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KWAZULU-NATAL PROVINCE

EDUCATION

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HIGHFLYERS MATERIAL

MATHEMATICS

2021

CURRICULUM GRADES 10 – 12

DIRECTORATE

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the content and give more guidance to the teachers.

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1.13.1 Solve for x if $k = 4$. (Leave the answer in the simplest surd form). (5)

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1.13.2 Write down the minimum value of k. (1)

1.14 Calculate the values of k, for which the equation $3x^2 + 2x - k + 1 = 0$ has real roots. (4)

1.15 The roots of the equation: $2x^2 - 12x + p = 0$ are in the ratio 5:7. Find p and the roots. (6)

1.16 The solution of a quadratic equation is: $x = \dfrac{-2 \pm \sqrt{13-2k}}{3}$. Find the largest integral value of k for which this this x value will be rational. (4)

1.17 The roots of the quadratic equation are $x = \dfrac{3 \pm \sqrt{13-2k}}{2}$ determine the values of k if the roots are real. (4)

1.18 For which values of k will $\dfrac{x-3}{(x-1)^2} = k$ have real roots (6)

1.19 The roots of the equation $(x + 2)(x + k) = 2 + 3x$ are non-real. Determine the possible values of k. (6)

1.20 Given: $f(x) = \dfrac{\sqrt{8x+1}}{x-4}$.

1.20.1 Determine the values of x for which f will have real roots. (3)

1.20.2 Solve for x if $f(x) = 1$. (4)

1.20.3 State the domain of f. (2)

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1.21.2 $\dfrac{\sqrt{36x^5}}{\sqrt[3]{x^6}}$ (3)

1.21.3 $\left(\sqrt[5]{\sqrt{35}+\sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35}-\sqrt{3}}\right)$ (3)

1.21.4 $\dfrac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}}$ (3)

1.21.5 $\dfrac{\sqrt{10^{1000}+10^{1001}}}{\sqrt{11^5.10^{1000}} - 21\sqrt{11.10^{1000}}}$ (3)

1.22 Calculate a and b if $\sqrt{\dfrac{7^{2014}-7^{2012}}{12}} = a(7)^b$, and '$a$' is not a multiple of 7 (4)

1.23 Solve for x: $x = \dfrac{a^2+a-2}{a-1}$ if a = 888 888 888 888 (3)

1.24 If $\dfrac{14}{\sqrt{63}-\sqrt{28}} = a\sqrt{b}$, determine, without using a calculator, the value(s) of a and b if a and b are integers. (4)

1.25 If $m^{\frac{1}{2}} + m^{-\frac{1}{2}} = 3$, calculate the value of $m + m^{-1}$. (3)

1.26 If $x = \dfrac{3-\sqrt{a}}{\sqrt{2}}$ and $y = \dfrac{4+\sqrt{a}}{\sqrt{2}}$, calculate the value of $(x+y)^2$. (3)

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1.27 The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1: 2 : 3. Calculate the length of the shortest side. (4)

2. NUMBER PATTERNS, SEQUENCES AND SERIES

QUESTION 1

1.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d,the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$ (4)

1.2 Calculate the value of: (4)

$$\sum_{k=1}^{50} (100 - 3k)$$

1.3 A quadratic sequence is defined with the following properties:

$T_2 - T_1 = 7$

$T_3 - T_2 = 13$

$T_4 - T_3 = 19$

1.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

1.3.2 Calculate the value of T_{69} if $T_{89} = 23\ 594$. (5)

[17]

QUESTION 2

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \cdots$

2.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

2.2 Explain why this series converges. (1)

2.3 Calculate the sum to infinity of the series. (2)

2.4 What is the smallest value of n for which $s_\infty - S_n < 1$? (5)

[11]

2.5 The sequence $3\ ;\ 9\ ;\ 17\ ;\ 27$; .... is a quadratic sequence.

2.5.1 Write down the next term. (1)

2.5.2 Determine an expression for the n^{th} term of the sequence. (4)

2.5.3 What is the value of the first term of the sequence that is greater than 269? (4)

[09]

QUESTION 3

The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$.

3.1 Prove , without the use of a calculator, that the sum of the series to infinity is $16 + 8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \cdots$ to 20 terms.

3.2.1 Write the series in sigma notation. (3)

3.2.2 Calculate the value of x. (3)

[10]

QUESTION 4

4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n + 9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of

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the first three terms of the arithmetic sequence. Determine **TWO** possible values for the common ratio, r of the geometric sequence.

[11]

QUESTION 5

5.1 Given the sequence : $4\ ;\ x\ ;\ 32$. Determine the value(s) of x if the sequence is

5.1.1 Arithmetic (2)

5.1.2 Geometric (2)

5.2 Determine the value of P if

$$\sum_{k=1}^{13} 3^{k-5}$$ (4)

[08]

QUESTION 6

The following sequence is a combination of an arithmetic and a geometric sequence:

$3\ ;\ 3\ ;\ 9\ ;\ 6\ ;\ 15\ ;\ 12\ ;....$

6.1 Write down the next TWO terms. (2)

6.2 Calculate $T_{52} - T_{51.}$ (5)

6.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

[09]

QUESTION 7

A quadratic pattern has a second term equal to 1 , a third term equal to -6 and a fifth term equal to -14.

7.1 Calculate the second difference of this quadratic pattern. (5)

7.2 Hence or otherwise, calculates the first term of the pattern. (2)

[07]

QUESTION 8

Given the arithmetic series: $-7 - 3 + 1 + \cdots + 173$.

8.1 How many terms are there in the series? (3)

8.2 Calculate the sum of the series. (3)

8.3 Write the series in sigma notation. (2)

[08]

QUESTION 9

9.1 Consider the geometric sequence:

$4\ ;\ -2\ ;\ 1\ ;$

9.1.1 Determine the next term of the sequence. (1)

9.1.2 2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

9.1.3 Calculate the sum to infinity of series $4 - 2 + 1$ (2)

9.2 If x is a real number, show that the following sequence can NOT be geometric:

$1\ ; x + 1\ ; x - 3$ .... (3)

[10]

QUESTION 10

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

10.1 Determine the values of r and s. (2)

10.2 Determine the values of a ; b and c. (3)

10.3 How far is the athlete from P when $n = 8$. (2)

10.4 Show that the athlete is moving towards P when $n < 5$, and away from P when $n > 5$. (3)

[10]

QUESTION 11

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11.1 $3x+1$; $2x$; $3x-7$ are the first three terms of an arithmetic sequence. Calculate the value of x . (2)

11.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

11.2.1 Calculate the 11th term of the sequence. (2)

11.2.2 The sum of the first n terms of this sequence is -560. Calculate n. (6)

[10]

QUESTION 12

12.1 Given a geometric series:

$$256 + p + 64 - 32 + \ldots\ldots$$

12.1.1 Determine the value of p. (3)

12.1.2 Calculate the sum of the first 8 terms of the series. (3)

12.1.3 Why does the sum to infinity for this series exist? (1)

12.1.4 Calculate S_∞. (3)

12.2 Consider the arithmetic sequence:

$$-8\ ;\ -2\ ;\ 4\ ;\ 10\ ;\ \ldots\ldots$$

12.2.1 Write down the next term of the sequence. (1)

12.2.2 If the n^{th} term of the sequence is 148 ; determine the value of n. (3)

12.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

[19]

QUESTION 13

13 Consider the sequence: 3 ; 9 ; 27 ; …

13.1 Jacob says that the fourth term of the sequence is 81. Vusi disagrees and says that the fourth term of the sequence is 57.

13.1.1 Explain why Jacob and Vusi could both be correct. (2)

13.2 Jacob and Vusi continue with their number patterns. Determine a formula for n^{th} term of:

13.2.1 Jacob's sequence (1)

13.2.2 Vusi's sequence. (4)

[07]

QUESTION 14

The values below are consecutive terms of a sequence that behaves consistently. The 4th term is 36.

.... ; ....; ..... ; 36 ; 54 ; 75 ; 99 ; ..... ; ..... ; ......

14.1 Determine the 1st; 2nd and 3rd terms of this sequence. (3)

14.2 Hence, determine a general formula for the nth term of this sequence. (4)

[07]

QUESTION 15

15.1 Prove that the sum to *n* terms of a geometric series, of which the first term is *a* and the common ratio is *r* , can be given as: (4)

$$S_n = \frac{a(r^n-1)}{r-1} \quad ;\ r \neq 1$$

15.2 Grace, who weighs 52kg, desperately seeks to gain weight. She follows a specific diet and training programme, in order to obtain her goal. She gains 2kg per week for the first two weeks. Thereafter, her weekly weight gain is 20% less than the weight gain in the previous week. Grace decides to follow this diet and fitness programme strictly, and to continue this pattern of weight gain indefinitely.

15.2.1 Write down, as a sequence, Grace's weight gained during the first FOUR weeks. (2)

15.2.2 Calculate Grace's weight after 15 weeks on this special programme. (5)

15.2.3 Calculate the maximum weight Grace will gain while following this programme. (3)

[14]

QUESTION 16

16.1 Given the geometric sequence: 7 ; x ; 63 ; ... Determine the possible value of x. (3)

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16.2 The first term of a geometric sequence is 15. If the second term is 10. Calculate:

16.2.1 T_{10} (3)

16.2.2 S_9 (2)

16.3 Given: $0\ ;\ -\frac{1}{2}\ ;0\ ;\ \frac{1}{2}\ ;0\ ;\ \frac{3}{2}\ ;\ 0\ ;\ \frac{5}{2}\ ;0\ ;\ \frac{7}{2}\ ;\ 0\ ;...$

Assume that this number pattern continues consistently.

16.3.1 Write down the value of the 191st term of this sequence. (1)

16.3.2 Determine the sum of the first 500 terms of this sequence. (4)

16.4 Given: $\sum_{k=2}^{20}(4x-1)^k$ (2)

16.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

16.4.2 18.4.2 For which values of x will $\sum_{k=2}^{\infty}(4x-1)^k$ exist? (3)

[21]

QUESTION 17

17.1 Write down the next term of the number pattern: $\frac{1}{2};\frac{8}{9};\frac{27}{28};\ldots$

17.1.1 Determine the general term. (2)

17.2 Given: $2\ ;6\ ;k;\ ...$ Write down the value of k if the sequence is:

17.2.1 Arithmetic (1)

17.2.2 Geometric (1)

17.3 Evaluate the sum of the infinite series: $5{,}6+3{,}36+2{,}016+1{,}2096$. (1)

17.4 Given: $0\ ;\ -1\ ;\ 1\ ;\ 6\ ;14$

17.4.1 Show that this sequence has a second difference. (2)

17.4.2 Determine a simplified expression for the nth term of the sequence. (4)

17.4.3 Find the 30th term. (2)

[13]

QUESTION18

The sum of the first n terms of a sequence is given by $S_n = 2^{n+2} - 4$.

18.1 Determine the sum of the first 24 terms. (2)

18.2 Determine the 24th term. (3)

18.3 Prove that the nth term of the sequence is 2^{n+1} (4)

[07]

QUESTION 19

Given the sequence: 5 ; 12 ; 21 ; 32 ; .....

19.1 Determine the formula for the n^{th} term of the sequence (4)

19.2 Determine between which two consecutive terms in the sequence is the first difference equal to 245. (5)

19.3 Sketch a graph to represent the second differences. (2)

[11]

QUESTION 20

20.1 Given: $16+8+4+2+\cdots$

20.1.1 22.1.1 Determine the sum of the first forty (40) terms of the series. (3)

20.1.2 Write the given series in sigma notation. (2)

20.1.3 Explain why the series converges. (2)

20.2 Calculate: $\sum_{k=3}^{350}(1-3k)+\sum_{t=1}^{200}\left(D_t\left[6t\right]\right)$ (6)

[13]

QUESTION 21

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The first term of a linear number pattern is 92 and the constant difference is -4.

21.1 Write down the values of the second and third terms of the number pattern (2)

21.2 Determine an expression for the n^{th} term of the number pattern. (2)

21.3 Determine the value of the eighteenth term. (2)

[06]

QUESTION 22

22.1 The following number pattern has a constant second difference.

$$41; 43; 47; 53; 61; 71; 83; 97; 113; 131; 151; 173; 197; 223; 251; ....$$

22.1.1 Write down the value of the constant difference. (2)

22.1.2 Determine the n^{th} term of the number pattern. (4)

22.2 The first forty terms of the number pattern are all prime numbers.

22.2.1 Determine the 41st term and show that it is not a prime number. (3)

[09]

QUESTION 23

23.1 Given the arithmetic series: $a + 13 + b + 27 + \cdots$

23.1.1 Show that $a = 6$ and $b = 20$ (2)

23.1.2 Calculate the sum of the first 20 terms of the series. (3)

23.1.3 Write the series in QUESTION 23.1.2 in sigma notation. (2)

23.2 Given the geometric series:

$(x-2) + (x^2-4) + (x^3+2x^2-4x-8) + \cdots$

23.2.1 Determine the value(s) of x for which the series converges. (4)

23.2.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series (4)

[15]

QUESTION 24

24 The first four terms of a quadratic number pattern are $-1\ ;\ 2\ ;\ 9\ ;\ 20$.

24.1 Determine the general term of the quadratic number pattern. (4)

24.2 Calculate the value of the 48th term of the quadratic number pattern. (2)

24.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)

24.4 If the sum of the first 69 first differences in Question 24.3 equals 9 591 (that is, $S_{69} = 9\ 591$), which term of the quadratic number pattern has a value of 9 590? (2)

[11]

QUESTION 25

25. Given the sequence: 2 ; 2 ; 5 ; 4 ; 8 ; 8 ; ....is a combination of a linear and geometric sequence.

25.1.1 If the pattern continues, then write down the next TWO terms. (2)

25.1.2 Calculate the sum of the first 40 terms of the sequence. (7)

25.2 Given the geometric series: $9x^2 + 6x^3 + 4x^4 + \cdots$

25.2.1 Determine a formula for T_n, the n^{th} term of the series. (2)

25.2.2 For which value(s) of x will the series converge? (3)

25.3 The sum to infinity of a geometric series with positive terms is 16 and the sum of the first two terms is 12. Determine the values of a and r. (8)

[22]

QUESTION 26

26.1 The twelfth term of an arithmetic sequence is 5 and the common difference of the sequence is 3.

26.1.1 Determine which term has a value of 47 (3)

26.1.2 Find the value of the first term (2)

26.2 The sum to n terms of an arithmetic series is $S_n = 4n^2 + 1$

26.2.1 Find the 15th term (3)

26.2.2 How many terms must be added to give a sum of 10001? (4)

26.3 The first term of a geometric series is 9 and the ratio of the sum of eight terms to the sum of the four terms is 97:81. Find the first three terms of the series, if it is given all the terms of the series are positive (8)

[20]

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QUESTION 27

27 For which value(s) of k will the series: $\left(\frac{1-k}{5}\right)+\left(\frac{1-k}{5}\right)^2+\left(\frac{1-k}{5}\right)^3+\ldots$ converge? (3)

[03]

QUESTION 28

28. The first two terms of an arithmetic series, A, and an infinite geometric series, B, are the same.

A: $-2+x+\cdots$ and

B: $-2+x+\cdots$ are given.

28.1 Write down in terms of x:

28.1.2 the third term of the geometric series, B (2)

28.1.3 the third term of the arithmetic series, A. (2)

28.2 If the sum of the first three terms in the arithmetic series A is equal to the third term of the geometric series B, then calculate the value of x (5)

28.3 If $x=-6$, does the geometric series B converge? (3)
Show calculations to support your answer.

QUESTION 29

29 Given:

$\sum_{k=1}^{n} T_k = n^2+4n$, where T_k is the general term of a series.

29.1 Calculate: $\sum_{k=1}^{250} T_k$ (2)

29.2 Calculate T_{100} (3)

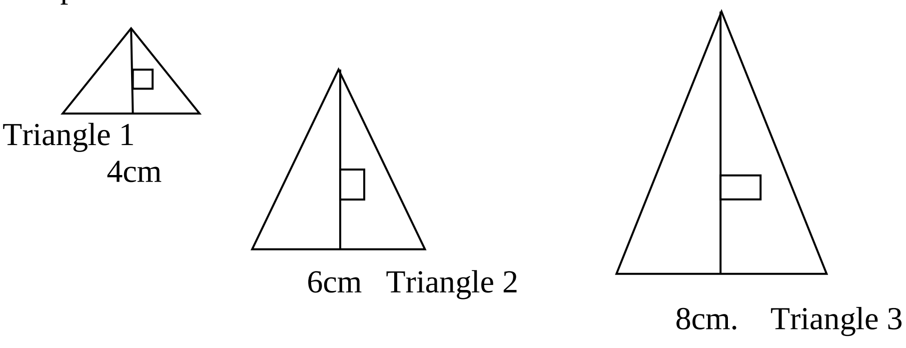
29.3 How many terms of the sequence must be added to give a sum of 1 440? (3)

[08]

QUESTION 30

A pattern of triangles is formed by increasing the base of the triangle by 2 *cm* and the perpendicular height by 1 *cm*, in each successive triangle. The first triangle has a base of 2 *cm* and a height of 2 *cm*.

The pattern continues in this manner.



30.1 Calculate the areas of the first four triangles. (2)

30.2 Calculate the area of the hundredth triangle in the pattern. (4)

[06]

QUESTION 31

31.1 The following is a combination of a linear and a geometric series: $3b-6+6b-10+12b-14+\cdots$

31.1.1 Write down the next two terms of the series. (2)

31.1.2 Determine in terms of, the sum of the first 30 terms of this series. (6)

31.2 Given the series $54+18+6+\cdots$

31.2.1 Determine the n^{th} term of this series. (2)

31.2.2 Hence, determine the 12^{th} term of the series. (2)

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31.2.3 Show that the sum to n terms of this series $81-81\left(\frac{1}{3}\right)^n$. (3)

31.2.4 Determine the maximum value of: $\sum_{n=0}^{10} 54\left(\frac{1}{3}\right)^n$. (6)

[21]

QUESTION 32

32 Given the quadratic sequence: $x ; y ; 16 ; 28 ; 42 ; 58 \ldots$

32.1 Determine the values of x and, the first two terms of the sequence. (2)

32.2 Determine the 45th term of this sequence. (5)

[07]

QUESTION 33

33.1 A water tank contains 216 litres of water at the end of day 1. Because of a leak, the tank loses one-sixth of the previous day's contents each day. How many litres of water will be in the tank at the end of:

33.1.1 the 2nd day? (2)

33.1.2 the 7th day? (3)

[05]

33.3 Consider the geometric series

$$2(3x-1)+2(3x-1)^2+2(3x-1)^3+\cdots$$

33.1.1 For which values of x is the series convergent? (3)

33.1.2 For which values of x is the series convergent? (3)

33.1.3 Calculate the sum to infinity of the series if $x=\frac{1}{2}$. (4)

33.1.4 $2 ; x ; 12 ; y ; \ldots$ are the first four terms of a quadratic sequence. If the second differences is 6, calculate the values of x and y. (4)

[19]

QUESTION 34

34.1 Determine the common difference and the first term of an arithmetic sequence in which the 8th term is -15 and the sum of the first eight terms is -8.

6)

34.2 Prove that the sum of the series $2^x+2^{x+1}+3.2^x+2^{x+2}+\ldots$ (15 terms) $=15.2^{x+3}$ (4)

34.3 If $(a+1)+(a-1)+(2a-5)+\cdots$are the first three terms of a convergent geometric series, calculate:

34.3.1 The value of a where $a>0$ (4)

34.3.2 The sum to infinity of the series (4)

[26]

QUESTION 35

35.1 Each time a photocopy is made from a previous photocopy, the quality of the print decreases by 11%. Determine how many times this photocopy can be done before the quality becomes less than 20% of the original. (4)

35.2 The sum of the first n terms of a sequence is: $S_n=3^{n-5}+2$. (3)
Determine the 80th term. Leave your answer in the form $a.b^p$
Where a ; b and p are all integers.

[11]

ADDITIONAL QUESTIONS

1. An arithmetic and geometric sequence have the first two terms the same. If the first term is 4 and the sum of the first 3 terms of the arithmetic sequence is equal to the 3rd term of the geometric sequence, determine the first three terms of both sequences. (7)

2. The first two terms of an arithmetic sequence and a geometric sequence are the same. The first term is 4 and is greater than the second term. The sum of the first three terms of the arithmetic

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sequence is 1 less than the sum of the first three terms of the geometric sequence. Calculate the value of the common difference of the arithmetic sequence.

(8)

3. An arithmetic sequence and a geometric sequence have the first term equal to 3. The first, second and fourth terms of the arithmetic sequence are the first three terms of the geometric sequence. Calculate the first three terms of the arithmetic sequence.

(6)

4. The common difference and common ratio of arithmetic and a geometric sequence is $\frac{1}{2}$ respectively. A new sequence is formed by adding corresponding terms of both sequences. The first term of the new sequence is 7 and the second term is $5\frac{1}{2}$.

4.1 Calculate the value of the 3rd and 4th terms of the new sequence. (3)

4.1 Determine an expression for the general term of the combined new sequence. (2)

5 Given the sequence: $4 ; 6 ; 9 ; 12 ; 14 ; 24 ; ...$

5.1 If the pattern continues, write down the next two terms of the sequence. (2)

5.2 Calculate the sum of the first 40 terms of the sequence. (3)

6. Given the combined sequence: $4 ; 8 ; 6 ; 18 ; 8 ; 32 ; 10 ; 50 ; ...$

6.1 Write down the names of the two sequences that are found in the combined sequence. (2)

6.2 Write down the next two terms. (2)

6.3 Calculate the value of the 80th term in the sequence. (2)

6.4 Determine the position of the term 202 in the sequence. (3)

6.5 Prove that the sequence will always have even terms. (4)

7. Given: $S_n = n^2 - 5n$, calculate the value of the 8th term. (3)

8. Given: $S_n = 4n^2 + \frac{2}{3}n$, calculate the value of the 5th term. (3)

[50]

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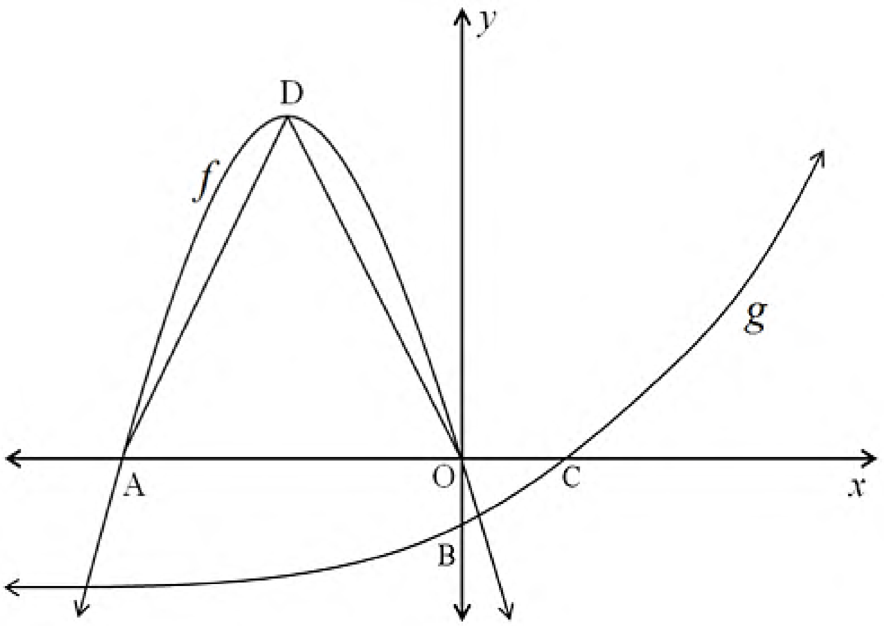
3. FUNCTIONS AND INVERSES

QUESTION 1

In the diagram below:

- $f(x) = -x^2 - 4x$
- $g(x) = 2^x - 6$
- Point D is the turning point of f
- Points A and C are the x-intercepts of f and g
- Point B is the y-intercept of g



1.1 Determine the area of ΔAOD. (6)

1.2 For what values of k will $-x^2 - 4x = 2^x + k$ have two real roots that are opposite in sign? (2)

1.3 For what values of p will $-(x-p)^2 - 4(x-p) = 2^x - 6$ have two real negative roots? (4)

March 2016

Determine the range of the function $y = x + \frac{1}{x}$, $x \neq 0$ and x is real. (6)

QUESTION 2

Given $f(x) = \frac{1}{2}x^2; x \geq 0$

2.1 Determine the equation of $g(x)$ if $h(x)$ is a reflection of $f(x)$ in the $x-axis$ and then $h(x)$ is reflected in the $y-axis$ to create $g(x)$. (3)

2.2 On the same system of axes sketch the graphs of $f(x)$ and $g(x.)$ (4)

2.3 From your graphs determine the average gradient of $h(x)$ between $x = -4$ and $x = 4$ if $h(x)$ is a combined graph of $f(x)$ and $g(x)$ (4)

NCS P1 OF JUNE 2021

QUESTION 3

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.

A and B are the x-intercepts of f. C is the turning point of f.

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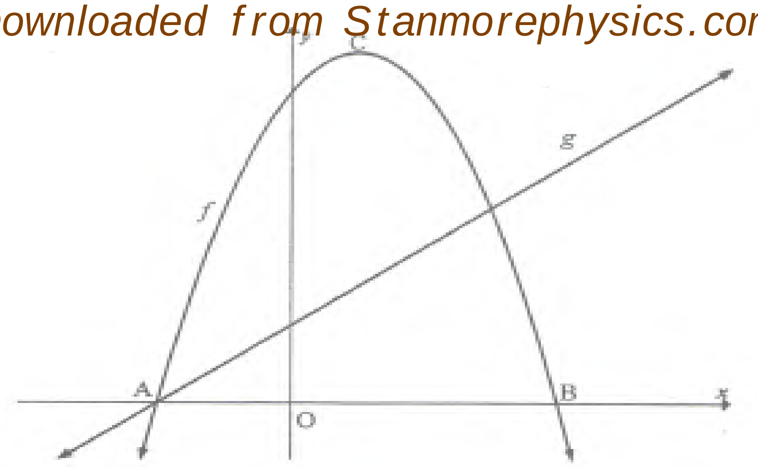

3.1 Calculate the coordinates of A and B. (3)

3.2 Determine the coordinates of C, the Turning point of f. (2)

3.3 Write down the range of m, if $m(x) = -f(x) - 1$ (2)

3.4 The graph of $h(x) = f(x+p)+q$ has a maximum value of 15 at $x = 2$
Determine the values of p and q. (3)

3.5 Determine the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$ (2)

3.6 For which value(s) of x will:

3.6.1 $g^{-1}(x).g(x) = 0$ (2)

3.6.2 $g'(x) > 0$ (1)

3.6.3 $f''(x) < 0$ (1)

3.6.4 $x.f'(x) < 0$ (3)

3.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT Intersect. (5)

[24]

QUESTION 4

Given $f(x) = \dfrac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate the intercepts of g with axes. (3)

4.3 Draw the graph of g, showing clearly the asymptote and the intercept with the axes. (3)

4.4 Determine the equation of line of symmetry that has a negative gradient in a form of $y = \ldots$ (3)

4.5 Determine the value(s) for which:

4.5.1 $\dfrac{6}{x+2} - 1 \geq x - 3$ (2)

4.5.2 $f'(x) < 0$ (2)

[15]

GAUTENG PRE-TRIAL 2021

QUESTION 5

The diagram below shows the graph of $f(x) = -x^2 + 5x + 6$ and $g(x) = x + 1$.

The graph of f intersects the x-axis at B and C and the y-axis at A. The graph of g intersects the graph of f at B and S. POR is perpendicular to the x-axis with points P and Q on and f and g respectively. M is the turning point of f.

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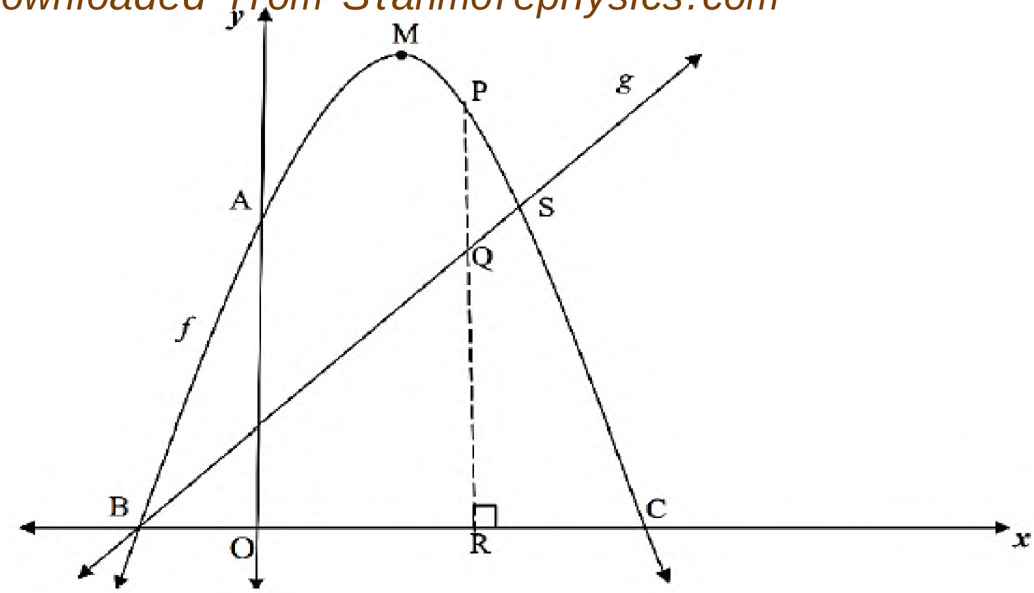

5.1 Write down the coordinates of A. (1)

5.2 S is the reflection of A about the axis of symmetry of f. Determine the coordinates of S. (2)

5.3 Calculate the coordinates of B and C. (3)

5.4 If PQ=5 units, calculate the length of OR. (5)

5.5 Calculate the:

5.5.1 Coordinates of M. (3)

5.5.2 Maximum length of PQ between B and S. (4)

5.5.3 Area of ΔBQR (3)

5.5.4 For which value(s) of x will $g'(x).f''(x)<0$ (2)

QUESTION 6

6.1 On the same system of axes sketch the graph of $f\colon x+y=4$; $g\colon x+3y=6$, where $x\geq 0$ and $y\geq 0$ (4)

6.2 Determine the value(s) of x for which $\dfrac{f(x)}{g(x)}\geq 1$ (3)

6.3 Find the distance between $f(x)$ and $g(x)$ when $y=\dfrac{1}{2}$ (3)

QUESTION 7

Given: $f(x)=\dfrac{x-3}{x+2}$

7.1 Show that $f(x)=1-\dfrac{5}{x+2}$ (1)

7.2 Write down the equations of the vertical and horizontal asymptotes of f. (2)

7.3 Determine the intercepts of the graph of f with the x-axes and y-axis. (2)

7.4 Write down the value of c if $y=x+c$ is a line of symmetry to the graph of f. (2)

[7]

QUESTION 8

Study the graphs of $f(x)=ax^2+bx+c$ and $g(x)=t^2$, where $a,b,c\in\mathbb{R}, a\sqrt{b^2-4ac}\neq 0$ and $t\in\mathbb{Z}$.
Graph f cuts x-axis at $x=-2$ and y-axis of 8.

