education
Department:
Education
PROVINCE OF KWAZULU-NATAL

KZN DEPARTMENT OF EDUCATION

MATHEMATICS
TRIAL & FINAL EXAM
REVISION MATERIAL
GRADE 12

2021

TABLE OF CONTENTS

QUESTION 1

1.1 Solve for x:

1.1.1 $9x^2 - 7x - 3 = 0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.2 $5x^2 - 10x > 0$ (3)

1.1.3 $4 - \sqrt{x+5} = x + 3$ (6)

1.2 If $(x-3)(y+4) = 0$ determine x if:

1.2.1 $y = 4$ (1)

1.2.2 $y = -4$ (1)

1.3 Solve simultaneously for x and y:

$2y + x = 1$ and $x^2 + y^2 = y - x$ (6)

1.4 Consider: $5x^2 - kx + 16 = (x+2).Q(x) + 10$ where k is a constant and $Q(x)$ is a polynomial in terms of x. Calculate k. (3)

[23]

QUESTION 2

2.1 Ann plans to start studying for her grade 12 final examination. On the first day she studies 1 hour (60 minutes) and plans to increase the study time with 15 minutes each day. As soon as Ann reaches 6 hours' study time, she will continue to study 6 hours each day thereafter.

2.1.1 Calculate the number of hours Ann will study on the 10^{th} day. (3)

2.1.2 Determine on which day Ann will study 6 hours for the first time. (2)

2.1.3 Calculate the total number of hours that Ann will study in the first 30 days. (4)

2.2 Prove that $x + y + z$ forms a geometric series if $\log x + \log y + \log z$ forms an arithmetic series. (4)

[13]

QUESTION 3

3.1 Consider the series: $64 + 32 + 16 + ...$

3.1.1 Determine the ninth term in the sequence. (3)

3.1.2 Determine the sum to infinite. (2)

3.2 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 2. The general term of the first differences is given by $6n + 8$.

3.2.1 Show that a = 3. (3)

3.2.2 Determine the general term T_n. (3)

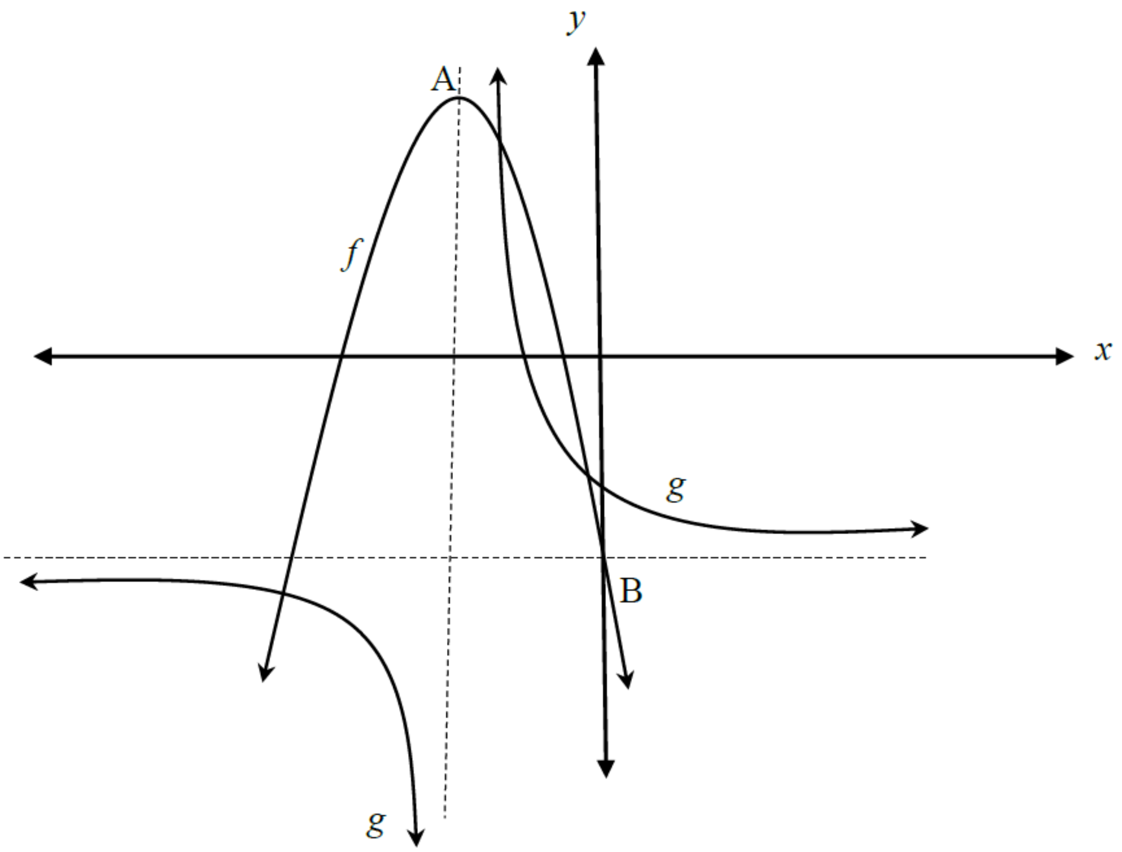
3.3 Given the series: $17p^8k^{15} + 20p^9k^{14} + 23p^{10}k^{13} + ... + 53p^{20}k^3$
Write the series in sigma notation. (4)

[15]

QUESTION 4

The graphs of $f(x) = -x^2 - 6x - 4$ and $g(x) = \dfrac{2}{x + p} + q$ are sketched below.

B is the y-intercept of f and A is the turning point of f. The vertical asymptote of g forms the axis of symmetry of f. The horizontal asymptote of g cuts the y-axis at B.



4.1 Determine the coordinates of A, the turning point of f. (3)

4.2 Determine the coordinates of B, the y-intercept of f. (1)

4.3 Determine the x-intercepts of f. (3)

4.4 Write down the equation of g. (2)

4.5 Determine the equation of the axis of symmetry of g that has a positive gradient. (2)

4.6 Determine the coordinates of the intersection of the axis of symmetry which is determined in QUESTION 4.5 and g, if $x > -3$. (5)

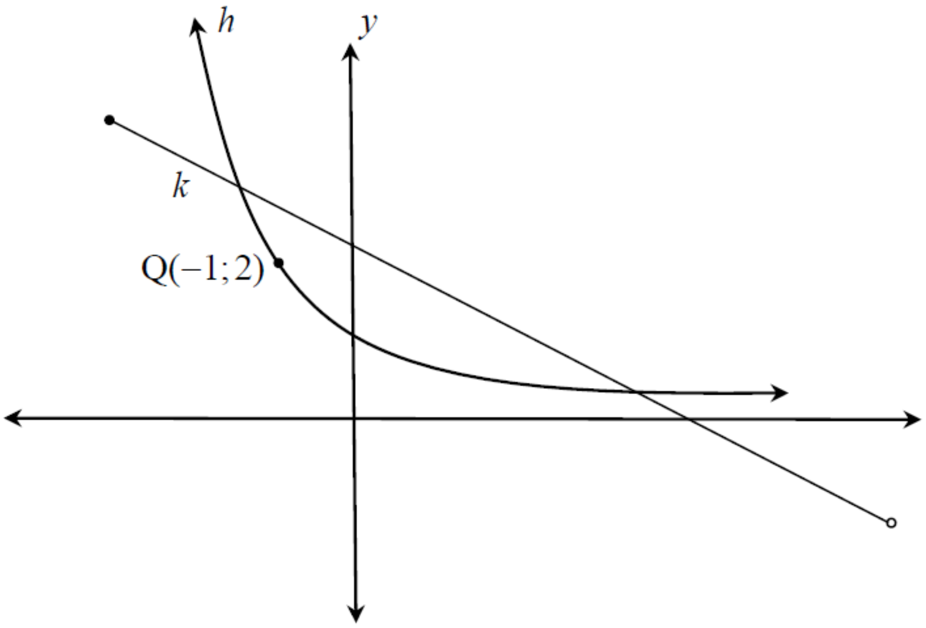
4.7 Determine the equation of m, the tangent to g, with the point where they touch, the intercept calculated in QUESTION 4.6. (3)

4.8 For which values of x will: $f(x).f^{/}(x) \geq 0$ (2)

[21]

QUESTION 5

Given: $k(x) = -\frac{2}{3}x + 3$ for $-4 \leq x < 6$ and $h(x) = 2^{-x}$. Q$(-1;2)$ is a point on h.



5.1 Determine the x-intercept of k. (2)

5.2 Determine the domain of k^{-1}. (2)

5.3 Determine the equation of h^{-1}. (2)

5.4 Give the coordinates of the x-intercept of h^{-1}. (2)

5.5 For which values of x is: $k^{-1}(x) < 0$? (2)

5.6 If $k(x) = q'(x)$, where q is a function defined for $-4 \leq x < 6$. Draw a neat sketch graph of q. Clearly show the x-values of the turning point(s) and end points. (3)

[13]

QUESTION 6

Patric takes out an annuity that he can live from after he retires in twenty years' time. He needs R3 000 000 in this annuity when he retires. The bank gives him an interest rate of 10% per annum compounded monthly.

6.1 Calculate his monthly instalment into the fund if he starts paying immediately and thereafter at the end of each month until his last payment in 20 years' time. (4)

6.2 After 20 years Patric retires, but decides not to let the R3 000 000 be paid out. Instead he decides to withdraw monthly amounts of R20 600 at the end of each month. He withdraws his first amount at the end of the fourth month. The interest that he earns over this period is 8% per year, compounded monthly. Determine how many months he will survive on his current lifestyle. (7)

6.3 How big is Patric's last withdrawal going to be? (4)

[15]

QUESTION 7

7.1 Given: $f(x) = -x^2 + 7x + 9$

Determine $f'(x)$ from first principles. (5)

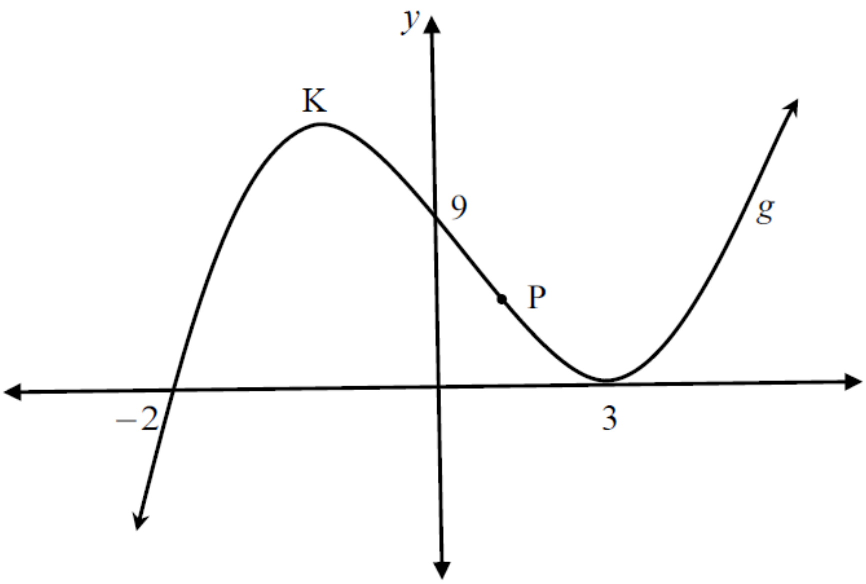
7.2 Determine $f'(x)$ if $f(x) = \dfrac{4}{x^2} + 3x^5$ (3)

7.3 Determine $\dfrac{dy}{dx}$ if: $\dfrac{y}{x-3} = 1 + x$ (3)

[11]

QUESTION 8

The graph of $g(x) = ax^3 + bx^2 + cx + d$ is sketched below. The graph of g intersects the x-axis at $x = -2$ and touches the x-axis at $x = 3$. K is a turning point of g. The graph g cuts the y-axis at (0; 9).



8.1 Show that $a = \frac{1}{2}$, $b = -2$, $c = -\frac{3}{2}$ and $d = 9$. (4)

8.2 Determine the x-coordinate of the turning point K. (4)

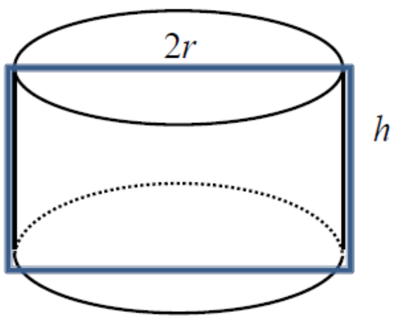
8.3 For which values of x is g concave up? (3)

8.4 Determine the coordinates of P if the gradient of the tangent to the graph at P is equal to $-\frac{7}{2}$. P touches g where g is concave up. (5)

[16]

QUESTION 9

Sam has a 16m steel cable to wrap around a cylindrical tank to strengthen (reinforce) the tank as shown in the shaded part of the sketch.



9.1 Show that the height can be written as $h = 8 - 2r$ in terms of the radius. (2)

9.2 Write the volume of the tank in terms of r. (3)

9.3 What must the radius and the height of the tank be so that the volume of the tank will be a maximum? (5)

[10]

QUESTION 10

Tom and Jerry enter the Ironman Competition. The probabilities that they will complete the race have been determined to be 0,85 and 0,67 respectively. The probability that Tom and Jerry will complete the competition is independent of each other. Determine the probability (correct to TWO decimal places) that:

10.1 Both will complete the Ironman Competition. (2)

10.2 Only Tom will complete the competition. (2)

10.3 At least one of the two will complete the competition. (3)

[7]

QUESTION 11

The digits 0 to 9 are used to create a 5-digit-number for a lucky draw for the grade 12 fundraising. The digits may repeat. The numbers lie between 10 000 and 20 000. These numbers are written on pieces of paper and thrown into a bottle.

To win a prize, you have to draw a 5-digit-number that has at least one six and the digits may not repeat.

11.1 Determine the number of papers in the bottle. (2)

11.2 What is the probability that a person who selects a paper randomly from the bottle, will win a prize? Show all your calculations. (4)

[6]

TOTAL: 150

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 7$ (correct to TWO decimal places) (4)

1.1.2 $2x^2 = 9x + 5$ (3)

1.1.3 $x^2 - 5x > -4$ (4)

1.1.4 $x - 3x^{\frac{1}{2}} = 4$ (6)

1.2 Show that the equation $2^{2x+1} + 7.2^x - 4 = 0$ has only ONE solution. (4)

1.3 Solve for x and y simultaneously:
$x = y - 13$ and $\sqrt{2-x} = y - 3$ (6)

[27]

QUESTION 2

2.1 Given the following quadratic sequence:

$1 \,;\, 7 \,;\, 15 \,;\, 25 \,;\, x \,;\, \ldots$

2.1.1 Write down the value of x in the sequence. (1)

2.1.2 Determine the expression for the n^{th} term of this sequence. (4)

2.1.3 W_n represents the general term of a sequence for the first differences.
Determine the value of the n^{th} term of the quadratic sequence if $W_n = 50$. (5)

2.2 Consider the following:

$0 \,;\, -\frac{1}{2} \,;\, 0 \,;\, \frac{1}{2} \,;\, 0 \,;\, \frac{3}{2} \,;\, 0 \,;\, \frac{5}{2} \,;\, 0 \,;\, \frac{7}{2} \,;\, 0\ldots\ldots$

Assume that the pattern continues consistently.

2.2.1 Write down the value of the 191st term of this sequence. (1)

2.2.2 Determine the sum of the first 500 terms of this sequence. (4)

2.3 Consider the following geometric series:

$$4\left(\frac{1-k}{5}\right)+8\left(\frac{1-k}{5}\right)^2+16\left(\frac{1-k}{5}\right)^3 \ldots$$

Determine the values of k. (4)

[19]

QUESTION 3

The first THREE terms of an infinite geometric sequence are 16, 8 and 4 respectively.

3.1 Determine ALL possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

3.2 Calculate the sum to infinity of this sequence. (2)

[5]

QUESTION 4

Given: $f(x)=\dfrac{6}{x+2}-1$

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Calculate:

4.2.1 the y-intercept of f. (2)

4.2.2 the x-intercept of f. (2)

4.3 Sketch the graph of f showing clearly the asymptotes and the intercepts of the axes. (3)

4.4 Determine the equation of the line of symmetry of f that has a negative gradient. Leave your answer in the form $y = \ldots$ (3)

[12]

QUESTION 5

Given: $p(x) = \log_3 x$

5.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

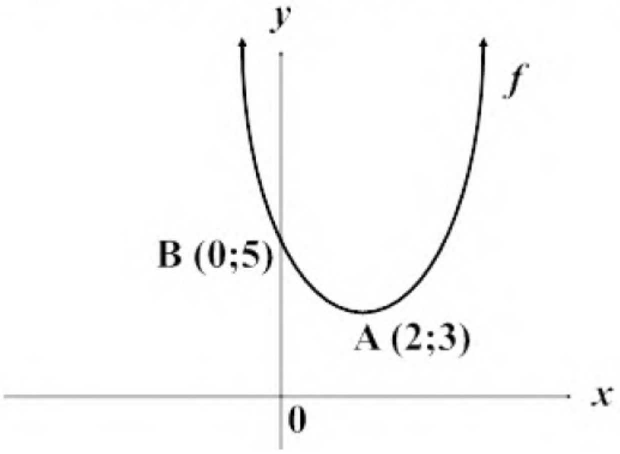
5.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axis. Show clearly the intercepts of the axes and ONE other point on each graph. (6)

5.3 Determine the values of x for which $p(x) \leq 2$. (3)

5.4 Write down the x-intercept of h if $h(x) = p(-x)$. (2)
[13]

QUESTION 6

The sketch below shows the parabola f with turning point A(2 ; 3) and y-intercept B(0 ; 5).



6.1 Show that the equation of the parabola f can be written as $y = \frac{1}{2}x^2 - 2x + 5$. (5)

6.2 WITHOUT calculating the discriminant, give an appropriate reason whether the discriminant of f is positive, negative or zero. (2)

6.3 Use the graph to determine the value(s) of k for which the equation $\frac{1}{2}x^2 - 2x + 5 = k$ has real and un-equal roots. (2)

6.4 The parabola is shifted vertically until the new y-intercept is the origin. Determine the equation of the NEW parabola. (1)
[10]

QUESTION 7

7.1 Sarah's investment earns interest at 11% p.a. compounded semi-annually.
Mary's investment earns an effective interest of 11,42% *p.a.*
Whose investment, Sarah's or Mary's, earns a higher rate of interest per annum. (3)

7.2 Buhle decided to start saving before retirement. She makes payments of R10 000 monthly into an account yielding 7,72% *p.a.* compounded monthly, starting on 1 November 2016 with a final payment on 1 April 2026.

7.2.1 Calculate how much will be in the savings account immediately after the last deposit is made. (4)

7.2.2 At the end of the investment period Buhle re-invested the full amount in order for her to be able to draw a monthly pension from the fund.

She re-invested the money at an interest rate of 10% p.a. compounded monthly. If she draws an amount of R30 000 per month from this investment, for how many full months will she be able to receive R30 000? (4)

7.2.3 After withdrawing R30 000 for 20 months Buhle requires R1 500 000. Determine whether she can access this amount of money from this annuity. (4)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from FIRST principles if
$f(x) = -2x^2 + 6x$ (4)

8.2 Evaluate:

8.2.1 $\frac{dy}{dx}$ if $y = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.2.2 $f'(x)$ if $f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4}$ (4)

8.3 The tangent to the curve of $y = 2x^2 - 3x - 5$ is drawn at the point $(2 ; -3)$. A straight line parallel to the tangent passing through the y-intercept of the curve is then drawn. Determine the equation of the straight line. (4)

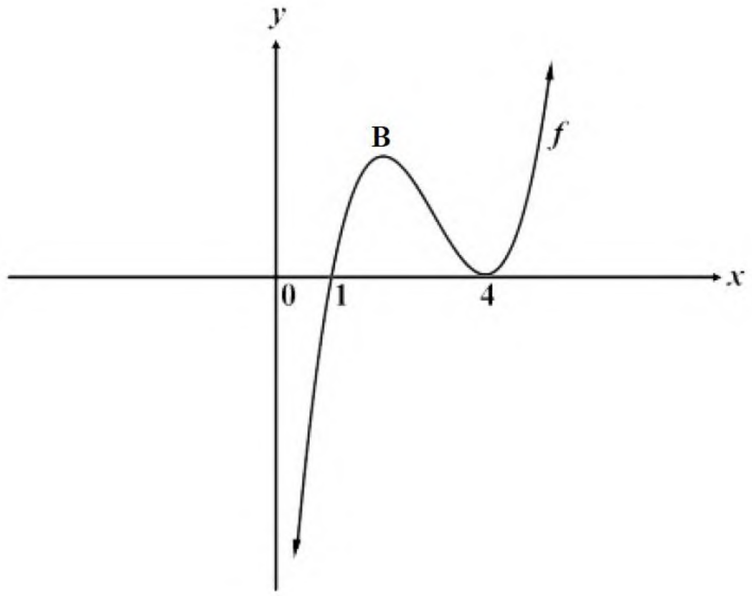
[14]

QUESTION 9

9.1 Determine the points on the curve of $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1. (5)

9.2 The graph of the cubic function with equation $y = x^3 + ax^2 + bx + c$ is drawn below.

- $f(1) = f(4) = 0$
- f has a local maximum at point B and a local minimum at $x = 4$.



9.2.1 Show that $a = -9$, $b = 24$ and $c = -16$. (2)

9.2.2 Calculate the coordinates of point B. (4)

9.2.3 Determine the value(s) of k for which $f(x) = k$ has ONLY negative roots. (2)

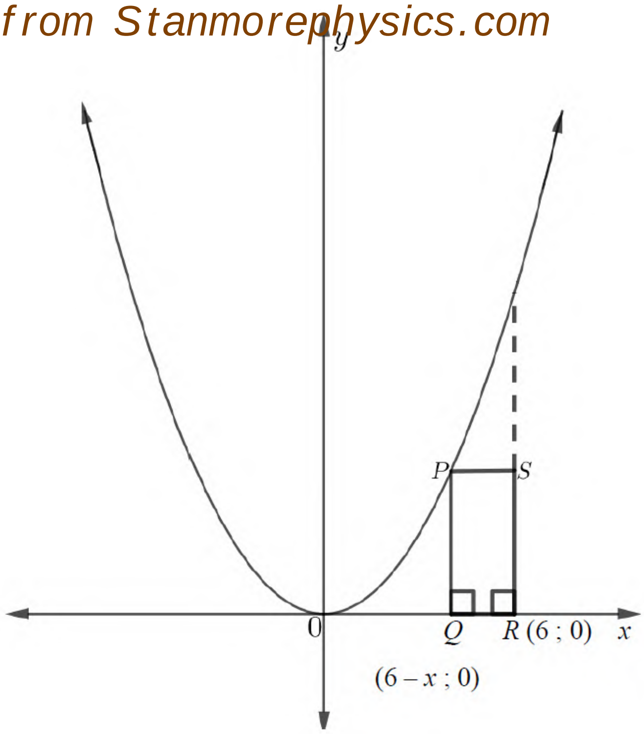
9.2.4 Determine the value(s) of x for which f is concave up. (2)

[15]

QUESTION 10

The diagram below represents the graph of $y = x^2$ and rectangle PQRS.

- Point P is a common point on both the graph and the rectangle.
- The graphs of $x = 6$ and $y = 0$ represent TWO adjacent infinite lines of rectangle PQRS.
- The coordinates of point R(6 ; 0) and point Q($6 - x$; 0) are given.



10.1 Write down the coordinates of point P in terms of x. (1)

10.2 Determine the maximum area of rectangle PQRS. (5)
[6]

QUESTION 11

11.1 Given that A and B are independent events.

- P(B only) = 0,3
- P(A and B) = 0,2
- P(A only) = x
- P(not A or B) = y

Determine the values of x and y. (4)

11.2 Six players of a volleyball team stand at random positions in a row before the game begins. X and Y are two players in this team.
Determine the probability that X and Y will NOT stand next to each other. (3)

11.3 Determine how many 4-digit numbers can be formed from 10 digits, 0 to 9, if:

11.3.1 repetition of digits is allowed (2)

11.3.2 repetition of digits is NOT allowed (3)

11.3.3 the last digit must be 0 and repetition of digits is allowed (2)
[14]

TOTAL: 150

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(x-3)(x+2)=0$ (2)

1.1.2 $x^2+x-3=0$ (correct to TWO decimal places) (3)

1.1.3 $(x-1)(x-2)\leq 6$ (4)

1.2 Solve simultaneously for x and y:
$x+y=8$ and
$x^2+5xy+6y^2=0$ (6)

1.3 Without solving the equation, show that $x^2+5mx+6m^2-1=0$ has real, unequal roots for all real values of m. (4)

1.4 If $\frac{1}{\sqrt{m}+\sqrt{m+1}}=\sqrt{m+1}-\sqrt{m}$, determine without the use of a calculator the exact value of: $\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{1680}+\sqrt{1681}}$ (3)

[22]

QUESTION 2

2.1 The first four terms of an arithmetic sequence are: -113 ; -120 ; -127 ; -134

2.1.1 Write down the next two terms in the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = bn + q$ (2)

2.1.3 Calculate the sum of the first 50 terms of the sequence. (2)

2.2 The following number pattern has a constant second difference.

41 ; 43 ; 47 ; 53 ; 61 ; 71 ; 83 ; 97 ; …

2.2.1 Write down the value of the constant second difference. (1)

2.2.2 Determine the n^{th} term of the number pattern in the form $T_n = an^2 + bn + c$ (4)

2.2.3 The first forty terms of the number pattern are all prime numbers. Show that T_{41} is not a prime number. (3)

[14]

QUESTION 3

3.1 Which term in the geometric series: $2 + 6 + 18 + \ldots$ has the value of 9 565 938? (4)

3.2 Given the infinite geometric series : $9x^2 + 6x^3 + 4x^4 + \ldots$

3.2.1 For which value(s) of x will the series converge? (3)

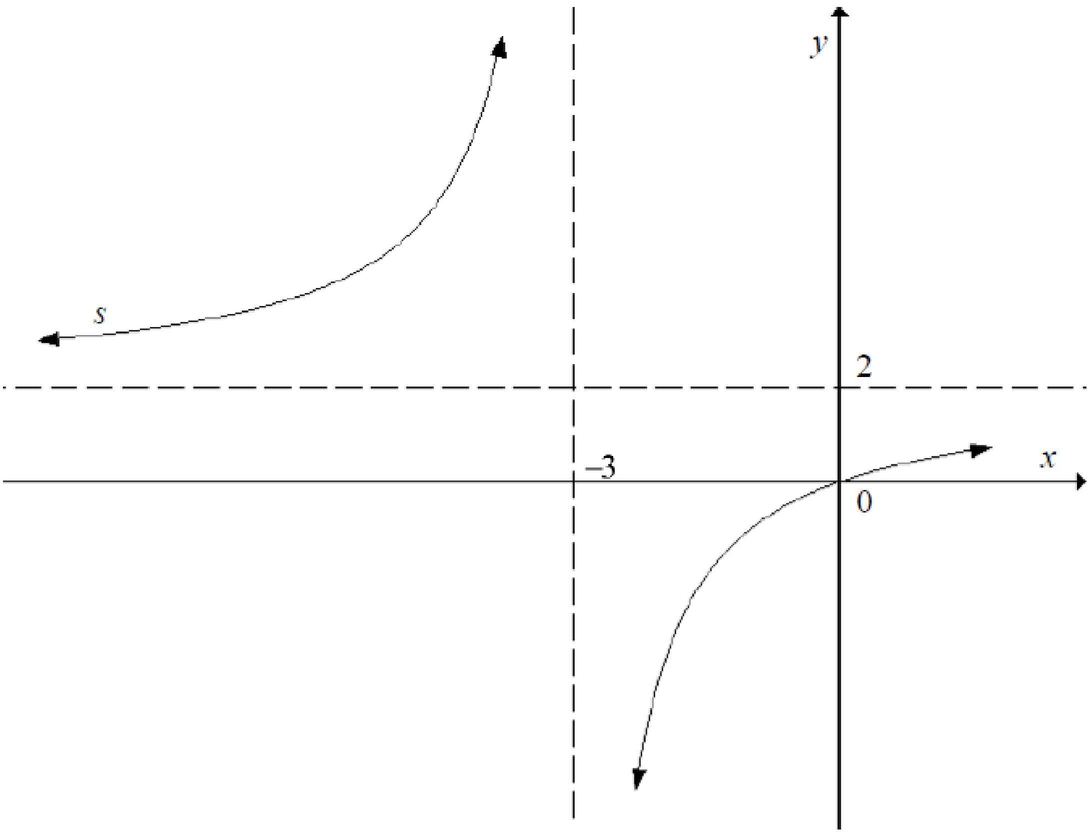
3.2.2 If $x = 1$, determine S_∞. (3)

3.3 The sum of the first k terms of an arithmetic series is given by $s(k) = 3k^2 + 4k$. If the sum of the last four terms is 640, determine the value of k. (5)

[15]

QUESTION 4

Given the graph $s(x)=\dfrac{a}{x+p}+q$ having the horizontal asymptote intersecting the y-axis at 2 and the vertical asymptote intersecting the x-axis at -3. The point (0 ; 0) lies on s.

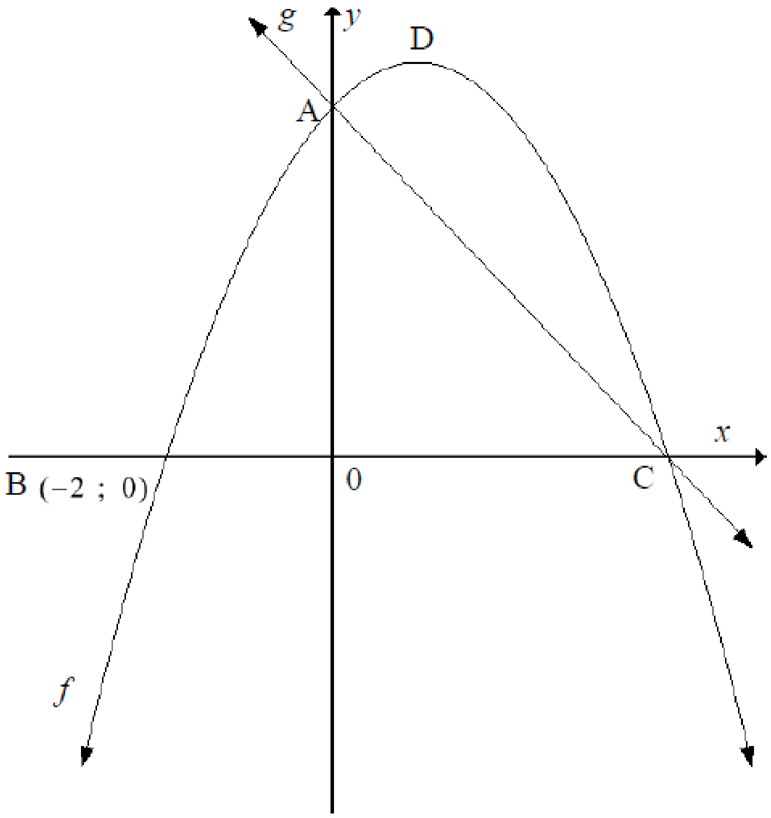


4.1	Write down the domain of s.	(1)
4.2	Write down the values of p and q.	(2)
4.3	Hence, determine the equation in the form $s(x)=\dfrac{a}{x+p}+q$.	(2)
4.4	A new graph is defined by the equation $b(x) = s(x) + 2$. Determine the equation of the axis of symmetry of b having a positive gradient.	(2)

[7]

QUESTION 5

In the diagram below, the graphs of $f(x) = -x^2 + 2x + 8$ and g are shown. Graph f intersects the x-axis at B$(-2\,;0)$ and C, intersects the y-axis at A and has a turning point at D. Graph g is a straight line passing through A and C.



5.1	Write down the coordinates of A.	(1)
5.2	Calculate the coordinates of D, the turning point of f.	(3)
5.3	Hence, or otherwise, write down the coordinates of C.	(2)
5.4	Determine the equation of g in the form $y = mx + c$.	(2)
5.5	A tangent to f has the equation $y = -2x + k$. Calculate the value of k.	(4)
5.6	Write down the value(s) of x for which $g(x) = -2x + j$ will intersect the graph f at two distinct positive x-values.	(2)
5.7	Determine the maximum value of h if $h(x) = 3^{f(x)-6}$	(3)

[17]

QUESTION 6

Consider the function $h(x) = 2.2^{x-2} - 4$

6.1 Write down the coordinates of the y-intercept of h. (1)

6.2 Calculate the coordinates of the x-intercept of h. (2)

6.3 Sketch the graph of h. Clearly show all the intercepts with the axes and asymptote, if any. (3)

6.4 Describe the transformation from h to j if $j(x) = -2^{x+3} + 4$. (2)

[8]

QUESTION 7

7.1 Sphe invested R20 000 at an interest rate of 8,5% p.a. compounded annually. How long will it take for this investment to grow to R36 000? (3)

7.2 Jack invested R1 000 at the end of each month for 8 years into an account paying interest at the rate of 12% p.a. compounded monthly.

7.2.1 Calculate the value of the investment after 8 years. (3)

7.2.2 Convert the nominal interest rate quoted in this question to an effective interest rate. (2)

7.3 Rosa is granted a home loan of R1 800 000 at an interest rate of 14% p.a. compounded monthly to be paid off by means of equal monthly payments over 20 years starting one month after the granting of the loan.

