kzn education

Department:
Education
KWAZULU-NATAL

MATHEMATICS

LEARNER ASSISTANCE

SPRING CLASSES

REVISION BOOKLET

GRADE 12

2021

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give guidance to teachers.

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ALGEBRA, EQUATIONS AND INEQUALITIES

SECTION	CONCEPT	EXAMPLE
Algebra, Equations and Inequalities ($\pm$25)	**Factorisation**	**1)** $x - 6x = 0$
		2) $(x-2)(x+2) = 0$
		3) $x - 3 = \frac{4}{x}$
		4) $3x^2 - 5x - 2 = 0$ **(*where a is greater than* 1)**
	Quadratic formula	1) $2x^2 + 3x - 1 = 0$ (ans corr to 2 decimal digits)
		2) $2x^2 + 3x - 1 = 0$ (ans in simplest surd form)
	Inequalities	1) $(x-4)(x+2) > 0$
		2) $(x+2)(2-x) < 0$
		3) $3^x(x-5) < 0$
		4) $x^2(x+5) < 0$
		4) $3x^2 - 5x - 2 \geq 0$ (for both a > **0** and a < 0)
	Exponential Equations	1) $2x^{-\frac{5}{3}} = 64$
		2) $2^{x+2} + 2^2 = 20$
		3) $2.3^x = 81 - 3^x$
	Surds	1) $\sqrt{x+1} = x - 1$
		2) $2 + \sqrt{2-x} = x$
	Simultaneous Equations	1) $y = x^2 - x - 6$ and $2x - y = 2$
		2) $2x - y + 1 = 0$ and $x^2 - 3x - 4 = y^2$
		3) $3^{x-10} = 3^{3x}$ and $y^2 + x = 20$

EXAMPLES:

Solve for x:

Example 1

$$(x-3)(x+5)=0$$
$$x=3 \text{ or } x=-5$$

Example 2

$$(x-3)(x+5)=9$$
$$x^2+5x-3x-15-9=0$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6 \text{ or } x=4$$

Example 3

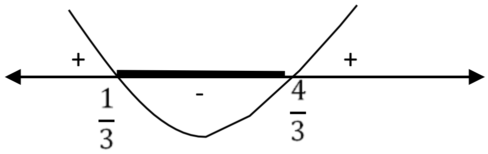
$$\sqrt{x-2}+x=4$$
$$\sqrt{x-2}=4-x$$
$$x-2=16-8x+x^2$$
$$x^2-8x+16-x+2=0$$
$$x^2-9x+18=0$$
$$(x-3)(x-6)=0$$
$$x=3 \text{ or } x=6$$

after checking both solutions

$x=3$ is the ONLY solution

Example 4

$$15x-4>9x^2$$
$$9x^2-15x+4<0$$
$$(3x-1)(3x-4)<0$$



$$\frac{1}{3}<x<\frac{1}{4}$$

PRACTISE EXERCISE

QUESTION 1			
1.1	Solve for x		
	1.1.1	$(x-4)=5$	(3)
	1.1.2	$4x^2-20x+1=0$	(4)
1.2	Solve simultaneously for x and y in the following system of equations: $y-x+3=0$ $x^2-x=6+y$		(6)

QUESTION 2			
2.1	Solve for x		
	2.1.1	$x^2-5x=-6$	(3)
	2.1.2	$(3x+1)(x-4)<0$	(3)
	2.1.3	$2x+\sqrt{x+1}=1$	(4)
	2.1.4	$12^{5+3x}=1$	(4)
2.2	Solve for x and y $2x-y=8$ $x^2-xy+y^2=19$		(7)

QUESTION 3			
3.1	Solve for x		
	3.1.1	$(x+2)^2 = 3x(x-2)$	(5)
	3.1.2	$x^2 - 9x \geq 36$	(4)
	3.1.3	$3^x - 3^{x-2} = 72$	(4)
3.2	Given $(2m-3)(n+5)=0$ Solve for:		
	3.2.1	n if $m = 1$	(1)
	3.2.2	m if $n \neq -5$	(1)
	3.2.3	m if $n = -5$	(2)

QUESTION 4			
4.1	Solve for x:		
	4.1.1	$(x-3)(x+1)=5$	(3)
	4.1.2	$9^{2x-1} = \frac{3x}{3}$	(3)
	4.1.3	$2\sqrt{2-7x} = \sqrt{-36x}$	(4)

QUESTION 5			
5.1	Solve for x:		
	5.1.1	$10x = 3x^2 - 8$	(3)
	5.1.2	$x + \sqrt{x-2} = 4$	(5)
	5.1.3	$x(2x-1) \geq 15$	(5)
5.2	Given $P = \frac{4^{x+3}+4^x}{8^{x+2}+8^x}$		
	5.2.1	Simplify P	(3)
	5.2.2	Hence solve for x: If $P = 3$	(2)
5.3	State whether the following numbers are rational, irrational or non-real		
	5.3.1	$\sqrt{3}$	(1)
	5.3.2	$\frac{22}{7}$	(1)
	5.3.3	The roots of $x^2 + 4 = 0$	(1)
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6.1	Solve for x:		
	6.1.1	$2x^2 + 11 = x + 21$	(3)
	6.1.2	$3x^3 + x^2 - x = 0$	(5)
	6.1.3	$2x + p = p(x+2)$ stating any restriction	(4)
	6.1.4	$x^{-1} - x^{-\frac{1}{2}} = 20$	(5)
6.2	Solve for x and y simultaneously in the following equations $2x^2 - 3xy = -4$ and $4^{x+y} = 2^{x+y}$		(6)

QUESTION 7			
7.1	Solve for x. Leave the answer in the simplest surd form where necessary		
	7.1.1	$(2x+5)(x^2-2)=0$	(3)
	7.1.2	$x^2-4\geq 5$	(4)
	7.1.3	$12^{2x}=8.36^x$	(4)
7.2	Solve for x, correct to TWO decimal places $2(x+1)^2=9$		(4)
7.3	Solve for x and y simultaneously: $y=-2x+7$ and $\frac{y+5}{x-1}=\frac{1}{2}$		
QUESTION 8			
8.1	Given: $x^2+2x=0$		
	8.1.1	Solve for x	(2)
	8.1.2	Hence, determine the positive values of x for which $x^2\geq -2x$	(3)
8.2	Solve for x: $2x^2-3x-7=0$ (Correct to 2 decimal places)		(4)
8.3	Given $k+5=\frac{14}{k}$		
	8.3.1	Solve for k	(3)
	8.3.2	Hence or otherwise, solve for x if $\sqrt{x+5}+5=\frac{14}{\sqrt{x+5}}$	(3)
8.4	Solve simultaneously for x and y: $\frac{1}{x}+\frac{1}{y}=3$ and $x-y=\frac{1}{2}$		(7)
8.5	The roots of a quadratic equation is given by $x=\frac{-2\pm\sqrt{4-20k}}{2}$. Determine the value(s) of k for which the equation will have real roots		(4)
	QUESTION 9		
9.1	Solve for x		
	9.1.1	$2x^2-5x-3=0$	(2)
	9.1.2	$(x-3)(x-4)\geq 12$	(5)
9.2	Consider $5x-\frac{3}{x}=1$		
	9.2.1	Solve for x correct to TWO decimal places.	(5)
	9.2.2	Hence, determine the value of y if $5(2y+1)-\frac{3}{2y+1}=1$	(3)
9.3	Solve simultaneously for x and y in the following set of equations: $y=x-1$ and $y+7=x^2+2x$		(5)
9.4	Calculate the value(s) of m if the roots of $3mx^2-7x+3=0$ are equal		(4)

QUESTION 10			
10.1	Solve for x in each of the following:		
	10.1.1	$x(x+5)=0$	(2)
	10.1.2	$2x^2-3x=7$	(4)
	10.1.3	$x^2-7-x-5=0$	(4)
	10.1.4	$\frac{1}{2}x(3x+1)<0$	(2)
10.2	Solve for x and y simultaneously: $2x+y=3$ and $x^2+y+x=y^2$		(6)
QUESTION 11			
11.1	Solve for x:		
	11.1.1	$4x^2=81$	(2)
	11.1.2	(a) $x^2-5x=2$, correct to TWO decimal places (b) Hence, or otherwise, solve $\left(x^2-2\right)^2-5\left(x^2-2\right)-2=0$	(4) (3)
	11.1.3	$(2-x)(x+4)\geq 0$	(3)
	11.1.4	$3^{x+1}-4+\frac{1}{3^x}=0$	(5)
11.2	Solve for x and y simultaneously: $x+y=3$ and $2x^2 2y^2=5xy$		
QUESTION 12			
12.1	Solve for x:		
	12.1.1	$3x^2+10x+6=0$ (correct to TWO decimal places)	(3)
	12.1.2	$\sqrt{6x^2-15}=x+1$	(5)
	12.1.3	$x^2+2x-24\geq 0$	(3)
12.2	Solve for x and y simultaneously: $5x+y=3$ and $3x^2-2xy=y^2-105$		(6)
	12.3.1	Solve for p if $p^2-48p-49=0$	(3)
	12.3.2	Hence, or otherwise, solve for x if $7^{2x}-48\left(7^x\right)-49=0$	(3)
QUESTION 13			
13.1	Solve for x:		
	13.1.1	$x^2+9x+14=0$	(3)
	13.1.2	$4x^2+9x-3=0$ (correct to TWO decimal places)	(4)
	13.1.3	$\sqrt{x^2-5}=2\sqrt{x}$	(4)
13.2	Solve for x and y if: $3x-y=4$ and $x^2+2xy-y^2=-2$		(6)
13.3	Given: $f(x)=x^2+8x+16$		

	Solve for x if: $f(x)=0$		
QUESTION 14			
14.1	Solve for x:		
	14.1.1	$(x-1)(x+8)=10$	(4)
	14.1.2	$4x+\frac{4}{x}+11=0;\ x\neq 0$ (Leave your answer correct to 2 decimal places)	(4)
	14.1.3	$6x<3x^2$	(5)
14.2	Solve for x and y: $3+x=2y$ and $x^2+4y^2=2xy+7$		(7)
QUESTION 15			
15.1	Solve for x:		
	15.1.1	$x(x-1)+2(x-1)=0$	(2)
	15.1.2	$1+3x^2-5x=0$	(3)
	15.1.3	$\sqrt{2x-1}=2x-3$	(4)
	15.1.4	$(2x)^{\frac{2}{3}}=64$	(3)
	15.1.5	$(2-x)(1-x)^2\leq 0$	(4)
15.2	Solve for x and y simultaneously: $y+3=2x$ and $x^2-xy+2y^2=4$		(5)
15.3	Given that $f(x)=bx^2+3x+4$ and $g(x)=-x+1$, calculate the value of b for which the graph of g will intersect the graph of f.		(5)

SEQUENCES AND SERIES

Arithmetic sequence/ linear

It is the sequence of numbers such that the difference between the consecutive terms is constant. i.e $5;\ (5+2);\ (5+2+2)....$ that forms $5;7;9....$ therefore $d=2$.

$d = T_2 - T_1 = T_3 - T_2$

$T_n = a + (n-1)d$ (a is a first term; d is common difference and n is number of term)

Arithmetic series

It is the sum of the terms of an Arithmetic sequence.

$a + (a+d) + (a+2d)...$

The sum of an arithmetic sequence can be calculated using the formula:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Geometric sequence

It is a number sequence with a common ratio between the consecutive terms. i.e. $5;\ 5\times 2;\ 5\times 2\times 2$ …that form

$5;\ 10;\ 20....$ therefore $r = 2$

$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$ but T_1 and $T_2 \neq 0$

$T_n = ar^{n-1}$ (a is a first term; r is a common ratio and n is number of term)

Geometric Series is the sum of terms of a geometric sequence

$a + (ar) + (ar^2) + ...$

Geometric sequence can be calculated using the formula:

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \text{or} \quad S_n = \frac{a(1-r^n)}{1-r} \quad \text{and } r \neq 1$$

A geometric series will converge if $-1 < r < 1$

The **sum of infinity** can be calculated of a geometric sequence/series if $-1 < r < 1$

$$S_\infty = \frac{a}{1-r}$$

Sigma notation $\left(\sum\ \right)$

It is a very useful notation for the sum of given numbers of a sequence. $\sum$ is the symbol used for (sum of)

Quadratic pattern

It is a sequence of numbers in which a second difference between any two consecutive terms is constant. i.e 1^2; 2^2; 3^2; 4^2…that form 1; 4; 9; 16;... therefore the first difference of the quadratic pattern forms an Arithmetic pattern.

Substitute the first five terms into the general term.

T_1	T_2	T_3	T_4	T_5
$a+b+c$	$4a+2b+c$	$9a+3b+c$	$16a+4b+c$	$25a+5b+c$

First differences: $3a+b$; $5a+b$; $7a+b$; $9a+b$

Second differences: $2a$; $2a$; $2a$

The substitution of five terms into the quadratic general formula helps us to arrive at a method to determine a, b and c of the general formula. Let's see how that works:

- The second constant difference is equal to $2a$. Therefore, with algebraic manipulation, one can state that $a = \frac{2^{nd} constant\ difference}{2}$.
- $T_2 - T_1 = 3a + b$
  $\therefore b = T_2 - T_1 - 3a$ ← Sub in T_1, T_2 and a
- $T_1 = a + b + c$
  $\therefore c = T_1 - a - b$ ← Sub in T_1, a and b

EXAMINATION QUESTIONS FROM PAST PAPERS

LIMPOPO SEPT 2013

QUESTION 1

1.1 In the sequence 3; p; q; 24 are the first four terms.

Determine the values of p and q if

1.1.1 The sequence is geometric. (3)L2

1.1.2 The sequence is arithmetic. (3)L2

1.2 Given that $S_n = 33n + 3n^2$

1.2.1 Determine the sum of 10 terms (1)L1

1.2.2 List the first three terms of this series (4)L2

1.3 Given $k=\frac{1}{3}x;\frac{1}{3}x^2;\frac{1}{3}x^3+\ldots$ which converges

1.3.1 Determine the value of x (2)L2

1.3.2 Determine K if $x=-2$ (3)L2

QUESTION 2

2.1 Given the sequence 3; 6; 13; 24; ….

2.1.1 Derive the general term of this sequence. (4)L2

2.1.2 Which term of this sequence is the first to be greater than 500. (5)L3

QUESTION 3: *MPUMALANGA SEPT 2013*

Given: 1; 11; 26; 46; 71; …….

3.1 Determine the formula for the general term of the sequence. (4)L2

3.2 Which term in the sequence has a value of 521? (4)L2

QUESTION 4			
4.1	Consider the geometric sequence: $2(2t-2);(2t-1)^2;\frac{1}{2}(2t-1)^3;\ldots\left(t\neq\frac{1}{2}\right)$ Calculate:		
	4.1.1	the common ratio r.	(2)L2
	4.1.2	the value(s) of t for which the sequence converges.	(3)L3
		the sum to infinity of the sequence, if $t=\frac{1}{4}$	(4)L2

QUESTION 5: *NORTH WEST TRIAL 2014*			
5.1	Evaluate $\sum_{n=3}^{20}(15-4n)$		(4)L2
5.2	A water tank contains 216 litres of water at the end of day 1. Because of a leak,the tank loses one-sixth of the previous day’s contents each day. How many litres of water will be in the tank by the end of:		
	5.2.1	the 2^{nd} day?	(2)L1

	5.2.2	the 7th day?	(3)L2
5.3	Consider the geometric series: $2(3x-1)+2(3x-1)^2+2(3x-1)^3+\ldots$ For which values of x is the series convergent?		(3)

QUESTION 6: ***WC METRO NORTH DISTRICT TRIAL 2014***			
6.1	The following arithmetic sequence is given: 20 ; 23 ; 26 ; 29; … ; 101		
	6.1.1	How many terms are there in this sequence?	(2)L1
	6.1.2	The even numbers are removed from the sequence. Calculate the sum of the terms of the remaining sequence.	(6)L2

QUESTION 7

The sequence 3 ; 9 ; 17 ; 27 ; … is quadratic.

7.1 Determine an expression for the n-th term of the sequence. (4)L2

7.2 What is the value of the first term of the sequence that is greater than 269? (4)L3

KZN TRIAL 2015

QUESTION 8

8.1 Given the combined arithmetic and constant sequences :

8.2 3 ; 2 ; 6 ;2 ; 9 ;2 ; ...

8.2.1 Write down the next **two** terms in the sequence. (2)L1

8.2.2 Calculate the sum of the first 100 terms of the sequence. (5)L2

Question 9

Given the geometric series: $\frac{24}{x}+12+6x+3x^2\ldots$

9.1 If $x = 4$, then determine the sum to 15 terms of the sequence. (4)L2

9.2 Determine the values of x for which the original series converges. (3)L2

9.3 Determine the values of x for which the original series will be increasing. (2)L4

QUESTION 10

Given the quadratic sequence: 5 ; 7 ; 13 ; 23 ; . . .

10.1 Calculate the nth term of the quadratic sequence. (4) L2

10.2 Determine between which two consecutive terms of the quadratic sequence the first difference will be equal to 2018. (3)L3

MARITZBURG COLLEGE TRIAL 2015

QUESTION 11

11.1 Given the arithmetic series: $1+4+7+...$

11.1.1 Determine the 65th term of the series. (1)L1

11.1.2 Derive a formula for T_n, the n^{th} term of this series. (2)L1

11.1.3 Calculate k if $1+4+7+...$ (to k terms) $=590$.

QUESTION 12

12.1 With reference to the sequence, $2;4;8;k$ give the value of k if:

12.1.1 the sequence is geometric. (1)L1

12.1.2 the sequence is quadratic. (2)L2

12.2 Given the quadratic sequence $6;3;-2;-9;\ldots$

12.2.1 Determine the n^{th} term of the sequence. (5)L2

12.2.2 The sum of two consecutive terms of this sequence is -827. Determine these terms. (4)L3

KZN TRIAL 2016

QUESTION 13

Given the quadratic sequence: 4; 4; 8; 16; . . .

13.1 Calculate the n^{th} term of the quadratic sequence. (4)L1

13.2 Between which two consecutive terms of the quadratic sequence, will the first difference be equal to 28088? (4)L3

QUESTION 14

14.1 Given the combined arithmetic and constant sequences:

6 ; 2 ; 10 ; 2 ; 14 ; 2 ;...

