle if 8 and 12 are legs.
- 1.5.6.2 Determine the area of the triangle if 8 and 12 are a leg and hypotenuse

**2. FUNCTIONS**

<b>TOPIC: Functions and graphs (Lesson 1)</b>								
<b>Sub-topics:</b>		Linear function						
<b>RESOURCES</b>								
Gr. 10 Textbooks : (Platinum Mathematics, Classroom Mathematics)								
<b>NOTES</b>								
<ul style="list-style-type: none"> <li>• Use the given points to plot on the Cartesian plane</li> <li>• Complete the given the table using substitution</li> </ul>								
<i>x</i>	-4	-3	-2	-1	0	1	2	3
<i>y = x</i>								
<ol style="list-style-type: none"> <li>1. Complete the table</li> <li>2. Plot the points on the Cartesian plane (graph sheet provided)</li> </ol>								

3. Join the points starting from left to right.
4. Write down the domain of the graph
5. Write down the range of the graph
6. Give the  $x$  and  $y$  intercepts of the graph



Domain:  $x \in \text{real numbers}$

Range:  $y \in \text{real numbers}$

$x$  int :  $x = 0$

$y$  int :  $y = 0$

### Defining a function.

- A function can be expressed in many ways e.g. mapping, table method or listing the set of ordered pairs (coordinates) where every value in the **domain** have **one and only one** value in **range**.

#### Notations (How to write the equation and co-ordinates)

- Since  **$x$  is an independent** variable and  **$y$  is a dependent** variable, the equation or function can be written as  $f(x) = \dots$  instead of  $y = \dots$
- For co-ordinates:  $(x; y) = (x; f(x))$  or  $(a; b) = (a; f(a))$  for function  $f$ .
- The point  $(0;0)$  is called **origin**, where  $x$  axis cuts or meets or intersect the  $y$  axis.
- The  $y$  axis is defined by the equation  $x = 0$ , the coordinates of any point on the  $y$  axis have zero as that the value of  $x$
- The  $x$  axis is defined by the equation line  $y = 0$ , so if you want to calculate the value of  $x$  or a coordinate that lie on the  $x$  axis just put a zero where there is  $y$  or  $f(x)$  and solve for  $x$ .
- For the point of intersection, the two graphs are sharing the same values or coordinate at that point. It means those graphs are equal at that point. To get those values or coordinates you have to equate the

functions or equations and solve for unknowns.

### ACTVITES/SSESSMENT

2.1.1. Draw the following graphs, on the same set of axis

- a)  $y = x$
- b)  $y = -x$
- c)  $y = 3x$
- d)  $y = -3x$

2.1.2. Draw the following graphs on the same set of axis and comment on the effect of  $a$  on the first graph.

- a)  $y = x$  and  $y = -x$
- b)  $y = x$  and  $y = 3x$
- c)  $y = 3x$  and  $y = -3x$

2.1.3 Draw the following pairs of linear graphs on the same set of axis.

- (a)  $y = x$  and  $y = x + 5$
- (b)  $y = 2x$  and  $y = 2x - 4$
- (c)  $y = -x$  and  $y = -x - 1$
- (d)  $y = x + 1$  and  $y = x + 4$

### TOPIC: Functions and graphs (Lesson 2)

**Sub-topics:** The effect of  $a$  in  $f(x) = ax + q$

### RESOURCES

Gr. 10 Textbooks (Platinum Mathematics and Classroom Mathematics)

Revision Documents (Previous Question Papers)

### NOTES

- Use the given equation of a line and draw a table to sketch to graph.
- From the previous lesson you have learnt about the intercepts. ( $x$  intercept and the  $y$  intercept)
- The range is  $y$  which an element of all real numbers is.
- The domain is  $x$  which an element of all real numbers is.

### EXAMPLES

- Draw following the graphs on the same set of **axis** equation using table method

- $y = x$
- $y = 2x$
- $y = \frac{1}{2}x$

Comment on the difference between the three graphs.

- If the equation of a straight line is represented in the form  $y = ax + q$ . Write down the value of  $a$  and  $q$  in the above equations

$$y = x \quad a = 1 \text{ and } q = 0$$

$$y = 2x \quad a = 2 \text{ and } q = 0$$

$$y = \frac{1}{2}x \quad a = \frac{1}{2} \text{ and } q = 0$$

Comment on how the change on the value of  $a$  affect the graph.

### ACTIVITY/ ASSESSMENT

2.2.1. Given the following equations, write down the values of  $a$  and  $q$  in each

(a)  $y = x + 1$

(b)  $y = 2x + 1$

(c)  $y = 3x - 2$

(d)  $y = \frac{1}{2}x + 1$

(e)  $y = -x + 1$

2.2.2. Sketch the following graphs on the same set of axes. Use the table method to determine the coordinates

(a)  $y = x$  and  $y = -x$

(b)  $y = 2x$  and  $y = -2x$

(c)  $y = \frac{1}{2}x$  and  $y = -\frac{1}{2}x$

### TOPIC: Functions and graphs (Lesson 3)

#### Sub-topics

The effect of  $q$  in the standard form  $y = ax + q$

#### RESOURCES

Gr. 10 Textbooks (Platinum Mathematics and Classroom Mathematics)

#### NOTES

#### EXAMPLE 1

Use the given two equations of to draw the graphs on the same set of axes using a table method.

$$f(x) = x$$

$$g(x) = x + 1$$

$$h(x) = x - 1$$

- Comment on the change you see on the graphs.
- Observe the graphs shifting up and down depending on the value of  $q$
- Try to make your own conclusions in relation to the value of  $q$  in relation to how the graph changes

#### EXAMPLE 2

Sketch the following graphs on the same set of axes

a)  $y = -x + 3$

b)  $y = -x - 3$

#### TRANSFORMATION

Transformation in simple terms means change.

**From there, it is where we need to tell them about the Transformation that is divided into four.**

**Translation – a change brought about by a vertical ( $y$ ) shift or horizontal ( $x$ ) shift or both**

- In grade 10 we only deal with vertical shift.

- If  $q$  is **positive** the graph is shifted **upwards** and if  $q$  is **negative** the graph is shifted **downward**.

**Reflection** – about x- axis and y- axis.

- Learners needs to observe two rules.
  - Rule 1: If the graph is reflected about **the x axis (line  $y=0$ )** the value of  $y$  changes the sign.  
 $(x; f(x))$  becomes  $(x; -f(x))$
  - Rule 2: If the graph is reflected about the **y axis (line  $x=0$ )** the value of  $x$  changes the sign.  
 $(x; f(x))$  becomes  $(-x; f(x))$
- **Enlargement and reduction** - expanding or reducing with the scale factor.  
 if we multiply function  $f$  with value  $m$ , all  $y$  values (coordinates) of function  $f$  will be multiplied by that value which is  $m$  ...i.e.  
 $y = m.f(x) \rightarrow (x; m.f(x))$

### ACTIVITY/ASSESSMENT

2.3.1 Draw the following graphs, on the same set of axis

- (a)  $y = x$  and  $y = -x$
- (b)  $y = x$  and  $y = 3x$
- (c)  $y = 3x$  and  $y = -3x$

2.3.2. comment on the effect of  $a$  on the graphs

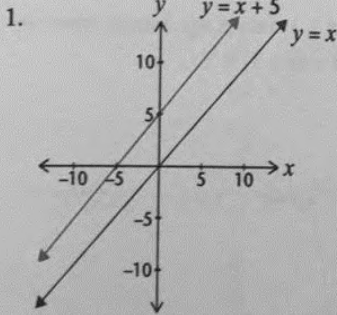
- (a)  $y = x$  and  $y = -x$
- (b)  $y = x$  and  $y = 3x$
- (c)  $y = 3x$  and  $y = -3x$

2.3.3. Draw the following pairs of linear graphs on the same set of axis.

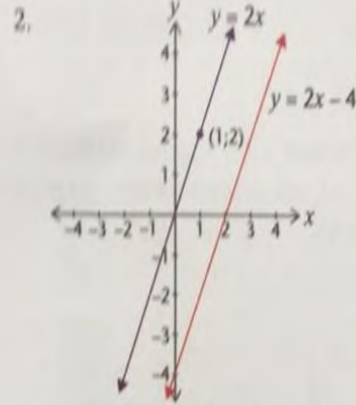
- (a)  $y = x$  and  $y = x + 5$
- (b)  $y = 2x$  and  $y = 2x - 4$
- (c)  $y = -x$  and  $y = -x - 1$
- (d)  $y = x + 1$  and  $y = x + 4$

### HOMEWORK

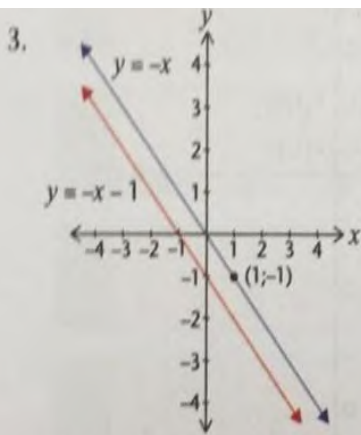
**EXERCISE 5**



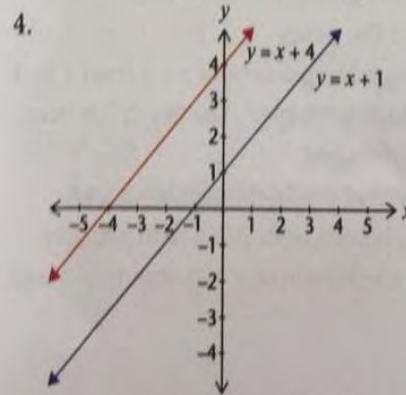
The graph of  $y = x$  moves **up** 5 units when it becomes  $y = x + 5$



The graph of  $y = 2x$  moves **down** 4 units when it becomes  $y = 2x - 4$ .



The graph of  $y = -x$  moves **down** 1 unit when it becomes  $y = -x - 1$ .



The graph of  $y = x + 1$  moves **up** 3 units when it becomes  $y = x + 4 = (x + 1) + 3$ .

**TOPIC: Functions and graphs (Lesson 4)**

**Sub-topics**

Restricted Linear and Finding the equation of a line

**RESOURCES**

Gr. 10 Textbooks (Platinum Mathematics and Classroom Mathematics)

Revision Documents (Previous Question Papers)

Casio calculators

**NOTES**

For Restricted Linear.

Note the following:

- The difference between the normal linear function and the restricted one is the range and domain.
- For the normal linear function, the range is the set of all real number and the range also is the set of all real numbers.
- But for the restricted straight line, the range and domain will be restricted too

For equation of a line

Two points. A(-1;2) and B(3;1) and use them to find the gradient and the equation of a linear.

- Then if you want to find the equation of a line, you need at least two have two points.
- Use the above activities to find the equation of the line.
- Use Analytical Geometry knowledge to find the equation of a line.
- Know that the standard form of a linear function is  $y = ax + q$ , but you can also use  $y - y_1 = m(x - x_1)$  to find the equation. From the first equation, **a** represents the gradient and **q** represents the y intercept.
- You must know how to find the equation of a line. To find the equation of a line we need two things, a **point** and a **gradient**.
- To find a gradient.
- Average gradient we need two points. (Gradient =  $m = \frac{\Delta y}{\Delta x}$  = change in y divided by change in x)

### ACTIVITIES./ASSESSMENT

- 2.4.1 What is the range of the linear functions  $f(x) = -1$  for the domain  $[-10 ; 10]$ ?
- 2.4.2 What is the domain of the linear function  $y = 2x - 5$  where the range is  $(-9; 7]$ ?
- 2.4.3 Study the sketch graphs and for each line segment:
- Write down the domain of the graph.
  - Write down the range of the graph.
  - Calculate the equation of the line segment sketched.

### TOPIC: Functions and graphs (Lesson 5)

Sub-topics

QUADRATIC FUNCTIONS (SKETCHING PARABOLA)  
Effect of a

### RESOURCES

Gr. 10 Platinum Textbook, calculators and worksheets.

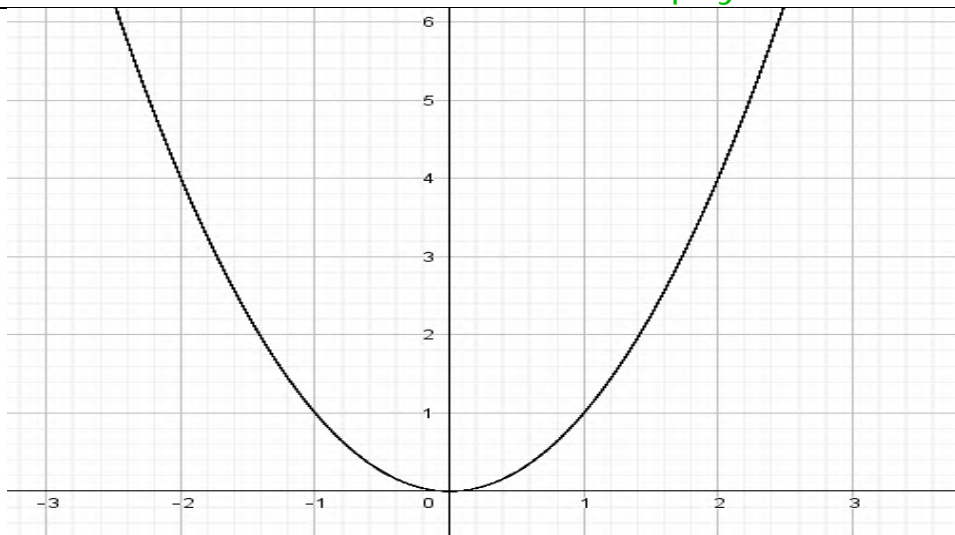
### NOTES

Investigating the effect  $a$

**EXAMPLE 1:** What happened when the coefficient of  $x^2$  is positive or negative?

1.  $y = x^2$ ,  $a > 0$

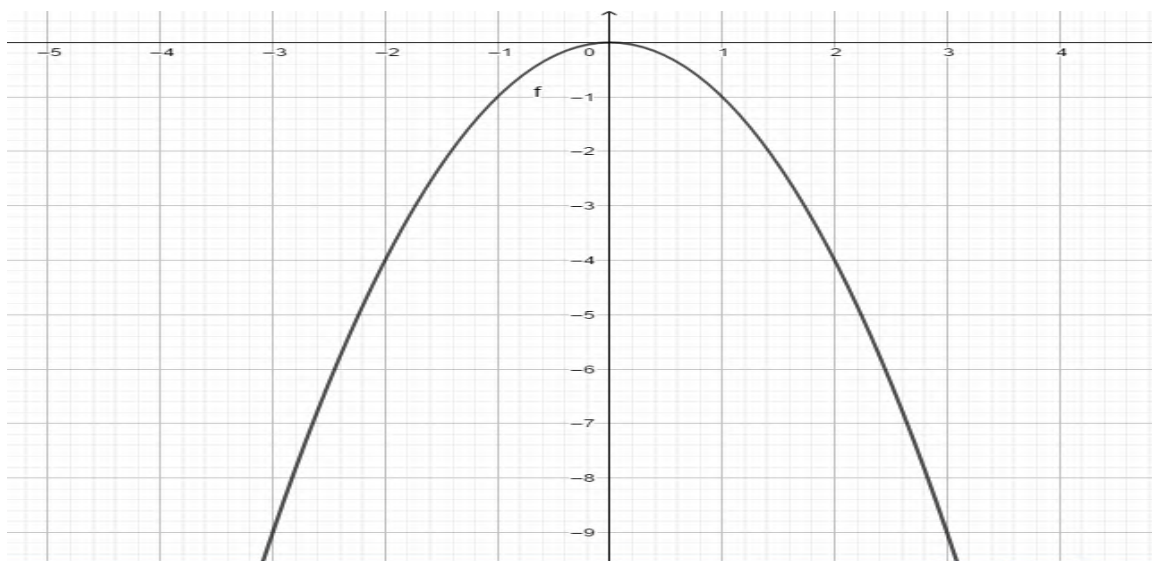
$x$	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