7.3.1 Calculate the monthly payments. (4)

7.3.2 If Rosa misses payments 121, 122 and 123, calculate the new monthly payments if she still wishes to pay off the loan in 20 years from the date the loan was granted. (5)

[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given $f(x) = -x^2 + 5$ (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 5x^3 - \frac{1}{4}x$ (2)

8.2.2 $D_x\left[\frac{7x^2-3x+1}{x^5}\right]$ (4)

8.2.3 $\frac{dy}{dp}$ if $y = 6(\sqrt[3]{x^2})$ and $8x = p^{-3}$ (4)

[15]

QUESTION 9

9.1 If $(x-3)$ is a factor of $f(x) = x^3 + tx^2 + 2x + 3$, determine the value of t. (2)

9.2 Given: $f(x) = x^3 - 4x^2 - 3x + 18 = (x+2)(x-3)^2$

9.2.1 Write down the x-intercepts of f. (2)

9.2.2 Show that $(3\,;0)$ and $\left(-\frac{1}{3};\frac{500}{27}\right)$ are the stationary points (turning points) of f. (3)

9.2.3 Sketch the graph of f, showing clearly the turning points and intercepts on the axes. (3)

9.2.4 Use the graph of f to determine the value(s) of x which satisfy $f(x) > 0$ and $f'(x) > 0$ simultaneously. (2)

[12]

QUESTION 10

10.1 If $f(x) = (3x - 2)^2$ and $f'(p) = 6$, determine the values of p and q if the tangent meets f at the point $(p; q)$. (4)

10.2 The distance, in metres, that an object travels in space (in t seconds) is given as: $s(t) = t^3 - 2t^2 + t + 6$.

10.2.1 Determine the speed of the object at 4 seconds. (3)

10.2.2 Calculate the value(s) of t for which the speed will be a minimum. (3)

[10]

QUESTION 11

11.1 The table below shows the results of an aptitude test taken at a company offering an apprenticeship in engineering.

	Male	Female	Totals
Pass	42	28	70
Fail	8	12	20
Total	50	40	90

11.1.1 If a person who participated in the aptitude test is chosen at random, determine the probability that the person is a male. (2)

11.1.2 Is passing the test independent of gender? Substantiate your answer with necessary calculations. (4)

11.2 Four-digit codes (not beginning with 0) are constructed from the following set of digits: {0 ; 1 ; 3 ; 4 ; 6 ; 7 ; 8}

11.2.1 How many four-digit codes can be constructed if repetition of digits is allowed? (2)

11.2.2 How many four-digit codes can be constructed if repetition of digits is not allowed? (2)

11.2.3 Calculate the probability of randomly constructing a four-digit code between 6000 and 7000 and which is divisible by five if repetition of digits is allowed. (3)

[13]

QUESTION1

1.1 Solve for x:

1.1.1 $3x^2 - 18x = 0$ (3)

1.1.2 $7x^2 - 4x = 5$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $(x+5)(x-2) > 0$ (2)

1.1.4 $26 - 5^{2x} = (5^x - 6)^2$ (6)

1.2 Solve simultaneously for$x$ and$y$:

$x - 4y = 5$ and $3x^2 - 5xy + 2y^2 = 25$ (6)

1.3 Solve for x if: $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$ (4)

[25]

QUESTION 2

2.1 Given the following arithmetic sequence: $-11 ; -4 ; 3 ; \ldots$

Determine the:

2.1.1 General term in the form $T_n = bn + c$. (2)

2.1.2 Value of the 60$^{\text{th}}$ term. (2)

2.1.3 Sum of the first 60 terms. (2)

2.2 Hence, or otherwise, write $-4 + 3 + 10 + \ldots + 486$ in sigma notation. (4)

2.3 This arithmetic sequence $-11 ; -4 ; 3 ; \ldots$ forms the first three first differencesof a quadratic sequence. Which term in this quadratic sequence will be the smallest? Show all your calculations. (5)

[15]

QUESTION 3

Consider the geometric series: $5(3x+1) + 5(3x+1)^2 + 5(3x+1)^3 + \ldots$

3.1 For which value(s) of x will the series converge? (3)

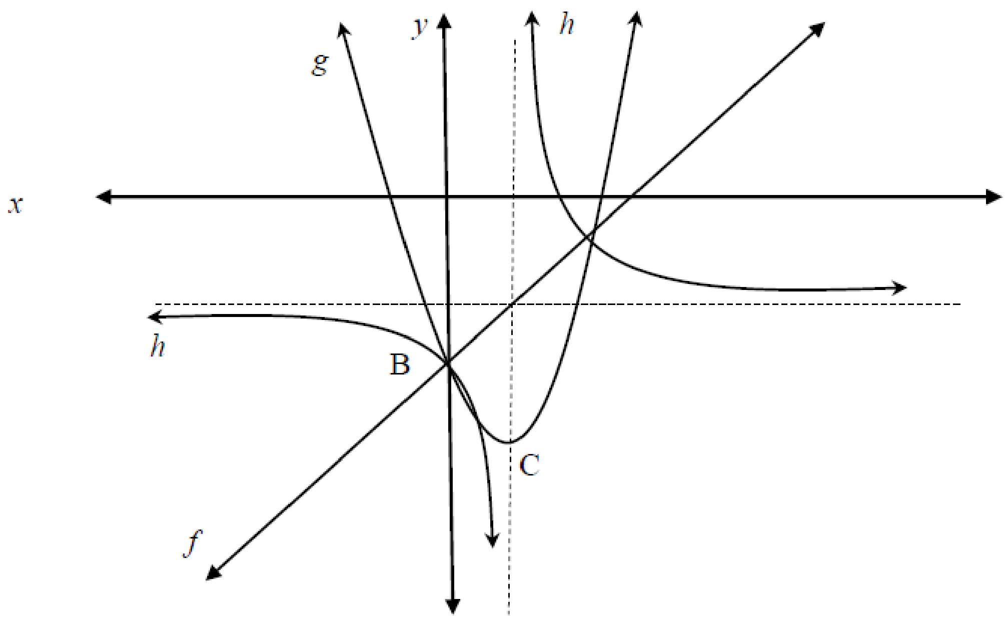
3.2 Calculate the sum to infinity of the series if $x = -\frac{1}{6}$ (4)

[7]

QUESTION 4

The graphs of $g(x) = \frac{1}{2}(x-2)^2 - 9$ and $h(x) = \dfrac{a}{x+p} + q$ are sketched below.

The axis of symmetry of graph g is the vertical asymptote of graph h. The line f is an axis of symmetry of graph h. B is the y-intercept of h, g and f.



4.1 Write down the coordinates of C, the turning point of g. (2)

4.2 Determine the coordinates of B. (2)

4.3 Write down the equation of f. (2)

4.4 Determine the equation of h. (5)

4.5 Write down the equations of the vertical and horizontal asymptotes of $k(x) = 3h(x) - 2$. (2)

4.6 Determine the x-intercept of h. (3)

4.7 For which values of x will:

4.7.1 $\dfrac{g'(x)}{h(x)} \geq 0$ (3)

4.7.2 $f^{-1}(x-1) < 2$ (4)

4.8 Calculate the value(s) of k for which $g(x) = f(x) + k$ has two unequal positive roots. (6)

[29]

QUESTION 5

5.1 Consider the function $f(x) = \left(\frac{5}{6}\right)^x$

5.1.1 Write down the equation of h, the reflection of f in the y-axis. (1)

5.1.2 Write down the equation of $f^{-1}(x)$ in the form $y = ...$ (2)

5.1.3 For which value(s) of x will $f^{-1}(x) \geq 0$? (2)

5.2 The function defined as $f(x) = ax^2 + bx + c$ has the following properties:

- $f'(-2,5) = 0$
- $f(1) = 0$
- $b^2 - 4ac > 0$
- $f(-2,5) = 6$

Draw a neat sketch graph of f. Clearly show all x-intercepts and turning point. (4)

[9]

QUESTION 6

On 1 July 2010, David bought a tractor for R2 000 000. On that day, he paid a deposit of 25% of the purchase price and the bank granted him a loan at an interest rate of 9,5% per annum, compounded quarterly, to pay off the balance of the purchase price. David agreed to pay quarterly instalments of R58 000, starting on 1 January 2011.

6.1 How much money did David borrow from the bank? (2)

6.2 How many quarterly instalmentsare required to pay off the loan? (6)

6.3 Calculate the amount owing on the loan immediately after David paid his quarterly instalment on 1 July 2016, i.e. six years after he bought the tractor. (4)

6.4 Hence, calculate the amount of interest that David had paid on this loan until immediately after paying his quarterly instalment on 1 July 2016. (4)

[16]

QUESTION 7

7.1 Given: $f(x) = -x^2 + 3x - 7$

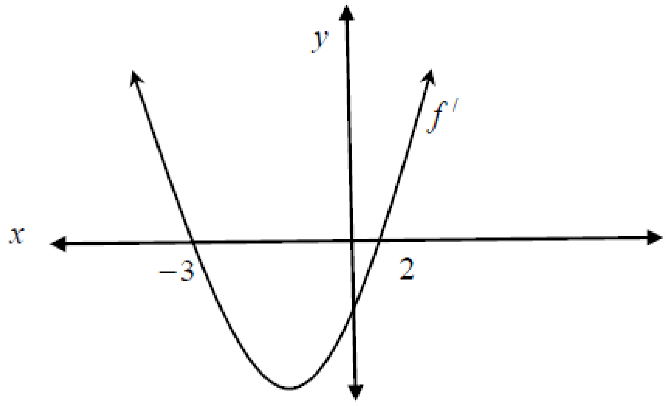
Determine $f'(x)$ from first principles. (6)

7.2 Determine: $D_x\left[15\sqrt[5]{x^4} - \frac{3x^7 + x}{4x^3}\right]$ (6)

[12]

QUESTION 8

The graph of $f'(x) = x^2 + bx + c$, where f is a cubic function, is sketched below.
The derivative function f' cuts the x-axis at $x = -3$ and $x = 2$.



8.1 For which values of x is graph f increasing? (2)

8.2 At which value of x does graph f have a local maximum value? (1)

8.3 Determine the equation of $f'(x)$. (2)

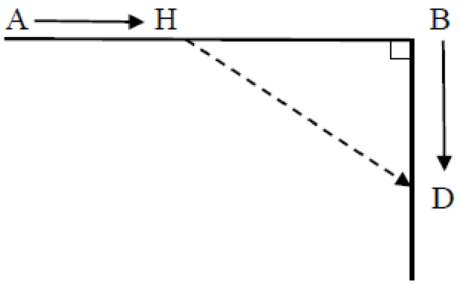
8.4 If $f(x) = px^3 + qx^2 + rx + 10$, show that $p = \frac{1}{3}$, $q = \frac{1}{2}$ and $r = -6$. (4)

8.5 For which value(s) of x is graph f concave down? (3)

[12]

QUESTION 9

A hunter was standing at point A, along the fence of a rectangular game enclosure, when he spotted a deer standing at point B, the corner of the rectangular enclosure. The distance from A to B is 1200m. At exactly the same time as the hunter started to move in an easterly direction towards B, the deer started to move in a southerly direction towards D. The hunter moves at 4metres per second and the deer moves at 5metres per second. After t seconds, the hunter is at a point H and the deer is at point D.



The hunter tries to shoot the deer but with his caliber rifle he must be at most 800m from the deer.

9.1 Show that the distance between the hunter and the deer (HD) at t seconds after they both started moving can be written as:

$$HD(t) = \sqrt{41t^2 - 9\ 600t + 1\ 440\ 000}$$ (4)

9.2 How long after they started walking, were they the nearest to one another? Show all calculations. (3)

9.3 The calibre of the hunter's rifle allows him to be at most 800m from his target. Was the hunter within shooting range of the deer at the time when they were nearest to each other? Show all calculations. (3)

[10]

QUESTION 10

The rules for the final game of the North West Hockey tournament specify that there must be a winner. In the event of a draw, the winner will be determined by a penalty-flick shootout.

On the day that the final game of the North West Hockey tournament takes place, there is a 45% chance that it could rain,a 32% chance that it could be cloudy or it could be sunny.The team from Taung, a low rainfall area, has an18% chance of winning the tournament on a rainy day, a 39% chance of winning on a cloudy day and a 63% chance of winning on a sunny day.

10.1 Draw a tree diagram to represent all outcomes of the above information. (2)

10.2 What is the probability of the Taung hockey team winning the final game of the tournament? (4)

[6]

QUESTION 11

A horse breeder has 9 single horse stables in a row next to each other. He has 4 stallions (male horses) and 5 mares (female horses), where one of the stallions is his breeding stallion and one of the mares his breeding mare. The horses are placed randomly in the stables.

11.1 In how many different ways can the 9 horses be placed in the 9 stables? (1)

11.2 In how many different ways can the 9 horses be placed if the breeder wants to place the breeding stallion and breeding mare next to each another? (2)

11.3 What is the probability that there will be a mare placed on both ends of the row stables? (3)

11.4 If 5 stables became unavailable due to renovations, in how many different ways can the breeder place his horses in the remaining single stables such that there will be at least one mare in these stables? (3)

[9]

TOTAL: 150

QUESTION 1

1.1 Solve for x:

1.1.1 $2x^2 + 3 = 8x$ (correct to TWO decimal places) (4)

1.1.2 $4x - 2x(x-3) \leq 0$ (4)

1.1.3 $2^x - 5.2^{x+1} = -144$ (4)

1.2 If $f(2) = 0$ and $f(-6) = 0$, determine an equation for $f(x)$ in the form $f(x) = x^2 + bx + c$. (2)

1.3 Solve for x and y simultaneously:

$$2x + y = 17 \quad \text{and} \quad xy = 8$$ (6)

1.4 Given: $2mx^2 = 3x - 8$ where $m \neq 0$.
Determine the value(s) of m for which the roots of the equation are non-real. (4)

[24]

QUESTION 2

Given the quadratic sequence: $-\frac{1}{2}$; 2 ; $\frac{11}{2}$; 10 ; …

2.1 Show that the nth term of this sequence can be written as $T_n = \frac{1}{2}(n^2 + 2n - 4)$. (4)

2.2 Determine the value of $T_{75} - T_{74}$. (2)

2.3 The first differences of the given sequence above forms another number sequence.

2.3.1 Is the sequence of the first differences arithmetic or geometric?
Give a reason for your answer. (2)

2.3.2 Which term in the sequence of the first differences will be equal to $\frac{151}{2}$? (1)

2.3.3 Calculate the value of the 30^{th} first difference. (2)

2.3.4 Calculate the number of terms in the quadratic sequence if the sum of the first n first differences is 2 176. (4)

[15]

QUESTION 3

3.1 The following geometric series is given:

$$2(3x-1)+2(3x-1)^2+2(3x-1)^3 \ \ldots$$

Determine the value(s) of x for which the series converges. (3)

3.2 The first two terms of a convergent geometric series are k and 6 respectively where $k \neq 0$.

The sum of the infinite series is 25.

Calculate the value(s) of k. (5)

3.3 Given the series:

$$(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+\ldots+(81\times 82)$$

Write the series in sigma notation. (It is not necessary to calculate the value of the series). (4)

[12]

QUESTION 4

4.1 How many years will it take for an investment of R3 000 to accumulate to R4 500, if it is invested at 8% p.a. compounded monthly? (4)

4.2 Bongani paid off a 20-year loan of R40 000. During the period of the loan the interest rate changed from 24% p.a. compounded monthly for the first five years to 18% p.a. compounded monthly for the remaining years.

4.2.1 Calculate the initial monthly payment before the interest rate changed. (4)

4.2.2 What is the outstanding balance of the loan after the FIRST five years? (4)

4.2.3 Determine the monthly payment after the interest rate changed. (4)

[16]

QUESTION 5

Given: $f(x) = \dfrac{3}{x-1} - 2$

5.1 Calculate the coordinates of the x-intercept of f. (2)

5.2 Calculate the coordinates of the y-intercept of f. (1)

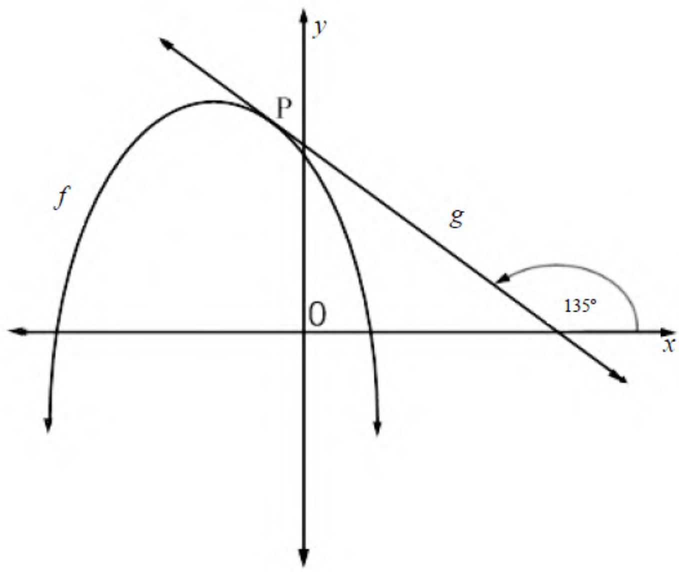
5.3 Sketch the graph of f in your ANSWER BOOK. Clearly show the asymptotes and the intercepts with the axes. (3)

5.4 ONE of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (3)

[9]

QUESTION 6

The graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$ are sketched below.
The angle of inclination of g is 135°. Graph g is a tangent to f at point P.



6.1 Calculate the coordinates of the turning point of f. (3)

6.2 Write down the range of f. (1)

6.3 Calculate the coordinates of point P, the point of contact of f and g. (4)

6.4 Determine the value(s) of k for which the straight line $y = k$ is NOT a tangent to $y = 2x^2 + 5x - 3$. (2)

[10]

QUESTION 7

Given $f(x) = a^x$, where $a > 0$, passing through the point $(2\ ;\ \frac{1}{4})$ and $g(x) = 4x^2$.

7.1 Prove that $a = \frac{1}{2}$. (2)

7.2 Determine the equation of $y = f^{-1}(x)$ in the form $y = \ldots$ (2)

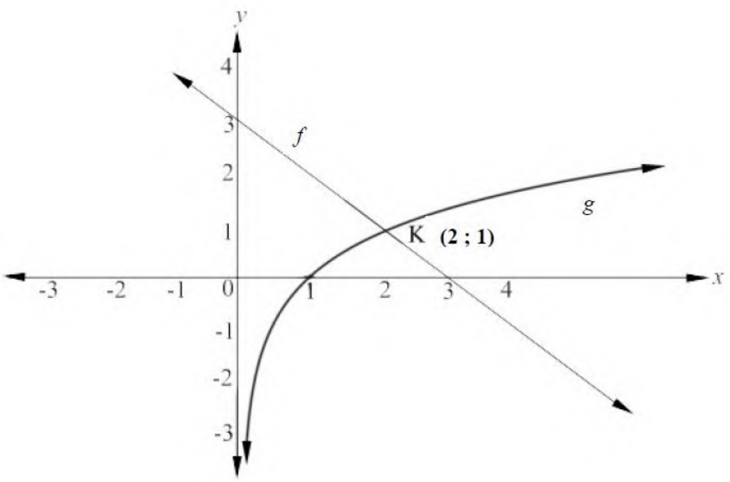
7.3 Determine the equation of $y = h(x)$ where h is the reflection of f in the x-axis. (1)

7.4 How must the domain of $g(x)$ be restricted so that $g^{-1}(x)$ will be a function? (2)

[7]

QUESTION 8

The graphs of $f(x) = -x + 3$ and $g(x) = \log_2 x$ are drawn below.
Graphs f and g intersect at point K(2 ; 1).



8.1 Write down value(s) of x for which:

8.1.1 $f(x) - g(x) > 0$ (2)

8.1.2 $g(x) \cdot g^{-1}(x) \leq 0$ (2)

8.2 8.2.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

8.2.2 Explain how you could use the given sketch to solve the equation $\log_2(3 - x) = x$. (2)

8.2.3 Write down the solution to $\log_2(3 - x) = x$. (1)

[9]

QUESTION 9

9.1 Given: $f(x) = 3x - x^2$

9.1.1 Determine $f'(x)$ from FIRST principles. (5)

9.1.2 Determine the average gradient of f between $x = 1$ and $x = 3$. (3)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = \frac{8 - 3x^6}{8x^5}$ (3)

9.2.2 $D_x\left[\sqrt[3]{x^2} + \frac{1}{x} + 2x\right]$ (4)

[15]

QUESTION 10

A cubic function has the following essential properties:

- $f(0) = 8$
- $f(4) = f(1) = 0$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

10.1 Sketch the graph of f in your ANSWER BOOK clearly indicating the turning point(s) and the points of intersection of the graph with the axes. (3)

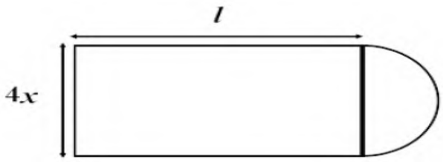
10.2 Show that the defining equation of f is $f(x) = -2x^3 + 12x^2 - 18x + 8$. (4)

10.3 Determine the value(s) of x for which graph of f is concave down. (3)

[10]

QUESTION 11

The diagram below shows a garden in the form of a rectangle and a semi-circle.



The rectangular section of the garden has the dimensions length (l) and width ($4x$).

The perimeter of the garden is $32m$.

11.1 Express the length (l) in terms of x. (3)

11.2 Show that the area of the garden can be written as $A(x) = -8x^2 - 2\pi x^2 + 64x$. (2)

11.3 Determine the value of x for which the area of the garden is minimum. (3)

[8]

QUESTION 12

12.1 For two events, A and B, it is given that:

- $P(A) = 0,3$
- $P(B) = 0,4$
- $P(A \text{ or } B) = 0,6$

Calculate $P(A \text{ and } B)$. (2)

12.2 A survey was completed among Grade 11 and Grade 12 learners at a certain school to establish the type of cell phone that each learner uses. Some of the results are shown in the table below.

	Gr. 11	**Gr. 12**	**Total**
Android	***A***	33	65
iPhone	53	***B***	101
Total	85	81	166

12.2.1 Calculate values for ***A*** and ***B*** in the table. (2)

12.2.2 If a learner from this group is selected at random, what is the probability that he/she will use an iPhone? (2)

12.2.3 All these Grade 11 and Grade 12 learners attend Mathematics lessons in a particular class at the school. At the end of the day, the Mathematics educator found an iPhone in this class.
What is the probability that the phone belongs to a Grade 12 learner? (2)

[8]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 3x - 4 = 0$ (3)

1.1.2 $2x^2 - x - 7 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5^{x+1} - 5^x = 2500$ (3)

1.1.4 $(x-3)(x+1) < 12$ (4)

1.2 Solve the following equations simultaneously:

$2x = y + 1$
$3x^2 - xy - y^2 = 1$ (6)

1.3 Given that $f(x) = x^2 - 2px + 8 + 2p$ has two equal roots and $p < 0$, determine the coordinates of the turning point of h, if $h(x) = f(x) - 3$. (5)

[24]

QUESTION 2

2.1 Given the quadratic number pattern: $3\ ;\ 1\ ;\ -3\ ; -9\ ; \ldots$

2.1.1 Write down the next 2 terms of the pattern. (1)

2.1.2 Determine T_n, the n^{th} term of the pattern, in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term of the pattern has a value of -809? (3)

2.2 Given the arithmetic sequence: $-1\ ;\ 1\ ;\ 3\ ;\ 5\ ; \ldots$

2.2.1 Determine T_{53}, the 53rd term of the sequence. (2)

2.2.2 Determine the sum of the first 29 terms of the sequence. (2)

2.2.3 Hence, write your answer in sigma notation. (2)

2.3 In an arithmetic sequence, $T_4 = 2x + y$ and $T_{10} = 8x - 2y$. Determine the first term of the sequence in terms of x and y. (5)

[19]

QUESTION 3

Given that: $p = \sum_{k=1}^{\infty}(x-1)^k$

3.1 Determine the values of x for which p converges. (2)

3.2 Calculate the value of p when $x = \frac{2}{3}$. (4)

[6]

QUESTION 4

Given: $f(x) = 1 + \frac{2}{x+3}$

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Calculate the x and y intercepts of f. (3)

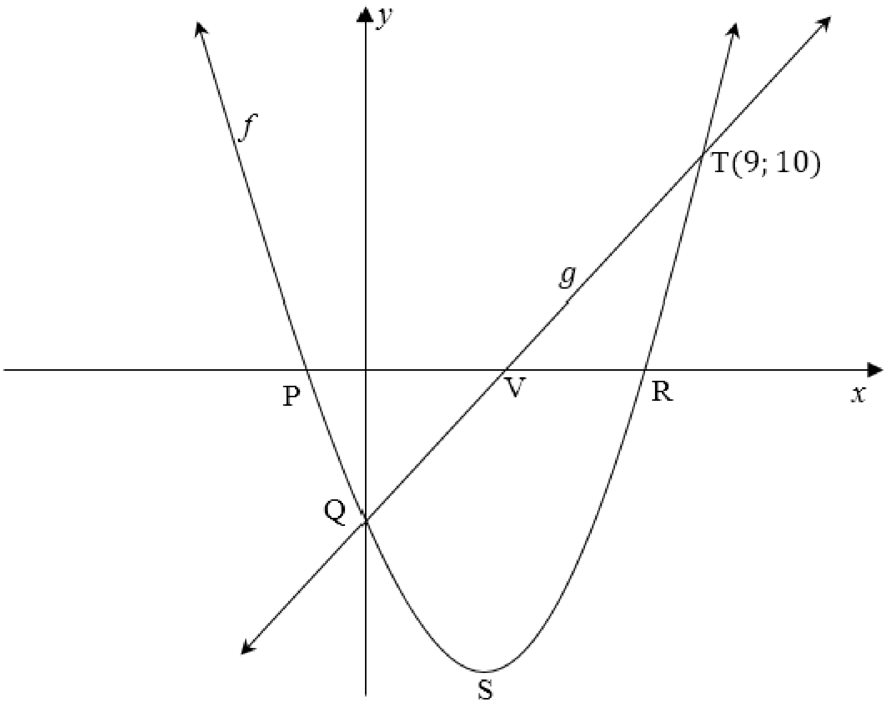
4.3 Draw a neat sketch of f, clearly indicating all intercepts with the axes and any asymptotes. (4)

4.4 Given that h is a reflection of f in the x-axis, determine the equation of the axis of symmetry of h having a positive gradient. (4)

[13]

QUESTION 5

The diagram below shows the graphs of $f(x) = x^2 - 7x - 8$ and $g(x) = mx + c$.
P and R are x-intercepts of f, and V is the x-intercept of g. S is the turning point of f.
f and g intersect on the y-axis at Q and also at T(9; 10).



5.1 Write down the coordinates of Q. (1)

5.2 Determine the equation of g. (3)

5.3 Write down the equation of f in the form $y = a(x + p)^2 + q$. (2)

5.4 Hence, or otherwise, determine the coordinates of S, the turning point of f. (2)

5.5 Determine the coordinates of a point W, on f, such that the average gradient between T and W is 1. (5)

5.6 Determine the values of x for which $f(x).g(x) < 0$. (4)

[17]

QUESTION 6

Given: $f(x) = \log_m x$

6.1 Determine the value of m, if the point $(64;3)$ lies on f. (2)

6.2 Determine the equation of f^{-1} in the form $y = \ldots$ (2)

6.3 Draw a neat sketch of f^{-1}, showing all intercepts with the axes. Indicate at least one other point on your graph. (2)

6.4 Write down the range of h if: $h(x) = f^{-1}(x) - 2$ (1)

[7]

QUESTION 7

7.1 Kamva bought a motorbike valued at R40 000. After 5 years the value of the motorbike had depreciated to R26 700, at a rate of r % p.a. on the reducing balance method, compounded annually. Calculate r, the rate of depreciation. (3)

7.2 A bank granted Nathan a loan for R1 200 000 to buy a house. He agreed to repay the loan over a period of 15 years at an interest rate of 11,5% p.a. compounded monthly. He made his first payment at the end of the first month after the loan was granted.

7.2.1 Calculate Nathan's monthly instalment. (3)

7.2.2 Due to unforeseen circumstances, Nathan could not pay his 76^{th}, 77^{th}, 78^{th}, 79^{th} and 80^{th} instalments. He resumed his payments at the end of the 81^{st} month.

(a) Calculate the outstanding balance at the end of the 80^{th} month. (5)

(b) If Nathan continues paying the same monthly instalment, how many months will it take him to pay the balance outstanding at the end of the 80^{th} month? (4)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3 - 2x^2$ (5)

8.2 Determine:

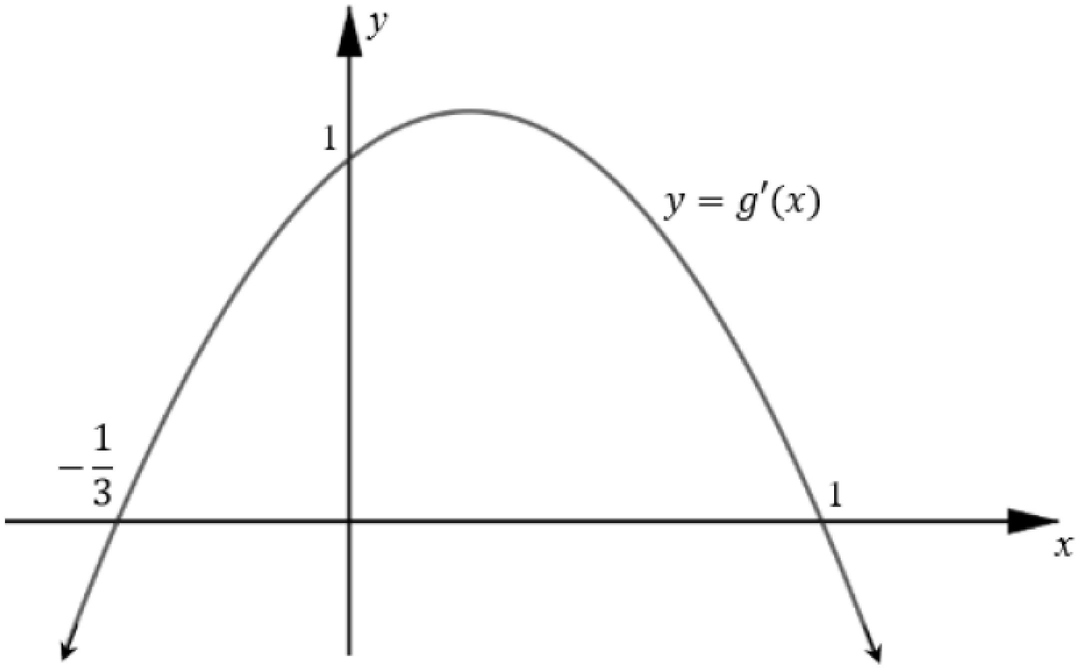
8.2.1 $D_x\left[x(x-2)^2\right]$ (4)

8.2.2 $\dfrac{dy}{dx}$ if $y = ax^{\frac{3}{7}} - \dfrac{2x}{\sqrt{x}} + 3$ (3)

[12]

QUESTION 9

The diagram below shows the graph of $y = g'(x)$ where $g(x) = ax^3 + bx^2 + cx + d$.
The graph $g'(x)$ cuts the y-axis at $(0; 1)$ and the x-axis at $(-\frac{1}{3}; 0)$ and $(1; 0)$.



9.1 Write down the x-coordinate(s) of the stationary point(s) of g. (2)

9.2 Determine the x-coordinate of the point of inflection of g. (2)

9.3 Determine the values of x for which g is an increasing function. (2)

9.4 Determine the equation of $g'(x)$ in the form: $g'(x) = px^2 + qx + r$. (4)

9.5 Given that:

- $g(x) + 1$ passes through $(0; 0)$ and
- $g'(x) = -3x^2 + 2x + 1$

Show that for $g(x)$, $a = -1, b = 1, c = 1$ and $d = -1$. (5)

[15]

QUESTION 10

Two numbers are such that their sum is 18. One of the numbers is multiplied by the square of the other. Calculate the numbers that make this product a maximum.

[7]

QUESTION 11

11.1 A school has 530 learners. Each learner is expected to choose his/her summer extra-curricular activity from the following:

- Athletics
- Cricket
- Tennis

The choices for 2019 were recorded in the following partially completed table:

	Athletics	Cricket	Tennis	**Total**
Girls	120	a	57	288
Boys	b	108	28	242
Total	226	219	85	530

11.1.1 Determine the values of a and b. (2)

11.1.2 A learner is chosen at random. Determine the probability that:

(a) It is a boy who plays cricket (2)

(b) It is a girl or **not** a tennis player (3)

11.2 Consider the letters of the word: NUMERATOR.

11.2.1 How many 9 letter word-arrangements can be formed, if repetition of letters is allowed? (1)

11.2.2 How many 9 letter word-arrangements can be formed, if all 4 vowels are never together and repetition of letters is not allowed? (3)

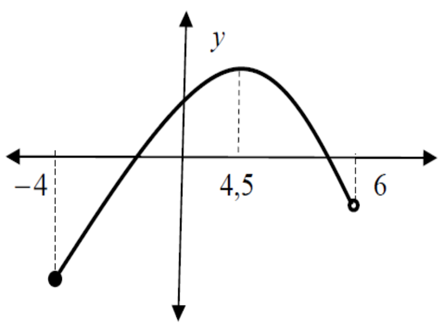
11.2.3 An 8 letter word-arrangement is made from the word NUMERATOR. All the vowels must be included in this word-arrangement and repetition of letters is not allowed. What is the probability that all odd-number spaces are occupied by vowels? (4)

[15]

TOTAL: 150

ANSWERS

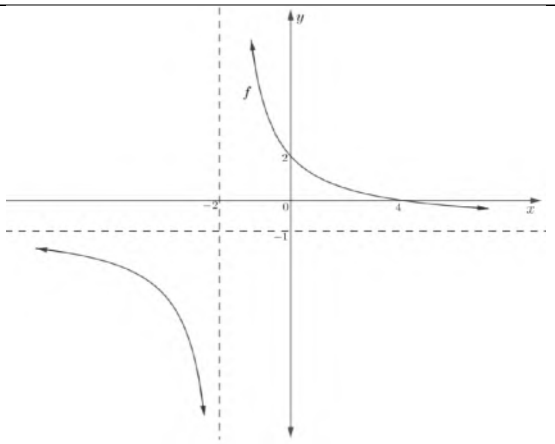
NORTHWEST PROVINCE 2021 PAPER 1

1.1.1	$x = 1{,}08\ or\ x = -0{,}31$
1.1.2	$x < 0\ or\ x > 2$
1.1.3	$x \neq 4\ \ or\ x = -1$
1.2.1	$x = 3$
1.2.2	$x \epsilon\ R$
1.3	$x = \frac{1}{5}; y = \frac{2}{5}$ or $x = -1; y = 1$
1.4	k = -13
2.1.1	3.25 hrs
2.1.2	21^{st} Day
2.1.3	127.5 hrs
2.2	Proof
3.1.1	$\frac{1}{4}$
3.1.2	128
3.2.1	Proof
3.2.2	$T_n = 3n^2 + 5n - 6$
3.3	$\sum_5^{17} (3r + 2)p^{r+3}k^{20-r}$
4.1	A(- 3; 5)
4.2	B (0 ; 5)
4.3	$x = -0{,}76\ or\ x = -5{,}24$
4.4	$g(x) = \frac{2}{x+3} - 4$
4.5	$y = x - 1$
4.6	(-1,59 ; - 2,59)
4.7	$y = - x - 4.18$
4.8	$-5.24 \leq x \leq -3\ or -0{,}76 \leq x$
5.1	x =4,5
5.2	$-1 < x \leq 5{,}67$
5.3	$y = \log_{\frac{1}{2}} x$
5.4	(1;0)
5.5	$3 < x \leq 5{,}67$
5.6	

6.1	R 3912,89
6.2	702 months
6.3	R11 322,72
7.1	$- 2x + 7$
7.2	$-8x^{-3} + 15x^4$
7.3	$2x - 2$
8.1	Proof
8.2	$x \neq 3\ or\ x = -\frac{1}{3}$
8.3	$x > \frac{4}{3}$
8.4	$x = \frac{2}{3}\ or\ x = 2$
9.1	$h = 8 - 2r$
9.2	$8\pi r^2 - 2\pi r^3$
9.3	$\frac{8}{3}$
10.1	$\approx 0{,}57$
10.2	$\approx 0{,}28$
10.3	$\approx 0{,}95$
11.1	9999
11.2	0,13

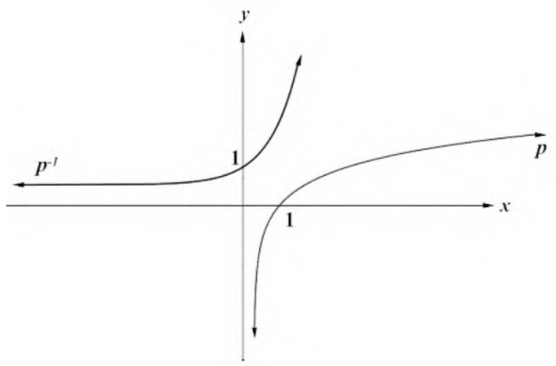
1.1.1	$x = 0{,}91 \text{ or } x = -2{,}57$
1.1.2	$x = -\frac{1}{2} \text{ or } x = 5$
1.1.3	$x < 1 \text{ or } x > 4$
1.1.4	$x \neq 1 \text{ or } x = 16$
1.2	$x = 1$
1.3	$x = -7; y = 6$
2.1.1	37
2.1.2	$T_n = n^2 + 3n - 3$
2.1.3	595
2.2.1	$T_{191} = 0$
2.2.2	310
2.3	$-\frac{3}{2} < x < \frac{7}{2}$
3.1	$n > 5 \text{ or } n \geq 6$
3.2	32
4.1	$x = -2 \quad y = -1$
4.2.1	$y = 2$
4.2.2	$x = 4$