14.1.1 Write down the next **TWO** terms in the sequence. (2)L1

14.1.2 Write down the sum of the first 50 terms of the constant sequence. (1)L1

14.1.3 Calculate the sum of the first 100 terms of the sequence. (4)L2

QUESTION 15: *LIMPOPO TRIAL 2016*		
The 7th term of a geometric series is $\frac{1}{128}$ and the 11th term is $\frac{1}{2048}$. If $r<0$.		
15.1	Determine the first term of the sequence.	(4)L3
15.2	Will this series converge? Explain	(2)L2
15.3	A new series is formed by taking $T_1+T_2+T_3+\ldots=\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\ldots$ from the above sequence. Calculate the sum to infinity of this new series	(4)L1
QUESTION 17: *WESTERN CAPE TRIAL 2016*		
17.1	Given $\frac{3x-1}{4};\frac{2x-1}{3};\frac{7x-5}{12}$	
	17.1.1 If $x=5$, determine the values of the first three terms	(1)L1
	17.1.2 What type of sequence is this? Give a reason for your answer.	(2)L1
	17.1.3 Which term will be equal to -445?	(3)L2
17.2	Given the series $18+6+2+\cdots$	
	17.2.1 What is the value of the first negative term, if any? Explain your answer.	(2)L1
	17.2.2 Determine the tenth term, T_{10}.	(2)L2
	17.2.3 Determine $S_\infty - S_{10}$.	(5)L3

QUESTION 18

18.1 Determine the value of: $\sum_{k=2}^{33}(1-2k)$ (3)L2

18.2 6; $5+x$; -6 and $6x$ form the first 4 terms of a quadratic sequence.

18.2.1 Show that $x=-3$. (4)L2

18.2.2 Determine an expression for the general term of the sequence. (4)L2

QUESTION 19

Given the quadratic sequence: 3; 5; 11; 21; x

19.1 Write down the value of x. (1)L1

19.2 Determine the value of the 48th term. (5)L2

19.3 Prove that the terms of this sequence will never consist of even numbers. (2)L3

19.4 If all the terms of this sequence are increased by 100, write down the general term of the new sequence (2)L2

WC WINELANDS DISTRICT TRIAL 2017

QUESTION 20

20.1 Which term in this sequence 36; 25; 14; . . . is equal to -52? (3)L2

20.2 In a quadratic pattern, with $T_n = an^2 + bn +$, the second term is equal to 8 and the first differences of the quadratic sequence are givenas: 6; 12; 18; . . . .

20.2.1 Write down the values of the first four terms of the quadratic sequence. (3)L2
20.2.2 Calculate the value of T_{40} of the quadratic sequence. (5)L3

QUESTION 21: *EDEN & CENTRAL KAROO DISTRICT TRIAL 2018*

21.1 Prove that in any arithmetic series of which the first term is a and where the constant difference is d, the sum of the first n terms is given by:

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

21.2 Given the following sequence: $-5\ ;\ -1\ ;3\ ;7\ ;\ldots\ldots\ldots\ldots;35$

21.2.1 Determine the number of terms in the sequence. (3)L2

21.2.2 Calculate the sum of the sequence. (2)L2

22.3 For an arithmetic series consisting of 15 terms, $S_n = 2n - n^2$
Determine:

22.3.1 the first term of the sequence. (2)L2

22.3.2 the sum of the last 3 terms. (3)L2

QUESTION 23			
23.1	A quadratic number pattern $T_n = an^2 + bn + c$ has a third term equal to -1, while the first differences of the quadratic sequence are given by: $-12; -8; -4$		
	23.1.1	Write down the values of the first four terms of the quadratic sequence.	(2)L2
	23.1.2	Calculate the value of a, b and c.	(3)L2
23.2	Consider the geometric series $4+p+\frac{p^2}{4}+\frac{p^3}{16}+\ldots$		
	23.2.1	Calculate the value(s) of p for which the series converges.	(2)L1
	23.2.2	Calculate the value of p if the sum to infinity is 3.	(3)L2

KZN TRIAL 2018

QUESTION 24

The first four terms of a quadratic sequence are $9\ ;19\ ;\ 33\ ;\ 51\ ;\ldots$
24.1 Write down the next TWO terms of the quadratic sequence. (2)L1
24.2 Determine the n^{th} term of the sequence. (4)L1
24.3 Prove that all the terms of the quadratic sequence are odd. (3)L3

QUESTION 25

$3-t; -t; \sqrt{9-2t}$ are the first three terms of an arithmetic sequence.
25.1 Determine the value of t. (4)L2
25.2 If $t=-8$, then determine the number of terms in the sequence that will be positive. (3)L

QUESTION 26

26.1 Given the infinite geometric series $(x-3)+(x-3)^2+(x-3)^3+\ldots$

27.1.1 Write down the value of the common ratio in terms of x. (1)L1

27.1.2 For which value(s) of x will the series converge? (3)L2

26.2 An arithmetic sequence and a geometric sequence have their first term as 3. Thecommon difference of the arithmetic sequence is p and the common ratio of the geometric sequence is p. If the tenth term of the arithmetic sequence is equal tothe sum to infinity of the geometric sequence, determine the value of p. (5)L3

FREE STATE TRIAL 2019

QUESTION 27

Given the quadratic sequence 1; 6; 15; 28; …

27.1 Write down the second difference. (1)L1

27.2 Determine the nth term. (4)L1

27.3 Calculate which term of the sequence equals 2701. (3)L2

QUESTION 28

Given the arithmetic series: $10 + 15 + 20 + 25 + \ldots + 185$

28.1 How many terms are there in the series? (3)L3

28.2 Calculate the sum of all the natural numbers from 10 to 185 that are NOT divisible by 5. (6)L3

QUESTION 30: *KZN TRIAL 2019*

The first four terms of a quadratic sequence are $8\,;15\,;\,24\,;\,35\,;...$

30.1 Write down the next TWO terms of the quadratic sequence. (1)L1

30.2 Determine the n^{th} term of the sequence. (4)L2

QUESTION 30

The first three terms of an arithmetic sequence are $2p-3\ ;\ p+5\ ;\ 2p+7$.

30.1 Determine the value(s) of p. (3)L2

30.2 Calculate the sum of the first 120 terms. (3)L2

30.3 The following pattern is true for the arithmetic sequence above:

$$T_1 + T_4 = T_2 + T_3$$
$$T_5 + T_8 = T_6 + T_7$$
$$T_9 + T_{12} = T_{10} + T_{11}$$
$$\therefore T_k + T_{k+3} = T_x + T_y$$

30.3.1 Write down the values of x and y in terms of k. (2)L2

30.3.2 Hence, calculate the value of $T_x + T_y$ in terms of k in simplest form. (4)L3

QUESTION 31

31.1 Given: $\sum_{k=1}^{\infty} 5(3^{2-k})$

31.1.1 Write down the value of the first TWO terms of the infinite geometricseries. (2)L2

31.1.2 Calculate the sum to infinity of the series. (2)L2

31.2 Consider the following geometric sequence: $\sin 30° \,; \cos 30° \,; \frac{3}{2} \,;.......; \frac{81\sqrt{3}}{2}$ Determine the number of terms in the sequence. (5)L3

FUNCTIONS

An overview of the four functions from previous grades

	Linear	**Quadratic**	**Hyperbolic**	**Exponential**
Equation	$y = ax$ $y = ax + c$	$y = ax^2 + bx + c$ $y = a(x+p)^2 + q$ $y = a(x - x_1)(x - x_2)$	$y = \frac{a}{x+p} + q$	$y = a.b^{x+p} + q$
Steps to sketch or draw If $a > 0$ $(+)$	Increases	conc. Up and has ***Min Value at*** y_{TP}	Sketched on the 1st and 3rd quad.	Sketched above $x - axis$
Steps to sketch or draw If $a < 0$ $(-)$	Decreases	conc. down and has ***Max Value at*** y_{TP}	Sketched on the 2nd & 4th quad.	Sketched below $x - axis$
Steps to sketch or draw If $a = 0$	Horizontal line			
Steps to sketch $x - intercept(s)$	let $y = 0$ $(x; 0)$	$(x_1; 0)$ or $(x_2; 0)$ or let $y = 0$	let $y = 0$ $(x; 0)$	let $y = 0$ $(x; 0)$
$y - intercept(s)$	$(0; c)$	$(0; c)$ or let $x = 0$	let $x = 0$ $(0; y)$	let $x = 0$ $(0; y)$
How to sketch	* **I**nt(s) (calc x and y $ints$) * Join 2 points • If it's only one term, then use table method	* **C**heck a for $shape$ * **I**nt(s) (calc x and y $ints$) * **T**.P (Turning Point) • If it's only one term, then use table method	* Draw both asymptote * **C**heck a * **I**nt(s) (calc x and y $ints$) • If it's only one term, then use table method	* Draw equation of asymptote * **C**heck a then b * **I**nt(s) (calc x and or y $ints$) • If it's only one term, then use table method
Axis of symmetry	No axis of symmetry	$x = \frac{-b}{2a}$	$y = \pm(x + p) + q$	No axis of symmetry
Turning Point	No turning point	T.P $(p; q)$ or $\left(\frac{-b}{2a}; f(\frac{-b}{2a})\right)$	No turning point	No turning point
	Linear	**Quadratic**	**Hyperbolic**	**Exponential**
Domain	$x \in R$	$x \in R$	$x \in R; x \neq p$	$x \in R$

Range	$y \in R$	$y \geq y_{TP}\ if\ \ a > 0$ $y \leq y_{TP}\ if\ \ a < 0$	$y \in R; y \neq q$	$y > q\ if\ a > 0$ $y < q\ if\ a < 0$
Determining Equation	✓ Determine the gradient ✓ Substitute gradient and another point to solve for c using $y = mx + c$ where m is a gradient	✓ **Given TP + another Point** Substitute TP in $y = a(x+p)^2 + q$, substitute point to get a. ✓ **Given any two points** Substitute both points separately and solve simultaneous. ✓ $\boldsymbol{x - intercepts}$ **+ point** Substitute to $y = a(x - x_1)(x - x_2)$ and then substitute a point to get a.	✓ Given Asymptotes Insert then in p and q then use any given point to solve for a.	*subst. the value of q *Take one point on the graph and subst.
** When interpreting the graph, it is important to classify given points				
Grade 12 (NEW)				
Inverse	$y = ax + c$	$y = ax^2$	$y = \frac{a}{x}$	$y = a^x$
	$y^{-1} = \frac{x}{a} - \frac{c}{a}$	$y^{-1} = \pm\sqrt{\frac{x}{a}}$; (domain)	$y^{-1} = \frac{a}{x}$	$y^{-1} = \log_a x$

Note!! Not every graph is a function

Function is a **1:1** mapping or **many:1**.

To test if a graph is a function a **vertical line test** can be used.

Restrict domain ($x \leq 0$ or $x \geq 0$) of the parabola for its inverse to be a functio

Vocabulary	Notations
• Function	• $f(x)\ vs\ f^{-1}(x)$
• Gradient or average gradient	• $f(x) > 0\ vs\ f(x) \geq 0$
• Line of symmetry or axis of symmetry	• $f(x) < 0\ vs\ f(x) \leq 0$
• Transformation (Reflection)	• $f'(x) > 0\ vs\ f'(x) < 0$
• Asymptote	• $f(x) = g(x)$
• Domain and Range	• $f(x) > g(x)\ vs\ f(x) < g(x)$
• Nature of roots	• $f(x) \geq g(x)\ vs\ f(x) \leq g(x)$
• x and y axis	• $f(x).g(x) > 0\ vs\ f(x).g(x) < 0$
• x and y intercepts	• $f(x).g(x) \geq 0\ vs\ f(x).g(x) \leq 0$
• Point (x and y co ordinate)	• $\frac{f(x)}{g(x)} > 0\ vs\ \frac{f(x)}{g(x)} < 0$
• Turning Point	• $\Delta = b^2 - 4ac$
• Point of intersection	• $m > 0\ vs\ m < 0$
• Max or Min value	

Practical Examples

EXAMPLE 1 (SEPT 2020)

Given: $g(x) = \dfrac{1}{2(x+3)} - 1$

a) Write down the equations of the vertical and horizontal asymptotes of g. (2)

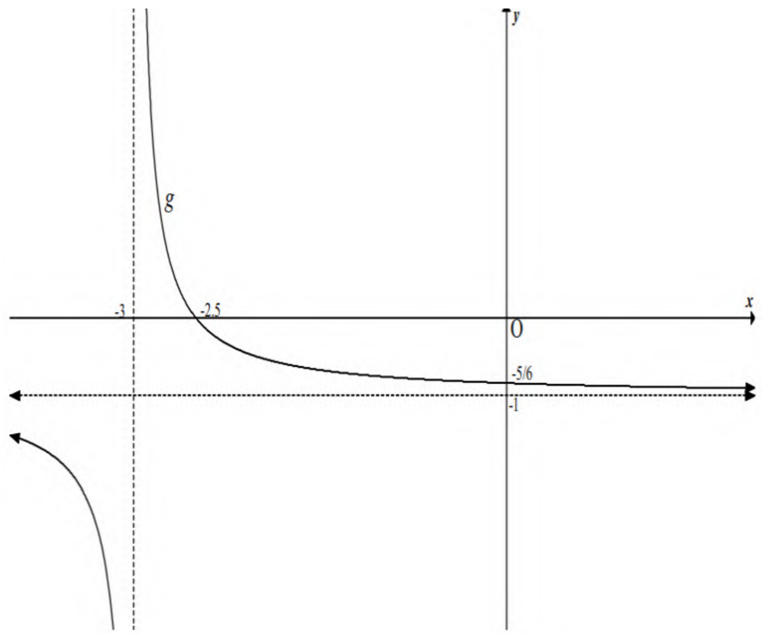
b) Determine the intercepts of the graph of g with the axes. (3)

c) Draw the graph of g. Show all intercepts with the axes as well as the asymptotes of the graph. (4)

d) Determine the equation of the axis of symmetry of g that has a negative gradient. (2)

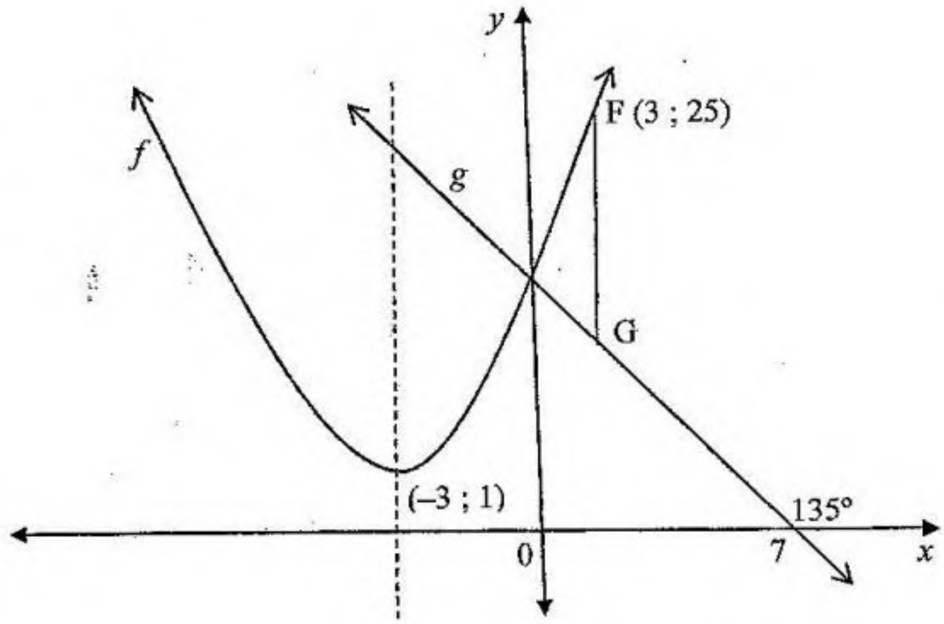
e) Write down the domain and the range of f if $f(x) = g(x-3)+1$. (3)

f) It is further given that k is the reflection of $g(x)$ about the $x-axis$. Determine the equation of $k(x)$. (2)

Solution (a) $x=-3$ and $y=-1$ (b) For y-intercept: Let $x=0$ $y=\frac{1}{2(0+3)}-1$ $y=-\frac{5}{6}$ $\left(0;-\frac{5}{6}\right)$ For x-intercepts: Let $y=0$ $\frac{1}{2(x+2)}-1=0$ $2x+6=1$ $x=-\frac{5}{2}$ $\left(-\frac{5}{2};0\right)$	(c) 
(d) $y=-(x+3)-1$ $y=-x-4$	**(e)** $f(x)=\frac{1}{2(x-3+3)}-1+1$ $f(x)=\frac{1}{2x}$ $Domain: x\in R; x\neq 0$ $Range: y\in R; y\neq 0$
(f) $-f(x)=\frac{1}{2(x+3)}-1$ $f(x)=-\frac{1}{2(x+3)}+1$	

EXAMPLE 2 (MP Sept 2020)

(i) In the diagram, $f(x) = ax^2 + bx + c$ *and* $g(x) = mx + c$ are drawn with an angle of inclination of g of $135°$. FG is parallel to the $y-axis$ with F$(3;\ 25)$. The turning point of f is $(-3;\ 1)$ and *the* $x-intercepts$ of g is (7; 0), f and g have the same $y-intercept$.



a) Determine the equation of g. (2)

b) Calculate the Calculate the co-ordinates of G. (2)

c) Determine the equation of f in the form $y = ax^2 + bx + c$ (4)

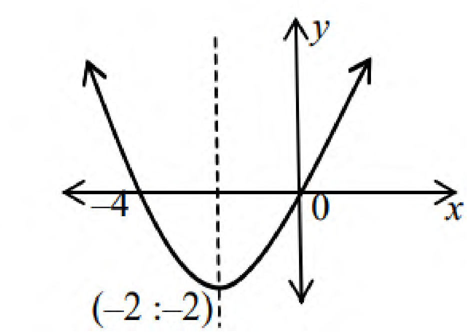
d) Describe the transformation from f to p if G is the turning of p. (2)

e) Write down the down the equation of symmetry of h if $h(x) = f(x-2)+3$. (2)

(ii) Draw the sketch of $f(x) = ax^2 + bx + c$ with the following properties:

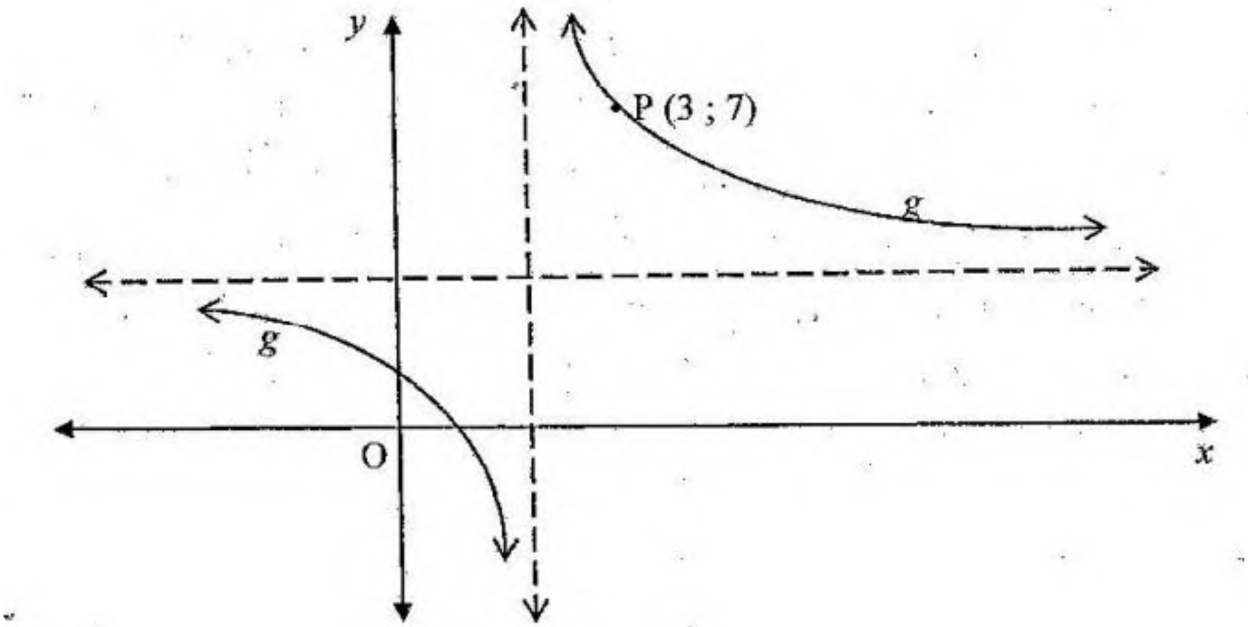
- Roots of $f(x) = 0$ differs by 4.
- $f'(-2) = 0$
- The range of f is $y \geq -2$ (4)

[16]

Solutions				
(i)	**(a)**	$y = mx + c$ $0 = -1(7) + c$ $c = 7$ $y = -1x + 7$	**(b)**	$y = -1x + 7$ $y = -1(3) + 7$ $y = 4$ $G(3;4)$
	(c)	$y = a(x+p)^2 + q$ $5 = a(0+3)^2 + 1$ $25 = a(3+3)^2 + 1$ $36a = 24$ $a = \frac{2}{3}$ $y = \frac{2}{3}(x+3)^2 + 1$ $y = \frac{2}{3}(x^2 + 6x + 9) + 1$ $y = \frac{2}{3}x^2 + 4x + 7$	**(d)**	Reflection over $y -$ $axis$. Move 3 units up.
	(e)	$f(x) = \frac{2}{3}(x+3)^2 + 1$ $h(x) = \frac{2}{3}(x+3-2)^2 + 1 + 3$ $h(x) = \frac{2}{3}(x+1)^2 + 4$ $\therefore x = 1$	**(ii)**	

EXAMPLE 3 (MP Sept 2020)

(i) In the diagram, the graph of, g: $(y-4)(x-2) = k$ is drawn. P (3; 7) is a point on g.



a) Write down the equation(s) of the asymptote of g. (2)

b) Determine the value of k. (2)

c) Determine the equation of f, if f is the equation where g moved three units downwards and one unit left. (2)

d) For which values of x is $x.g(x) \leq 0$ (3)

(ii) Given: $f(x) = \left(\frac{1}{2}\right)^x$

a) Sketch the graph of h. Show at least 2 points on the graph. (3)

b) Write down the equation of $h^{-1}(x)$, the inverse of h. (2)

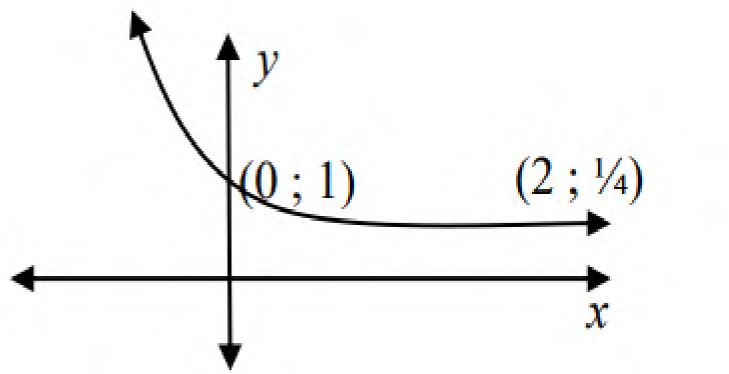
c) For which values of x is:

$$\left(\frac{1}{3}\right)^x > \left(\frac{1}{2}\right)^x$$ (2)

[16]

Solutions			
(a)	$y = 4 \ \ and \ \ x = 2$	**(b)**	$(y-4)(x-2) = k$ $(7-4)(3-2) = k$ $k = 3$

(c)	$f(x)=\dfrac{3}{x-2+1}+4-3$ $f(x)=\dfrac{3}{x-1}+1$	**(d)**	$(0-4)(x-2)=3$ $x=\dfrac{5}{4}$ $\dfrac{5}{4}\leq x<2$

(ii)	(a)	
	(b)	$h^{-1}(x)=\log_2 x$
	(c)	$\left(\dfrac{1}{3}\right)^x>\left(\dfrac{1}{2}\right)^x$ $3^{-x}>2^{-x}$ $x<0$

ACTIVITIES

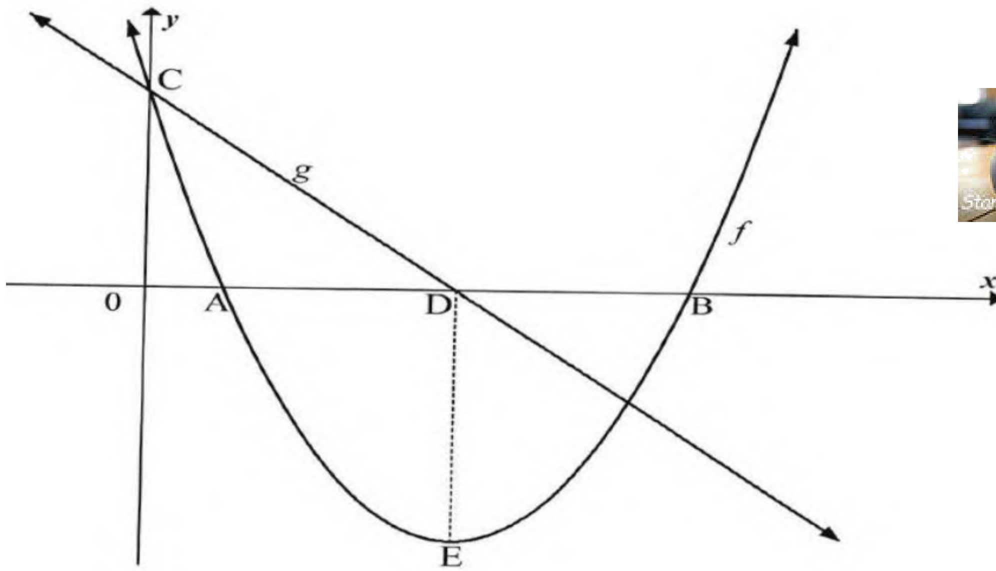
QUESTION 1 (METRO EAST SEP 2018)

Sketch the graph of $f(x)=\dfrac{k}{x+p}+q$ if:

- The domain is given as: $x\in R; x\neq -1$.
- The range is given as: $y\in R; y\neq 2$
- $k<0$
- $x-\text{int}\,ercept:(-\frac{1}{2};0)$
- $f(0)=1$

QUESTION 2 (FEB/MAR 2018)

Below are the graphs of $f(x)=(x-4)^2-9$ and a straight line g. A and B are x-intercepts of f and E is the turning point of f. C is the y-intercept of both f and g. The x-intercept of g is D. DE is parallel to the $y-axis$.



2.1	Write down the coordinates of E.	(2)
2.2	Calculate the coordinates of A.	(3)
2.3	M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M.	(3)
2.4	Determine the equation of g in the form $y = mx + c$.	(3)
2.5	Write down the equation of g^{-1} in the form $y = \cdots$ ...	(3)
2.6	For which values of x will $x.f(x)\le 0$?	(4)

[18]

QUESTION 3 (SEPT 2018)

Given: $f(x)=\dfrac{x-3}{x+2}$

3.1	Show that $f(x)=1-\dfrac{5}{x+2}$.	(1)
3.2	Write down the equations of the vertical and horizontal asymptotes of f.	(2)

3.3 Determine the intercepts of the graph of f with the $x - axes$ and $y - axes$. (2)

3.4 Write down the value of c if $y = x + c$ is a line of symmetry to the graph of f. (2)

3.5 Determine the equation of k if $k(x) = f(x) - 1$. (1)

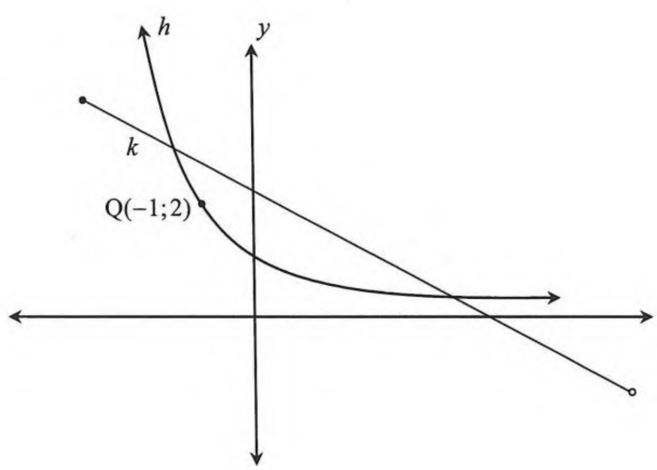
3.6 Hence or otherwise sketch the graph of k showing ALL the asymptotes and the intercepts with the axis. (3)

3.7 Determine the domain and the range of k. (2)

[10]

QUESTION 4 (NW SEPT 2020)

Given: $k(x) = -\frac{2}{3}x + 3 \ \ for \ -4 \le x < 6 \ and \ h(x) = 2^{-x}$. $Q(-1;2)$ is a point on h.



4.1 Determine the $x - intercept$ of k. (2)

4.2 Determine the domain of k^{-1}. (2)

4.3 Determine the equation of h^{-1}. (2)

4.4 Give the coordinates of the $x - intercept$ of h^{-1}. (2)

4.5 For which values of x is: $k^{-1}(x) < 0$? (2)

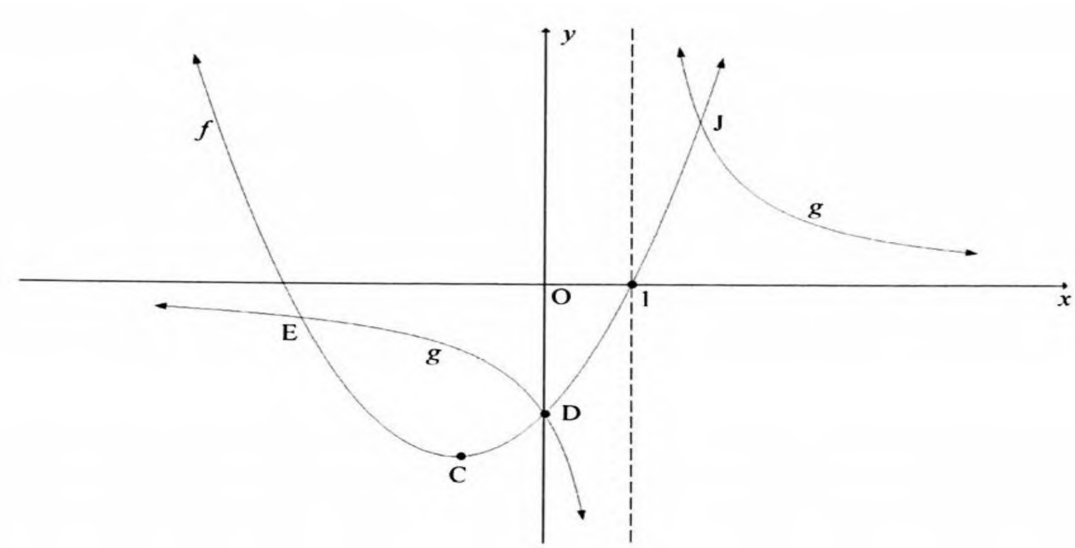
4.6 If $k(x) = q'(x)$, where q is a function defined for $-4 \le x < 6$. Draw a neat sketch graph of q. Clearly show the $x -$ values of the turning point(s) and end points. (3)

[13]

QUESTION 5 (NOV 2019)

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x + p}$

- f has a turning point at C and passes through the $x - axis$ at (1;0).
- D is the y −intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



5.1 Write down the value of p. (1)

5.2 Show that $a = 3$ and $b = 2$. (3)

5.3 Calculate the coordinates of C. (4)

5.4 Write down the range of f. (2)

5.5 Determine the equation of the line through C that makes an angle of 45^0 with the positive x-axis. Write your answer in the form $y = ...$ (3)

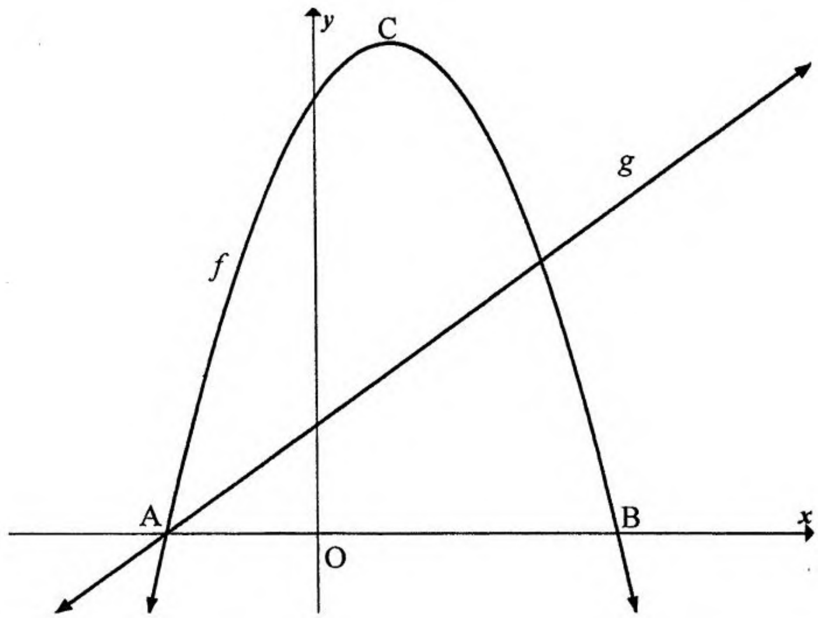
5.6 Is the straight line, determined in QUESTION 5.5, a tangent of f? Explain your answer. (2)

5.7 The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q. (4)

[19]

QUESTION 6 (MAY-JUNE 2021)

Sketched below are graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$. A and B are the $x -$ intercepts of f. C is the turning point of f.



6.1 Calculate the coordinates of A and B. (3)

6.2 Determine the coordinates of C, the turning point of f. (2)

6.3 Write down the range of f. (1)

6.4 The graph of $h(x) = f(x+p)+q$ has a maximum value of 15 at $x=2$.

Determine the values of p and q. (3)

6.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \cdots$ (2)

6.6 For which value(s) of x will $g^{-1}(x).g(x)=0$? (2)

6.7 If $p(x) = f(x)+k$, determine the value(s) of k for which p and f will NOT intersect. (2)

6.8 If $p(x) = f(x)+k$, determine the value(s) of k for which p and g will NOT intersect. (5)

6.9 It is further given that f is the graph of $h'(x)$.

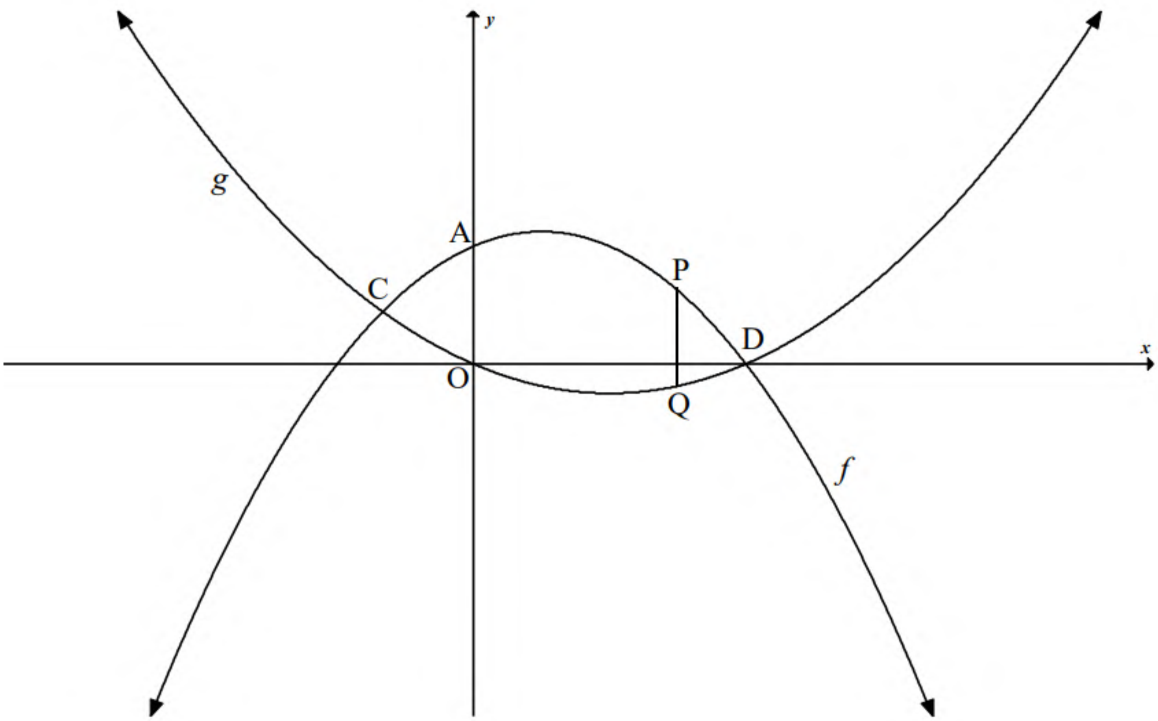
6.9.1 For which values of x will the graph of h be concave up? (2)

6.9.2 Sketch the graph of h, clearly showing the x-values of the turning point of inflection. (3)

[25]

QUESTION 7 (SEPT 2020)

In the diagram, the graphs of $f(x) = -x^2 + x + 2$ *and* $g(x) = \frac{1}{2}x^2 - x$ are drawn below. f and g intersect at C and D. A is the y – intercept of f. P and Q are any points on f and g respectively. PQ is parallel to the y – axis.