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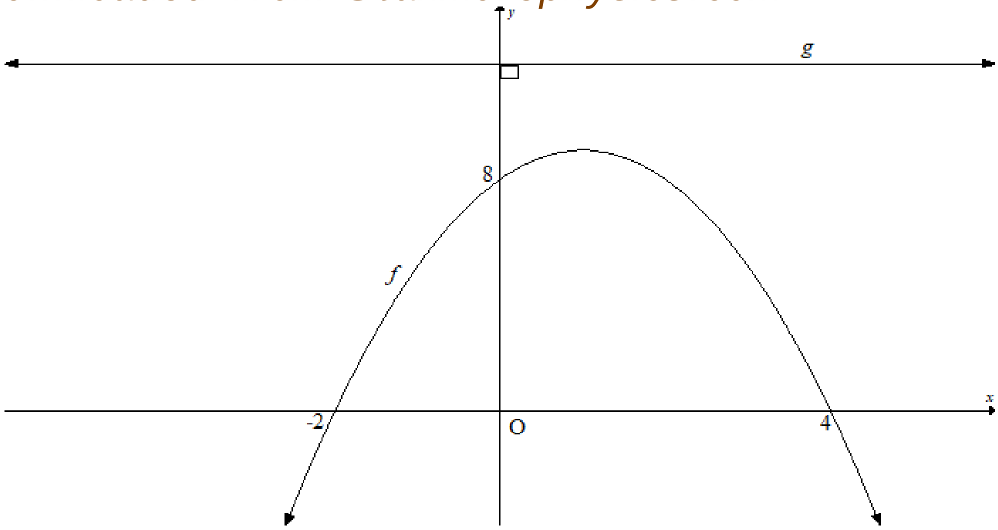


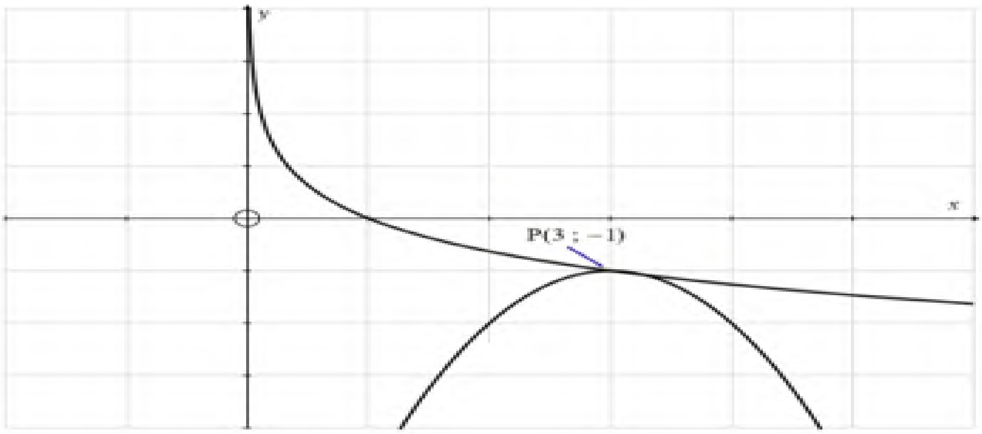
8.1 Show that $a = -1$ and $b = 2$. (3)

8.2 Determine the value(s) of t for which $f(x) - g(x) = 0$ will have non-real roots. (4)

QUESTION 9 IEB EXEMPLAR 2014

Refer to the diagram below

The graph of $g(x) = \log_a x$ and $y = h(x) - (x-3)^2 - 1$ are given. The point $P(3;-1)$ lies on the graph of both g and h.



9.1 Determine:

9.2 The value of a. (2)

9.3 The equation which defines $g^{-1}(x)$ in the form of $y = \ldots$ (2)

9.4 The x-values for which $1 \leq g^{-1}(x) \leq 3$ (2)

9.5 a possible restriction that could be placed on $h(x)$ to ensure that $h^{-1}(x)$ is a function. (1)

the values of x for which $g(x).f(x) \leq 3$ (2)

[9]

IEB SUPPLEMENTARY EXAMINATION 2016

QUESTION 10

10.1 Given: $f(x) = \dfrac{2}{x^2} + 1$ (4)

Determine $f(x^{-1}) - x^2 f(-1)$

Simplify your answer fully.

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10.2 Given $g(x) = x^2$ with $x \geq 0$.

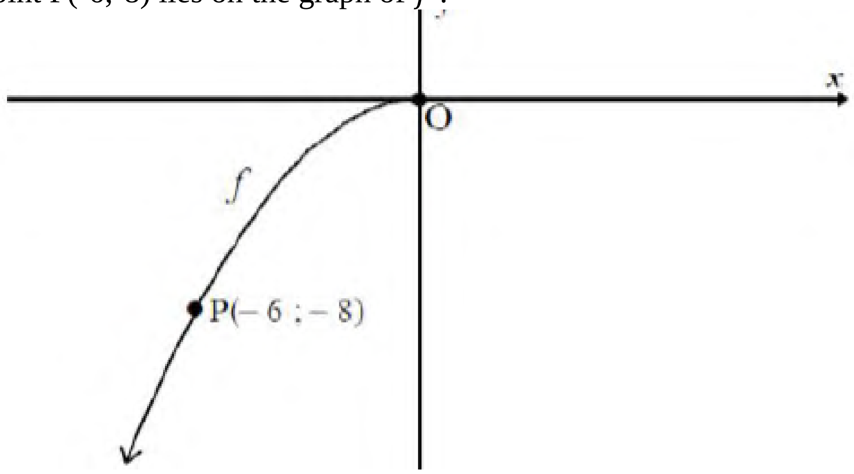
10.2.1 Write down the range of $g^{-1}(x)$ (1)

10.2.2 On the same set of axes, draw sketch graphs of $y = g(x)$ and $y = g^{-1}(x)$ clearly labelling intercepts with axes. (4)

[9]

QUESTION 11

The graph of $f(x) = ax^2, x \leq 0$ is sketched below.
The point P(-6;-8) lies on the graph of f .



11.1 Calculate the values of a (2)

11.2 Determine the equation of f^{-1}, in the form y=… (3)

11.3 Write down the range of f^{-1}. (1)

11.4 Draw the graph of f^{-1}. Indicate the coordinates of a point on the graph different from (0;0) (2)

11.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axes. Determine the equation of the new function in the form $y = \cdots$ (3)

[11]

QUESTION 12

Given $f = -x + 3$ and $g(x) = \log_2 x$.

12.1 On the same set of axes sketch the graphs of $f(x)$ and $g(x)$, clearly show the intercepts with all the axes. (4)

12.2 Write down the equation of $g^{-1}(x)$, the inverse of g in a form of $y = \ldots$ (3)

12.3 Explain how you will use the QUESTION 12.1 and/or QUESTION 12.2 to solve the equation $\log_2(3-x) = x$ (2)

12.4 Write down the solution to $\log_2(3-x) = x$ (1)

[10]

QUESTION 13

$f(x) = \log_p x$ and $g(x) = ax^2 + bx$ are sketched below. A is the turning point of f and B is the common x-intercepts of f and g . The point C(2;-1) lies on the graph of f.

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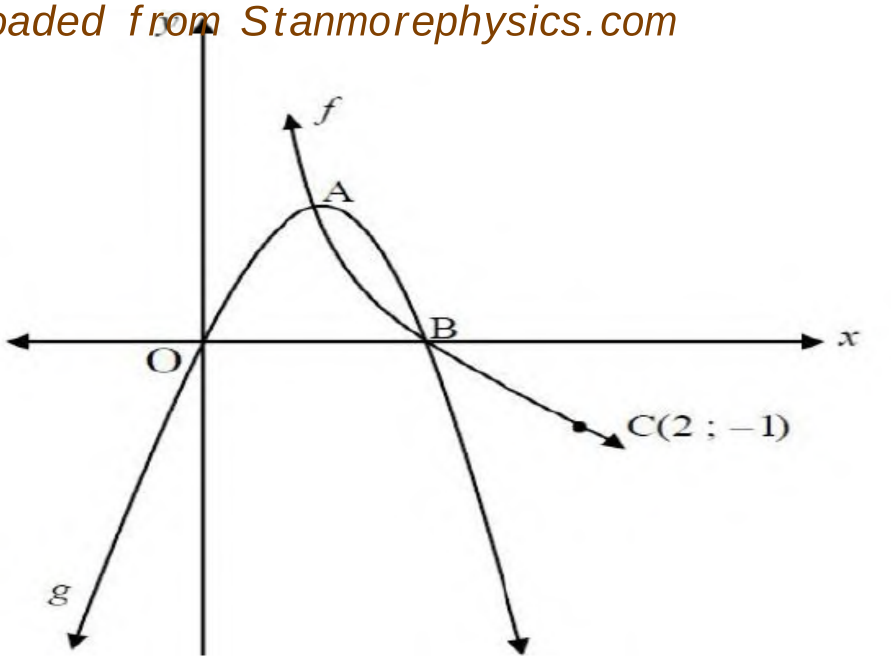

13.1	Calculate the value of p.	(2)
13.2	Write down the coordinates of B.	(1)
13.3	If $p = \frac{1}{2}$ calculate the coordinates of A.	(3)
13.4	Determine the values of a and b.	(4)
13.5	Write down the equation of f^{-1}, inverse of f, in the form of y=…	(2)
13.6	Determine the values of x for which $f(x) \geq -1$.	(2)
13.7	Determine the values of x for which $f(x).g'(x) \leq 0$	(2)
		[16]

QUESTION 14

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

14.1	Write down the range of g.	(1)
14.2	Determine the range of g^{-1} in a form of y=…	(2)
14.3	Is the graph of g^{-1} a function? Justify your answer.	(2)
14.4	The point M(a;2) lies on g^{-1}. Calculate the value of a.	(2)
14.5	M', the image of M, lies on g. Write down the coordinates of M'.	(1)
14.6	If $h(x) = g(x+3)+2$, write down the coordinates of the image of M' on h.	(3)
		[11]

QUESTION 15

Given $t(x) = 8^x$

15.1	Write down the equation of t^{-1},the inverse of t, in the form y=…	(2)
15.2	Show that $t\left(x+\frac{1}{3}\right) = 2.t(x)$	(3)
15.3	Sketch t and t^{-1}in the same system of axes, showing the line of reflection and intercept with the axes.	(4)
		[9]

QUESTION 16

Sketched below is the graph of $f(x) = k^x; k > 0$. The point (4;16) lies on f.

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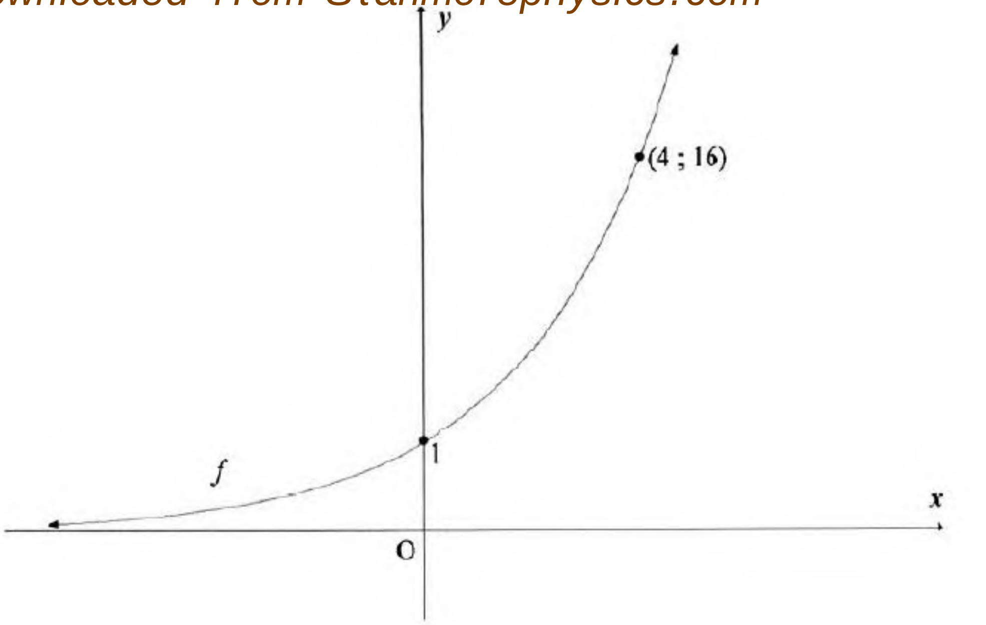

16.1	Determine the value of k.	(2)
16.2	Graph $g$is obtained by reflecting graph f about the line $y = x$. Determine the: equation of g in the form of y=…	(2)
16.3	Sketch the graph g. Indicate on your graph the coordinates of two points on g .	(4)
16.4	Use the graph to determine the value(s) of x for which:	
	16.4.1 $f(x) \times g(x) > 0$.	(2)
	16.4.2 $g(x) \leq -1$.	(2)
16.5	If $h(x) = f(-x)$.calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$.	(4)

[16]

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4. CALCULUS

The definition of a derivative is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

THE RULES OF DIFFERENTIATION

You are expected to know and understand the following rules

- if $f(x)$ is a constant, then $f'(x) = 0$.
- if $f(x) = x^n$, then $f'(x) = nx^{n-1}$.
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$.
- $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$.

USES OF THE DERIVATIVE

The derivative can be used to

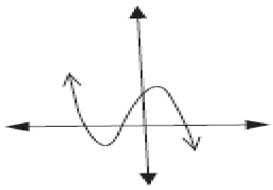
- find the gradient of the equation of a tangent line
- identify stationary points on a graph
- find a maximum or minimum value
- describe rates of change
- draw graphs of cubic functions. (Function of the form $f(x) = ax^3 + bx^2 + cx + d$)

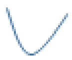
CURVE SKETCHING

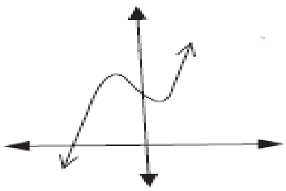
To sketch graphs of cubic functions, you should determine:

- local max / min (stationary / turning points): $f'(x) = 0$.
- point of inflection: $f''(x) = 0$.
- zeros: $ax^3 + bx^2 + cx + d = 0$.

- $a > 0$



- **concavity:**
- concave up in the interval  : $f''(x) > 0$
- concave down in the interval  : $f''(x) < 0$



- $a < 0$

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First principle

QUESTION 1

1.1 Given $f(x)=\frac{2}{x}$

1.1.1. Determine $f^{/}(x)$ by using first principles. (5)

1.1.2. Find the equation of the tangent to $f^{/}(x)$ at the point where $x = 2$. (5)

1.1.3. Determine whether $f^{/}(1)+f^{/}(2)=f^{/}(1+2)$. (5)

[15]

1.2

1.2.1 Determine the derivative, from first principles, of. $f(x)=-\frac{1}{3}x^3$ (5)

1.2.2 Hence, calculate the co-ordinates of the point at which the gradient of the tangent of *f* is -9 if $x < 0$. (4)

[9]

1.3. Given: $f(x)=-2x^2+1$

1.3.1. Show that the average gradient of the graph of f between the point where $x = 3$ and $x = 3 + h, (h \neq 0)$, is $-12-2h$. (4)

1.3.2. Use your answer in question 1.3.1 to calculate $f^{/}(3)$ from first principles. (2)

1.3.3. Determine the numerical value of the gradient of the graph of f at $x = 0$ (1)

[7]

1.4. Differentiate $f(x)=\frac{x^2}{3}+1$ from the first principle. (5)

1.5 Given the curve with equation $f(x)=x+\frac{12}{x}$ passes through the point A (2; *b*).

1.5.1. Determine $f^{/}(x)$ from first principles. (5)

1.5.2. Determine the equation of the line perpendicular to the tangent to the curve at A. (4)

[9]

1.6 Lungisani determines $g^{/}(x)$ the derivative of a certain g at $x = a$, and arrives at the answer; $\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$

Write down the equation of g and the value of a. (2)

1.7 $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x=1$. Calculate the values of a and b. (5)

1.8 The tangent to the curve $f(x)=2x^3+px^2+qx-7$ is perpendicular to line g at $x = 1$. Line g makes an angle of 45° with the positive x-axis $g(0)=-8$. Calculate the values of p and q. (8)

1.9 Given that f is a tangent to the curve $g(x)=x^3-ax^2+bx-4$. f is parallel to h where $h^{/}(x)=-3$. If $f(1)=g(1)=2$ (7)

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Determine $g'(x)$

Rules of differentiation

QUESTION 2

2.1 Determine (leaving your answers with positive exponents):

2.1.1. $g'(x)$ if $g(x)=\dfrac{5x^2-5x}{1-x}$ (3)

2.1.2. $D_x\left[\dfrac{3x^3-7x^2}{x^2}\right]$ (3)

2.1.3. $\dfrac{dy}{dx}$ if $y=\dfrac{64-x^{\frac{3}{2}}}{\sqrt{x}-4}$ (5)

2.1.4. $\dfrac{dz}{dx}$ if $z=\sqrt{\dfrac{4}{x}+\dfrac{x}{8}}$ (3)

2.1.5. $\dfrac{dy}{dx}=$ if $\sqrt{y}=x-2$ (4)

2.1.6. $\dfrac{dy}{dx}=$ if $7x^2-\dfrac{3}{\sqrt[3]{x}}+2^{-1}$ (4)

2.1.7. $p'(x)$ if $p(x)=\left(\dfrac{1}{x^3}+4x\right)^2$ (4)

2.1.8. $\dfrac{dy}{dx}$ if $y=\dfrac{x^2+x^{\frac{3}{2}}-6x}{\sqrt{x}+3}$ (5)

[31]

2.2 Given: $y=8x^3$ and $\sqrt{a}=y^{\frac{2}{3}}$. Determine:

2.2.1 $\dfrac{dy}{dx}$ (1)

2.2.2 $\dfrac{da}{dy}$ (2)

2.2.3 $\dfrac{da}{dx}$ (3)

[6]

2.3 Given: $y = 4(\sqrt[3]{x^2}) - 2x$ $y=4\left(\sqrt[3]{x^2}\right)-2x$ and $x=w^{-3}$. Determine $\dfrac{dy}{dw}$ (4)

Cubic function

QUESTION 3

3.1 Given $f(x)=(x-1)^2(x+3)$

3.1.1 Determine x and y intercepts of f. (3)

3.1.2 Determine the turning points of f. (5)

3.1.3 Draw a neat sketch of f showing all intercept with the axes as well as the turning points. (4)

3.1.4 Determine the x co-ordinate of the point where the concavity of f changes. (3)

3.1.5 Determine the equation of the tangent to f that is parallel to the line $y=-5x$ if $x<0$.

3.1.6 Determine the value(s) of k, for which $f(x) = k$ has three distinct roots. (6)

(2)

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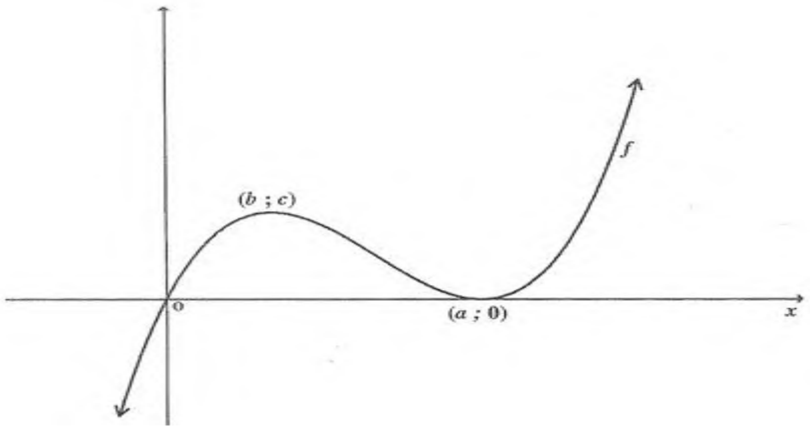
3.1.7 Determine the value(s) of k, for which x^3+x^2-5x-2 has one real root. (4)

3.1.8 Determine the value(s) of k, for which $f(x)+k=0$ has two unequal positive roots and one negative root. (4)

[31]

3.2 The sketch below represents the graph of a cubic function f defined by the equation $f(x)=x^3-4x^2+4x+k$. The graph passes through the origin, has a local maximum at $(b;c)$ and a local minimum at $(a;0)$.



3.2.1 Explain why *k=0* (1)

3.2.2 Using this value of k, determine the values of a and b. (6)

3.2.3 The graph of g with equation $g(x) = mx$ is a tangent to f at the point $(0;0)$ calculate the value of *m*. (2)

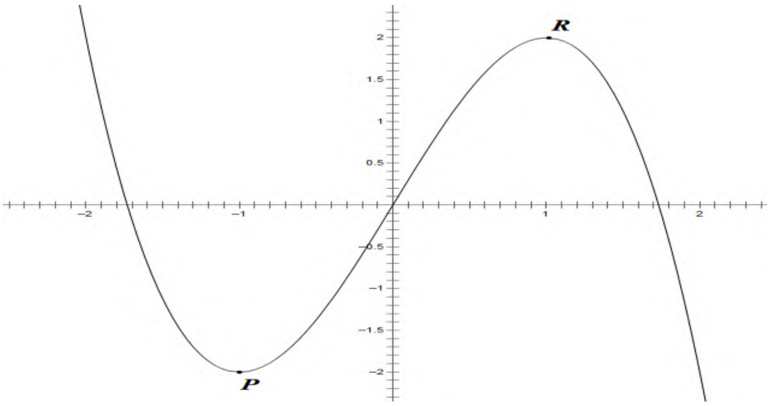
3.2.4 Make use of the graph, or any other way, to determine the value(s) of p for which $x^3+4x^2=4x-2=p$ has three unequal positive roots. (4)

3.2.5 For which value(s) of x is $f(x)$ concave down? (2)

3.2.6 For which values of x is $f(x)$ is $x.f'(x)\leq 0$ (2)

[17]

3.3 The sketch represents the graphs of $f(x)=ax^3+cx$. $P(-1;-2)$ and R are turning points of *f*.



3.3.1 Calculate the values of *a* and *c* if *f* has a minimum value at (-1; -2). (5)

3.3.2 Determine the coordinates of point R where the function has a local maximum value. (3)

3.3.3 For which values of x will $f'(x)\geq 0$? (3)

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3.3.4 For which values of x will $\frac{f'(x)}{f(x)} < 0$? (6)

3.3.5 Write down the x-coordinates of the turning points of h if $h(x) = f(x+3)$. (2)

[19]

3.4 Given the equation: $f(x) = ax^3 + bx^3 + cx + d$

The gradient at any point $(x; f(x))$ is given by $(18x^2 + 14x - 8)$, and $f(0) = -7$. Determine the values of $a, b,$ c and d (5)

3.5 Given the equation of cubic function $f(x) = -2x^3 + ax^2 + bx + c$ The turning point of f are T (2; -9) and S. f is concave down at $x > \frac{7}{2}$.

3.5.1 Determine the x-coordinate of f in terms of a where the concavity of f changes. (3)

3.5.2 Hence calculate the values of $a; b$ and c (6)

3.5.3 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

3.5.4 Determine the co-ordinates of a point where the tangent cut the curve $f(x)$ again. (5)

[19]

3.6 The following information about a cubic polynomial, $y = f(x)$ is given:

- $f(-1) = f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- If $x < -1$ then $f'(x) > 0$
- If $x > 1$ then $f'(x) > 0$

3.6.1 Use this information to draw a neat sketch graph of f. (5)

3.6.2 For which value(s) of x is f strictly decreasing? (3)

3.6.3 For which value(s) of x is f concave up? (2)

[10]

3.7 $f(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $a < 0$
- $d = 0$
- $f(-3) = f(8) = 0$
- $f'(1) = f'(5) = 0$

3.7.1 Draw a sketch graph of f using the information given above. (5)

3.7.2 Choose from the following two graphs the one that represents $f'(x)$ Only write A or B.

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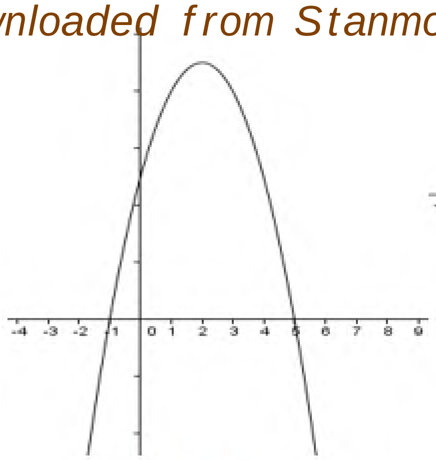
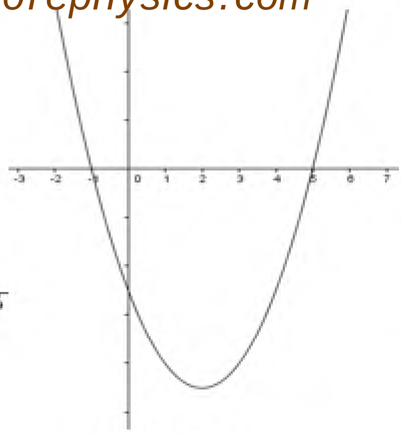


Graph A



Graph B

(1)

3.7.3 Use your graph in 3.7.1 and your choice in 3.7.2 to determine the values of x for which $f'(x).f(x) \geq 0$ (3)

[9]

3.8 Given that $y - x - 4 = 0$ is the equation of a tangent to the curve $f(x) = ax^3 + bx$. If the point of contact is (– 1; 3). Determine the values of a and b. (5)

QUESTION 4

4.1 For a certain function f passes the $y - axis$ if $f(x) = -18$ and the local minimum occurs at $-3x = -1$. The second derivative of f is given as $2y - 12x = 16$.

4.1.1 Calculate the x – coordinate of the maximum turning point of f. (4)

4.1.2 For which values of x is f concave down? (2)

4.1.3 Determine the values of x for which f is strictly increasing. (2)

4.1.4 Given that a gradient of the tangent of f at $x = 0$ is -3, determine the equation of. $f'(x) = ax^2 + bx + c.$ (5)

4.1.5 Hence Determine the equation of f. (5)

[18]

4.2 The graphs of $y = g'(x) = ax^2 + bx + c.$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c.$ is the derivative graph of a cubic function g. The graphs of h and g' have a common y-intercept at E. C(– 2 ; 0) and B(6 ; 0) are the x-intercepts of the graph of $f'(x)$ $g'(x)$ A is the x-intercept of h and D is the turning point of $g'(x)$.AB || y-axis.

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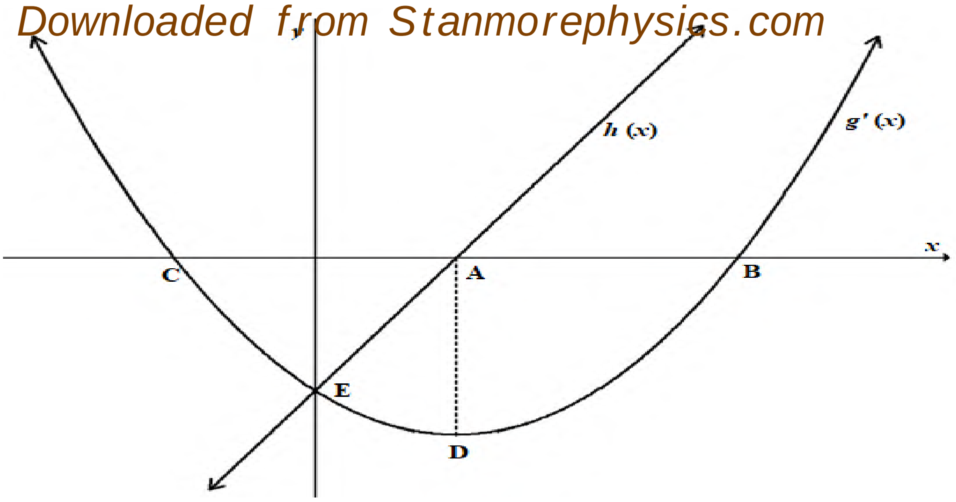


4.2.1 Write down the coordinates of E (2)

4.2.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$ (4)

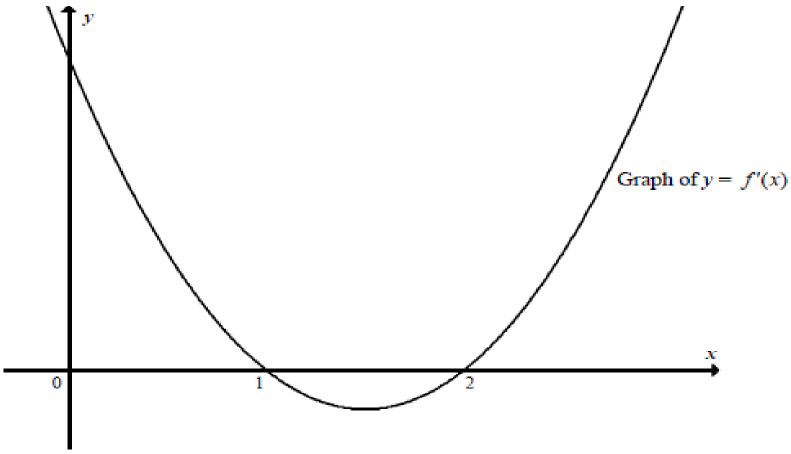
4.2.3 Write down the x–coordinates of the turning points of g (2)

4.2.4 Write down the x-coordinates of the point of inflection of the graph of g . (2)

4.2.5 Explain why g has local maximum at $x = -2$ (2)

[12]

4.3 In the sketch below, the graph $y = ax^2 + bx + c$ represent the derivative of f where f is a cubic function.



4.3.1 Write down the stationery points of f (2)

4.3.2 State whether each stationery point in QUESTION 4.3.1 is a local maximum or a local minimum. Substantiate your answer (4)

4.3.3 Determine the x co-ordinate of the point of inflection of f. (1)

4.3.4 Hence or otherwise draw a sketch graph of f (2)

[9]

4.4 The graph of $y = f'(x)$ where f is a cubic function, is sketched below. Use the graph to answer the following question:

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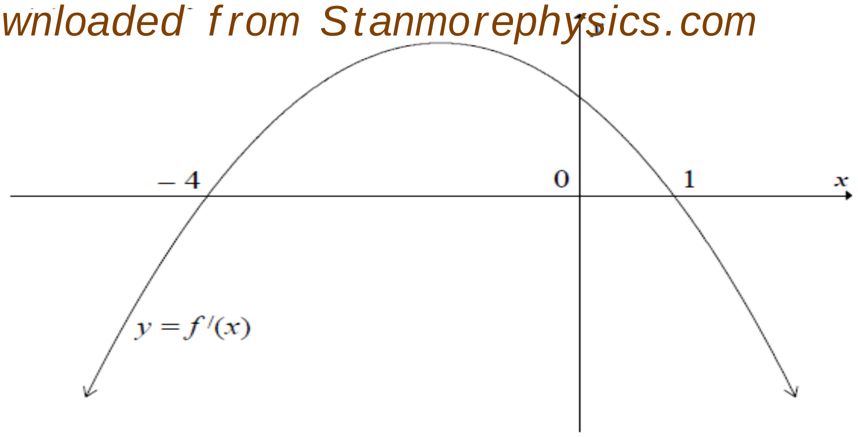


4.4.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

4.4.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

4.4.3 For which values of x is f concave up. (1)

4.4.4 Write down the value of the gradient of a tangent at f if $x = -4$. (1)

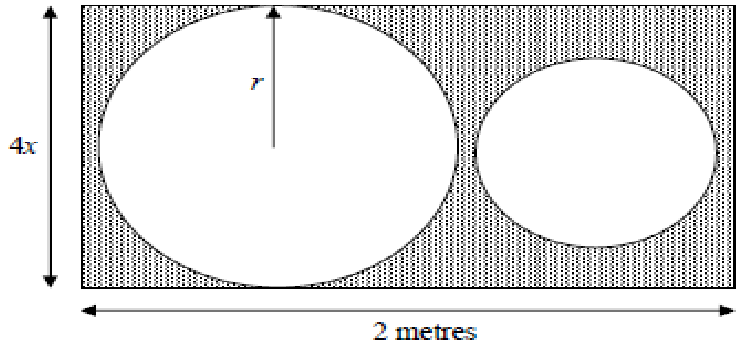
[5]

Optimisation

Question 5

5.1 Devan wants to cut two circles out of a rectangular piece of cardboard of 2 metres long and $4x$ metres wide. The radius of the larger circle is half the width of the cardboard and the smaller circle has a radius that is $\frac{2}{3}$ the radius of the bigger circle.