4.3

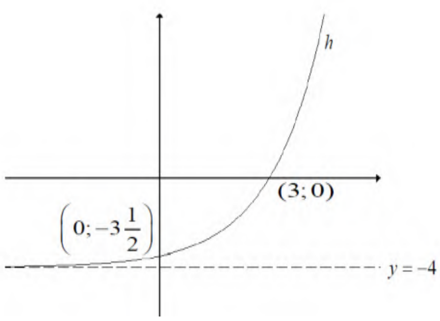


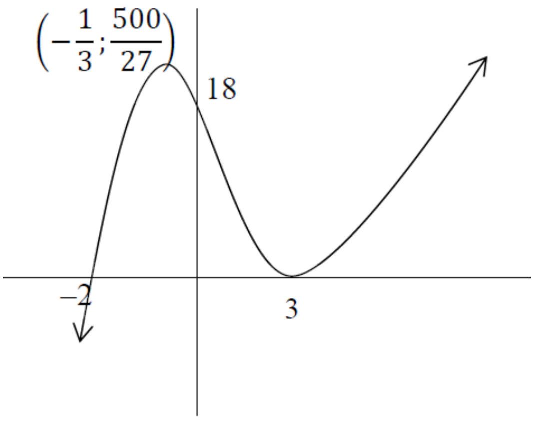
4.4	$y = -x - 3$
5.1	$y = 3^x$

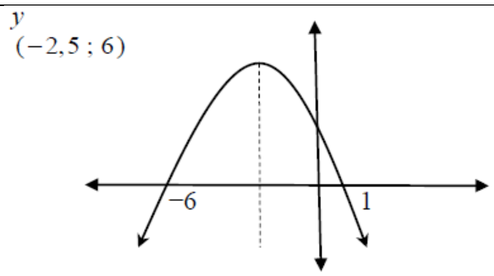
5.2

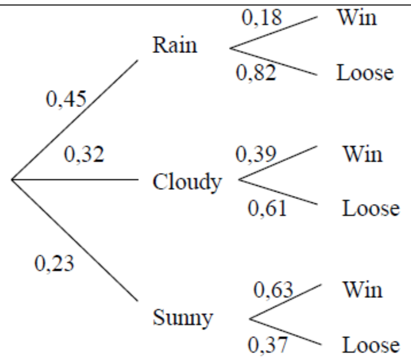


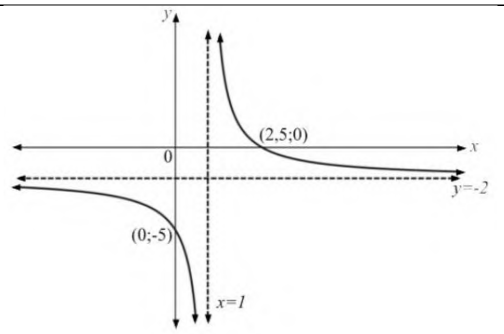
5.3	$0 < x \leq 9$
5.4	*x-intercept* of *p* is (1;0) *x-intercept* of *h* is (-1;0)
6.1	$y = \frac{1}{2}x^2 - 2x + 5$
6.2	$\Delta < 0$
6.3	$k > 3$
6.4	$y = \frac{1}{2}x^2 - 2x$
7.1	11,30%
7.2.1	R1 674 501.44
7.2.2	75 months
7.2.3	P_v = R1 319 260,60 …..No
8.1	$-4x + 6$
8.2.1	$4x + 2x^3$
8.2.2	$2x - 1$
8.3	$y = 5x - 5$
9.1	(- 2; - 2) and (2, 2)
9.2.1	$y = x^3 - 9x^2 + 24x - 16$
9.2.2	B (2; 4)
9.2.3	$k < -16$
9.2.4	$x > 3$
10.1	P [$6 - x$; $(6 - x)^2$]
10.2	32
11.1	$y = 0{,}3$
11.2	$\frac{2}{3}$
11.3.1	9000
11.3.2	4536
11.3.3	4536

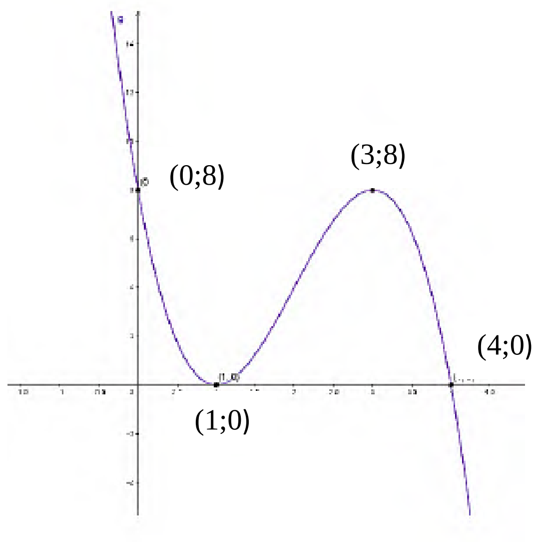
1.1.1	3 or - 2
1.1.2	1,3 or – 2, 3
1.1.3	$-1 \leq x \leq 4$
1.2	$x = 16; y = -8$ or $x = 12; y = -4$
1.3	$m^2 + 4 > 0$
1.4	39
2.1.1	$-141;\ -148$
2.1.2	$-7n - 106$
2.1.3	-14225
2.2.1	2
2.2.2	$n^2 - n + 41$
2.2.3	1681 has more than two factors
3.1	$n = 15$
3.2.1	$-\frac{3}{2} \leq x \leq \frac{3}{2}$
3.2.2	27
3.3	$k = 28$
4.1	$x \in R\ x \neq -3$
4.2	$p = 3\ and\ q = 2$
4.3	$y = \frac{-6}{x+3} + 2$
4.4	$y = x + 7$
5.1	(0;8)
5.2	D (1; 9)
5.3	$C\ (4{:}\ 0)\ by\ symmetry$
5.4	$y = -2x + 8$
5.5	$k = 12$
5.6	$0 < x < 4 \quad x \neq 2$
5.7	$y = 27$
6.1	$\left(0;\ -3\frac{1}{2}\right)$
6.2	(3;0)
6.3	
6.4	Translation 4 units to the left and reflection in x - axis
7.1	7,21 years
7.2.1	R159 927,29

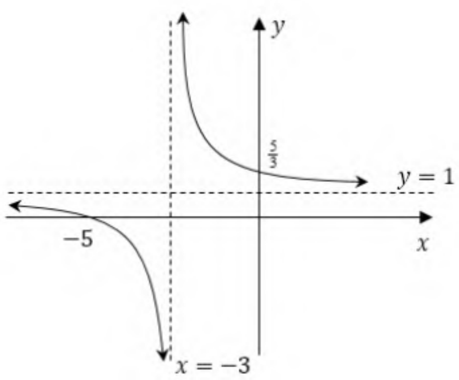
7.2.2	12,68% pa compounded monthly
7.3.1	R22 383,37
7.3.2	R23 450,73
8.1	- $2x$
8.2.1	$15x^2 - \frac{1}{4}x$
8.2.2	$-21x^{-4} + 12x^{-5} - 5x^{-6}$
8.2.3	$\frac{1}{8p^3}$
9.1	t = - 4
9.2.1	-2 or -3
9.2.2	$(3; 0)\ \left(-\frac{1}{3}; 18\frac{14}{27}\right)$
9.2.3	
9.2.4	$x \in \left(-2; -\frac{1}{3}\right) \cup (3; \infty)$
10.1	$p = 1\ and\ q = 1$
10.2.1	33m.s^{-1}
10.2.2	$t = \frac{2}{3} sec$
11.1.1	$\frac{5}{9}$
11.1.2	Not independent
11.2.1	2085
11.2.2	720
11.3	$\frac{8}{343}$

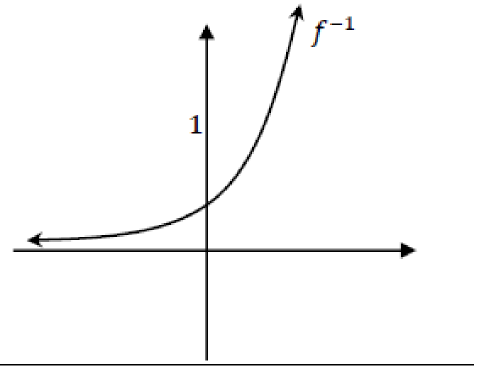
1.1.1	0 or 6
1.1.2	1,18 or – 0,61
1.1.3	x < -5 or x > 2
1.1.4	No Real Solution
1.2	$x=\frac{7}{3}; y=-\frac{2}{3}$ or $x=-5; y=-\frac{5}{2}$
1.3	$x=4 \; or \; x\neq -3$
1.4	$x=\frac{9}{4}$
2.1.1	$7n-18$
2.1.2	402
2.1.3	11 730
2.2	$\sum_{n=2}^{72}(7n-18)$
2.3	Term 3 is the smallest term
3.1	$-\frac{2}{3}<x<0$
3.2	5
4.1	C (2 ; 9)
4.2	B (0 ; -7)
4.3	$f(x)=x-7$
4.4	$y=\frac{4}{x-2}-5$
4.5	x =2 *and* y = -17
4.6	x =2,8
4.7.1	x < 2,8 $x\neq 2$
4.7.2	x < - 4
4.8	$-4\frac{1}{2}<k<0$
5.1.1	$h(x)=\left(\frac{6}{5}\right)^x$
5.1.2	$y=\log_{\frac{5}{6}} x \quad x>0$
5.1.3	$0<x\leq 1$
5.2	

6.1	R 1 500 000
6.2	n = 42,22 therefore 43 quarterly payments
6.3	R 886 790,15
6.4	R 720 790,15
7.1	- 2x + 3
7.2	$12x^{\frac{-1}{5}}-3x^3+\frac{1}{2}x^{-3}$
8.1	$x\leq -3 \; or \; x\geq 2$
8.2	x = - 3
8.3	x^2+x-6
8.4	$p=\frac{1}{3} \; or \; q=\frac{1}{2}$
8.5	$x<-\frac{1}{2}$
9.1	$HD=\sqrt{41t^2-9600t+1440000}$
9.2	t = 117,07 sec
9.3	937,04m No, the Hunter is outside the shooting range.
10.1	
10.2	35,07%
11.1	362 880
11.2	80 640
11.3	$\frac{5}{18}$
11.4	3000 ways

1.1.1	3,58 or 0,42
1.1.2	$x \leq 0$ or $x \geq 5$
1.1.3	$x = 4$
1.2	$f(x) = x^2 + 4x - 12$
1.3	$x = \frac{1}{2}; y = 16$ or $x = 8; y = 1$
1.4	$m > \frac{9}{64}$
2.1	$\frac{1}{2}(n^2 + 2n - 4)$
2.2	$\frac{151}{2}$
2.3.1	Arithmetic . Constant diff betw terms
2.3.2	74^{th}
2.3.3	$\frac{63}{2}$
2.3.4	$n = 65$
3.1	$0 < x < \frac{2}{3}$
3.2	$k = 10\ or\ k = 15$
3.3	$\sum_{n=1}^{21} 16n^2 - 20n + 6$
4.1	$n = 61{,}02\ months\ accept\ 62$ $n = 5{,}09\ years\ accept\ 5{,}17$
4.2.1	$x =$ R806,96
4.2.2	R 39 205,67
4.2.3	R631,38
5.1	$\left(\frac{5}{2}; 0\right)$
5.2	$(0; -5)$
5.3	
5.4	$y = -2x + 8$
6.1	$\left(-\frac{5}{4}; \frac{49}{8}\right)$
6.2	$y \leq \frac{49}{8}$
6.3	$(-1; 6)$
6.4	$k < -\frac{49}{8}\ OR\ k > -\frac{49}{8}$

7.1	$a = \frac{1}{2}$
7.2	$y = \log_{\frac{1}{2}} x$
7.3	$h(x) = -\left(\frac{1}{2}\right)^x$
7.4	$x \leq 0$ OR $x > 0$ OR $x < 0$ OR $x \geq 0$
8.1.1	$x < 2$
8.1.2	$0 < x \leq 1$
8.2.1	$g^{-1}(x) = 2^x$
8.2.2	Point of intersection of g^{-1} and f
8.2.3	$x = 1$
9.1.1	$3 - 2x$
9.1.2	- 1
9.2.1	$-5x^{-6} - \frac{3}{8}$
9.2.2	$\frac{2}{3}x^{-\frac{1}{3}} - x^{-2} + 2$
10.1	
	
10.2	$-2x^3 + 12x^2 - 18x + 8$
10.3	$x > 2$
11.1	$l = 16 - \pi x - 2x$
11.2	$-8x^2 - 2\pi x^2 + 64x$
11.3	$x = 2{,}24$m
12.1	P(A and B) = 0,1
12.2.1	A = 32 B = 48
12.2.2	P (iPhone) = $\frac{101}{166}$
12.2.3	P (iPhone/Gr12) = $\frac{48}{101}$
13.1	362 880
13.2	1728
13.3	Two ways

1.1.1	$x = -1$ or $x = 4$
1.1.2	$x = 2{,}14$ or $x = -1{,}64$
1.1.3	$x = 4$
1.1.4	$-3 < x < 5$
1.2	$x = \frac{2}{3}; y = \frac{1}{3}$ or $x = 1; y = 1$
1.3	$TP\ (-2; -3)$
2.1.1	$-17\ and -27$
2.1.2	$-n^2 + n + 3$
2.1.3	$n = 29$
2.2.1	103
2.2.2	783
2.2.3	$\sum_{n=1}^{29}(2n-3) = 783$
2.3	$a = \frac{5}{2}y - x$
3.1	$0 < x < 2$
3.2	$\infty = -\frac{1}{4}$
4.1	$x = -3\ y = 1$
4.2	$x = -3\ ;\ y - \frac{1}{4}$
4.3	
4.4	$y = x + 2$
5.1	$(0; -8)$
5.2	$y = 2x - 8$
5.3	$y = \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$
5.4	$\left(\frac{7}{2}; -\frac{81}{4}\right)$
5.5	$W(-1; 0)$
5.6	$x < -1\ or\ 4 < x < 8$
6.1	$m = 4$
6.2	$y = 4^x$

6.3	
6.4	$y > -2$
7.1	$r = 7{,}77$ %p.a.
7.2.1	R14 018,28
7.2.2a	R970 637,89
7.2.2b	$n = 114{,}213$months or 115 payments
8.1	$-4x$
8.2.1	$3x^2 - 8x + 4$
8.2.2	$\frac{3}{7}ax^{\frac{-4}{7}} - x^{\frac{-1}{2}}$
9.1	$x = -\frac{1}{3}\ and\ x = 1$
9.2	$x = \frac{1}{3}$
9.3	$-\frac{1}{3} < x < 1$
9.4	$g'(x) = -3x^2 + 2x + 1$
9.5	$d = -1$
10.1	*The two numbers are* 12 *and* 6
11.1.1	a = 111 and b = 106
11.1.2a	$\frac{108}{530} = \frac{54}{265}$
11.1.2b	$\frac{502}{530} = \frac{251}{265}$
11.2.1	9^9 or 387 420 489
11.2.2	$9! - (6! \times 4!) = 345\ 600$
11.2.3	$\frac{1}{126}$

NORTH WEST PROVINCE SEPTEMBER 2020 Paper 2

QUESTION 1

During the 2019 Rugby World Cup the South African Springbok players played seven matches. The score obtained in each match by the Springbok players are given below:

During the 2019 Rugby World Cup the South African Springbok players played seven matches. The score obtained in each match by the Springbok players are given below:

k	19	$2k$	32	49	57	66

The average score obtained by the Springbok players during the seven matches they played in the 2019 Rugby World Cup, was 37,43.

1.1 Show that $k = 13$. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 The two highest scores lie outside p standard deviation of the mean. Calculate the maximum value of p. (3)

1.4 Suppose EACH score is increased by adding a value of y to each score.

1.4.1 Write down the interquartile range of the new data set. (1)

1.4.2 Calculate the value of y if the lower quartile of the new data set is 30. (1)

[9]

QUESTION 2

A town has two popular municipal swimming pools, namely the Madiba swimming pool and the Cronje swimming pool. The table below indicates the number of people that visited the **Madiba** swimming pool over a period of eight days. The corresponding maximum daily temperatures (in °C) for each day is also given in the table.

Maximum daily temperature (°C)	18	24	22	28	34	30	26	20
Number of visitors	180	210	200	350	410	300	280	195

2.1 Determine the equation of the least squares regression line of the data. (3)

2.2 Calculate the value of the correlation coefficient. (1)

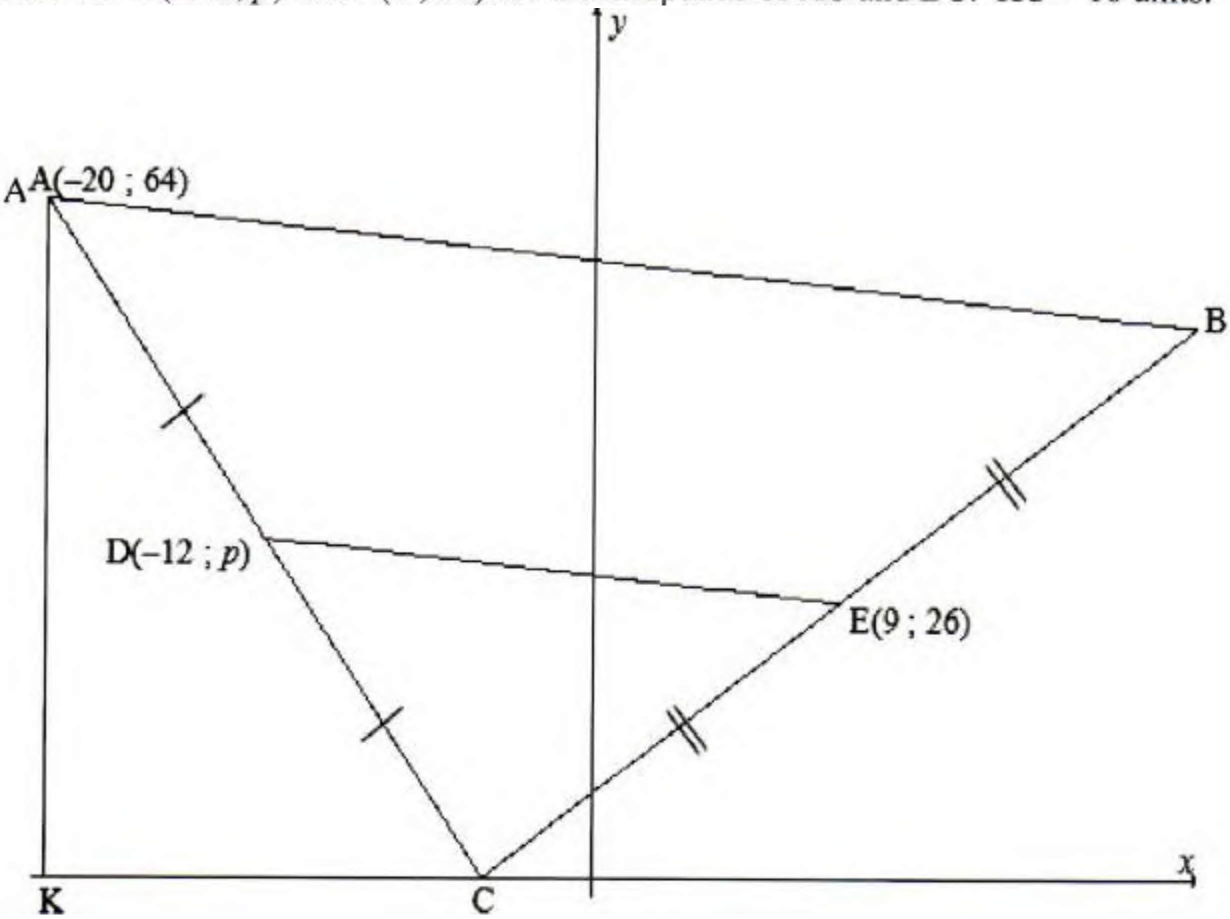
2.3 The manager of the **Madiba** swimming pool asks a grade 12 Mathematics learner, Jan-Breet, to predict how many visitors can be expected on a certain day with a maximum daily temperature of 32 °C. Jan-Breet predicts that 365 visitors can be expected on that day. Should the manager take this prediction seriously and prepare for 365 vistors? Motivate your answer. (3)

2.4 Over the very same period of eight days, with the exact same daily maximum temperatures, the **Cronje** swimming pool also record the number of visitors for each day. The equation for the least squares regression line for the **Cronje** swimming pool is given as $\hat{y} = k + 12{,}85x$.
Which swimming pool had the highest number of visitors over the period of eight days? Motivate, **without any calculations**, your answer. (3)

[10]

QUESTION 3

In the diagram below, A(–20 ; 64), B and C are the vertices of Δ ABC, with C a point on the *x*-axis. D(–12 ; *p*) and E(9 ; 26) are the midpoints of AC and BC. KC = 16 units.



3.1 The gradient of AK is undefined. Write down the equation of the line AK. (1)

3.2 Calculate the x-intercept of the line AC. (1)

3.3 Calculate the length of AC. (2)

3.4 Show that $p = 32$. (1)

3.5 Determine the equation of the line AB. (5)

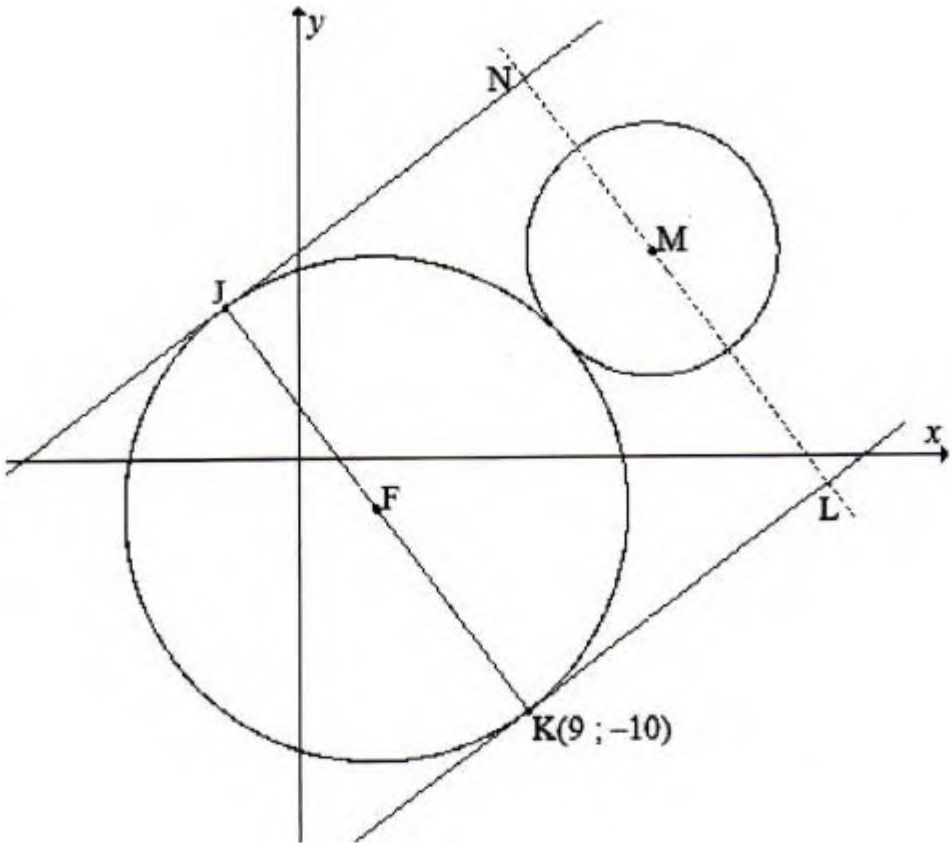
3.6 Calculate the size of $K\hat{A}B$ (5)

3.7 If $K\hat{A}C = 38,67°$, determine the area of ΔABC. (6)

[21]

QUESTION 4

In the diagram below, the equation of the circle with centre F is $x^2 + y^2 - 6x + 4y = 87$.
The equation of the circle with centre M is $(x - 14)^2 + (y - h)^2 = 25$.
The point K(9 ; –10) lies on the circle with centre F. JN and KL are tangents to the circle F at the points J and K. The two circles touch externally.



4.1 Write down the length of the radius of the circle with centre M. (1)

4.2 Determine the coordinates of F. (3)

4.3 Write down the length of the radius of the circle with centre F. (2)

4.4 Show, with calculations, that $h = \sqrt{104} - 2$. (4)

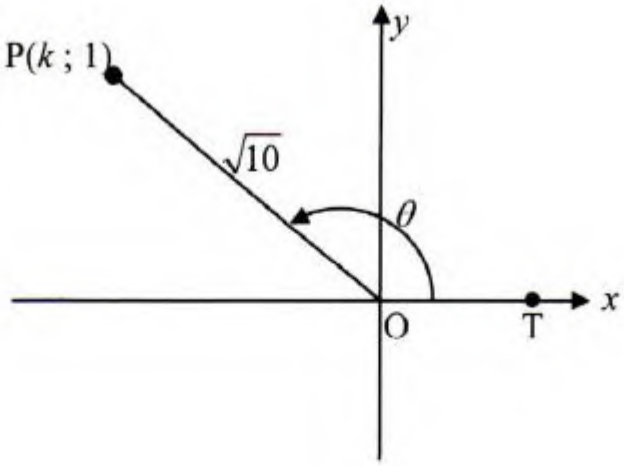
4.5 Show that the equation of the tangent JN is $y = \frac{3}{4}x + \frac{33}{4}$, if J(–3 ; 6) is given. (3)

4.6 A line NL, passes through the centre M, such that NL || JK. The circle with centre M, is translated alongside the line NL and between the two tangents of circle F, for the interval $x \in [p;q]$. Give the minimum value of p, such that the circle M will not intersect the tangents to the circle with centre F, when translated alongside NL. (6)

[19]

QUESTION 5

5.1 In the diagram, $P(k\,;1)$ is a point in the 2nd quadrant and is $\sqrt{10}$ units from the origin. T is a point on the positive x-axis and obtuse $\hat{POT} = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $-\cos\theta$ (1)

(b) $\sqrt{1-\sin(180°+\theta).\sin(360°-\theta)}$ (4)

5.1.3 Show, **without using a calculator**, that

$$\sin\left(\frac{\theta}{2}-15°\right).\cos\left(\frac{\theta}{2}-15°\right)=\frac{\sqrt{3}+3}{4\sqrt{10}}$$ (5)

5.2 Given: $\sin(\theta+60°)=d$

5.2.1 Write down the values of d such that $\sin(\theta+60°)$ is defined. (2)

5.2.2 If $d=\sin\theta.\cos 60°+\sqrt{\frac{3}{8}}$, the general solution of $\sin(\theta+60°)=d$, is $\theta=\pm m°+n.360°$. Calculate, **without the use of a calculator**, the value of m. (5)

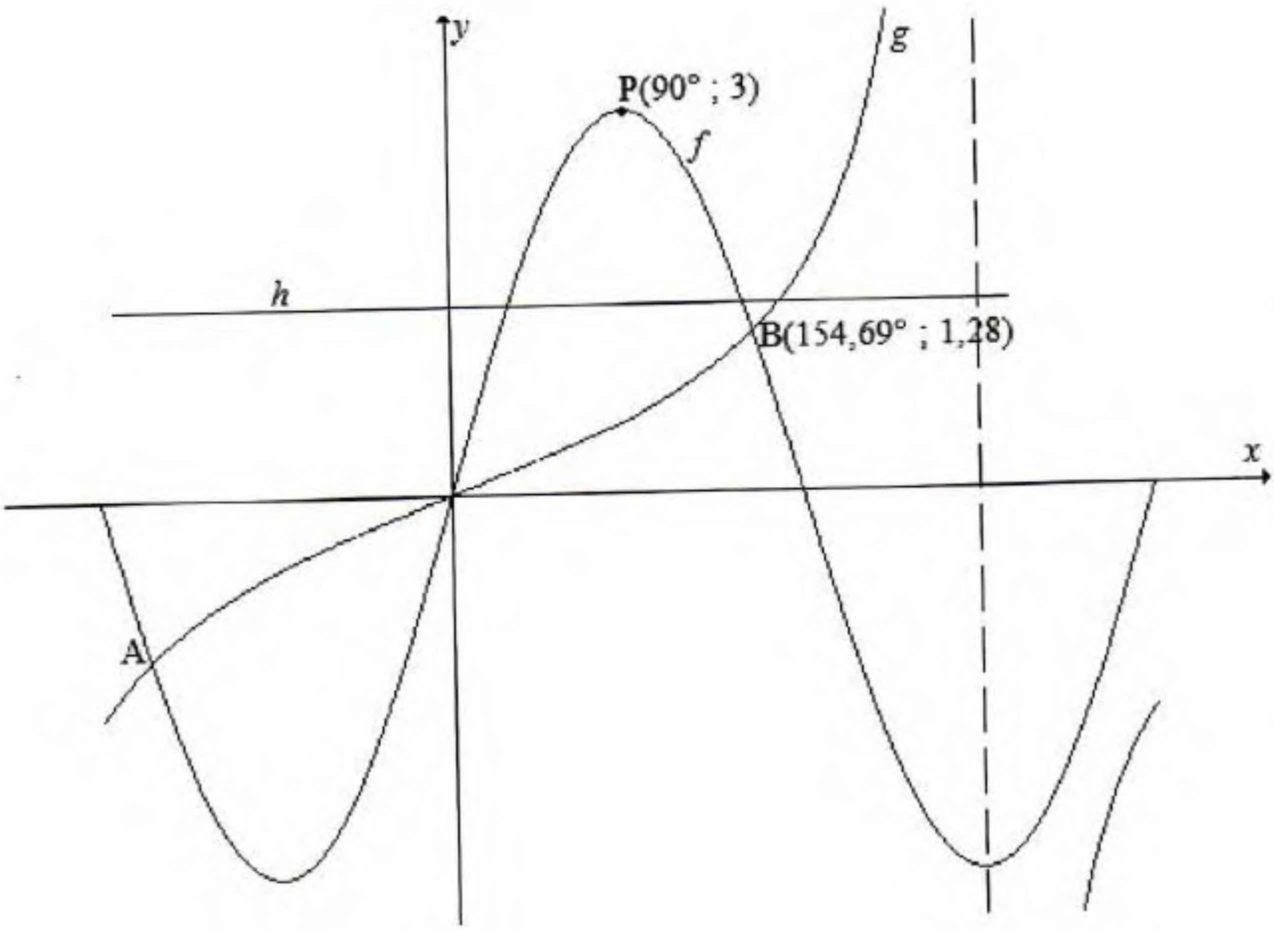
5.3 It is given that $A=2^{\sin(a-m).\sin(b-m).\sin(c-m)...\sin(x-m)}$

Calculate the value of A. (3)

[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-180° ; 360°]$. The point P(90° ; 3) is on the graph of *f*. The function $h(x) = \frac{3}{2}$ intersects the graphs of *f* and *g*. The asymptote of *g* goes through one of the turning points of *f*. A and B(154,69° ; 1,28) are points of intersections of the graphs *f* and *g*.



6.1 Write down the values of *a* and *b*. (2)

6.2 Write down the period of *g*. (1)

6.3 Write down the coordinates of A. (2)

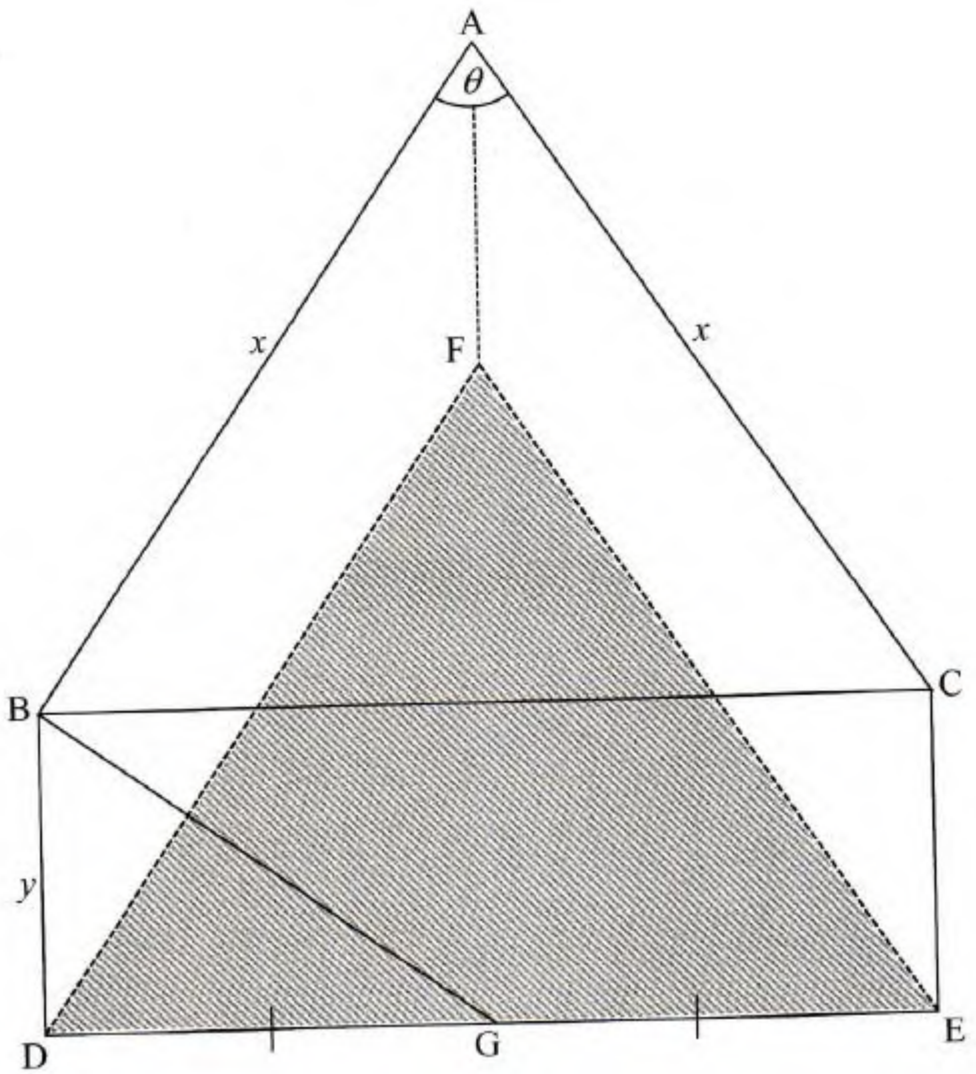
6.4 It is given that $2\tan 56,31° - 6\sin 168,93° = t + 5$. Calculate in terms of *t*, the vertical distance between *f* and *g*, at the point where *h* intersects *g*. (4)

[9]

QUESTION 7

In the diagram below, steel poles AF, BD and CE are equal in length and vertical to the horizontal plane FDE. The steel poles hold ΔABC that forms the roof of a storage area. G is the midpoint of DE. BCED is in the same vertical plane.

AB = AC = x, BD = y and $B\hat{A}C = \theta$.