7.1 Write down the co-ordinates of A. (1)

7.2 Calculate the coordinates of C and D. (5)

7.3 Determine the values of x for which $f(x) \leq g(x)$. (2)

7.4 Calculate the maximum length of PQ where line PQ is between C and D. (4)

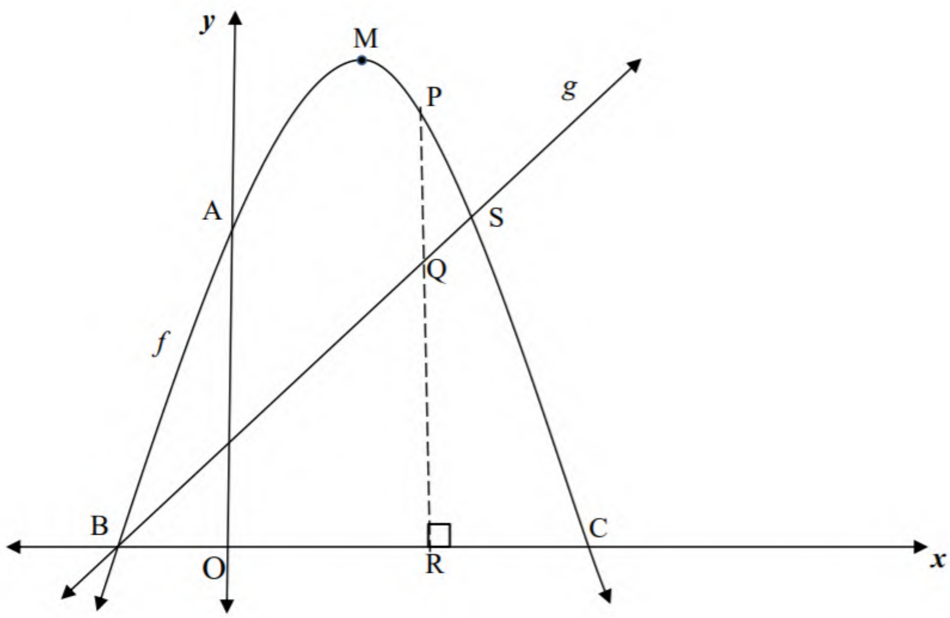
7.5 Calculate the values of x where the gradient of f is equal to 3. (3)

7.6 Determine the values of k for which $f(x) = k$ has two positive unequal roots. (4)

[19]

QUESTION 8 (Sept 2019)

In the diagram, the graphs of $f(x) = -x^2 + 5x + 6$ and $g(x) = x + 1$ are drawn below. The graph of f intersects the x – *axis* at B and C and the y – *axis* at A. The graph of g intersects the graph of f at B and S. PQR is perpendicular to the x – *axis* with points P and Q on f and g respectively. M is the turning point of f.



8.1 Write down the co-ordinates of A. (1)

8.2 S is the reflection of A about the axis of symmetry of f. Calculate the coordinates of S. (2)

8.3 Calculate the coordinates of B and C. (3)

8.4 If PQ=5 units, calculate the length of OR. (4)

8.5 Calculate the:

8.5.1 Coordinates of M. (4)

8.5.2 Maximum length of PQ between B and S. (4)

[18]

QUESTION 9 (Sept 2019)

Given $f(x)=\dfrac{-1}{2-x}-1$

9.1 Write down the equations of the vertical and horizontal asymptotes of f. (2)

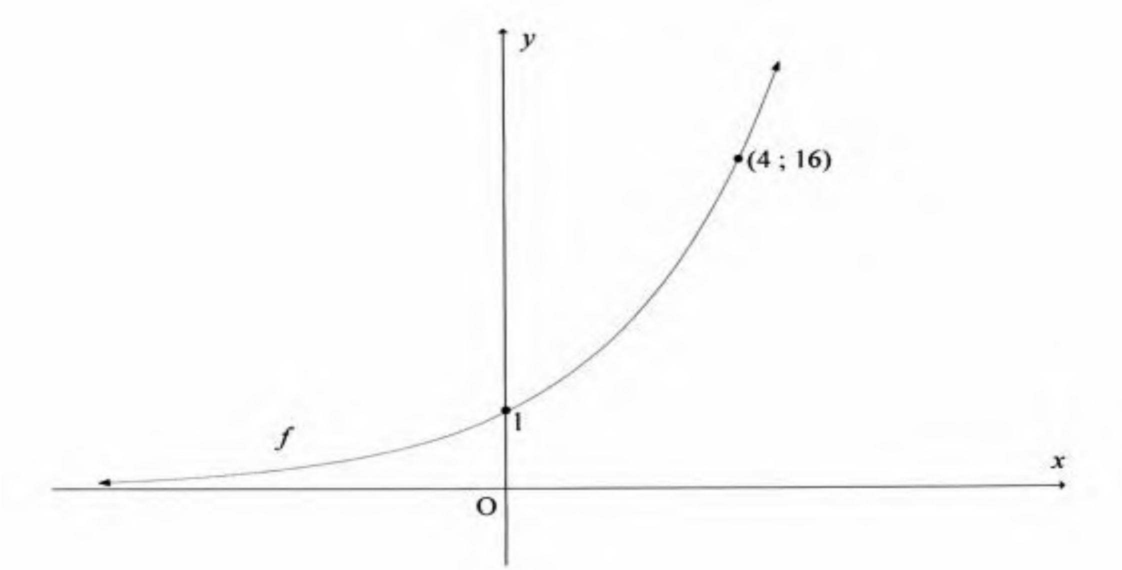
9.2 Determine the intercepts of the graph of f with the axes. (3)

9.3 Draw the graph of f. Show all the intercepts with the axes as well as the asymptotes of the graph. (4)

[9]

QUESTION 10 (Nov 2019)

Sketched below is the graph of $f(x) = k^x; k > 0$. The point $(4;16)$ lies on f.



10.1 Determine the value of k. (2)

10.2 Graph g is obtained by reflecting graph of f about the line $y = x$. Determine the equation of g in the form $y = ...$ (2)

10.3 Sketch the graph of g. Indicate on your graph the coordinates of two points on g. (4)

10.4 Use your graph to determine the value(s) of x for which:

10.4.1 $f(x) \times g(x) > 0$ (2)

10.4.2 $g(x) \leq -1$ (2)

10.5 if $h(x) = f(-x)$, calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$ (4)

[16]

CALCULUS

1. DERIVATIVE OF A FUNCTION

The derivative of a function at x is given by:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

(Provided the limit exist)

It is called the first principles

TIPS ON HOW TO APPLY THE FIRST PRINCIPLES

1. Find $f(x+h)$ separately
2. Write down the formula

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

3. Substitute $f(x)$ and $f(x+h)$ into the formula
4. Be careful with the exponent of $(x+h)$ as well as the signs
5. Simplify and remember to cancel out from the denominator
6. Remind yourself of method of taking out an LCD (for a hyperbolic function)
7. Solve [at this stage we no longer write $\lim_{h\to 0}$]

2. RULES FOR DIFFERENTIATION

2.1 Constant Rule

$\frac{d}{dx}[c] = 0$ i.e. the derivative of a constant is zero

Power Rule

$$\frac{d}{dx}[x^n] = n.x^{n-1}$$

For any given function $y = x$; **the derivative:** $y' = nx^{n-1}$

Example: $y = 3x^2$

$$\therefore \frac{dy}{dx} = 3.2x^{2-1} = 6x$$

2.2 Sum and Difference Rule

$$\frac{d}{dx}[f(x) + g(x) = f'(x) + g'(x)]$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

Example: If: $y = x^n + x$

$$y' = y'(x^n) + y'(x)$$

$$y = x^2 + 5x$$

$$\therefore y' = 2x + 5$$

❖ REMEMBER: The derivative of f at $x = c$ is equivalent to the gradient of the tangent line to the curve of f at $x = c$

2 (a) NOTATION OF DERIVATIVE

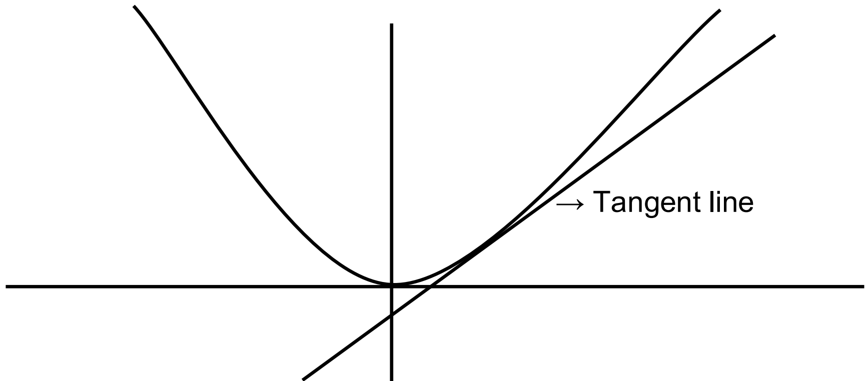
Ways to write the derivative of $y = f(x)$

NOTE $\frac{dy}{dx}$ **does** not mean dy divide by dx

Function	**Derivative**
$f(x)$	$f'(x)$ or $\frac{df(x)}{dx}$
f	f' or $\frac{df}{dx}$
y	y' or $\frac{dy}{dx}$

3. THE EQUATIONS OF A TANGENT TO THE CURVE

A tangent is a straight line that touches a curve at one point.



A tangent is determined by two conditions. The two conditions are a gradient and a point of contact.

The equation of a tangent is in the form $y = mx + c$. The gradient (m) is determined from the derivative i.e.

$$m = \frac{dy}{dx} \text{ or } f'(x)$$

The curve and its tangent line has the same gradient at the point of contact. The two – point form for finding the equation of a tangent is given by:

$$y - f(a) = f'(a)(x - a)$$

where a is the value of x at a point of contact, $f(a)$ is the value of y at a point of contact and $f'(a)$ is the slope/gradient at the point of contact

TIPS ON HOW TO FIND THE EQUATION OF A TANGENT TO THE CURVE:

If $f(x)$ is a function:

1. Find the derivate of a function ,i.e. $f'(x)$

2. Then substitute the x-value given into the derivative *(that will give you gradient at that point)*
3. Substitute the x-value given into the original function *f* equation to get the corresponding y-value *(that will give you the co-ordinates of the point)*
4. Substitute the x- and y-values (refer to 3) and the gradient in the straight line formula: $y = mx + c$ or $y - y_1 = m(x - x_1)$

4. CUBIC GRAPHS

A cubic graph is of the form:

$$f(x) = ax^3 + bx^2 + cx + d$$

where $a \neq 0$

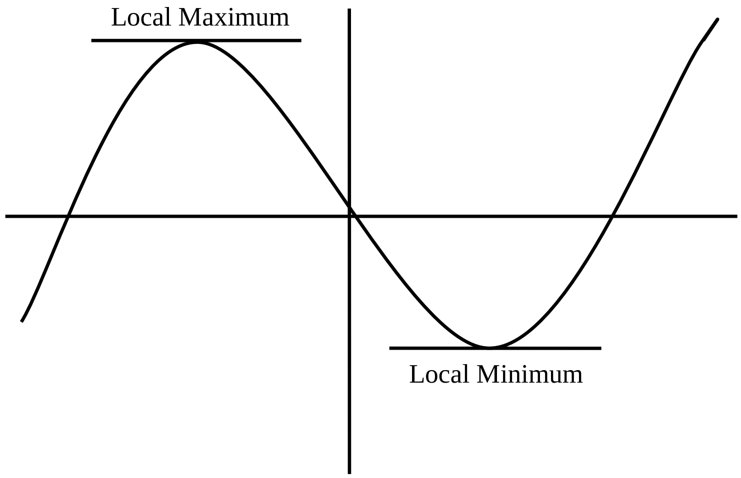
NOTE:

- The graph of f is **concave upward** on the interval if f' is increasing on the interval
- The graph of f is **concave downward** on the interval if f' is decreasing on the interval

TIPS ON HOW TO FIND THE MAIN PROPERTIES OF A GRAPH

1. y-intercepts ($x = 0$)
2. x – intercepts ($y = 0$)
    - Use the factor theorem to find the first factor of a cubic expression.
    - Use long division, synthetic division or the inspection method to find the other (quadratic) factor.
    - Factorise the quadratic factor into two linear factors.
    - Write down the three roots (solutions) of the cubic equation.
3. **Stationary points: Minimum and Maximum**
   Where a horizontal line is tangent to the curve and it is calculated by equating the first derivative to zero and solving for x.

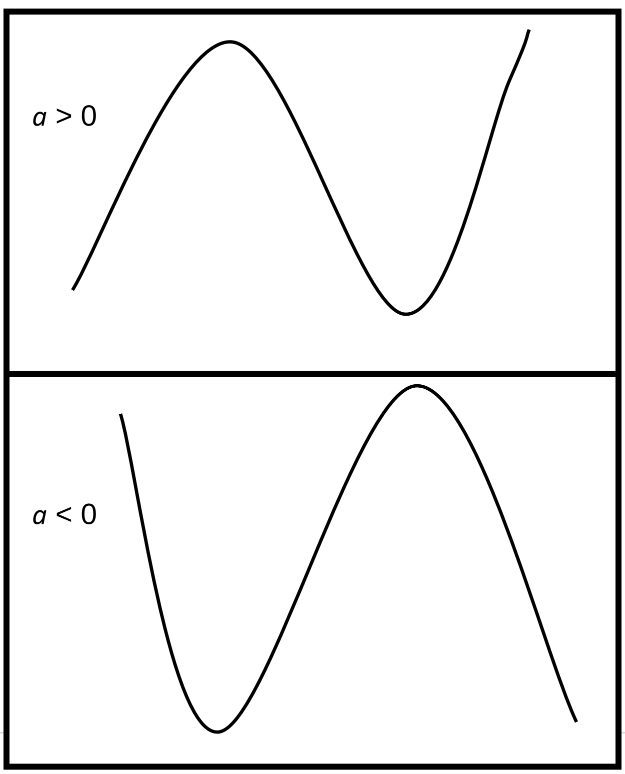
i.e. $f'(x) = 0$



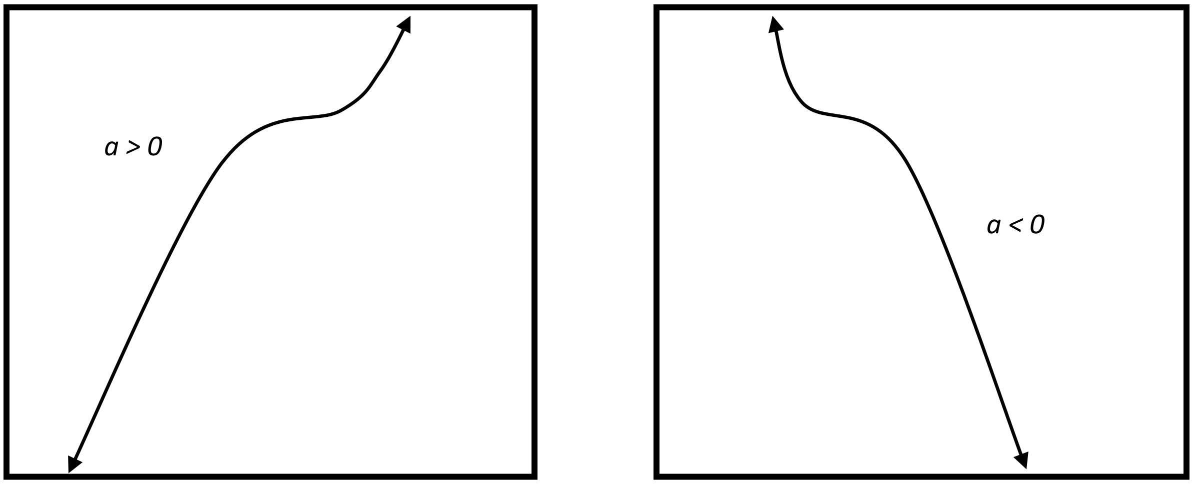
- $f'(x) = 0$
- Solve for x
- Substitute the x-values into the original equation to find the corresponding values of y

SUMMARY OF A CUBIC GRAPH

- **For 2 stationary points**



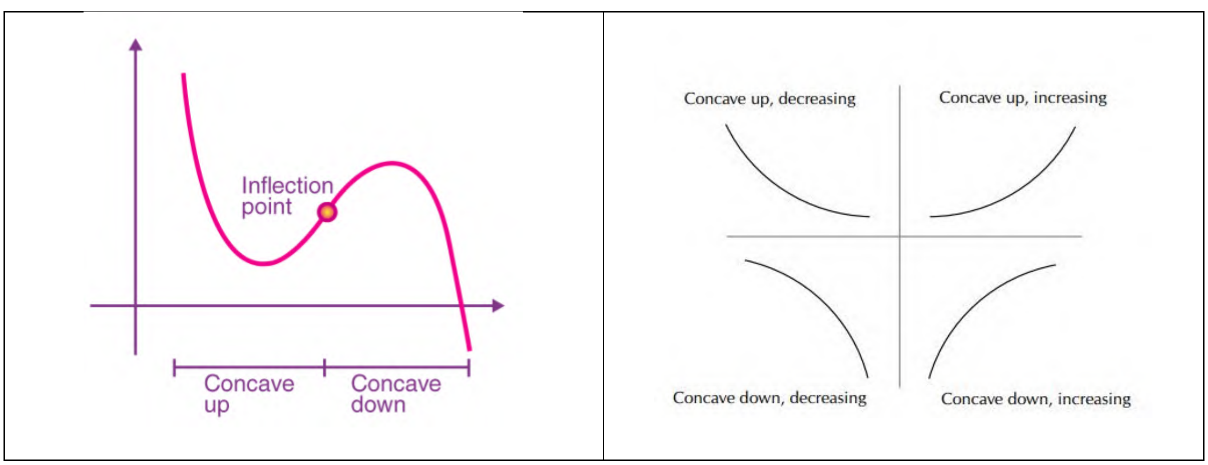
❖ **For 1 stationary point**



5. POINTS OF INFLECTION

The point at which the functions changes its concavity at this point

$f''(x) = 0$ **i.e. The second derivative is equal to zero.**



[Byjus.com] *[Siyavula, Grade 12 Mathematics]*

The direction of bending changes an inflection point. The graph is concave down on one side of an inflection and concave up on the other side of an inflection. The second derivative tells about change is slope.

We can also use the fact that the x-coordinate of the point of inflection is half way between the two critical values of the graph of f . So $x = \frac{x_A + x_B}{2}$, if A and B are turning points of $f(x)$.