The parabola is concave up

2.  $y = -x^2$ ,  $a < 0$

$x$	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9



The parabola is concave down

**EXAMPLE 2**

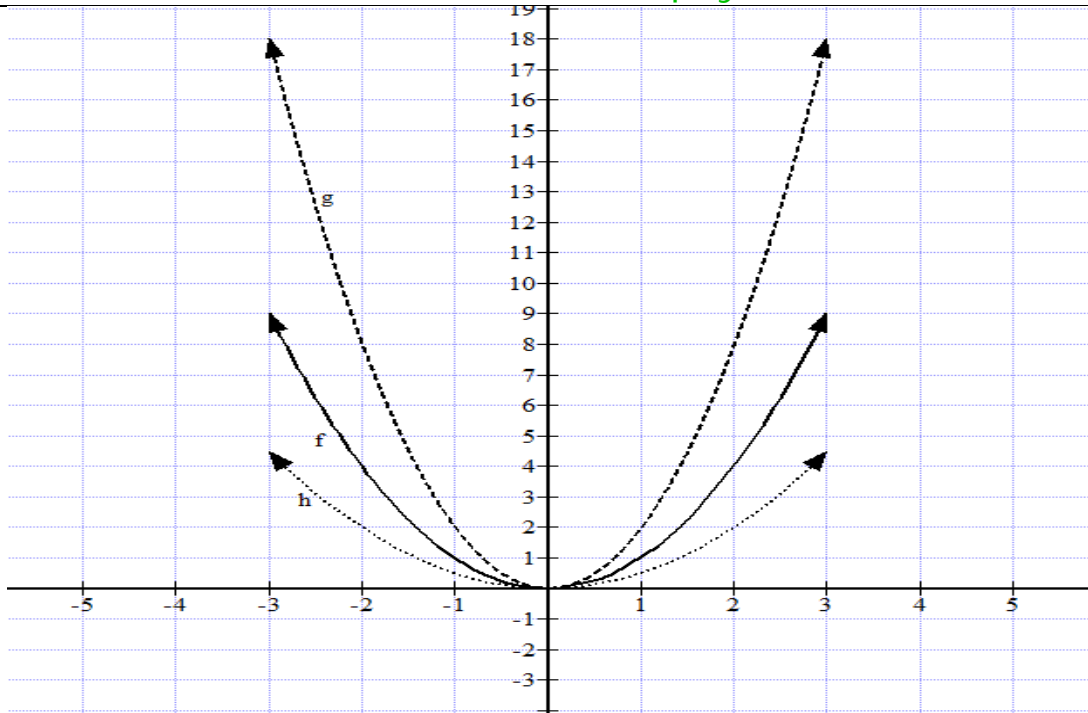
Sketch the following functions in the same set of axes

$$f(x) = x^2$$

$$g(x) = 2x^2$$

$$h(x) = \frac{1}{2}x^2$$





**Conclusion:**

- If  $a$  is positive the graph faces upward (concave up), and if  $a$  is negative the curve face downward (concave down)
- If  $a$  increases, the graph becomes narrower or stretches. As  $a$  decreases the graph becomes wider or flatter.
- As  $a$  decreases the graph becomes narrower or stretches. As  $a$  increases the graph becomes wider or flatter.

**1. ACTIVITIES/ASSESSMENT**

Plot the following functions

2.5.1  $y = x^2$  and  $y = 3x^2$

2.5.2  $y = -x^2$  and  $y = -\frac{1}{2}x^2$

**TOPIC: Functions and graphs (Lesson 6)**

<b>Sub-topics</b>	<b>Parabolic or Quadratic function</b>
	<b>Effect of <math>q</math></b>

**RESOURCES**

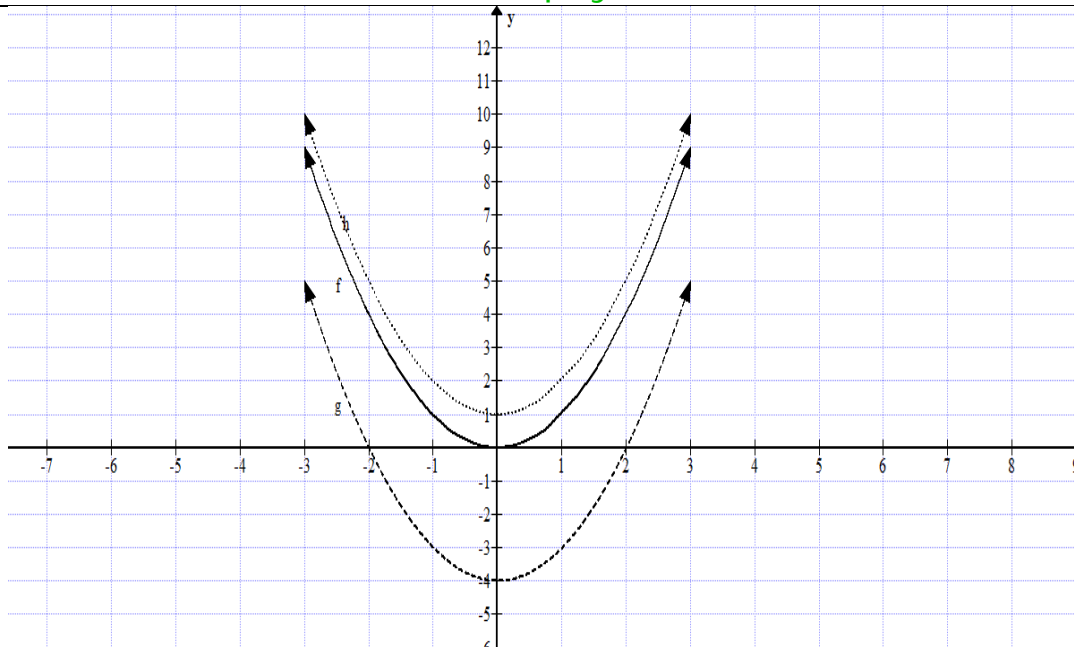
Gr. 10 Platinum Textbook, calculators and worksheets.

A Revision exercise on Gr. 10 Functions, e.g. Classroom Mathematics Gr. 10 page 139– 153.

**NOTES**

Draw the graphs of  $y = x^2$ ,  $y = x^2 + 1$  and  $y = x^2 - 4$ , on the same Cartesian planes

$y = x^2$	-4	-3	-2	-1	0	1	2	3	4
$y = x^2 + 1$	-4	-3	-2	-1	0	1	2	3	4
$y = x^2 - 4$	-4	-3	-2	-1	0	1	2	3	4



- Compare the mother graph with the graphs you have drawn

### ACTIVITIES/ASSESSMENT

The value of  $q$  in the equation:  $y = ax^2 + q$  determines the shift of the graph of  $y = ax^2$  up or down. It also represents the  $y$  intercept of the graph of  $y = ax^2 + q$

2.6.1 Given :  $y = -2x^2 + 8$  and  $y = -x^2 - 2$

- Sketch the two graphs on the same set of axis.
- For these graphs, determine algebraically the coordinates of intercepts with the axis.

### TOPIC: Functions and graphs (Lesson 7)

#### Sub-topics

#### Parabolic or Quadratic function (further characteristics)

#### RESOURCES

Gr. 10 Platinum Textbook, calculators and worksheets.

#### NOTES

Note the following concepts, namely:

- Intercepts
- Domain and Range
- Turning point
- Maximum and minimum values
- Increasing and decreasing
- For further practice (assessment), give learners similar sums to do. Intercepts, turning point, axis of symmetry.
- Investigating the domain and range, minimum and maximum values, increasing and decreasing values.
- For the graph of a given function, if the  $y$  values increase as  $x$  values increase, then the graph is said to be increasing.
- If the  $y$  values decrease as the  $x$  values increase, then the graph is said to be decreasing.

- In conclusion, the graph is increasing if it moves upwards when moving from the left to right and decreasing if it moves downwards when moving from left to right.

**ACTIVITIES/ASSESSMENT**

2.7 Sketch the graphs and find the domain, Range, Minimum or Maximum values and the value of  $x$  where the graph increases and decreases.

2.7.1  $y = -x^2 + 9$

2.7.2  $y = 3x^2 - 12$

2.7.3  $y = -2x^2 + 8$

**TOPIC: Functions and graphs (Lesson 8)**

**Sub-topics** | **Parabolic or Quadratic function**

**RESOURCES**

Gr. 10 Platinum Textbook, calculators and worksheets.

A Revision exercise on Gr. 10 Functions, e.g. Classroom Mathematics Gr. 10 page 139– 153.

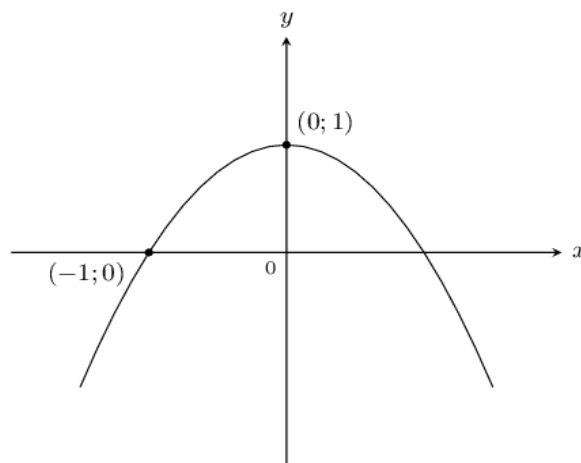
**ACTIVITIES/ASSESSMENT**

2.8.1 Find the equation of the parabola in a form of :  $y = ax^2 + q$  if it passes through points (2; 7) and (0;3)

2.8.2 Find the equation of the parabola in a form of  $y = ax^2 + q$  if it passes through points (2; 5) and (0;-20)

2.8.3 Find the equation of the parabola in a form of :  $y = x^2 + q$  if it passes through points (2; 0) and (-2;0)

2.8.4 Use the sketch below to determine the values of a and q for the parabola of the form  $y = ax^2 + q$



**TOPIC: Functions and graphs (Lesson 9)**

**Sub-topics** | **Parabolic or Quadratic function**

**RESOURCES**

Gr. 10 Platinum Textbook, calculators and worksheets.

A Revision exercise on Gr. 10 Functions, e.g. Classroom Mathematics Gr. 10 page 139– 153.

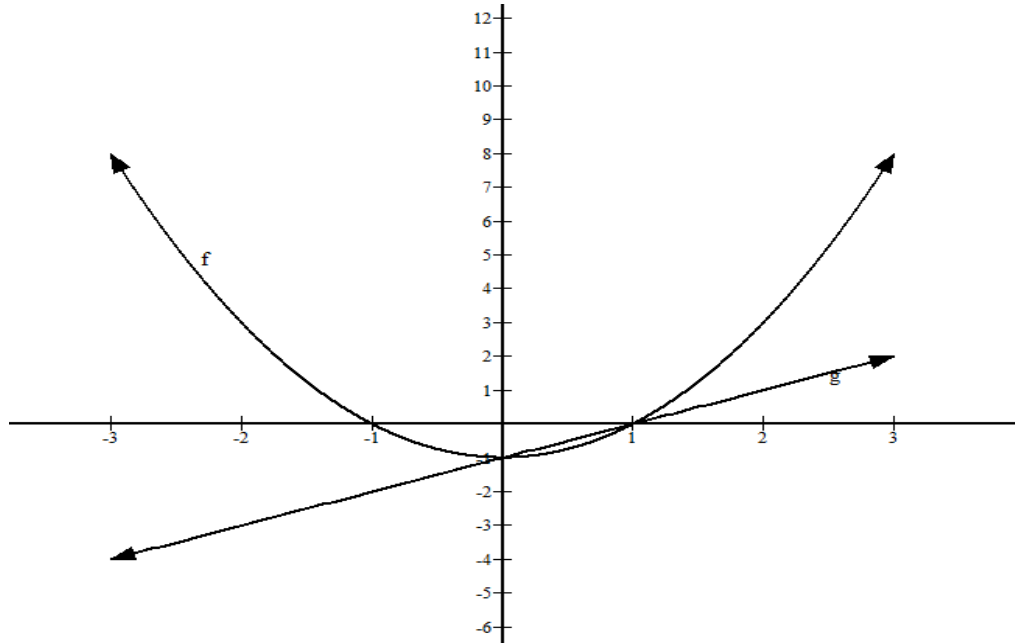
**NOTES**

**Example**

Given:  $f(x) = x^2 - 1$  and  $g(x) = x - 1$

- (a) Sketch the graph of  $f$  and  $g$  on the same set of axes.
- (b) Determine the domain and range of  $f$
- (c) If  $h(x) = f(x) + 3$ , write down the coordinates of the turning point of  $h$
- (d) Determine the maximum value of  $p$  if  $p(x) = -h(x)$ .
- (e) Sketch the graphs of  $h$  and  $p$  on the same set of axes.
- (f) Determine the y-intercept of the graph of  $y = -g(x)$ .

(a)



(b) Domains of  $f : x \in R$

Range of  $f : y > -1, y \in R$

(c)  $h(x) = x^2 - 1 + 3$

$h(x) = x^2 + 2$

Turning point: (0;2)

(d)

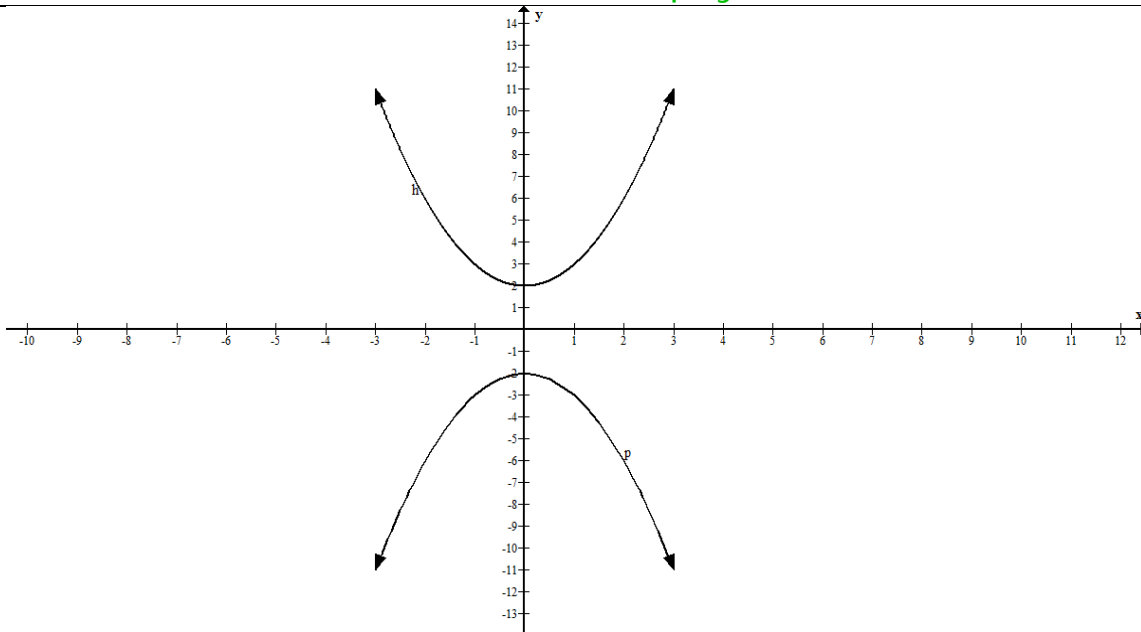
$p(x) = -h(x)$

$p(x) = -(x^2 + 2)$

$p(x) = -x^2 - 2$

Maximum value:  $y = -2$

(e)



**ACTIVITIES/ASSESSMENT**

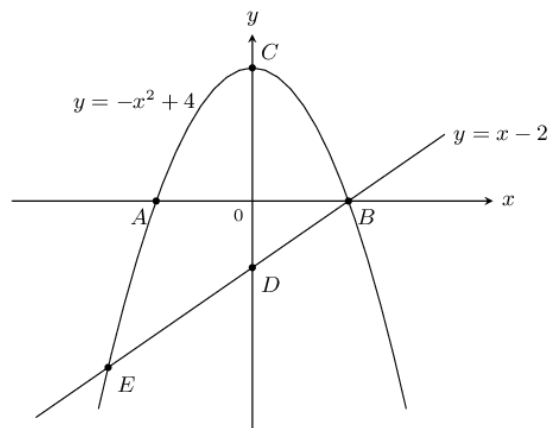
2.9.1 Given the graph of  $f(x) = 2x^2 - 2$ . The graph cuts the x- axes at B and C and the y-axis A. Line h is parallel to the x-axis and passes through A.