7.1 Show that $BC = x\sqrt{2(1-\cos\theta)}$ (3)

7.2 Write down the size of $B\hat{D}G$. (1)

7.3 Show that $BG = \sqrt{\dfrac{2y^2 + x^2(1-\cos\theta)}{2}}$ (3)

7.4 It is given that $y = \dfrac{8}{3}$ metres and $x = \dfrac{15}{2}$ metres.

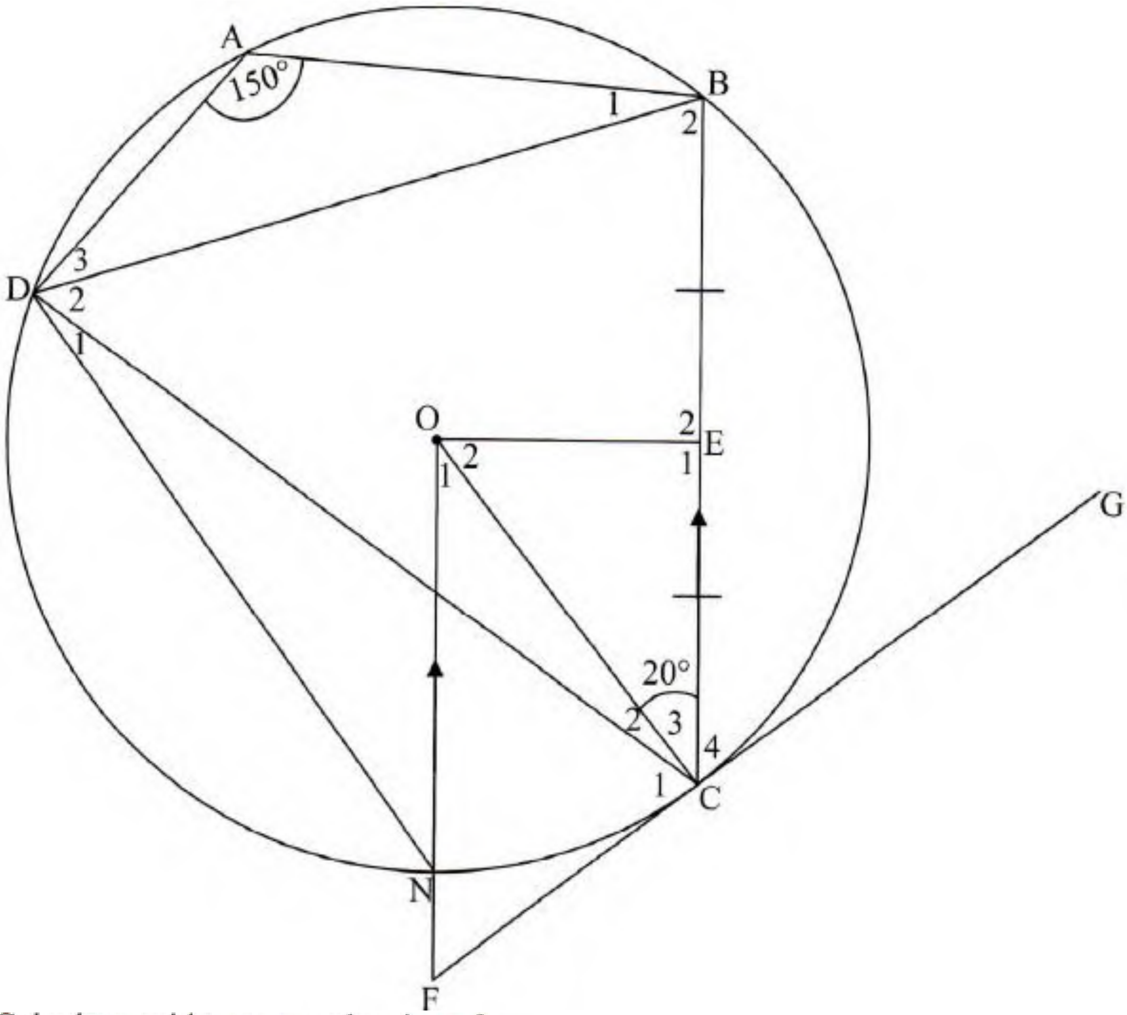
Calculate the maximum length of BG. (3)

[10]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

In the diagram, O is the centre of the circle with radius OC. ABCD is a cyclic quadrilateral. N is another point on the circle such that ON || BC. Chord DN is drawn. OE bisects chord BC at E. GC is a tangent to the circle at C. ON and GC are produced to meet in F. $O\hat{C}E = 20^\circ$ and $D\hat{A}B = 150^\circ$.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{O}_1$ (1)

8.1.2 $\hat{D}_1$ (2)

8.1.3 $\hat{C}_2$ (2)

8.1.4 $\hat{D}_2$ (4)

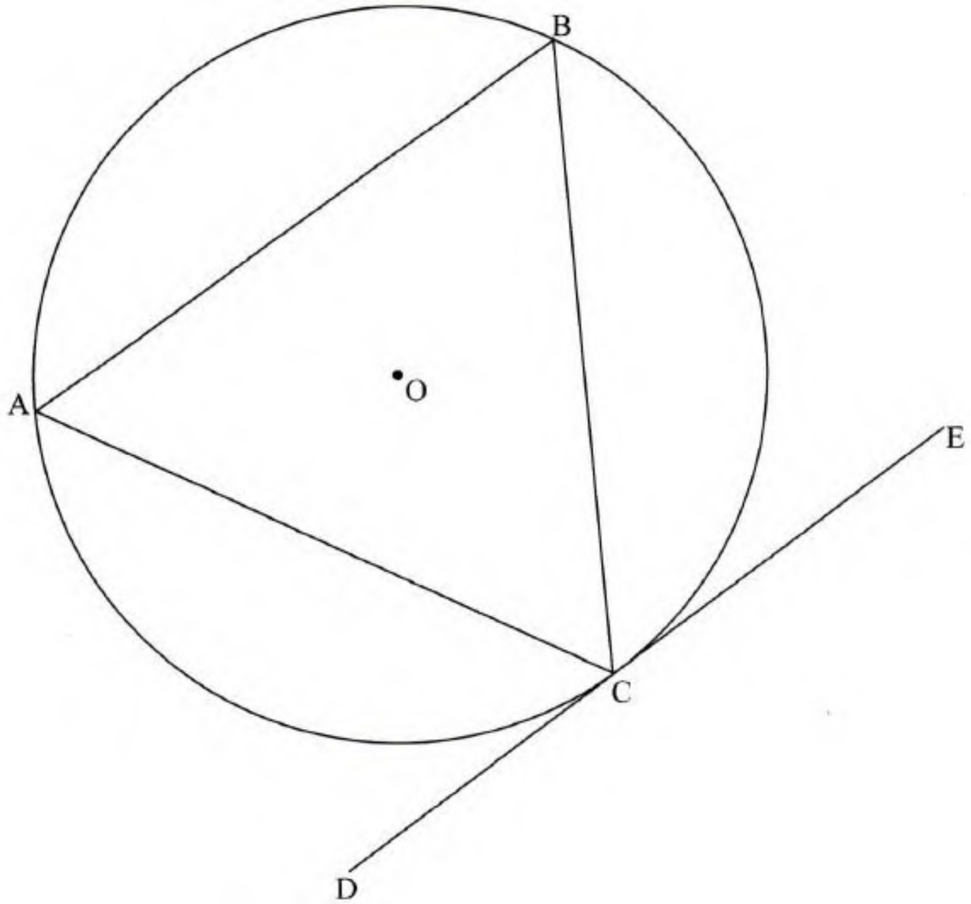
8.2 It is given that $OF = \sqrt{3}y$ and $EC = 2x$. The area of OECF is $\frac{3}{4}y^2 - x^2$.

Determine the length of OE in terms of x and y. (5)

[14]

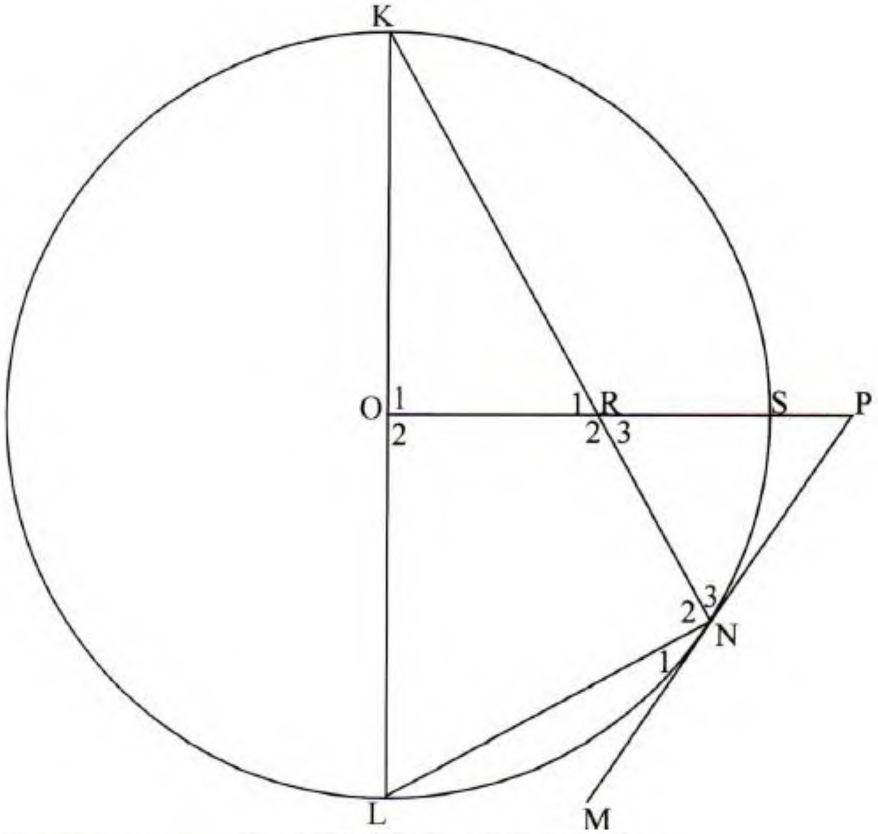
QUESTION 9

9.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is the tangent to the circle at C.



Prove the theorem which states that $A\hat{C}D = A\hat{B}C$ (5)

9.2 In the diagram below, the centre of the circle is O with the diameter KL. MNP is a tangent to the circle at N. Chords KN and LN are drawn. R is a point on KN and ORSP forms a straight line. OP = 15 units.



9.2.1 Give the length of OR if OR : RP = 2 : 3. (1)

9.2.2 It is given that $KR^2 - KO^2 = 36$. Prove that $OR \perp KL$. (2)

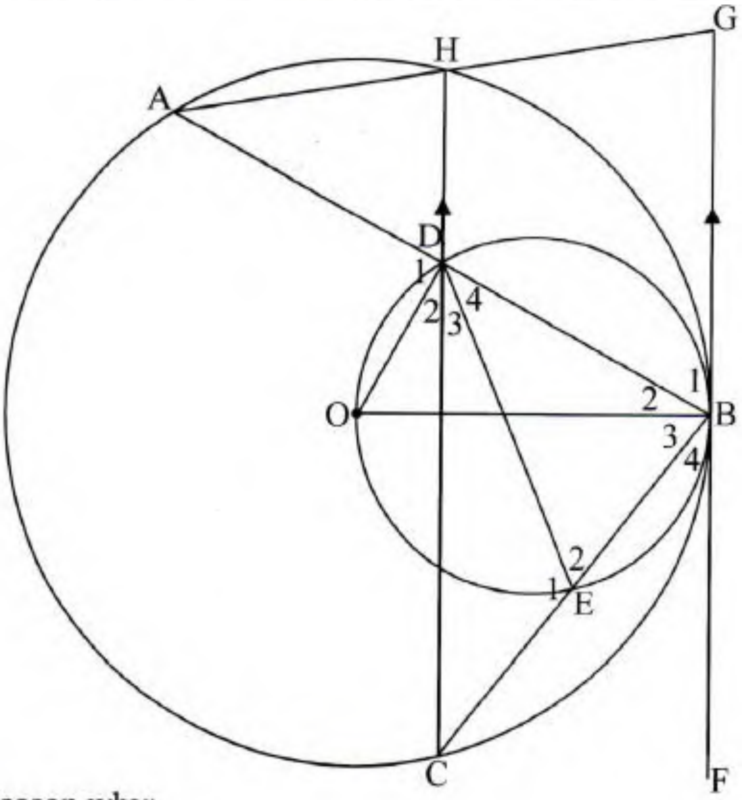
9.2.3 Prove that ORNL is a cyclic quadrilateral. (3)

9.2.4 MNP is translated to the left side of the circle, such that M' falls outside of the circle and N' inside of the circle. The point P' coincides with the point S. The points O, L and S remain fixed as indicated on the original diagram and MNP || $M'N'P'$. Is it possible for $M'N'P'$ to be a tangent to the circle drawn through O, L and S? Motivate your answer. (4)

[15]

QUESTION 10

Two circles touch each other internally at B. O, the centre of the bigger circle, lies on the circumference of the smaller circle. FBG is a common tangent. AB and BC are chords of the bigger circle and intersect the smaller circle at D and E respectively. AHG forms a straight line. Chords OD, DE and OB of the smaller circle are drawn. AH:AG = 1:2.



10.1 Give a reason why:

10.1.1 $\hat{B}_4 = H\hat{C}B$ (1)

10.1.2 $\hat{B}_4 = \hat{D}_4$ (1)

10.2 Prove that:

10.2.1 $\Delta DBE \ ||| \ \Delta DBC$ (3)

10.2.2 D is the midpoint of AB (3)

10.2.3 $\frac{AB^2}{4} = BC.BE$ (4)

10.2.4 CE:BC = 5:9 if DB = 3 units and BE = 2 units. (3)

10.3 Calculate the ratio of $\frac{\text{area } \Delta AHD}{\text{area } \Delta HGB}$. (6)

[21]

TOTAL: 150

QUESTION 1

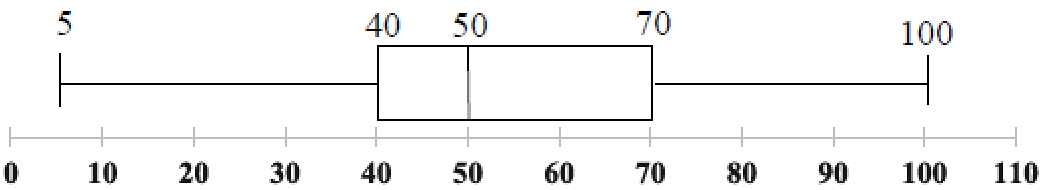
The A-Rithmetic High School decided to compare the results of 31 Grade 12 learners in Mathematics and Physical Sciences in the 2019 Preparatory Examination.

- The Mathematics results are recorded in the table below.
- The box and whisker plot below illustrates the results of Physical Sciences.
- Marks are recorded as percentages.

Mathematics Results

7	11	15	19	19	23	28	28	31	38	39
40	41	48	48	52	53	55	57	59	59	64
67	72	76	83	85	87	89	92	96	-	-

Physical Sciences Results



1.1 Calculate the mean mark of the Mathematics learners. (2)

1.2 Comment on the skewness of the Mathematics data. (1)

1.3 Determine which subject performed better in the 2019 Preparatory Examination. Give a reason for your conclusion. (2)

1.4 Write down a possible mark for a learner who achieved the tenth lowest mark in Physical Sciences. (2)

1.5 A learner scored the fourth highest in both subjects. The learner obtained the GREATEST possible difference between both subjects.
Calculate the learner's mark in Physical Sciences. (2)

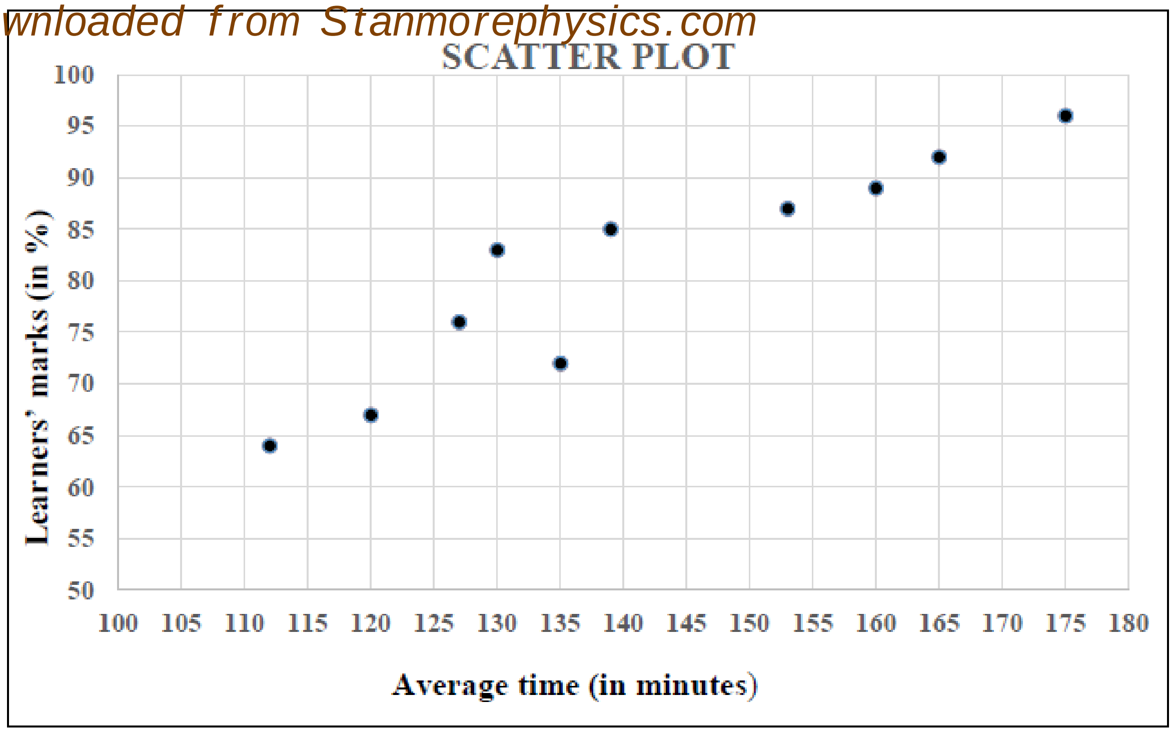
[9]

QUESTION 2

A question raised by many educators is whether the results that a learner achieves in an examination is dependent on the time that the learner takes to complete the examination.

The average time taken by each of the top 10 Mathematics learners was recorded. The data is represented in the table and scatter plot below.

Average time (in minutes)	175	165	160	153	139	130	127	135	120	112
Learners' marks (in %)	96	92	89	87	85	83	76	72	67	64



2.1 Calculate the equation of the least squares regression line for the data. (3)

2.2 A learner completed the exam in 2,5 hours. Predict the mark that the learner achieved. (2)

2.3 Explain within the context why the regression line is not reliable. (1)

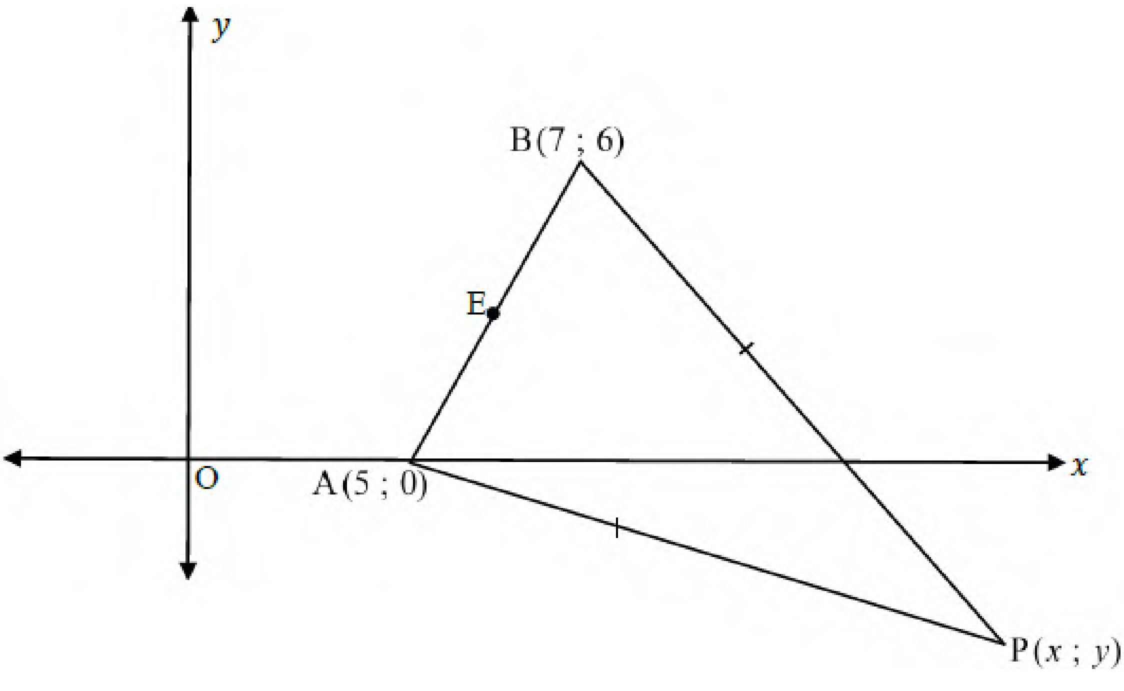
2.4 Calculate the standard deviation of the top 10 Mathematics learners. (2)

2.5 It is further given that $(p\ ;\ 103{,}59)$ is the interval of 15 random learners' marks within ONE standard deviation of the mean. If $\bar{x} = 63{,}96$, calculate the value of p. (3)

[11]

QUESTION 3

In the diagram below, points $A(5\ ;\ 0)$, $B(7\ ;\ 6)$ and $P(x\ ;\ y)$ form a triangle.
BP = AP and E is the midpoint of AB.



3.1 Determine the coordinates of E. (2)

3.2 Determine the equation of line BA. (3)

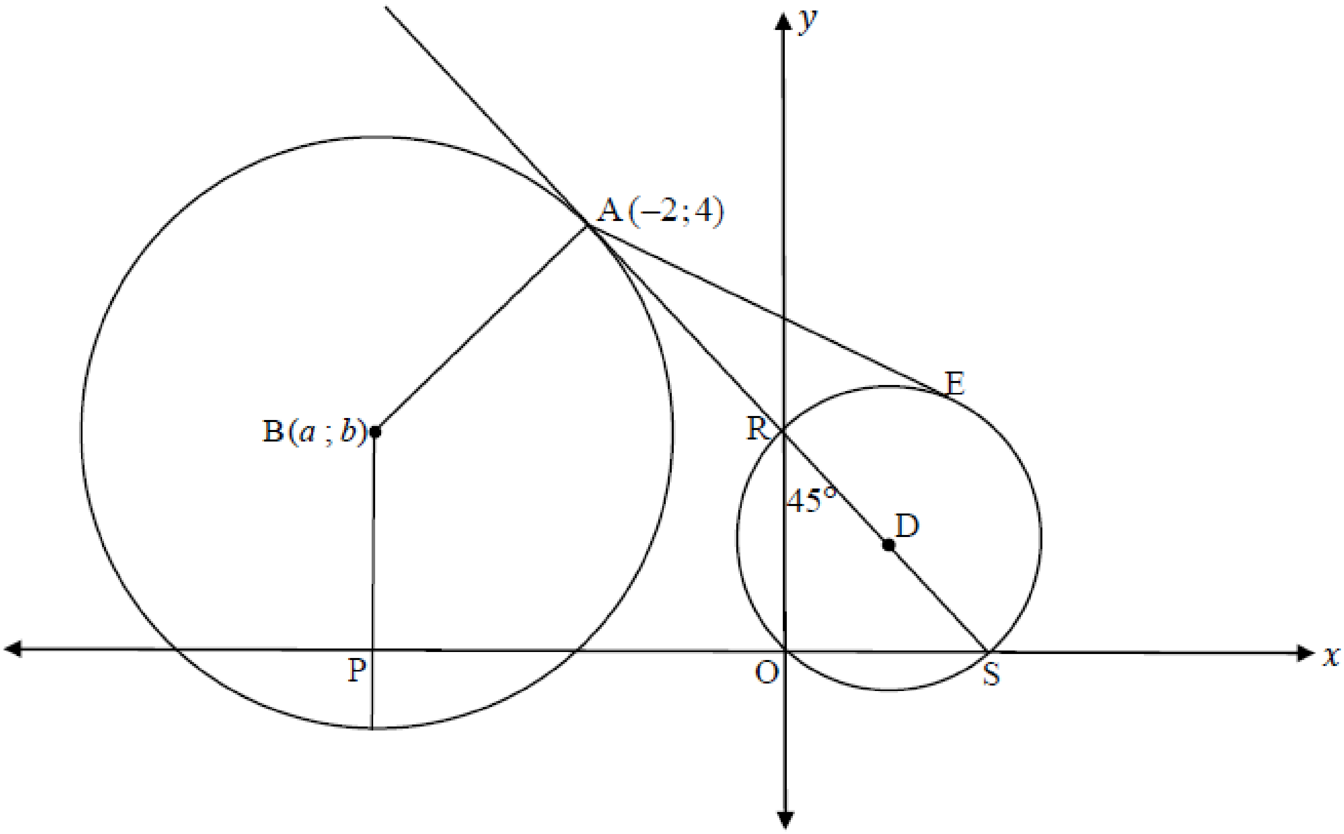
3.3 Line BA is parallel to the straight line with equation $rx - 3y + 5 = 0$.
Calculate the value of r. (3)

3.4 If the area of $\Delta AOP = 10$ units^2 and $y < 0$, calculate the coordinates of P. (7)

[15]

QUESTION 4

The diagram below shows a circle with centre $B(a\,;b)$. BP is parallel to the y-axis with P on the x-axis. AS is a tangent to circle B at $A(-2\,;\,4)$ and intersects the x-axis at S and the y-axis at R. AE is a tangent to the smaller circle with centre D and touches the circle at E. $O\hat{R}S = 45°$.



4.1 Determine the equation of tangent AS. (4)

4.2 If OP = 4 units, determine the values of a and b, the centre of the larger circle. (4)

4.3 Determine the equation of the circle with centre B. (3)

4.4 The equation of the smaller circle with centre D is $x^2 - 2x + y^2 - 2y = 0$.
Write this equation in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.5 Write down the coordinates of D, the centre of the smaller circle. (1)

4.6 Calculate the length of AE, the tangent to circle D at E. (6)

[21]

QUESTION 5

5.1 Calculate the value of $1-4\sin^2 15°$ without the use of a calculator. (5)

5.2 Simplify without the use of a calculator:

$$\frac{\sqrt{3}\sin x.\sin^2 72° + \sin^2 198°.\sqrt{3}\cos(x-90°)}{\tan 120°.\sin x}$$ (6)

5.3 Determine the general solution of the following:

$6\sin x.\cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0$ (7)

5.4 Prove that:

$$(1-\tan A)\left(\frac{\cos A}{\cos 2A}\right) = \frac{1}{\cos A + \sin A}$$ (4)

5.5 If $\sin 2\theta = k$ and $0° < 2\theta < 90°$, determine in terms of k:

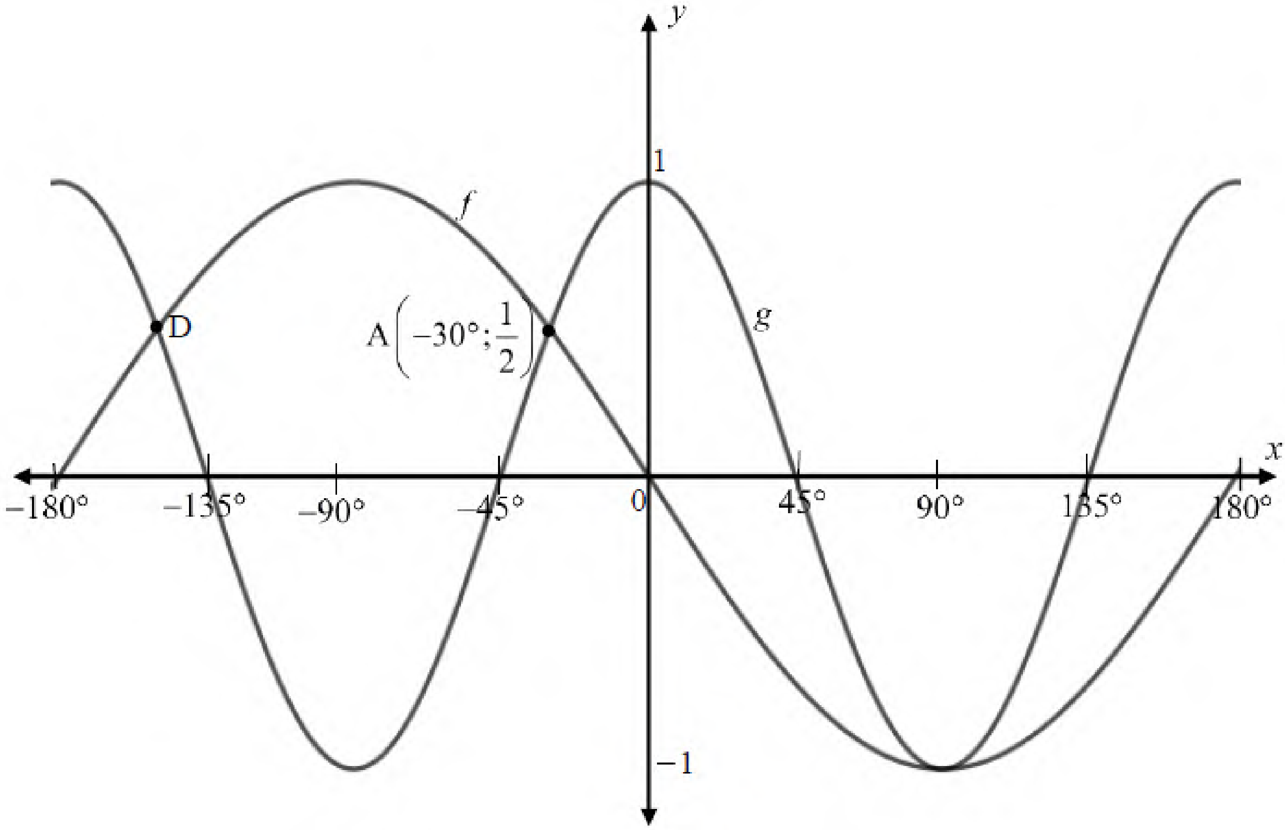
5.5.1 $\cos 2\theta$ (2)

5.5.2 $\dfrac{\sin 2\theta}{\tan\theta}$ (5)

[29]

QUESTION 6

The sketch below shows the graphs of $f(x) = a\sin x$ and $g(x) = \cos dx$ for $x \in [-180° ; 180°]$. $A\left(-30° ; \frac{1}{2}\right)$ is a point of intersection of f and g.



6.1 Write down the values of a and d. (2)

6.2 Determine the coordinates of D. (1)

6.3 For which value(s) of x is:

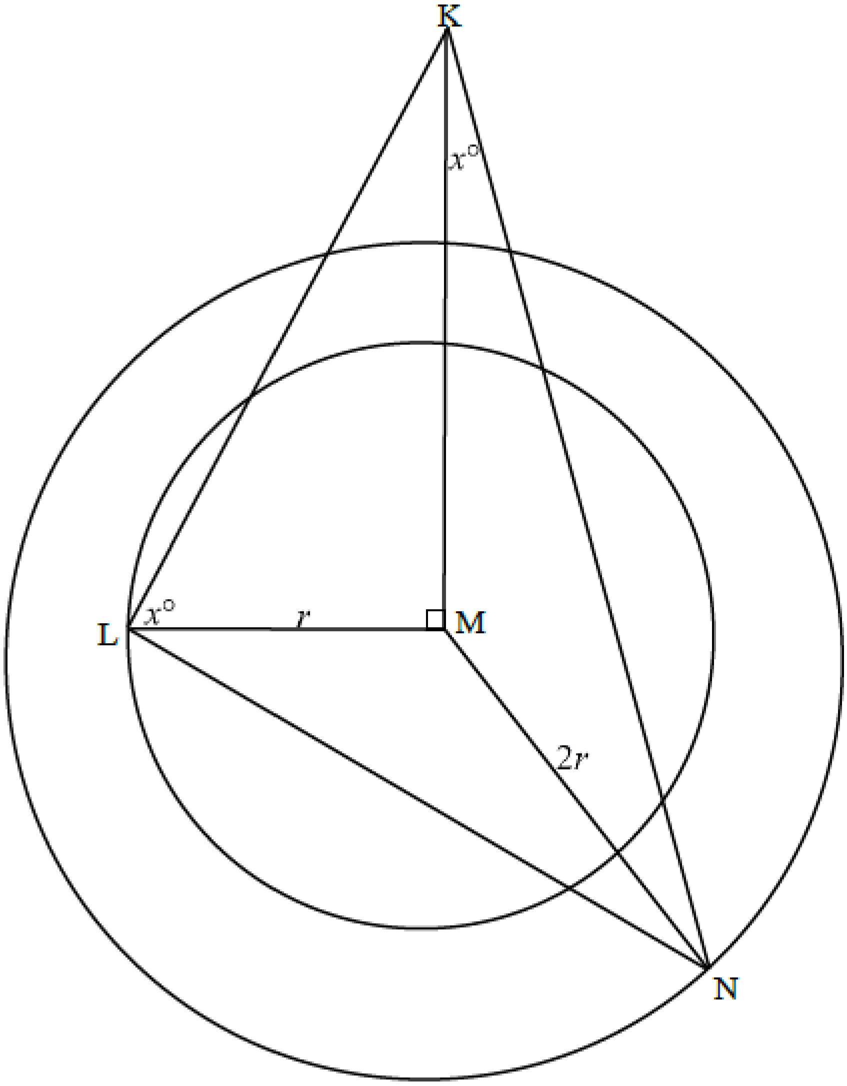
6.3.1 f decreasing for $x \in [-180° ; 180°]$? (2)

6.3.2 $f(x) \cdot g(x) < 0$ for $x \in [-180° ; 0°]$? (2)

[7]

QUESTION 7

In the figure below, KM is a vertical flag post set in the centre of two circles which lie on the same horizontal plane. $M\hat{K}N = M\hat{L}K = x°$. The radius of the inner circle $ML = r$ units and the radius of the outer circle $MN = 2r$ units.



7.1 Calculate the value of x. (6)

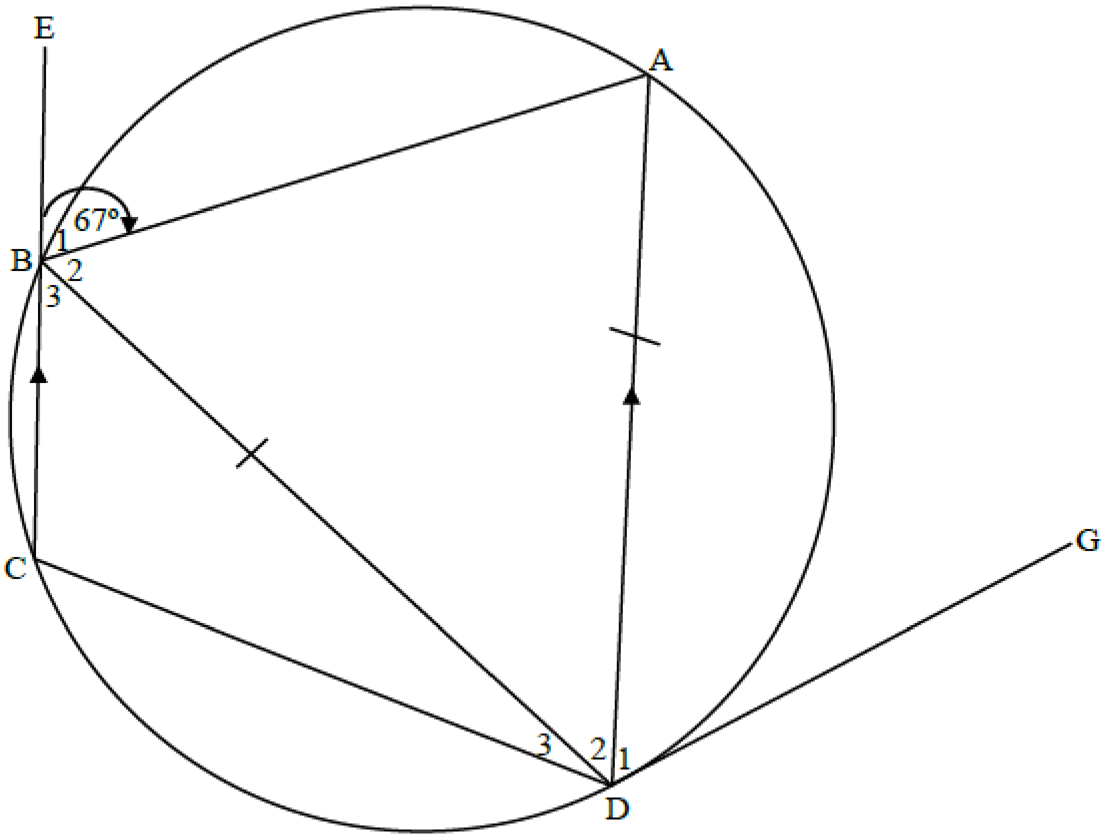
7.2 If $r = 5m$ and $L\hat{M}N = 110°$, calculate the length of LN. (2)

[8]

QUESTION 8

In the diagram below, points A, B, C and D lie on the circumference of a circle with AD || EC. CB is produced to E. GD is a tangent to the circle at D and DB = AD.

$E\hat{B}A = 67°$.