$A = lb$	$A = \pi r^2$	$P = 2(l + b)$	$C = 2\pi r$



5.1.1. Show that the area of the shaded region is $A(x) = 8x - \frac{52\pi}{9}x^2$. (5)

5.1.2. Determine the value of x. such that the area of the shaded region is a maximum. (3)

5.1.3. Calculate the total area of the circles, if the area of the shaded region is to be a maximum (2)

[10]

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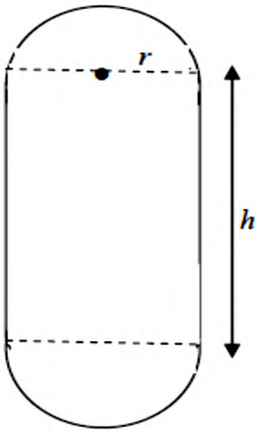
A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

5.2

5.2.1. Show that $h = \frac{1}{6r^2} - \frac{4r}{3}$. (3)

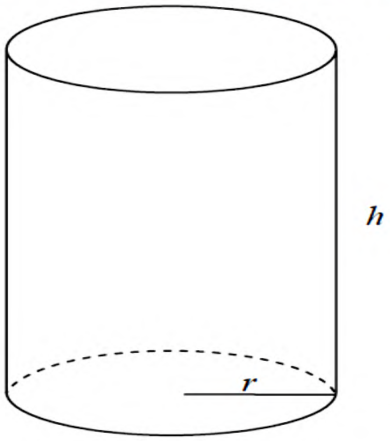
5.2.2. Hence, show that the outer surface area of the satellite can be given as $S = \frac{4\pi r^2}{3} - \frac{\pi}{3r}$. (3)

5.2.3. Calculate the maximum outer surface area of the satellite. (6)

[12]

5.3

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



5.3.1. Show that the height of the glass, h, can be expressed as $h = \frac{200}{\pi r^2}$ (2)

5.3.2. Show that the total surface of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$. (2)

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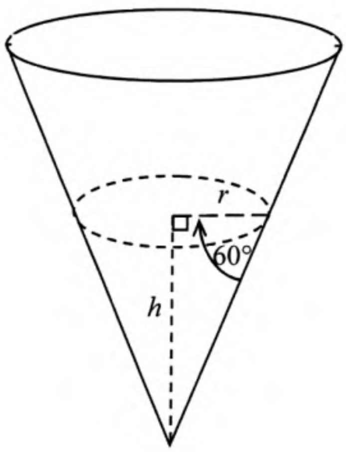
5.3.3. Hence determine the value of h for which the total surface area of the glass is a minimum (5)

[9]

5.4

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad$ $V = \frac{4}{3}\pi r^3$

5.4

5.4.1. Determine r in terms of h. Leave your answer in surd form. (2)

5.4.2. Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

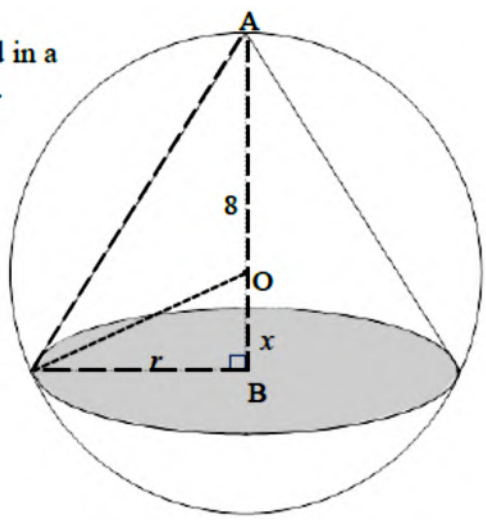
[7]

5.5

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB = x.

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of cone = $\frac{1}{3}\pi r^2 h$



5.5

5.5.1. Calculate the volume of the sphere. (1)

5.5.2. $r^2 = 64 - x^2$ (1)

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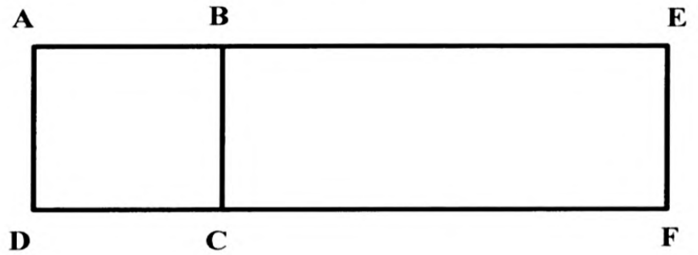
5.5.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. **(7)**

[9]

5.6

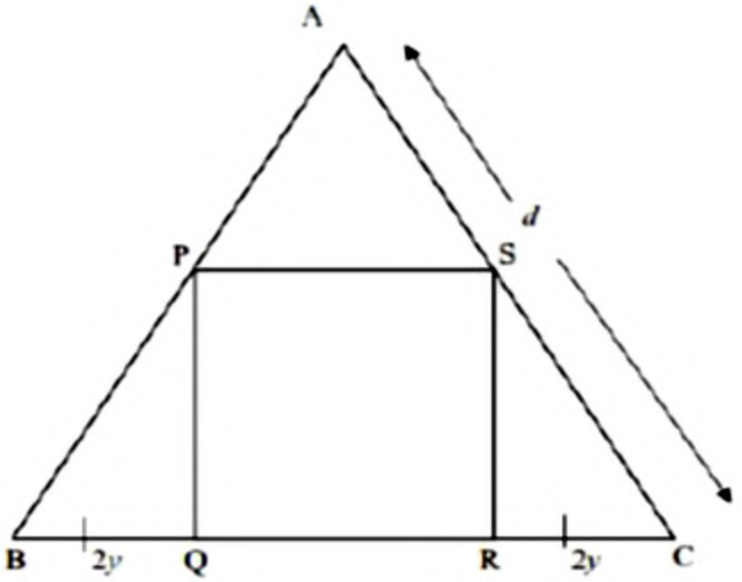
A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

5.7

In the diagram below, ΔABC is an equilateral triangle with sides d units long. P and S are points on sides AB and AC respectively. Q and R are points on BC such that PQRS is a rectangle. BQ = RC = $2y$ units.



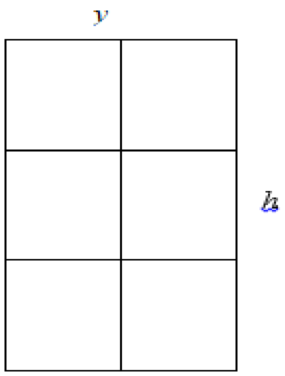
5.7.1. Show that the area of the rectangle PQRS is given by $A = 2\sqrt{3}\,y(d - 4y)$ (4)

5.7.2. Determine the maximum area of the rectangle in terms of d. (6)

[10]

5.8 Downloaded from Stanmorephysics.com

A window frame with dimensions $y \times h$ is illustrated below. The frame consists of six smaller frames.



5.8.1. If 12m of material is used to make the entire frame, show that $y = \frac{1}{4}(12 - 3h)$ (2)

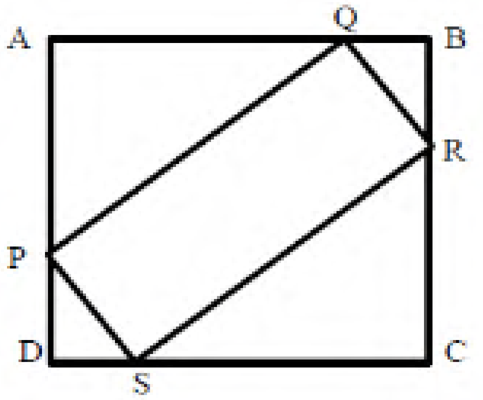
5.8.2. Show that the area of the window is given by $A = 3h - \frac{3}{4}h^2$ (3)

5.8.3 Find $\frac{dA}{dh}$ and hence the dimensions, h and y, of the frame so that the area of the window is a maximum. (5)

[10]

5.9

ABCD is a square with sides 20 mm each. PQRS is a rectangle that fits inside the square such that QB = BR = DS = DP = k mm



5.9.1. Prove that the area of PQRS = $-2k(k-20) = 40k - 2k^2$. (4)

5.9.2. Determine the value of k for which the area of PQRS is a maximum. (4)

[8]

5.10

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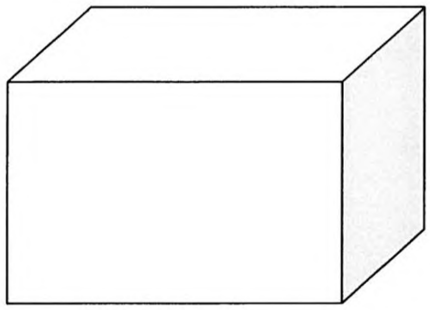
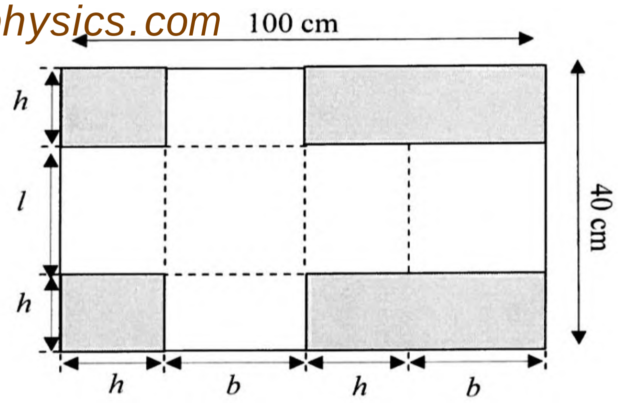




A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

5.10.1 Express the length l in terms of the height h (1)

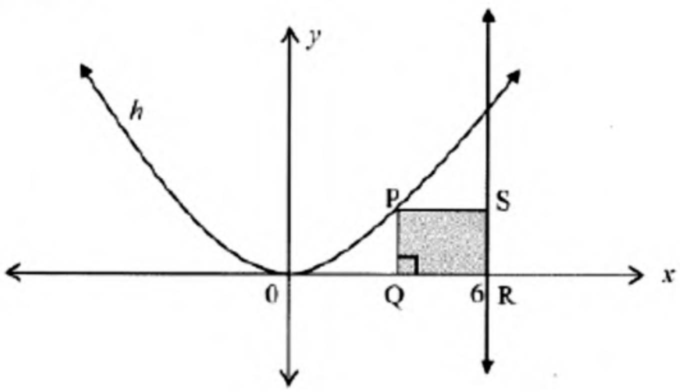
5.10.2. Hence prove that the volume of the box is given by
$v = h(50 - h)(40 - 2h)$ (3)

5.10.3. For which value of h will the volume of the box be a maximum? (5)

[9]

QUESTION 6

PQRS is a rectangle with P on the curve $h(x) = x^2$ and with the x-axis and the line $x = 6$ as boundaries.



6.1.1 Show that the area of rectangle PQRS can be expressed as $A = 6x^2 - x^3$. (3)

6.1.2. Determine the largest possible area for rectangle PQRS. Show all your calculations (4)

[7]

6.2

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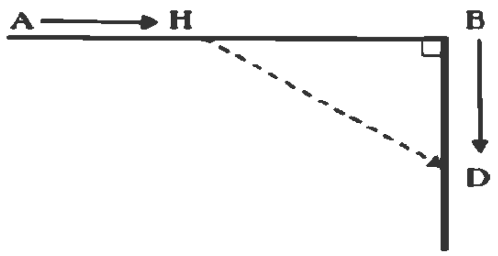
A hunter was standing at point A, along the fence of a rectangular game enclosure. when he spotted a deer standing at point B, the corner of the rectangular enclosure. The distance from A to B is 1200m. At exactly the same time as the hunter started to move in an easterly direction towards B, the deer started to move in a southerly direction towards D. The hunter moves at 4metres per second and the deer moves at 5metres per second. After t seconds, the hunter is at a point H and the deer is at point D.



The hunter tries to shoot the deer but with his caliber rifle he must be at most 800m from the deer.

6.2.1 Show that the distance between the hunter and the deer (HD) at t seconds after they both started moving can be written as:

$$HD(t) = \sqrt{41t^2 - 9\,600t + 1\,440\,000}$$ (4)

6.2.2 How long after they started walking, were they the nearest to one another? Show all calculations. (3)

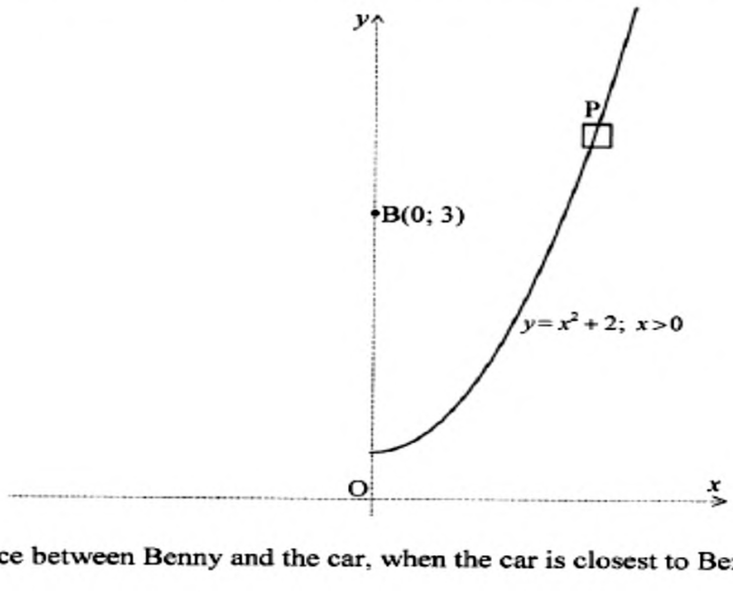
6.2.3 The calibre of the hunter's rifle allows him to be at most 800m from his target. Was the hunter within shooting range of the deer at the time when they were nearest to each other? Show all calculations. (3)

[10]

6.3

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

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QUESTION 7

7.1

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

7.1.1 What is the initial volume of the water in the tank? (1)

7.1.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)

7.1.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)
[6]

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

7.2.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

7.2.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

7.2.3 After how many seconds will the particle be closest to the fixed point? (2)
[6]

7.3

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time.

The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0,2t^2 + 10t$$

where t is measured in seconds.

7.3.1 After how long will the water be flowing at the maximum rate? (3)

7.3.2 After how many seconds does the water stop flowing? (3)
[6]

7.4 Downloaded from Stanmorephysics.com

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

7.4.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

7.4.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

7.4.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)

[8]

7.5

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

7.5.1 At what height above the floor did the insect start to crawl? (1)

7.5.2 How many times did the insect reach the floor? (3)

7.5.3 Determine the maximum height that the insect reached above the floor. (4)

[8]

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QUESTION 1

1.1 Melokuhle invests R15 500 for t years at a compound interest rate of 9% p.a. compounded quarterly. At the end of t years, his investment is worth R40 000. Calculate the value of t (4)

1.2 Dela bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 18% p.a. compounded monthly.

1.2.1 Calculate the annual effective interest rate of the loan. (3)

1.2.2 Calculate Dela's monthly instalments (4)

1.2.3 Dela decided to pay R12 200 each month as his repayment. Calculate the outstanding balance of the loan after 3 years. (4)

1.2.4 At the end of 3 years, the market value of Dela's car has reduced to R208 400. Calculate the rate of depreciation on the diminish value. (2)

[17]

QUESTION 2

2.1 How long will it take for the lump sum of money to be doubled at 4,5% p.a. interest compounded monthly? (3)

2.2 A loan of R50 000 is amortised over a period of 5 years. Payments are made monthly starting six months after the loan is granted. The interest rate is 10,5% p.a. compounded monthly.

2.2.1 Calculate the monthly repayments (6)

2.2.2 Calculate the outstanding balance after 2 years the loan was granted. (6)

[15]

QUESTION 3

Mrs Naidoo plans to buy a flat. She requires a mortgage bond of R800 000. The interest rate on the bond is 9% p.a compounded monthly. Mrs Naidoo plans to repay the loan with equal monthly payments starting one month after the loan is granted.

3.1 If Mrs Naidoo pays R6 500 per month until the bond is cleared; calculate the number of payments required to amortise the loan (4)

3.2 Calculate Mrs Naidoo's final payment. (5)

3.3 Determine how much interest Mrs Naidoo paid. (2)

3.4 If Mrs Naidoo want to pay R1500 per month, decide whether the bank will allow her to takeout the bond under these conditions. (Justify your answer with calculations) (6)

[17]

QUESTION 4

A farmer buys a tractor for R2,2 million.

4.1 Determine the book value of a tractor at the end of 5 years if the depreciation is calculated at 14% p.a. on a reducing balance method. (3)

4.2 Determine the expected cost of buying a new tractor in five years' time if the average rate of inflation is expected to be 6% p.a. (3)

4.3 The farmer decides to replace the old tractor in five years' time. He will trade in the old tractor. Calculate the sinking fund. (3)

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4.4 Calculate the monthly payment into the sinking fund if payments commenced one month after he bought the tractor if the interest rate is 7% per annum compounded monthly. (4)

[13]

QUESTION 5

5.1 Daniel buys a house for R 450 000. He pays a 10% deposit and takes out a loan called a bond from the bank to pay off the balance. The bank charges 7,2% p.a. compounded monthly and He takes it out over a 25-year period.

5.1.1 Determine the value borrowed from the bank. (1)

5.1.2 What is his monthly repayments? (4)

5.1.3 After 11 years, He inherits money from his grandmother, and decides to pay off the rest of his bond. What is the outstanding balance that he needs to settle at the end of 11 years? (3)

5.2 At the beginning of October 2016 Lungile opened a savings account with a single deposit of R10000. She then made 24 monthly deposits of R1600 at the end of every month starting at the end of October 2016. She earns 15% p.a. interest compounded monthly in her account. Calculate the amount that should be in his savings account immediately after she makes the last deposit.

(5)

[13]

QUESTION 6

6.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

6.1.1 Determine the scrap value of the machine at the end of 5 years (3)

6.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

6.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000 into which equal monthly instalments must be paid, is set up. Interest on this is 8,5% per annum, compound monthly. The first payment will be made immediately, and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payment into the sinking fund. (5)

6.2 Nesta receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compound monthly.

She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

For how many months will she be able to live from her investment? (6)

[17]

QUESTION 7 IEB NOV 2018

7.1 Victor opened a bank account 15 years ago, with the intention of saving money for when he retires.

The bank offered him an interest rate of 16% per annum compounded monthly for the first 5 years and thereafter changed the interest rate to 11% per annum compounded annually.

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Victor made an immediate deposit of R 300 000 upon opening the account and withdrew R 500 000 at the end of 13 years.
Calculate how much money he would have in this account at the end of the 15th year. (5)

7.2 If instead, Victor had taken a retirement annuity over the same period of 15 years, and the insurance company had offered him 8% per annum compounded monthly, what would his monthly payments have been if he were to save an amount of R 1 270 000 at the end of the 15th year.

QUESTION 8

8.1 Joe invested a sum of R50 000 in a bank. The investment remained in the bank for 15 years, earning interest at a rate of 6% p.a. compounded annually. Calculate the amount at the end of 15 years.

8.2 Nobuhle took a mortgage loan of R850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8% p.a. compounded monthly (2)

8.2.1 Show that her monthly instalment was R6 237 (4)

8.2.2 Calculate the outstanding balance on her loan at the end of the first year. (3)

8.2.3 Hence calculate how much of the R74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged. (3)

[12]

QUESTION 9

9.1 Two colleagues each receives an amount of R8 000 to invest for a period of 6 years. They invest money as follows:

- Zinhle: 7,5% p.a. simple interest. At the end of 6 years, she will receive a bonus of exactly 5% of the principal amount.
- Ntando: 7,0% p.a. compounded quarterly.

Who will have a bigger investment after 6 years? Justify your answer with appropriate calculations. (6)

9.2 How much will Thulani's investment worth at the end of 3 years, if he invests R4 million into earning interest of 6% per annum, compounded annually? (3)

9.3 Tom invests R900 000 into an account earning interest of 6,5% per annum, compounded monthly.

9.3.1 He withdraws an allowance of R20 000 per month. The first withdrawal is exactly one month after he has deposited R900 000. How many such withdrawals will Tom be able to make? (6)

9.3.2 If Tom withdraws R10 000 per month, how many withdrawals will he be able to make? (3)

[18]

QUESTION 10

Jake takes out a bank loan of R600 000 to pay for his new car. He repays the loan with monthly instalments of R9 000, starting one month after the granting of the loan. The interest rate is 13% per annum, compounded quarterly

10.1 How many instalments of R9 000 must be paid? (5)

10.2 What will the final payment be? (5)

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10.3 What did the car cost Jake by the time it is paid off (2)

[12]

QUESTION 11

Due to load shedding, a restaurant buys a large generator for R227851. It depreciates at 23% per annum on a reducing balance. A new generator is expected to appreciate in value at a rate of 17% per annum. A new generator will be purchased in five years' time.

11.1 Find the scrap value of the old generator in five years' time. (3)

11.2 Find the cost of a new machine in five years. (3)

11.3 The restaurant will use the money received from the sale of the old machine (at scrap value) as part payment for the new one. The rest of the money will come from a sinking fund that was set up when the old generator was bought. Monthly payments which started one month after the purchase of the old generator, have been paid into a sinking fund account paying 11,4% per annum compounded monthly. The payments will finish three months before the purchase of a new machine. Calculate the monthly payments into the sinking fund that will provide the required money for purchasing of the new machine. (6)

[12]

QUESTION 12

Mrekza takes out a loan of R450 000 at an effective interest rate of 14% p.a. in order to purchase a town house. She repays the loan with equal monthly instalments of R7500, starting one month from the granting of the loan. The interest is compounded monthly.

12.1 Show that the nominal interest rate is approximately 13,17% p.a. (3)

12.2 Calculate:

12.2.1 The number of payments to payed up the loan. (4)

12.2.2 The value of the last payment (less than R7500). (7)

[14]

QUESTION 13

A loan of R180 000 is to be repaid over 20 years by means of equal monthly payments, starting 3 months after the loan is granted. The interest rate is 16% p.a. compounded monthly.

13.1 Calculate the monthly repayments. (4)

13.2 Calculate the outstanding balance after 10 years. (3)

[7]

QUESTION 14

A young man decides to invest money each month into a pension fund, starting on his 30^{th} birthday and ending on his 60^{th} birthday. He wants to have R1,5 million on retirement. If the interest rate is 14% p.a., compounded monthly, what will be his monthly payments? (4)

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QUESTION 15

A company has an excavator which they have purchased for R1.5 million rand. It will depreciate on a reducing balance at 10% p.a. and it is anticipated that it will need to be replaced after 6 years. Over this period, it is predicted that inflation will run at 7% p.a.

15.1 Calculate the scrap value of the existing elevator after 6 years. (2)

15.2 Calculate the price of a new elevator in 6 years' time. (2)

15.3 Assuming that proceeds from the sale of the old excavator will be put towards the new one, determine how much money should be invested in a sinking fund today in order that the company will be able to replace the excavator in 6 years time. Assume that the sinking fund will earn 12% p.a. compounded monthly. (4)

[8]

QUESTION 16

Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing- balance method. The book value of Sandile's car is currently R79 866,96.

16.1 How many years ago did Sandile buy the car? (3)

16.2 At exactly the same time that Sandile bought the car, Anile deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Anile accumulated enough money in his savings account to buy Sandile's car now? (3)

[6]

QUESTION 17

17.1 A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method. (3)

17.2 Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly. How many months will it take Musa to repay the loan, if the monthly instalment is R1 900? (4)

17 .3 Neil set up an investment fund. Exactly 3 months later and every 3 months thereafter he deposited R3 500 into the fund. The fund pays interest at 7,5% p.a., compounded quarterly. He continued to make quarterly deposits into the fund for 6½ years from the time that he originally set up the fund. Neil made no further deposits into the fund but left the money in the same fund at the same rate of interest. Calculate how much he will have in the fund 10 years after he originally set it up. (6)

[13]

QUESTION 18

Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

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18.1 Calculate Piet's monthly instalment. (4)

18.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[10]

QUESTION 19

A bank granted Zabelungu a loan of R800 000 at an interest rate of 10,25% compounded monthly. The bank stipulated the loan:

- Must be repaid over 20 years.
- Must be repaid by means of monthly repayments of R7 853,15 starting one month after the loan was granted.

19.1 How much did Zabelungu owe immediately after making her 6th repayment? (4)

19.2 Due to financial difficulties, Zabelungu missed the 7th, 8th and 9th payments. She was able to make payments from the end of the 10th month onwards. Calculate Zabelungu's increased monthly payment in order to settle the loan in the original 20 years. (5)

[10]

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6. COUNTING PRINCIPLE AND PROBABILITY

QUESTION 1

Ryan packs his suitcase for his holiday with 3caps, 5 shirts, 3 pairs of jeans and 2 pairs of takkies:

1.1 How many different outfits can he put together if when he dresses, he must wear a shirt, a pair of jeans, a pair of takkies and a cap? (2)

1.2 Ryan reaches his destination and hangs all the 5 shirts and the three pairs of jeans (each item separately) on a different hanger, on the rail in the cupboard

a) How many different arrangements are possible? (2)

b) What is the probability that the shirts are all hanging together next to each other in the cupboard? (3)

c) While on holiday Ryan decides to buy a pair of sandals in addition to his outfit items, on a given day what is the probability that Ryan will wear a pair of sandals or a pair of takkies? (3)

d) Find the number of different arrangements of the letters DDD EE F G, if all the letters must be used and there are no restrictions. (3)

QUESTION 2

2.1 Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.
How many personal identity numbers (PINs) can be made if:

2.1.1 Digits are repeated? (2)

2.1.2 Digits cannot be repeated? (2)

2.2 Suppose a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4)

QUESTION 3

Each of the digits 1; 1; 2; 3; 4; 7 is written on a separate card. The cards are then placed next to each other to make a 6-digit number.

3.1 How many different 6-digit numbers can be formed from these digits? (2)

3.2 How many numbers start and end with the same digit? (2)

3.3 What is the probability of getting a number that starts and ends with the same digit? (1)

3.4 Find the probability that a number is 112347 or 743211 (2)

QUESTION 4

In Gauteng the number plates consists of 3 alphabets, excluding the five vowels, next to each other followed by 3 digits from 0 to 9. All number plates end with GP. An example: TDG 234 GP. The alphabets and digits are allowed to repeat.

4.1 Determine the number of unique number plates (2)

4.2 Determine the probability that the number plate starts with a Y. (3)

4.3 Calculate the probability that the number plate contains an E (2)

4.4 Determine the number of number plates that will contain one 5. (3)

QUESTION 5

5.1 Consider the word "SIMPLIFY"

5.1.1 How many six letter words can be made? (2)

5.1.2 Calculate the probability of the word starting and ending with the same letter. (3)

5.2 Six cars are parked alongside each other, three are silver. How many ways can the cars be arranged if the silver cars have to be next to each other. (3)

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QUESTION 6

Three men (Peter, Jabu and Les) and two women (Liz and Kate) are to stand in a straight line to have their group photograph taken. Find the probability that Peter stands next to Liz and Jabu stands next to Kate. (5)

QUESTION 7

A music group plans a concert tour in South Africa. They will perform in Cape Town, Port Elizabeth, Pretoria, Johannesburg, Bloemfontein, Durban, and East London.

7.1 In how many different orders can they plan their tour if there are no restrictions? (1)

7.2 In how many different orders can they plan their tour if tour begins in Cape Town and ends in Durban? (1)

7.3 If the tour cities are chosen at random, what is the probability that their performance in Cape Town, Port Elizabeth, Durban, and East London happen consecutively? Give your answer correct to 3 decimal places (3)

QUESTION 8

Landline telephone numbers are 10 digits long. Numbers begin with a zero followed by 9 digits chosen from the digits 0 to 9. Repetitions

8.1 How many different phone numbers are possible? (1)

8.2 The first three digits of a number form an area code. The area code for Cape Town is 021. How many different phone numbers are available in the Cape Town area? (1) (1)

8.3 What is the probability of the second digit being an even number? (2)

8.4 Ignoring the first digit, what is the probability of a phone number consisting of only odd digits? Write your answer correct to three decimal places. (2)

(3)

QUESTION 9

The code to a safe consists of 10 digits chosen from 0 to 9. None of the digits are repeated. Determine the probability of a code where the first digit is odd and none of the first three digits may be a zero. Write your answer as a percentage correct to two decimal places.

QUESTION 10

The data below was obtained from the financial aid office at a university.

	Receiving financial aid	**Not receiving financial aid**	**Total**
Undergraduates	4 222	3 898	8 120
Postgraduates	1 879	731	2 610
Total	6 101	4 629	10 730

10.1 Determine the probability that the student selected at random is…

10.1.1 receiving financial aid. (2)

10.1.2 a postgraduate student and not receiving financial aid. (2)

10.1.3 an undergraduate student and receiving financial aid. (2)

10.2 Are the events of being an undergraduate and receiving financial aid independent? Show ALL relevant workings to support your answer. (4)

10.3 Are the events of being an undergraduate and receiving financial aid mutually exclusive? Justify your answer (2)

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QUESTION 11

Each of the 200 employees of a company wrote a competency test. The results are indicated in the table below.

	Pass	**Fail**	**Total**
Males	46	32	78
Females	72	50	122
Total	118	82	200

11.1 Are the events Pass and Fail mutually exclusive? Explain your answer. (2)

11.2 Is passing the competency test independent of gender? Substantiate your answer with the necessary calculations. (4)

(5)

QUESTION 12

Three cards are selected at random (without replacement) from a standard full pack of playing cards. There are 52 cards in the pack, jokers are excluded. Find the probability that the cards are all the same colour.