Determine:

- (a) the coordinates of A, B and C
- (b) the values of x for which  $f$  is increasing
- (c) the values of x for which  $f$  is decreasing
- (d) the minimum value of  $f$
- (e) the turning point of  $f$
- (f) the equation of the axis of symmetry of  $f$
- (g) the domain and range of  $f$
- (h) the equation of  $h$  and the value of its gradient
- (i) the domain and range of  $h$

2.9.2 The graphs of  $y = -x^2 + 4$  and  $y = x - 2$  are given. Calculate the following:

- (a) coordinates of A, B, C, D
- (b) coordinates of E
- (c) distance CD



**TOPIC** : **Functions and graphs (Lesson 10)**

**SUB-TOPIC**

- The Hyperbola (The effect of  $a$ )

**RESOURCES**

- Grade 10 Platinum Mathematics Textbook (name the page)
- Calculators

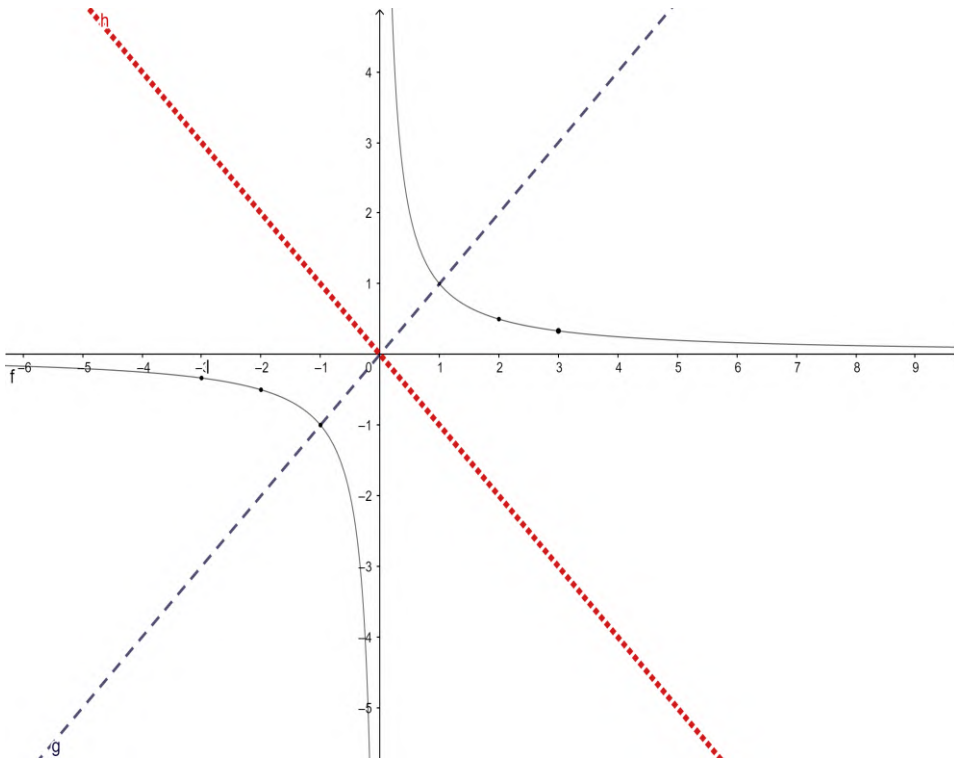
**EXAMPLE**

Redraw the table below and complete it using the calculator:

$x$	-3	-2	-1	0	1	2	3
$f(x) = \frac{1}{x}$	$\frac{-1}{3}$	$\frac{-1}{2}$	$\frac{-1}{1}$	$\frac{1}{0}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$

Division by zero as a concept will be highlighted

Sketch the graph of  $f$ , on the grid provided and use free hand to draw. Answer the questions that follow



1. Which two quadrants are taken by the graph of  $f$ ?  
**Quadrants 1 and 3**
2. Does the function intersect the x axis  
**NO**
3. Hence write down the domain of  $f$   
 $x \in R, x \neq 0$
4. Write down the range of  $f$ .  
 $y \in R, y \neq 0$
5. Write down the equations of the lines the function does not touch at all  
 $x = 0$  **and**  $y = 0$
6. Indicate by a dotted line how the function can be divided into two parts (halves)

**7. Determine the equation of the line of symmetry**

**Line passing through the origin (0,0) and (1,1)**

$$m = \frac{y_B - y_A}{x_B - x_A}$$

$$= \frac{1 - 0}{1 - 0}$$

$$= 1$$

$$y = mx + c$$

$$0 = 1(0) + c$$

$$c = 0$$

Equation:  $y = x$

Points (0,0) and (-1,-1) should be used for another line

**CONCLUSION**

- The basic graph of the hyperbola should not touch the axis, which result in both axis being the asymptotes of the graph.
- Then learners should know that if a line is an asymptote, the graph can approach that line but it won't touch that line.
- If the value of  $a$  is positive, the graph will be in the first and third quadrants.

**ACTIVITIES/ASSESSMENT**

2.10.1 Redraw the table below and complete it using the calculator:

$x$	-2	-1	0	1	2
$f(x) = \frac{1}{x}$					

Sketch the graph of  $f$ , on the grid provided and use free hand to draw.

2.10.2 Given :  $h(x) = -\frac{1}{x}$

(a) Redraw and complete the table below

$x$	-2	-1	0	1	2
$h(x) = -\frac{1}{x}$					

(b) Sketch the graph of  $h$  on the same set of axis.

2.10.3 Redraw the table below and complete it using the calculator:

$x$	-3	-2	-1	0	1	2	3
$y = \frac{2}{x}$							
$y = \frac{3}{x}$							
$y = \frac{4}{x}$							

1. Sketch the graphs of  $y$ , on the same set of *axis*.
2. What changes do you observe as the value of  $a$  keeps on increasing?

**TOPIC : Functions and graphs (Lesson 11)**

**SUB-TOPIC**

- The Hyperbola ( The effect of  $q$  )

**RESOURCES**

- Grade 10 Platinum Mathematics Textbook (name the page)
- Calculators

**ACTIVITIES/ASSESSMENT**

2.11.1 Redraw the table below and complete it using the calculator:

$x$	-2	-1	0	1	2
$g(x) = \frac{1}{x} + 1$					

- Write the horizontal asymptote of  $g$ .
- Sketch the graphs of  $g$ . Clearly show all the intercepts with the axis and the horizontal asymptote.
- What change(s) do you observe compared to the first graph you drew which is  $y = \frac{1}{x}$ ?
- Write the equation of  $h$ , if  $h(x) = -g(x)$ .
- By comparing the equation of  $h$  and  $g$ , what type of a **reflection** that resulted to the equation of  $h$ ?
- Write down the equation of  $k$ , if  $k$  is the reflection of  $g$  about the y-axis.
- If the graph of  $g$  is shifted 3 units upwards, write the new horizontal asymptote.

2.11.2 Given:  $f(x) = \frac{3}{x} + 2$

- Redraw and complete the table below

$x$	-2	-1	0	1	2
$f(x) = \frac{3}{x} + 2$					

- Write down the equation of the horizontal asymptote of  $f$ .
- Sketch the graph of  $f$ , clearly show the intercepts with the axis and the horizontal asymptote.
- Write down the equation of  $h$ , if  $h$  is the reflection of  $f$  about the x-axis.
- If  $g$  is the result of  $f$  shifted 2 units up, therefore write down the equation of  $g$ .

2.11.3 Given :  $h(x) = \frac{1}{x} - 1$

- Redraw and complete the table below

$x$	-2	-1	0	1	2
$h(x) = \frac{1}{x} - 1$					

- Write down the equation of the horizontal asymptote of  $h$ .



- (c) Sketch the graph of  $h$ , clearly show all the intercepts with the axis and the horizontal asymptote.
- (d) Compare the graph of  $h$  and the graph of  $y$  in the diagram sheet provided.
- (e) Write down the equation of  $g$ , if  $g(x) = h(-x)$ .
- (f) What type of the reflection is  $g$  from  $h$ ?

2.11.4 Given :  $f(x) = \frac{4}{x} - 3$

- (a) Draw a table and complete the table, where  $x \in [-2, 2]$ .
- (b) Sketch the graph of  $f$  in your exercise book, clearly show all the intercepts with the axis and the horizontal asymptote.
- (c) The equation of  $g$  is obtained by reflecting  $f$  about the x-axis, then write down the equation of  $g$ .

**TOPIC : Functions and graphs (Lesson plan 12)**

**SUB-TOPIC**

- Exponential Functions ( $f(x) = b^x$ , where  $b > 1$  and  $0 < b < 1$ )

**RESOURCES**

- Grade 10 Mind Action Series Mathematics Textbook
- Calculators

**ACTIVITIES/ASSESSMENT**

Redraw the table below and complete it using the calculator:

$x$	-3	-2	-1	0	1	2	3
$f(x) = 3^x$							
$g(x) = 2^x$							
$h(x) = \left(\frac{1}{3}\right)^x$							
$k(x) = \left(\frac{1}{2}\right)^x$							

- (a) Sketch the graphs of  $f$  and  $h$ , on the same set of axis and the graphs of  $g$  and  $k$  on another set of axis.
- (b) What do you observe if the value of  $b > 1$  and when  $0 < b < 1$ ?
- (c) Which two graphs are increasing and which two are decreasing?
- (d) What are the values of the y-intercept in each graph?
- (e) Is there a graph that cuts the x-axis? If so, where does it cut?
- (f) Write down the domain and range of each graph.
- (g) Is there a graph that cuts the x-axis? If so, where does it cut?
- (h) Write down the domain and range of each graph.

**TOPIC : Functions and graphs (Lesson 13)**

**SUB-TOPIC**

- Exponential Functions the effect of  $q$  if  $y = b^x + q$

**RESOURCES**

- Grade 10 Mind Action Series Mathematics Textbook (name the page)
- Calculators

**ACTIVITIES/ASSESSMENT**

1. Redraw the table below and complete it using the calculator:

$x$	-3	-2	-1	0	1	2	3
$f(x) = 3^x + 1$							
$g(x) = 2^{x+2}$							
$h(x) = 3^x - 1$							
$k(x) = 2^x - 2$							

2. Sketch the graphs of  $f$  and  $h$ , on the same set of  $axis$  and  $g$  and  $k$  on another set of axis
3. What do you observe if the value of  $q$  is no more zero?
4. What are the values of the  $y$ -intercept in each graph?
5. Is there a graph that cuts the  $x$ -axis? If so, where does it cut?
6. Describe in words, a change (transformation) that has happened in each graph compared to the mother graph that you drew on the previous lesson.
7. Write down the equation of  $m$ , if  $m(x)=f(x)+2$

**CONCLUSION**

The effect of  $q$  is that it shifts the graph up or down. A positive value of  $q$  shifts the graph upward and a negative value shifts the graph downward.

**TOPIC : Functions and graphs (Lesson 14)**

**SUB-TOPIC**

- Exponential Functions: The effects  $a$  if  $y = a \cdot b^x$

**RESOURCES**

- Grade 10 Mind Action Series Mathematics Textbook
- Calculators

**ACTIVITY**

1. Redraw the table below and complete it using the calculator:

$x$	-3	-2	-1	0	1	2	3
$f(x) = 3^x$							
$g(x) = 2^x$							
$h(x) = 2 \cdot 3^x$							
$k(x) = 3 \cdot 2^x$							

2. Sketch the graphs of  $f$ ,  $g$ ,  $h$ , and  $k$ , on the same set of  $axis$ .
3. What do you observe as the value of  $a$  changes?
4. What are the values of the  $y$ -intercept in each graph?
5. Is there a graph that cuts the  $x$ -axis? If so, where does it cut?
6. Describe in words, a change (transformation) that has happened in each graph compared to the mother graphs.

**TOPIC : Functions and graphs (Lesson 15)**

**SUB-TOPIC**

- Investigate the effect of  $a$  on the graphs of  $y = a \sin x$  and  $y = a \cos x$

**RESOURCES**

- Grade 10 Platinum Mathematics Textbook (page 102 to103)
- Calculators

**NOTES**

- Given the basic graphs (mother graphs)  $y = \sin x$  and  $y = \cos x$

$x$	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$	$210^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$330^\circ$	$360^\circ$
$y$													

- Fill in the table for each of the functions and sketch the graphs.

**ACTIVITIES/ASSESSMENT**

Sketch the graphs of the following functions on the separate system of axes, for interval  $x \in [0^\circ; 360^\circ]$ . Compare their amplitudes, maximum and minimum with that of the basic graphs (mother graphs).

1.1  $f(x) = 2\cos x$

1.2  $h(x) = 3\sin x$

1.3  $g(x) = \frac{1}{2}\sin x$

1.4  $t(x) = \frac{1}{4}\cos x$

- What changes do you observe as the value of  $a$  increase?
- What changes do you observe as the value of  $a$  decreases?

**TOPIC : Functions and graphs (Lesson 16)**

**SUB-TOPIC**

- Investigate the effect of  $q$  on the graphs of  $y = \sin x + q$  and  $y = \cos x + q$

**RESOURCES**

- Grade 10 Platinum Mathematics Textbook (name the page)
- Calculators

**ACTIVITIES/ASSESSMENT**

Draw the graphs of the following functions and compare their maximum values, minimum values as well as their intercepts with axes with that of the mother graphs drawn above. The interval is  $0^\circ \leq x \leq 360^\circ$

2.16.1  $h(x) = \sin x + 1$

2.16.2  $t(x) = \sin x - 2$

2.16.3  $p(x) = \cos x + 1$

2.16.4  $q(x) = \cos x - 3$

**TOPIC : Functions and graphs (Lesson 17)**

**SUB-TOPIC**

- Trigonometric functions(reflection and translation)

**RESOURCES**

- Grade 10 Platinum Mathematics Textbook (page 139 to 143)
- Calculators

**NOTES**

Recall a reflection about the  $x$ -axis and  $y$ -axis.