8.1 Calculate, with reasons, the size of the following angles:

8.1.1 $A\hat{D}C$ (2)

8.1.2 $\hat{C}$ (1)

8.1.3 $\hat{A}$ (1)

8.1.4 $\hat{D}_2$ (3)

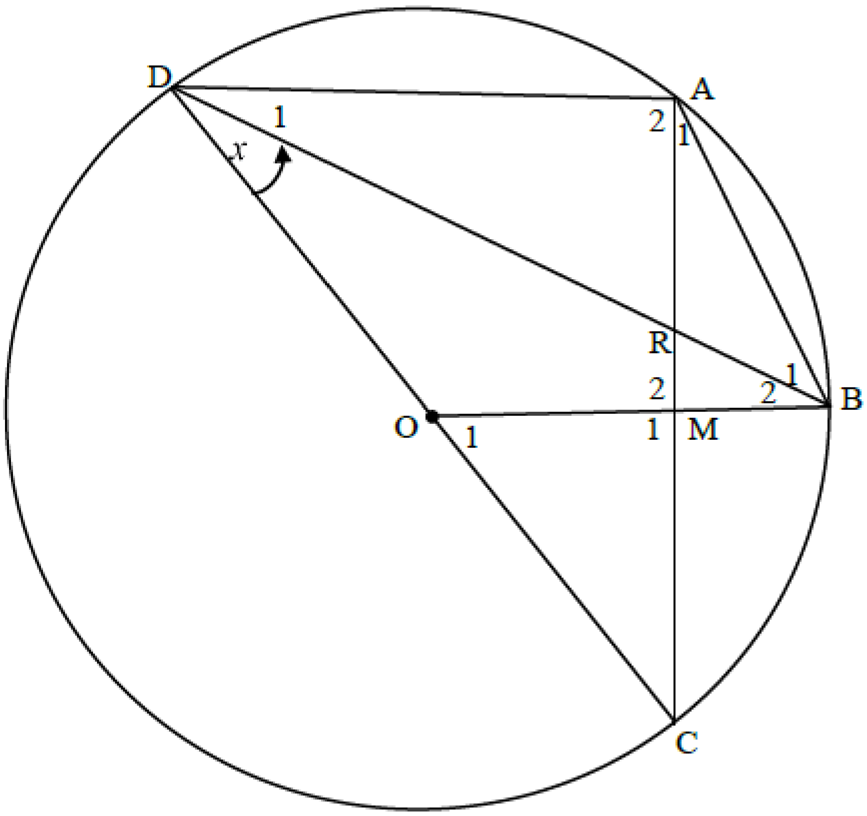
8.1.5 $B\hat{D}G$ (2)

8.2 Prove that AB = CD. (2)

[11]

QUESTION 9

9.1 In the diagram below, A, B, C and D are points on a circle with centre O. OB intersects AC at M, the midpoint of chord AC.
Let $B\hat{D}C = x$.



9.1.1 Determine, with reasons, in terms of x:

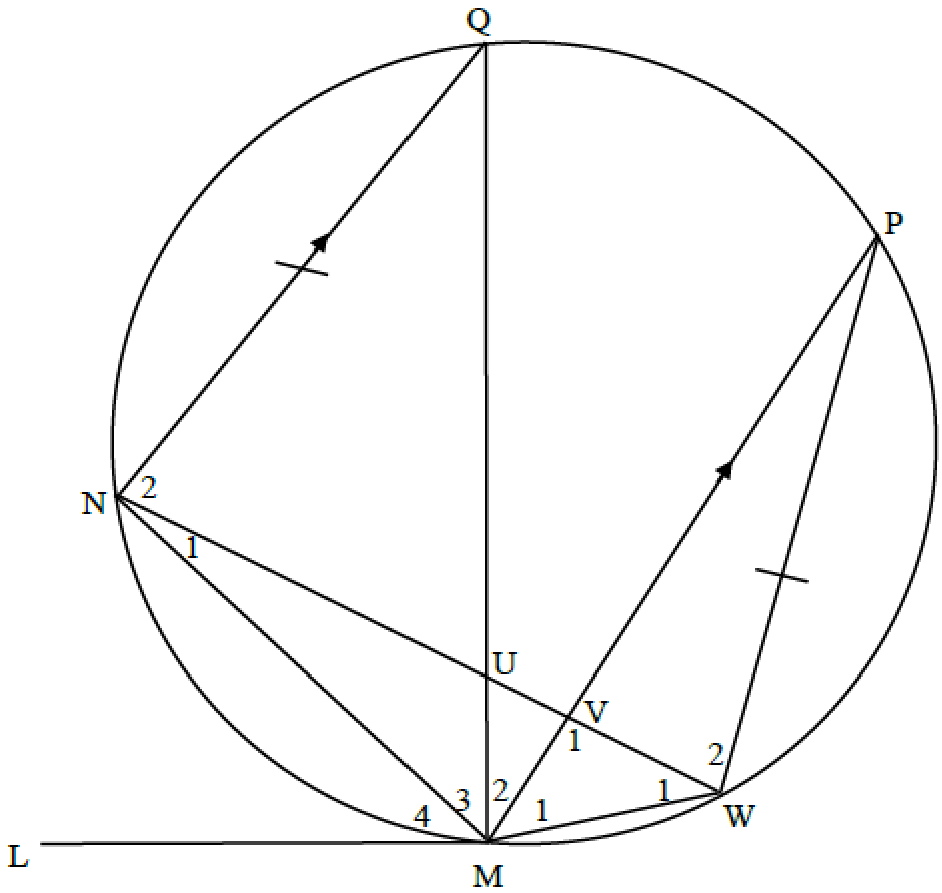
(a) $\hat{O}_1$ (1)

(b) $A\hat{B}O$ (4)

9.1.2 Prove that AB is a tangent to the circle that passes through points A, D and R. (6)

9.1.3 Prove that $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$. (4)

9.2 In the diagram below, LM is a tangent to circle QNMWP at M. NW cuts QM and PM at U and V respectively.
NQ = WP and NQ || MP.



9.2.1 State, with reasons, THREE angles equal to $\hat{M}_2$. (3)

9.2.2 Prove that $\Delta WMV \parallel\!| \Delta QMN$. (3)

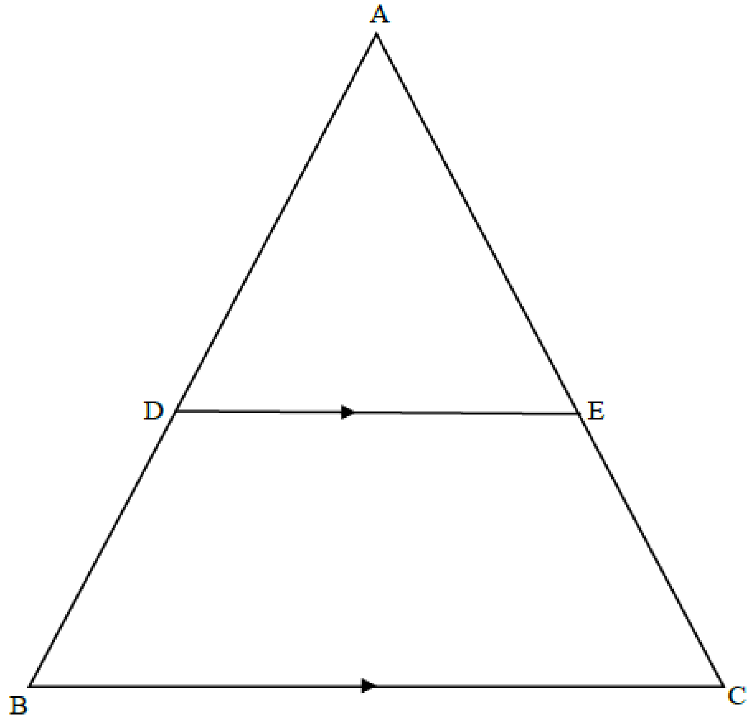
9.2.3 Prove that $\frac{MV}{WV} = \frac{MN}{PW}$. (3)

[24]

QUESTION 10

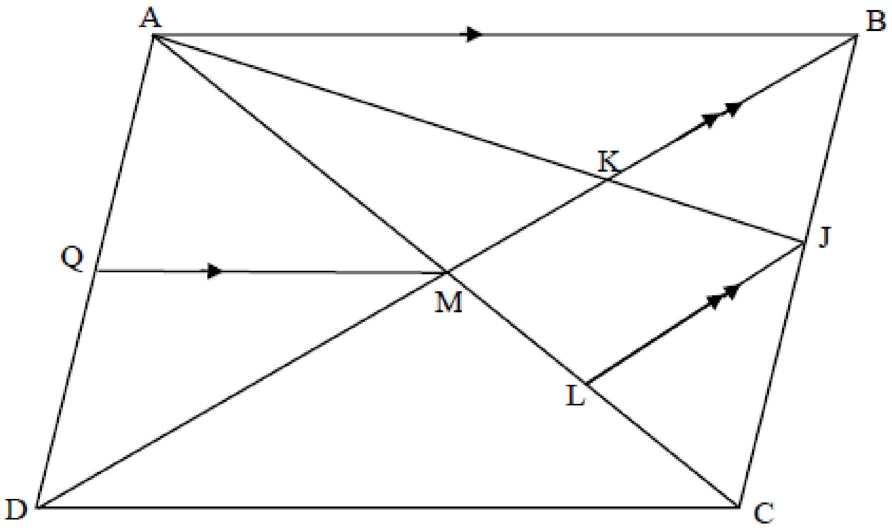
10.1 In ΔABC below, D and E are points on sides AB and AC respectively such that DE || BC.

Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 ABCD is a parallelogram with diagonals that intersect at M. J is a point on BC. BJ : JC is 2 : 3. AJ meets BD at K. BD || JL and JL meets AC at L. Q is a point on AD such that AB || QM.



10.2.1 Determine, with reasons, the following ratios:

(a) $\frac{ML}{LC}$ (2)

(b) $\frac{AK}{KJ}$ (3)

10.2.2 If $AB = \sqrt{10}$ units and $BC = \frac{2}{3}AB$.

Calculate the length of AQ. (4)

[15]

TOTAL: 150

WESTVILLE BOYS HIGH SCHOOL SEPTEMBER 2019 PAPER 2:

QUESTION 1

1.1 The ages (in years) of 18 people who visited the local municipal library in an hour on a particular Monday afternoon were recorded as follows:

12	14	15	16	18	18
18	22	25	29	29	31
33	34	37	44	53	68

1.1.1 Calculate:

(a) The mean of their ages. (2)

(b) The median of their ages. (2)

(c) The interquartile range of their ages. (2)

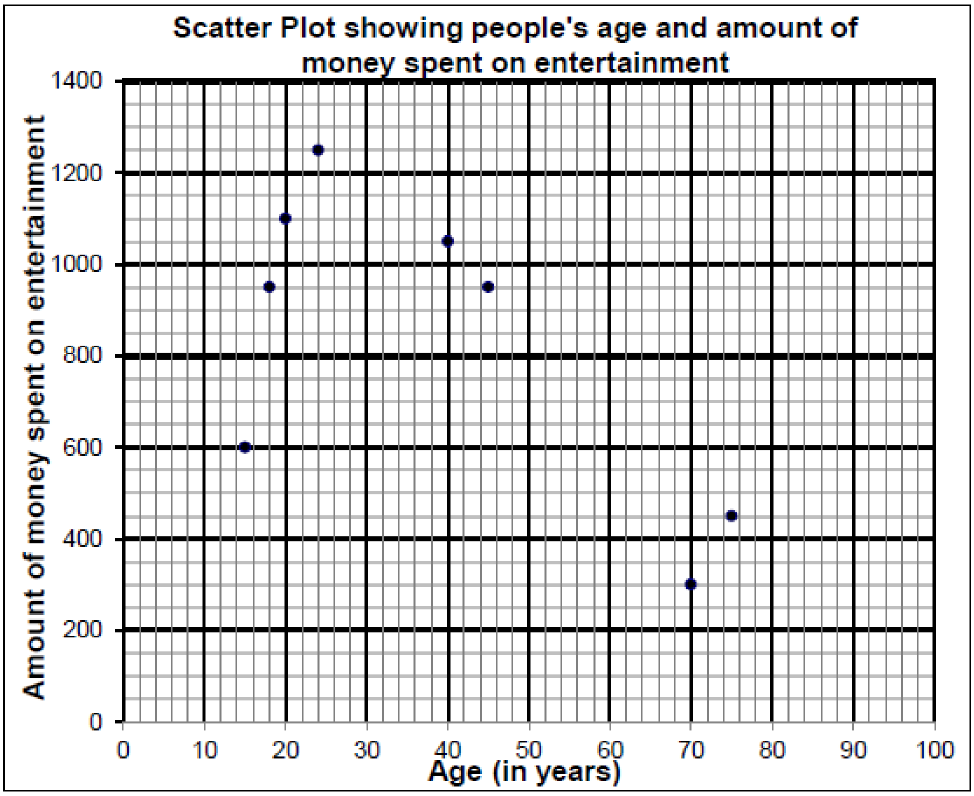
1.1.2 Write down the standard deviation of their ages. (1)

1.1.3 Hence, calculate how many people who visited the library were older than one standard deviation of the mean age. (3)

1.1.4 15 People visited the same local municipal library in an hour on a particular Friday afternoon. The five-number summary of the ages of the 15 people is given as: (8 ; 16 ; 29 ; 36 ; 49).
Which of the two groups, Monday or Friday, would have a higher upper quartile? (1)

1.2 The age of a person (in years) and the amount of money (in Rands) that they spent on entertainment in a certain month was recorded. The information is shown in the table and scatter plot below:

Age (in years)	15	18	20	24	40	45	70	75
Money spent on entertainment	600	950	1 100	1 250	1 050	950	300	450



1.2.1 Write down the correlation coefficient, r. (2)

1.2.2 Comment on the correlation between the person's age and the amount of money they spent on entertainment. (1)

1.2.3 The equation of the least squares regression line for the data is given as $\hat{y} = 1201{,}74 - 9{,}6546x$. Predict, correct to two decimal places, how much a 55 year old person would have spent on entertainment in that month. (2)

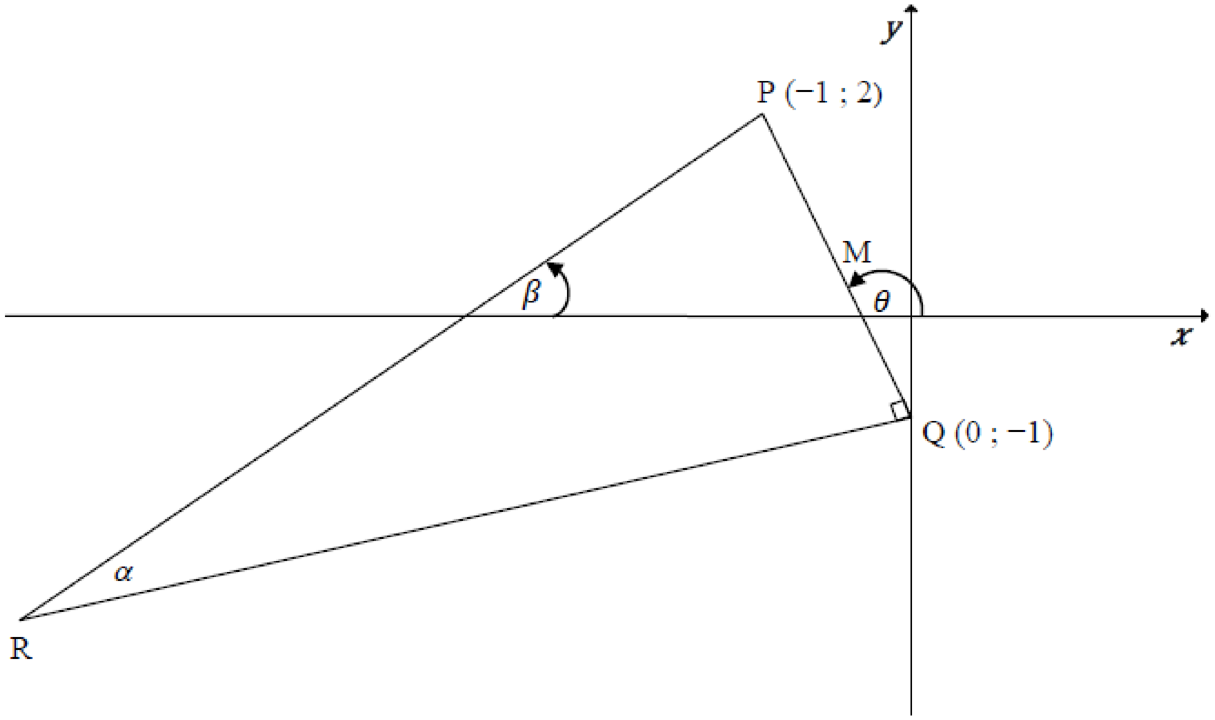
1.2.4 Draw the least squares regression line on the grid provided in your answer book. (2)

1.2.5 Explain why the prediction in QUESTION 1.2.3 may not be reliable. (1)

[19]

QUESTION 2

In the diagram below, $P(-1;2)$, $Q(0;-1)$ and R are the vertices of ΔPQR. The equations of PR and QR are $y = x + 3$ and $3y - x + 3 = 0$ respectively. θ is the angle of inclination of line PQ and β is the angle of inclination of line PR. $P\hat{R}Q = \alpha$ and $P\hat{Q}R = 90°$.



2.1 Calculate the length of PQ. Leave your answer in simplified surd form. (2)

2.2 Determine the coordinates of M, the midpoint of PQ. (2)

2.3 Determine the gradient of PQ. (2)

2.4 Given that T is the point $(2;-7)$, determine if the points P, Q and T are collinear. (3)

2.5 Determine the coordinates of R. (3)

2.6 W is a point so that PQRW, in that order, forms a parallelogram. Determine the coordinates of point W. (3)

2.7 Calculate the size of α. (4)

[19]

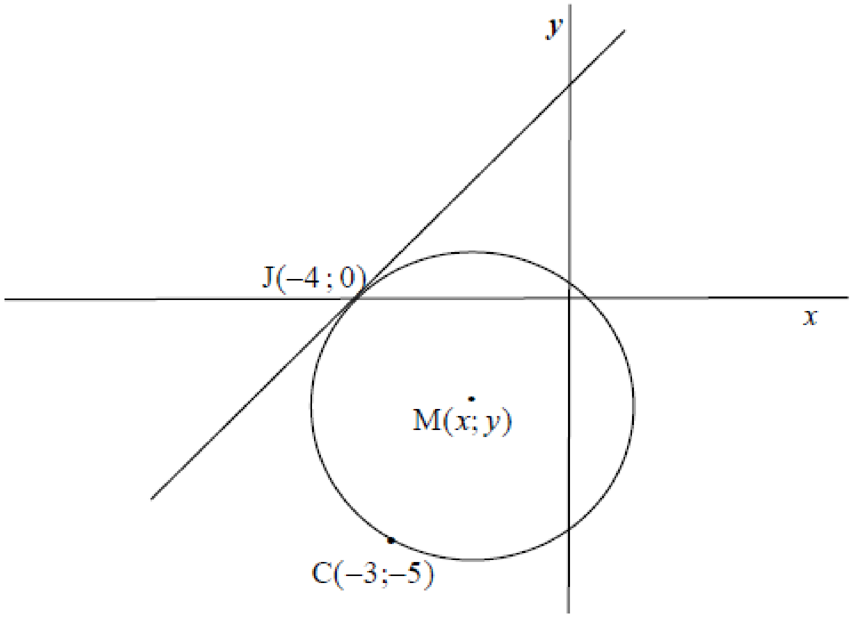
QUESTION 3

3.1 The equation of the circle with centre O, is given by $x^2 + y^2 + 6x - 8y = 33$.

3.1.1 Write down the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

3.1.2 If P(4 ; 7) is a point on the circle, determine the equation of the radius OP. (3)

3.2 In the diagram below, M $(x; y)$ is the centre of the circle that passes through the point C$(-3; -5)$. The tangent with equation $y = \frac{3}{2}x + 6$ touches the circle at the point J$(-4; 0)$.



3.2.1 Determine the equation of JM in the form $y = mx + c$. (4)

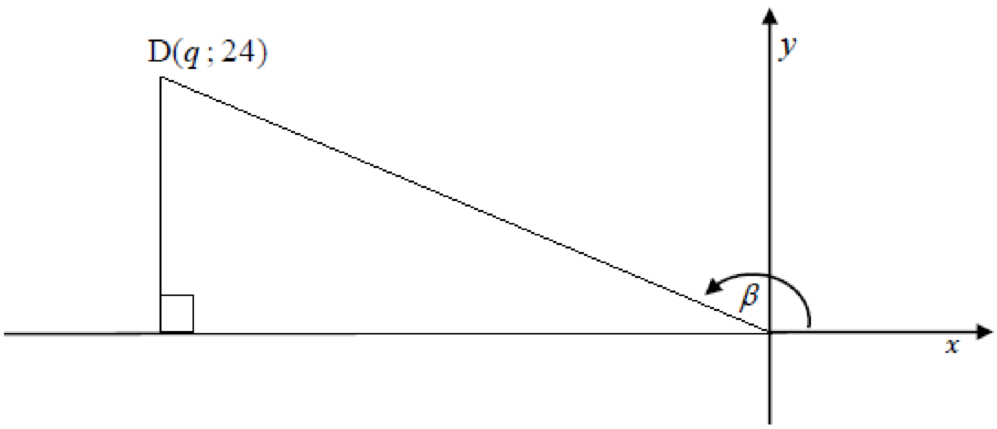
3.2.2 Show, by calculation, that the coordinates of M are $(-1; -2)$. (6)

3.2.3 Hence, determine the equation of the circle in the form: $(x-a)^2 + (y-b)^2 = r^2$. (3)

[18]

QUESTION 4

4.1 In the diagram, D is the point $(q\ ; 24)$. Also, $\sin\beta = \frac{24}{26}$. β is an obtuse angle.



4.1.1 Determine the value of q. (2)

4.1.2 **Without using a calculator**, calculate the value of:

a) $\tan\beta$ (1)

b) $\sin(\beta + 60°)$ (4)

4.2 Given: $\dfrac{\sin(B-360°).\cos(90°+B)}{\cos(90°-B).\tan(-B)}$

Simplify the above expression to a single trigonometric ratio in terms of B. (5)

4.3 Prove the following identity: $\dfrac{\sin\theta - \sin 2\theta}{1-\cos\theta + \cos 2\theta} = -\tan\theta$ (5)

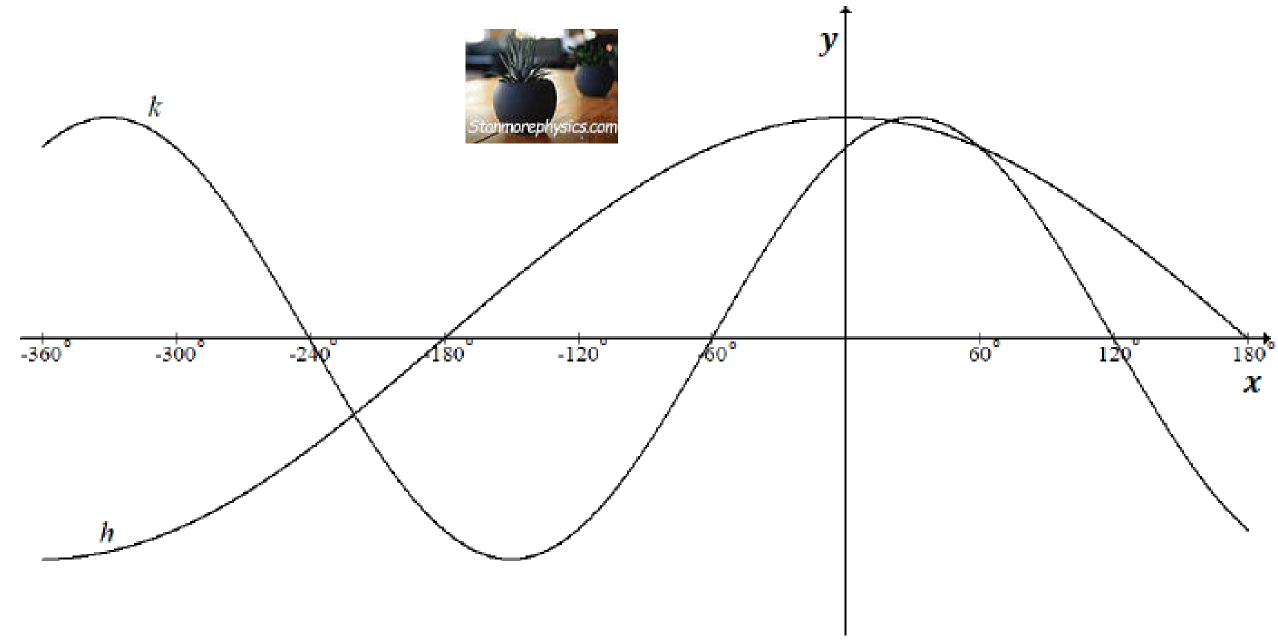
4.4 Determine the general solution of: $\cos 2\theta - \sin\theta = 0$ (5)

4.5 It is given that $f = \cos\alpha + \sin\alpha$ and $g = \cos\alpha - \sin\alpha$.
Determine tan α, in terms of f and g. (4)

[26]

QUESTION 5

The diagram below shows the graphs of $h(x) = \cos px$ and $k(x) = \sin(x+q)$ for $x \in [-360°; 180°]$.



5.1 Write down the period of h. (1)

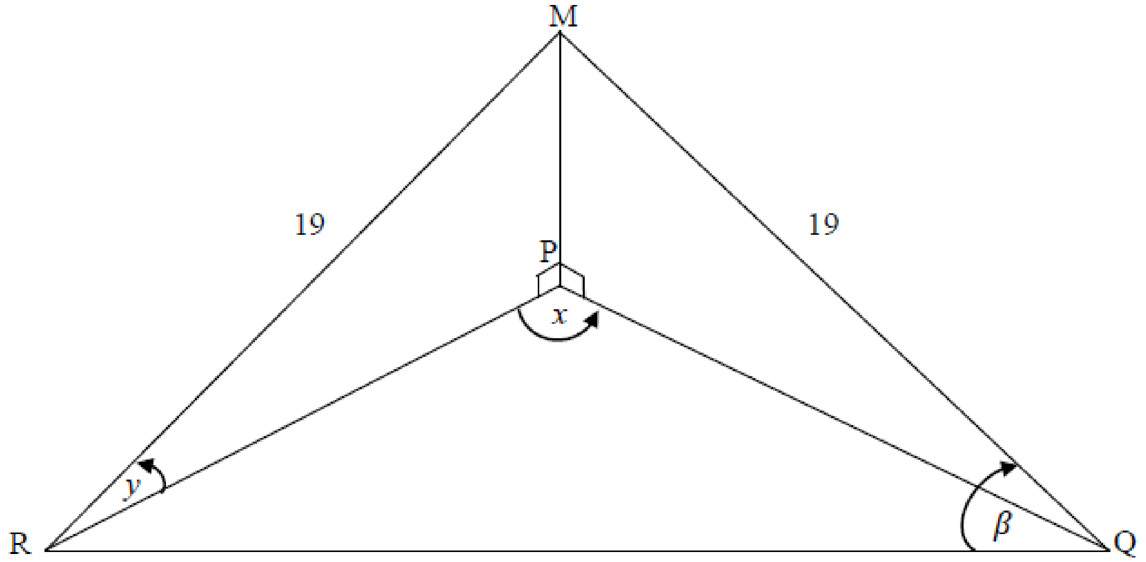
5.2 Determine the values of p and q. (2)

5.3 For which values of x in the interval $x \in [-360°; -60°]$ is $\dfrac{h(x)}{k(x)} \leq 0$? (3)

[6]

QUESTION 6

In the figure, MP is a vertical tower. MQ and MR are wire ropes used to stabilise MP and are each 19 metres in length. R, P and Q are in the same horizontal plane. The angle of elevation of M from R is y. $R\hat{P}Q = x$ and $M\hat{Q}R = \beta$.



Prove that:

6.1 Area of $\Delta PQR = \dfrac{361 \sin x . \cos^2 y}{2}$ (5)

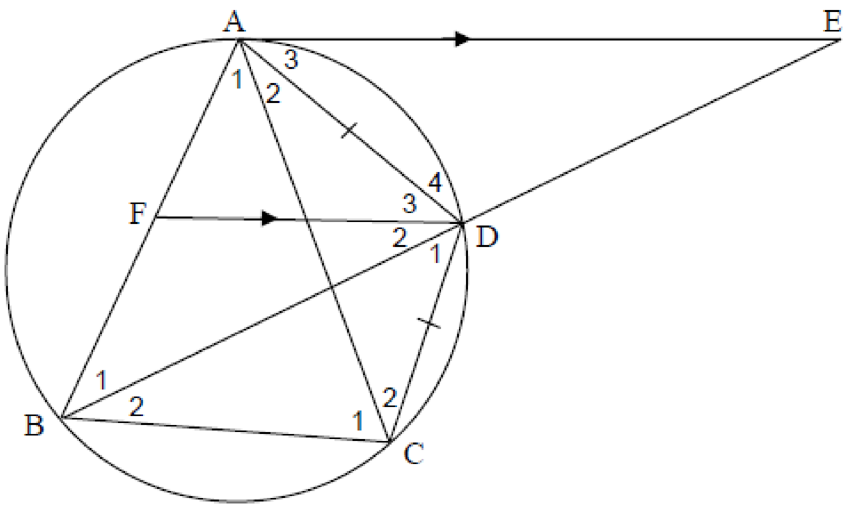
6.2 $RQ = 38 \cos \beta$ (6)

[11]

Give reasons for ALL statements and calculations in Questions 7, 8 and 9.

QUESTION 7

7.1 In the diagram below, ABCD is a cyclic quadrilateral. AE is a tangent to circle ABCD at A. Chord AD = chord CD and AE || FD.



7.1.1 Provide a reason why each of the given statements is true:

(a) $\hat{B}_1 = \hat{A}_3$ (1)

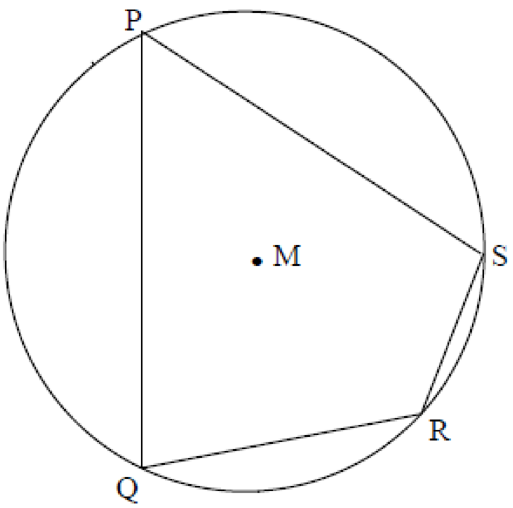
(b) $\hat{B}_1 = \hat{B}_2$ (1)

7.1.2 If $\hat{D}_3 = x$ and $\hat{E} = y$, determine the size of $B\hat{A}D$ in terms of x and y. (3)

7.2 7.2.1 Complete the following statement:

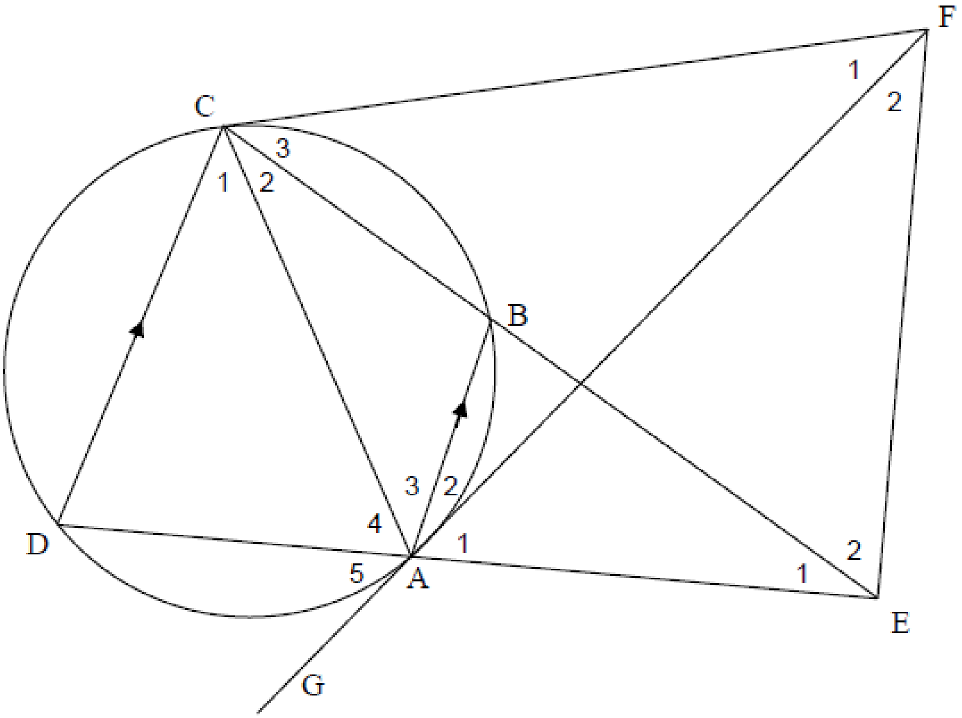
The angle subtended by an arc (or chord) of a circle at the centre is ______ the angle subtended by the same arc (or chord) at any point on the circle. (1)

7.2.2 In the diagram below, cyclic quadrilateral PQRS is drawn with M as the centre of the circle.



Use the diagram above to prove that $\hat{P} + \hat{R} = 180°$. (5)

7.3 In the diagram below, CF is a tangent to the circle CBAD at C. Chords DA and CB are produced to meet at E. FAG is a straight line. $\hat{A}_1 = \hat{A}_3$ and AB || DC. CA is drawn.



Prove that:

7.3.1 AEFC is a cyclic quadrilateral. (3)

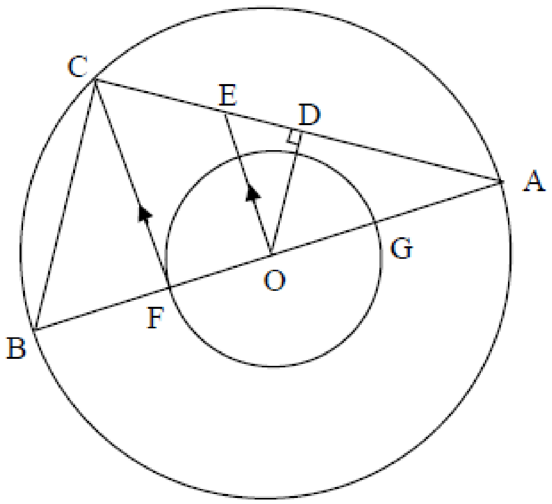
7.3.2 FAG is a tangent to the circle CBAD. (4)

[18]

QUESTION 8

8.1 In the diagram below, O is the centre of two concentric circles.

- AOB is a diameter of the larger circle with AB = 26 units.
- FOG is a diameter of the smaller circle with FG = 8 units.
- Chord BC = 10 units.
- OD $\perp$ AC.
- OE $\parallel$ CF.

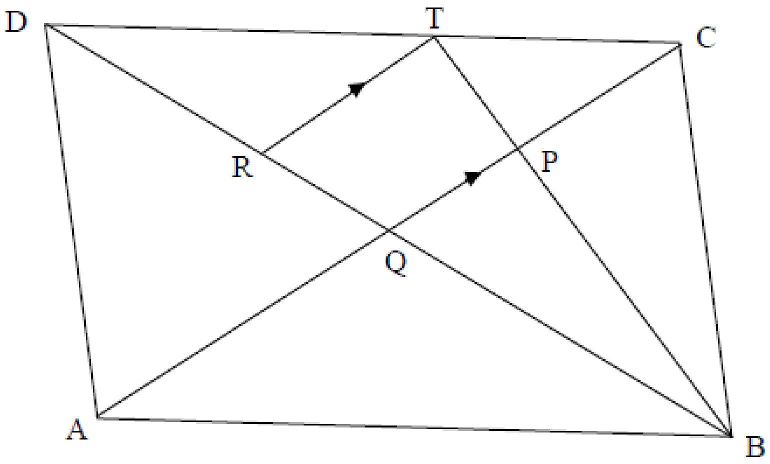


8.1.1 Calculate, with reasons, the length of AC. (2)

8.1.2 Calculate, with reasons, the value of $\frac{CE}{EA}$. (2)

8.1.3 Hence or otherwise, determine with reasons, the length of DE. (4)

8.2 In the diagram below, ABCD is a parallelogram.
The diagonals of ABCD intersect at Q. T is a point on DC such that DT : TC = 5 : 3.
R is a point on DQ such that RT $\parallel$ AC. BQ = $\frac{8}{3}$QR. TB and QC intersect at P.