QUESTION 13

A study of numbers of male and female offspring in a certain population is being carried out. It is found that the first child in any family is equally likely to be male or female, but that for any subsequent offspring, the probability that they will be of the same sex as the previous child is $\frac{3}{5}$. No twins, triplets etc., are possible.

13.1 Find the probability that the first child of a family will be female. (1)

13.2 Find the probability that the first two children of a family will be female. (1)

13.3 Find the probability that a family will have two females followed by two males (in that order). Leave your answer in simplified fraction form. (2)

QUESTION 14

There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

14.1 Calculate the probability that the first learner chosen is a boy. (1)

14.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate on your diagram ALL possible outcomes (4)

14.3 Calculate the probability that a boy, then a girl and then another boy is chosen in that order. (3)

14.4 Calculate the probability that all three learners chosen are girls. (2)

14.5 Calculate the probability that at least one of the learners chosen is a boy. (3)

14.6 What is the probability that 5 learners chosen are of the same gender? (4)

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QUESTION 15 (5)

There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag.

QUESTION 16 (6)

There are four black balls and y yellow balls in a bag. Thandi takes out a ball, notes its colour and then puts it back in the bag. She then takes out another ball and also notes its colour. If the probability that both balls have the same colour is $\frac{5}{8}$, determine the value of y.

QUESTION 17

At a school for boys there are 240 learners in Grade 12. The following information was gathered about participation in school sport.

122 boys play rugby (R)
58 boys play basketball (B)
96 boys play cricket (C)
16 boys play all three sports
22 boys play rugby and basketball
26 boys play cricket and basketball
26 boys do not play any of these sports
Let the number of learners who play rugby and cricket only be x.

17.1 Draw a Venn diagram to represent the above information. (5)

17.2 Determine the number of boys who play rugby and cricket. (2)

17.3 (Leaving your answer(s) correct to THREE decimal places.)
Determine the probability that a learner in Grade 12 selected at random:

17.3.1 Does not play cricket (2)

17.3.2 Participates in at least 2 of these sports (2)

QUESTION 18

Given that:

P(A only)=x
P(A and B)=0,1
P(B)=0,4
P(not (A or B))=y

18.1 Represent this information in a Venn Diagram. (4)

18.2 If A and B are independent events, find the values of x and y. (5)

QUESTION 19 (6)

Given that:

- A and B are independent events
- P(B) = 2P(A)
- P(AorB) = 0,6

Calculate P(B)

QUESTION 20

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Given that: $P(A) = \frac{3}{5}$; $P(B) = \frac{2}{5}$; $P(C) = \frac{3}{10}$

A and B are independent events
B and C are independent events

20.1 Calculate P (A or B) (2)

20.2 Calculate P(C only) (4)

QUESTION 21 (4)

In a Physical Science quiz, two teams work independently on a problem. They are allowed a maximum of 10 minutes to solve the problem. The probabilities that each team will solve the problem are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Calculate the probability that the problem will be solved in the ten minutes allowed.

QUESSTION 22

A local club has facilities that include tennis courts and a golf course.
A survey of the club members indicated that 504 regularly use the golf course and 336 regularly use the tennis courts. Some members regularly use both while 56 use neither of the facilities. The club has 700 members.

22.1 Determine the number of members that regularly use at least one of the facilities. A Venn diagram may be useful (2)

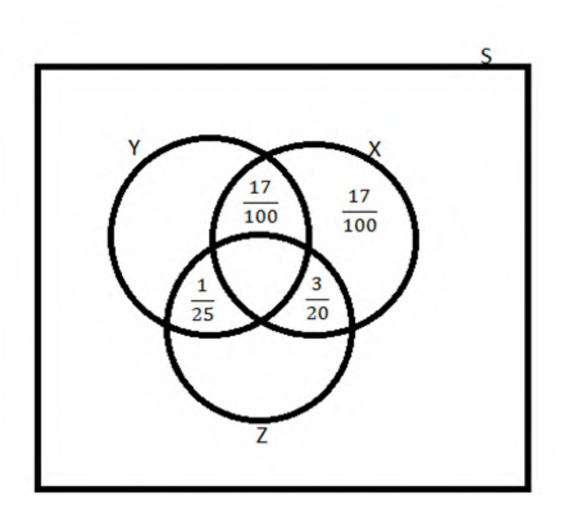
22.2 What is the probability that a club member selected at random uses exactly (only) one facility? (3)

22.3 Given that: P (using the golf course) X P (using the tennis courts) = 0,3456. Validate statistically whether these events are independent or not. (2)

(4)

QUESTION 23

The Venn diagram below shows probabilities of 3 events.



Complete the Venn diagram using the additional information provided.

P(Z and(not Y)) = 31/100
P(Y and X) =23/100
P(Y) =39/100

After completing the Venn diagram, compute P(Z and not (X or Y))

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QUESTION 24

24.1 N and M are wo events. P(N) =0,3;P(M) =0,4 and P(M or N)=0,6.

24.1.1 Sketch a Venn-diagram to represent the events. Sketch a Venn-diagram to represent the events. (5)

24.1.2 Are the events N and M independent? Motivate your answer by showing all relevant calculations. (5)

24.2 A five-digit code is created by using digits 0 to 9.Digits may not be repeated. How many different codes are possible if the code must be a multiple of 5 and the code must start with an 8? (4)

QUESTION 25

25.1 The digits 0 to 9 are used to form codes.

25.1.1 Determine the number of different 6-digit codes that can be formed If repetition of digits is allowed. (1)

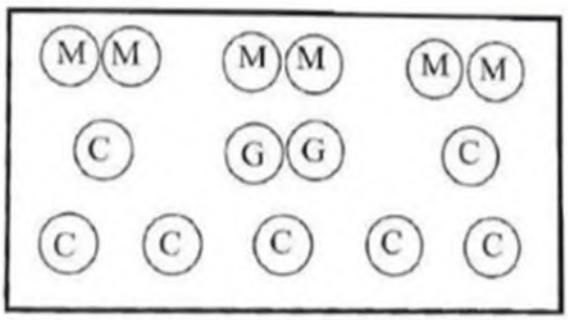
25.1.2 2 Determine the number of 6-digit codes that can be formed that starts with a 9 and ends with a 2 if repetition of digits is not allowed (2)

25.2 The digits 0 to 9 are used to form10-digit codes. Determine the number of 10-digit codes that can be formed if the 2 and the 3 may not appear next to each other and if repetition of digits is not allowed. (3)

(5)

QUESTION 26

The Ngcobo family takes family photos. The photographer arranges three married couples, seven children and two grandparents as follows:
The couples stand husband and wife together at the back, the grandparents in the middle and the children in the other positions as shown in the diagram below.



M	Married couples
G	Grandparents
C	Children

How many different ways can the Ngcobo family be arranged for the photo?

QUESTION 27
Events A,B and C occur as follows where and B are independent events:

- P(A)= 0,38
- P(B)=0,42

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- P(A and B)=0,1596
- P(C)= 0,28
- There are 456 people in event A

27.1 Are A and B mutually exclusive? Motivate your answer. (2)

27.2 By using an appropriate formula , show that the value of P(A or B) =0,64 (2)

27.3 Calculate the number of people in the sample space. (2)

27.4 Determine n(C). (4)

QUESTION 28

Five boys and four girls go to the movies. They are all seated next to each other in the same row

28.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)

28.2 (3)

QUESTION 29

Four digits codes (not beginning with 0), are to be constructed from the set of digits {1;3;4;6;7;8;0}

29.1 How many four - digit codes can be constructed, if repetition of digits is allowed? (2)

29.2 How many four - digit codes can be constructed, if repetition of digits is not allowed? (2)

29.3 Calculate the probability of randomly constructing a four-digit code which is divisible by 5 if repetition of digits is allowed. (3)

QUESTION 30 (5)

A survey was conducted asking 60 people with which hand they write and what colour hair they have. The result are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	Total
HAIR COLOUR	Light	a	b	20
	Dark	c	d	40
	Total	48	12	60

The survey concluded that the 'hand used for writing 'and 'hair colour' are independent events. Calculate the values of a, b, c and d.

QUESTION 31 (5)

The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny,the probability that Pitso plays soccer is 4/5. If it is not sunny, the probability that Pitso plays soccer is 2/5 .Determine the probability that Pitso does not play soccer.

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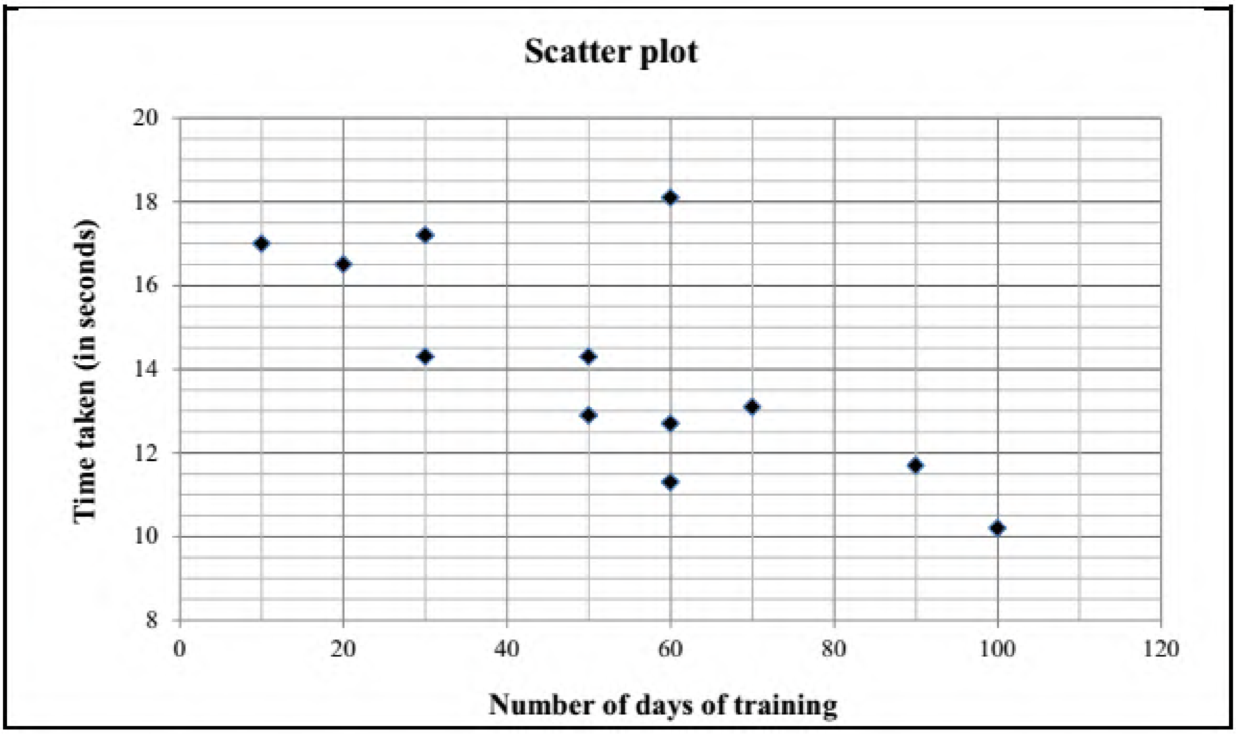
QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Discuss the trend of the data collected.
1.2 Identify any outlier(s) in the data. (1)
1.3 Calculate the equation of the least squares regression line. (4)
1.4 Draw the regression line. (2)
1.5 Use the equation of the regression line to predict the time taken to run the 100 m sprint for an athlete training for 45 days. State whether this is interpolation or extrapolation. (2)
1.6 Calculate the correlation coefficient. (2)
1.7 Comment on the strength of the relationship between the variables. (1)

1.8 The point (60; 18) was wrongly captured, it was supposed to be (60; 15). If this point is corrected on the scatter, what effect does it have on the value of r and give a reason for your answer. (2)

QUESTION 2

The weights (in kilogram) of the 20 boys in the hockey squad of School A are given below:

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69	59	59	66	64	58	63	58	62	61
57	53	60	51	60	48	47	60	40	60

.1 Determine the mean and variance for the weights of the School A squad. (3)

2.2 2.2.1 How many boys are in the School B squad? (1)

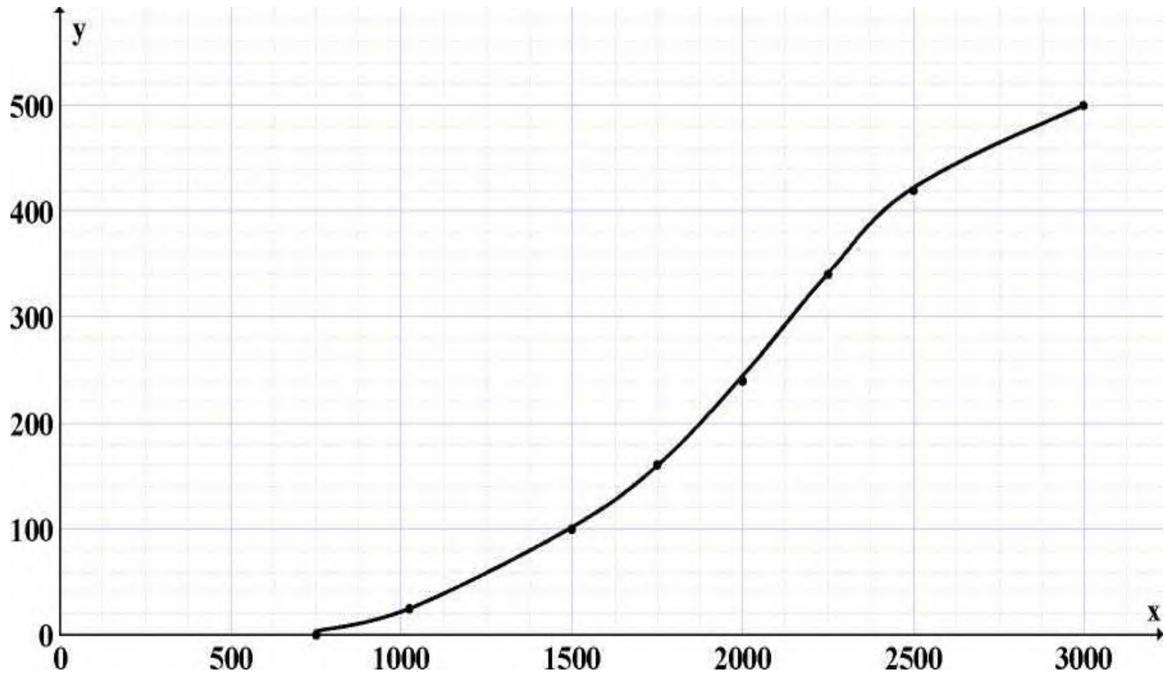
2.2.2 Determine the mean weight for the School B squad (2)

2.2.3 Determine the standard deviation for the School B squad. (2)

2.2.4 If five boys of equal weight are added to the squad of School A so that the means of both schools are the same, what must be the weight of each boy? (2)

QUESTION 3

The lifetime of electric light bulbs was measured in a laboratory. The results are shown in the cumulative



frequency curve below.

3.1 The number of light bulbs that were tested? (2)

3.2 The median lifetime of the electric light bulbs tested. (2)

3.3 The percentage of light bulbs that have a lifetime of between 1500hrs to 2300hrs (2)

3.4 The interquartile range. (2)

3.5 The modal class (2)

3.6 The number of electric light bulbs with a lifetime of between 1750 and 2000 hours. (2)

3.7 The amount spent on purchasing the light bulbs that lasted longer than 2 500 hours if the cost of one light bulb is R5.00. (2)

QUESTION 4

The owner of Harvey Tours uses the following data to illustrate the relationship between the annual advertising expenditure and the annual profit of the business. (All data is in THOUSAND of Rands.)

Annual advertising expenditure	12	14	17	21	26	30
Annual Profit	60	70	90	100	100	120

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4.1 Draw a scatter plot to represent the data. Use the grid provided in the answer sheet (2)
4.2 Determine the equation of the least squares line for the data (2)
4.3 Draw the least squares regression line on your scatterplot diagram. (1)
4.4 Predict the annual profit if the annual expenditure is R25 000 (2)
4.5 Calculate the correlation coefficient. (2)
4.6 Describe the strength of the relationship between the annual profit and the annual advertising expenditure (2)

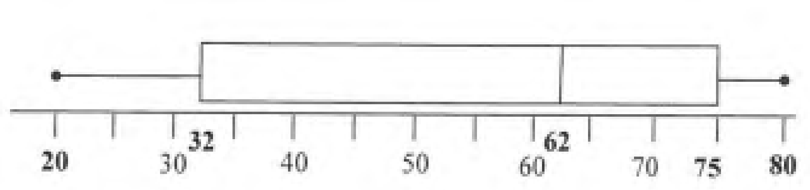
QUESTION 5

The following data relates to the scores for a Maths test for Grade 12 learners at Bluebell high.

25	56	78	67	89	90	43	55	77	87	52	67	89	53	05	34
22	34	65	75	75	67	75	76	88	43	56	78	54	75	84	32

5.1 Determine the five number summary from the data above. (4)
5.2 Use the diagram sheet to draw a box and whisker diagram for the learner scores. (3)
5.3 Comment on the skewness of the scores. (1)
5.4 Determine the semi-interquatile range. (2)
5.5 How many scores lie within one standard deviation from the mean? (3)
5.6 Determine from the mean.the probability that a learner chosen at random scored a mark outside one standard deviation from the mean. (2)

QUESTION 6



The box and whisker diagram above shows the marks (out of 80) obtained in a History test by a group of learners.

6.1 Comment on the skewness of the data. (1)
6.2 Write down the range of the marks obtained. (1)
6.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (1)

QUESTION 7

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x < 20$	2
$20 < x < 30$	8
$30 < x < 40$	5
$40 < x < 50$	10
$50 < x < 60$	12
$60 < x < 70$	18
$70 < x < 80$	3
$80 < x < 90$	2

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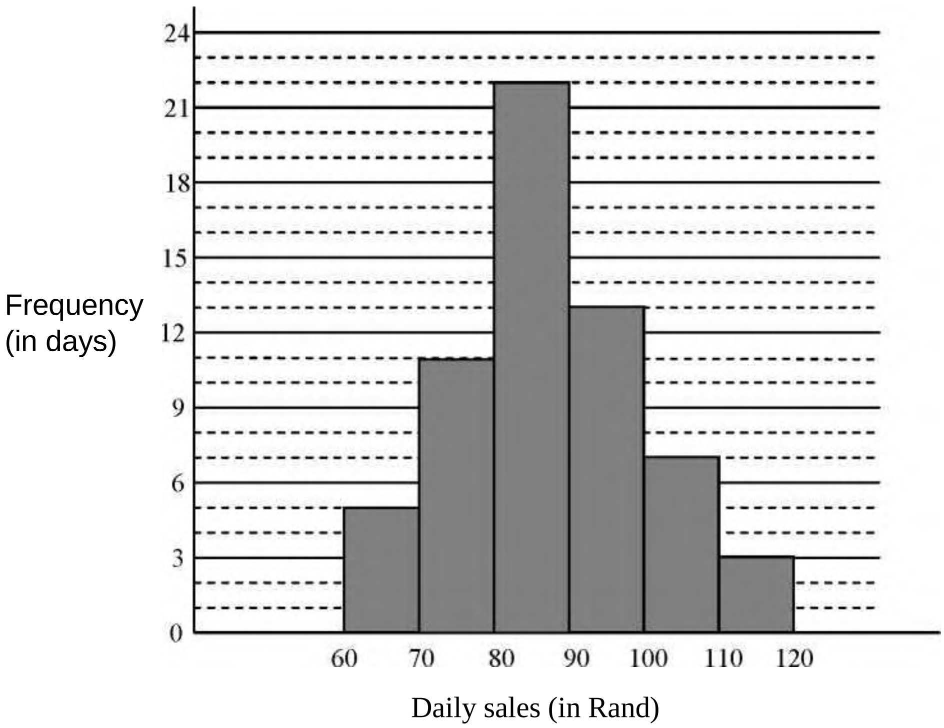
7.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places (3)
7.2 Determine the median class (2)
7.3 Calculate the standard deviation of the data. (2)
7.4 Draw a cumulative frequency graph (ogive) of the data (4)
7.5 Hence, estimate the percentage number of days on which 65 or more messages were sent. (2)

QUESTION 8

A street vendor has kept a record of sales for November and December 2007. The daily sales in Rand is shown in the histogram below.



8.1 Complete the cumulative frequency table for the sales over November and December. (3)
8.2 Draw an ogive for the sales over November and December (3)
8.3 Use your ogive to determine the median value for the daily sales. (1)
8.4 Estimate the interval of the upper 25% of the daily sales. (2)

QUESTION 9

The price of 95-octane unleaded petrol in Gauteng for the period January 2007 to July 2008 is shown below. The price is in South African cents per litre.

January 2007	598	February 2007	575	March 2007	599
April 2007	667	May 2007	701	June 2007	724
July 2007	716	August 2007	701	September 2007	691
October 2007	701	November 2007	704	December 2007	747
January 2008	747	February 2008	764	March 2008	825
April 2008	891	May 2008	946	June 2008	996
July 2008	1 070				

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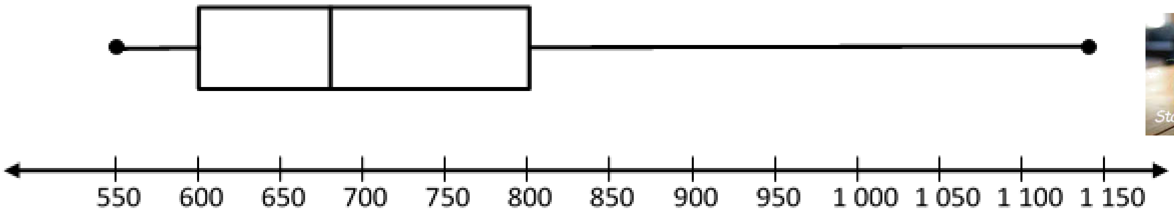
9.1 Determine the median, lower quartile and upper quartile for the data. (4)

9.2 Draw a box and whisker diagram (2)

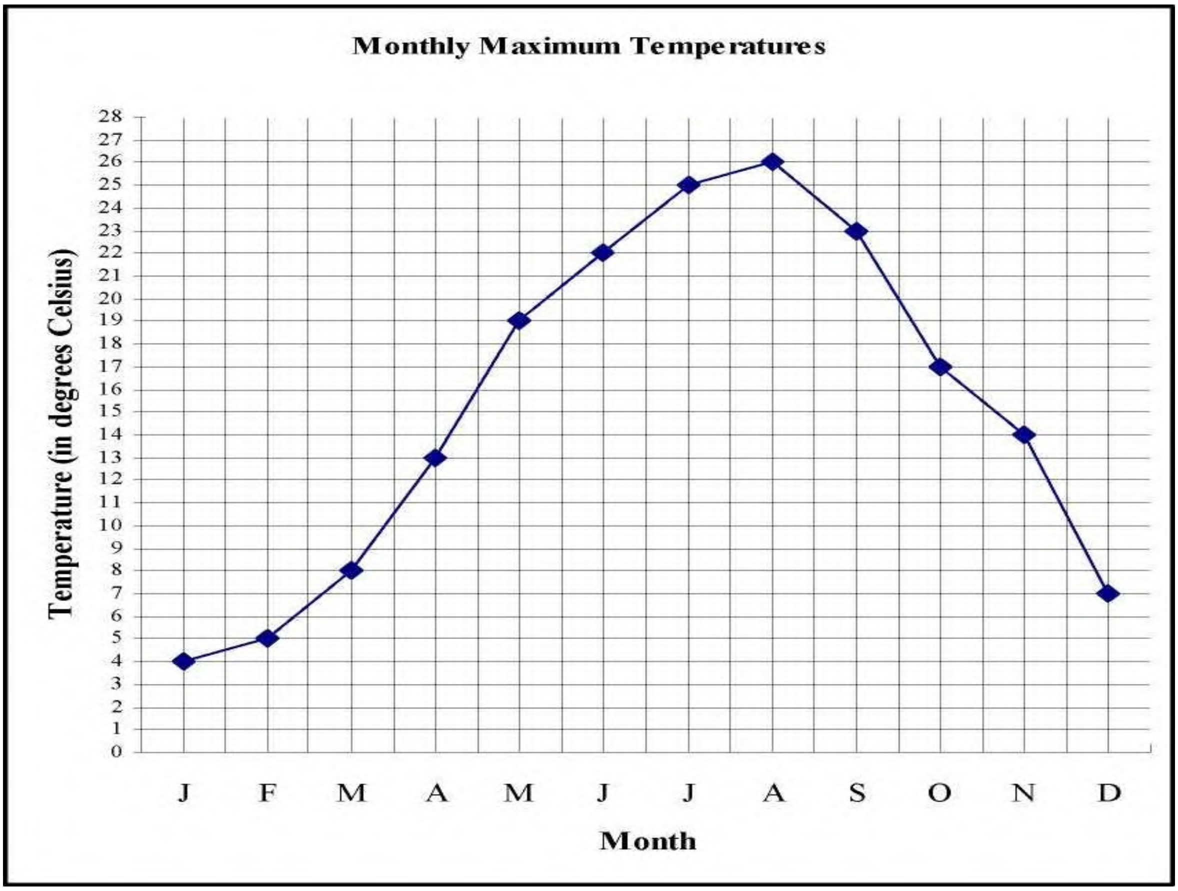
9.3 The box and whisker diagram for the price of diesel for the same period as above is shown below. The lower quartile is 600 and the upper quartile is 800. (2)



How many data points are there, strictly between 600 and 800?

QUESTION 10

The graph below shows the monthly maximum temperatures in a certain city



10.1 What is the range of the monthly maximum temperatures? (2)

10.2 Calculate the mean monthly maximum temperature. (3)

10.3 Calculate the standard deviation of the monthly maximum temperature. (2)

10.4 It is predicted that one hundred years from now, global warming is likely to increase the city's monthly maximum temperature by 5° C in December, January and February. It will also result in an increase of 1° C in the other months of the year.

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	10.4.1	By how much does the mean increase?	(2)
	10.4.2	Describe the effect that the predicted increases in temperature will have on the standard deviation. Justify your answer.	(2)
10.5	Learners at Phambili High School travel from three different neighbourhoods, Neighbourhood A, B and C. The table below shows the number of learners from each neighbourhood, and their mean travelling times from home to school.		(4)

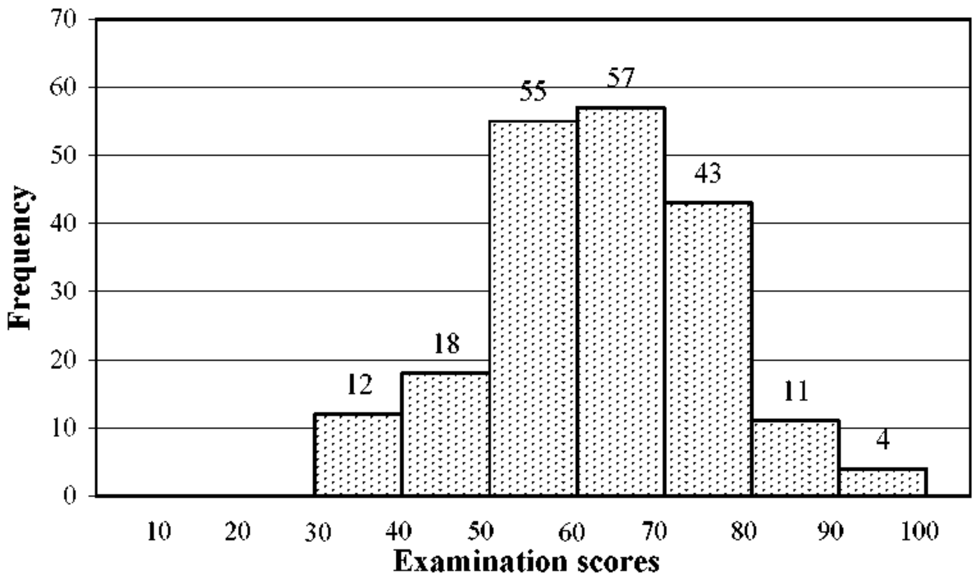
Neighbourhood	**A**	**B**	**C**
Number of learners	135	225	200
Mean travelling time (in min.)	24	32	x

The mean travelling time for learners living in Neighbourhood C is the same as the mean travelling time for all 560 learners.

Calculate the mean travelling time for Neighbourhood C.

QUESTION 11

The histogram below shows the distribution of examination scores for 200 learners in Introductory Statistics



11.1	Complete the cumulative frequency table for the above data provided	(3)
11.2	Draw an ogive of the above data	(2)
11.3	Use the ogive to estimate how many learners scored 75% or more for the examination.	(5)

QUESTION 12

A researcher suspects that airlines, whose planes arrive on time, are less likely to lose the luggage of their passengers. Information gathered from 10 airline companies is summarised in grid below

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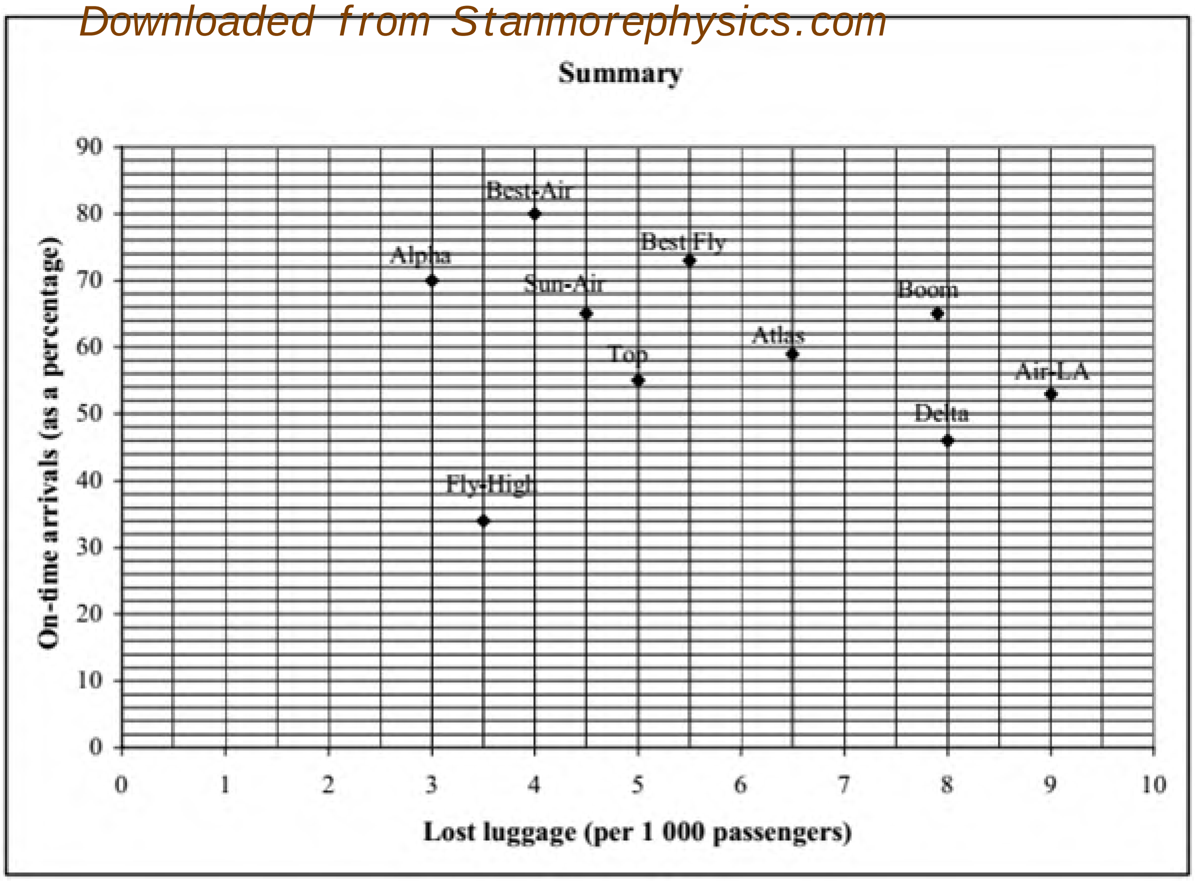

12.1 Which airline has the worst record for on-time arrivals? (1)

12.2 Is the following statement likely to be TRUE? Motivate your answer. Of 5 120 passengers transported by Boom airlines, 40 passengers lost their luggage. (2)

12.3 Does the data confirm the researcher's suspicions? Justify your answer. (2)

12.4 Which ONE of the 10 airlines would you prefer to use? Give a reason for your answer (2)

QUESTION 13

A parachutist jumps out of a helicopter and his height above ground level is estimated at various times after he opened his parachute. The following table gives the results of the observations where y measures his height above ground level in metres and t represents the time in seconds after he opened his parachute.