- Reflection about the  $x$ -axis ( $y = 0$ ),  $A(x; y)$  to  $A(x; -y)$
- Or  $y = f(x)$  to  $y = -f(x)$
- Reflection about  $y$ -axis ( $x = 0$ )  $B(x; y)$  to  $B(-x; y)$
- Or  $y = f(x)$  to  $y = f(-x)$

**ACTIVITY/ASSESSMENT**

2.19.1 Given  $f(x) = \sin x$  and  $g(x) = \cos x$

Fully describe the transformation that took place in the following functions

- $p(x) = -\sin x$
- $q(x) = -2g(x)$
- $h(x) = -\frac{1}{3}f(x)$

2.19.2 What will happen if you reflect  $f(x) = \cos x$  about the  $y$ -axis

- reflect  $g(x)$  about  $y$ -axis or  $x$ -axis
- reflect  $g(x)$  about the  $y$ -axis then vertically stretch the result by a factor of 2
- reflect  $f(x)$  about the  $x$ -axis, then vertically shrink the result by a factor of  $\frac{1}{3}$

**TOPIC: Functions and graphs (Lesson plan 20)****Sub-topics**

Trig. Function ( $\tan \theta$ )

**RESOURCES**

Gr. 10 Textbooks (Platinum Mathematics and Classroom Mathematics)  
Revision Documents (Previous Question Papers)  
Calculators

**ACTIVITY****QUESTION 5**

5.1 Consider the function  $f(x) = -3 \tan x$ .

5.1.1 Sketch, on the grid provided in the ANSWER BOOK, the graph of  $f$  for  $0^\circ \leq x \leq 360^\circ$ . Clearly show ALL the intercepts and asymptotes. (3)

5.1.2 Hence, or otherwise, write down the:

- Period of  $f$  (1)
- Equation of  $h$  if  $h$  is the reflection of  $f$  about the  $x$ -axis (1)

**DRAW THE FOLLOWING FUNCTIONS.**

$$y = \tan \theta$$

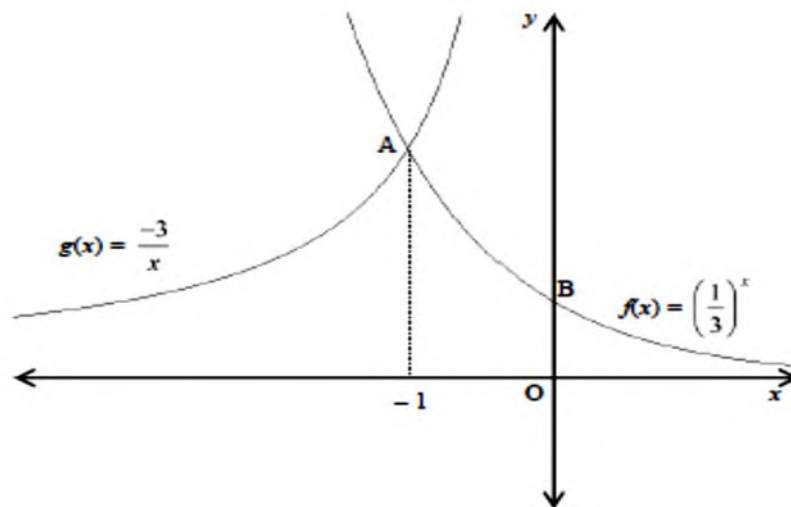
$$y = \tan \theta + 1$$

$$y = \tan \theta - 2$$

## ADDITIONAL ACTIVITIES

**QUESTION 5**

The graphs of  $f(x) = \left(\frac{1}{3}\right)^x$  and  $g(x) = \frac{-3}{x}$  where  $x < 0$  are represented below:



Answer the following questions by using the graphs:

- 5.1 Write down the co-ordinates of A and B. (2)
- 5.2 Write down the domain of  $f(x)$ . (1)
- 5.3 Write down the equation of the reflection of  $f$  in the  $y$ -axis. (2)
- 5.4 Give the equations of the asymptote of the graph of  $y = g(x) + 2$ . (2)
- [18]

**QUESTION 12**

- 12.1 Draw a sketch graph of  $y = \frac{4}{x} + 2$ . Clearly indicate the asymptote(s) and the intercept(s) with the axes. (3)
- 12.2 Given the two functions  $f(x) = -x^2 + 9$  and  $g(x) = -x + 3$ :
- 12.2.1 Sketch the graph of  $f$  and  $g$  on the same axes, showing the Co-ordinates of all the intercepts with the axes. (6)
- 12.2.2 Calculate the co-ordinates of the points at which  $f(x) = g(x)$  (4)
- 12.2.3 Use your graph to write down the values of  $x$  for which  $f(x) > 0$
- 12.2.4 Draw a dotted line on your graph, showing the graph of  $y = \frac{1}{2}f(x)$ . The intercepts on the axes must be shown. (3)
- 12.3 Determine the equation of a linear function  $f(x) = mx + c$ , if  $f(0) = -7$  and  $f(2) = 0$ . (3)

## 3. DATA HANDLING

TOPIC: Data Handling/Statistics (Lesson 1)	Weighting	$\pm 15\%$	Grade	10										
Sub-topics	Organize and represent grouped and ungrouped data.													
<b>RESOURCES</b>														
Grade 9 and 10 textbooks including these: <ul style="list-style-type: none"> <li>Mathematics Today (grade 9); pages (282 - 284)</li> <li>Platinum (grade 9); pages (273 ; 278 ; 278 ; 284 ; 286)</li> <li>Mind Action Series (grade 10); pages (219 - 223)</li> <li>Grade 10 previous examination question papers.</li> </ul>														
<b>Example 1:</b>														
In a survey conducted in a shopping mall in 2014, people of different ages were asked what type of phone they were using. The following table shows ages of thirty people between ages of 15 and 60 that were using smartphones.														
16	17	17	17	17	18	18	25	25	27	28	28	28	28	28
28	32	34	34	34	46	46	48	54	55	56	56	56	56	56
<p>(a) Copy and complete the frequency table given below for this data.</p> <p>(b) Represent the data in a stem – and – leaf plot.</p> <p>(c) Draw a frequency bar graph for this data.</p> <p>(d) Write down the age with the highest frequency?</p> <p>(e) Determine the percentage represented by the age mentioned in <b>d)</b> above.</p> <p>(f) Group the data into four class intervals.</p> <p>(g) Draw a histogram.</p> <p>(h) Determine the modal class of the above data.</p>														
<b>Age</b>	<b>Tally</b>		<b>Frequency</b>											
16														
17														
18														
25														
27														
28														
32														
34														
46														
48														
54														
55														
56														

**Solutions to example 1**

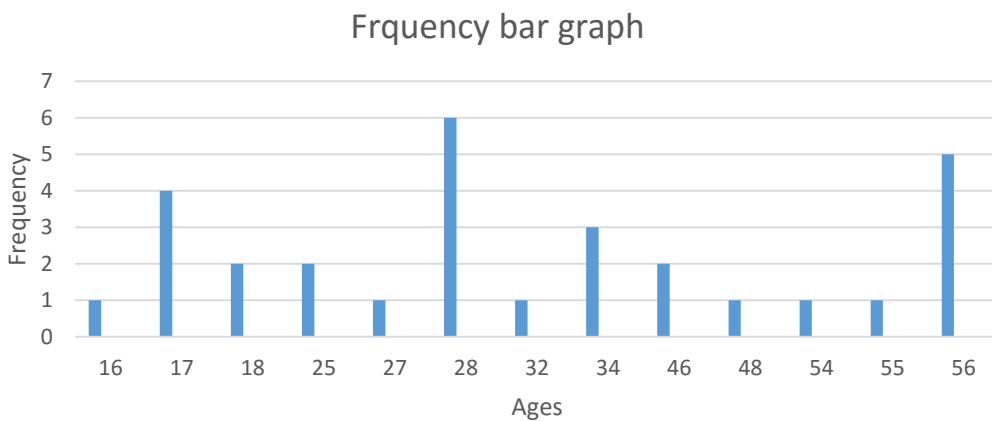
(a)

Age	Tally	Frequency
16		1
17		4
18		2
25		2
27		1
28		6
32		1
34		3
46		2
48		1
54		1
55		1
56		5
		$n = 30$

(b) In a stem – and – leaf plot, the tens digit is used as a “stem” and the units as a “leaf”.  
Ensure that the “leaves” are evenly spaced.

1	6	7	7	7	7	8	8		
2	5	5	7	8	8	8	8	8	8
3	2	4	4	4					
4	6	6	8						
5	4	5	6	6	6	6	6		

(c) The individual ages are the independent variable (horizontal axis) and the frequency on the dependent variable (vertical axis).



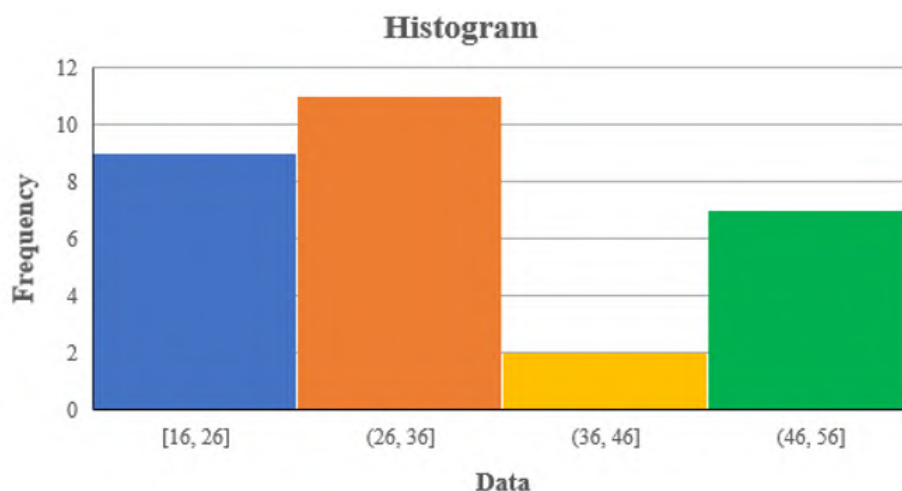
(d) Age – 28

(e) Percentage =  $\frac{6}{30} \times 100 = 20\%$

(f)

Class intervals	Frequency
$16 \leq x \leq 26$	9
$26 < x \leq 36$	11
$36 < x \leq 46$	2
$46 < x \leq 56$	7

(g)



(h)  $(26, 36]$  or  $26 < x \leq 36$

**ACTIVITIES/ASSESSMENT**

3.1.1 The number of air conditioners sold by fifty sales representatives in the year 2015 are recorded below.

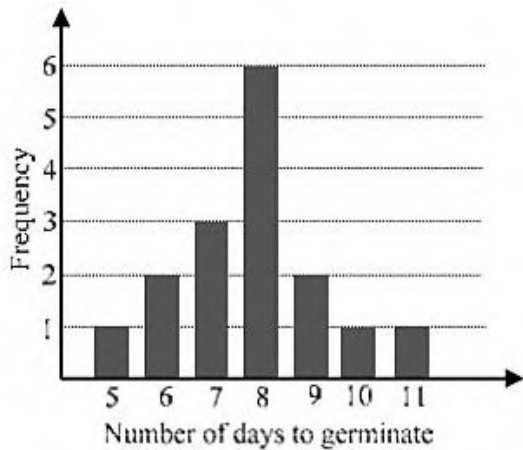
25	22	19	27	27	19	23	21	14	12
13	13	9	4	21	18	30	31	28	21
20	3	7	14	14	9	7	27	21	39
18	22	27	30	23	14	14	14	8	1
3	14	4	18	5	24	20	8	10	8



- Draw a stem – and – leaf plot for this data.
- Draw a frequency bar graph for this data.
- How many agents sold twenty or more air conditioners?
- What percentage of the agents sold less than 20 air conditioners?
- Draw a histogram for this data.



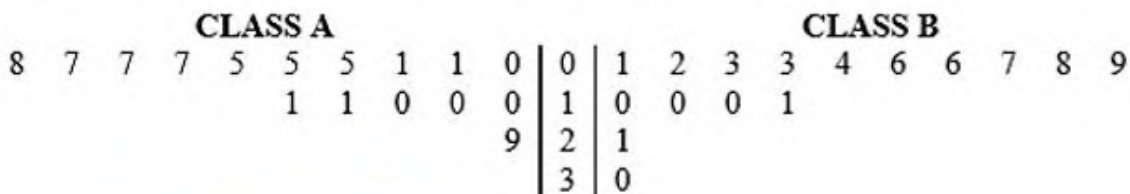
3.1.2 Seeds of a particular variety of flower were sown. All germinated, but not at the same time. The following bar graph shows how many seeds germinated after various number of days.



- How many seeds were sown?
- After how many days did the first seed germinate?
- What percentage of seeds germinated within the first 8 days?

### 3.1.3

The following back-to-back stem-and-leaf diagram shows the average number of hours spent per week on social networking websites by learners from two different classes.



- How many learners are there in Class A?
- How many learners are there in Class B?
- How many learners in Class A spend exactly one hour per week on a social networking website?
- How many learners in Class B spend more than five hours per week on a social networking website?
- Which class spends more time, in total, on a social networking website?

TOPIC: Data Handling/Statistics (Lesson 2)	Weighting	$\pm 15\%$	Grade	10
Sub-topics	Measures of central tendency (Values that tell you where the middle (center) of the data set lies).			
<b>RESOURCES</b>				
Grade 10 textbooks including these: <ul style="list-style-type: none"> <li>• Mind Action Series; (pages: 223 – 228)</li> <li>• Platinum; (pages: 224 – 226)</li> <li>• Grade 10 previous examination question papers</li> </ul>				
<b>Example 1</b>				
1.1 Determine the: <ol style="list-style-type: none"> <li>mean(average),</li> <li>median and,</li> <li>mode of the following sets of data: <ol style="list-style-type: none"> <li>1.1.1 4 ; 13 ; 5 ; 7 ; 9 ; 6 ; 5</li> <li>1.1.2 9 ; 1 ; 4 ; 4 ; 2 ; 8 ; 6 ; 2 ; 6 ; 6</li> </ol> </li> </ol>				
<b>Solutions to example 1:</b>				
1.1.1 <b>Remember!</b> Arrange the data in ascending order. 4 ; 5 ; 5 ; 6 ; 7 ; 9 ; 13				
a) Mean $(\bar{x}) = \frac{4+5+5+6+7+9+13}{7} = 7$				
b) <b>There is an odd number of values and therefore the median will be part of the data set.</b>				
The position of the median = $\frac{1}{2}(7+1) = 4$ th position.				
The median is <b>6</b> , dividing the data into two equal halves. 4 ; 5 ; 5 ; 6 ; 7 ; 9 ; 13				
c) Mode is 5. This data item appears twice. (which is more than any other data item).				
1.1.2 <b>Remember!</b> Arrange the data in ascending order. 1; 2 ; 2 ; 4 ; 4 ; 6 ; 6 ; 6 ; 8 ; 9				
a) Mean $(\bar{x}) = \frac{1+2+2+4+4+6+6+6+8+9}{10} = 4,8$				
There is an even number of data items.				
The position of the median = $\frac{1}{2}(10+1) = 5,5$ th position. This means the median is the average between the 5 <sup>th</sup> and 6 <sup>th</sup> data items.				

1; 2; 2; 4; 4; 6; 6; 6; 8; 9

- b) Median =  $\frac{4+6}{2} = 5$ . It is not part of the original data set but it divides the data set into two equal halves.
- c) Mode is 6. Most frequently appearing data item.

### ACTIVITY 3.2

3.2.1 A dairy farmer has 32 cows for sale. The weights of these cows in kilograms are recorded below. The total weight of the cows is 5 060 kg.

80	82	83	83	84	85	85	86
86	87	87	88	88	89	90	92
92	93	94	95	97	153	153	154
155	321	371	376	377	381	382	391



- a) Calculate the mean and the median.
- b) The farmer describes the cows to a buyer and states that the average weight is over 158 kg. Which measure of central tendency did the farmer use to describe the cows, and does this measure describe the cows fairly?

3.2.2 The following stem-and-leaf diagram represents the ages of forty people who donated blood. Refer to exercise 1 number c). The total of all the ages is 1 544.