6. FINDING THE EQUATION OF THE CUBIC FUNCTION

- If the 3 x-intercepts of the graph are known:
  - start with the equation: $y = a(x-x_1)(x-x_2)(x-x_3)$ where $x_1; x_2$ and x_3 represents $x-$ intercepts.
  - Expand the binomials
  - Substitute the co-ordinates of another point on the graph to determine a

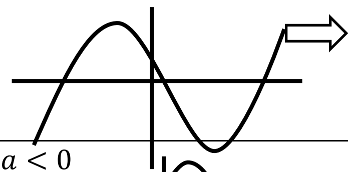
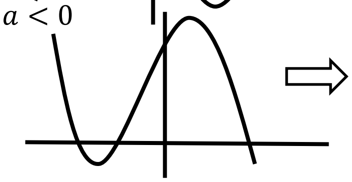
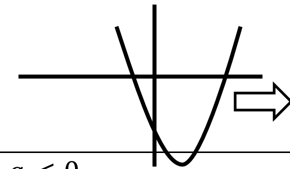
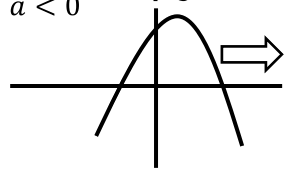
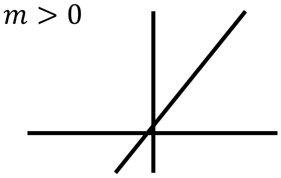
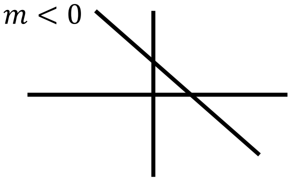
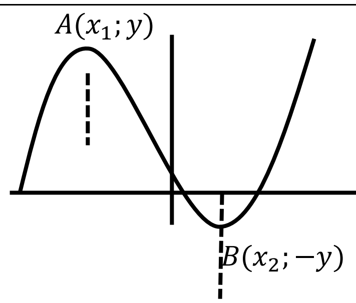
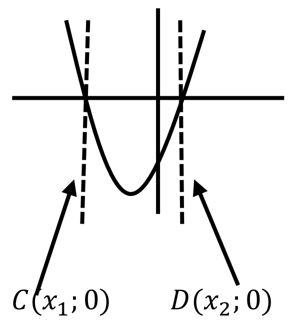
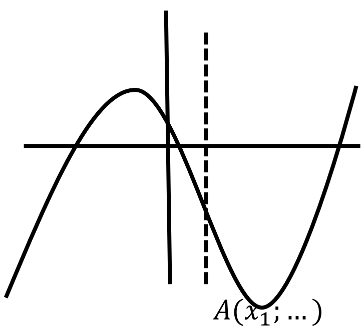
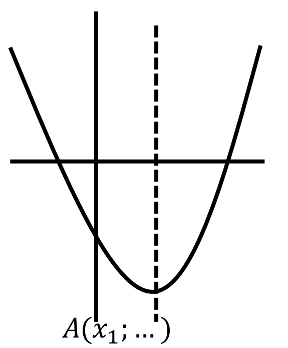
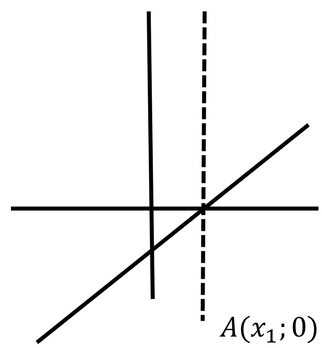
- If the graph has a turning point on one of the x-intercepts, use the equation: $y = a(x-x_1)(x-x_2)^2$

- If the turning points are known, substitute into:
  $f'(x) = 0 \quad in\ the\ form:$
  $a(x-x_1)(x-x_2) = 0$ where x_1 and x_2 are the x-values of the turning point

7. GRAPH OF A DERIVATIVE

IMPORTANT FACTS ABOUT THE GRAPH OF $f(x),\ f'(x),\ f''(x)$

Graph	$f(x) = ax^3 + bx^2 + cx + d$ where a $\neq 0$	$f'(x) = ax^2 + bx + c$ where a $\neq 0$	$f''(x) = mx + c$ where m $\neq 0$

Shape	$a > 0$  $a < 0$ 	$a > 0$  $a < 0$ 	$m > 0$  $m < 0$ 
(a) x-values of TP of $f(x)$ (b) x-intercepts of $f'(x)$ *FOLLOW THE DOTTED LINES.*			The x-values of the turning points of $f(x)$ are the x-values of the x-intercepts of $f'(x)$.
(a) Point of inflection of $f(x)$ (b) x-value of the TP of $f'(x)$ (c) x-intercepts of $f''(x)$ *FOLLOW THE DOTTED LINE.*			
	x-value of the point of inflection of $f(x)$	It is now the x-value of the turning point of $f'(x)$	It is now the x-intercept of the function $f''(x)$

DIFFERENTIAL CALCULUS

First Principles

1. $f(x) = 2x^2 + 4$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0}\frac{2(x+h)^2 + 4 - (2x^2 + 4)}{h}$$
$$= \lim_{h\to 0}\frac{2x^2 + 4xh + 2h^2 + 4 - 2x^2 - 4}{h}$$
$$= \lim_{h\to 0}\frac{4xh + 2h^2}{h}$$
$$= \lim_{h\to 0}\frac{h(4x + 2h)}{h}$$
$$= \lim_{h\to 0} 4x + 2h$$
$$= 4x$$

2. $f(x) = 2x^2 - 5x$

$$f(x+h) = 2(x+h)^2 - 5(x+h)$$
$$f(x+h) = 2(x^2 + 2xh + h^2) - 5x + 5h$$
$$f(x+h) = 2x^2 + 4xh + 2h^2 - 5x + 5h$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0}\frac{2x^2 + 4xh + 2h^2 - 5x - 5h - (2x^2 - 5x)}{h}$$
$$= \lim_{h\to 0}\frac{4xh + 2h^2 - 5h}{h}$$
$$= \lim_{h\to 0}\frac{h(4x + 2h - 5)}{h}$$
$$= \lim_{h\to 0} 4x + 2h - 5$$
$$= 4x + 2(0) - 5$$
$$= 4x - 5$$

3. $f(x) = \frac{-2}{x}$

$$f(x+h) = \frac{-2}{x+h}$$

$$f'(x) = \lim_{h\to 0}\frac{\frac{-2}{x+h} - (-\frac{2}{x})}{h}$$
$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{-2}{x+h} + \frac{2}{x}\right)$$
$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{-2x + 2x + 2h}{x(x+h)}\right)$$
$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{2h}{x(x+h)}\right)$$
$$= \lim_{h\to 0}\frac{2}{x^2 + xh}$$
$$= \frac{2}{x^2}$$

WHEN DETERMINING THE DERIVATIVE USING RULES, TAKE NOTE OF:

- Subject of the formula

  When determining $\frac{dy}{dx}$ make y the subject of the formula first with respect to x

Examples

(a)

$$y = 3x^5 - 4x^3 + 2x^2 - 5$$

$$\frac{dy}{dx} = 15x^4 - 12x^2 + 4x$$

(c)

$$\sqrt{y} = 2x + 1$$

$$(\sqrt{y})^2 = (2x+1)^2$$

$$y = 4x^2 + 4x + 1$$

$$\frac{dy}{dx} = 8x + 4$$

$$xy = x^2 + y - 1$$

$$xy - y = x^2 - 1$$

(b) $y(x-1) = (x+1)(x-1)$

$$y = x + 1$$

$$\frac{dy}{dx} = 1$$

- **Multiplication (Products)**

  Determine the product first

  e.g.

(a)

$$y = 3x^2.4x^3$$

$$y = 12x^5$$

$$\frac{dy}{dx} = 60x^4$$

(b)

$$f(x) = (3x^2 - 2)^2$$

$$f(x) = 9x^4 - 12x^2 + 4$$

$$f'(x) = 36x^3 - 24x$$

Surds

Change the surds into exponential form then differntiate

$$\sqrt[a]{x^n} = x^{\frac{a}{n}}$$

Example

$$D_x\left(\sqrt[3]{x^4} + 8\sqrt{x}\right)$$

$$= D_x\left(x^{\frac{4}{3}} + 8x^{\frac{1}{2}}\right)$$

$$= \frac{4}{3}x^{\frac{1}{3}} + 4x^{\frac{-1}{2}}$$

❖ **Variable in the denominator (TWO or more terms)**	❖ **Variable in the denominator (ONE term)**
$y=\frac{3x^2-2x-1}{x-1}$ $y=\frac{(3x+1)(x-1)}{x-1}$ $y=3x+1$ $\frac{dy}{dx}=3$	*Divide each term of the numerator by the denominator e.g.* $g(x)=\frac{6x^5-4x^2+8x-7}{2x^2}$ $g(x)=\frac{6x^5}{2x^2}-\frac{4x^2}{2x^2}+\frac{8x}{2x^2}-\frac{7}{2x^2}$ $g(x)=3x^3-2+4x^{-1}-\frac{7}{2}x^{-2}$ $g'(x)=9x^2-4x^{-2}+7x^{-3}$

		PRACTICE EXERCISES	
1		**Determine the derivative from the first principles**	
	1.1	$f(x)=5-3x$	(5)
	1.2	$f(x)=3x^2+7$	(5)
	1.3	$f(x)=-2x^2+x$	(5)
	1.4	$f(x)=2x^3$	(5)
	1.5	$f(x)=\frac{2}{x}$	(6)
	1.6	$f(x)=-\frac{3}{x}$	(6)
2.1		**Use the rules of differentiation to differentiate the following:**	
		CASE 1: Making *y* the subject of the formula	
	2.1.1	$xy=x^3+2x^2-5x$	(3)
	2.1.2	$xy+4y=x^2-16$	(3)
	2.1.3	$xy=x^2+5x-y+4$	(4)
	2.1.4	$y=8x^3-2xy+1$	(5)

2.2	**CASE 2: Products**		
	2.2.1	$D_x\left[(2x-5)^2\right]$	(3)
	2.2.2	$y=(2x-3)(4x^3+5)$	(4)
	2.2.3	$y=\left(\frac{1}{x}-x\right)^2$	(4)
2.3	**CASE 3: Surds**		
	2.3.1	$g(x)=\sqrt[3]{x^2}+5x^3$	(3)
	2.3.2	$h(x)=\sqrt[5]{x}-4\sqrt{x^4}$	(3)
	2.3.3	$y=5\sqrt{x}.2\sqrt[5]{x^3}$	(4)
2.4	**CASE 4: Variable in the denominator (one term)** $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$		
	2.4.1	$y=\frac{2x^4-3x^3+4x}{x}$	(3)
	2.4.2	$y=\frac{2x^2-4x+3\sqrt{x}}{x^2}$	(4)
	2.4.3	$y=\frac{3x^3+6x^2-15x+2}{3x}$	(4)
2.5	**CASE 5: Variable in the denominator (two or more terms)**		
	2.5.1	$y=\frac{3x^2-2x-5}{x+1}$	(3)
	2.5.2	$D_t\left[\frac{t^2-1}{2t+2}\right]$	(4)
	2.5.3	$y=\frac{x^3+8}{x^2-2x+4}$	(4)
MIXED QUESTIONS FROM PAST PAPERS			
1	Differentiate the following:		
	1.1	$y=\frac{4}{\sqrt{x}}-\frac{x^3}{9}$	(4)
	1.2	$y=(1+\sqrt{x})^2$	(4)
	1.3	$y=\frac{8-3x^6}{8x^5}$	(4)

	1.4	$p(x)=\left(\frac{1}{x^3}+4x\right)^2$	(4)
	1.5	$D_x\left[\frac{x^3-1}{x-1}\right]$	(3)
	1.6	$\sqrt[3]{y}=x+1$	(4)
	1.7	$y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$	(3)
	1.8	$\frac{dy}{da}, if, y=ax^2+a$	(2)

EQUATION OF A TANGENT

Example 1. Given: $f(x)=-2x^2+1$. Determine the equation of the tangent to *f* at $x=-1$

$f'(x)=-4x$

$\therefore f'(-1)=-4(-1)=4$

$\therefore m=4$

$f(x)=-2x^2+1$

$\therefore f(-1)=-2(-1)^2+1=-1$

Substitute $(-1;-1)$

$y-(-1)=4(x-(-1))$

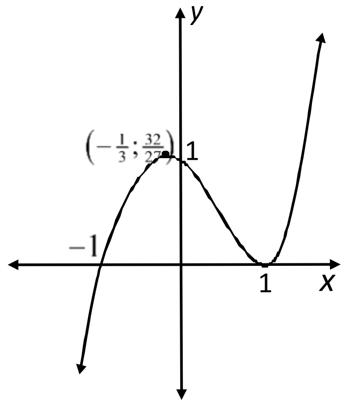
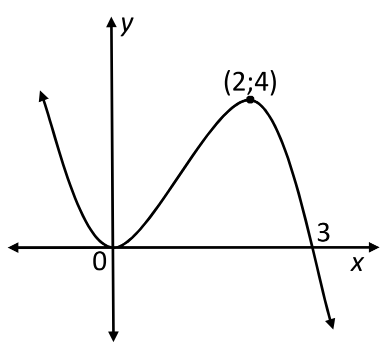
$\therefore y+1=4(x+1)$

$\therefore y+1=4x+4$

$\therefore y=4x+3$

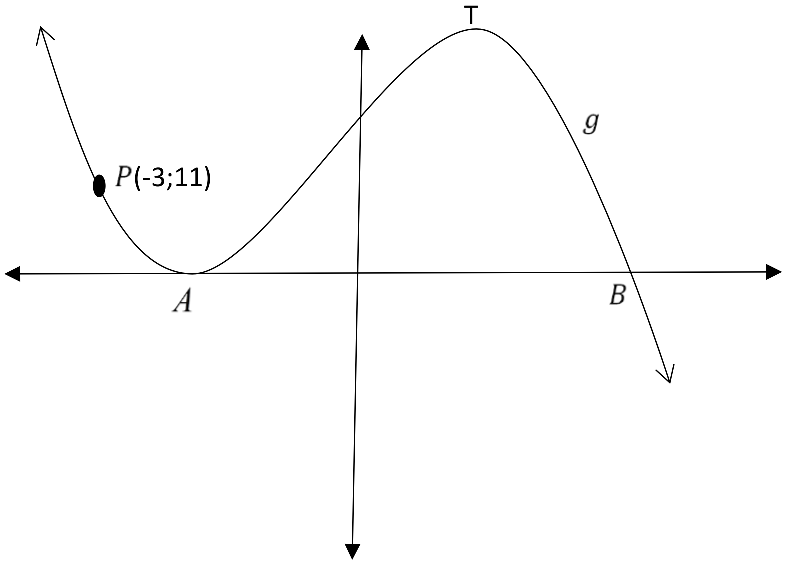
ACTIVITY

1.1	Determine the equation of the tangent to $f(x)=x^3-6x^2-6x+5$ at $x=2$	
1.2	The function defined by $g(x)=x^2-8x+20$ is given. Determine: • The point on the curve of g where the gradient of the curve is 4. • The equation of the tangent at this point	

CURVE SKETCHING		
	Sketch the following curve: $y = x^3 - x^2 - x + 1$ y-intercept: (0,1) *Use factor theorem.* $f(1) = 1^3 - 1^2 - 1 + 1 = 0$ $\therefore (x-1)$ is a factor $x^3 - x^2 - x + 1 = (x-1)(x^2 + mx - 1)$ $mx^2 - x^2 = -x^2 \Rightarrow m = 0$ $(x-1)(x^2-1) = 0$ $(x-1)(x-1)(x+1) = 0$ $(1;0), (-1;0)$ Stationary points $f'(x) = 0 \qquad 3x^2 - 2x - 1 = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 1 = \frac{32}{27}$  	
	Example 2. Sketch the graph of $f(x) = -x^3 + 3x^2$ **Solution: y-intercept (0,0)** **x-intercepts:** $-x^3 + 3x^2 = 0$ $-x^2(x-3) = 0$ $x = 0$ $x = 3$ **(3,0) , (0,0)** $f'(x) = 0 \quad -3x^2 + 6x = 0$ $x(-3x+6) = 0$ $x = 0$ **or** $x = 2$ **When x = 0,** $y = -(0)^3 + 3(0)^2 = 0$ **(0,0)** **When x=2,** $y = -(2)^3 + 3(2)^2 = 4$ **(2,4)**  	
	ACTIVITY	
1	Solve for x:	
	1.1 $\quad x^3 - 12x^2 + 36x = 0$	
	1.2 $\quad x^3 - 2x^2 - 4x + 8 = 0$	

	1.3	$x^3 - x^2 - 10x - 8 = 0$	
2	Determine the coordinates of the turning point of the following functions		
	2.1	$h(x) = x^3 - 12x^2 + 36x$	
	2.2	$f(x) = x^3 - 2x^2 - 4x + 8$	
	2.3	$g(x) = x^3 - x^2 - 10x - 8$	
3	Sketch the following functions:		
	3.1	$h(x) = x^3 - 12x^2 + 36x$	
	3.2	$f(x) = x^3 - 2x^2 - 4x + 8$	
	3.3	$g(x) = x^3 - x^2 - 10x - 8$	
	3.4	$p(x) = -x^3 - 4x^2 + 3x + 18$	
MIXED PROBLEMS			
	QUESTION 1		
1.1	Given $f(x) = 1 - 4x^2$		
	1.1.1	Determine $f'(x)$ from first principle	(5)
	1.1.2	Hence, calculate the gradient of a tangent of at $x = 2$	(2)
1.2	Determine:		
	1.2.1	$\frac{dy}{dx}$ if $y = (2 - x)^2$	(3)
	1.2.2	$f'(x)$ if $f(x) = \sqrt[3]{x} + \frac{1}{4x^4}$	(4)
	QUESTION 2		
2.1	2.1	Determine $f'(x)$ from first principle if ' $f(x) = 3x^2 - x$	(5)
2.2	Determine $\frac{dy}{dx}$ if:		
	2.2.1	$y = (x + x^{-2})^2$	(4)
	2.2.2	$y = \sqrt[3]{x^4} - \frac{1}{10}x^5$	(3)
2.3	Given: $f(x) = x^2 - \frac{4}{x^2}$		
	2.3.1	Determine the gradient of the tangent to f at the point where $x = 2$	(3)
	2.3.2	Determine the equation of the tangent to f at $x = 2$	(3)
QUESTION 3			
3.1	Given: $f(x) = 5 - 2x^2$		
	3.1.1	Determine $f'(x)$ from first principles	(5)

	3.1.2	The line $g(x) = -\frac{1}{8}x + p$ is a tangent to the graph of f at the point A. Determine the coordinates A.	(4)
	3.2	Given: $f(x) = x^3 - 2x^2$. Determine the equation of the tangent of f at the point where $x = 2$	(6)
	3.3	It is given that $f(x) = ax^3 - 24x + b$ has a local minimum at (-2,17). Calculate the values of a and b.	(4)
QUESTION 4			
	Given: $f(x) = (x-1)^2(x+2)$		
	4.1	Determine the turning points of f.	(5)
	4.2	Draw a neat sketch of f showing all intercepts with the axes as well as the turning points	(4)
	4.3	Determine the coordinates of the point where the concavity of f changes.	(3)
	4.4	Determine the value(s) of k, for which $f(x) = k$ has three distinct roots.	(4)
	4.5	Determine the equation of the tangent to f that is parallel to the line $y = -5x$ if $x < 0$	(6)
QUESTION 5			
	Given: $f(x) = -x^3 + x^2 + 8x - 12$		
	5.1	Calculate the x-intercepts of the graph of f.	(5)
	5.2	Calculate the coordinates of the turning points of the graph of f.	(5)
	5.3	Sketch the graph of f, showing clearly all the intercepts with the axes and the turning points.	(3)
	5.4	Write down the x-coordinate of the point of inflection of f.	(2)
	5.5	Write down the coordinates of the turning points of $h(x) = f(x) - 3$	(2)
QUESTION 6			
	Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x-5)(x+2)^2$ A and T are turning points of g. A and B are the x-intercepts of g. P(-3;11) is a point on the graph.		