2	3	4	5	6	7	8
500	300	200	120	70	40	20

13.1 Draw a scatter plot for the above information

13.2 Describe the curve of bestfit (1)

13.3 Use the scatter plot to estimate the height of the parachutist 5,5 seconds (1)

QUESTION 14

The following marks were obtained from Mr Dlamini's 7 Mathematics learners.

It is further given that:

Range of the scores is 65

The difference between Q_1 and Q_2 is 11

Semi IQR is 17.5

The average score is 50.71

One of the learners scored a mark which is coincidentally the mean of Q_2 and Q_3

23	a	B	45	c	d	e

14.1 Calculate the value of a, b, c, d and e.

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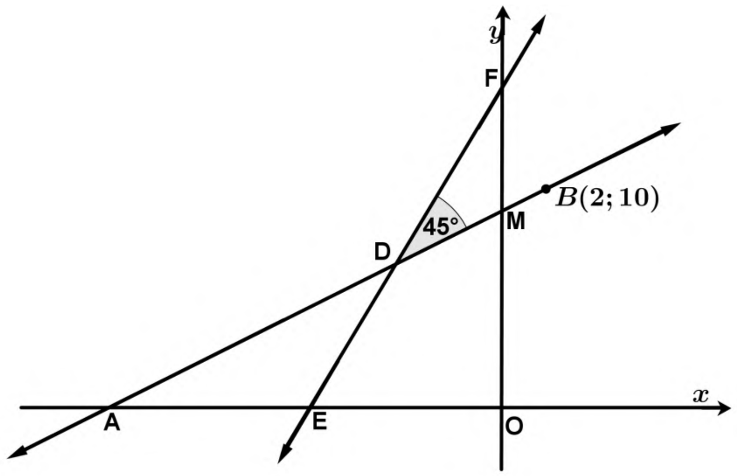
QUESTION 1

In the given diagram, E and F are the *x* and *y* intercepts of the line having equation $y = 6x + 16$. The line through B (2; 10) making an angle of 45° with EF, as shown below, has *x* and *y* intercepts A and M.



.

1.1. Determine the coordinates of E. (2)

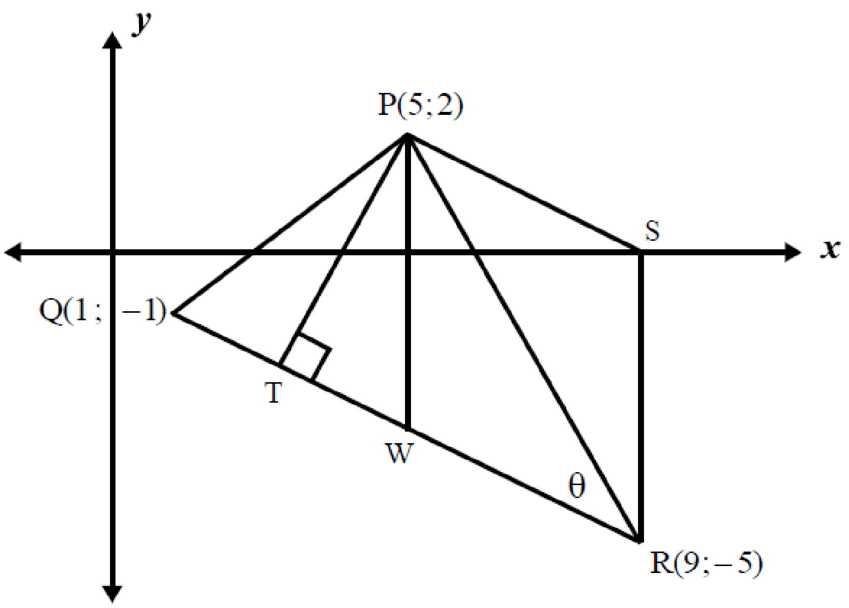
1.2. Calculate the size of $D\hat{A}E$. (3)

1.3. If AB intersect EF at D, Calculate the coordinates of D (6)

1.4. Calculate the area of quadrilateral DMOE (6)

QUESTION 2

In the diagram, PQRS is a trapezium with vertices P (5; 2), Q(1;1), R(9;5)and S. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and $R\hat{P}S = \beta$



2.1 Determine the equation of PW if W is the mid-point of QR (2)

2.2 Determine the equation of PS (4)

2.3 Determine the equation of PT. (3)

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2.4 Show that $QT = \frac{1}{3}TR$ (5)

2.5 Calculate the size of β rounded off to two decimal places (5)

QUESTION 3

Consider the following points on a Cartesian plane:

$A(1\,;2)$, $B(3\,;1)$, $C(-3\,;k)$ and $D(2\,;\,-3)$. Determine k, if:

3.1 $(-1\,;3)$ is the midpoint of AC (3)

3.2 AB is parallel to CD (3)

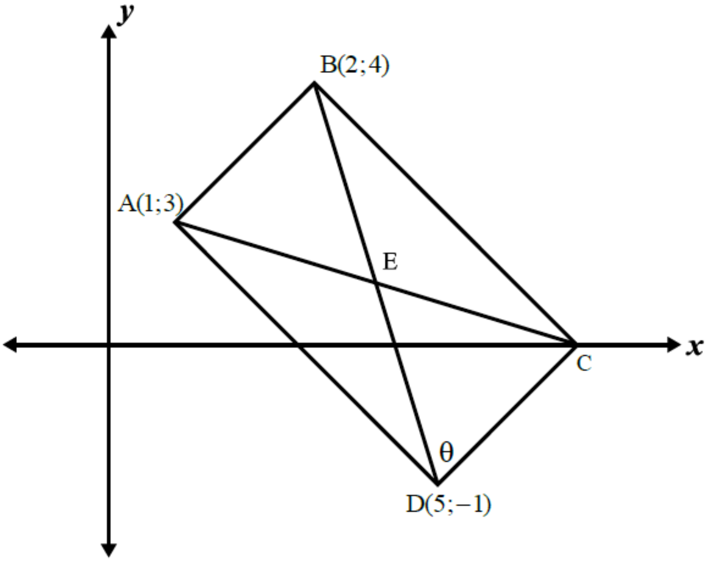
3.3 AB ⊥ AC (3)

3.4 A, B and C are collinear. (3)

3.5 $CD = 5\sqrt{2}$ (5)

QUESTION 4

ABCD is a parallelogram with vertices $A(1\,;3), B(2\,;4), C$ and $D(5\,;\,-1)$



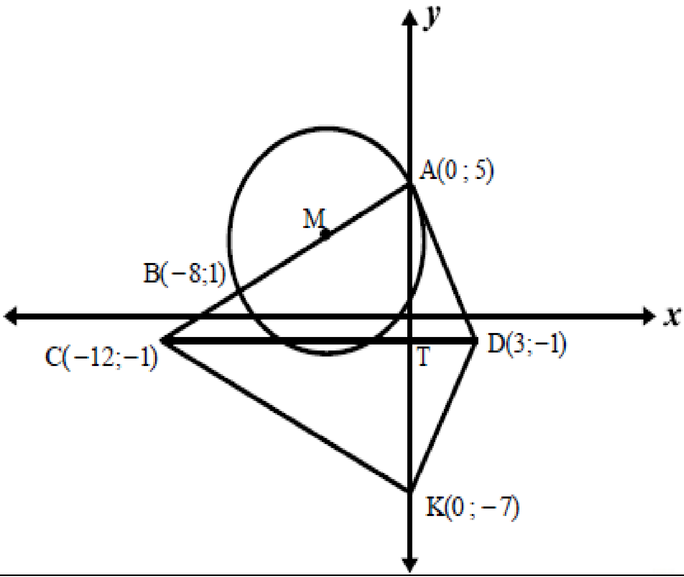
4.1 Determine the coordinates of C (1)

4.2 Show that ABCD is a rectangle. (3)

4.3 Determine the area of ABCD. (4)

QUESTION 5

A(0 ; 5) and B(-8 ; 1) are two points on the circumference of the circle centre M, in a Catersian plane. M lies on AB. DA is a tangent to the circle at A. Points D (3 ; -1) and C (-12 ; -1) are joined. K is the point (0 ; -7). CTD is a straight line



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5.1 Determine the equation of CD. (1)

5.2 Determine the equation of the tangent AD. (4)

5.3 Determine the length of AM. (3)

5.4 Determine the equation of the circle centre M in the form: (4)

$$ax^2 + by^2 + cx + dy + e = 0$$

5.5 Quadrilateral ACKD is one of the following: parallelogram; kite, rhombus or rectangle. Which one is it? Justify your answer. (4)

QUESTION 6 In the diagram below, trapezium ABCD with AD// BC is drawn. The coordinates of the vertices are $A(1;7); B(p;q); C(-2;-8)$ and $D(-4;-3)$. BC intersects the x-axis at F.$D\hat{C}B = \alpha$.

6.1 Calculate the gradient of AD (2)

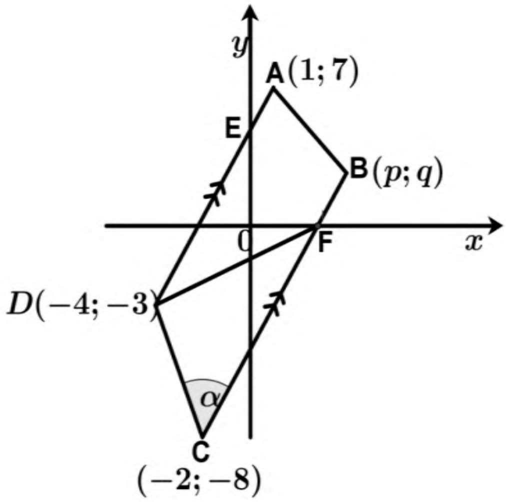
6.2 Determine the equation of BC in the form of $y = mx + c$ (3)

6.3 Determine the coordinates of F (2)

6.4 Show that $\alpha = 48{,}37°$ (4)

6.5 Calculate the area of Δ DCF (6)

6.6 Given that ABCD is a parallelogram, determine the coordinates of B. (5)



QUESTION 12

12.1 Equations of circles with centres A and B respectively, are given below. (2)

Circle A: $(x-2)^2 + (y-3)^2 = 9$ and Circle B: $(x-1)^2 + (y+1)^2 = 16$

Without solving for x and y, show that the circles intersect each other at two points.

(Show all your arguments)

12.2 The circle defined by the equation $x^2 + y^2 - 2x + 8y - 71 = 10$ has the centre M and the circle defined by the equation $(x-2)^2 + y^2 = 5$ has centre N

12.2.1 Determine the coordinates of the centres M and N. (4)

12.2.2 Calculate the radii of Circle M and Circle N (2)

(8)

QUESTION 13

Refer to the figure below. The point B is on the y-axis and the coordinates of A are (1;-1) the equations of the sides. BC and AC are $x - 3y + 6 = 0$ and $x - y - 2 = 0$ respectively

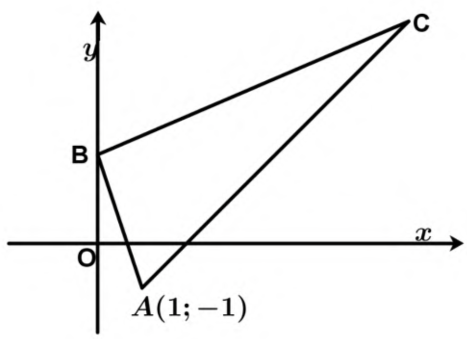
13.1 Show that the coordinates of B are (0;2) (2)

13.2 Determine the gradient of BC (2)

13.3 Prove that $A\hat{B}C = 90°$ (3)

13.4 Determine the coordinates of C (4)

13.5 Calculate the length of AC (2)



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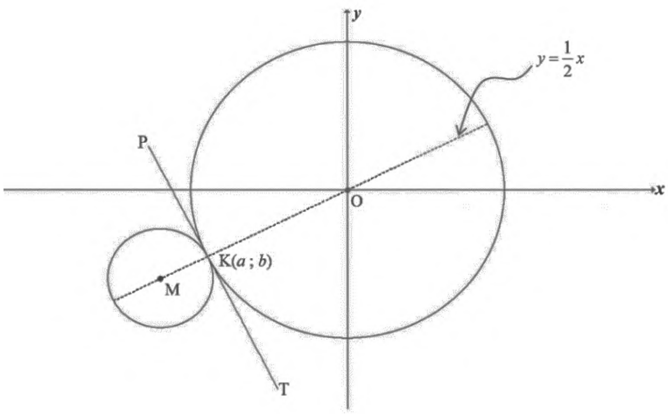
QUESTION 14

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



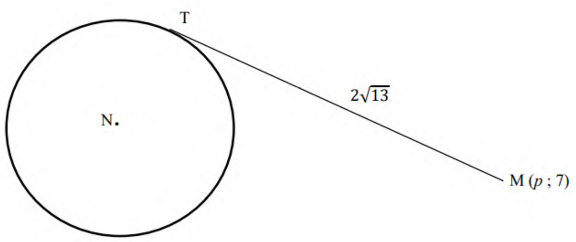
14.1 Write down the length of OK in surd form.
14.2 Show that K is the point (-12 ; -6).
14.3 Determine:
14.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$
14.3.2 The coordinates of M
14.3.3 The equation of the smaller circle in the form $(x - a)^2 + (y - b)^2 = r^2$

14.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

14.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn.
Prove by calculation that this circle does NOT cut the circle with centre M(-16 ; -8). (5)

QUESTION 15

Given $x^2 + y^2 - 6x - 2y + 1 = 0$ is the equation of the circle, centre N. $M(p\ ;\ 7)$ is a point outside the circle and the length of the tangent to the point T on the circle is $2\sqrt{13}$ units.



15.1.1. Determine the coordinates of the centre and the length of the radius, TN , of the circle. (4)

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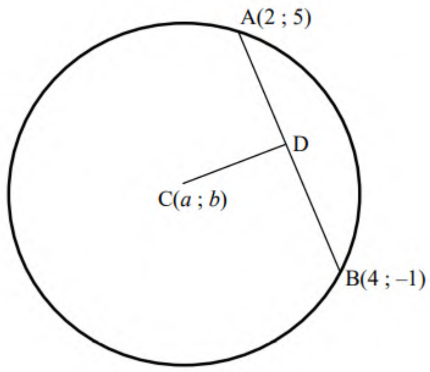
15.1.2. Calculate the value(s) of p. (6)

15.2. A(2 ; 5) and B(4 ; – 1) are two points on a circle. $C(a;b)$ is the centre of the circle. The centre of the circle lies on the line $2x - y + 1 = 0$. AB is a chord of the circle with $AD = \frac{1}{2}AB$.



15.2.1. Give a reason as to why CD ⊥ AB. (1)

15.2.2. Determine the equation of the tangent to the circle at B. (5)

15.2.3. Determine the equation of the circle. (5)

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8. TRIGONOMETRY

1.1 Show that the value of the following expression is independent of the value of A: (4)

$\sin(A+40°)\cos(A+30°)-\cos(A+40°)\sin(A+30°)$

1.2 Given: $\sin A\cos A = k$ and k is acute.

1.2.1 Determine the value of $\tan A + \frac{1}{\tan A}$ in terms of k (8)

1.2.2 Prove that $\sin A + \cos A = \sqrt{1+2k}$. (4)

1.3 Given that P and Q are both acute, solve for P and Q if:

$\sin P\sin Q - \cos P\cos Q = \frac{1}{2}$ and $\sin(P-Q) = \frac{1}{2}$.

1.4 Determine the value of the following: $\left[\sin\left(22\frac{1}{2}\right)° + \cos\left(22\frac{1}{2}\right)°\right]^2$ (4)

1.5 Simplify: $\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ (5)

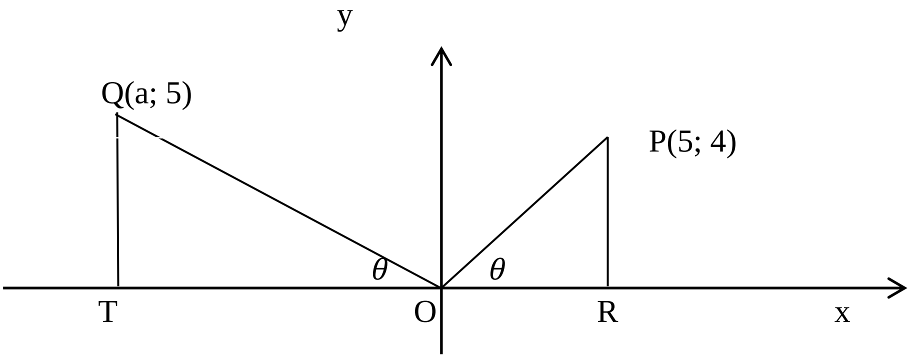
1.6 Simplify: $\dfrac{\cos(-x).\sin(x-180°).\tan x}{\sin 960°.\cos^2(x-90°).\sin 270°}$ (7)

1.7 If $\tan\theta = 1{,}5$ and $90° \leq \theta \leq 360°$, calculate without using a calculator and by using a diagram, the value of $\sin 2\theta$. (4)

1.8 Simplify: $\dfrac{\sin 6A}{\sin 2A} - \dfrac{\cos 6A}{\cos 2A}$ (7)

1.9 If $\cos\beta + \sin\beta = T$, express $\dfrac{\cos 2\beta}{\sin(\beta - 45°)}$ in terms of T. (5)

1.10 In the diagram below, similar triangles ΔOPR and ΔOQT are drawn. O is the origin. R and T are points on the x-axis.



Determine, leaving answers in surd form if necessary:

(a) $\cos(90° + \theta)$ (2)

(b) The value of a (3)

1.11 Simplify the following expression as far as possible.

(a) $\dfrac{\sin(180°-\theta).\cos(90°-\theta)-1}{\cos(-\theta)}$ (4)

(b) Hence determine for which value(s) of θ, and $\theta \in [0°;\ 360°]$ (4)

$\sqrt{\dfrac{\sin(180°-\theta).\cos(90°-\theta)-1}{\cos(-\theta)}}$ will be real.

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1.12 Given $\tan\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta < 0$. Without the use of a calculator, express each of the following in its simplest form:

(a) $\sin\theta$ and $\cos\theta$ (2)

(b) $\sin 2\theta$ (2)

(c) $cos^2(90° + \theta)$ (3)

1.13 Simplify fully: $\frac{\tan(180° - \theta).\cos(-\theta).\sin 390°}{(\cos 300°.\sin\theta) - \cos 450°}$

1.14 If $\cos A + \sin A = k$, express the following in terms of k:

(a) $\cos(A - 45°)$ (4)

(b) $1 + \sin 2A$ (3)

1.15 $4\tan\theta + 5 = 0$ and $\theta \in [0°;\ 180°]$. Determine, without the use of a calculator, the value of (5)

$\sqrt{41}\cos\theta - 4\sin(-150°).\cos 180°$

1.16 If $\tan 50° = k$, evaluate $\frac{4\cos^2 25° - 2}{2\sin 25°.\cos 25°}$ in terms of k. (4)

1.17 Given the expression: $\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A}$

(a) If $A \in [0°;\ 360°]$, for what value(s) of A is the expression undefined? (4)

(b) Prove that $\frac{\sin 2A + \cos 2A - (\sin A + \cos A) + 1}{\sin A + \cos A} = 2\cos A - 1$ (5)

1.18 Given: $\sin(A+B) = \sin A\cos B + \cos A\sin B$, $\cos(A+B) = \cos A\cos B - \sin A\sin B$ and $\tan\theta = y$, then determine

(a) $\sin 2\theta$ (4)

(b) $\cos 2\theta$ (3)

1.19 Simplify: $\frac{\cos 10°.\cos 340° - \sin 190°.\sin(-20°)}{\sin 80°.\cos 20° + \cos 100°.\cos 70°}$ (6)

1.20 (a) Prove that : $\cos(A - B) - \cos(A+B) = 2\sin A\sin B$ (3)

(b) Hence find without the use of a calculator the value of $\cos 15° - \cos 75°$ (7)

1.21 Prove without the use of a calculator: $\cos 80° + \cos 40° = \cos 20°$ (4)

1.22 If $\sin\frac{x}{2} = p$, express the following in terms of p:

(a) $\cos x$ (4)

(b) $\sin x$ (3)

(c) $\tan x$ (3)

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1.23 If $\sin 2A = \frac{2\sqrt{6}}{5}$ where $A > 45°$, determine with the aid of a sketch, the value of $\sin A$ (4)

1.24 (a) Deduce that $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$ (5)

(b) Use 1.24 (a) to simplify $\frac{\sin 3\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$ (3)

1.25 Given: $\cos D = 2p$ and $\cos 2D = 7p$

(a) Calculate the value(s) of p (3)

(b) If $\hat{D} \in [0°;\ 360°]$, calculate the values of $\hat{D}$. (3)

1.26 If $q \sin 61° = p$, express the following in terms of p and q

(a) $\cos 151°$ (3)

(b) $\cos 1°$ (4)
$\sin 122°$

(c) $\sin 122°$ (3)

(d) $\cos 40°.\cos 8° + \sin 40° \sin 8°$ (4)

2. PROVING IDENTITIES

$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

2.1 Prove that $\cos(A+45°) = \frac{\cos A - \sin A}{\sqrt{2}}$. (4)

Then solve for A if $\cos A - \sin A = \frac{1}{\sqrt{2}}$ and $0° \le A \le 90°$.

2.2 Prove that $\sqrt{3}\sin(x+60°) - \sin(x+30°) = \cos x$. (5)

2.3 Prove the following identities:

(a) $1 + \sin 2B = (\sin B + \cos B)^2$ (4)

(b) $\cos A - \sin A = \frac{\sin 2A - 1}{\sin A - \cos A}$. (6)

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2.4 (a) Prove the identity $\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$ (4)

(b) Hence, or otherwise, determine the maximum value of $\frac{(1+\tan\theta)^2}{1+\tan^2\theta}$ (3)

2.5 Prove the following identity: $\tan\theta.\sin\theta+\cos\theta=\frac{1}{\cos\theta}$ (4)

2.6 Prove that $\frac{\cos(A-45°)}{\cos(A+45°)}=\frac{1+\sin 2A}{\cos 2A}$ (9)

2.7 Prove the following identity: $\frac{\sin 3\alpha}{\sin\alpha}=3-4\sin^2\alpha$ (9)

2.8 Prove that: $\frac{\cos 3x}{\cos x}=2\cos 2x-1$ (7)

2.9 Prove that $\sin 2x+2\sin^2(45°-x)=1$ and hence deduce, without the use of a calculator,

deduce that $\sin^2 15°=\frac{2-\sqrt{3}}{4}$.

2.10 Given the following identity: $\frac{\cos x-\sin x\sin 2x}{\cos 2x}=\cos x$

(a) Prove the identity. (4)
(b) For which values of x is the identity undefined? (5)
Give your answer in general solution form.

2.11 Prove: $\frac{2\sin^2 x}{2\tan x-\sin 2x}=\frac{1}{\tan x}$ (7)

3. GENERAL SOLUTION, SOLVING TRIG EQUATIONS, GIVEN DOMAIN

- **Compound and double angles, in disguise, quadratic trig form,**
- ***Sin x* and *cos x*, period is 360° and 180° (*tan x*), so k.180° or k.360°**
- **When a rational function is undefined**

Where 2 trig functions intersect graphically

3.1 If $\cos\theta=2\sin 75°\sin 15°$; $\theta\in[-360°;360°]$, determine θ without using a calculator (8)

3.2 (a) Solve for A if $\tan A=\tan 135°$ and (4)

(b) $180°<A<360°$ (5)
(c) $360°<A<720°$

3.3 Determine the general solution to $3\sin\theta\sin 22°=3\cos\theta\cos 22°+1$

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3.4 Determine the general solution to $\tan\theta.\sin\theta+\cos\theta=\frac{3}{\sin\theta}$ (7)

3.5 Determine the general solution to $\frac{\sin 3\alpha}{\sin\alpha}=2$ (9)

3.6 Consider: $\cos 6x+\cos 2x=2\cos 4x.\cos 2x$

(a) Show that $\cos 6x+\cos 2x=2\cos 4x.\cos 2x$ (8)

(b) Hence otherwise, write down the general solution of the equation (5)

$\cos 6x+\cos 2x+\cos 4x=0$

3.7 If $A\in[0°;360°]$, for what value(s) of A is the expression below undefined? (7)

$$\frac{\sin 2A+\cos 2A-(\sin A+\cos A)+1}{\sin A+\cos A}$$

3.8 Calculate the values of x if : $4\sin^2 x+6\sin x\cos x-2\sin x-3\cos x=0$ for $-360°\leq x\leq 0°$. Round off the answer to 2 decimal digits, if necessary. (9)

3.9 Determine the general solution of the equation $2\sin A\cos A-0{,}8=0$ (5)

3.10 If $\theta\in[-180°;180°]$, determine the value(s) of θ if:

(a) $\sin 5\theta\cos 20°-\cos 5\theta\sin 20°=1$ (10)

(b) $2\cos 3\theta\cos 30°-2\sin 3\theta\sin 30°=1$ (5)

3.11 Calculate the value of x between 0° and 360° if: $\cos 2x+\sin x=0$. (8)

3.12 Determine the general solution:

(a) $\sin x=2\cos^2 15°-1$ (7)

(b) $\cos 2x=\sin x-2$ (7)

(c) $\sin 3x\cos x-\cos 3x\sin x=\sin x$ (8)

(d) $\sin x\cos 320°+\cos x\sin 320°=-1$ (9)

(e) $2\sin 2x+\cos 2x+2=0$ and $\tan 71{,}6°=3$ (8)

3.13 Find the values of x between $-180°$ and 180° if: $7\sin(x-30°)+2=0$

3.14 (a) Prove for any angles A and B: $\frac{\sin A}{\sin B}-\frac{\cos A}{\cos B}=\frac{2\sin(A-B)}{\sin 2B}$ (4)

Hence, show without using a calculator

(b) $\frac{\sin 5B}{\sin B}-\frac{\cos 5B}{\cos B}=4\cos 2B$ (4)

(c) $\frac{1}{\sin 18°}=4\cos 36°$ (3)

(d) $\sin 18°$ is a solution of the cubic equation $8x^3-4x+1=0$

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3.14 Prove the following identity: $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$ (6)

3.15 Given that $\cos 314° = t$. Calculate, with the aid of a sketch

(a) $\sin 46$ (4)

(b) $\tan 88°$ (3)

(c) $\cos 134°$ (3)

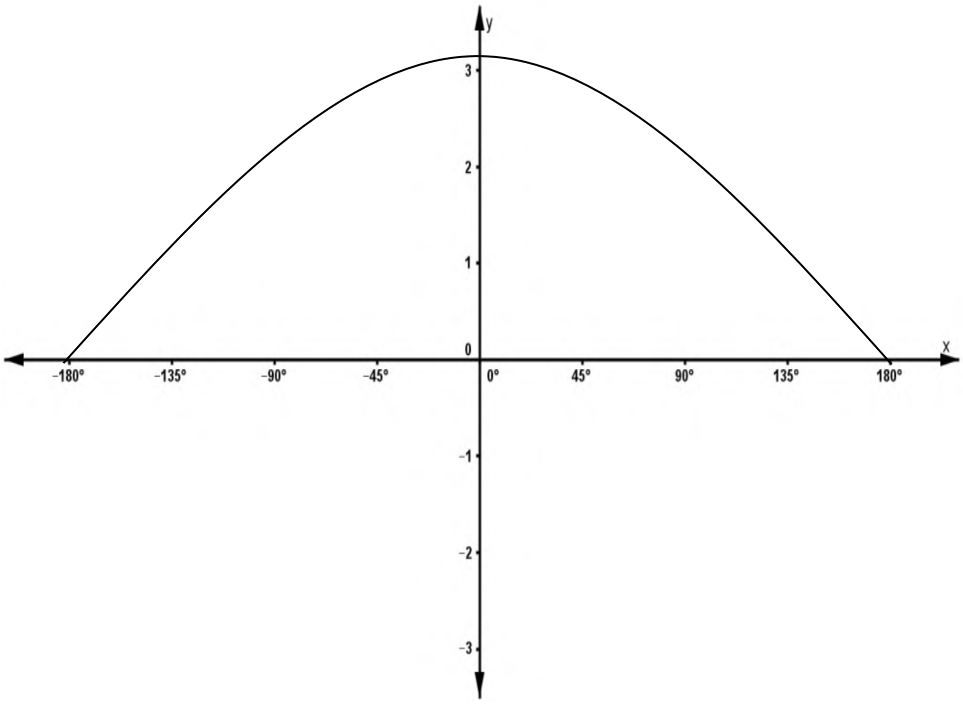
3.16 Determine, without using a calculator, the numerical value of: (4)

$tan1° \times tan2° \times tan3° \times tan4° \times \ldots \ldots .. \times tan87° \times tan88° \times tan89°$

4. TRIGONOMETRIC GRAPHS

- **Domain**
- **Range**
- **Determine equations**
- **Amplitude**
- **Intersection between TWO graphs**
- **Increasing and decreasing graphs**
- **Inequalities**
- **Distance between curves**
- **Transformation of functions**

4.1 On the axes the graph of $f(x) = a\cos bx$ for $-180° \leq x \leq 180°$ is sketched



(a) Write down the values of a and b. (4)

(b) Write down the period of f. (2)

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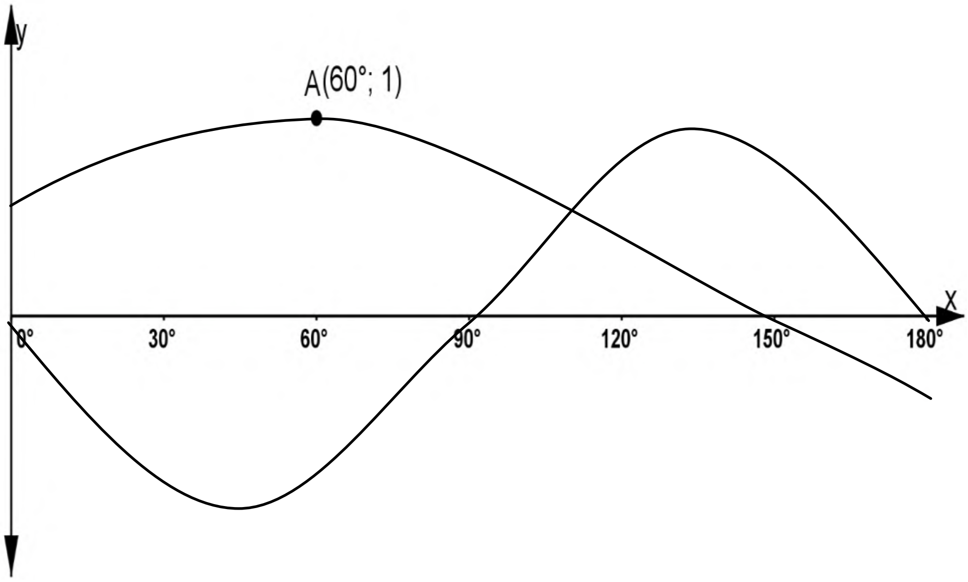
(c) Show on the x-axis where you would read off the solution to $a\cos bx = 2$ for $-180° \leq x \leq 180°$. (Use letters A and B on the x-axis.) (2)

(d) On the same set of axes, draw a second graph which would allow you to read off the solution of the equation $a\cos bx = 1 + \sin(x - 45°); -180° \leq x \leq 180°$ (4)

4.2 The figure shows the graph $f(x) = \cos(x + \theta)$ and $g(x) = -\sin 2x$ for $x \in [0°;\ 180°]$.



(a) Write down the range of g. (2)

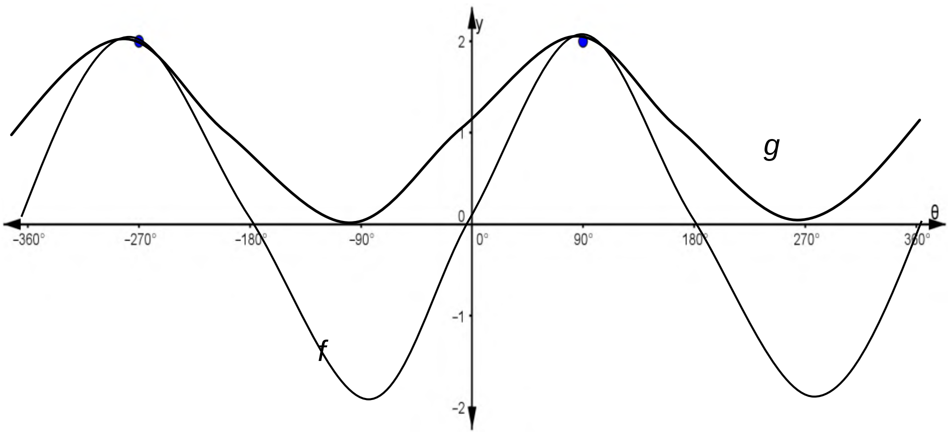
(b) Determine the value of θ (2)

(c) $C(x; y)$ is the point of intersection of the two graphs. Solve for x. (3)

(d) For which values of x is $f(x).g(x) > 0$?