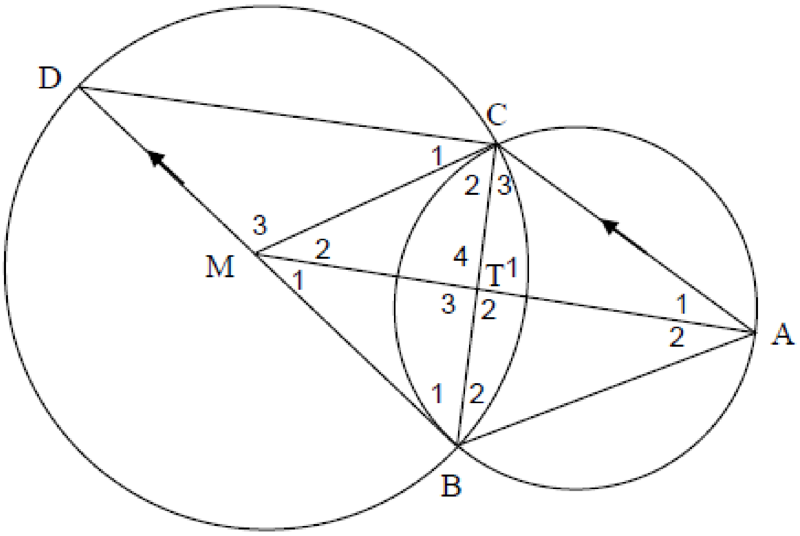


Calculate, with reasons, the value of $\frac{\text{Area } \Delta DTR}{\text{Area } \Delta ABC}$ (6)

[14]

QUESTION 9

9.1 In the diagram below, M is a point on DB. DB and CM are tangents to the smaller circle at B and C respectively. CB intersects MA at T. MA bisects $B\hat{A}C$. DB || CA.



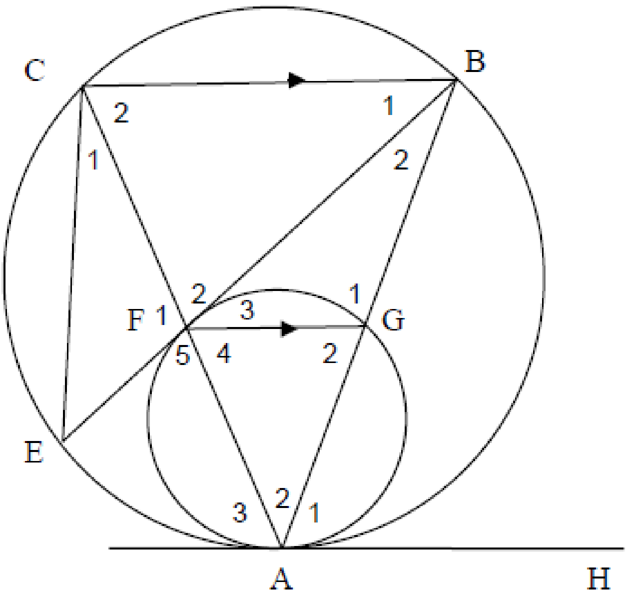
Prove that:

9.1.1 ΔBTM ||| ΔCTA (3)

9.1.2 AB . AT = AC . TM (3)

9.1.3 M is the centre of the larger circle, if BD is a diameter. (4)

9.2 In the diagram below, AH is a common tangent to the two circles. AFC and AGB are chords in both circles. Chord EB of the larger circle is a tangent to the smaller circle at F. Chords CB, CE and FG are drawn. CB || FG.



Prove that:

9.2.1 BCFG is a cyclic quadrilateral, if AF = AG. (3)

9.2.2 $\frac{BF^2}{BA^2} = \frac{CF}{CA}$ (6)

[19]

TOTAL MARKS: [150]

NORTH WEST SEPTEMBER 2019 PAPER 2:

QUESTION 1

The time (in seconds) between the consecutive landings of aeroplanes at an airport on day 1 was recorded. The data is given in the Cumulative Frequency table below.

Time in seconds	Number of aeroplanes (Frequency)	Cumulative Frequency
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	k	p
$210 < t \leq 240$	7	80

1.1 Show that $k = 10$. (1)

1.2 Write down the value of p. (1)

1.3 Calculate the estimated mean time between the landings of two consecutive aeroplanes. (3)

1.4 It is given that $(q\,;\,186{,}89)$ is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.

1.4.1 Write down the estimated standard deviation of the time between the consecutive landings of the aeroplanes. (2)

1.4.2 Calculate the value of q. (1)

1.5 On day 2, the same number of aeroplanes that landed on day 1, land at the airport. The elapsed time between all the consecutive landings of all the aeroplanes is m seconds shorter than the time that is given in the table above.

If an ogive is to be drawn of the data of day 2, the following will be true:

- The ogive will be grounded at (57 ; 0)
- The maximum value of the ogive will be at (237 ; 80)

Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same. (2)

[10]

QUESTION 2

The marks, in percentage, obtained in an Accounting and Mathematics test by a group of ten Grade 12 learners is shown in the table below.

Accounting Test	76	65	88	68	70	79	51	66	59	74
Mathematics Test	80	69	93	19	76	85	57	79	62	78

2.1 Identify an outlier of the above data. (1)

2.2 Determine the equation for the least squares regression line after ignoring the outlier in the above data. (3)

2.3 Another learner in the same class obtained 83% in the Accounting test, but due to illness could not write the Mathematics test.Use the equation established in 2.2 to predict the learner's mark for the Mathematics test. (2)

2.4 The teacher decided to award the learner who was absent the predicted mark obtained in 2.3 for the Mathematics test. Other learners in the class felt that it was unfair.

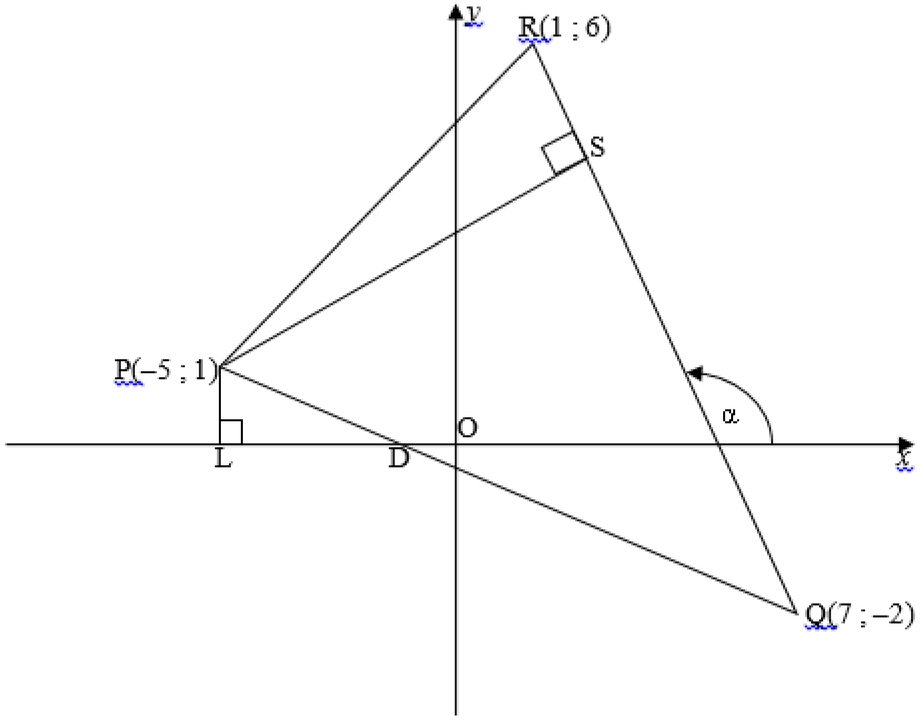
Motivate to these learners why the predicted mark is a good indication of what the learner may have scored in the Mathematics test. (2)

2.5 After the Mathematics subject advisor has moderated the answer books of the Mathematics tests, she decides to lower every test mark by p%. Explain, **without any calculations,** what influence the lowering in the marks of the Mathematics test has on the slope of the least squares regression line of the above data when the outlier is ignored. (2)

[10]

QUESTION 3

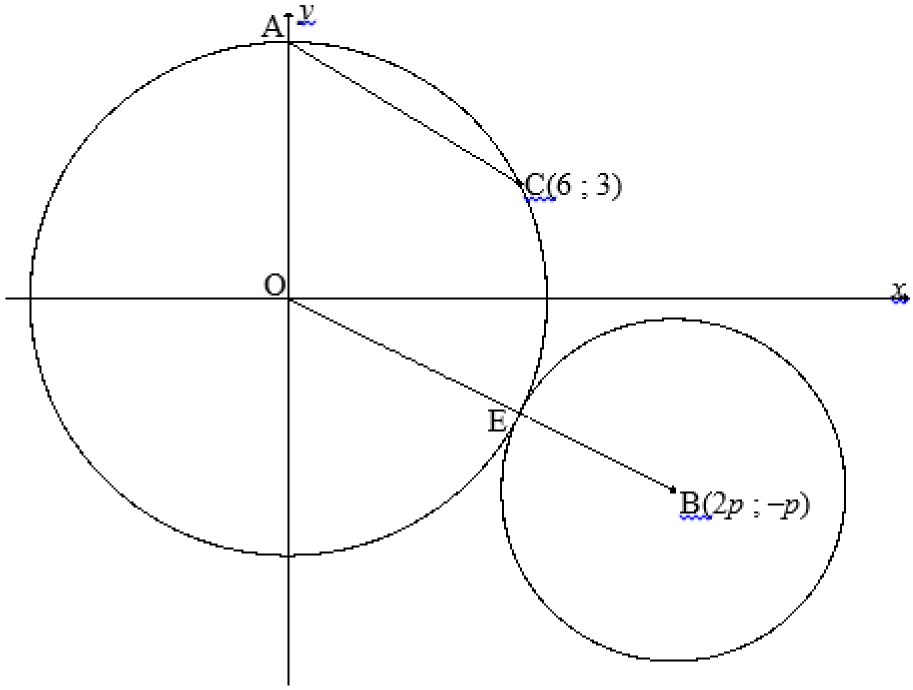
In the diagram below, P (–5; 1), Q(7; –2) and R(1; 6) are the vertices of ΔPQR. PQ intersects the x-axis at D. The angle of elevation of QR is α.
PS $\perp$ RQ and L lies on the x-axis such that PL $\perp$ x – axis.



3.1	Write down the equation of the line PL.	(1)
3.2	Calculate the gradient of QR.	(2)
3.3	Determine the equation of the line PS.	(4)
3.4	Calculate the size of the angle of inclination of PQ.	(3)
3.5	Calculate the size of P$\hat{Q}$S.	(4)
3.6	It is given that the areas of Δ PRS = $4x^2$ and Δ PQS = $16x^2$. Calculate the length of SQ, WITHOUT calculating the coordinates of S.	(5) **[19]**

QUESTION 4

In the diagram below, two circles are given. Circle O, having the origin as centre, intersects the y-axis at A and passes through the point C(6 ; 3). The circle having centre B(2p ;$-p$) touches circle O externally in point E. The centres of the two circles are joined by the line OB.



4.1 Determine the equation of the circle having centre O. (2)

4.2 Determine the coordinates of A. (2)

4.3 Determine the equation of AC. (3)

4.4 Calculate the value(s) of k for which the line $y = \frac{1-\sqrt{5}}{2}x + k$ will intersect the circle having centre O at two points, one of which has a positive x-value and the other a negative x-value. (2)

4.5 It is given that the length of EB $= \sqrt{20}$.

4.5.1 Write down, in terms of p, the equation of circle B in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.5.2 Determine the value of p if $p > 0$. (5)

4.6 Suppose a third circle with the following equation is given:
$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$

Determine the maximum length that the radius of this circle can be for any value of θ. (6)

[22]

QUESTION 5

5.1 Simplify each of the following **without the use of a calculator.** Show ALL calculations.

5.1.1 $\dfrac{\sin 110°.\tan 60°}{\cos 540°.\tan 250°.\sin 380°}$ (7)

5.1.2 $(1-\sqrt{2}\sin 22,5°)(\sqrt{2}\sin 22,5°+1)$ (4)

5.2 Given the expression: $\dfrac{\cos 2x.\tan x}{\sin^2 x}$

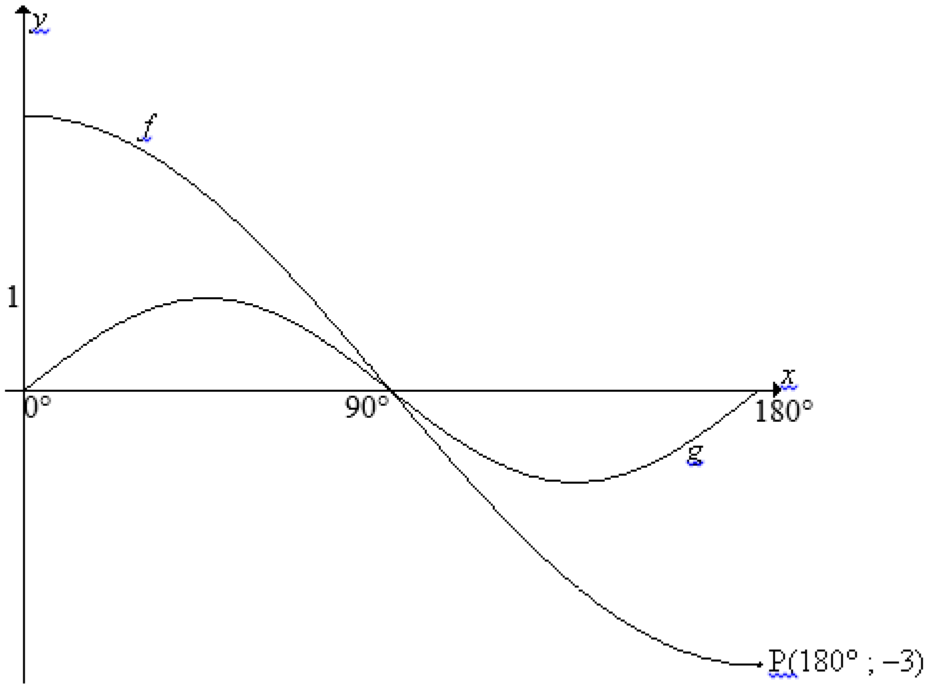
5.2.1 For which value(s) of x, in the interval $x \in [0°\,;180°]$, will this expression be undefined? (3)

5.2.2 Prove that $\dfrac{\cos 2x.\tan x}{\sin^2 x} = \dfrac{\cos x}{\sin x} - \tan x$ (5)

[19]

QUESTION 6

In the diagram below, the graphs of $f(x) = a\cos x$ and $g(x) = \sin bx$ are drawn for the interval $x \in [0°\,;180°]$. The point P(180° ; −3) is on the graph of f.



6.1 Write down the values of a and b. (2)

6.2 Write down the period of f. (1)

6.3 Write down the range of $g(x) + 3$ (2)

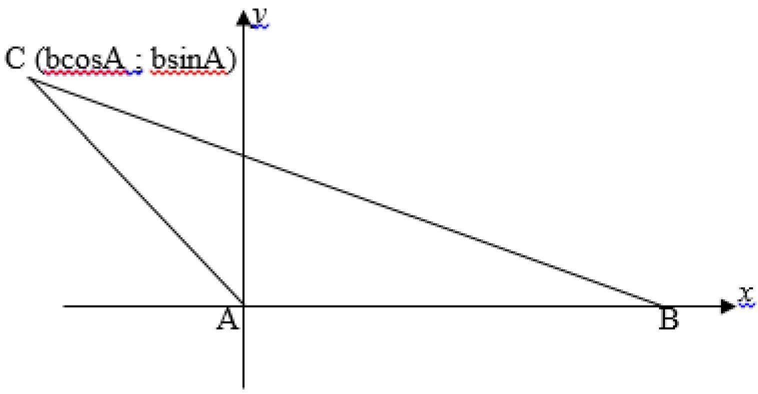
6.4 For which values of x, in the given interval, is $f(x).g'(x) > 0$ (3)

6.5 When the graph of g is shifted $q°$ to the left, it coincides with the function $y - \cos^2 x = -\sin^2 x$. Determine the value of q. (3)

[11]

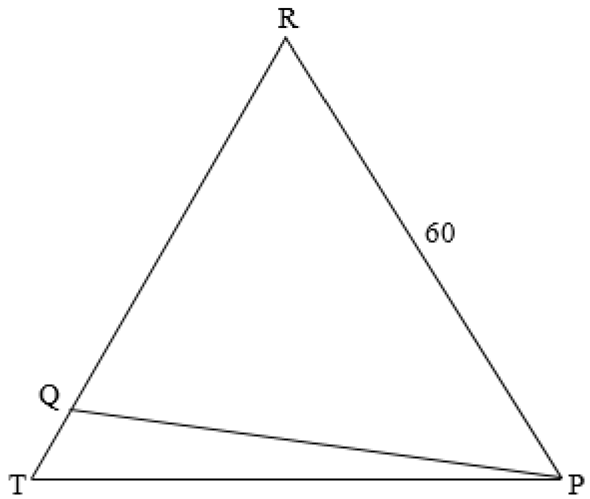
QUESTION 7

7.1 In the diagram below, Δ ABC is drawn having A at the origin, B on the x-axis and the vertex C has the coordinates (bcosA ; bsinA).



Use the above diagram to prove that $a^2 = b^2 + c^2 - 2bc\cos A$ (4)

7.2 In the diagram below, ΔTPR is equilateral with PR = 60 units. Q is a point on RT such that RQ:QT = 5:1.



7.2.1 Show, by calculations, that PQ = 55,68 units. (4)

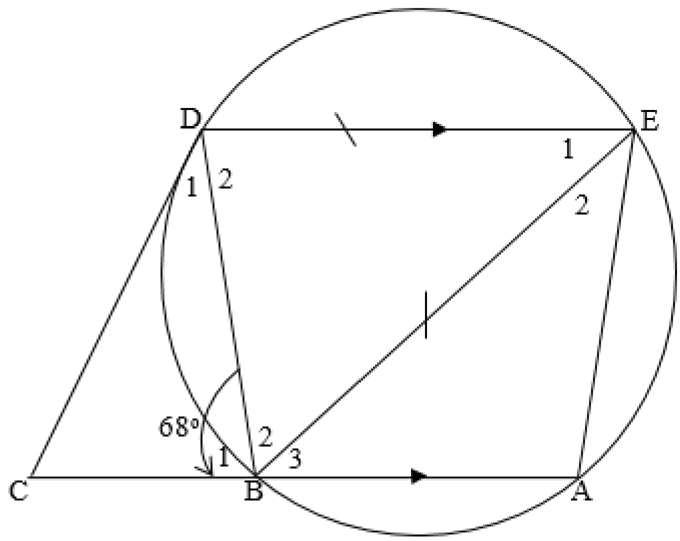
7.2.2 It is given that S is any point on the straight line PQ. Calculate the distance QS when S is the nearest to R. (4)

[12]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

In the diagram below, BAED is a cyclic quadrilateral with BA ∥ DE. BE = DE and $D\hat{B}C = 68°$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

8.1.1 $D\hat{E}A$ (2)

8.1.2 $\hat{A}$ (1)

8.1.3 $\hat{D}_2$ (2)

8.1.4 $\hat{B}_2$ (1)

8.1.5 $\hat{D}_1$ (3)

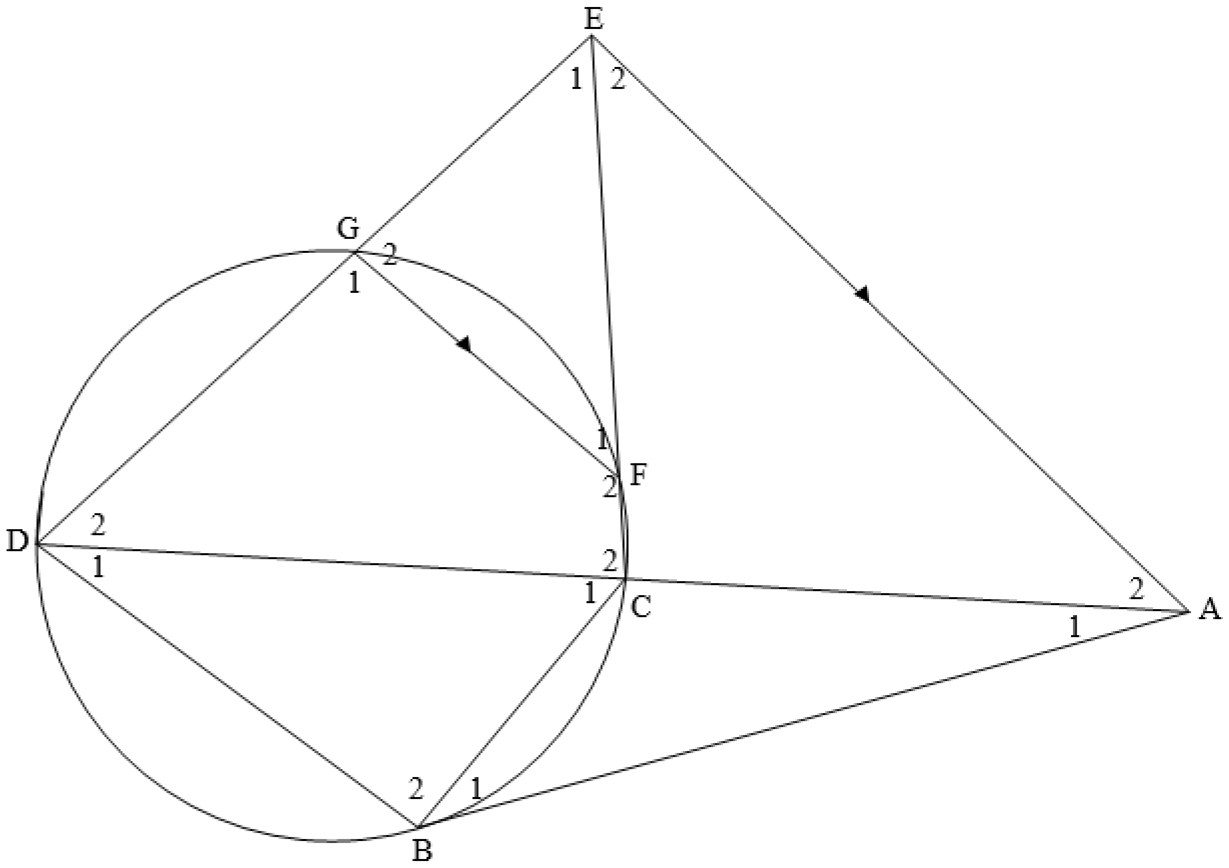
8.2 Prove that Δ BDC is isosceles. (2)

8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)

[13]

QUESTION 9

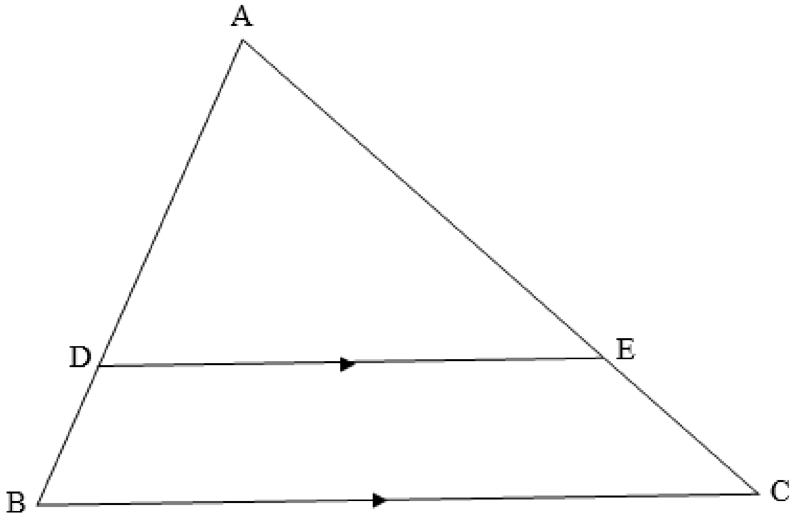
In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. EA | | GF



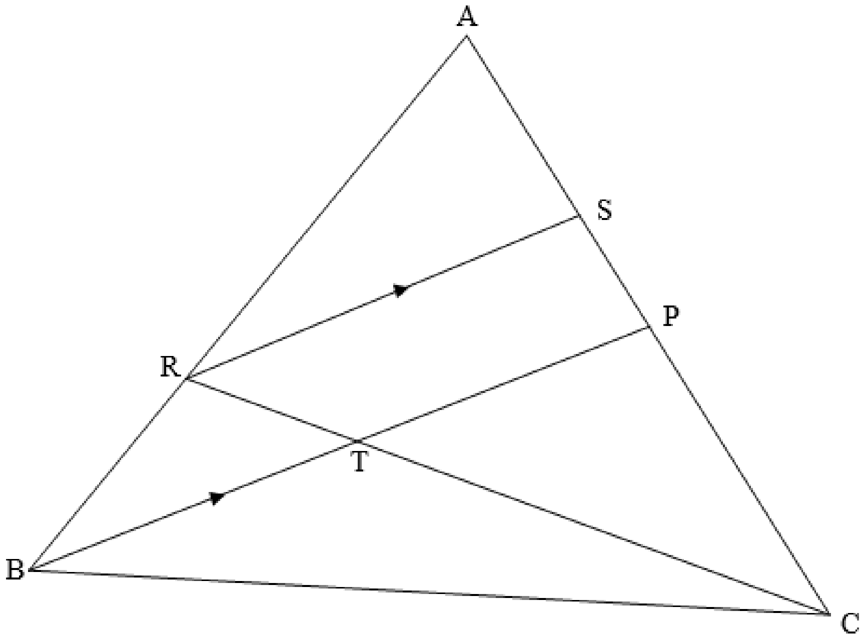
9.1	Give a reason why $\hat{B}_1 = \hat{D}_1$.	(1)
9.2	Prove $\Delta ABC \mid\mid\mid \Delta ADB$.	(3)
9.3	Prove $\hat{E}_2 = \hat{D}_2$	(4)
9.4	Prove $AE = \sqrt{AD \times AC}$.	(5)
9.5	Hence, show that AE = AB.	(3) **[16]**

QUESTION 10

10.1 In ΔABC below, D and E are points on AB and AC respectively such that $DE \parallel BC$. Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



10.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that $RS \parallel BP$ and $\frac{AR}{AB} = \frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$ (4)

10.2.2 $\frac{RT}{TC}$ (3)

10.2.3 $\frac{\text{Area of } \Delta RAS}{\text{Area of } \Delta RSC}$ (2)

10.2.4 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ (3)

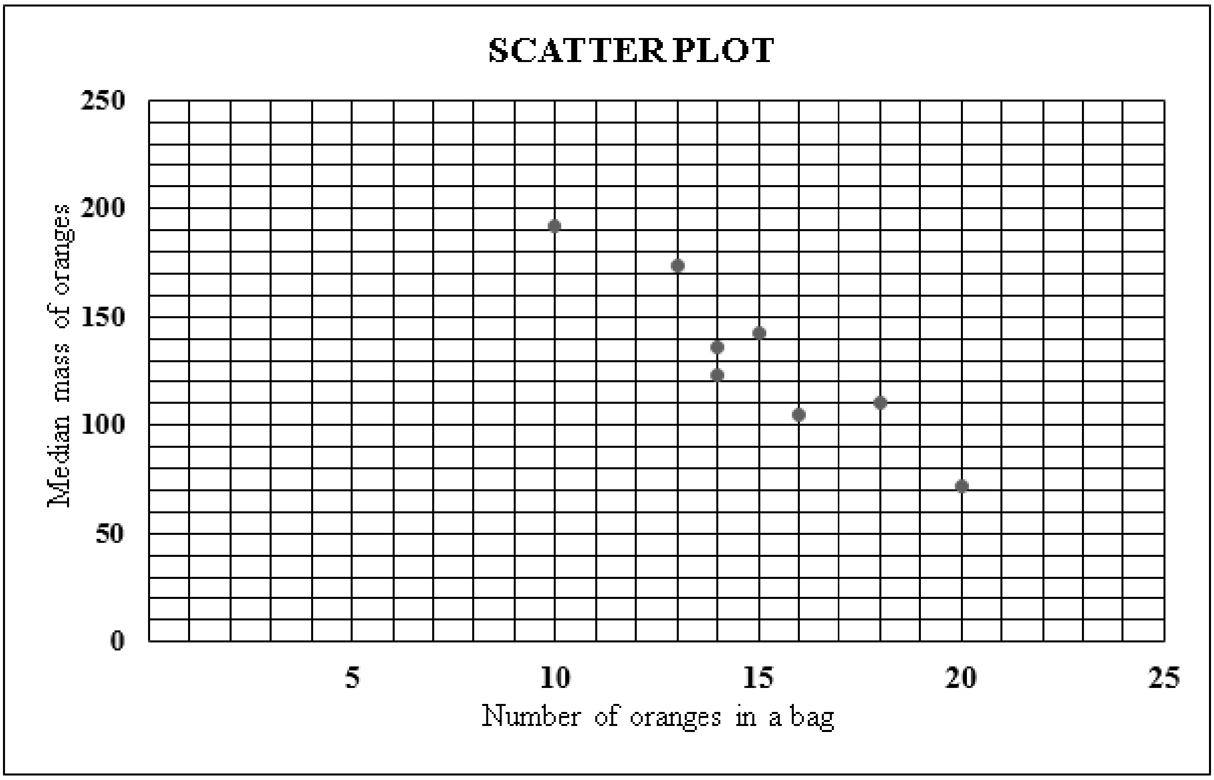
[18]

TOTAL: 150

QUESTION 1

A student is investigating the number of oranges in a bag in relation to the median mass of the oranges filled in the same bag. The findings are recorded in the table below.

Number of oranges in a bag	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram)	110	105	72	142	123	174	136	192



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Write down the correlation coefficient of the data. (1)

1.3 Draw the least squares regression line on the scatter plot given in your ANSWER BOOK. (2)

1.4 Comment on the strength of the relationship between the number of oranges in the bag and the median mass of the oranges. (1)

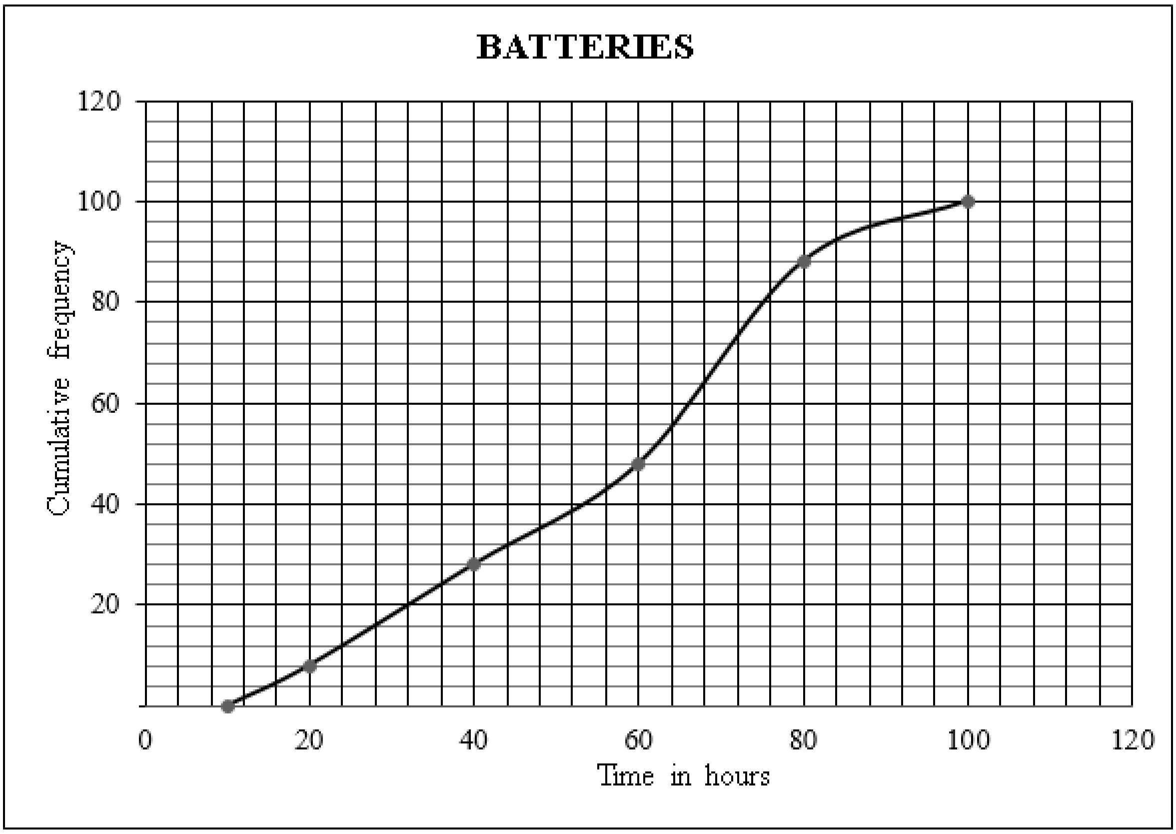
1.5 Determine the possible median mass of oranges in a bag, if there are 12 oranges in that bag. (2)

[9]

QUESTION 2

2.1 Batteries are used in everyday life. The Grade 12 Physical Sciences learners investigated the life span of batteries under constant test conditions.

The ogive (cumulative frequency graph) below shows the lifespan (in hours) of the batteries.



2.1.1 How many batteries were tested for this investigation? (1)

2.1.2 Use the graph to estimate the median time for the life span (in hours) of the batteries. (2)

2.1.3 The minimum lifespan of batteries is 10 hours and the maximum lifespan is 100 hours. Use the cumulative frequency graph to draw a box and whisker diagram in your ANSWER BOOK. (3)

2.1.4 Comment on the skewness of the distribution of the lifespan of the batteries. (1)

2.2 The table below represents values in a data set written in increasing order. None of the values in the data set are repeated.

5	a	19	b	c	d	35

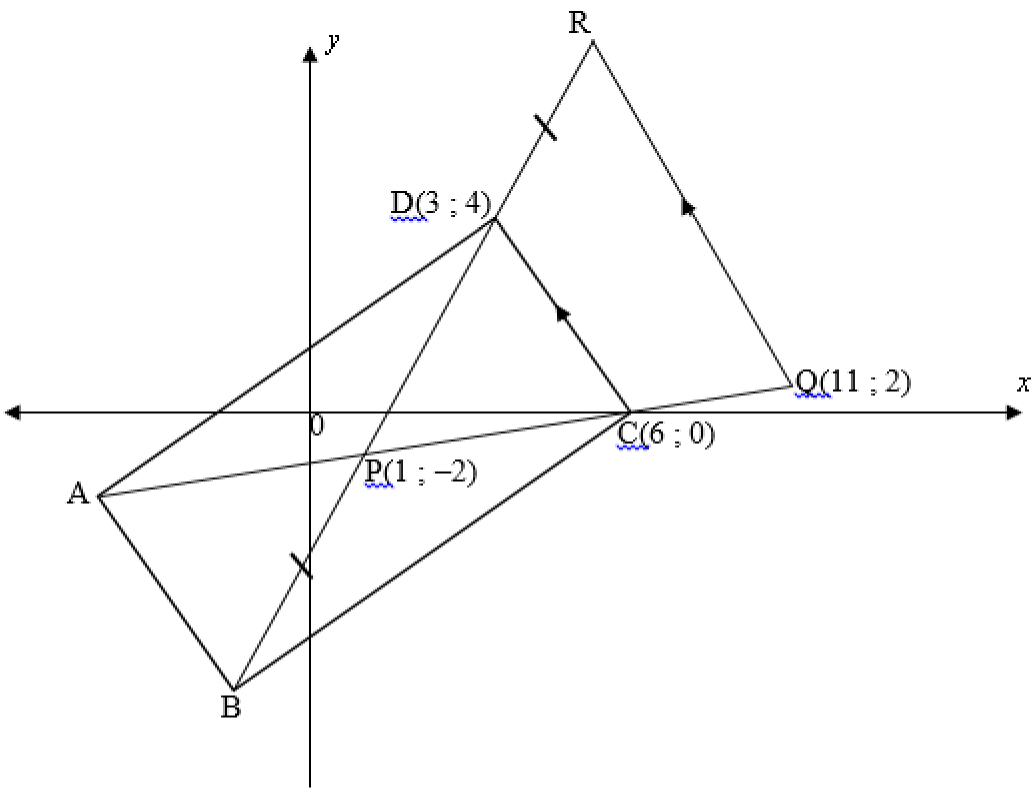
Determine the values of a, b, c, and d if:

- The median is 20.
- The semi interquartile range is 8.
- The upper quartile is twice the lower quartile.
- The mean is 22.

(4)
[11]

QUESTION 3

3.1 In the diagram below, A, B, C (6; 0) and D (3; 4) are the vertices of rectangle ABCD. Diagonals AC and BD bisect each other at P(1 ; – 2). AC is produced to Q(11; 2) and BD is produced to R such that BP = DR and CD ∥ QR.