6.1	Determine the length of AB	(3)
6.2	Determine the x-coordinate of T.	(3)
6.3	Determine the equation of the tangent to g at P(-3;11), in the form y =…..	(4)
6.4	Determine the value(s) of k for which $-2x^3-3x^2+12x+20=k$ has three distinct roots.	(4)
6.5	Determine the x-coordinate of the point of inflection	(4)

STATISTICS

Definition:
Data Handling is a process during which data (information) is collected, recorded, and presented.

NB: All learners should be able to use a calculator to do statistics' calculations.

Key Concepts:

- Data – information that is being analysed.
  - **Population** – data is collected on the entire group of elements.
  - **Sample** – data is collected on a specified set from a larger group of elements.
  - **Ungrouped data** – a set of random data elements gathered for analysis.
  - **Grouped data** – data elements aggregated into different classes or intervals.
  - **Univariate data** – single set of data that distinguished by specific characteristics.
  - **Bivariate data** – data set that compares two related variables.
- Measures of central tendency
  - The **Mean**, also known as the average, is the sum of all the data values in a set, divided by number of all elements in the set.
  - The **Median,** (Q_2), is the middle data item in an ordered data set.

    Position of median $= \frac{1}{2}(n+1)$
  - The **Mode** is the most frequent data item in a set.

- Measures of dispersion
  - The **Range** is the difference between the maximum and the minimum data values in a given data set $[Range = Max.value - Min.value]$
  - The **Inter-Quartile-Range** (IQR) is the difference between the third and first quartiles $[IQR = Q_3 - Q_1]$
  - **Standard Deviation** (σ) is a measure of how dispersed data is around the mean. The square of the standard deviation is the **variance** (σ^2).

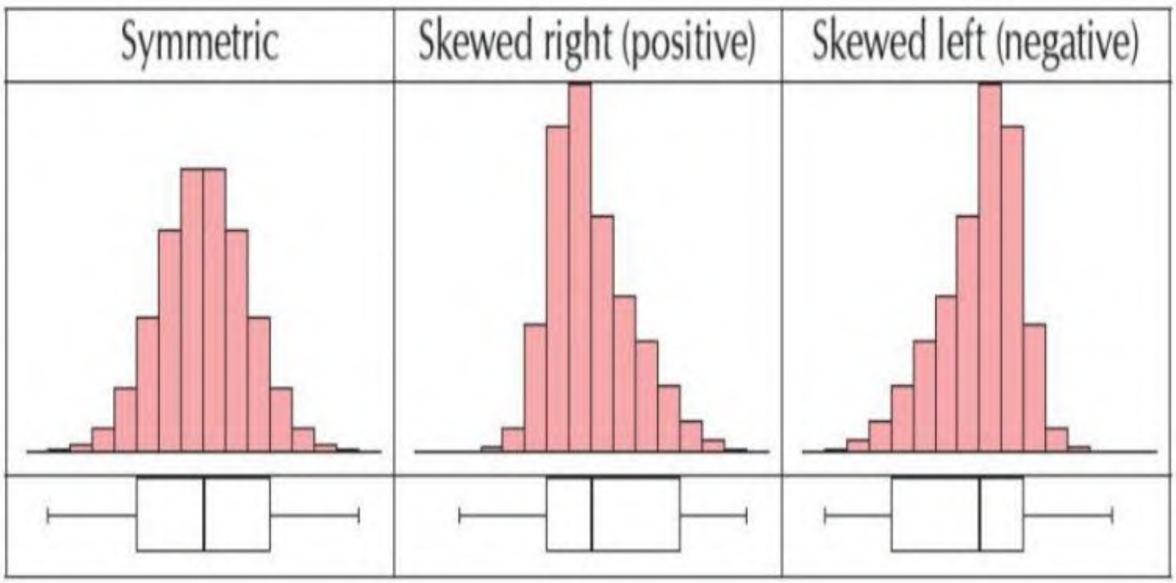
- Quartiles – numbers that divide data into quarters in an ordered data set.
  - **Lower quartile**, (Q_1) is a data item below which a quarter of the data lies.

    Position of median $= \frac{1}{4}(n+1)$
  - **Upper quartile**, (Q_3), is a data item above which a quarter of the data lies.

    Position of median $\frac{3}{4}(n+1)$
- Percentiles – numbers below which a certain percentage of data item lies.
  - Position of percentile $= \frac{percentile}{100} \times$ Number of data items.
- Five Number Summary – five numbers that separate a data set into quarters.
  - Minimum value

- ❖ Lower quartile (Q_1)
- ❖ Median (Q_2)
- ❖ Upper quartile Q_3
- ❖ Maximum value

- Box – and – Whisker Diagram (drawn using the five number summary)
  - ❖ It is important in identifying whether data in a set is symmetrical or skewed.
  - ❖ If mean – median = 0, then the distribution is symmetric.
  - ❖ If mean – median > 0, then the distribution is positively skewed.
  - ❖ If mean – median < 0, then the distribution is negatively skewed.

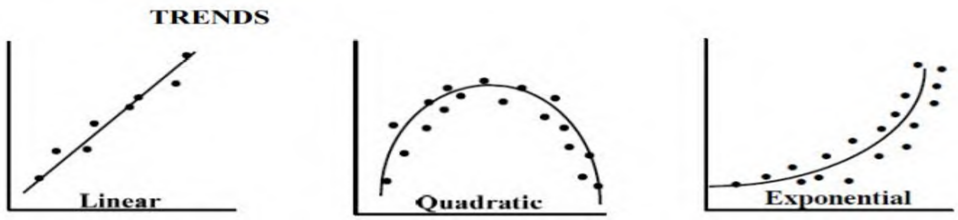


  - ❖ In a symmetrical data set approximately 68% of the data will fall within one standard deviation of the mean $(\bar{x}-\sigma;\bar{x}+\sigma)$ and approximately 95% of the data will lie within two standard deviations of the mean $[\bar{x}-\sigma;\bar{x}+\sigma]$

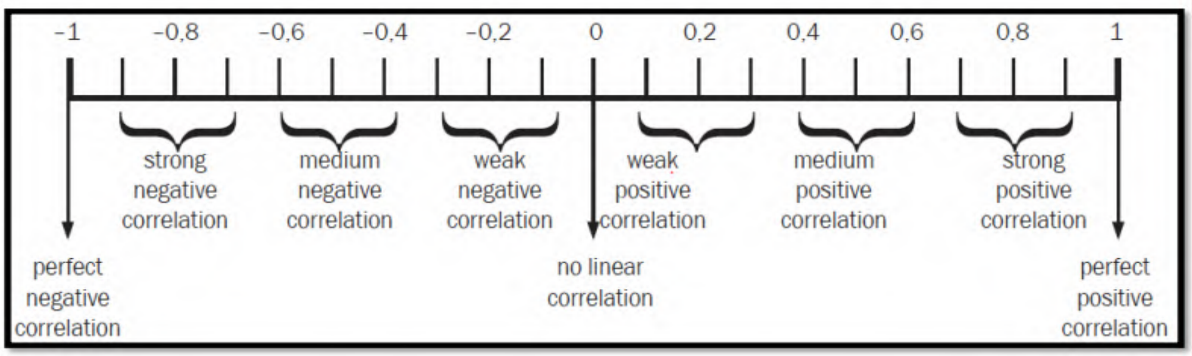
- Outliers – data items that are a lot bigger or smaller than the rest of the elements in the data set. They are determined as follows:
  - ❖ Outlier $< Q_1 - 1{,}5 \times IQR$
  - ❖ Outlier $> Q_3 + 1{,}5 \times IQR$
- Graphical representations
  - ❖ **Histogram** – represents grouped data as condensed bars whose widths and lengths represent class intervals and frequency respectively.
  - ❖ **Ogive (Cumulative Frequency Curve)** – an *s*-shaped smooth curve drawn by plotting upper limits of class intervals of a grouped data against cumulative frequency of a data set.
  - ❖ **Scatter plot** – representation of bivariate data as discrete data points.

- Bivariate data summaries

- **Regression line (line of best fit) -** a line drawn on the scatter plot that shows a general trend that bivariate data seem to follow.



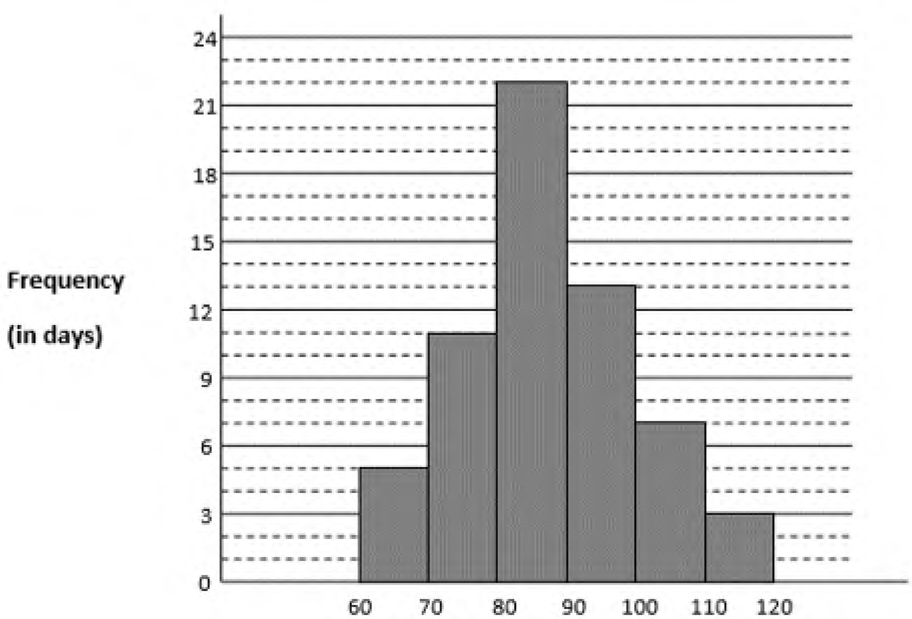
- **Least squares regression line** – is a straight line that passes through the mean point $(\bar{x};\bar{y})$ relating bivariate data.
- **Correlation Coefficient (r)** – indicates the strength of the relationship between the variables in bivariate data. It lies between -1 and 1.



Example 1

A street vendor has kept a record of sales for November and December 2007.

The daily sales in rands is shown in the histogram below.



Daily sales (in Rands)

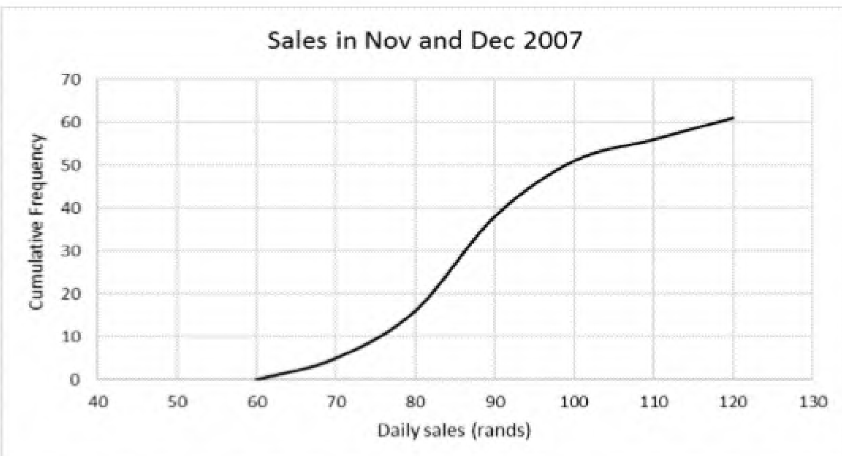
a) Complete the comulative frequency table for the sales over November and December.
b) Draw an ogive for the sales over November and December.
c) Use your ogive to determine the median value for the daily sales. Explain how you obtain your answer.
d) Estimate the interval of the upper 25% of the daily sales.

Solutions

(a)

Daily sales (in Rand)	Frequency	Cumulative Frequency
$60 \le rand < 70$	5	5
$70 \le rand < 80$	11	16
$80 \le rand < 90$	22	38
$90 \le rand < 100$	13	51
$100 \le rand < 110$	7	58
$110 \le rand < 120$	3	61

(b



Hints:

- x-coordinate - use upper limit of each interval
- y-coordinate – cumulative frequency
- if the frequency of the first interval is not 0, then include an interval before the given one and use 0 as its frequency

(c) *Median = R87. There are 61 data points, so the median in on the 31st position. On the y-axis put a ruler at 31; move horizontally until you touch the graph, then move vertically down to read the x-coordinate.*

(d) *The upper 25% lies above 75%. 75% of 61 = 45,75. Read 45,75 from the y-axis across to the graph and down to the x-axis. Therefore the upper 75% of sales lies in the :*

$96 \leq rand < 120$

Example 2

The data below shows the energy levels, in kilocalories per 100 g, of 10 different snack foods.

440 520 480 560 615 550 620 680 545 490

(a) Calculate the mean energy level of these snack foods.
(b) Calculate the standard deviation.
(c) The energy levels, in kilocalories per 100 g, of 10 different breakfast cereals had a mean of 545,7 kilocalories and a standard deviation of 28 kilocalories. Which of the two types of food show greater variation in energy levels? What do you conclude?

Solution

(a) Mean $= \frac{5500}{10} = 550$

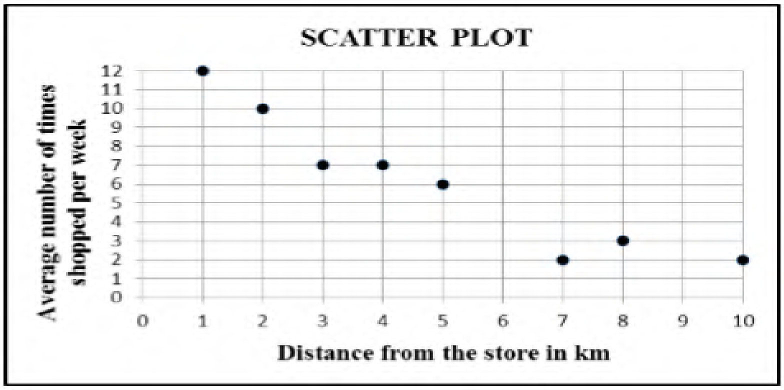
(b) $\sigma = 69{,}03$ kilocalories

(c) Snack foods have a greater variation. The standard deviation for snack foods is 69,03 kilocalories whilst the standard deviation for breakfast cereals is 28 kilocalories. i.e. energy levels of breakfast cereals is spread closer to the mean than in those of the snack food.

Example 3 DBE Nov 2016

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



(a) Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week.

(b) Calculate the correlation coefficient of the data.

(c) Calculate the equation of the least squares regression line of the data.

(e) Sketch the least squares regression line on the scatter plot.

Solutions

a) Strong

(b) $r = -0{,}95\ (-0{,}9462..)\ldots$

Using CASIO fx 82 ZA plus calculator

- Mode
- 2: STAT
- 2: A + B x
- Enter the x values first:
  1 =; 2 =; 3 =; 4 =; 5 =; 7=; 8=; 10=
- Use arrows to move right to y column and up to start next to 1.
- Enter y values:
  12 =; 10 =; 7 =; 7=; 6=; 2=; 3=; 2=
- Press (orange) AC button
- Press SHIFT STAT (at 1)
- Press 5: Reg

Press 3: r = and get $r = -0{,}95\ (-0{,}9462..)\ldots$

To get equation of regression line:

- Press (orange) AC button
- Press SHIFT STAT (at 1)
- Press 5: Reg
- Press 1: A = and get 11,7132…

This is the y-intercept of the regression line

- Press orange AC button
- Press SHIFT STAT
- Press 5: Reg
- Now press 2: B = and get -1,1176…

This is the gradient of the regression line

<u>Answer</u>:

The least squares regression line:

$\hat{y} = -1{,}12x + 11{,}71$

Using SHARP EL-W53HT

- Mode
- 1: STAT
- 1: LINE
- Enter the values in coordinate form:
- 1 (x,y) 12change; 2 (x,y) 10 change;
- 3 (x,y) 7 change; 4 (x,y) 7 change;
- 5 (x,y) 6 change; 7 (x,y) 2 change;
- 8 (x,y) 3 change; 10 (x,y) 2 change

Press On: It goes back to Stat 1 (LINE)

Press ALPHA (÷): r _ appears on the screen

Press =: the value of r appears on the screen.