(e) For which values of x is $f'(x).g'(x) > 0$?

4.4 The sketch shows the curves of $f = \{(x; y)/\ y = a\sin x\}$ and g for $x \in [-360°; 360°]$



Answer the following questions with the aid of the graph:

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(a) The maximum value of f is . . . (1)

(b) The value of a is . . . (1)

(c) The amplitude of g is . . . (1)

(d) The equation of g is . . . (2)

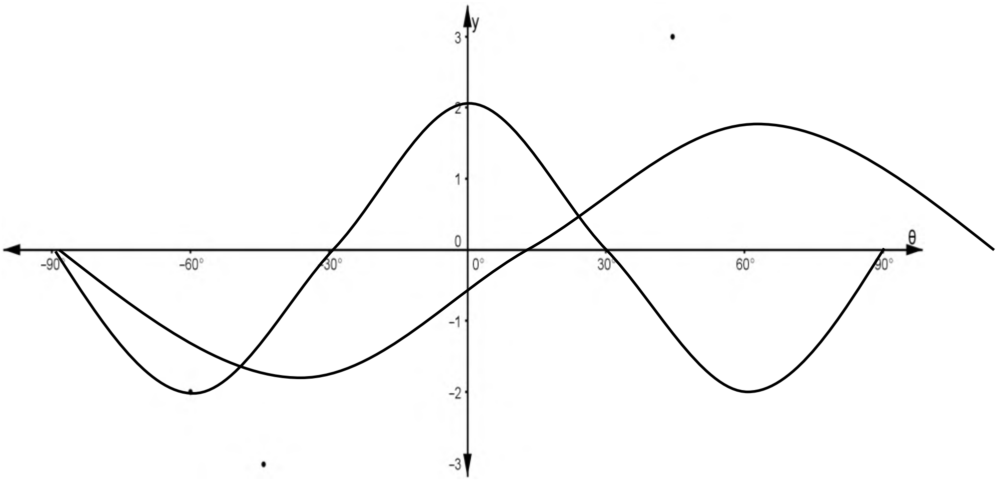
(e) Write down two values of x for which $\sin x = \frac{1}{2}\sin x + \frac{1}{2}$ (4)

(f) For which negative values of x will g decrease if x increase? (2)

(g) For which values of x is $\frac{f(x)}{g(x)}$ undefined? (3)

4.5 In the figure are sketch graphs of the functions

$y = n\sin 2x$ and $y = 2\cos mx$ for $x \in [-90°; 90°]$



(a) Use the sketch graphs to answer the following questions: (2)
Determine the value of m and n

(b) Write down the range of $\{(x; y / 2\cos mx \le y \le n\sin 2x)\}$ for $x \in [-90°; 90°]$ (2)

4.6 Given: $f(x) = \sin(x + 30°)$ and $g(x) = \cos 2x$ $x \in [-180°; 180°]$

(a) Draw neat sketch graphs of $f(x)$ and $g(x)$ on the same set of axes for

(b) For which value(s) of x is $f(x).g(x) > 0$ for $x \in [-180°; -30°]$

(c) Write down the period of $h(x) = g\left(\frac{x}{2}\right)$

(d) Write down the new equations of the transformations if f is moved 60^0 to the right and g is moved 2 units up.

Give your answers in the form $f'(x) = . . .$ and $g'(x) = . . .$

4.7 Given: $f(x) = \cos 2x$ and $g(x) = -\sin x$, for $x \in [-180°; 180°]$

(a) Calculate the values of x for which $f(x) = g(x)$ for $x \in [-180°; 180°]$

(b) Sketch, on the same set of axes, the graphs of f and g showing all intercepts with the axes as well as the turning points for $x \in [-180°; 180°]$.

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(c) Write down the period of f

(d) Determine the values of x for which $f(x) - g(x) \leq 0$ for $x \in [-180°;180°]$

(e) Hence, determine the maximum value of $\cos 2x + \sin x$ on the interval $[-180°;180°]$

(f) $g(x)$ is reflected about the x-axes and then shifted 1 unit down to $h(x)$. Write down the equation of $h(x)$.

4.8 Given: $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30°)$

(a) On the same set of axes, draw sketch graphs of the curves of f and g for $x \in [-120°;120°]$. Show clearly all intercepts with the axes, coordinates of all turning points and coordinates of all end points of both curves.

(b) Use the graphs drawn in (a) to determine for which value(s) of $x \in [-120°;60°]$ is:

(c) $\cos(60° - x) < 0$

(d) $f(x) - g(x) > 0$

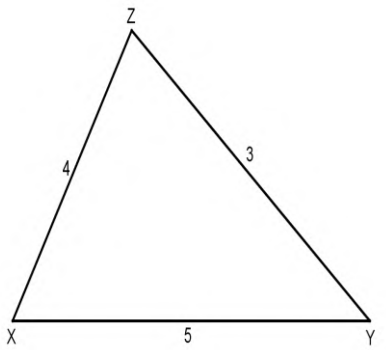
(e) $\frac{f(x)}{g(x)}$ undefined?

4.9 (a) Determine the general solution of $\sin 2x = \cos(x + 60°)$

(b) Hence, solve for x if $\sin 2x = \cos(x + 60°)$ and $x \in [-90°;180°]$.

2D AND 3D TRIGONOMETRY

5.1 ΔXYZ has lengths 4, 5 and 6 as shown in the diagram. (7)



Using the Cosine rule, show that $\cos\hat{Y} + \cos\hat{Z} = \frac{7}{8}$

5.2 The diagram represents a triangular car park PQS and a building RQ of height 70 metres. T is a point on PS such that PT : TS = 5 : 3
PS = 144 metres, $Q\hat{P}T = 63°$ and $S\hat{T}Q = 114°$.

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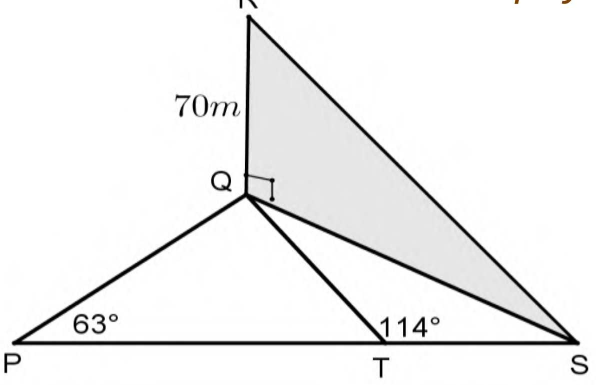


(a) Show that QT = 103 metres rounded to the nearest whole number. (6)

(b) Determine the angle of elevation of R from S. Round off your final answer to the nearest whole number.

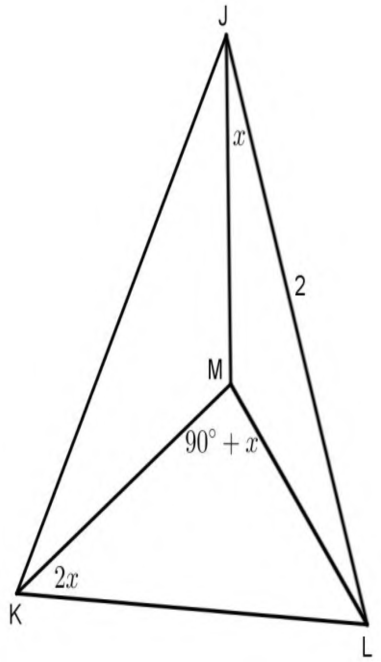
5.3 JM is a vertical tower and points K and L are in the same horizontal plane as point M, the foot tower.

$M\hat{J}L = x$

$K\hat{M}L = 90° + x$

$M\hat{K}L = 2x$

$JL = 2\ units$



(a) Show that KL = 1 (3)

(b) Show that $MK = 2\cos 2x - 1$. (4)

(c) Find the values of x for which MK exists. (5)

5.4 Two identical rhombuses ABCD and EFCD are placed at right-angles against each other. $A\hat{D}C = 60°$, AD = h units and $AK \perp DC$,

Calculate the following lengths in terms of h

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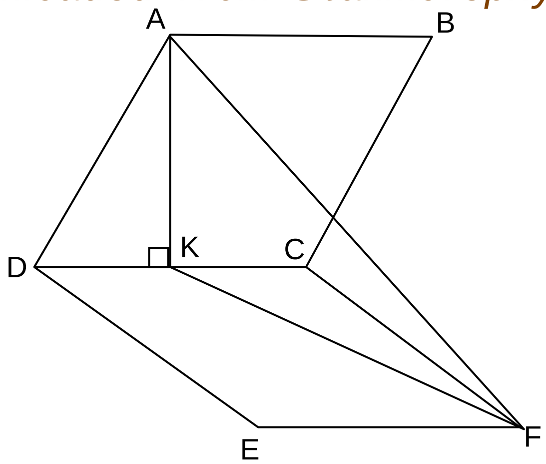


(a) AK (2)

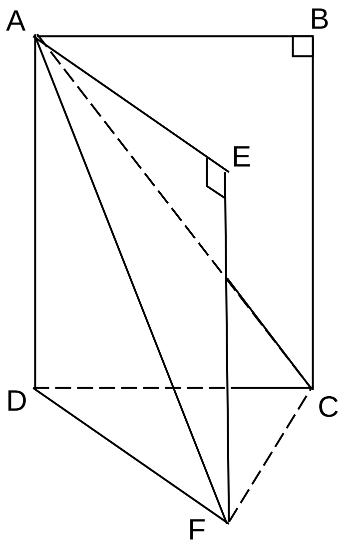
(b) DK (2)

(c) KF (4)

(d) If the two rhombuses are pressed against each other so that the angle between them is < 90° the area of ΔAKF becomes $\frac{\sqrt{21}h^2}{16}$ (4)

Calculate the angle between the two rhombuses.

5.5 The cover of a book EABCDF stands upright as in the figure. AC and AF are the diagonals of identical rectangles ABCD and AEFD respectively. AB = p units and CF = q units.



(a) Show that $\cos C\hat{D}F = 1 - \frac{q^2}{2p^2}$ (7)

(b) If p = 15 and q = 12, calculate (3)

i. the size of $C\hat{D}F$

ii. the length of AC if $F\hat{A}C = 27,8°$

5.6 PQ is a vertical flagpole of length x metres, with Q at the foot of the flagpole. R, Q and S are three points on the same horizontal surface. If RQ = RS, $Q\hat{S}R = \alpha$ and $P\hat{S}Q = \theta$

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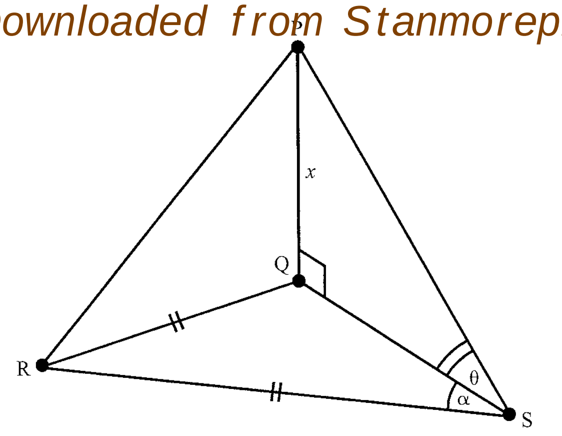
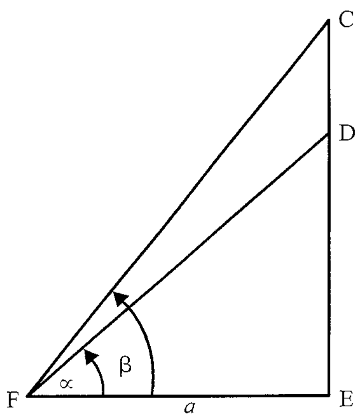


(a) Show that: $QS = \dfrac{x}{\tan\theta}$ (3)

(b) Prove that: $RS = \dfrac{x}{2\tan\theta\cos\alpha}$ (4)

(c) If $\theta = 45°$ and $\alpha = 60°$ and $x = 4$ metres, calculate the length of RS. (3)

5.7 CD is a flagstaff on the top of a building, ED. The angle of elevation from point F, *a* metres away from the base of the building, to the top of the building is α. The angle of elevation from F to the top of the flagstaff is β.



(a) Determine $F\hat{C}E$. (3)

(b) Prove: $CD = \dfrac{a\sin(\beta - \alpha)}{\cos\alpha\cos\beta}$ (4)

(c) If $a = 5$ metres, $\alpha = 30°$ and $\beta = 60°$, determine without a calculator, the length of the flagstaff (3)

5.8 The rectangular wall ABCD has a length that is twice as long as its height. Let the height equal a length of x units and $E\hat{B}C = 90° - \theta$. The angle of elevation of A from E is θ.

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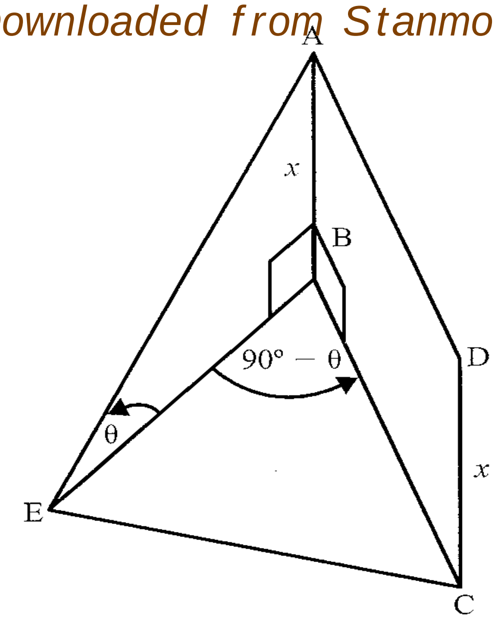
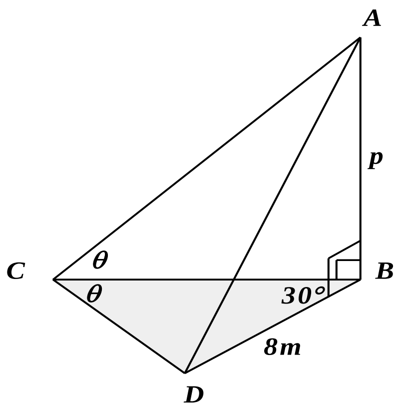


(a) Find the length of EB in terms of x and θ. (5)

(b) Prove that: $EC = x\sqrt{\dfrac{1}{\sin^2\theta} - 4\cos\theta + 3}$ (7)

5.9 In the sketch below, B, C and D are three points in the same horizontal plane. AB is a vertical pole p metres high. The angle of elevation of A from C is θ, $B\hat{C}D = \theta$, $C\hat{B}D = 30^0$ and $BD = 8$ metres.



(a) Express $C\hat{D}B$ in terms of θ. (3)

(b) Express BC in terms of p and a trigonometric ratio of θ (3)

(c) Hence or otherwise, show that $P = 4(1 + \sqrt{3}tan\theta)$ (4)

5.10 In the diagram, DA represents a vertical tower. B and C are two points in the same horizontal plane as A, the foot of the tower. The angle of elevation of D, as measured from B, is equal to α and $B\hat{A}C = 90°$.
It is further given that $BD = BC,\ AD = AC$ and $AB = x$ units.

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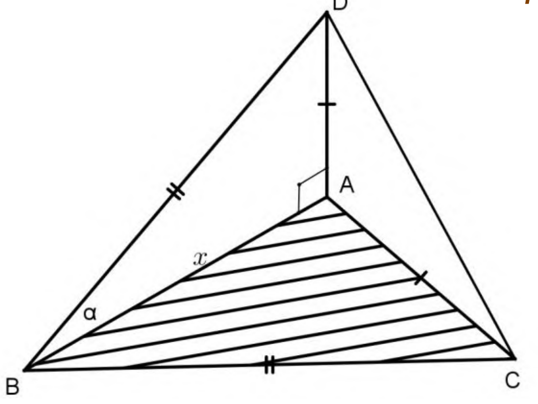


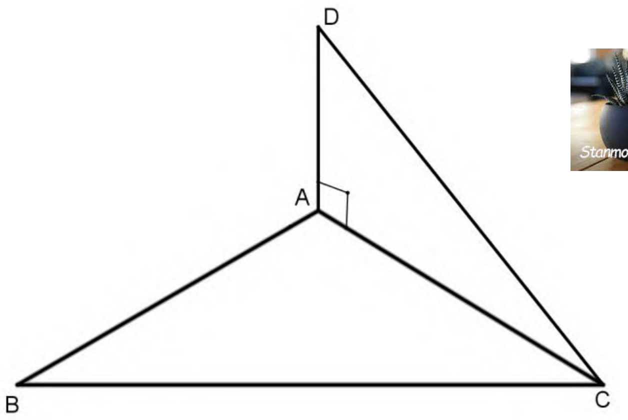
(a) Express AC and BC in terms of x and α (3)

(b) Express CD^2 in terms of x and α (4)

(c) Hence prove that $\cos C\hat{B}D = cos^2\alpha$ (4)

5.11 In the figure, A, B and C are three points in the same horizontal plane. DA is perpendicular to the horizontal plane at A and D is joined to C.

$AB = \frac{1}{2}BC = a$ and $A\hat{C}D = \frac{1}{2}A\hat{B}C = \alpha$.



(a) Show that $AD = a.\tan\alpha.\sqrt{1 + 8sin^2\alpha}$ (4)

(b) Hence calculate the value of AD if $a = 89\ mm$ and $\alpha = 35°$. (3)
(Round the answer off to one decimal digit.)

5.12 In the diagram below, an acute-angled triangle ABC is drawn: (5)

- A line PQ is drawn, where P lies on the line BC and Q lies on the line AC.
- The length of PQ is 14 units and the length of AB is 18 units.
- $\hat{A} = 68°$ and $\hat{C} = 50°$.

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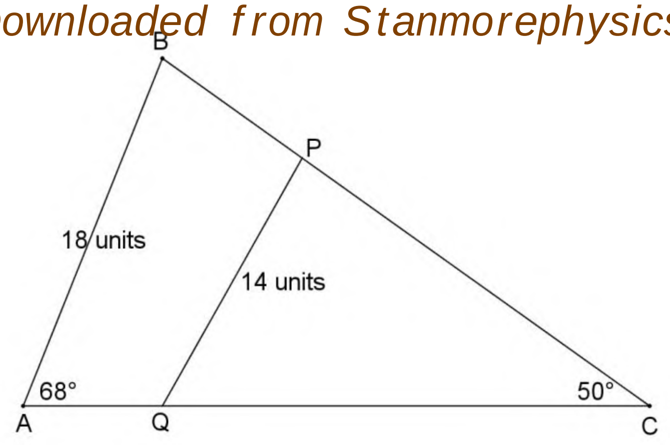
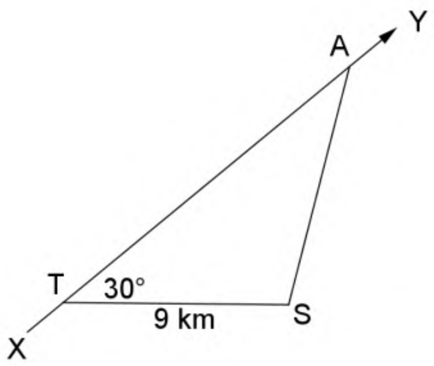


If the ratio of BP : PC is 2 : 3 determine the size of $P\hat{Q}C$.

5.13 In the figure, S represents the position of a stationary submarine which is involved in target practice. A target vessel is steering a straight course along the path XY. When the target vessel is at T, it is 9 kilometres from S. The submarine is armed with torpedoes which have a maximum range of 7,5 kilometres. $S\hat{T}Y = 30°$.



(a) If A is the furthest point along XY that can be reached by a torpedo fired from the submarine at S, calculate the size of $T\hat{A}S$ to the nearest degree. (3)

(b) Hence calculate the total length of the path XY that can be brought under fire from the submarine at S. (3)

(b) As the size of $B\hat{A}C$ increase, so the area of ΔABC increases. (4)

5.14 Prove that for any $\Delta PQR,$ its area A, is given by $A = \dfrac{p^2 \sin\theta \sin R}{2\sin P}$ (6)

5.15 In the figure, M is the centre of semicircle PRQ and r is the radius. PM is the diameter of semicircle PTM.
$\hat{Q} = x.$

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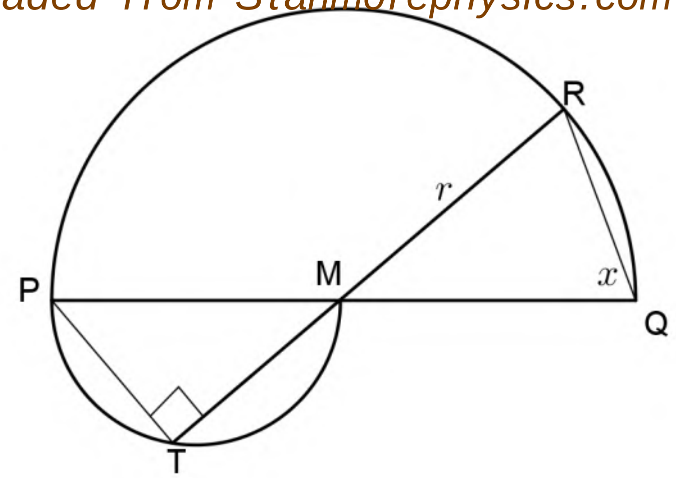
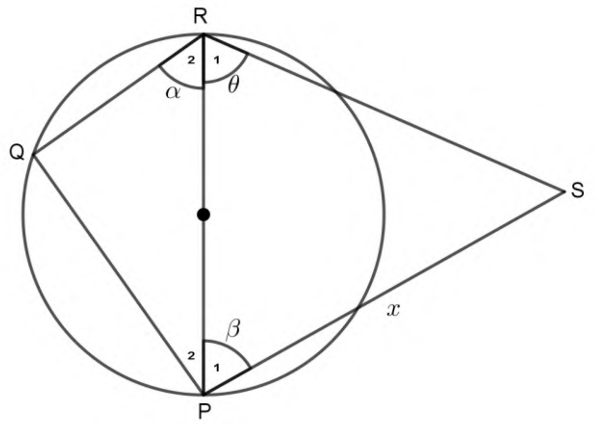


(a) Determine RQ in terms of r and x and simplify the expression as far as possible. (4)

(b) Determine the area of ΔPTM in terms of r and x. (4)

5.16 In the circle below, PR is a diameter of the circle, passing through P, Q and R. S is a point outside of the circle. RS and PS are drawn. $PS = x$. $P\hat{R}S = \theta$. $R\hat{P}S = \beta$ and $P\hat{R}Q = \alpha$.



(a) Prove that $PR = \dfrac{x\sin(\theta+\beta)}{\sin\theta}$ (4)

(b) Prove that $QR = \dfrac{x\cos\alpha.\sin(\theta+\beta)}{\sin\theta}$ (6)

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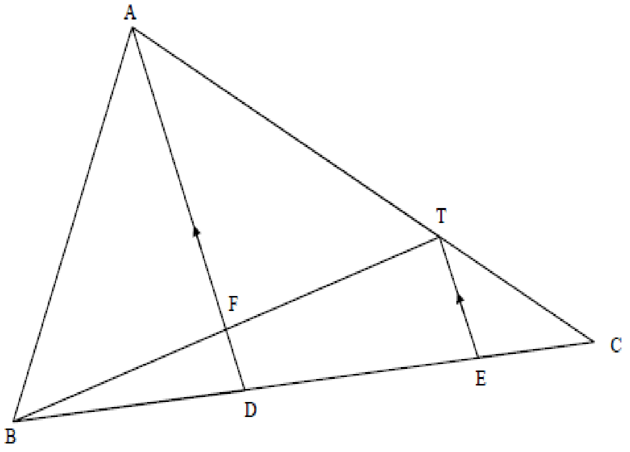
1. In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD || TE.



1.1 Write down the numerical value of $\dfrac{CE}{ED}$ (1)

1.2 Show that D is the midpoint of BE. (2)

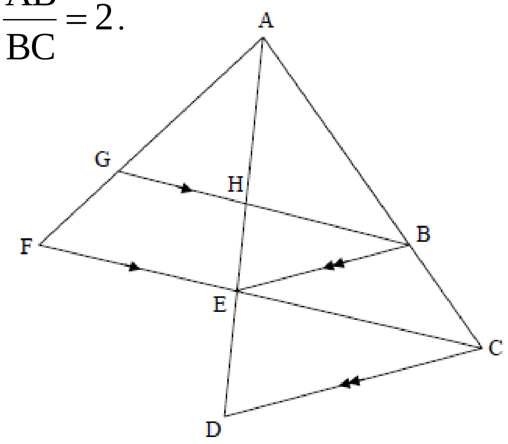
1.3 If FD = 2 cm, calculate the length of TE. (2)

1.4 Calculate the numerical value of:

1.4.1 $\dfrac{\text{area of } \Delta\ ADC}{\text{area of } \Delta\ ABD}$ (1)

1.4.2 $\dfrac{\text{area of } \Delta\ TEC}{\text{area of } \Delta\ ABC}$ (3)

2. In the figure below, GB || FC and BE || CD. AC = 6 cm and $\dfrac{AB}{BC} = 2$.



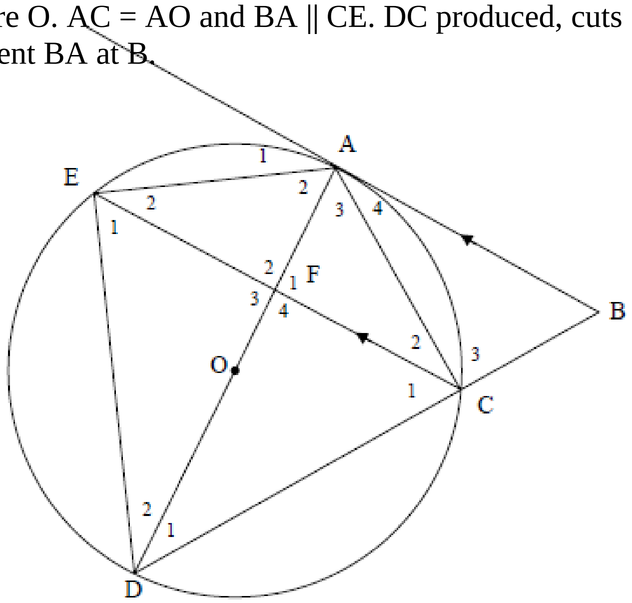
Calculate with reasons:

2.1.1 AH:ED (4)

2.1.2 $\dfrac{BE}{CD}$ (2)

2.2 If $HE = 2cm$, calculate the value of $AD \times HE$ (2)

3. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



3.1 Show that $\hat{C}_2 = \hat{D}_1$ (3)

3.2 Prove that $\Delta ACF\ |||\ \Delta ADC$. (3)

3.3 Prove that $AD = 4AF$ (4)

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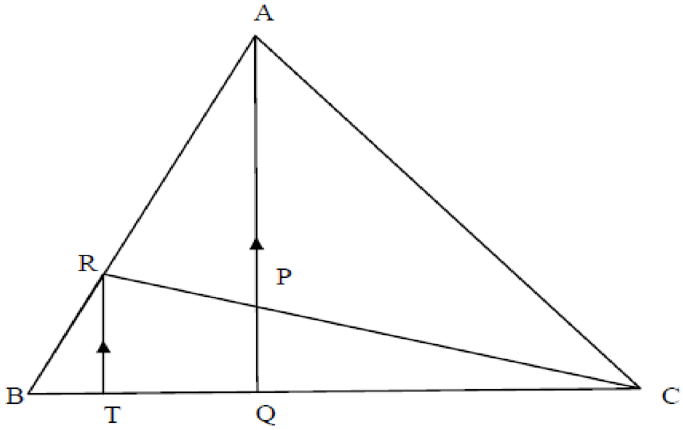
4. In the figure AQ || RT, $\frac{BQ}{QC}=\frac{3}{5}$ and $\frac{BR}{RA}=\frac{1}{2}$

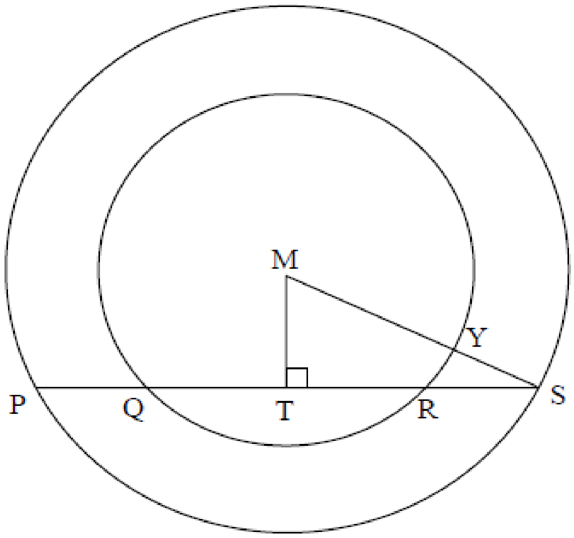


4.1. If BT = k, calculate TQ in terms of k. (2)

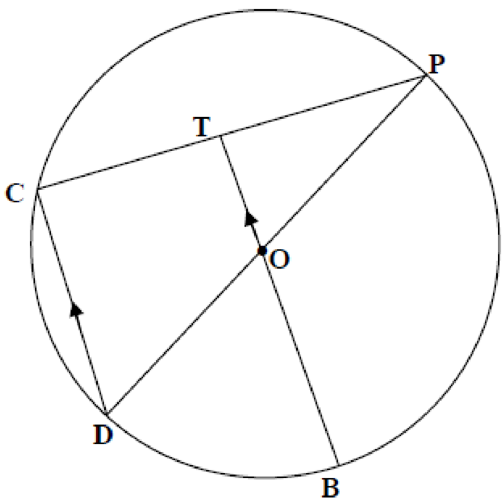
4.2. Hence, or otherwise, calculate the numerical value of:

4.2.1 $\frac{CP}{PR}$ (3)

4.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC}$ (3)



5. In the diagram alongside, two concentric circles with centre at M and with radii 5 cm and 8,5 cm are given. PQRS is a chord of the larger circle cutting the smaller circle at Q and R. MYS is a straight line with Y on the smaller circle. QR = 6 cm.
Calculate, with reasons, the length of PS. (7)



In the diagram alongside, O is the centre of circle PBDC. BOT is drawn with T on chord PC. PD and BT intersect at O. CD // TB.
PD = 10x units and OT = 3x units.