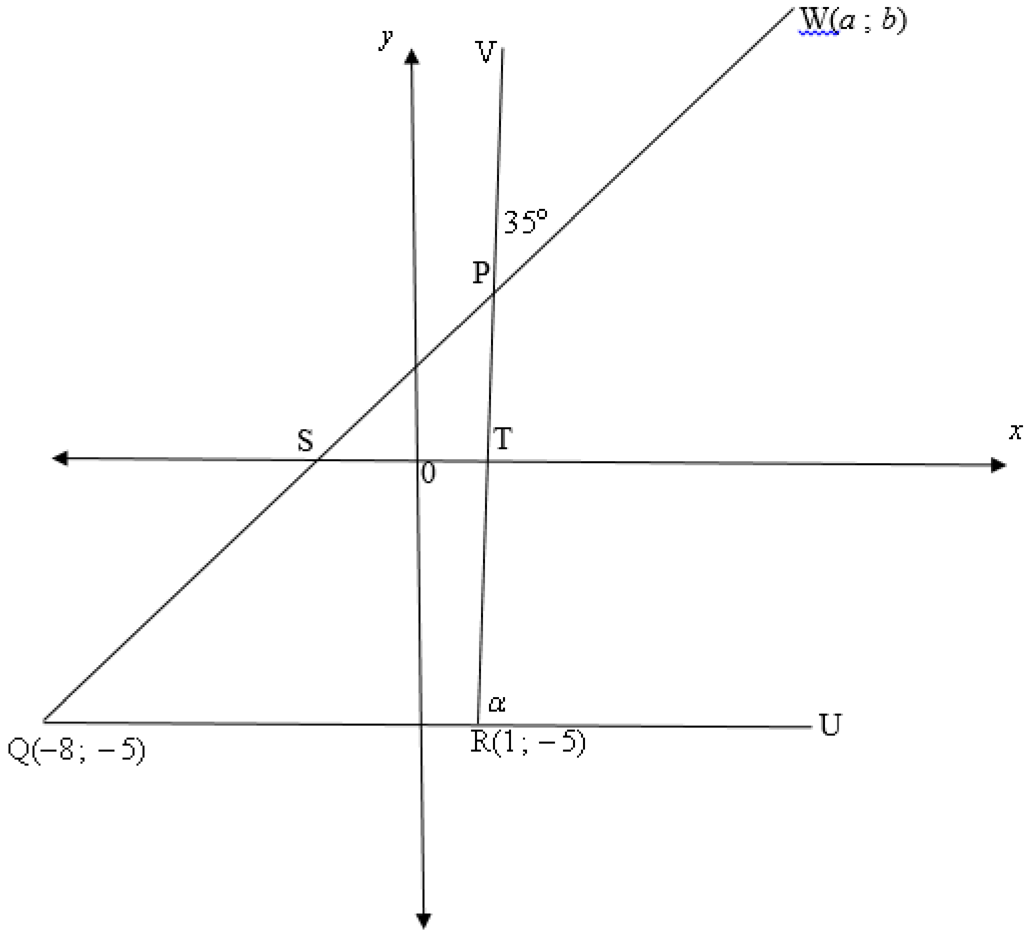
3.1.1 Calculate the coordinates of B. (3)

3.1.2 Determine the gradient of CD. (2)

3.1.3 Show that the equation of QR is $y = -\frac{4}{3}x + \frac{50}{3}$. (2)

3.1.4 If K(4 ; y) is a point in the 4th quadrant such that PK = RQ, calculate the value of y. (6)

3.2 In the diagram below, P, $Q(-8\,;-5)$ and $R(1\,;-5)$ are the vertices of $\triangle PQR$. RP is produced to V and QP is produced to $W(a\,;b)$ such that $V\hat{P}W = 35°$. The equation of QW is $y = x + \frac{2}{3}$. QR is produced to U and $U\hat{R}V = \alpha$. QW and RV intersect the x-axis at S and T respectively.



3.2.1 Calculate the size of α. (5)

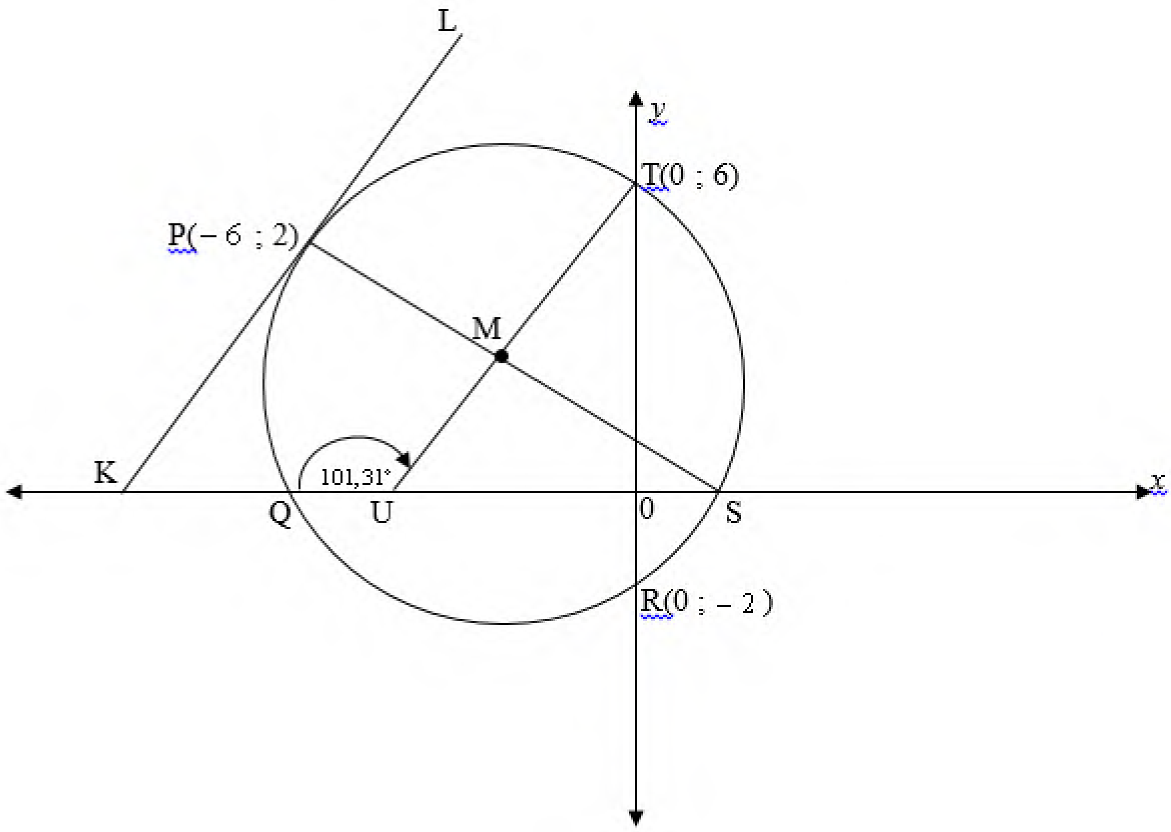
3.2.2 It is further given that $QU \perp WU$ and R is the midpoint of QU. Calculate the area of ΔQWU. (6)

[24]

QUESTION 4

In the diagram below, a circle with centre M, cuts the x-axis at Q and S and the y-axis at T(0 ; 6) and R(0 ; − 2). The equation of diameter SMP is $y = \frac{-1}{5}x + \frac{4}{5}$.

KPL is a tangent to the circle at P(−6 ; 2). TM produced cuts the x-axis at U. $Q\hat{U}T = 101,31°$.



4.1 Determine the equation of TU. (3)

4.2 Calculate the coordinates of M. (3)

4.3 If the coordinates of M are (−1 ; 1), determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.4 Prove that KL is parallel to TU. (3)

4.5 Is the point $V\left(-\frac{1}{2} ; 7\right)$ inside the circle? Support your answer with calculations. (3)

[15]

QUESTION 5

5.1 If $\sin 16° = \dfrac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a calculator.**

5.1.1 $\tan 16°$ (2)

5.1.2 $\cos 32°$ (3)

5.2 Simplify the following expression.

$$\frac{\cos(90°+x)\,\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$ (6)

5.3 Calculate the value of the following, **without the use of a calculator**.

$\cos 75°.\cos 45° - \cos 15°.\cos 45°$ (4)

5.4 Given: $\tan\theta\left(\sin 2\theta + \dfrac{3\cos^2\theta}{\sin\theta}\right) = -2\cos^2\theta + 3\cos\theta + 2$

5.4.1 Prove the identity. (3)

5.4.2 Determine the general solution of:

$$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right) = 0$$ (4)

5.5 Solve for a and b:

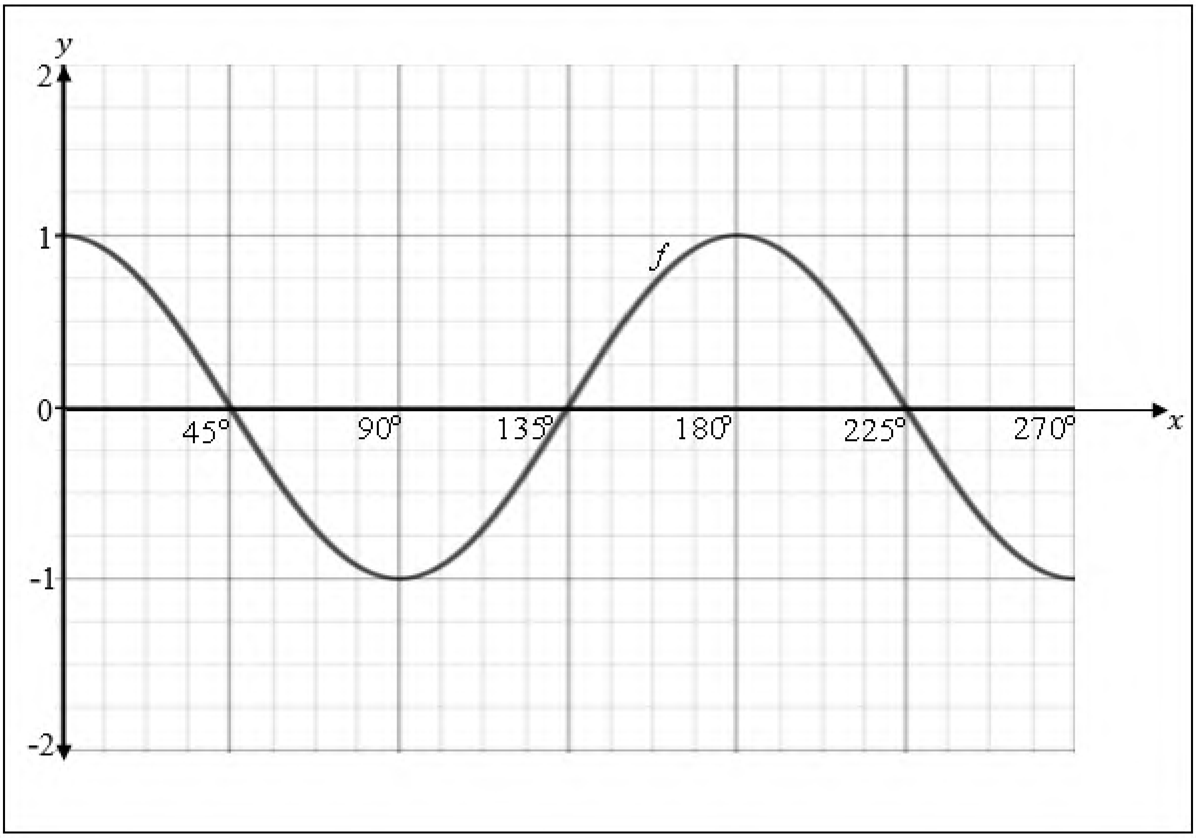
$\cos(a+b) = -\dfrac{\sqrt{2}}{2}$ if $a+b \in [0° ; 180°]$

$\cos(a-2b) = \dfrac{1}{2}$ if $a-2b \in [0° ; 180°]$ (4)

[26]

QUESTION 6

In the diagram below, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [0°;\ 270°]$.



6.1 Draw the graph of $g(x) = -\frac{1}{2}\tan x$ for the interval $x \in [0°\ ;\ 270°]$ on the grid provided in the ANSWER BOOK. Show all intercepts with the axes and asymptotes. (4)

6.2 Write down the range of $h(x) = 3 - f(x)$. (1)

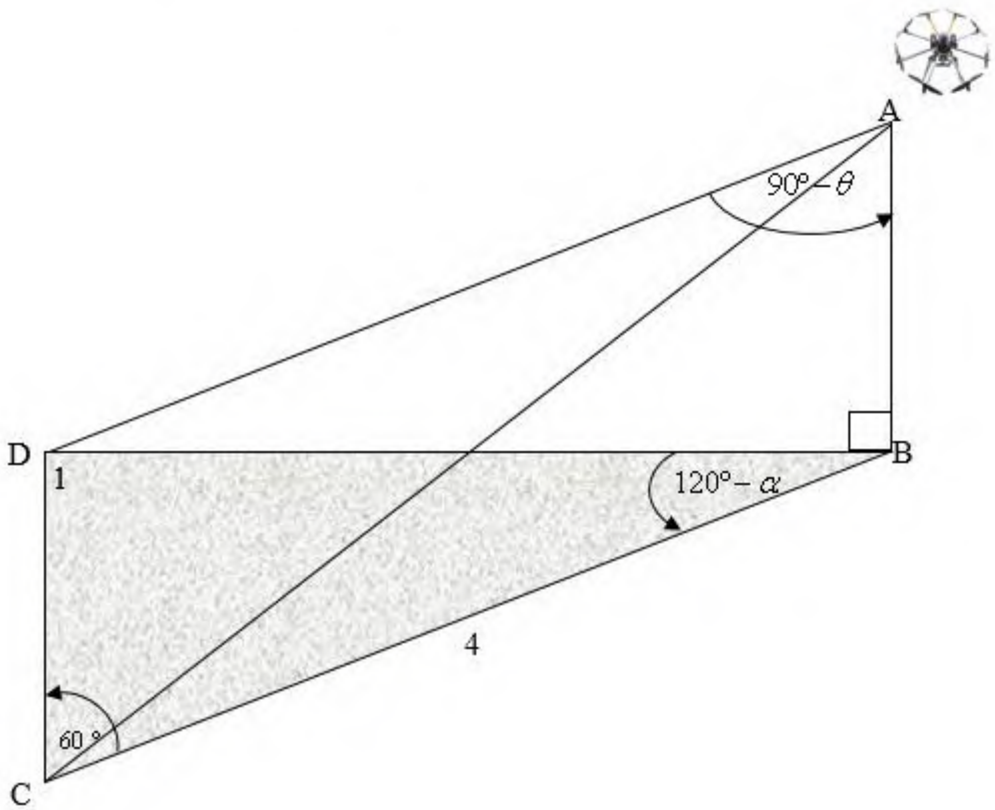
6.3 Use the graph to determine the value(s) of x in the interval $x \in [135°\ ;\ 270°]$ for which $\frac{f(x)}{g(x)} \geq 0$. (2)

[7]

QUESTION 7

In the diagram below, B, C and D are in the same horizontal plane. A drone positioned at A, captures the images of two objects at B and C. B is directly below the drone and C is 4 units away from B.

$D\hat{C}B = 60°$; $D\hat{B}C = 120° - \alpha$ and $D\hat{A}B = 90° - \theta$.



7.1 Determine $\hat{D}_1$ in terms of α . (2)

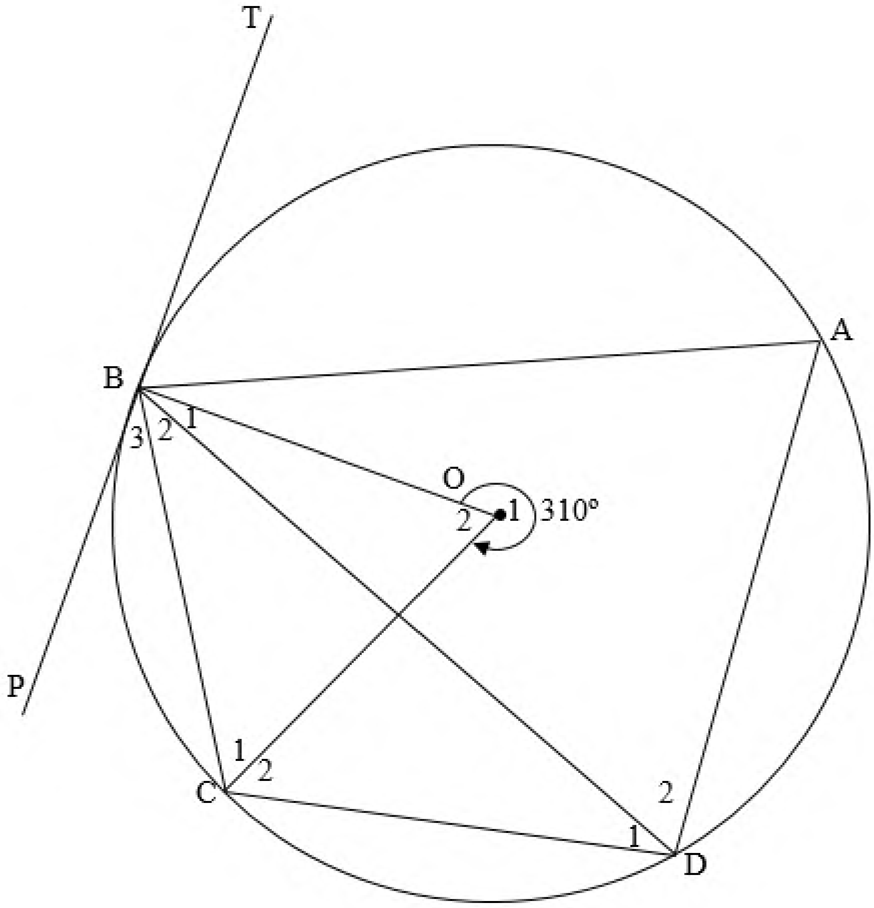
7.2 **Without the use of a calculator**, determine BD in terms of α . (3)

7.3 Show that $AB = \dfrac{2\sqrt{3}\tan\theta}{\sin\alpha}$. (3)

[8]

QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O.
PBT is a tangent to the circle at B.
Reflex $B\hat{O}C = \hat{O}_1 = 310°$ as shown in the diagram below.



Calculate, giving reasons, the size of:

8.1 $\hat{D}_1$ (3)

8.2 $\hat{B}_3$ (2)

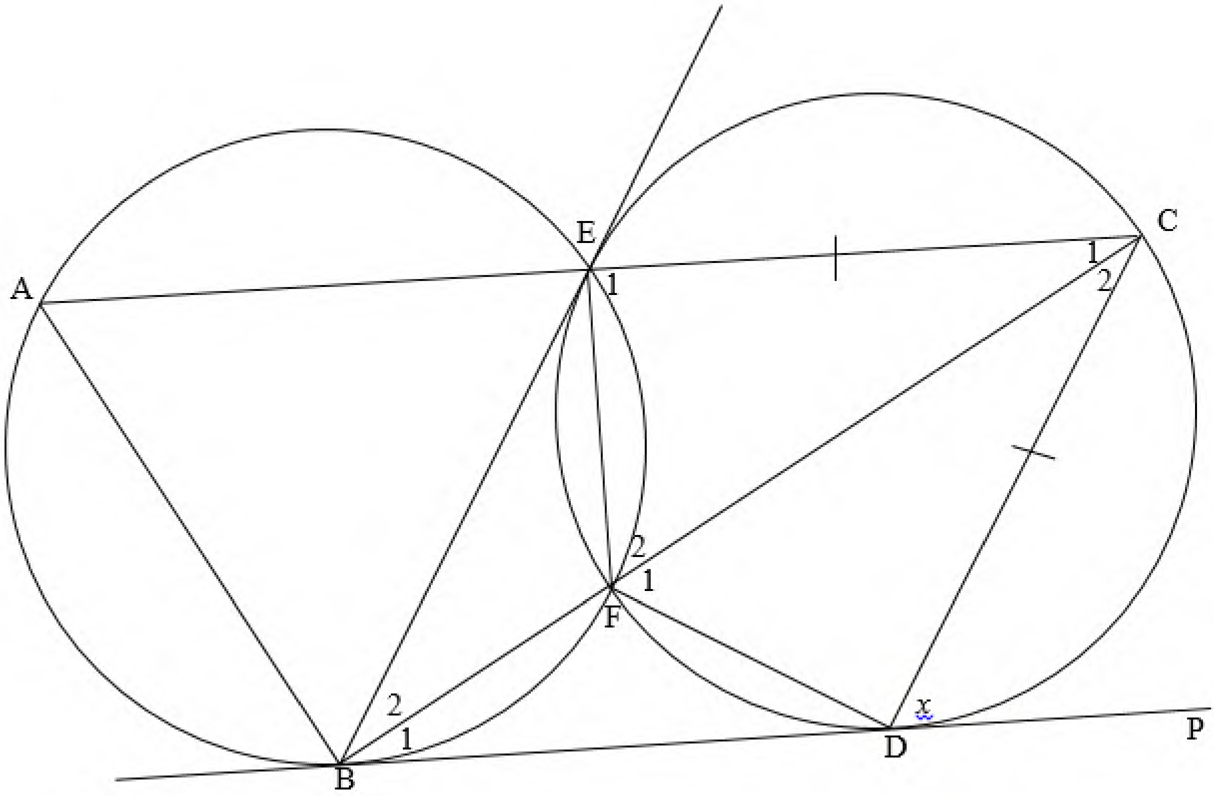
8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60°$. (4)

[9]

QUESTION 9

9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are … (1)

9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. EC = CD.
Let $C\hat{D}P = x$



Prove, giving reasons, that:

9.2.1 $\hat{F}_1 = \hat{F}_2$. (3)

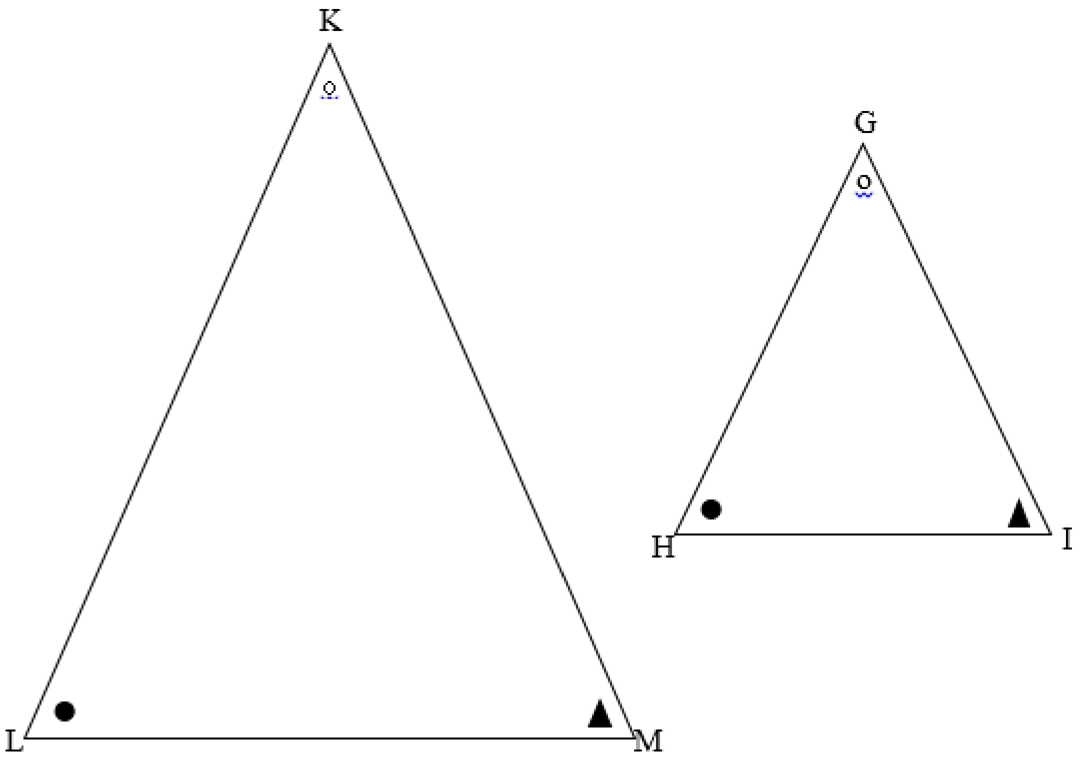
9.2.2 ABDC is a cyclic quadrilateral. (3)

9.2.3 BE || CD. (2)

9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)
[14]

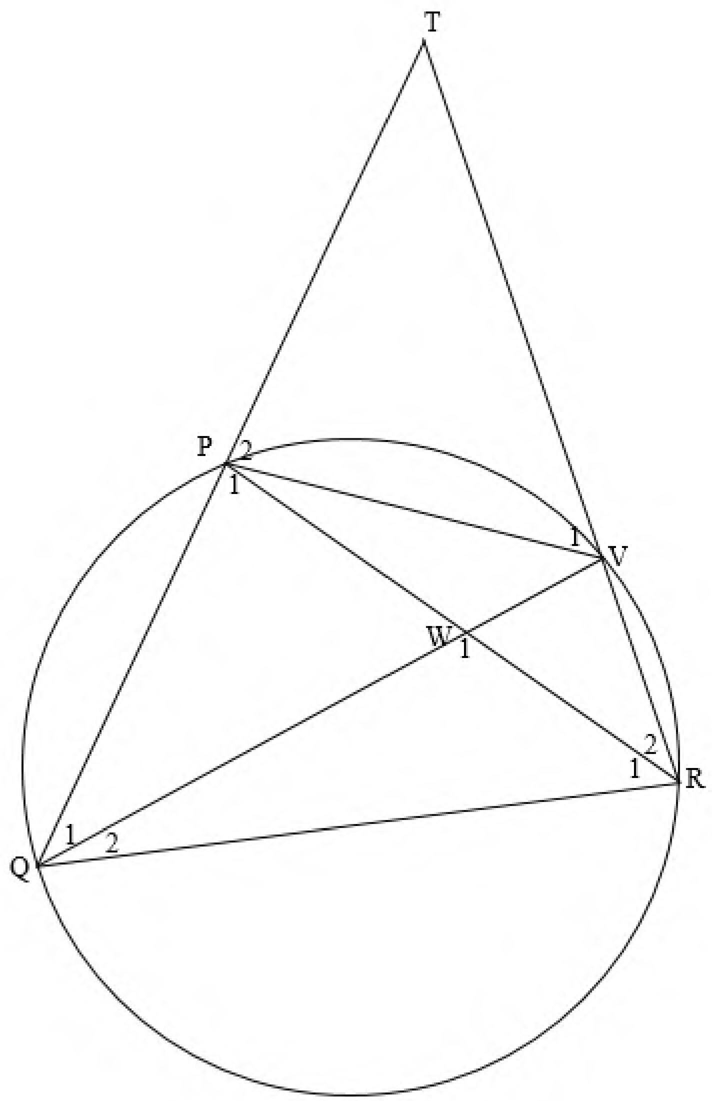
QUESTION 10

10.1 In the diagram below, ΔKLM and ΔGHI are drawn such that $\hat{K} = \hat{G}$, $\hat{L} = \hat{H}$ and $\hat{M} = \hat{I}$. Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. prove that $\frac{KL}{GH} = \frac{KM}{GI}$.



(5)

10.2 In the diagram below, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T. PR and QV intersect at W.



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = T\hat{R}Q$ (3)

10.2.2 $\Delta TQR ||| \Delta QRW$ (3)

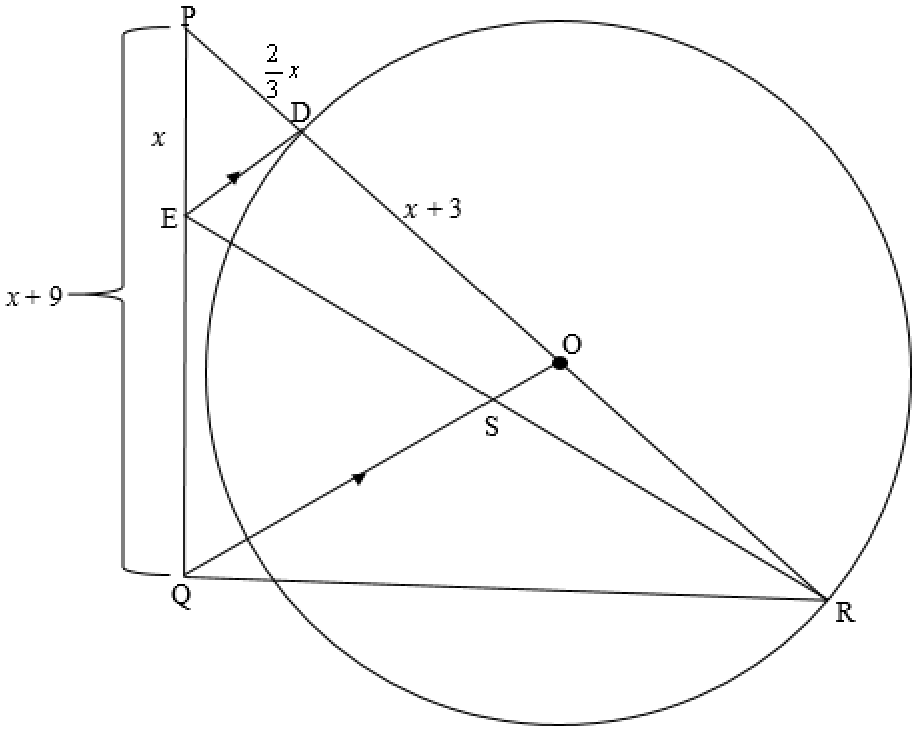
10.2.3 $\frac{PT}{QW} = \frac{PV}{WR}$ (6)

[17]

QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ.
RD is produced to P and PQ is joined.

PE = x units, PQ = $x + 9$ units, PD = $\frac{2}{3}x$ units and DO = $x + 3$ units.



11.1 Calculate the length of RO. (4)

11.2 If OS = 1,4 units and S is the midpoint of ER, determine the length of DE. (2)

11.3 If the area of $\Delta PED = 2{,}7 \text{ units}^2$, find the area of ΔPER. (4)

[10]

TOTAL: 150

QUESTION 1

The table below shows the height, in metres, of 80 giraffes.

Height in m	Frequency
$4{,}6 \leq h < 4{,}8$	4
$4{,}8 \leq h < 5{,}0$	7
$5{,}0 \leq h < 5{,}2$	15
$5{,}2 \leq h < 5{,}4$	33
$5{,}4 \leq h < 5{,}6$	17
$5{,}6 \leq h < 5{,}8$	4

[Image from *Stuff You Should Know*]

1.1 Calculate the estimated mean height of the giraffes. (2)

1.2 The height range of $4{,}8 \leq h < 5{,}0$ m, consisted entirely of 7 young male giraffes. The mean of the heights of the 7 young males was calculated to be 4,86 m. However, it was noticed that the height of one of the young males was incorrectly recorded as 4,98 m but should have been 4,89 m. Calculate the new mean of the 7 young males. (3)

1.3 Would you expect the standard deviation of the individual heights of the 80 giraffes to be more or less than the standard deviation of the individual heights of the 7 young male giraffes? Explain your answer. (2)

[7]

QUESTION 2

Cocoa solids are used to make chocolate.
A student is investigating the relationship between the percentage of cocoa solids in a 100g slab of chocolate and the price of the slab (in rands). The data obtained is shown in the table below.

Chocolate brand	A	B	C	D	E	F	G	H
x (% of cocoa)	10	20	30	35	40	50	60	70
y (price in rand)	6,50	10,20	9,40	24,00	11,20	16,80	20,50	24,20

2.1 Use the grid in the ANSWER BOOK to represent this data in a scatter plot. (2)

2.2 Determine the equation of the least squares regression line for this data. (3)

2.3 Draw the least squares regression line in the ANSWER BOOK on the same grid used in QUESTION 2.1. (2)

2.4 Calculate the correlation coefficient for the percentage of cocoa solids and the price of the chocolate slab. (1)

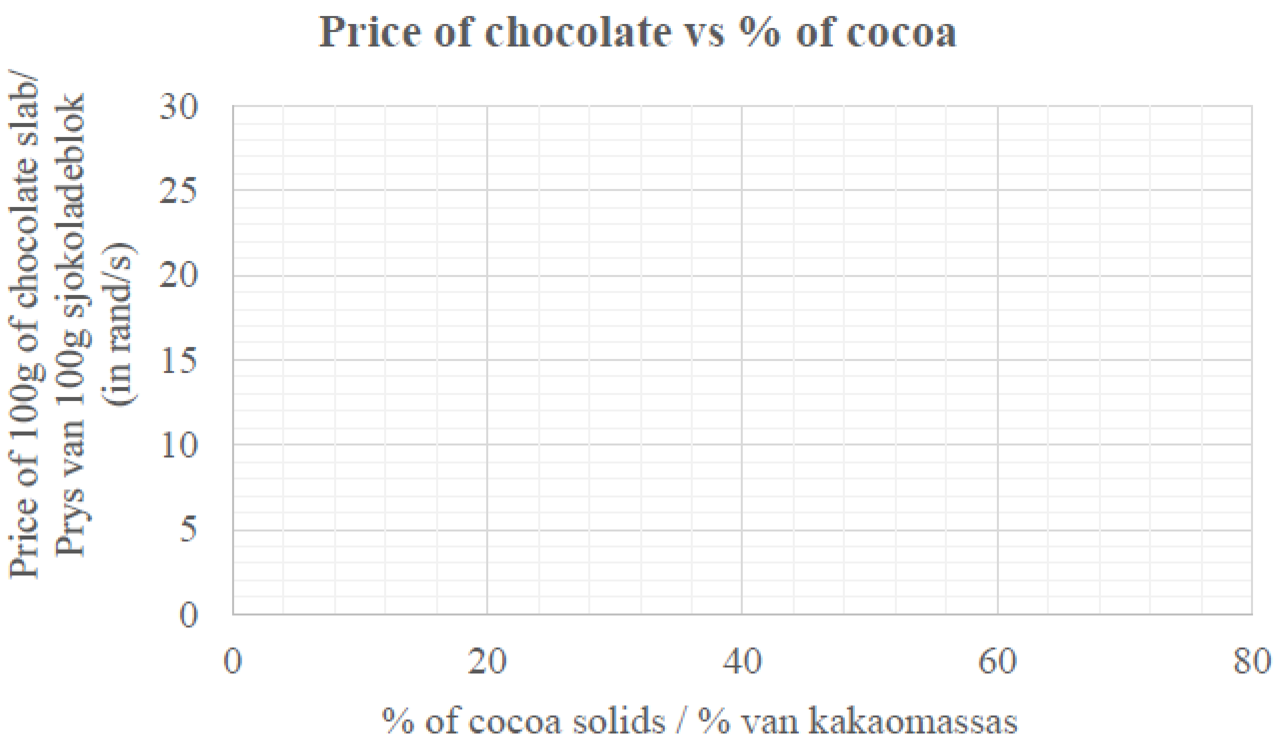
2.5 Comment on the relationship between the percentage of cocoa solids and the price of the chocolate slab. (1)

2.6 The student believes one brand of chocolate is overpriced.

2.6.1 Identify the chocolate brand which is over-priced. (1)

2.6.2 Estimate by how much more this slab of chocolate is overpriced. (3)

[13]

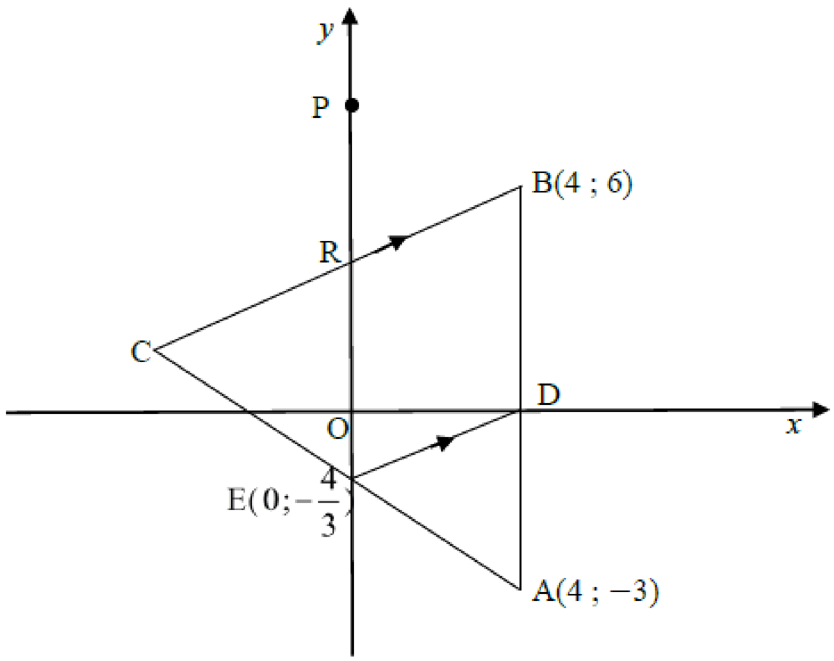


QUESTION 3

In the diagram below, ΔABC is shown with coordinates of A (4; – 3) and B (4 ; 6) given. AB intersects the x-axis at D and AC intersects the y-axis at E.