1		7	7	8	8	9	9				
2		0	2	4	4	6	7	7			
3		4	5	5	5	5	5	5	9	9	
4		1	2	2	3	7	8				
5		0	3	3	4	5	7				
6		3	4	5	6	6					

- a) Calculate the mean, median and mode for this data.
- b) Comment on the usefulness of these measures of central tendency.
- 3.2.3
- a) The mean of 3; 4; 8; 9;  $x$ , is 7. Determine  $x$ .
- b) The median of five consecutive natural numbers is 12. What is the mean?
- c) The numbers 4; 6; 8; 9;  $x$  are arranged from smallest to largest. If the mean and the median are equal, determine  $x$ .
- d) The mean of five numbers is 27. The numbers are in the ratio 1: 2 : 3 : 4 : 5. Determine the five numbers.
- e) Write down three possible sets of five numbers such that the median is 4, the mean

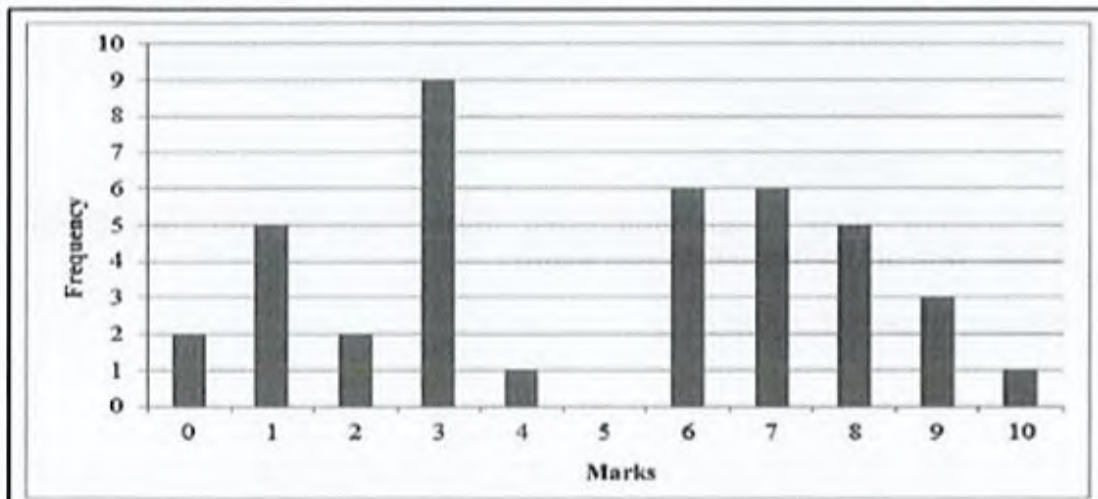
is 5 and the mode is 3.

- f) The mean of six numbers is 44 and the mean of five of these numbers is 46. What is the sixth number?

**ASSESSMENT 3.2 (DBE Nov 2019, Question 1.2)**

3.2.1 Learners in a certain class wrote a Mathematics test that had a maximum mark of 10. The teacher represented the marks obtained by the learners of this class in the bar graph below.

**Bar graph showing distribution of marks scored in Mathematics test**



- How many learners scored 8 marks out of ten for the test?
- How many learners are in this class?
- Calculate the range of the marks scored in the test.
- If the pass mark for the test was fifty percent, what percentage of learners failed the test?
- Calculate the mean mark scored in the test.

TOPIC: Data Handling/Statistics (lesson 3)		Weighting	±15%	Grade	10
Sub-topic(s)	<ul style="list-style-type: none"> <li>• Quartiles</li> <li>• Percentiles</li> </ul>				
<b>RESOURCES</b>					
Grade 10 textbooks including these: <ul style="list-style-type: none"> <li>• Mind Action Series; (pages: 228 – 233)</li> <li>• Platinum; (pages: 231 – 233)</li> <li>• Previous question papers.</li> </ul>					
<b>Examples:</b>					
1. Determine the quartiles for the following sets of data: <ol style="list-style-type: none"> <li>1 4 6 7 11 13 15 15 20</li> <li>6 7 9 9 10 12 16 21 23 26 27 28 30 32</li> </ol>					
<b>Solutions:</b>					

a) The position of the median  $= \frac{1}{2}(9+1) = 5$

The median is 11.

The lower half of the data set is: 1 ; 4 ; 6 ; 7

The lower quartile is the median of the lower half. The position of the lower quartile  $= \frac{1}{2}(4+1) = 2,5$ . The lower quartile is the average between the 2<sup>nd</sup> and 3<sup>rd</sup> values.

$$Q_1 = \frac{4+6}{2} = 5$$

The upper half of the data set is: 13 ; 15 ; 15 ; 20

The upper quartile is the median of the upper half. The position of the upper quartile  $= \frac{1}{2}(4+1) = 2,5$ . The upper quartile is the average between the 2<sup>nd</sup> and 3<sup>rd</sup> values.

$$Q_3 = \frac{15+15}{2} = 15$$

a) The position of the median  $= \frac{1}{2}(14+1) = 7,5$

The median is the average between the 7<sup>th</sup> and 8<sup>th</sup> values.

$$Q_2 = \frac{16+21}{2} = 18,5$$

The lower half of the data set is: 6 ; 7 ; 9 ; 9 ; 10 ; 12 ; 16

The lower quartile is the median of the lower half. The position of the lower quartile  $= \frac{1}{2}(7+1) = 4$ .

$$Q_1 = 9$$

The upper half of the data set is: 21 ; 23 ; 26 ; 27 ; 28 ; 30 ; 32

The upper quartile is the median of the upper half. The position of the upper quartile  $= \frac{1}{2}(7+1) = 4$

$$Q_3 = 27$$

2.

A Maths professor at a university posted a list of marks, without names, on the notice board outside his office. The students were informed as to the percentile they were in. There are 45 students in his class and the marks are as follows:

66	86	65	78	32	52	69	85	87	28	90	98	73	64	56
58	78	65	50	36	67	55	72	57	64	70	92	95	33	32
24	42	54	55	54	68	65	88	80	84	68	61	75	76	82

- (a) Jaco scored in the 70th percentile. What is his mark?  
 (b) Michael scored in the 20th percentile. What is his mark?  
 (c) Dimpho scored in the 50th percentile. What is her mark?

**Solutions:**

- (a) Arrange the marks in ascending order:

24 28 32 32 33 36 42 50 52 54 54 55 55 56 57  
 58 61 64 64 65 65 65 66 67 68 68 69 70 72 73  
 75 76 78 78 80 82 84 85 86 87 88 90 92 95 98

$$i = \frac{70}{100} \times (45) = 31,5$$

All we now do is round this number up to 32 and the 70th percentile is the 32nd mark which is 76%. Jaco therefore obtained 76% and scored better than 70% of all students.

- (b)
- $i = \frac{20}{100} \times (45) = 9$

The 20th percentile is the average between the 9th and 10th mark:  $\frac{52+54}{2} = 53$

There is no score of 53 in the data and therefore Michael will have obtained 54% and will have scored better than 20% of all students.

- (c)
- $i = \frac{50}{100} \times (45) = 22,5$

The 50th percentile is the 23rd mark which is 66.

Dimpho obtained 66% and scored better than 50% of all students.

**ACTIVITY 3.3:**

- 1.3.1 A teacher records the following results for an examination out of 100:

98 63 79 76 58 71 86 78 91 87  
 89 41 19 88 41 99 97 83 78 90

Determine the quartiles for this data.

- 3.3.2 Five data values are represented as follows:
- $2x$
- ;
- $x+1$
- ;
- $x+2$
- ;
- $x-3$
- ;
- $2x-2$

- Determine the value of  $x$  if the mean of the data is 15.
- Calculate the quartiles.

- 1.3.2 Tobacco use is a leading cause of death in the United States. Nicotine found in tobacco is rapidly metabolized in the liver to a substance called cotinine. The levels of cotinine in the body are measured in nanograms per milliliter (ng/ml). A nanogram is one – billionth of a gram. Consider the following cotinine levels of 50 smokers:

5	6	6	8	22	40	43	44	48	86
88	103	113	122	123	130	131	149	165	168
174	174	198	208	210	223	224	227	233	245
249	250	253	265	267	277	280	284	286	289
290	313	313	314	350	360	401	460	476	490

- Calculate the 25<sup>th</sup>, 50<sup>th</sup> and 75<sup>th</sup> percentiles.
- Calculate the 30<sup>th</sup> percentile.
- Calculate the 65<sup>th</sup> percentile.
- Calculate the 80<sup>th</sup> percentile.

<b>TOPIC: Data Handling/Statistics (Lesson 4)</b>	<b>Weighting</b>	<b>±15</b>	<b>Grade</b>	<b>10</b>
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<b>Sub-topics</b>	Five number summary and Box – and – Whisker plots
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**RESOURCES**

- Grade 10 textbooks including these:
- Mind Action Series; (pages: 234 -236)
  - Classroom Mathematics; (pages: 313 – 317)
  - Platinum; (pages: 236 – 237)
  - Grade 10 previous examination papers

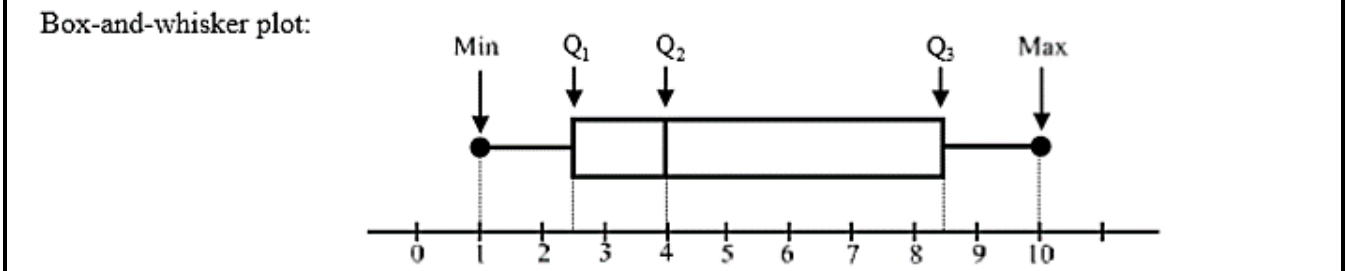
**Example 1:**

Determine the five-number summary and draw a box-and-whisker plot for the following data:  
 1 2 2 2 3 3 4 4 4 6 7 8 8 9 10 10 10

**Solution**

Five number summary:

Minimum: 1                      Maximum: 10  
 Lower Quartile ( $Q_1$ ):                       $Q_1 = \frac{2+3}{2} = 2,5$   
 Median (M or  $Q_2$ ):                       $Q_2 = 4$   
 Upper Quartile ( $Q_3$ ):                       $Q_3 = \frac{8+9}{2} = 8,5$

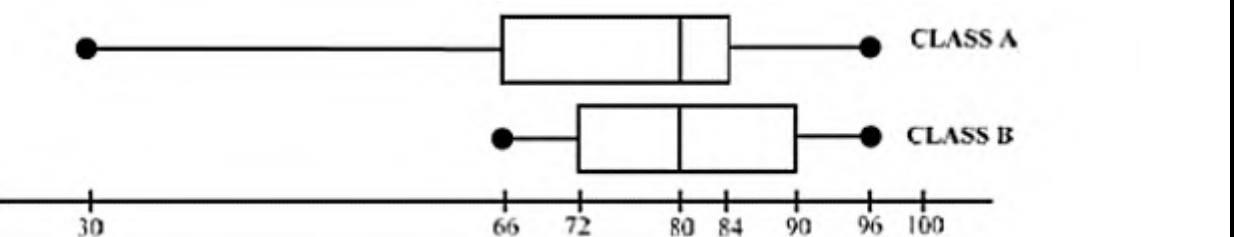


**ACTIVITY 3.4**

3.4.1 Consider the following data: 10 8 1 6 8 8 10 2 8 10 8 8

- Determine the five-number summary.
- Draw a box-and-whisker diagram.
- Calculate the mean.
- How is the data distributed? Explain.

3.4.2 The box-and-whisker plots below summarize the final test scores for two Mathematics classes from the same grade.



- a) Describe the features in the scores that are the same for both classes.
- b) The head of department considers the median of each class and reports that there is no significant difference in the performance between them. Is this conclusion valid? Support your answer with reasons.

TOPIC: Data Handling/Statistics (lesson 5)	Weighting	±15%	Grade	10
Sub-topic(s)	<ul style="list-style-type: none"> <li>Measures of dispersion (values that show how spread out or grouped the data are).</li> <li>Outliers (a value or measurement that is significantly higher or lower than the rest of the data).</li> </ul>			
<b>RESOURCES</b>				
Grade10 textbooks including these: <ul style="list-style-type: none"> <li>Mind Action Series; (pages:237 – 238)</li> <li>Classroom Mathematics; (pages:307 – 312)</li> <li>Platinum; (pages:231 – 235)</li> <li>Previous exam papers</li> </ul>				
<b>Example(s):</b>				
1. Determine the range, interquartile range and semi – interquartile range for the following data set. 10 8 8 2 1 10 8 8 6 8 10 8				
<b>Solutions:</b> 1 2 6 8 8 8 8 8 8 10 10 10 Range = max – min = 10 – 1 = 9 Lower quartile: $Q_1 = \frac{6+8}{2} = 7$ Upper quartile: $Q_3 = \frac{8+10}{2} = 9$ IQR = $Q_3 - Q_1 = 9 - 7 = 2$ Semi – interquartile range = $\frac{1}{2}(Q_3 - Q_1) = \frac{1}{2}(2) = 1$				
<b>ACTIVITY 3.5:</b>				
3.5.1. Determine the range, interquartile range and semi – interquartile range for the following data set. 2 2 2 4 4 6 6 8 8 10 10 10				
3.5.2. Consider the two classes A and B in the previous activity no <b>b</b> ). a) Compare the classes in terms of the range and interquartile range. b) Compare the classes in terms of the semi – interquartile range.				
3.5.3 Six data values are represented as follows: $3x; x + 4; 2x + 2; 5x; 4x + 1; 6x + 2$ a) Calculate the value of $x$ if the mean is 12. b) Determine the interquartile range.				
3.5.4 The table below contains the mean, median and range of the Mathematics final exam for a large group of students.				



Mean	Median	Range
56	51	86


The Mathematics teacher added 3 marks to each of the students' marks. Write down the mean, median and the range of the new set of Mathematics.

**ASSESSMENT 3.5: DBE Nov 2015**

Nineteen girls were required to complete a puzzle as quickly as possible. Their times (in seconds) were recorded and are shown in the table below.

14	15	16	16	17	17	18	18	19	19
19	20	21	21	22	23	24	24	29	

- 3.5.1 Identify the median time taken by the girls to complete the puzzle.
- 3.5.2 Determine the lower and upper quartiles for the data.
- 3.5.3 Draw a box – and – whisker diagram to represent the data.
- 3.5.4 The five-number summary of the time (in seconds) taken by 19 boys to complete the same puzzle is (15 ; 19 ; 23 ; 26 ; 30)
  - a) Calculate the Interquartile range for the time taken by the boys.
  - b) If only one boy took 19 seconds to complete the puzzle, what percentage of the boys took at least 19 seconds to complete the puzzle?
- 3.5.5 In which group, the girls or the boys, did a larger number of learners complete the puzzle in less than 23 seconds? Justify your answer.