6.1 Determine TB : TO (2)

6.2 Prove that BT $\perp$ PC (3)

6.3 Express the length of PC in terms of x (5)

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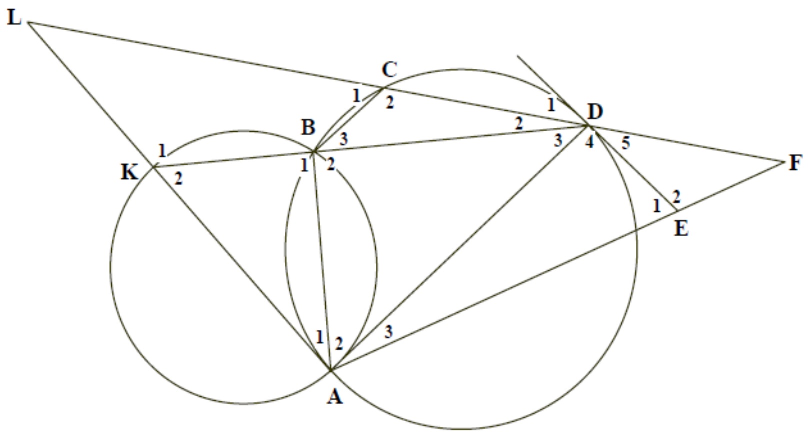
7. In the diagram below, two circles intersect each other at A and B. ED is a tangent to circle ABCD. DA is a tangent to circle AKB. DBK is a straight line. AK and DC are produced to meet at L. LCD and AE are produced to meet at F. CD = DF



Prove that :

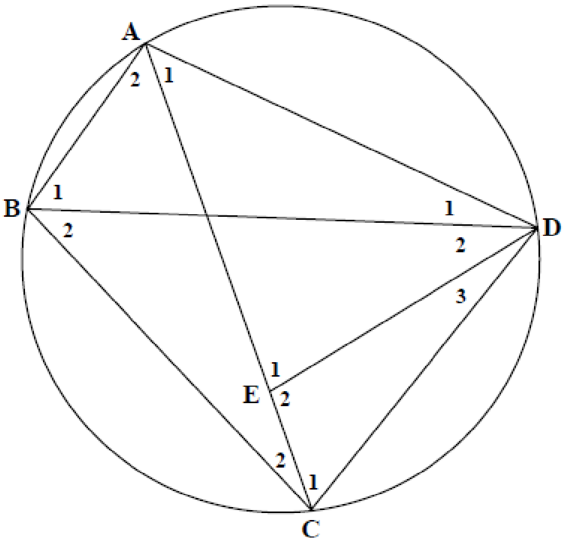
7.1 LKBC is a cyclic quadrilateral. (5)

7.2 $\hat{B}_2 = L\hat{A}D$ (3)

7.3 DE // LA (5)

7.4 CD. FA = FE . FL (4)

8. In the diagram alongside, ABCD is a cyclic quadrilateral. E is a point on the diagonal AC such that $\hat{D}_1 = \hat{D}_3$.



Prove that:

8.1 EC . BD = AB . CD (3)

8.2 AE . BD = BC . AD (5)

8.3 AC . BD = AB . CD + BC . AD (3)

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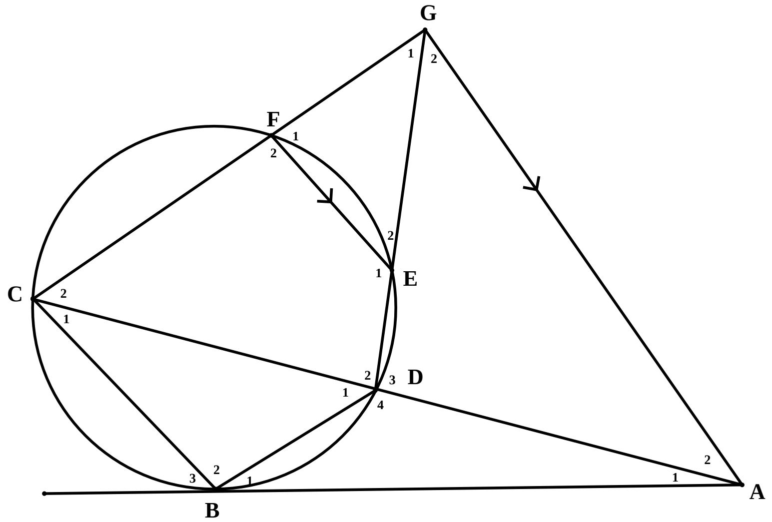
9. In the diagram FE // GA. BA is a tangent at B.



Prove that :

9.1 $AB^2 = AD \cdot AC$ (3)

9.2 $\Delta ADG \text{ /// } \Delta AGC$ (5)

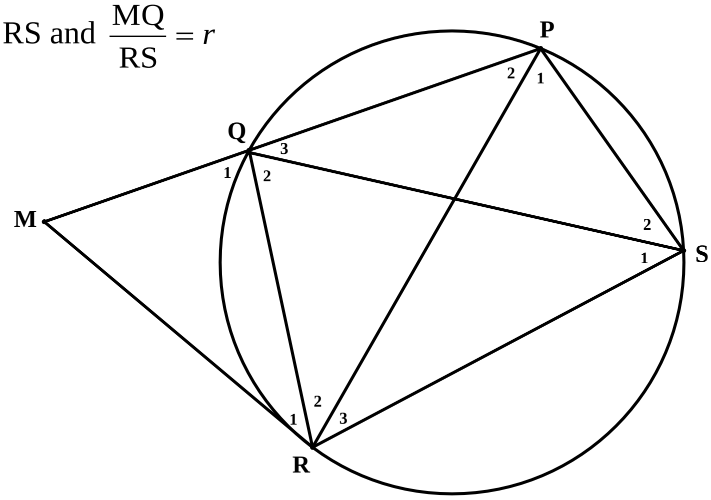
9.3 AG = AB (2)

9.4 If AG = 6 units and AD = 4 units
Calculate:

9.4.1 The length of DC (2)

9.4.2 The length of GD correct to two decimal places, CE is the diameter of the circle. (3)

10. In the diagram PQRS is a cyclic quadrilateral. QM.PS = QR. RS and $\frac{MQ}{RS} = r$

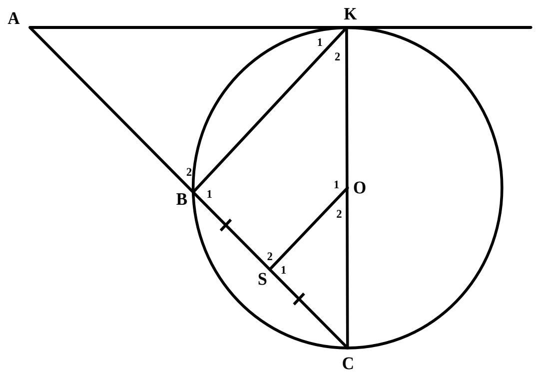


10.1 Prove that $\frac{QR}{SP} = r$ (2)

10.2 Hence prove that $\frac{MR}{RP} = r$ (3)

10.3 Deduce that $\Delta MQR \text{ /// } \Delta RSP$ (2)

11. In the diagram below KOC is the diameter of circle O. KA is a tangent. AC intersects the circle at B. BS = SC. KB and OS are joined.



Prove:

11.1 CO : CS = CA : CK (3)

11.2 $\Delta COS \text{ /// } \Delta KAB$ (4)

11.3 $2SO^2 = CS \cdot BA$ (3)

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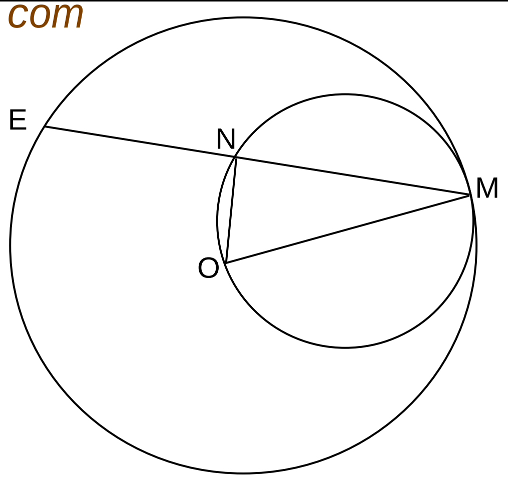
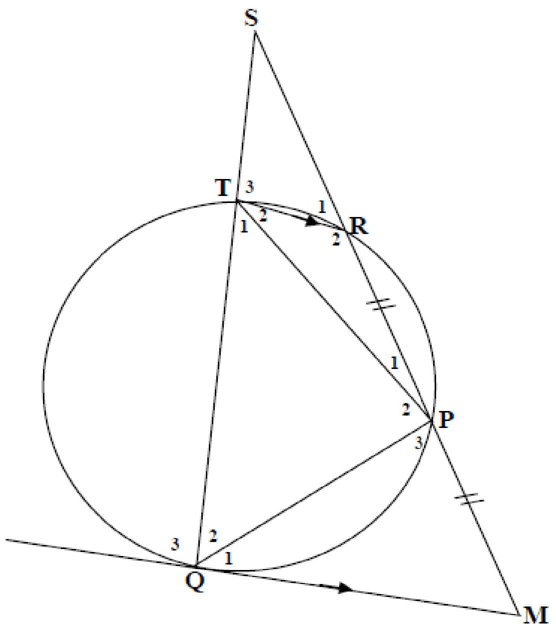
12. In the diagram alongside, OM is the radius of the larger circle and also the diameter of the smaller circle. Chord EM of the larger circle cuts the smaller circle at N. If EM = $(2x^2 - 2)$ units and ON = $2x$ units, express, giving reasons, the length of the radius of the larger circle in terms of x. (6)



13. In the diagram below, circle QPRT intersects SQ at T and SM at R and P. QM // TR. MP = PR and 2ST = TQ ST = a units and SR = b units



13.1 Prove that :

13.1.1 Δ SQP /// ΔSMQ (6)

13.1.2 $\frac{QM}{QP} = \frac{3a}{2b}$ (6)

13.2 It is further given that Δ QPT /// ΔMPQ

Prove:

13.2.1 QM is a tangent to circle QPRT (3)

13.2.2 $TP = \frac{4b}{3}$ (4)

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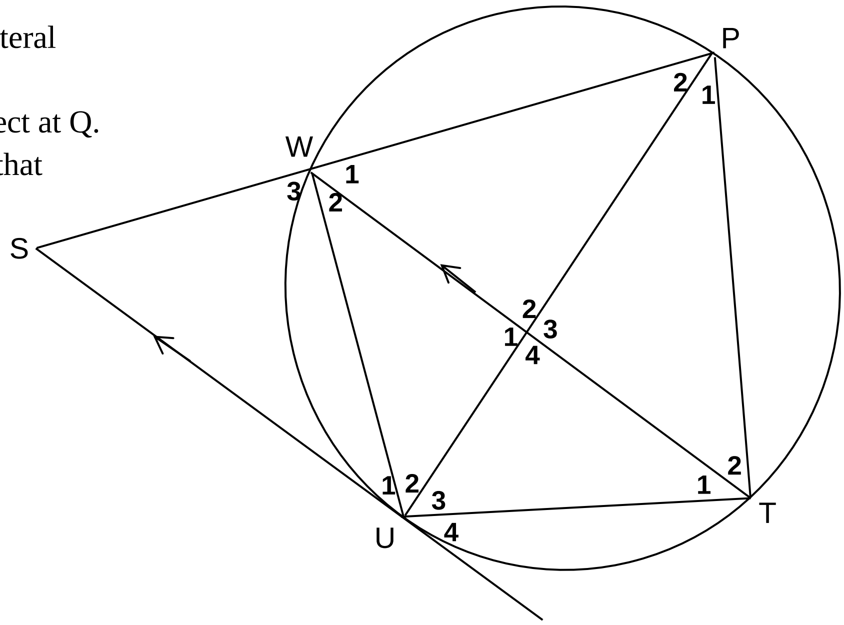
14. In the diagram alongside,
PWUT is a cyclic quadrilateral
with WU = TU.
Chords WT and PU intersect at Q.
PW is extended to S such that
US ∥ TW.



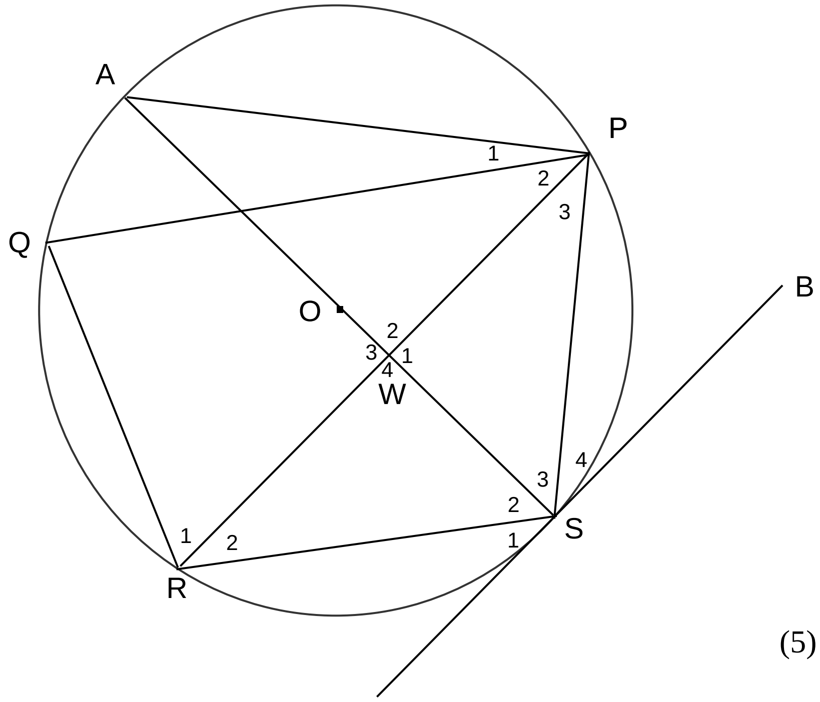
Prove that:

14.1 US is a tangent to circle PWUT at U (5)

14.2 Δ SPU ||| Δ SUW (4)

14.3 $SU^2 . QU = PU . SW^2$ (6)

15. In the diagram below, P, A, Q, R and S lie on the circle with centre O.
SB touches the circle at S and RW = WP.
AOWS and RWP are straight lines.



Prove that:

15.1 SB ∥ RP (5)

15.2 $RS^2 = WS . AS$ (8)

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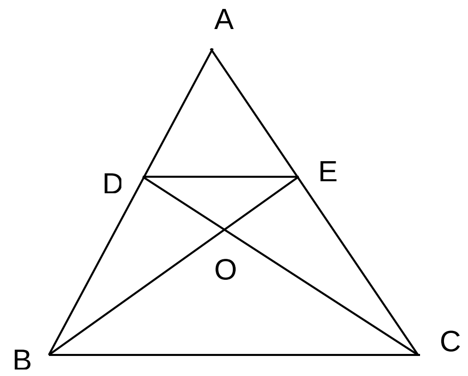
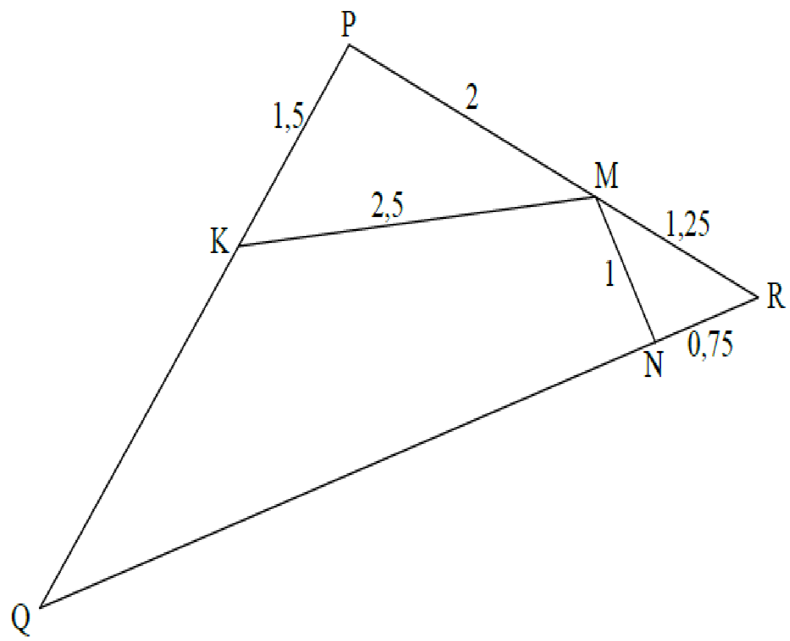
15.3 $AS = \frac{RW^2}{WS} + WS$ (4)

16. In the figure D is the midpoint of AB and E is the midpoint of AC. DC and EB intersect at right angles at O.
Prove that: $AB^2 + AC^2 = 5BC^2$ (8)



17. In the diagram alongside
K, M and N respectively are points on sides PQ, PR and QR of ΔPQR.
KP =1.5 ; PM = 2 ;KM =2.5 ;MN=1
MR=1.25 and NR= 0.75.
17.1 Prove that ΔKPM/// ΔRNM (3)
17.2 Determine the length of NQ (6)

17.3 Determine the numerical value of
$\frac{Area\ of\ \Delta KPM}{Area\ of\ \Delta RNM}$ (2)



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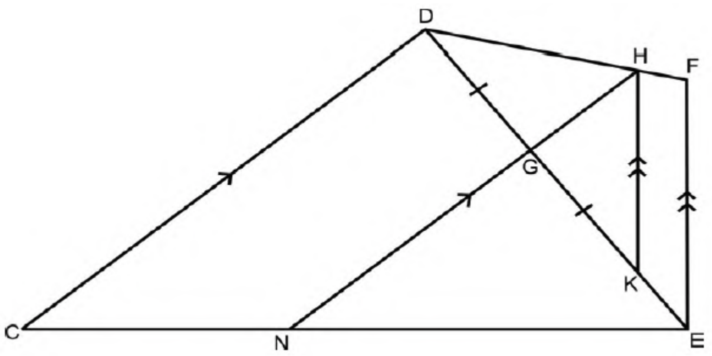
ADDITIONAL EXERCISES

Question 1

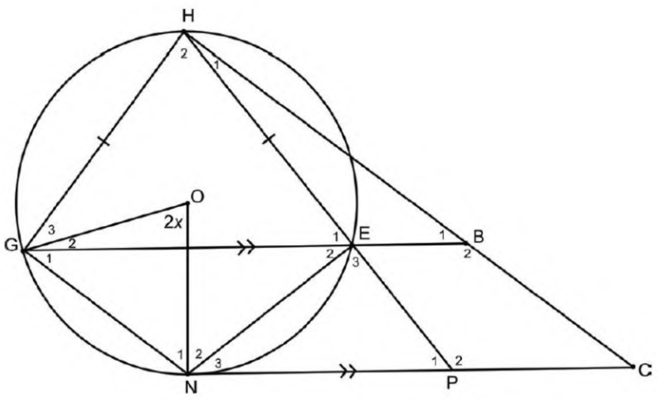


In the diagram alongside:

- CD//NH
- NH intersects DE at G.
- K is a point on DE with DG = GK
- HK // FE.
- CN:CE = 2:5

1. Determine $\frac{EK}{KG}$ (show all working) (3)
2. Calculate $\frac{\text{Area of } \Delta DGH}{\text{Area of } \Delta DEF}$ (4)

Question 2



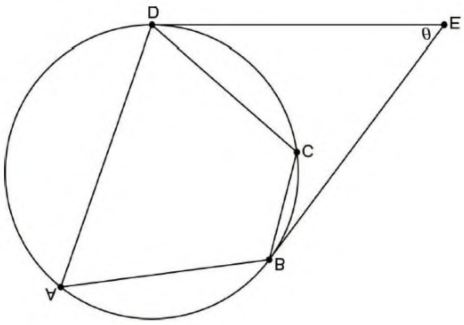
In the diagram alongside, H;E;N and G lie on the circle with centre at O.

- NC is a tangent at N and passes through P.
- B lies on HC with GB//NC.
- HP and GB intersect at E.
- HG = HE
- $G\hat{O}N = 2x$

Prove that:

1. GN = NE (6)
2. $\Delta GON /// \Delta GHE$ (5)
3. $\frac{ON \times GE}{GN} = \frac{HB \times EP}{BC}$ (3)

Question 3

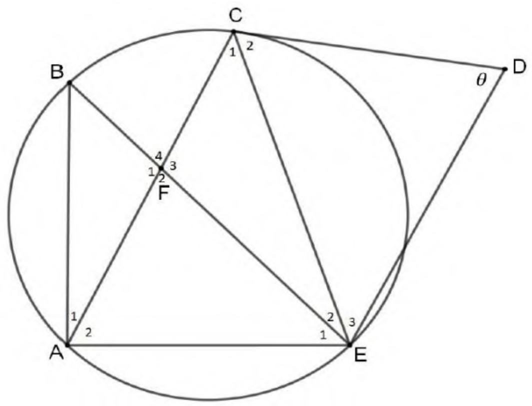


In the diagram alongside, a circle is drawn passing through A, B, C and D.

- $B\hat{E}D = \theta$.
- BE and ED are tangents at B and D respectively.

Prove that $B\hat{C}D = 90° + \frac{\theta}{2}$ (6)

Question 4



In the diagram below, A,B,C and E are points on a circle. AC and BE intersect at point F. D is a point outside the circle. CD and DE are drawn:

- $A\hat{B}E = x$
- $B\hat{E}C = y$
- $C\hat{D}E = \theta$

1. If $\theta = x + y$ then prove that FCDE is a cyclic quadrilateral. (5)

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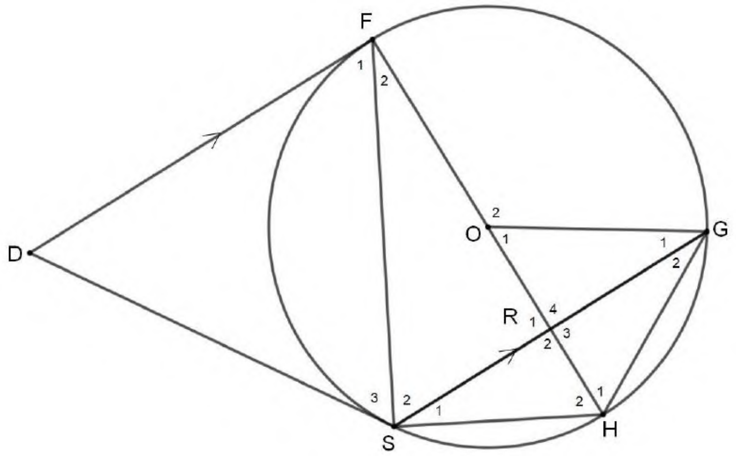
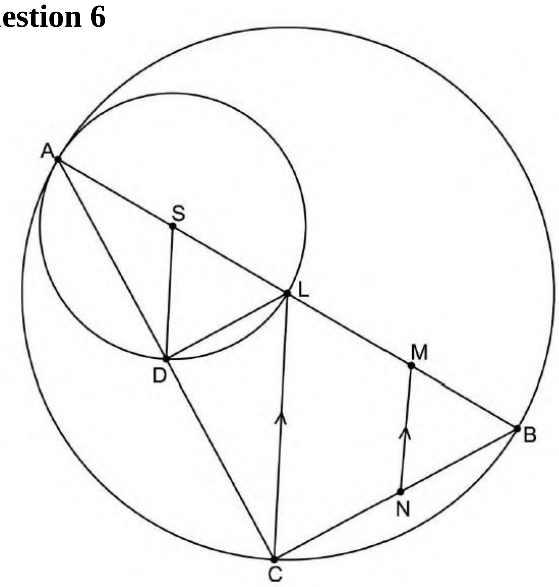
	2. If AB = AE, then prove that line AE is a tangent to the circle that goes through F, C, D and E. (4)
Question 5 	In the diagram alongside, circle centre O passes through F, S, H and G. • DF and DS are tangents to the circle at F and S respectively. • DF//SG. • FOH intersects SG at R. 1. Prove that $\Delta DSF /// \Delta OHG$ (7) 2. $2 \times DF = \frac{SF \times FH}{HG}$ (3)
Question 6 	In the diagram alongside, two circles touch internally at A. • AB is the diameter of the larger circle and AL is the diameter of the smaller circle. • S and L are the centre of the circles. • D is a point on the smaller circle and C is a point on the larger circle. ADC is a straight line. • M is a point on LB so that MN || LC 1. Prove that DL || CB. (4) 2. Prove that 2SD = LC. (3) 3. Determine the value of $\frac{SL}{AB}$ (2) 4. If AB = 30 units and $\frac{BN}{NC} = \frac{7}{9}$ then determine the length of LM. (3)

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ALGEBRA AND EQUATIONS ANSWERS

QUESTION 1			QUESTION 2		
1.1			1.2		
	1.1.1	$x = 9\ ;\ x \neq 6$		1.2.1	$x = -1$ or $x = 0$
	1.1.2	$x = 1\ ;\ x \neq -4$		1.2.2	$x = 4$ only
	1.1.3	$x = 2\ ;\ x \neq \frac{13}{2}$	1.3	1.3.1	$x = 2\ ;\ x \neq 3$
	1.1.4	$x = \frac{1}{8}$		1.3.2	$p = -1$ or $p = 2$
	1.1.5	$x = \frac{1}{3}$ or $x = 1$	1.4	$c = -4$	
	1.1.6	$x = 5{,}37$ or $x = -0{,}37$	1.5	max is 4	
	1.1.7	$x = 5{,}65$ or $x = 0{,}35$	1.6	1.6.1	$x = \frac{1}{2}$ or $x = 1$ $y = 3$ or $y = 2$
	1.1.8	$x = 3\ ;\ x \neq -1$		1.6.2	$x = \frac{5}{3}\ ;\ y = \frac{2}{3}$
	1.1.9	$x = 0$ or $x = -3$		1.6.3	$x = 5\ ;\ y = 15$
	1.1.10	$x = 1{,}46$ or $x = 1$	1.7		
	1.1.11	$x = 3$		1.7.1	
	1.1.12	$x = -1$ or $x = 0$		1.7.2	$y = -\frac{1}{2}$ or $y = 3$ $x = -\frac{3}{2}$ or $x = 2$
	1.1.13	$x = -1$ or $x = 3$	1.8		
	1.1.14	$x = 2$		1.8.1	$\frac{x}{y} = -3$ or $\frac{x}{y} = -2$
	1.1.15	$x = -\frac{8}{27}$ or $x = 125$		1.8.2	$y = -4$ or $y = -8$ $x = 12$ or $x = 16$
	1.1.16	$x = -4$	1.9		
	1.1.17	$x = 4$		1.9.1	$2^x = -y + 4$
	1.1.18	$x < 1$ or $x > 4$		1.9.2	$x = 3$
	1.1.19	$-\frac{1}{2} < x < \frac{4}{3}$		1.9.3	$x = 0$
	1.1.20	$x \leq -1$ or $x \geq 0$	1.10		
	1.1.21	$\frac{1}{2} \leq x \leq \frac{5}{2}$		1.10.1	$x = 0{,}88$ or $x = -0{,}68$
	1.1.22	$x < -2$		1.10.2	$y = -0{,}06$ or $y = -0{,}84$
	1.1.23	$x < \frac{2}{3}$	1.11	$x + y = 4$	
	1.1.24	$x < 3$	1.12	$\frac{1}{27}$	
	1.1.25	$x < 5$		1.13.1	$x = \pm 2\sqrt{5} - 1$
1.18	$k \leq \frac{1}{8}$			1.13.2	$k = 0$
1.19	$1 < k < 9$		1.14	$k \geq \frac{2}{3}$	
	1.20.1	$x \geq -\frac{1}{8}\ ;\ x \neq 4$	1.15	$p = \frac{35}{2}\ ;\ x = \frac{7}{2}$ or $x = \frac{5}{2}$	
	1.20.2	$x = 15\ ;\ x \neq 1$	1.16	$k = 6$	
	1.20.3	$x \geq -\frac{1}{8}\ ;\ x \neq 4$	1.17	$k \leq \frac{13}{2}$	
1.21			1.22	$a = 2\ ;\ b = 1006$	
	1.21.1	$\frac{1}{7.2^{a+1}}$	1.23	$x = 888\ 888\ 888\ 890$	
	1.21.2	$6\sqrt{x}$	1.24	$a = 2\ ;\ b = 7$ or $a = 1\ ;\ b = 28$	
	1.21.3	2	1.25	$m + m^{-1} = 7$	