To get equation of regression line:

Press ALPHA(a): a appears on the screen

Perss =: the value of a appears 11,7132…

This is the y-intercept of the regression line

To get equation of regression line:

Press ALPHA(b): b appears on the screen

Perss =: the value of b appears -1,1176…

This is the gradient of the regression line

<u>Answer</u>:

The least squares regression line:

$\hat{y} = -1{,}12x + 11{,}71$ (correct to 2 decimal places)

(c) $a = 11{,}71 \ (11{,}7132\ldots)$

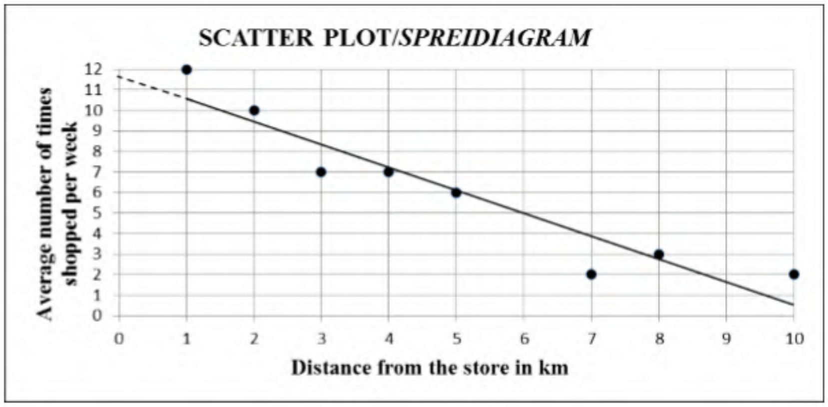
$b = -1{,}12 \ (-1{,}1176\ldots)$

$\hat{y} = -1{,}12x + 11{,}71$

(d) $\hat{y} = -1{,}12(6) + 11{,}71$

$= 5$ times

(e)



EXERCISES

QUESTION 1: North –West Sept 2019

1.1 The time (in seconds) between the consecutive landings of aeroplanes at an airport on day 1 was recorded. The data is given in the Cumulative Frequency table below:

Time (in seconds)	Number of Aeroplanes (Frequency)	Cumulative Frequency
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	K	p
$210 < t \leq 240$	7	80

1.1.1 Show that $k = 10$. (1)

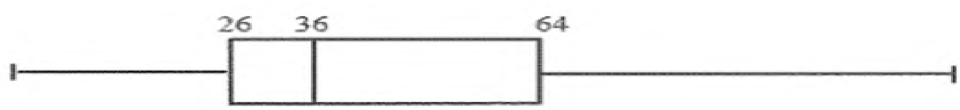
1.1.2 Write down the value of p. (1)

1.1.3 Calculate the estimated mean time between the landings of the two consecutive aeroplanes. (3)

1.2	It is given the $(q;186{,}89)$ is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.		
	1.2.1	Write down the estimated standard deviation of the time between the consecutive landings of aeroplanes.	(2)
	1.2.2	Calculate the value of q.	(1)
1.3	On day 2, the same of aeroplanes that landed on day 1, land at the airport. The elapsed time between all the consecutive landings of all the aeroplanes is *m* seconds shorter than the time that is given in the above table above. If an ogive is to be drawn of the data of day 2, the following will be true: • The ogive will be grounded at $(57;0)$ • The maximum value of the ogive will be at $(237;80)$ Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same.		(2)

QUESTION 2: Limpopo Sept 2019

Some of the test results of 21 learners are given below. There was only one result of 26 marks and only one result of 64 marks.



What information is omitted on the above diagram?

The results were read to the learners in ascending order. If the fifth learner was 26, which leaner obtained a result of 64?

One of the learners was arguing that the distribution of data was not symmetrical. Is the learner correct? Give a reason for a learner's remark.

The class calculated the following using the test results: $\bar{x} = 45{,}5$ and $\sigma = 19{,}2$

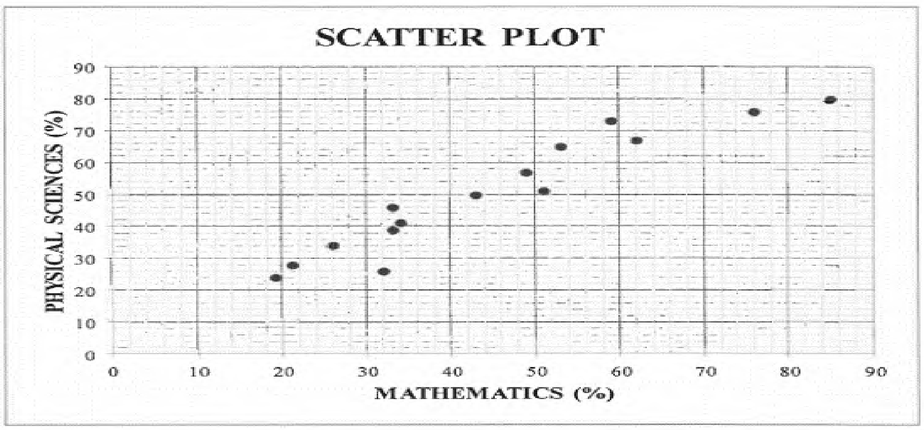
Use the above information and determine the number of learners whose results fall outside ONE standard deviation of the mean.

If the marks of each learner would increase by 5 marks, what effect would it have on the mean and standard deviation?

QUESTION 3: DBE Nov 2020

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

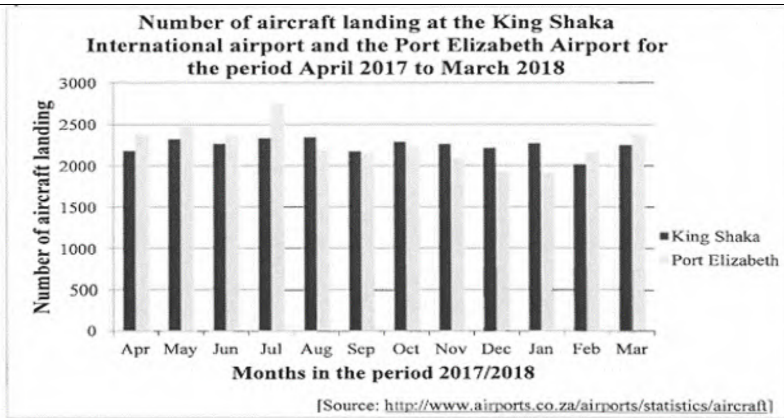
Mathematics (As %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
Physical Sciences (As %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80



3.1	Determine the equation of the least squares regression line for the data.	(3)
3.2	Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK.	(2)
3.3	Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics.	(2)
3.4	Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data.	(1)
3.5	Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data.	(1)
3.6	What trend did the teacher observe between the results of the two subjects?	(1)

QUESTION: 4 DBE Nov 2020

The number of aircraft landing at the King Shaka International and the Port Elizabeth Airport for the period starting in April 2017 ending in March 2018, is shown in the double bar graph below.



4.1	The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During Which month is this difference the greatest?	(1)
	The number of aircraft landing at the King Shaka International Airport during these months are:	

2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2018	2 254

4.2	Calculate the mean for the data.		(2)
4.3	Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period.		(2)
4.4	Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean.		(3)
4.5	Which one of the following statements is CORRECT?		(1)
	a)	During December and January, there were more landings at the Port Elizabeth Airport than that at the King Shaka International Airport.	
	b)	There was a greater variation in the number of aircraft landings at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.	
	c)	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport.	

QUESTION 5: DBE May/June 2021

5.1 Sam recorded the amount of data (in MB) that she had used on each of the first 15 days in April. The information is shown in the table below.

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

5.1.1	Calculate the: a) Mean for the data set. b) Standard deviation for the data set.	 (2) (1)
5.1.2	Determine the number of days on which the amount of data used was greater than one standard deviation above the mean.	(2)
5.1.3	Calculate the maximum total amount of data that Sam must use for the remainder of the month if she wishes for the overall mean of April to be 80% of the mean for the first 15 days.	(3)

5.2	The wind speed (in km per hour) and temperature (in ^{0}C) for a certain town were recorded at 16:00 for a period of 10 days. The information is shown in the table below.	
5.2.1	Determine the equation of the least squares regression line for the data.	(3)
5.2.2	Predict the temperature at 16:00 if, on a certain day, the wind speed of this town was 9 km per hour.	(2)
5.2.3	Interpret the value of ***b*** in the context of the data.	(1)

Wind speed in km/h (x)	2	6	15	20	25	17	11	24	13	22
Temperature in ^{0}C (y)	28	26	22	22	16	20	24	19	26	19

QUESTION 6: DBE Nov 2019

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below.

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

6.1	How many people paid R 200 or less on their monthly cellphone contracts?	(1)
6.2	Use the information above to show that $a = 24$ and $b = 16$.	(5)
6.3	Write down the modal class for the data.	(1)
6.4	Determine the estimated: a) Mean b) Standard deviation	 (3) (2)
6.4	On the grid provided in the ANSWER BOOK, draw an Ogive (cumulative frequency graph) to represent the data.	(4)
6.5	Determine how many people paid more than R420 per month for their cellphone contracts.	(2)
		[18]

QUESTION 7 DBE Nov 2010

QUESTION 8 DBE May/June 2016

On a certain day a tour operator sent 11 tour buses to 11 different destinations.

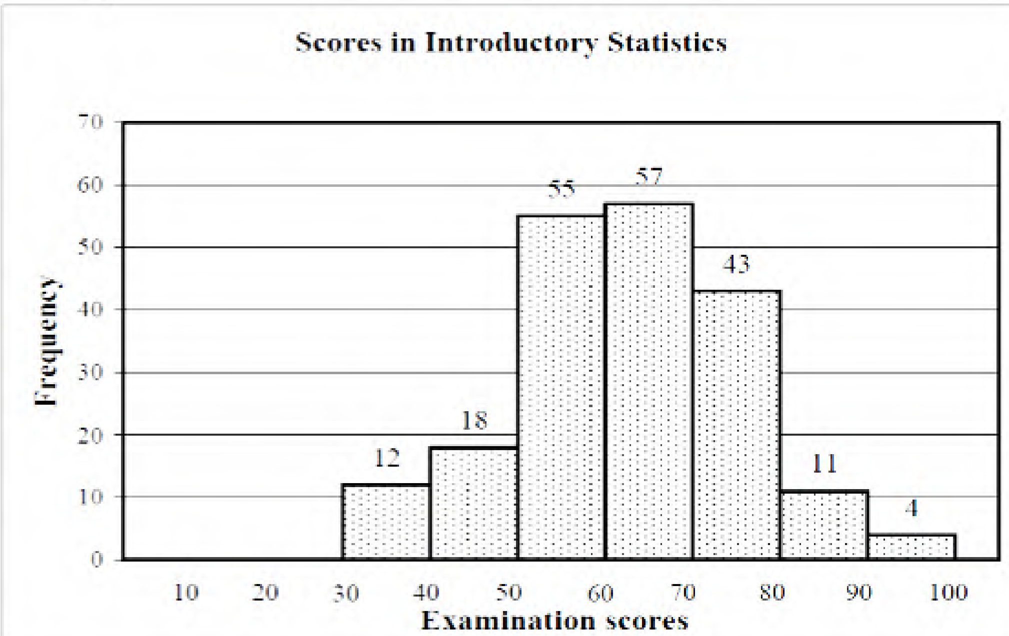
The table below shows the number of passengers on each bus.

8	8	10	12	16	19	20	21	24	25	26

8.1	Calculate the mean number of passengers travelling in a tour bus.	(2)
8.2	Write down the five number summary of the data.	(3)
8.3	Draw a box and whisker diagram for the data.	(3)
8.4	Refer to the box and whisker diagram and comment on the skewness of the data set.	(1)

8.5	Calculate the standard deviation of this data set.	(2)
8.6	A tour is regarded as popular if the number of passengers on a tour bus is one standard deviation above the mean. How many destinations were popular on this particular day?	(2)

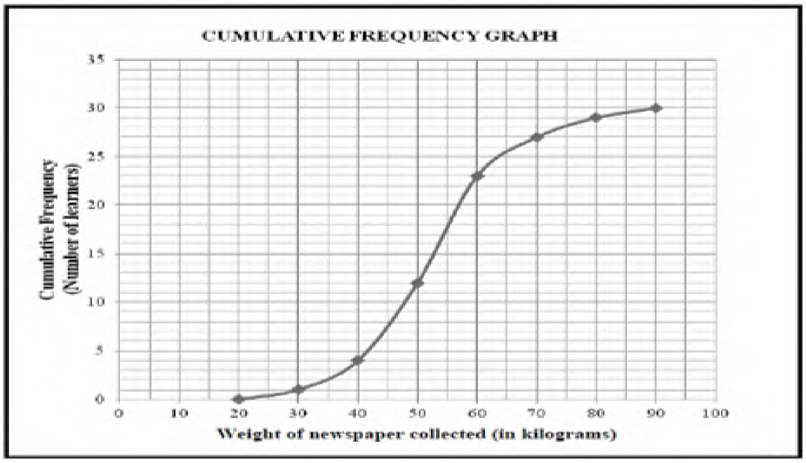
The histogram below shows the distribution of examination scores for 200 learners in Introductory Statistics.



7.1	Comment on the skewness of the data.	(1)
7.2	Complete the cumulative frequency table for the above data provided on your answer book.	(2)
7.3	Draw an ogive of the above data on the grid provided on your answer book.	(5)
7.4	Use the ogive to estimate how any learners scored 75% or more for the examination.	(1)

QUESTION 9: DBE Nov 2012

As part of an environmental awareness initiative, learners of Greenside High School were requested to collect newspapers for recycling. The cumulative frequency graph (ogive) below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by 30 learners.



9.1	Determine the modal class of the weight of the newspaper collected.	(1)
9.2	Determine the median, lower quartile and upper quartile weight of the newspapers collected by this group of learners.	(3)
9.3	How many learners collected more than 60 kilograms of newspaper?	(2)
		[6]

QUESTION 10: DBE Feb/March 2012

In the grid below, *a, b, c, d, e, f,* and *g* represent values in a data set written in an increasing order. No value in the data set is repeated.

A	*b*	*c*	*d*	*e*	*f*	*g*

Determine the value of *a, b, c, d, e, f and g* if :

- The maximum value is 42
- The range is 35
- The median is 23
- The difference between the median and the upper quartile is 14
- The interquartile range is 22
- $e = 2c$
- The average is 25

		[7]

QUESTION 11 DBE Nov 2015

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below. Determine:

Sum of the values	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

The mean of the data (2)

The median. (2)

The standard deviation. (2)

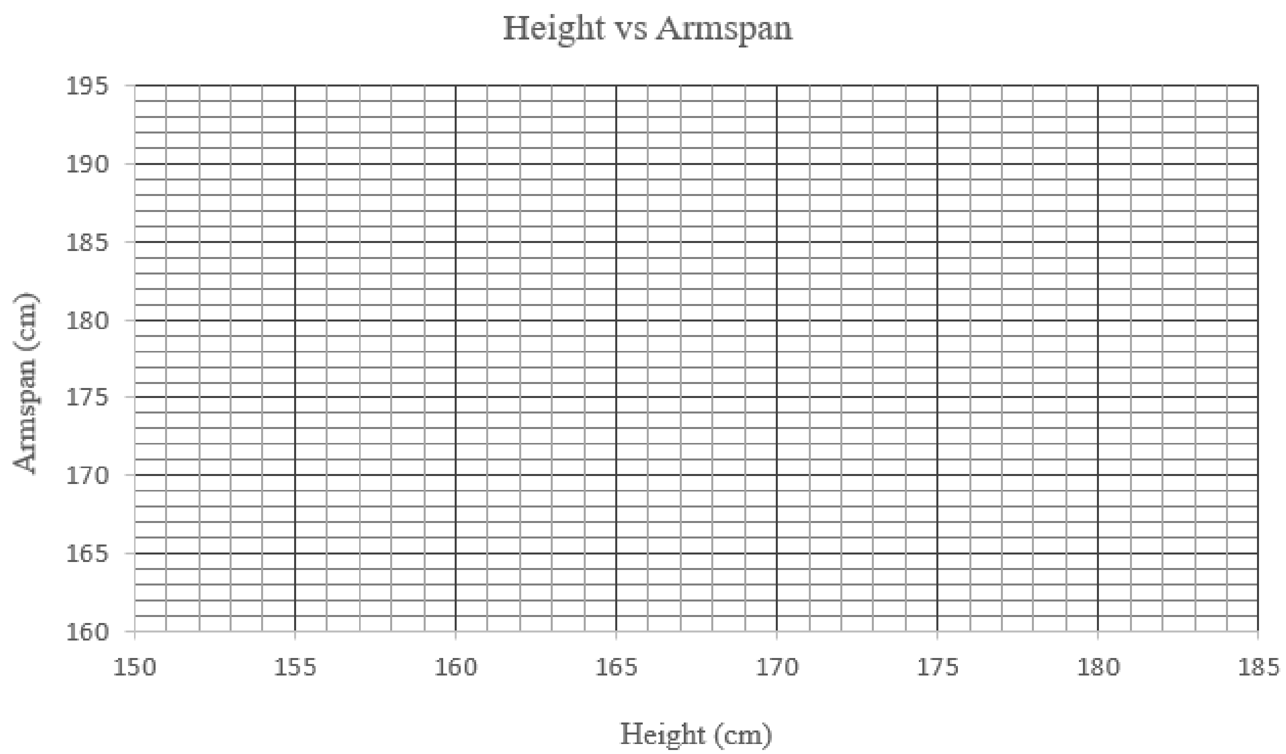
The number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)

QUESTION 12 Sept 2015

Ten athletes took part in is a javelin throwing competition. Their height, in *cm*, and their arm span, in *cm*, is shown in the table below.

Athlete	1	2	3	4	5	6	7	8	9	10
Height (in *cm*)	156	173	181	174	167	170	169	174	177	168
Arm span (in *cm*)	164	181	193	178	172	178	165	183	190	173

12.1 Represent the height and arm span for each athlete on the scatter plot on your answer book.

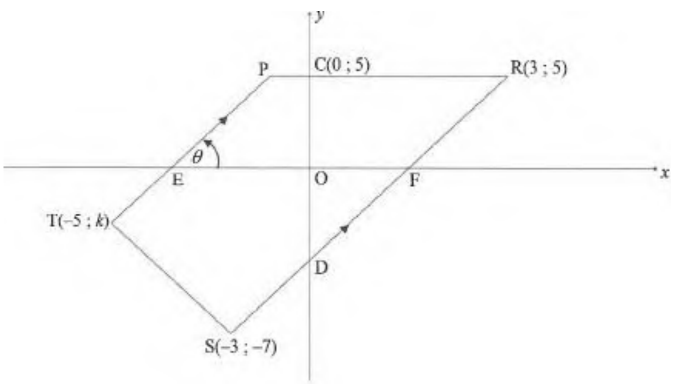


12.2	Determine the equation of the least square regression line.	(4)
12.3	Use the equation in 12.2 to estimate the arm span of an athlete whose height is 176 cm.	(2)
12.4	The correlation coefficient for this set of data is 0, 89. Comment on the strength of the relationship between height and arm span.	(1)

ANALYTICAL GEOMETRY

QUESTION 1 **DBE NOV 2019**

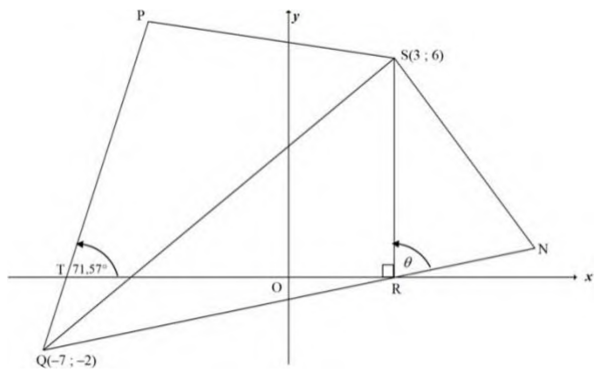
1. In the diagram P, R(3; 5), , S(−3; −7) and T(−5; k) are vertices of trapezium PRST and PT ∥ RS. RS and PR cut the y − axis at D and C(0; 5) respectively. PT and RS cut the x− axis at E and F respectively. $P\hat{E}F = \theta$.