<b>TOPIC: Data Handling/Statistics (Lesson 6)</b>	<b>Weighting</b>	<b>± 15%</b>	<b>Grade</b>	<b>10</b>																
<b>Sub-topics</b>	Grouped data																			
<b>RESOURCES</b>																				
Mind action Series; (pages: 239 – 241); Platinum; (pages: 226 – 230); Classroom Mathematics; (pages: 303 – 306)																				
<b>Example 1:</b>																				
<p>Medical science has always recognized human growth and height as an important measure of the health and wellness of individuals. Research into the average height of people in different counties revealed that the tallest race of humans is the Nilotic peoples of Sudan having an average height of 1,83 m. The tallest man currently living is Sultan Kosen from Turkey who measures 2,51 m. The average heights (ranging from 150 – 185 cm) of people in 135 countries have been grouped into class interval</p>																				
<table border="1"> <thead> <tr> <th>Class intervals (average heights in cm)</th> <th>Frequency (number of countries)</th> </tr> </thead> <tbody> <tr> <td><math>150 \leq x &lt; 155</math></td> <td>12</td> </tr> <tr> <td><math>155 \leq x &lt; 160</math></td> <td>15</td> </tr> <tr> <td><math>160 \leq x &lt; 165</math></td> <td>19</td> </tr> <tr> <td><math>165 \leq x &lt; 170</math></td> <td>25</td> </tr> <tr> <td><math>170 \leq x &lt; 175</math></td> <td>33</td> </tr> <tr> <td><math>175 \leq x &lt; 180</math></td> <td>22</td> </tr> <tr> <td><math>180 \leq x &lt; 185</math></td> <td>9</td> </tr> </tbody> </table>		Class intervals (average heights in cm)	Frequency (number of countries)	$150 \leq x < 155$	12	$155 \leq x < 160$	15	$160 \leq x < 165$	19	$165 \leq x < 170$	25	$170 \leq x < 175$	33	$175 \leq x < 180$	22	$180 \leq x < 185$	9			
Class intervals (average heights in cm)	Frequency (number of countries)																			
$150 \leq x < 155$	12																			
$155 \leq x < 160$	15																			
$160 \leq x < 165$	19																			
$165 \leq x < 170$	25																			
$170 \leq x < 175$	33																			
$175 \leq x < 180$	22																			
$180 \leq x < 185$	9																			

- (a) Calculate an estimated value for the mean.
- (b) What is the modal class?
- (c) In which class interval does the median lie?

**Solution to example 1:**

(a)

Class intervals	Frequency	Midpoint	Freq × Midpt
$150 \leq x < 155$	12	$\frac{150+155}{2} = 152,5$	$12 \times 152,5 = 1\ 830$
$155 \leq x < 160$	15	$\frac{155+160}{2} = 157,5$	$15 \times 157,5 = 2\ 362,5$
$160 \leq x < 165$	19	$\frac{160+165}{2} = 162,5$	$19 \times 162,5 = 3\ 087,5$
$165 \leq x < 170$	25	$\frac{165+170}{2} = 167,5$	$25 \times 167,5 = 4\ 187,5$
$170 \leq x < 175$	33	$\frac{170+175}{2} = 172,5$	$33 \times 172,5 = 5\ 692,5$
$175 \leq x < 180$	22	$\frac{175+180}{2} = 177,5$	$22 \times 177,5 = 3\ 905$
$180 \leq x < 185$	9	$\frac{180+185}{2} = 182,5$	$9 \times 182,5 = 1\ 642,5$
<b>Totals</b>	<b>135</b>		<b>22 707,5</b>

$$\text{Estimated mean} = \frac{22\ 707,5}{135} = 168,2037037 \text{ cm}$$

**Note:** The upper boundary of the class interval  $150 \leq x < 155$  can have a value that is extremely close to 155. There may be a height of 154,999999...  
It therefore makes sense that the average of the class interval is the average of the lower and upper boundaries.

- (b) Since  $170 \leq x < 175$  contains the highest frequency of heights, this class interval will be the **modal class**.
- (c) It is not possible to determine the actual median. There are 135 values and therefore the position of the median is  $\frac{1}{2}(135+1) = 68$ . The 68th value lies in the class interval  $165 \leq x < 170$  (there are 46 values below 165 and 71 below 170).  
This class interval is called the **median class**.

**ACTIVITY 3.6**

3.6.1

In a research survey, a gym measured the weights (in kg) of a number of members.

Class intervals (weights in kg)	Frequency (number of members)
$30 \leq x < 35$	11
$35 \leq x < 40$	13
$40 \leq x < 45$	15
$45 \leq x < 50$	17
$50 \leq x < 55$	19
$55 \leq x < 60$	26
$60 \leq x \leq 65$	36



- a) Calculate the estimated mean.
- b) What is the modal class?
- c) In which class interval does the median lie?

3.6.2

a) The raw data below shows an athlete's different times in seconds in the 400 m.

43,0 43,1 45,3 44,8 44,9 46,3 44,8 46,3 46,1  
 45,4 44,7 43,1 44,9 45,3 45,2 45,5 45,6 45,0  
 45,1 46,2 45,9 43,2 43,3 43,8 43,9 43,7 45,3  
 45,7 44,7 46,2 45,7 44,9 45,0 45,5 46,0 46,9

- i) Draw a stem-and-leaf diagram for this data.
- ii) Complete the following table.

Class interval	Frequency
$43,0 \leq x < 44,0$	
$44,0 \leq x < 45,0$	
$45,0 \leq x < 46,0$	
$46,0 \leq x < 47,0$	

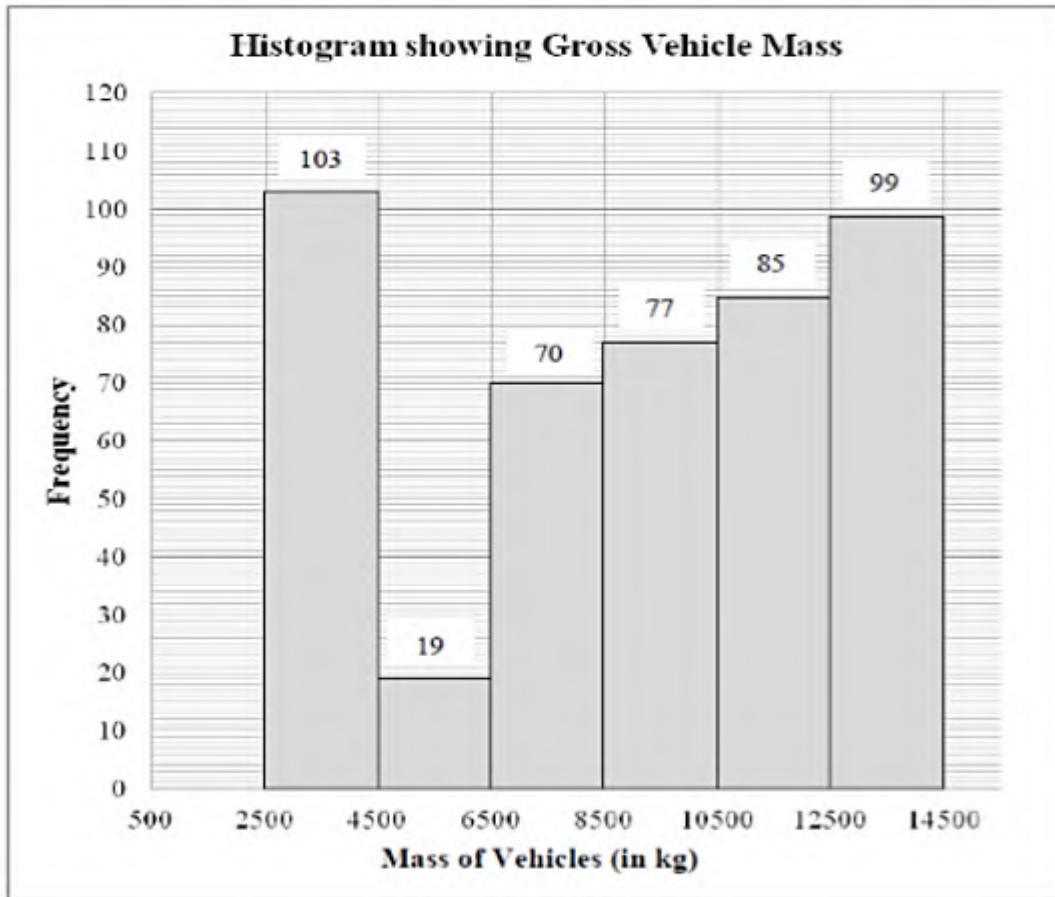


- iii) Calculate the actual mean, median and mode for this data.
- iv) Calculate the range and the inter-quartile range.
- v) Draw a box-and-whisker plot for the data.

### ASSESSMENT 3.6

#### 3.6.1 DBE EXAMPLAR 2012, QUESTION 2

The traffic authorities are concerned that heavy vehicles (trucks) are often overloaded. In order to deal with this problem, a number of weigh bridges have been set-up along the major routes in South Africa. The gross (total) vehicle mass is measured at these weigh bridges. The histogram below shows the data collected at a weigh bridge over a month.



- Write down the modal class of the data.
- Estimate the mean gross vehicle for the month.
- Which measures of central tendency, the modal class or the estimated mean, will be most appropriate to describe the data set? Explain your choice.

3.6.2 DBE Nov 2012, QUESTION 2

The intelligence quotient score (IQ) of a grade 10 class is summarized in a table below.

IQ INTERVAL	FREQUENCY
$90 \leq x < 100$	4
$100 \leq x < 110$	8
$110 \leq x < 120$	7
$120 \leq x < 130$	5
$130 \leq x < 140$	4
$140 \leq x < 150$	2

- Write down the modal class of the data.
- Determine the interval in which the median lies.
- Estimate the mean IQ score of this class of learners.

## 4. PROBABILITY

TOPIC : Probability 1	Weighting		Grade	9
Sub-topics	Introductory Probability			
<b>RESOURCES</b>				
<ul style="list-style-type: none"> <li>National Education Collaboration Trust (NECT)</li> </ul>				
<b>NOTES</b>				
<b>Probability</b> = $\frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$				
<b><u>FACTS ABOUT PROBABILITY</u></b>				
<ul style="list-style-type: none"> <li>The probability of an event that cannot occur is 0</li> <li>The probability of an event that is certain to occur is 1</li> <li>Every probability is a number between 0 and 1 inclusive. the probability assigned probability <b>can never be negative.</b></li> <li>The sum of the probabilities of all possible outcomes in experiment is <b>always 1</b></li> <li>A probability of 0.5 or 50%, means the event is just <b>likely</b> to happen as not to happen</li> </ul>				
<b><u>THE PROBABILITY SCALE</u></b>				
<p>The sum of probabilities of all possible outcomes of an experiment is equal to one (1)</p>				
<b><u>WAYS TO DESCRIBE PROBABILITY</u></b>				
<ol style="list-style-type: none"> <li><b>Certain</b> – the event is definitely going to happen <i>e.g The probability that the day after Monday is Tuesday is 1.</i></li> <li><b>Likely</b> – the event will probably happen, but not definitely</li> <li><b>Unlikely</b> – the event will probably not happen, but it might</li> <li><b>Impossible</b> – the event is definitely not going to happen <i>e.g The probability of getting a 7 when you throw an ordinary dice is 0.</i></li> </ol>				
<b><u>HOW TO EXPRESS PROBABILITY (NUMERICAL)</u></b>				
<ol style="list-style-type: none"> <li>As a fraction, e.g. <math>\frac{1}{2}</math></li> <li>As a decimal number, e.g. 0,5</li> <li>As a percentage, e.g. 50 %</li> </ol>				

**TYPES OF PROBABILITIES**

**Relative Frequency (or Experimental Probability) & Theoretical**

- Sometimes we calculate probability and sometimes we estimate probability.
- Probability that is calculated is called **theoretical probability** or just probability.

✓ **1. Theoretical probability** =  $\frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

- Probability that is estimated is calculated after performing a very large number of trials of an experiment or conducting a survey involving a very large number of items, and is called **relative frequency**.

**2. Relative frequency probability** =  $\frac{\text{Number of times the event happens}}{\text{The total number of trials in experiment}}$

**ACTIVITIES / ASSESSMENT**

- 4.1.1 Write down how you can calculate the theoretical probability.
- 4.1.2 Write down how you can calculate relative frequency probability.
- 4.1.3 Briefly define what term probability is.
- 4.1.4 Define the sample space.
- 4.1.5 What do we mean by impossible events?

<b>TOPIC :</b> Probability 2	Weighting		Grade	9
<b>Sub-topics</b>	Relative Frequency			

**RESOURCES**

Clever Gr 9

**NOTES**

Identify the sample space.  
Identify the different events.

**Example:**

Seven cards consist of 3 red and 4 yellow. The card is picked and replaced in a box.

- 1.1 Find the probability of picking a yellow card.
- 1.2 In the 15 trials (picking of cards), how many times will you expect the yellow card?
- 1.3 If the trials are 24, how many times will you expect the red card?

**Solutions :**

1.1  $P(Y) = \frac{4}{7}$

1.2  $\frac{4}{7} \times 15$   
= 8,57  
 $\cong 9$  times

1.3  $\frac{3}{7} \times 24$   
= 10,29  $\cong 10$  times

**ACTIVITIES / ASSESSMENT**

- 4.2.1 A coin is flipped 500 times. Discuss the relative frequency on the coin falling on heads.

4.2.2 A die is tossed 240 times. Determine the relative frequency of the die landing on:

4.2.2a) a six

4.2.2b) an even number

4.2.2c) an odd number

4.2.2d) a number smaller than 4

4.2.2e) a number larger than 3.

4.2.3 A bag contains 50 coloured buttons:

15 green, 5 white, 17 brown, 3 red and 10 blue buttons. A button is taken from the bag. The colour of the button is noted down and it is replaced. This is done 300 times.

4.2.3a) How many times do you expect a button taken from the bag to be a red button?

4.2.3b) What do you expect the relative frequency of taking out a brown button to be?

4.2.3c) What is the probability of a button taken from the bag not being a white button?

4.2.3d) How many times do you expect a button taken from the bag will be a green or a white button?

<b>TOPIC :</b> Probability 3	<b>Weighting</b>		<b>Date :</b>	
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<b>Sub-topics</b>	Relative Frequency versus Theoretical Probability
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**RESOURCES**

Clever Gr 9

**NOTES**

A bag contains 5 red balls, 6 black balls, 3 yellow balls, and 6 purple balls. A ball is taken out of the bag. The colour is noted down and the ball is replaced. This is done 100 times. The following frequency table is completed.

<b>Outcome</b>	<b>Frequency</b>
Red	24
Black	30
Yellow	12
Purple	34
<b>Total</b>	<b>100</b>

1.1 Calculate the theoretical probability of a purple ball being removed from the bag.

1.2 Calculate the relative frequency (experimental probability) for a ball taken from the bag being purple.

1.3 Explain the relationship between these two numbers.

1.4 Calculate the theoretical probability of drawing a red, black or yellow ball from the bag. What is the answer

when you add together the probabilities of taking out a red, black, yellow and purple ball?

1.5 Calculate the experimental probabilities of the red, black or yellow balls. Add these probabilities together. What is the answer?

Solutions :

$$1.1 P(\text{purple}) = \frac{6}{20}$$

$$= \frac{3}{10} \text{ OR } (0,3)$$

$$1.2 \text{ Relative frequency (purple)} = \frac{34}{100}$$

$$= \frac{17}{50} \text{ OR } (0,34)$$

1.3 The two numbers are almost the same even though they are NOT exactly the same. The

experimental answer gives exactly what happened in a number of trials. The theoretical one just gives the chances that are possible to take place BUT not meaning that chances are certain chances.

$$1.4 \quad P(\text{red}) = \frac{5}{20} \\ = \frac{1}{4}$$

$$P(\text{black}) = \frac{6}{20} \\ = \frac{3}{10}$$

$$P(\text{yellow}) = \frac{3}{20}$$

Adding all the answers:  $\frac{1}{4} + \frac{3}{10} + \frac{3}{20} + \frac{3}{10} = 1$

$$1.5 \quad P(\text{red}) = \frac{24}{100} \\ = \frac{6}{25}$$

$$P(\text{black}) = \frac{30}{100} \\ = \frac{3}{10}$$

$$P(\text{yellow}) = \frac{12}{100} \\ = \frac{3}{25}$$

Adding all the answers:  $\frac{6}{25} + \frac{3}{10} + \frac{3}{25} + \frac{17}{50} = 1$

### ACTIVITIES / ASSESSMENT

4.3.1 A die is cast 60 times. The following frequency table is completed during the experiment.

Outcome	Frequency
1	9
2	10
3	11
4	11
5	8
6	11

4.3.1a) What is the theoretical probability of the die landing a six?