The coordinates of E are $(0\,;-\frac{4}{3})$. BC intersects the y-axis at R. P is a point on the y-axis.

DE ǁ BC.



3.1 Write down the coordinates of D. (1)

3.2 Determine the gradient of DE. (2)

3.3 Calculate $P\hat{R}B$. (4)

3.4 Calculate the length of DE in simplest surd form. (2)

3.5 Determine the ratio AD: AB in its simplest form. (2)

3.6 Hence, or otherwise, calculate the length of BC. (3)

3.7 Determine:

3.7.1 The midpoint of DE (2)

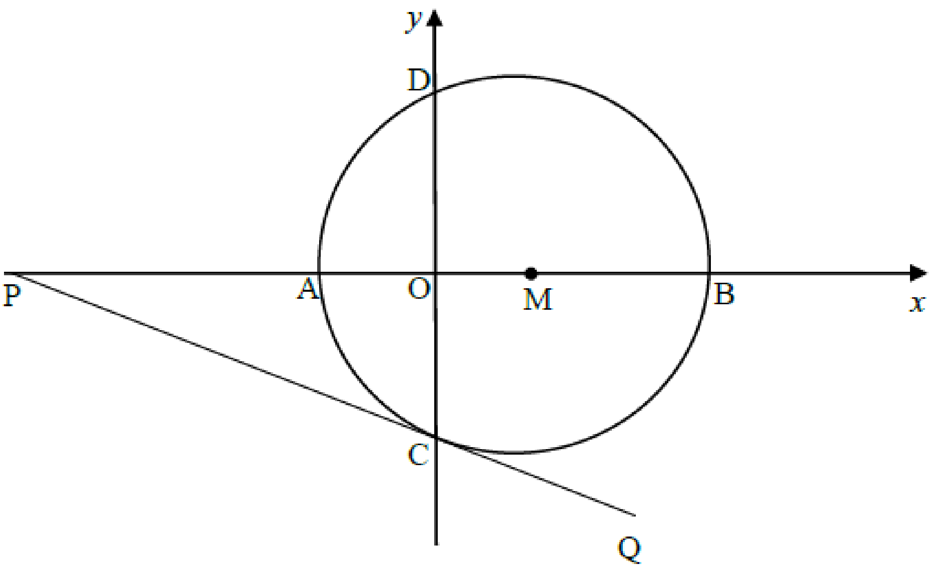
3.7.2 The equation of the perpendicular bisector of DE in the form $y = mx + c$ (3)

3.8 Does this perpendicular bisector pass through A? Justify your answer. (2)

[21]

QUESTION 4

The diagram below shows a circle having centre M which intersects the x-axis at A and B and the y-axis at D and C. PCQ is a tangent to the circle at C, the point of contact on the y-axis. P lies on the x-axis.
The equation of the circle is: $x^2 + y^2 - 6x - 16 = 0$



4.1 Show that the coordinates of M are (3; 0) and that the radius is 5 units. (3)

4.2 Determine:

4.2.1 The coordinates of B (3)

4.2.2 The coordinates of C (2)

4.3 If the length of PM is $8\frac{1}{3}$ units, calculate the length of PC. (3)

4.4 Calculate the angle subtended by the chord DC at B, i.e. find $D\hat{B}C$. (4)

4.5 If the given circle is moved 2 units right and 1 unit up, determine the equation of the tangent to the circle in its new position passing through point C'. (4)

[19]

QUESTION 5

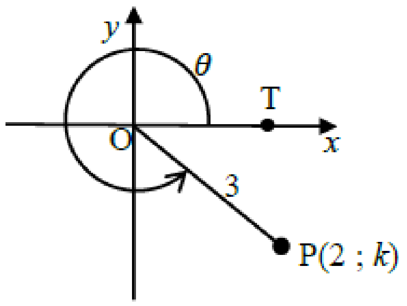
DO NOT USE A CALCULATOR FOR THIS QUESTION.

5.1 Complete the following identities:

5.1.1 $\cos^2 A + \sin^2 A = \ldots$ (1)

5.1.2 $\cos^2 A - \sin^2 A = \ldots$ (1)

5.2 P(2 ; k) is a point in the Cartesian plane such that OP = 3 units and reflex angle $T\hat{O}P = \theta$, as shown in the diagram below.



5.2.1 Calculate the value of k (leave your answer in surd form). (2)

5.2.2 Hence, determine the value of the following:

(a) $\tan(\theta - 180^\circ)$ (2)

(b) $\dfrac{1 - \sin^2 2\theta}{1 - 2\sin^2 \theta}$ (4)

5.3 Determine, **without the use of a calculator**, the value of:

$\sin(-200^\circ).\cos 310^\circ + \tan(-135^\circ).\cos 380^\circ\ .\sin 230^\circ$ (6)

5.4 Prove the following identity:

$\sin 2\theta + \cos(2\theta - 90^\circ) = 4\sin\theta\cos\theta$ (3)

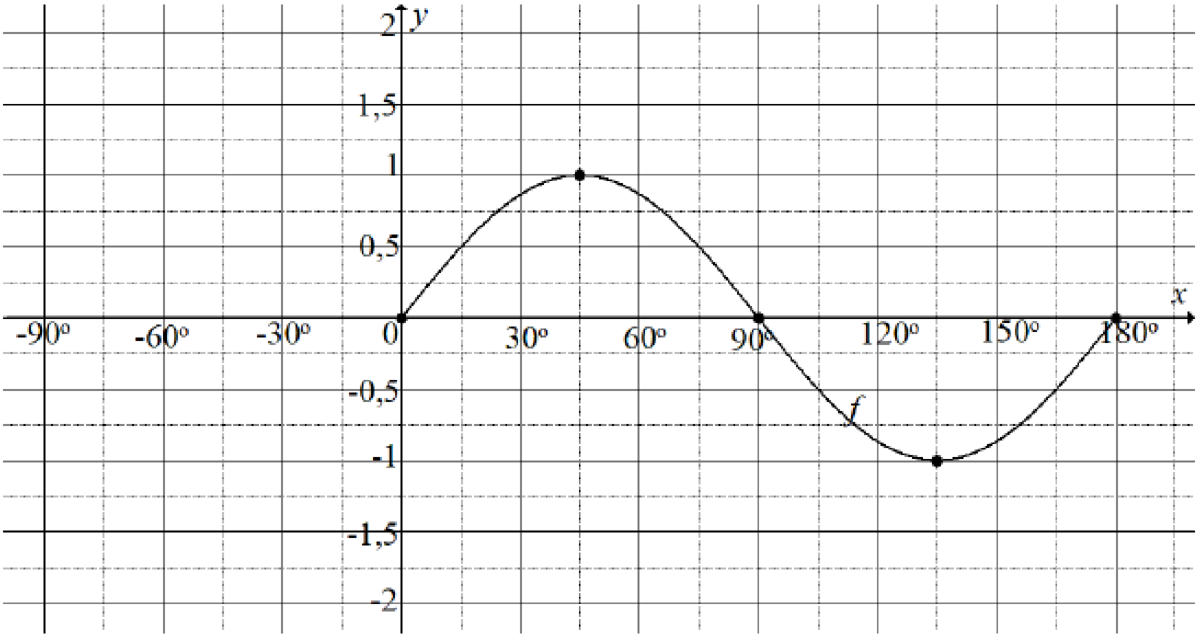
5.5 Solve for x if:

$10^{\sin x} + 10^{\sin x + 1} = 110 \quad \text{for } -360^\circ \le x \le 360^\circ$ (5)

[24]

QUESTION 6

The graph below shows part of the function $f(x) = \sin 2x$ for $0° \leq x \leq 180°$.



6.1 On the grid provided in the ANSWER BOOK, complete the graph of f for the interval $-90° \leq x \leq 180°$. (1)

6.2 On the same grid, draw the graph of $g(x) = \cos(x - 30°)$ for the interval $-90° \leq x \leq 180°$. Clearly show the intercepts with the axes, the coordinates of the turning points and the end points of the graph. (4)

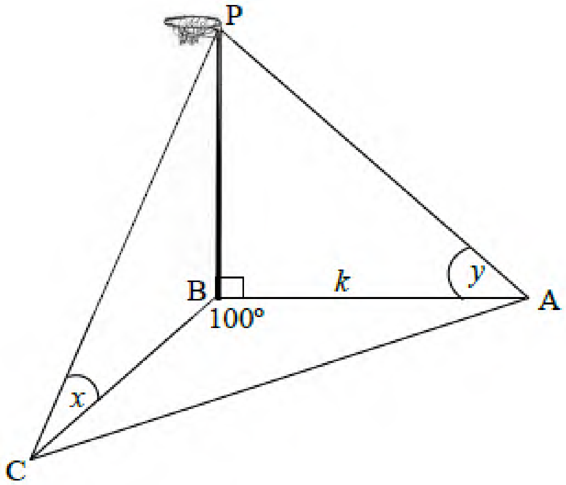
6.3 Calculate the solutions to the equation:

$$\sin 2x = \cos(x - 30°) \quad \text{for} \quad -90° \leq x \leq 90°$$ (6)

[11]

QUESTION 7

The diagram below shows a vertical netball pole PB. Player A is standing on the base line of the court and the angle of elevation from A to the top of the pole P is $y°$.
A second player is standing in the court at C, and the angle of elevation from C to P is $x°$.
Points A, B and C are in the same horizontal plane. BA is k metres; $A\hat{B}C = 100°$.



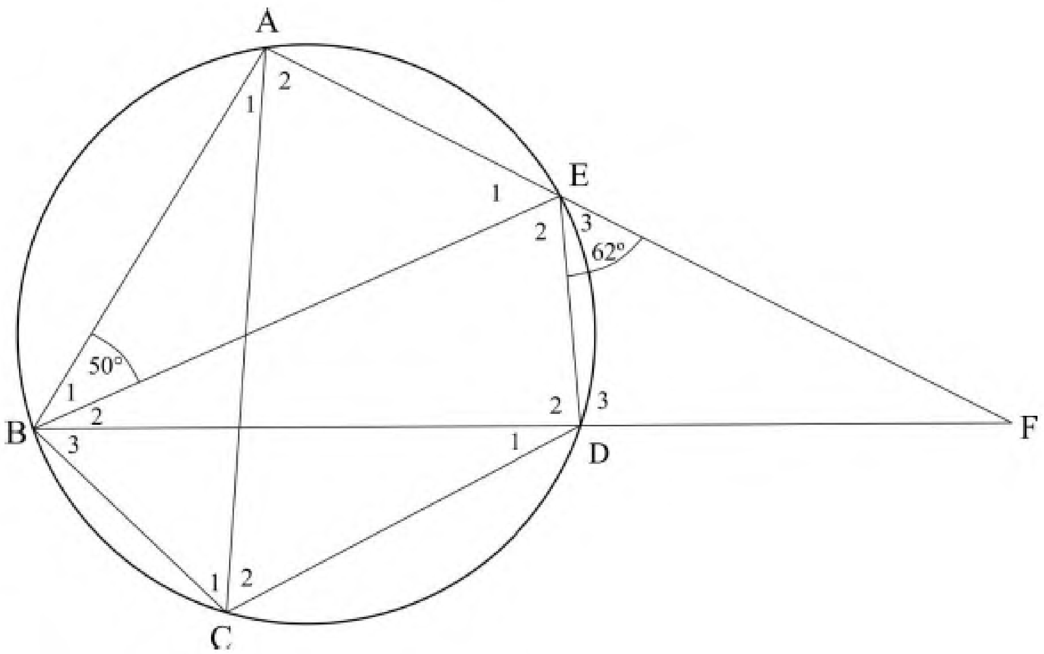
7.1 Show that $BC = \dfrac{k.\tan y}{\tan x}$ (3)

7.2 Calculate the length of AC if BC = 4,73 m and $k = 3$ m. (3)

[6]

QUESTION 8

8.1 In the given diagram, A, B, C, D and E are points on the circle.
BE is a diameter. $\hat{E}_3 = 62°$ and $\hat{B}_1 = 50°$.
BD produced meets AE produced at F.



Determine, with reasons:

8.1.1 $B\hat{A}E$ (2)

8.1.2 $\hat{E}_1$ (2)

8.1.3 $\hat{C}_1$ (2)

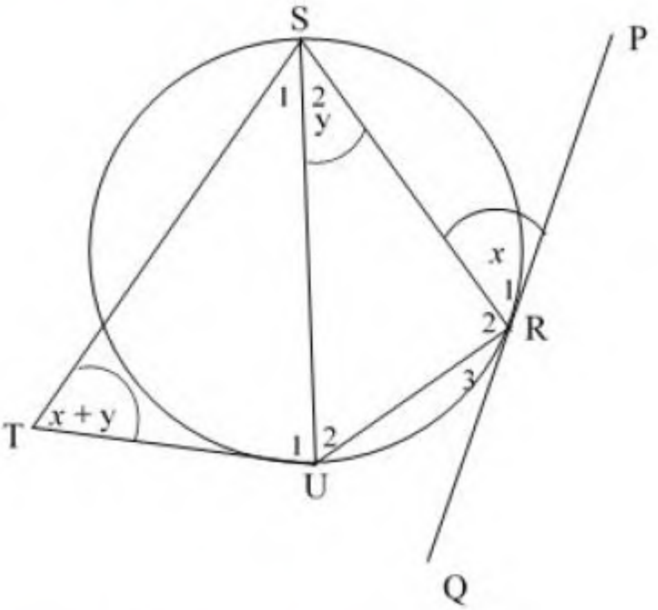
8.1.4 $\hat{C}_2$ (2)

8.1.5 $A\hat{B}D$ (2)

8.2 Complete the following theorem statement:

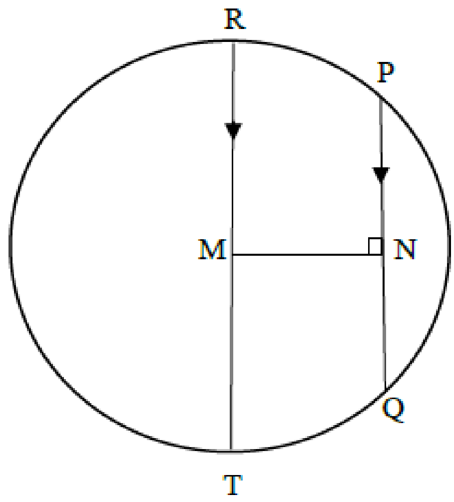
The angle between the tangent to a circle and the chord drawn from the point of contact is … (1)

8.3 In the diagram below, PRQ is a tangent to the circle SUR at R. SU, SR and UR are drawn. Lines from S and U produced meet at T outside the circle.
$\hat{R}_1 = x$; $\hat{S}_2 = y$ and $\hat{STU} = x + y$



Prove that STUR is a cyclic quadrilateral. (5)

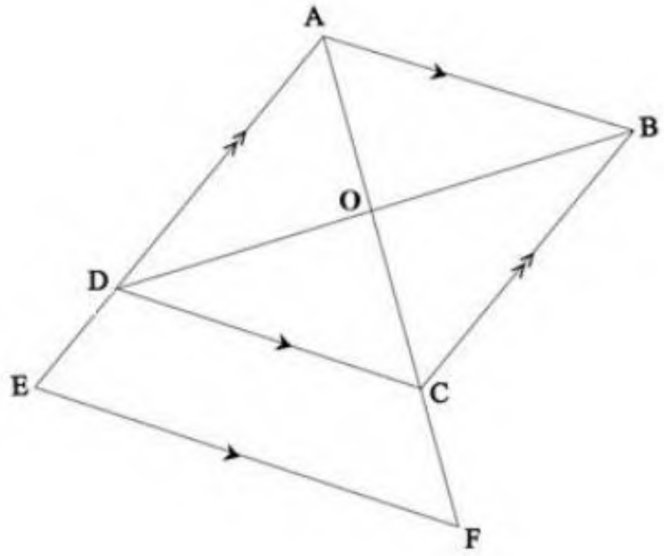
8.4 The diagram below shows a circle centre M passing through the points R, P, Q and T.
RT is the diameter. PQ is a chord such that PQ ∥ RT and MN ⊥ PQ.
PQ = 16 units, MN = 6 units.



Determine the length of RT. (4)
[20]

QUESTION 9

In the diagram below, ABCD is a parallelogram. AD and AC are produced to E and F respectively so that EF || DC. AF and DB intersect at O.
AD = 12 units; DE = 3 units; DC = 14 units; CF = 5 units.



9.1 Calculate, giving reasons, the length of:

9.1.1 AC (3)

9.1.2 AO (1)

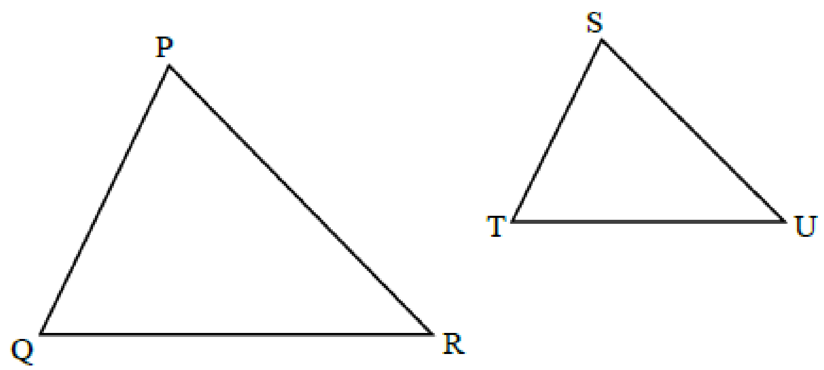
9.1.3 EF (3)

9.2 Prove that $\frac{\text{area } \Delta ADO}{\text{area } \Delta AEF} = \frac{8}{25}$ (3)

[10]

QUESTION 10

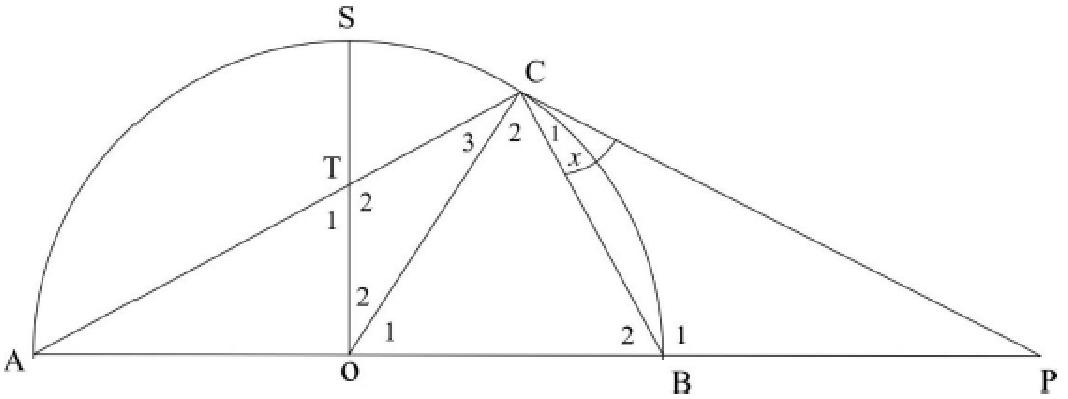
10.1 For the triangles shown below,



prove that if $\hat{P} = \hat{S}$ and $\hat{Q} = \hat{T}$ and $\hat{R} = \hat{U}$ then

$$\frac{ST}{PQ} = \frac{SU}{PR}$$ (6)

10.2 In the diagram below, O is the centre of a semi-circle ACB. S is a point on the circumference and T lies on AC such that STO ⊥ AB. Diameter AB is produced to P, such that PC is a tangent to the semi-circle at C. Let $\hat{C}_1 = x$.



10.2.1 Write down, with reasons, 2 other angles equal to x. (3)

10.2.2 Prove that $\Delta TOC \,|||\, \Delta BPC$ (5)

10.2.3 Prove that $TO.PC = OB.BP$ (2)

10.2.4 If $BP = OB$, show that $3OC^2 = PC^2$ (3)

[19]

TOTAL: 150

PAPER 2 ANSWERS

NORTH WEST 2020 TRIALS PAPER 2:

1.2.1 18,63

1.3 1,05

1.4.1 38

1.4.2 11

2.1 $y = -106,95 + 14,76x$

2.2 0,94

2.3 Yes (365,37 visitors using the equation)

2.3 $r = 0,98$

2.4 Madiba pool; gradient value is higher

3.1 $x = -20$

3.2 $C(-4;0)$

3.3 65,97 units

3.5 $y = -\frac{2}{7}x + \frac{408}{7}$

3.6 $74,05^\circ$

3.7 834,22 units^2

4.1 5 units

4.2 $F(3;-2)$

4.3 10 units

4.6 8,94

5.1.1 -3

5.1.2 a) $\frac{3}{\sqrt{10}}$

5.1.2 b) $-\frac{3}{\sqrt{10}}$

5.2.1 $-1 \leq d \leq 1$

5.2.2 45°

5.3 1

6.1 $a = 3$ and $b = \frac{1}{3}$

6.2 540°

6.3 $A(-154,69^\circ; -1,28)$

6.4 $\frac{t+5}{2}$

7.2 90°

7.4 7,96 units

8.1.1 20°

8.1.2 10°

8.1.3 10°

8.1.4 70°

8.2 $\frac{\sqrt{3}}{2}y - x$

9.2.1 6 units

9.2.4 No. For $M'N'P'$ to be a tangent to circle LOS, $L\hat{S}M'$ should be equal to $L\hat{O}S$, but that is not possible, since $L\hat{O}S = 90^\circ$, and $L\hat{S}M' < 90^\circ$.

10.1.1 alternate $\angle$s; HC || GF

10.1.2 tan-chord theorem

10.3 $\frac{1}{2}$

GAUTENG 2020 TRIALS PAPER 2:

1.1 51

1.2 skewed to the right (positively skewed)

1.3 Physical Sciences performed better; $Q_1 = 40\%$ in Physical Sciences and 28% in Mathematics; this indicates that the lowest 25% of the class performed much better in Physical Sciences than in Maths.

1.4 any mark between 40 and 50 is acceptable

2.1 $y = 12{,}41 + 0{,}49x$

2.2 $85{,}91 \approx 86\%$ (if substituted); or $85{,}17 \approx 85\%$ (directly from calculator).

2.3 y-intercept is 12,41, which means that a learner who did not even start the exam is getting a mark of 12,41%.

2.4 10,28

2.5 24,33

3.1 $E(6;3)$

3.2 $y = 3x - 15$

3.3 9

3.4 $P(27;-4)$

4.1 $y = -x + 2$

4.2 $a = -4$ and $b = 2$

4.3 $(x+4)^2 + (y-2)^2 = 8$

4.4 $(x-1)^2 + (y-1)^2 = 2$

4.5 $D(1;1)$

4.6 4 units

5.1 $\sqrt{3} - 1$

5.2 -1

5.3 $x = 210^\circ + k.360^\circ$ or $x = 330^\circ + k.360^\circ$ or $x = 56{,}31^\circ + k.180^\circ$, $k \in Z$

5.5.1 $\sqrt{1-k^2}$

5.5.2 $\sqrt{1-k^2} + 1$

6.1 $a = -1$ and $d = 2$

6.2 $D\left(-150^\circ; \frac{1}{2}\right)$

6.3.1 $-90^\circ < x < 90^\circ$

6.3.2 $-135^\circ < x < -45^\circ$

7.1 $54{,}74^\circ$

7.2 12,62 m

8.1.1 67°

8.1.2 113°

8.1.3 67°

8.1.4 46°

8.1.5 113°

9.1.1 a) $2x$

9.1.1 b) $90^\circ - x$

9.2.1 $\hat{Q}$, $\hat{M}_4$ and $\hat{W}_1$

10.2.1 a) $\frac{2}{3}$

10.2.1 b) $\frac{5}{2}$

10.2.2 $\frac{\sqrt{10}}{3}$ units

WESTVILLE BOYS HIGH PAPER 2:

1.1.1 (a) 28,67

1.1.1 (b) 27

1.1.1 (c) 34 – 18 = 16

1.1.2 14,41

1.1.3 3 people

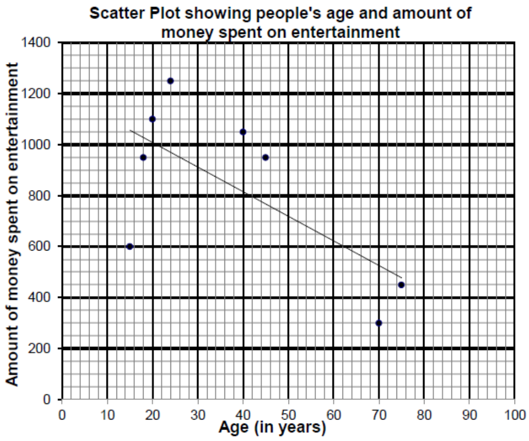
1.1.4 Friday people

1.2.1 $-0{,}672$

1.2.2 moderate negative correlation

1.2.3 R670,74

1.2.4



1.2.5 the correlation is not very strong (only moderate)

2.1 $\sqrt{10}$

2.2 $M\left(-\frac{1}{2};\frac{1}{2}\right)$

2.3 -3

2.4 P, Q and T are collinear

2.5 $R(-6;-3)$

2.6 $W(-7;0)$

2.7 $26{,}57^{\circ}$

3.1.1 $(x+3)^2+(y-4)^2=58$

3.1.2 $y=\frac{3}{7}x+\frac{37}{7}$

3.2.1 $y=-\frac{2}{3}x-\frac{8}{3}$

3.2.3 $(x+1)^2+(y+2)^2=13$

4.1.1 -10

4.1.2 (a) $-\frac{12}{5}$

4.1.2 (b) $\frac{12-5\sqrt{3}}{26}$

4.2 $\cos B$

4.4 $\theta=30^{\circ}+360.k$ or $150^{\circ}+360.k$ or $270^{\circ}+360.k$, $k\in Z$

4.5 $\frac{f-g}{f+g}$

5.1 720°

5.2 $p=\frac{1}{2}$; $q=60^{\circ}$

5.3 $-360^{\circ}\le\theta<-240^{\circ}$ or $-180^{\circ}\le\theta<-60^{\circ}$

7.1.1 (a) tan-chord-theorem

7.1.1 (b) = chords subtend = angles

7.1.2 $180^{\circ}-2x-y$

7.2.1 double (or: twice)

8.1.1 24 units

8.1.2 $\frac{4}{13}$

8.1.3 $\frac{108}{17}$ or 6,35 units

8.2 $\frac{25}{128}$

NORTH WEST SEPT. 2019 PAPER 2:

1.2 $p = 73$

1.3 149,25

1.4.1 $\sigma = 37{,}64$

1.4.2 $q = 111{,}61$

1.5 146,25

2.1 (68 ; 19)

2.2 $y = 7{,}35 + 0{,}98x$

2.3 88,69% (88,35% with calculator)

2.4 $r = 0{,}96$

very strong correlation

2.5 no influence on the gradient

3.1 $x = -5$

3.2 $-\frac{4}{3}$

3.3 $y = \frac{3}{4}x + \frac{19}{4}$

3.4 $165{,}96^{\circ}$

3.5 $39{,}09^{\circ}$

3.6 8 units

4.1 $x^2 + y^2 = 45$

4.2 $A(0; \sqrt{45})$

4.3 $y = \frac{1-\sqrt{5}}{2}x + \sqrt{45}$

4.4 $-\sqrt{45} < k < \sqrt{45}$

4.5.1 $(x-2p)^2 + (y+p)^2 = 20$

4.5.2 $p = 5$

4.6 $\sqrt{13}$

5.1.1 $-\sqrt{3}$

5.1.2 $\frac{1}{\sqrt{2}}$

5.2.1 0° ; 90° ; 180°

6.1 $a = 3;\ b = 2$

6.2 360°

6.3 $2 \leq y \leq 4$

6.4 $0^{\circ} < x < 45^{\circ}$ or $90^{\circ} < x < 135^{\circ}$

6.5 $q = 45^{\circ}$

7.2.2 17,96 units

8.1.1 68°

8.1.2 112°

8.1.3 68°

8.1.4 68°

8.1.5 44°

9.1 tan-chord-theorem

10.2.1 $\frac{3}{7}$

10.2.2 $\frac{2}{5}$

10.2.3 $\frac{3}{7}$

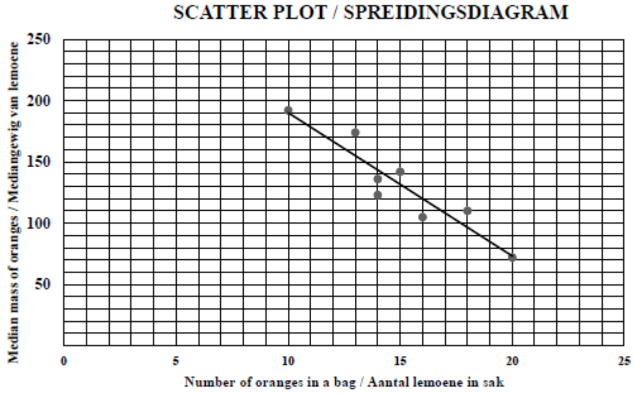
10.2.4 $\frac{25}{49}$

GAUTENG SEPT. 2019 PAPER 2:

1.1 $y = 307{,}20 - 11{,}7x$

1.1 $r = -0{,}93$

1.3



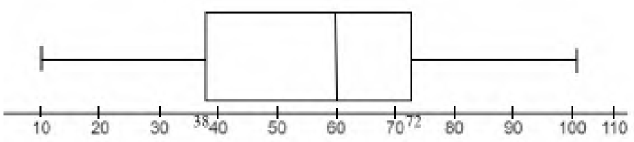
1.4 strong negative association

1.5 166,8

2.1.1 100

2.1.2 accept 61 or 62

2.1.3



2.1.4 skewed to the left

2.2 $a = 16;\ b = 20;\ c = 27;\ d = 32$

3.1.1 $B(-1;-8)$

3.1.2 $-\dfrac{4}{3}$

3.1.4 $y = -\dfrac{4}{3}x + \dfrac{50}{3}$

3.1.5 $-11{,}54$

3.2.1 $\alpha = 80^{o}$

3.2.2 141 units^2

4.1 $y = 5x + 6$

4.2 $M(-1\ ;\ 1)$

4.3 $(x+1)^2 + (y-1)^2 = 26$

4.5 does not lie within the circle

$6{,}02 > 5{,}1$

5.1.1 $\dfrac{1}{k}$

5.1.2 $2\left(\dfrac{k}{\sqrt{1+k^2}}\right)^2 - 1$ or $1 - 2\left(\dfrac{1}{\sqrt{1+k^2}}\right)^2$ or

$\left(\dfrac{k}{\sqrt{1+k^2}}\right)^2 - \left(\dfrac{1}{\sqrt{1+k^2}}\right)^2$

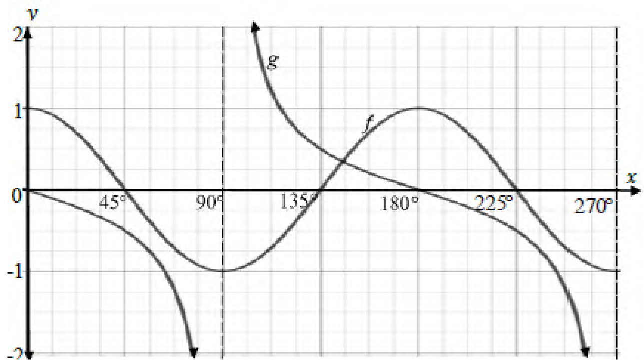
5.2 -1

5.3 $-\dfrac{1}{2}$

5.4.2 $\theta = 120^{o} + 360.k$ or $\theta = 2400^{o} + 360.k$; where $k \in Z$

5.5 $a = 110^{o}$; $b = 25^{o}$

6.1



6.2 $2 \le y \le 4$

6.3 $135^{o} \le x \le 180^{o}$ or $225^{o} \le x \le 270^{o}$

7.1 $\hat{D}_1 = \alpha$

7.2 $BD = \dfrac{2\sqrt{3}}{\sin\alpha}$

8.1 25^{o}

8.2 25^{o}

8.3 30^{o}

9.1 equal

11.1 6 units

11.2 2,8 units

11.3 18,9 units^2

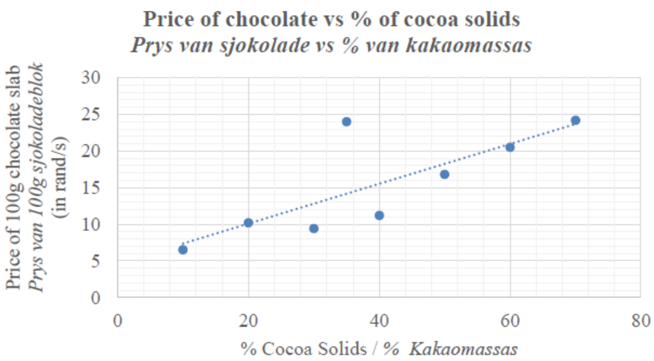
EASTERN CAPE SEPT. 2019 PAPER 2:

1.1 5,26 m

1.2 4,85 m

1.3 a bigger standard deviation; bigger spread of heights in a mixed group consisting of babies, juveniles and adults – than in a group of just 7 young males.

2.1 and 2.3



2.2 $y = 4{,}64 + 0{,}27x$

2.4 $r = 0{,}78$

2.5 fairly strong correlation between the % of cacao and the price.

2.6.1 Brand D or (35 ; 24)

2.6.2 R24,00 – R14,09 = R9,91

3.1 $D(4;0)$

3.2 $\frac{1}{3}$

3.3 $71{,}57^\circ$

3.4 $\frac{4\sqrt{10}}{3}$

3.5 $\frac{1}{3}$

3.6 $4\sqrt{10}$

3.7.1 $\left(2;-\frac{2}{3}\right)$

3.7.2 $y = -3x + \frac{16}{3}$

3.8 No; Show that $y = -3 \neq -3(4) + \frac{16}{3}$

4.2.1 $B(8;0)$

4.2.2 $C(0;-4)$

4.3 PC = 6,67

4.4 $53{,}14^\circ$

4.5 $y = -\frac{3}{4}x - \frac{3}{2}$

5.1.1 1

5.1.2 cos 2A

5.2.1 $-\sqrt{5}$

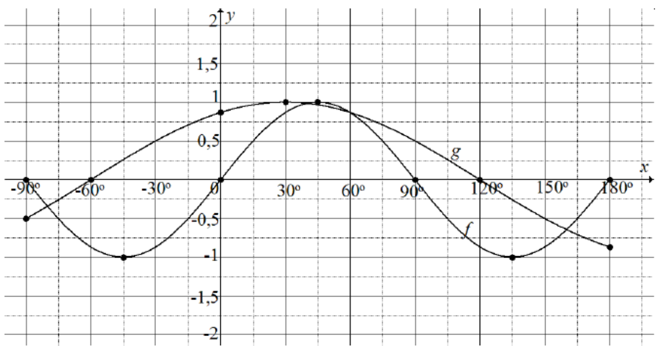
5.2.2 (a) $\frac{-\sqrt{5}}{2}$

(b) $-\frac{1}{9}$

5.3 $-\frac{1}{2}$

5.5 -270°; 90°

6.1 and 6.2



6.3 -80°; 40°; 60°

7.2 6,03 m

8.1.1 90°

8.1.2 40°

8.1.3 40°

8.1.4 62°

8.1.5 62°

8.2 ..equal to the angle in the alternate segment

8.4 20 units

9.1.1 20

9.1.2 10

9.1.3 17,5

10.2.1 $\hat{A}$; $\hat{C}_3$

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$