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	1.21.4	$\frac{10}{99}$	1.26	$\frac{49}{2}$
	1.21.5	$\frac{1}{100}$	1.27	$x = 8$

NUMBER PATTERNS, SEQUENCES AND SERIES ANSWERS					
QUESTION 1			**QUESTION 2**		
1.1	*Bookwork*		2.1	$r = 0.9; T_{12} = 14.12$	
1.2	$S_{50} = 1175$		2.2	$-1 < 0,9 < 1$	
1.3.1	(a)	$T_2 - T_4 = 25$	2.3	$S_\infty = 450$	
1.3.1	(b)	$T_{70} - T_{69} = 415$	2.4	$n > 57,98 \therefore n = 58$	
1.3.2	$T_{69} = 14154$		2.5	2.5.1	39 ;53
				2.5.2	$T_n = n^2 + 3n - 1$
QUESTION 3					
3.1	$r = \frac{1}{\sqrt{2}}$; $S_\infty = \frac{a}{1-r}$		**QUESTION 4**		
3.2	3.2.1	$\sum_{n=1}^{20} 5(3)^{n-1}$	4.1	4.1.1	$S_{23} = 1426$
	3.2.2	$x = S_{20} = 871696100$		4.1.2	$S_{22} = 1309 \quad \therefore \quad T_{23} = 117$
			4.2	$r = \frac{3}{2}$ or $r = \frac{1}{2}$	
QUESTION 5					
5.1	5.1.1	Arithmetic: $x = 18$	**QUESTION 6**		
	5.1.2	Geometric: $x = \pm\sqrt{128}$	6.1	21; 24	
5.2	$P = S_{13} = 99645{,}125$				
QUESTION 7			**QUESTION 8**		
7.1			8.1	$n = 46$	
7.2			8.2	$S_{46} = 3818$	
			8.3	$\sum_{n=1}^{46}(4n - 11)$	
QUESTION 9			**QUESTION 10**		
9	9.1.1	$\frac{-1}{2}$	10.1	$r = 1; s = 2$	
	9.1.2	$n = 9$	10.2	$a = 1; b = -10; c = 26$	
	9.1.3	$S_\infty \frac{8}{3}$	10.3	$T_8 = 10m$	

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CALCULUS ANSWERS

QUESTION 1			**QUESTION 2**		
1.1			2.1		
	1.1.1	$f'(x) = \frac{-2}{x^2}$		2.1.1	$g'(x) = -5$
	1.1.2	$y = \frac{-1}{2}x + 2$		2.1.2	$= 3 + \frac{14}{x^3}$
1.2				2.1.3	$\frac{dy}{dx} = \frac{-2}{\sqrt{x}} - 1$
	1.2.1	$f'(x) = -x^2$		2.1.4	$\frac{dz}{dx} = -x^{\frac{-3}{2}} + \frac{1}{8}$
	1.2.2	(-3;9)		2.1.5	$\frac{dy}{dx} = 2x - 4$
1.3				2.1.6	$\frac{dy}{dx} = 14x + x^{\frac{4}{3}}$
	1.3.1	-12-2h		2.1.7	$p'(x) = \frac{-24}{x^7} - \frac{32}{x^3} + 32x$
	1.3.2	-12		2.1.8	$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2$
	1.3.3	0			
			2.2		
	1.4	$f'(x) = \frac{2}{3}x$		2.2.1	$\frac{dy}{dx} = 24x^2$
1.5				2.2.2	$\frac{da}{dy} = \frac{4}{3}y^{\frac{1}{3}}$
	1.5.1	$f'(x) = 1 - \frac{12}{x^2}$		2.2.3	$\frac{da}{dx} = 4096x^4$
	1.5.2	$y = \frac{1}{2}x + 7$			
			2.3		$\frac{dy}{dw} = \frac{-8}{w3} + \frac{6}{w^4}$
1.6		$g(x) = \sqrt{x}$ and $a = 4$			
1.7					
1.2					
1.3					
QUESTION 3			**QUESTION 4**		
3.1			4.1		
	3.1.1	$x = 1$ or $x = -3$ and $y = 3$		4.1.1	$x = -3$
	3.1.2	(1;0) and $\left(\frac{-5}{2}; \frac{256}{27}\right)$		4.1.2	$x < -\frac{4}{3}$
	3.1.4	$x = -\frac{1}{3}$		4.1.3	$x < -3$ or $x \geq \frac{1}{3}$
	3.1.5	$y = -5x + 85/27$		4.1.4	$f'(x) = 3x^2 + 8x - 3$
	3.1.6	$0 < k < \frac{256}{27}$		4.1.5	$f(x) = x^3 + 4x^2 - 3x - 18$
	3.1.7	$k + 5 > \frac{256}{27}$; $k > \frac{121}{27}$ OR $k + 5 < 0$; $k < -5$	4.2		
	3.1.8	$0 < k < 3$		4.2.1	E(0; -4)

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3.2				4.2.2	$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$
	3.2.2	$a = 2\,; b = \frac{2}{3}$		4.2.3	$x = -2$ or $x = 6$
	3.2.3	m=4		4.2.4	$x = 2$
	3.2.4	$0 < p < c$ or $0 < p < 2$	**4.3**		
	3.2.5	$x < \frac{4}{3}$		4.3.1	$x = 1$ or $x = 2$
	3.2.6	$x \leq 0$ or $\frac{2}{3} \leq x \leq 2$		4.3.2	$x = 1$ local min $x = 2$ local max
3.3				4.3.3	$x = \frac{3}{2}$
	3.3.1	$A = -1;\ c = 3$	**4.4**		
	3.3.2	$R(1; 2)$		4.4.1	$x > -\frac{3}{2}$
	3.3.3	$-1 \leq x \leq 1$		4.4.2	$x = -4$
	3.3.4	x-intercepts are $\sqrt{3}$ or $-\sqrt{3}$ $x < -\sqrt{3}; -1 < x < 0$ or $1 < x < \sqrt{3}$		4.4.3	$x < -\frac{3}{2}$
	3.3.5	$x = -4$ or $x = -2$		4.4.4	mt=0
3.4		$a = 6, b = 7$ and $c = -18$	**3.6**	3.6.2	$x < -1$ or $x > 1$
3.5				3.6.3	$x > 0$
	3.5.1	$x = \frac{a}{6}$			
	3.5.2	$a = 21; b = -60$ and $c = 43$		3.7.2	Graph A
	3.5.3	$y = -24x + 26$		3.7.3	$-3 \leq x \leq -1; 0 \leq x \leq 5$ or $x \geq 8$
	3.5.4	$\left(\frac{17}{2}; -178\right)$			
			3.8	$a = 2$ and $b = -5$	

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CALCULUS ANSWERS					
QUESTION 5			**QUESTION 7**		
5.1			7.1		
	5.1.2	$x = 022m$		7.1.1	V =100l
	5.1.3	A=0.88m^2		7.1.3	K=9
5.2			7.2		
	5.2.3	$A = 3{,}14m^2$		7.2.1	$-18m.s^{-1}$
5.3				7.2.2	$s''(t) = 4m.s^{-2}$
	5.3.3	r =3,99 cm		7.2.3	$t = \frac{9}{2}s$
5.4					
	5.4.1	$r = \frac{h}{\sqrt{3}}$	7.3		
	5.4.2	$V'(x) = 27\pi cm^3/cm$		7.3.1	$t = 25s$
5.5				7.3.2	$t = 50s$ or $t = 0s$
	5.5.1	$V = 2144.66cm^3$	7.4		
	5.5.3	0,3		7.4.1	216 Molecules
				7.4.2	M'(t)=72
5.6		$x = 4$		7.4.3	t =1 hr
			7.5		
5.7		$A = \frac{\sqrt{3}d^2}{8}$ Unit square		7.5.1	36 cm
5.8				7.5.2	t = 6 sec
	5.8.3	$h = 2m$ and $y = \frac{3}{2}m$		7.5.3	h=52cm
5.9			6.1		
	5.9.2	$k = 10mm$		6.1.2	A =32 square unit
5.10			6.2		
	5.10.1	$l = 40 - 2h$		6.2.2	t =117.07s
	5.10.3	$h = 8.80cm$	6.3	BP =0.87 units	

FINANCIAL MATHEMATICS ANSWERS					
QUESTION 1			**QUESTION 2**		
1.1	n = 42,61 therefore 43 quarters		2.1	185,19	
1.2			2.2		
	1.2.1	19,56%		2.2.1	R1 200,38
	1.2.2	R12 696,71		2.2.2	R36 931,99
	1.2.3	R244 370,95	**QUESTION 4**		
	1.2.4	25,3%	4.1	R1 034 939,44	
QUESTION 3			4.2	R3 120 742,05	
3.1	n = 343,2741252 344 months		4.3	R2 085 802,61	
3.3	R1 786,65		4.4	R29 134,25	
3.4	R1 431 286,65		**QUESTION 5**		
3.4	Log of negative number is undefined. Which means the bank won't allow her because she will not finish the loan.		5.1		
QUESTION 6				5.1.1	R405 000

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6.1				5.1.2	R2 914,33
	6.1.1	R74 883,86		5.1.3	R307 923,73
	6.1.2	R168 306,21	5.2	R57 934,44	
	6.1.3	R1 208,99	**QUESTION 7**		
6.2	n = 66, 04316664 67 months		7.1	R1 269 726,92	
QUESTION 8			7.2	R3 670,11	
8.1	R119 827,91		**QUESTION 9**		
8.2			9.1	Zinhle R12 000 Ntando R12 131,54 Therefore it is Ntando	
	8.2.1	R6 237	9.2	R4 764 064	
	8.2.2	R842 899,56	9.3		
	8.2.3	R67 743,56		9.3.1	n = 51,7180… therefore 52 months
QUESTION 10				9.3.2	n = 123,7409…. Therefore 124
10.1	n = 125,5611473 therefore 125 for R9 000		**QUESTION 11**		
10.2	R5 062,86		11.1	R61 674,35	
10,3	R1 130 062,86		11.2	R499 551,48	
QUESTION 12			11.3	R5 661,61	
12.2			**QUESTION 14**		
			14.1	R273,08	
	12.2.1	n = 98,4321354 therefore 99	**QUESTION 15**		
	12.2.2	R3 251,06	15.1	R797 161,50	
QUESTION 13			15.2	R2 251 095,53	
13.1	R 2504,26		15.3	R13 885,35	
13.2	R149 496,35		**QUESTION 16**		
QUESTION 17			16.1	n = 4,999 therefore 5 years	
17.1	12,80%		16.2	R 80 292,21 Yes, He has enough money to buy Sandile's car.	
17.2	n = 32,77 therefore 33 months		**QUESTION 18**		
17.3	R140 471,48		18.1	R5 536,95	
QUESTION 19			18.2	R23 739,60	
19.1	R793 749,25				
19.2	R8 089,20				

PROBABILITY ANSWERS				
Question 1			Question 4	
1.1	90		4.1	9261000
1.2			4.2	0,048
	a)	40320	4.3	0,136
	b)	0,07	4.4	0,243
	c)	1	Question 5	
	d)	420	5.1	

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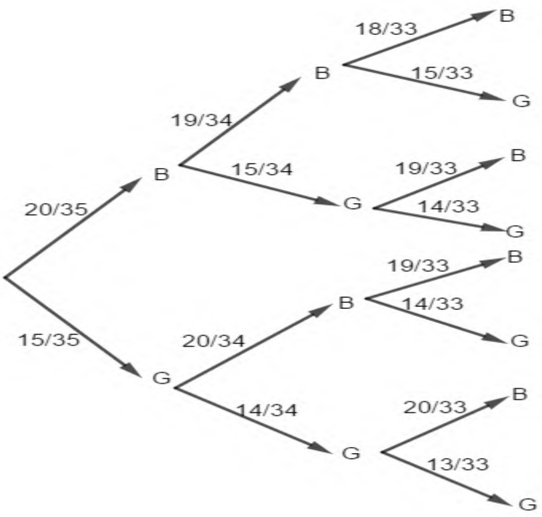
Question 2		
2.1		
	2.1.1	100 000
	2.1.2	30240
2.2	$\frac{40951}{100000}$	
Question 3		
3.1	360	
3.2	24	
3.3	0,067	
3.4	$\frac{1}{180}$	
Question 9		
0,39		
Question 10		
10.1		
	10.1.1	0,57
	10.1.2	0,07
	10.1.3	0,39
10.2	Not independent	
10.3	Not mutually exclusive	
Question 13		
13.1	$\frac{1}{2}$	
13.2	$\frac{1}{4}$	
13.3	0,063	
Question 17		

	5.1.1	
	5.1.2	
5.2		
Question 6		
0,40		
Question 7		
7.1	5040	
7.2	120	
7.3	0,114	
Question 8		
8.1	1000000000	
8.2	100000000	
8.3	0,50	
8.4	$\frac{1}{512}$	
Question 11		
11.1	Yes, P(Pass and Fail)=0	
11.2	Yes, P(F and P)= P(F) x P(P)	
Question 12		
$\frac{572}{8925}$		
Question 14		
14.1	$\frac{4}{7}$	
14.2	Tree diagram 	
14.3	$\frac{190}{1309}$	
14.4	$\frac{13}{187}$	

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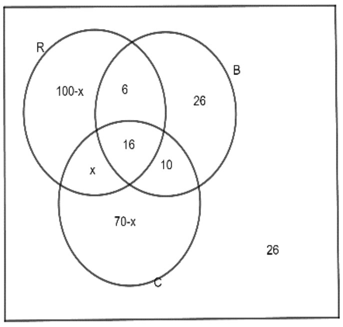
17.1



17.2	$x = 14$	
17.3		
	17.3.1	P(Not Cricket)= 0,600
	17.3.2	P(at least Two)= 0,192

Question 18

18.1	Venn diagram
18.2	$x = 0{,}15$ $y = 0{,}45$

Question 19

P(B)= 0,48

14.5	$\frac{174}{187}$
14.6	$\frac{597}{10472}$

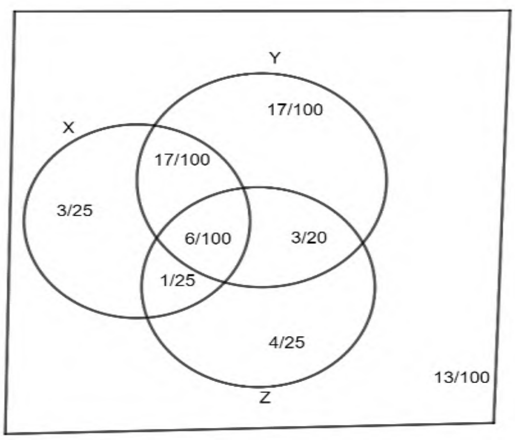
Question 15

3

Question 16

12

Question 23



$\frac{4}{25}$

Question 24

24.1

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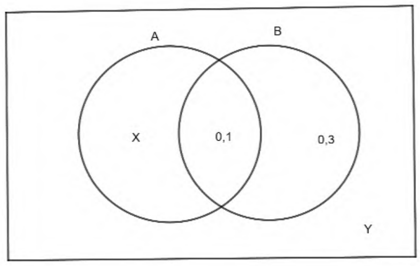
Question 20			24.1.1	
20.1	P(A Or B) =0,76		24.1.2	Not independent
20.2	P(C only)=0,18	24.2	672	
Question 21		**Question 25**		
		25.1		
Question 22			25.1.1	1000000
22.1	Venn diagram		25.1.2	1680
22.2	0,64	25.2	2903040	
22.3	Not independent	**Question 26**		
Question 27		252		
27.1	Not mutually exclusive	**Question 29**		
27.2	P(A or B)= 0,64	29.1	2058	
27.3	1200	29.2	720	
274	336	29.3	0,142	
Question 28		**Question 30**		
28.1	20160		16	
28.2	0,047		**4**	
			32	
			8	
		Question 31		
		0,467		

STATISTICS ANSWERS				
QUESTION 1		**QUESTION 2**		
1.1	The greater number of days an athlete trained, the shorter the time s/he ran the 100m Or Any answer in line with the given answer	2.1	$\bar{x} = 57.75 \text{ and } \sigma^2 = 45.4$	
1.2	(60;18.1)	2.2	2.2.1	22
1.3	$y = 17{,}82 - 0{,}07x$		2.2.2	60
1.4	refer to the diagram		2.2.3	6.78
1.5	14,7		2.2.4	$69kg$

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1.6	$r = -0,74$
1.7	moderate negative correlation
1.8	The value of r will decrease, because when the point was captured as (60;18.1) it pulled line of best towards it
QUESTION 3	
3.1	500
3.2	$Q_2 = 2000$
3.3	50%
3.4	IQR = 800
3.5	$2000 \leq x \leq 2250$
3.6	80
3.7	R400
QUESTION 5	
5.1	Min = 05 $Q_1 = 47.5$ $Q_2 = 67$ $Q_3 = 77.5$ Max = 90
5.2	DIAGRAM
5.3	Skewed to the left
5.4	15
5.5	20 scores
5.6	$\frac{6}{32}$
QUESTION 7	
7.1	$\bar{x} = 51.33$
7.2	$50 \leq x \leq 60$
7.3	$\sigma = 17.03$
7.4	DIAGRAM
7.5	14 days
QUESTION 9	
9.1	$Q_2 = 716$; $Q_1 = 691$; $Q_3 = 825$
9.2	DIAGRAM
9.3	9
QUESTION 11	

QUESTION 4		
4.1	DIAGRAM	
4.2	$y = 30.65 + 2.97x$	
4.3	refer to the diagram in 4.1	
4.4	105	
4.5	$r = 0.95$	
4.6	Very strong positive correlation	
QUESTION 6		
6.1	Skewed to the left	
6.2	60	
6.3	25% failed the test	
QUESTION 8		
8.1	(see table below)	

Daily sales	F	CF
$60 < x < 70$	5	5
$70 < x < 80$	11	16
$80 < x < 90$	22	38
$90 < x < 100$	13	51
$100 < x < 110$	7	58
$110 < x < 120$	3	61

8.2	DIAGRAM	
8.3	$Q_2 = 87$	
8.4	$96 \leq sales \leq 120$	
QUESTION 10		
10.1	22	
10.2	$\bar{x} = 15.25$	
10.3	$\sigma = 7.6$	
10.4	10.4.1	$2°C$ per month
	10.4.2	The range will decrease and the standard deviation as well
10.5	$\bar{x} = 5800$	
QUESTION 12		
12.1	Fly High	
12.2	$\frac{5120}{1000} \times 7.9 = 40.45$ Yes likely to be true	
12.3	Yes, the points are showing a weak negative correlation	
12.4	Alpha, 70% on time of arrival and fewer luggage losses	
QUESTION 14		

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11.1	Examination	F	C.F	14.1	$a = 34$
	$30 < x < 40$	12	12		$b = 44.47$
	$40 < x < 50$	18	30		$c = 51.5$
	$50 < x < 60$	55	85		$d = 69$
	$60 < x < 70$	57	142		$e = 88$
	$70 < x < 80$	43	185		
	$80 < x < 90$	11	196		
	$90 < x < 100$	4	200		
11.2	DIAGRAM				
11.3	refer to the diagram in 11.2				
QUESTION 13					
13.1	DIAGRAM				
13.2	refer to the diagram				
13.3	refer to the diagram				

ANALYTICAL GEOMETRYANSWERS			
QUESTION 1		**QUESTION 2**	
1.1	$E\left(\frac{-8}{3};0\right)$	2.1	$x = 5$
1.2	$D\hat{A}E = 35{,}5^\circ$	2.2	$y = \frac{-1}{2}x + \frac{9}{2}$
1.3	$D(-1{,}4\,;7{,}6)$	2.3	$y = 2x - 8$
1.4	$DM = 1{,}4\ units$	2.4	$QW = WR \ldots W\ is\ a\ midpt\ of\ QR$ $QT = \frac{1}{2}QT$ $QR = 4QT$ $TR = 3QT$ $QT = \frac{1}{3}TR$
		2.5	$\beta = 33{,}6^\circ$
QUESTION 3		**QUESTION 4**	
3.1	$k = 4$	4.1	$C(6\,;0)$
3.2	$k = \frac{-1}{2}$	4.2	$mAB = 1$ $mBC = -1$ $\therefore AB \perp BC$ $\therefore ABCD\ is\ a\ Rectangle \ldots \angle s\ of\ rect = 90^\circ$
3.3	$k = -6$	4.3	$AD = \sqrt{32}$ and $AB = \sqrt{2}$ $\therefore Area\ of\ rect = 8\ square\ units$
3.4	$k = 4$		
3.5	$k = -8\ or\ k = 2$		
QUESTION 5		**QUESTION 6**	
5.1	$y = -1$	6.1	$mAD = 2$
5.2	$y = -2x + 5$	6.2	$y = 2x - 4$
5.3	$AM = \sqrt{20}$	6.3	$F(2\,;0)$
5.4	$x^2 + y^2 + 8x - 6y - 25 = 0$	6.4	$\beta = 111{,}80^\circ\ , \theta = 63{,}43^\circ$ $\therefore \alpha = 111{,}80^\circ - 63{,}43^\circ = 48{,}37^\circ$
5.5	$AT = 6\ units = TK$ $\therefore One\ diagonal\ bisects\ the\ other\ one.$	6.5	$Area\ of\ \Delta DCF = 18\ square\ units$

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		6.6	$B(3;2)$
QUESTION 7		**QUESTION 8**	
7.1	$mPQ = 1$	8.1	$mAQ = 2$ $\therefore mAD = 2 \ldots (A, Q \text{ and } D..collinear)$
7.2	$y = x - 11$	8.2	$mBC = \frac{-1}{2} \ldots BC \perp AD$
7.3	$M(2 ; \frac{-3}{2})$	8.3	$y = \frac{-1}{2}x + \frac{19}{2}$
7.4	$a = 4 ; b = -7$	8.4	$M(6;1)$
7.5	$E\left(\frac{-18}{5} ; \frac{-16}{5}\right)$	8.5	$y = x - 5$
7.2*	$\theta = 45^{\circ}$	8.6	$C\left(\frac{29}{3} ; \frac{14}{3}\right)$
7.4*	$\alpha = 153.43^{\circ}$	8.7	$B\hat{A}D = 8{,}14^{\circ}$
7.6	$Area\ of\ PQRS = 30\sqrt{10}$ square units		
QUESTION 9		**QUESTION 10**	
9.1	$a = 3$	10.1	$P(2;4)$
9.2.1	$A(0;6)$	10.2	$mAQ = 2$
9.2.2	$C(10;0)$	10.3	$\theta = 116{,}57^{\circ}$
9.2.3	$B(10;6)$	10.4	$Area\ of\ \Delta AQO = 2{,}0\ square\ unit$
9.3.1	$p = 3$	10.5	$mQG = \frac{y2 - y1}{x2 - x1}$ $2 = \frac{b - 0}{a - 2}$ $2a - 4 = b$
9.3.2	$p = 2$	10.6	$Q\hat{A}T = 53{,}14^{\circ}$
9.3.3	$p = 0$	10.7	$B\left(2 ; \frac{4}{3}\right)$
QUESTION 11		**QUESTION 12**	
11.1	$K(4;3)$	12.1	$r = 3 , r = 4$ $centre(2;3) centre(1; -1)$
11.2	$(x-8)^2 + (y-6)^2 = 25$	12.2.1	$N(2;0), M(1;-4)$
11.3	$(x-8)^2 + (y-6)^2 = 100$	12.2.2	$r = 7\sqrt{2}, r = \sqrt{5}$
11.4	$y = \frac{-4}{3}x + \frac{25}{3}$		
11.5	$OB = \frac{25}{3}\ units\ \therefore OC = \frac{25}{4}\ units$ $\therefore OB \neq OC$		
QUESTION 13		**QUESTION 14**	
13.1	$B(0;2)$	14.1	$r = \sqrt{180}$
13.2	$mBC = \frac{1}{3}$	14.2	$y = \frac{1}{2}x$ $x^2 + y^2 = 180$ $x^2 + \left(\frac{1}{2}x\right)^2 = 180$ $x^2 + \frac{x^2}{4} = 180$ $x = \pm 12$ $y = \frac{1}{2}(-12)$

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			$y = -6$ $K(-12\,;\,-6)$
13.3	$mBC \times mAB = -1 \;\therefore mBC \perp mAB$	14.3.1	$y = -2x - 30$
13.4	$C(6\,;4)$	14.3.2	$M(-16\,;\,-8)$
13.5	$AC = \sqrt{50}$	14.3.3	$(x+16)^2 + (y+8)^2 = 20$
		14.4	$6\sqrt{5} < r < 10\sqrt{5}$
		14.5	The circle will never cut the circle with centre M as they have the same centre (concentric circles) but unequal radii.
QUESTION 15			
15.1.1	Centre $(3\,;1)$ *and Radius* : 3 *units*		
15.1.2	$p = 8$ *or* $p = -2$		
15.2.1	Line from centre to midpoint of chord.		
15.2.2	$x^2 + (y-1)^2 = 20$		
15.2.3	$y = 2x - 9$		

TRIGONOMETRY ANSWERS

1.1		$\sin 10°$
1.2	(a)	$\frac{1}{k}$
	(b)	Proof
1.3		
1.4		$\frac{2\sqrt{2}}{2}$
1.5		4
1.6		$\frac{-2}{3}tan^2x$
1.13		-1
1.14	(a)	$\frac{\sqrt{2}}{2}k$
	(b)	$1 + 2k$
1.15		-6
1.16		$\frac{2}{k}$
1.17	(a)	$A = 135° + k.180, k \in z$
	(b)	
1.18	(a)	$2\sin\theta\cos\theta$
	(b)	$cos^2\theta - sin^2\theta$
1.22	(a)	$1 - 2P^2$
	(b)	$\sqrt{\frac{1}{2} - \frac{1}{2}(1-2P^2)}$
	(c)	$\frac{\sqrt{\frac{1}{2}-\frac{1}{2}(1-2P^2)}}{1-2P^2}$

1.7		$\frac{12}{13}$
1.8		2
1.9		$-T$
1.10	(a)	
	(b)	
1.11	(a)	-1
	(b)	
1.12	(a)	$\frac{-2}{\sqrt{7}}$
	(b)	$\frac{12}{7}$
	(c)	$\frac{3}{7}$
1.19		1
1.20	(a)	Proof
	(b)	$\frac{\sqrt{2}}{2}$
1.21		Proof
1.26	(a)	$-\frac{p}{q}$
	(b)	$\frac{\sqrt{3}}{2q}p + \frac{\sqrt{q^2-p^2}}{2q}$
	(c)	$\frac{2p\sqrt{p^2-q^2}}{q^2}$
	(d)	$\frac{2p\sqrt{q^2-p^2}}{q}$

2.PROVING IDENTITIES

All proofs

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3. GENERAL SOLUTIONS SOLVING TRIG EQUTION, GIVEN DOMAIN

3.1		$x\varepsilon[-300;-60;60;300]$
3.2	(a)	$A\varepsilon[-45;135]$
	(b)	$A\varepsilon[495;675]$
3.3		
3.4		$\theta = 71{,}56 + k.180, k\epsilon z$ or $\theta = 251{,}56 + k.180, kez$
3.5		$\alpha = \pm 30° + k.360°, k\epsilon z$ or $\alpha = 300° + k.360°, k\varepsilon z$ or $\alpha = 150° + k.360°, k\epsilon z$ or $\alpha = 210° + k.360°, k\varepsilon z$
3.6	(a)	
	(b)	
3.7		
3.8		$x = 30° + k.360°, k\varepsilon z$ or $x = 150° + k.360°, k\varepsilon z$ $x = 123{,}69 + k.180°, k\varepsilon z$
3.15	3.15.1	$\sqrt{1-y^2}$
	3.15.2	
	3.18.3	$-t$

3.9		$26{,}57° + k.180°, k\varepsilon z$ or $63{,}44 + k.180°, k\varepsilon z$
3.10	(a)	$\theta\varepsilon[-50°;-122°;22°;94°;166°$
	(b)	$\theta\varepsilon[-110°;10°;130°]$
3.11		$x = 330° + k.360°, k\varepsilon z$ or $90° + k.360°, k\varepsilon z$
3.12	(a)	$60° + k.360°, k\varepsilon z$ or $120° + k.360, k\varepsilon z$
	(b)	$x = 90° + k.360°, k\varepsilon z$
	(c)	
	(d)	
	(e)	
3.13		
3.14		Proof
	3.14.2	(a) Proof
	(a)	Proof
	(b)	$8sin^3 18 - 4sin18° + 1 = 0$

4. Trigonometry

4.1	(a)	$f(x) = 3cos\frac{x}{2}$
	(b)	720°
	(c)	$x = 90°$
	(d)	DIAGRAM
4.2	(a)	$y\varepsilon[-2;2]$
	(b)	$\theta = 60°$ $f(x) = \cos(x - 60°)$
	(c)	$C(30°;0{,}87)$
	(d)	$x\varepsilon[90°;150°]$
	(e)	$x\varepsilon(135°;180°)$
4.3	(a)	Max. at 2
	(b)	2
	(c)	Ampl.=2
	(d)	$g(x) = cosx$
	(e)	$x\varepsilon[-270°\& 90°]$
	(f)	$x\varepsilon[-270°;-90°]$
	(g)	$x\varepsilon[-270°;-90°;90°;270°]$
4.9	(a)	$x = 10° + k.120°, k\varepsilon z$ or $x = 10° + k.120°, k\varepsilon z$ or $x = -210° - k.360°, k\varepsilon z$
	(b)	$x\varepsilon[10°;130° \&150°]$

4.5	(a)	$n = 2$ $m = 2$
	(b)	$-2 \leq y \leq 2$
4.6	(a)	DIAGRAM
	(b)	$x\varepsilon(-90°;-60°)$
	(c)	360°
	(d)	$f(x) = \sin(x - 30) + 2$
4.7	(a)	$x\varepsilon[-150°;30°;90]$
	(b)	DIAGRAM
	(c)	180°
	(d)	$x\varepsilon(-180°' - 150°)$ and $x\varepsilon(-30°;90°)$ $x\varepsilon(90°;180°)$
	(e)	1
	(f)	$h(x) = \sin x - 1$
4.8	(a)	DIAGRAM
	(b)	
	(c)	DIAGRAM
	(d)	$x\varepsilon(-120{,}0)$
	(e)	$x = -30°$ or$x = 0°$

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5. 2D AND 3D TRIGONOMETRY

5.1		Proof
5.2	(a)	Proof
	(b)	
5.3	(a)	Proof
	(b)	Proof
	(c)	
5.4	(a)	$AK = \frac{\sqrt{3}}{2}h$
	(b)	$DK = \frac{1}{2}h$
	(c)	$KF = \frac{h^2}{2}$
	(d)	
5.5	(a)	Proof
	(b)	i. $C\hat{D}F = 47.16°$
		ii.
5.11	(a)	Proof
	(b)	
5.12	(a)	
	(b)	
	(c)	
5.13		Proof

5.6	(a)	Proof
	(b)	Proof
	(c)	$RS = 4$
5.7	(a)	
	(b)	
	(c)	
5.8	(a)	
	(b)	Proof
5.9	(a)	
	(b)	
	(c)	
5.10	(a)	
	(b)	
5.14	(a)	
	(b)	
5.15	(a)	Proof
	(b)	Proof