4.3.1b) What is an experimental probability (relative frequency) of the die landing on a six according to the table?

4.3.1c) What is the relationship between these numbers?

4.3.2

4.3.2a) What is the theoretical probability of a new born baby being a boy?

4.3.2b) In a hospital, 500 babies are born, of which 210 were girls. What is the experimental probability of a baby being a boy in this hospital?

<b>TOPIC</b> : Probability 4	Weighting		Grade	
<b>Sub-topics</b>	COMPOUND EVENTS (Tree Diagram)			
<b>RESOURCES</b>				
<ul style="list-style-type: none"> <li>Maths Handbook and Study Guide Gr 9, by Kevin Smith</li> </ul>				

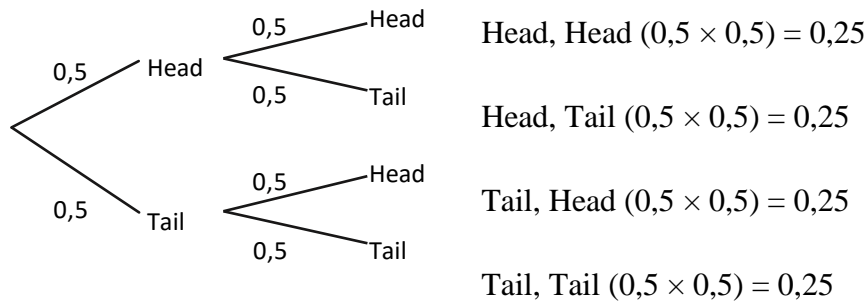


**NOTES**

- When drawing a tree diagram each branch represents the probability of a certain outcome. Each set of branches represents an event.

Example

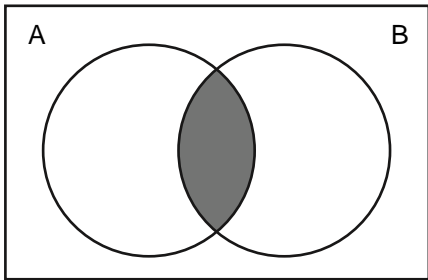
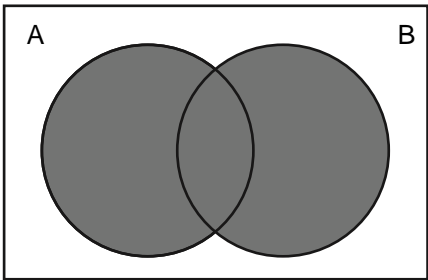
A fair coin is tossed twice. Draw a tree diagram to show all the possible outcomes.

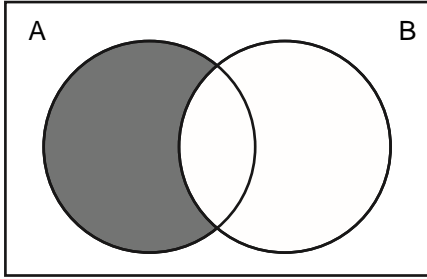
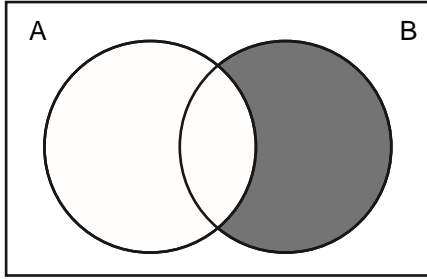
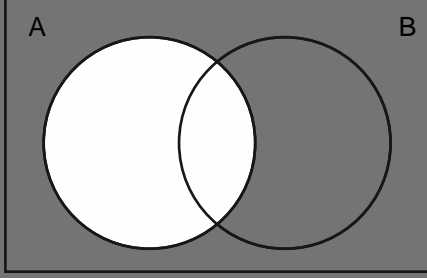
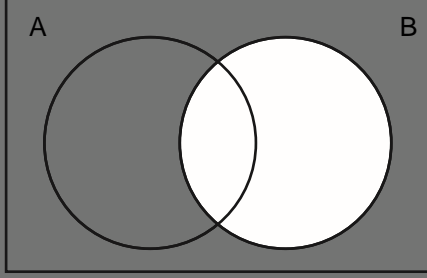
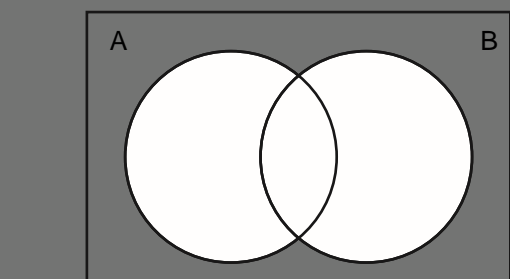


**ACTIVITIES/ ASSESSMENT**

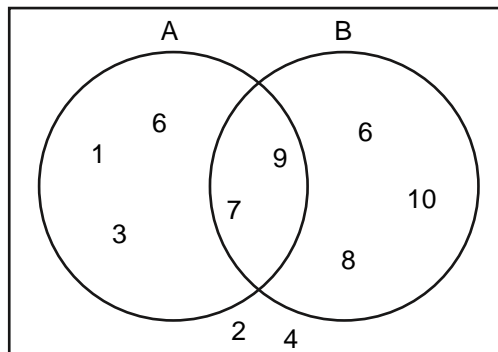
- 4.4.1 A woman would like to have two children. What is the probability that she will have two boys? (Use a tree diagram to determine your answer).
- 4.4.2 The coin is tossed followed by spinning the spinner. The coin is weighted so that it is more likely to land on heads than tails. The probability of it landing on heads is 0,75. The spinner is equally likely to land on 1 or 2.
- 4.4.2a) What is the probability of obtaining a tail in a single toss?
- 4.4.2b) Draw a tree diagram to show all the possible outcomes.
- 4.4.2c) What is the probability of obtaining a 1 in the above experiment?
- 4.4.2d) What is the probability of obtaining a tail or a 1 in the above experiment?
- 4.4.3 Zama has two bags. One bag contains 5 purple balls, 3 green balls and 2 red balls. The other bag contains 5 balls with the letter A and 3 balls with the letter B on it. A ball is drawn from the bag with coloured balls and then from the bag with lettered balls.
- 4.4.3a) If a ball is randomly selected from bag 1, determine the probability of obtaining purple ball.
- 4.4.3b) If a ball is randomly selected from bag 2, determine the probability of obtaining the letter A and also that of obtaining letter B.
- 4.4.3c) Draw a tree diagram to show all the possible outcomes of the experiment.
- 4.4.3d) Determine the probability of obtaining a red ball.
- 4.4.3e) Determine the probability of obtaining the letter A or a green ball.
- 4.4.4 There is a 25% chance that it will rain on Saturday. If it rains on Saturday there is an 80% chance that it will rain on Sunday. If it does not rain on Saturday there is a 10% chance that it will rain on Sunday.
- 4.4.4a) What is the probability that it does not rain on Saturday?
- 4.4.4b) Draw a tree diagram to show all the possibilities of rain on Saturday and Sunday.
- 4.4.4c) What is the probability that it does not rain on Sunday?

<b>TOPIC</b> : Probability 5	Weighting		Grade	
<b>Sub-topics</b>	COMPOUND EVENTS (Two-way table)			
<b>RESOURCES</b>				
Clever Gr 9				
<b>NOTES</b>				
<u>Example:</u> A woman would like to have two children. Represent this information in a two-way table and find the probability that the two children will be a girl and a boy in any order.				
<u>Solution</u>				
		Boy (B)	Girl (G)	
Boy (B)		BB	BG	
Girl (G)		GB	GG	
$P(G, B) = \frac{2}{4}$				
$P(G, B) = \frac{1}{2}$				
<b>ACTIVITIES/ASSESSMENT</b>				
4.5.1 A coin is spun twice. Use a two-way table to determine the following probabilities:				
4.5.1a) Head followed by head				
4.5.1b) Head followed by tail.				
4.5.1c) Tail followed by tail.				
4.5.2 Consider a fair die being thrown twice.				
(a) Draw a two-way table to show all the possible outcomes.				
(b) Find a probability that a die lands on six in two-way consecutive throws.				

<b>TOPIC</b> Probability 6	Weighting		Grade	
<b>Sub-topics</b>	Venn Diagram			
<b>RESOURCES</b>				
<ul style="list-style-type: none"> <li>National Education Collaboration Trust (NECT)</li> </ul>				
<b>NOTES</b>				
<ul style="list-style-type: none"> <li>The following diagrams are used to discuss the terms: intersection, union, A only, B only, not A, not B and not A or B.</li> </ul>				
Intersection (A and B)		Union (A or B)		
				

<p>A only</p> 	<p>B only</p> 
<p>not A or (A')</p> 	<p>not B or (B')</p> 
<p>not (A or B) or (A or B)'</p> 	

**ACTIVITIES/ASSESSMENT**

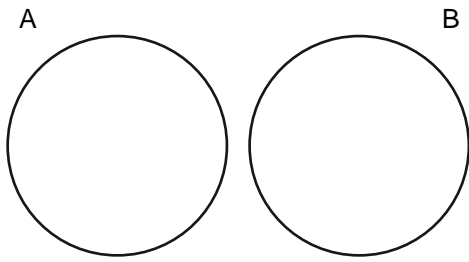


Complete a table such as the one below as the discussion progresses.

Notation	What does it mean?	Answer using the above Venn diagram
$n(S)$	The number of elements in the sample space, S.	4.6.1
$n(A)$	The number of elements in the set, A.	4.6.2
$n(B)$	The number of elements in the set, B.	4.6.3
$n(A \text{ or } B)$	The number of elements in the set, A or B.	4.6.4
$n(A \text{ and } B)$	The number of elements in the set, A and B.	4.6.5
$n(A')$	The number of elements NOT in the set, A.	4.6.6
$n(B')$	The number of elements NOT in the set, B.	4.6.7
$n(A \text{ or } B)'$	The number of elements NOT in the set, A or B.	4.6.8
$n(A \text{ and } B)'$	The number of elements NOT in the set, A and B.	4.6.9
<b>Probability</b>		
$P(S)$	The probability of an element being in the sample space	4.6.10
$P(A)$	The probability of an element being in set A	4.6.11
$P(B)$	The probability of an element being in set B	4.6.12
$P(A \text{ or } B)$	The probability of an element being in the set, A or B.	4.6.13
$P(A \text{ and } B)$	The probability of an element being in the set, A and B.	4.6.14
$P(A')$	The probability of an element NOT being in set, A.	4.6.15
$P(B')$	The probability of an element NOT being in set, B.	4.6.16
$P(A \text{ or } B)'$	The probability of an element NOT being in set, A or B	4.6.17
$P(A \text{ and } B)'$	The probability of an element NOT being in the set, A and B.	4.6.18

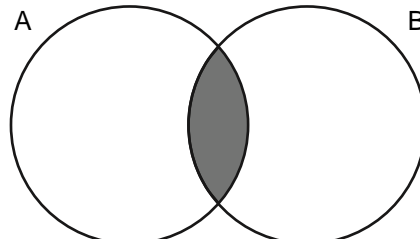
<b>TOPIC Probability 7</b>	Weighting		Grade	
<b>Sub-topics</b>	Mutually exclusive events			
<b>RESOURCES</b>				
<ul style="list-style-type: none"> <li>National Education Collaboration Trust (NECT)</li> </ul>				
<b>NOTES</b>				

Mutually exclusive events



- $P(A \text{ or } B) = P(A) + P(B)$

Non-mutually exclusive events/Inclusive events



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- $P(A \text{ and } B) = 0$

Consider the set of whole numbers from 1 to 10:

"1 2 3 4 5 6 7 8 9 10"

We will define two sets taken from this group of numbers:

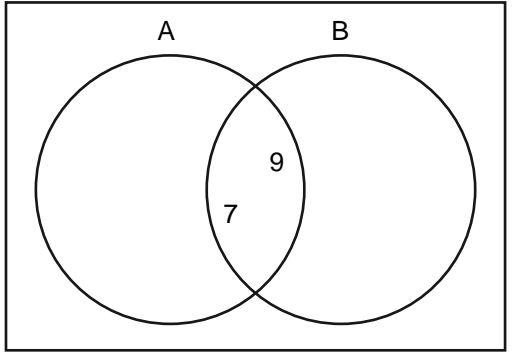
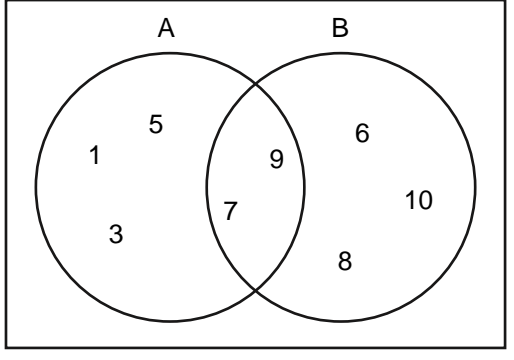
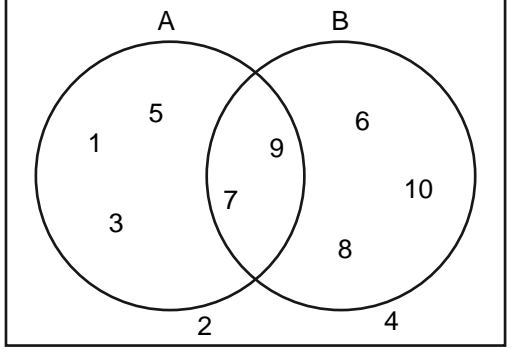
Set A = the odd numbers in the group = {1, 3, 5, 7, 9}

Set B = the numbers which are 6 or more in the group = {6, 7, 8, 9, 10}

Represent the information on a Venn diagram

<b>Step 1</b>	<p>Draw a frame to represent the sample space.</p> <p>This is important as it will represent the entire sample space – in this case, the set of whole numbers from 1 to 10.</p> <p>When the diagram is complete, all 10 numbers should have been used.</p>	
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<b>Step 2</b>	<p>There are two sets.</p> <p>Draw two overlapping circles inside the sample space frame.</p>	
<b>Step 3</b>	<p>Label the sets (circles) A and B.</p>	

<p><b>Step 4</b></p>	<p>Always start with the common elements of the two sets.</p> <p>Consider the numbers that belong in both set A and set B.</p> <p>Write the common numbers in the overlapping part – the intersection.</p>	
<p><b>Step 5</b></p> <p><b>Step 6</b></p>	<p>Complete set A by writing the other (three) numbers into set A.</p> <p>Complete set B by writing the other (three) numbers into set B.</p>	
<p><b>Step 7</b></p>	<p>Complete the diagram by writing any left-over numbers from the sample space which did not belong in either of the two sets.</p>	

**ACTIVITIES / ASSESSMENT**

4.7.1 In a certain class of 42 boys:

- 27 play hockey (H)
- 32 play soccer (S)
- 7 do not play hockey or soccer
- 14 play both hockey and soccer

4.7.1(a) Represent the information on the Venn diagram

4.7.2 In a small town of 1 500 people, there are two main banks, Acorn Savings and Nest Egg Bank. Most of the people in the town bank with one or both of these banks. 600 people bank with Acorn Savings and 950 bank with Nest Egg Bank. 235 people in the town do not use either Acorn Savings or Nest Egg Bank.

- (a) Draw a Venn diagram to illustrate the information given above.
- (b) What is the probability that a person from the town uses both banks?

- (c) What is the probability that a person from the town does not use Nest Egg Bank?
- (d) What is the probability that a person from the town uses Acorn Savings but not Nest Egg Bank?

<b>TOPIC Probability 8</b>	Weighting		Grade
<b>Sub-topics</b>	Complementary Events		

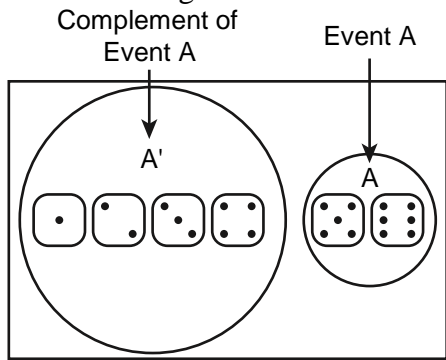
**RESOURCES**

- National Education Collaboration Trust (NECT)

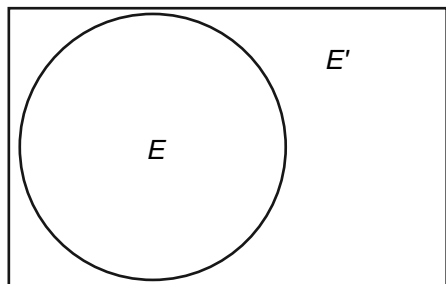
**NOTES**

- Example of throwing a die is used to illustrate further, that when the die is thrown, you would like to get a 5 or a 6.

Draw this diagram on the board:



- The notation used to show complementary events using the following diagram to illustrate:

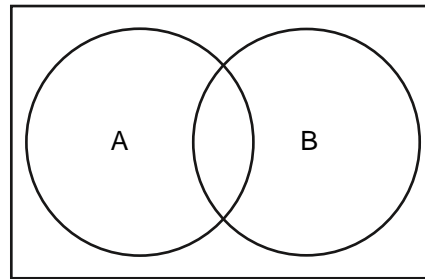
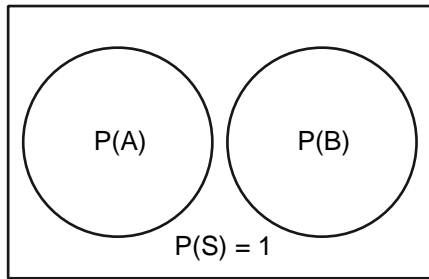


- The complement of event E is shown as E' – this means the set of all outcomes in the sample space that are not in event E.

$$P(E) + P(E') = 1$$

$$P(E) = 1 - P(E') \text{ OR } P(E') = 1 - P(E)$$

The following Venn diagrams to explain further:



Both situations represent exhaustive events even though one has an intersection and the other doesn't.

The key aspect is that there are no left-over elements in the sample space: all elements are in one set or the other.

Note: For events to be complementary, they must be mutually exclusive and exhaustive

**ACTIVITIES**

4.8.1 10 balls 1 to 10 are placed in a bag and one is chosen at random:

- 4.8.1a) Use the letter O to represent the event of getting an odd number and represent it in a set notation
- 4.8.1b) Use the letter E to represent the event of getting an even number and represent it in a set notation
- 4.8.1c) Draw a Venn diagram to represent the sample space(S) and the above events.
- 4.8.1d) Determine the probability of getting an even number
- 4.8.1e) Determine the probability of getting an odd number
- 4.8.1f) Are event O and E mutually exclusive? Justify your answer.
- 4.8.1g) Are event O and E complementary events? Justify your answer.

4.8.2 The numbers from 1 to 20 are written on cards and placed in a box. You choose a number out of the box at random.

- 4.8.2a) What is the probability that you choose a number greater than 20?
- 4.8.2b) What is the probability that you will choose a number larger than 15?
- 4.8.2c) What is the probability that you will choose a multiple of 3?

4.8.3 Answer the following questions about mutually exclusive events.

- 4.8.3a) What are mutually exclusive events? Give a simple example of two events that are mutually exclusive.
- 4.8.3b) If A and B are mutually exclusive and complementary events, then what is  $P(A) + P(B)$ ?
- 4.8.3c) If  $P(A) = P(B) = 0,4$ ,  $P(A \text{ or } B) = 0,9$  prove that A and B are not mutually exclusive.

<b>TOPIC Probability 9</b>	Weighting		Grade	
<b>Sub-topics</b>	Rules			

**RESOURCES**

- National Education Collaboration Trust (NECT)

**NOTES**

- What does mutually exclusive mean? (no intersection)



- What statement can be made from the identity learned about  $P(A \text{ or } B)$ , keeping in mind that there is no intersection?
- $P(A \text{ or } B) = P(A) + P(B)$
- Introduce  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$  using the following as an example:
- Example:
  1. Complete the following statement: If A and B are two mutually exclusive events, then  $P(A \text{ and } B) = \dots$
  2. Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7. Calculate  $P(A \text{ or } B)$ .

Solution:

1.  $P(A \text{ and } B) = 0$
2.  $P(B) = 1 - P(B')$   
 $1 - 0,7 = 0,3$ ,

$$\begin{aligned}
 P(A \text{ or } B) &= P(A) + P(B) \\
 &= 0,55 + 0,3, \\
 &= 0,85
 \end{aligned}$$

### ACTIVITIES

- 4.9.1 Complete the following statement: If A and B are two mutually exclusive events, then  $P(A \text{ and } B) = \dots$
- 4.9.2 Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7. Calculate  $P(A \text{ or } B)$ .
- 4.9.3 Given the following probabilities:  $P(A) = 0,45$ ;  $P(B) = 0,3$  and  $P(A \text{ or } B) = 0,615$
- (a) Determine whether or not events A and B are mutually exclusive.
  - (b) Hence, determine the probability of A and B.
  - (c) Are events A and B complementary?
- 4.9.4 Siphon is an engineer. He has recently designed a new computer chip. At the moment, two types of fault can occur: from a power surge or from overheating. Siphon has calculated  $P(\text{power surge}) = 0,05$  and  $P(\text{overheating}) = 0,12$ . He has also found that a quarter of the times that overheating happens, it is because there has also been a power surge.
- (a) What is the probability of overheating and a power surge occurring?
  - (b) Determine the probability of a fault occurring.
  - (c) What is the probability of no fault occurring with the chip?
  - (d) What is the probability of a power surge but no overheating occurring?
- 4.9.5 Answer the following questions relating to probability.
- (a) A and B are events that can both occur together. If you know that  $P(A \text{ or } B) = 0,8$  and  $P(A) = P(B) = 0,7$ , find  $P(A \text{ and } B)$ .
- 4.9.6 Two events A and B are mutually exclusive
- $P(B') = 0,3$
  - $P(A \cup B) = 0,8$
- Calculate  $P(A)$

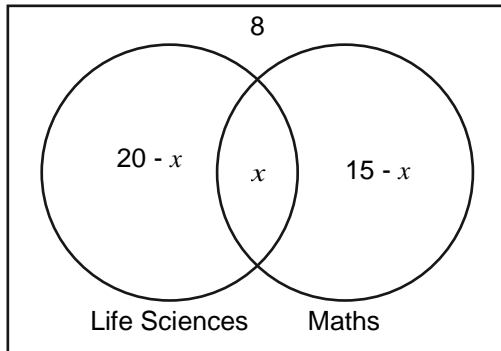
<b>TOPIC Probability 10</b>	Weighting		Grade
<b>Sub-topics</b>	Mixed problem for further practice of the Venn diagrams and rules		
<b>RESOURCES</b>			
<ul style="list-style-type: none"> <li>• Maths handbook by Kevin Smith</li> <li>• National Education Collaboration Trust (NECT)</li> </ul>			

**NOTES****Example**

- There are 40 learners in a class; 15 do Mathematics; 20 do Life Sciences and 8 do neither. Determine the number of learners who do both subjects.

**Solution**

- First represent the given information on the Venn diagram



How many learners are there altogether? (40)

Make an equation of all the learners represented adding up to make 40.

$$20 - x + x + 15 - x + 8 = 40$$

$$43 - x = 40$$

$$-x = -3$$

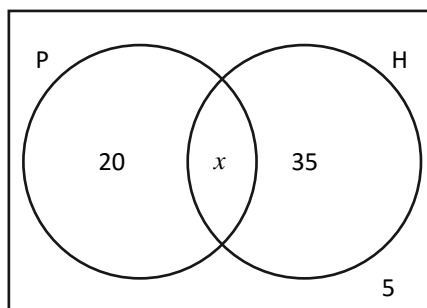
$$x = 3$$

**ACTIVITIES**

4.10.1 The information is represented in the Venn diagram below:

160 Grade 10 learners are selected at random and asked which fast food they eat.

- (P) pizza
- (H) hamburgers



4.10.1.1 Determine the value of  $x$

4.10.1.2 How many learners that:

- Did not eat any of the above
- eat pizza
- eat pizza only

- (d) eat pizza and hamburger
- (e) eat pizza or hamburger

4.10.1.3 Calculate the probability that:

- (a) A learner selected eats none of the fast foods
- (b) A learner selected eats at least one of the fast foods
- (c) A learner selected eats pizza

4.10.2 For two events A and B are not Mutually exclusive, you are given the following information:

- $P(B) = 0,45$
- $P(A) = 0,55$
- $P(\text{not } A \text{ or not } B) = 0,2$

Let the value of  $P(A \text{ and } B)$  be  $x$

- (a) Draw a Venn diagram based on the information given above.

Use your diagram to determine the following probabilities:

- (b)  $P(A \text{ and } B)$
- (c)  $P(A \text{ or not } B)$
- (d)  $P(B \text{ only})$

4.10.3 Learners in grade 10 were surveyed about their subject choices

- 150 were surveyed
- 125 did life Sciences
- 85 did Science
- 55 did Life science, but not Science
- 10 did neither

Draw a Venn diagram representing the above information. Let A be Life Sciences and B be Science

## SOLUTIONS

### TOPIC: DATA HANDLING/STATISTICS

#### ACTIVITY 3.1

NO	Sub - Question	ANSWERS																																																																													
3.1.1	a)	<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">9</td> <td style="padding: 2px 5px;">9</td> <td colspan="5"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">9</td> <td style="padding: 2px 5px;">9</td> <td colspan="2"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">8</td> <td colspan="2"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">9</td> <td colspan="14"></td> </tr> </table>	0	1	3	3	4	4	5	7	7	8	8	8	9	9						1	0	2	3	3	4	4	4	4	4	4	4	8	8	8	9	9			2	0	0	1	1	1	1	2	2	3	3	4	5	7	7	7	7	8			3	0	0	1	9														
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	b)	
	c)	21 Agents
	d)	58%
	e)	<p style="text-align: center;"><b>Histogram</b></p>
3.1.2.	a)	16 seeds
	b)	5 days
	c)	75%

**ACTIVITY 3.2**

NO	Sub – Question	ANSWERS
3.2.1	a)	Mean is 158; Median is 92
	b)	Mean
3.2.2	a)	Mean is 38,6; Median is 35; Mode is 35
	b)	All three measures are useful since most of the ages are in the 30 – year group.
3.2.3	a)	$x = 11$
	b)	Mean is 12
	c)	$x = 11$
	d)	9 ; 18 ; 27 ; 36 ; 45
	e)	3 ; 3 ; 4 ; 5 ; 10    3 ; 3 ; 4 ; 6 ; 9    3 ; 3 ; 4 ; 7 ; 8
	f)	34

**ACTIVITY 3.3**

NO	Sub – Question	ANSWERS
3.3.1		Median is 81; $Q_1 = 67$ ; $Q_3 = 89,5$
3.3.2	a)	$x = 11$
	b)	Median is 13; $Q_1 = 10$ ; $Q_3 = 21$
3.3.3	a)	The 25 <sup>th</sup> percentile is 113 The 50 <sup>th</sup> percentile is 216,5 The 75 <sup>th</sup> percentile is 284
	b)	The 30 <sup>th</sup> percentile is 126,5
	c)	The 65 <sup>th</sup> percentile is 253
	d)	The 80 <sup>th</sup> percentile is 289,5

**ACTIVITY 3.4**

NO	Sub – Question	ANSWERS
3.4.1	a)	Min is 1; Max is 10; Median is 8; $Q_1 = 7$ ; $Q_3 = 9$
	b)	
	c)	Mean is 7,25
	d)	Negatively skewed
3.4.2	a)	Maximum values and medians are the same.
	b)	Not Valid. Although both classes have the same median which means that half the scores are above 80 and half are below 80, none of the scores for Class B are below 66. For Class A, one quarter of the scores are below 66, therefore Class B performed better than Class A.

**ACTIVITY 3.5**

NO	Sub – Question	ANSWERS
3.5.1		Range is 8; IQR is 6; Semi – IQR is 3
3.5.2	a)	Class A – Range is 66; IQR is 18 Class B – Range is 30; IQR is 18 The range for Class A is greater than the range for Class B. The maximum value for Class A is an outlier and has affected its range. Both classes have the same interquartile range.
	b)	Semi – IQR for both classes is 9.
3.5.3	a)	$x = 3$
	b)	IQR is 7
3.5.4		Mean is 59; Median is 54; Range is 86

**ACTIVITY 3.6**

NO	Sub – Question	ANSWERS																																																
3.6.1	a)	51,33																																																
	b)	$60 \leq x \leq 65$																																																
	c)	$50 \leq x < 55$																																																
3.6.2	a)	<table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">43</td> <td>0</td><td>1</td><td>1</td><td>2</td><td>3</td><td>7</td><td>8</td><td>9</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">44</td> <td>7</td><td>7</td><td>8</td><td>8</td><td>9</td><td>9</td><td>9</td><td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">45</td> <td>0</td><td>0</td><td>1</td><td>2</td><td>3</td><td>3</td><td>3</td><td>4</td><td>5</td><td>5</td><td>6</td><td>7</td><td>7</td><td>9</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">46</td> <td>0</td><td>1</td><td>2</td><td>2</td><td>3</td><td>3</td><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> </table>	43	0	1	1	2	3	7	8	9	44	7	7	8	8	9	9	9		45	0	0	1	2	3	3	3	4	5	5	6	7	7	9	46	0	1	2	2	3	3	9							
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	b)	<table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Class interval</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td><math>43,0 \leq x &lt; 44,0</math></td> <td>8</td> </tr> <tr> <td><math>44,0 \leq x &lt; 45,0</math></td> <td>7</td> </tr> <tr> <td><math>45,0 \leq x &lt; 46,0</math></td> <td>14</td> </tr> <tr> <td><math>46,0 \leq x &lt; 47,0</math></td> <td>7</td> </tr> </tbody> </table>	Class interval	Frequency	$43,0 \leq x < 44,0$	8	$44,0 \leq x < 45,0$	7	$45,0 \leq x < 46,0$	14	$46,0 \leq x < 47,0$	7																																						
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	c)	Actual mean is 45,01; Median is 45,15; The modes (bimodal) are 44,9 and 45,3																																																
	d)	Range is 3, 9; IQR is 1																																																
	e)																																																	

**ANSWERS TO ASSESSMENT**

**ASSESSMENT 3.1**

NO	ANSWERS
3.1.1	16 learners
3.1.2	16 learners
3.1.3	2 learners
3.1.4	11 learners
3.1.5	Class B

**ASSESSMENT 3.2**

NO	ANSWERS
3.2.1	5 learners
3.2.2	40 learners
3.2.3	10
3.2.4	47.5%
3.2.5	4.88

**ASSESSMENT 3.5**

NO	Sub-Question	ANSWERS
3.5.1		19 seconds
3.5.2		Lower quartile is 17 ; Upper quartile is 22
3.5.3		
3.5.4	a)	IQR is 7
	b)	75 % of the boys took at least 19 seconds to complete the puzzle.
3.5.5		About 50% but not more than 75% of the boys completed the puzzle in less than 23 seconds. More than 75% of the girls completed the puzzle in less than 23 seconds. Therefore more girls completed the puzzle in less than 23 seconds.

**ASSESSMENT 3.6**

NO	Sub - Question	ANSWERS
3.6.1	a)	$2500 \leq x < 4500$
	b)	Estimated mean is 8908,39kg
	c)	Estimated mean, It is more at the centre of the data set. The modal class is found at the extreme left – hand side of the data set.
3.6.2	a)	$100 \leq x < 110$
	b)	$110 \leq x < 120$
	c)	116