1.1	Write down the equation of PR.	(1)
1.2	Calculate the:	
	1.2.1 Gradient of RS.	(2)
	1.2.2 Size of θ	(3)
	1.2.3 Coordinates of D	(3)
1.3	If it is given that, TS=$2\sqrt{5}$ calculate the value of k.	(4)
1.4	Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N.	(3)
1.5	ΔPRD is reflected in the y − axis to form ΔP′R′D′. Calculate the size of $\angle RDR'$.	(3)
		[19]

QUESTION 2 **DBE Feb-Mar 2018**

In the diagram P, Q(−7; −2), R and S(3; 6) are vertices of a quadrilateral. R is a point on the x −axis. QR is
produced to N such that QR = 2RN. SN is drawn. $\angle PTO = 71{,}57°$ and $\angle SRN = \theta$.
Determine:



2.1 The equation of SR (1)

2.2 The gradient of QP to the nearest integer. (2)

2.3 The equation of QP in the form $y = mx + c$ (2)

2.4. The length of QR. Leave your answer in **surdform.** (2)

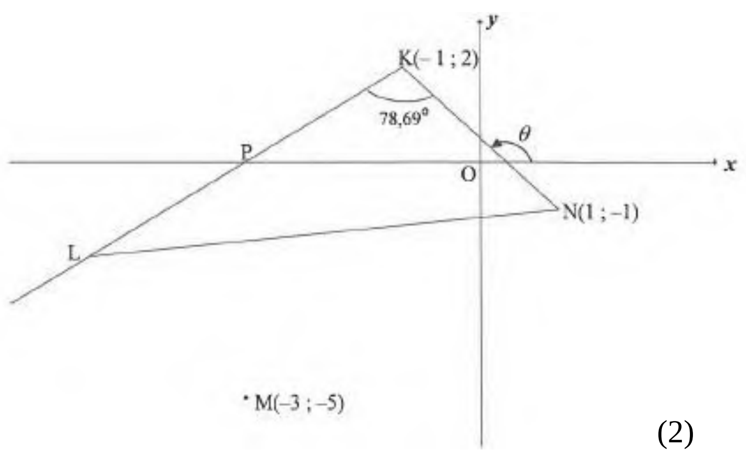
2.5 $\tan(90° - \theta)$ (3)

2.6 The area of Δ RSN, **without using a calculator.** (6)

QUESTION 3

In the diagram K(−1;2), L and N(1;−1) are vertices of Δ KLN such that ∠LKN = 78,69°. KL intersects the x − axis at P. KLis produced. The inclination of KN is θ.

The coordinates of M are (−3; −5).



3.1 Calculate

3.1.1 The gradient of KN. (2)

3.1.2 The size of θ, the inclinationof KN (2)

3.2 Show that the gradient of KL is equal to -1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

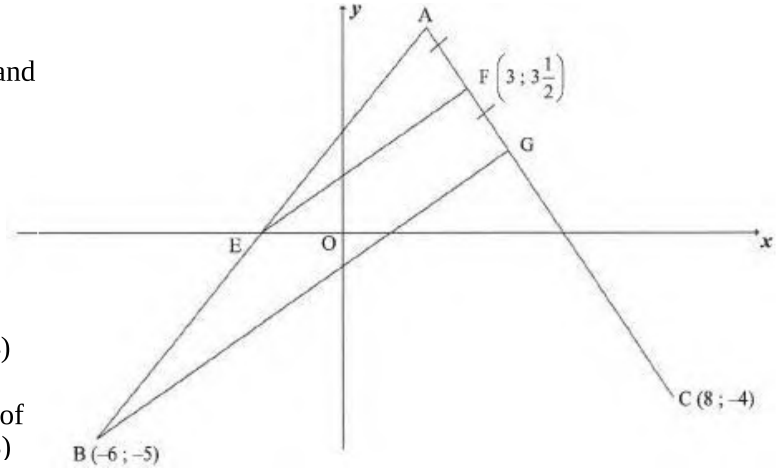
3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN (4)

[22]

QUESTION 4: DBE NOV 2017

In the diagram, A, B(−6; −5) and C(8; −4) are points in the Cartesian plane. $F\left(3;3\frac{1}{2}\right)$ and G are points on the line AC such AF=FG.

E is the x-intercept of AB.



4.1 Calculate:

4.1.1 The equation of AC, in the form $y = mx + c$. (4)

4.1.2 The coordinates of G if theequation of BG is (3)

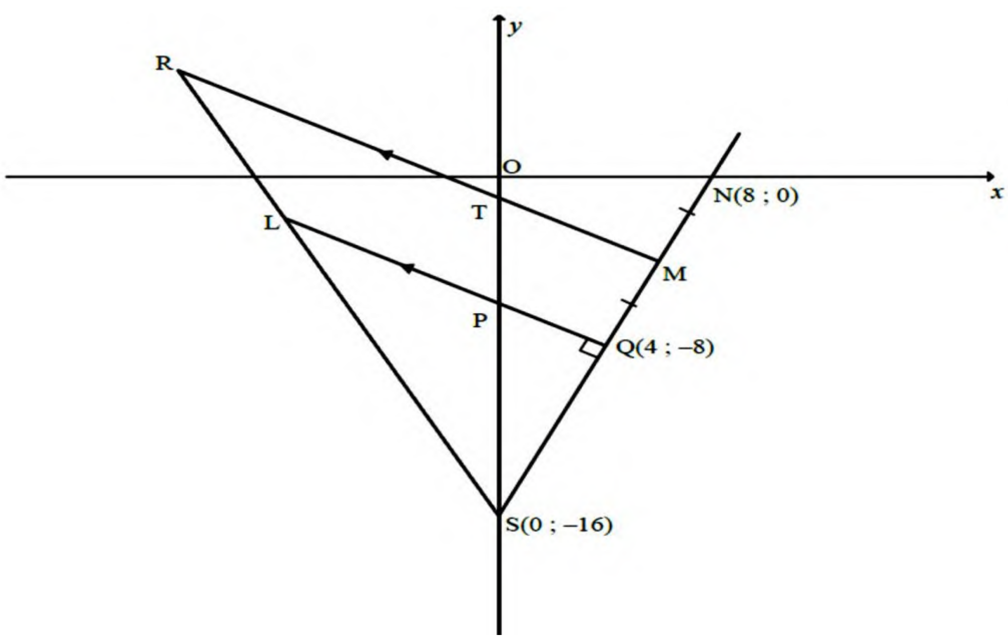
4.2 Show by calculation that the coordinates of A is (2; 5). (2)

4.3 Prove that $EF \parallel BG$. (4)

4.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D.(4)

QUESTION 5: DBE May/June 2021

In the diagram, S(0 ; –16), L and Q(4 ; –8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM || LQ. SM produced cuts the x-axis at N(8 ; 0). QM = MN. T and P are the y-intercepts of RM and LQ respectively.



5.1 Calculate the coordinates of M. (2)

5.2 Calculate the gradient of NS. (2)

5.3 Show that the equation of line LQ is $y = -\frac{1}{2}x - 6$. (3)

5.4 Determine the equation of a circle having centre at O, the origin, and also passing through S. (2)

5.5 Calculate the coordinates of T. (3)

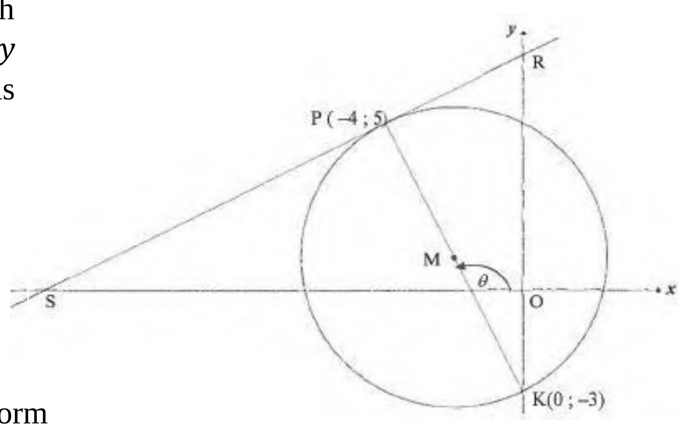
5.6 Determine $\frac{LS}{RS}$. (3)

5.7 Calculate the area of PTMQ. (4)

[19]

QUESTION 6 **DBE NOV 2017**

In the diagram, P$(-4; 5)$ and K$(0; -3)$ are the end points of the diameter of the circle with centre M. Sand R are respectively the $x-$ and $y-$ intercepts of the tangent to the circle at P. θ is the inclinationof PK with the positive x −axis.



6.1 Determine

6.1.1 The The gradient of SR (4)

6.1.2 The equation of SR in the form $y = mx + c$ (3)

6.1.3 The equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (4)

6.1.4 The size of ∠PKR (3)

6.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$. (2)

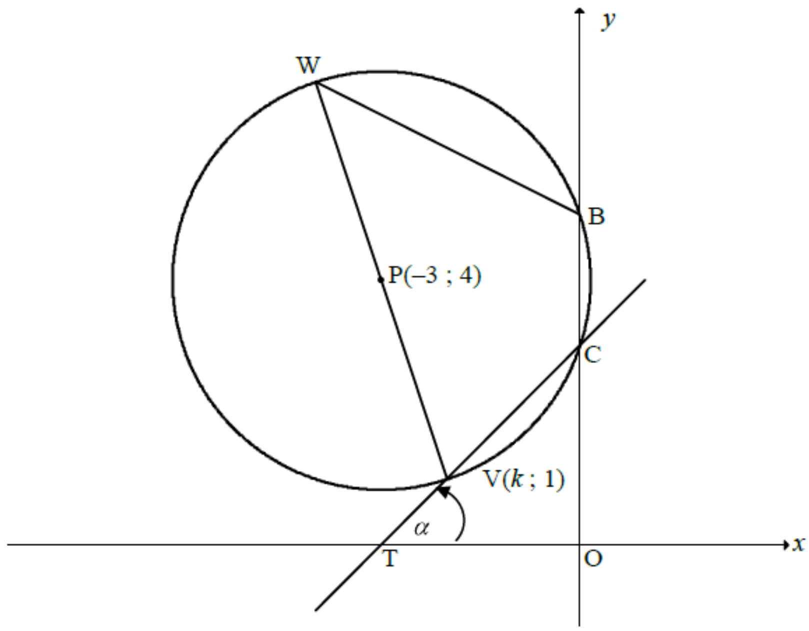
6.2 Determine the value(s) of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

6.3 Calculate the area of ΔSMK. (5)

[23]

QUESTION 7: DBE May/June 2021

In the diagram, P(–3 ; 4) is the centre of the circle. V(k ; 1) and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



7.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

7.2 The equation of the circle is given as $x^2 + 6x + y^2 - 8y + 15 = 0$. Calculate the length of BC. (4)

7.3 If $k = -2$, calculate the size of:

7.3.1 α (3)

7.3.2 $V\hat{W}B$ (2)

7.4 A new circle is obtained when the given circle is reflected about the line $y = 1$.

Determine the:

7.4.1 Coordinates of Q, the centre of the new circle (2)

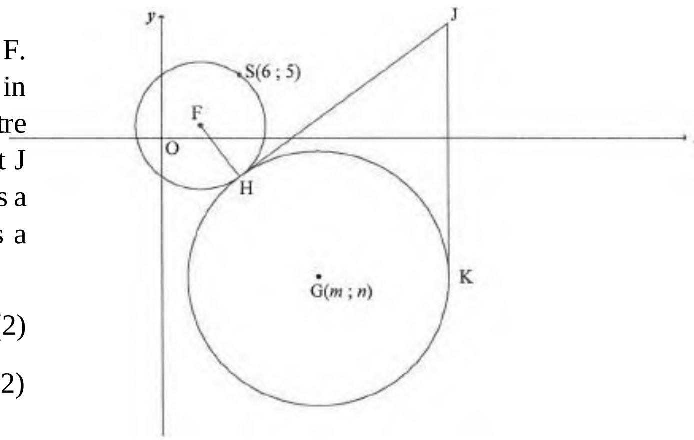
7.4.2 Equation of the new circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

7.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

QUESTION 8: DBE NOV 2018

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6; 5), is a point on the circle with centre F. Another circle with centre $\mathrm{G}(\mathrm{m};\mathrm{n})$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2 The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



8.1 Write down the coordinates of F. (2)

8.2. Calculate the length of FS. (2)

8.3 Write down the length of HG. (1)

8.4 Give a reason why $\mathrm{JH}=\mathrm{JK}$. (1)

8.5 Determine:

8.5.1 The distance FJ, with reasons, if it is given that $\mathrm{JK}=20$. (4)

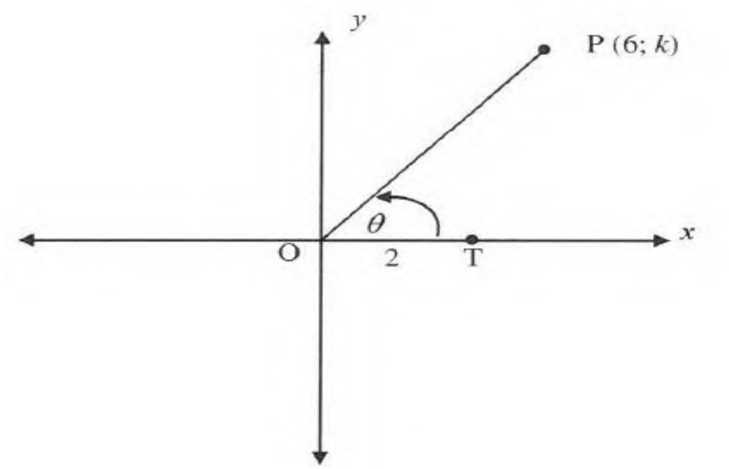
8.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

8.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

TRIGONOMETRY

QUESTION 1(Using a Sketch)

In the diagram, P (6 ; k) is a point in the first quadrant. $P\hat{O}T = \theta$ and $OT = 2$. It is further given that $\sqrt{5}\cos\theta - 2 = 0$.



Determine without the use of a calculator

1.1 $\tan\theta$ in terms of k (1)

1.2 The value of k (5)

QUESTION 2

If $8\sin\theta + 5 = 0$ and $\tan\theta > 0$, determine the value of each of the following without a calculator:

2.1 $\tan(-\theta)$ (4)

2.2 $\sin\left(180^0 + 2\theta\right)$ (4)

QUESTION 3

3.1 Given: $\sin 56° = q$

Determine without using a calculator, the value of the following in terms of q:

3.1.1 $\cos 146°$ (2)

3.1.2 $\sin 112°$ (3)

3.1.3 $\cos 17^0$ (4)

3.2 Simplify

$$\frac{\sin(450 - x).\tan(x - 180).\sin 23^0 \cos 23^0}{\cos 44^0 \sin(-x)}$$ (6)

QUESTION4: (REDUCTION FORMULA)

Simplify, without the use of a calculator, the following expression to a single trigonometric ratio:

$$\frac{\sin(90^0 - x).\tan(180^0 - x)}{\cos(-x).\sin(180 + x)}$$ (6)

QUESTION 5

5.1.1 Simplify: $\dfrac{\sin 40^\circ.\tan(-315^\circ)}{\cos 230^\circ.\sin 420^\circ}$ (5)

5.1.2 Simplify: $\dfrac{\sin 15^\circ \cos 15^\circ}{\cos(45^\circ - x)\cos x - \sin(45^\circ - x)\sin x}$ (5)

QUESTION 6

6.1 Show that $\cos 15^\circ = \dfrac{\sqrt{\sqrt{3}+2}}{2}$ (4)

6.2 Simplify to a single trig ratio: $\dfrac{\sin(180^\circ - x)\cos^2(-180^\circ + x)\cos 35^\circ}{\tan(540^\circ - x)\sin 235^\circ.\sin(90^\circ - x)}$

(7)

QUESTION7

7.1 Reduce the expression below to a single trigonometric ratio of one angle, without using a calculator. $\dfrac{\tan 43^\circ \sin 47^\circ.2\cos 137^\circ}{2\cos 317^\circ \sin 133^\circ - 1}$

(4)

7.2 Simplify $\dfrac{\cos 3\theta.\sin\theta - \sin 3\theta.\cos\theta}{\sin\theta.\cos\theta}$ (5)

5.2 PROVING IDENTITIES

QUESTION 8

8.1 Prove that: $\dfrac{-\cos^2 x + \sin(-2x)}{\sin x - 2\cos x} = \sin x$

(4)

8.2 Prove that: $\dfrac{\sin 2x}{\sin(90^\circ - x) - \cos(180^\circ - x)} = \sin x$ (3)

(3)

8.3 Prove that : $(3\sin x + 3\cos x)^2 = 9\sin 2x + 9$

8.4 Given: $\dfrac{\tan x - \sin x}{\sin^3 x} = \dfrac{1}{\cos x(1+\cos x)}$

8.4.1 Prove the identity. (7)

8.4.2 For which values of x, $x \in [0^\circ; 360^\circ]$, will $\dfrac{1}{\cos x(1+\cos x)}$ be defined? (4)

8.5 Prove the identity $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

8.6 Prove that: $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$ (4)

5.3 GENERAL SOLUTION AND SOLVING TRIGONOMETRIC EQUATIONS.

QUESTION 9

Determine the general solution of:

9.1 $6\sin^2\theta + \cos\theta = 4$ (6)

9.2 $3\tan^2 x + 5\tan x - 2 = 0$ (6)

9.3 $6\cos^2 x + \sin x - 5 = 0$ (7)

9.4 $1 + 4\sin^2 x - 5\sin x + \cos 2x = 0$ (7)

9.5 $2\cos 2x.\sin x = \cos 2x$ (6)

QUESTION 10

10.1 If $\cos\theta = 2\sin 75°\sin 15°$; $\theta \in [-360°; 360°]$, determine θ without using a calculator.

10.2 Solve for A if $\tan A = \tan 135°$ and

a) $180° < A < 360°$

b) $360° < A < 720°$

10.3 Determine the general solution to $3\sin\theta\sin 22° = 3\cos\theta\cos 22° + 1$ (4)

10.5 Determine the general solution to: $\dfrac{\sin 3\alpha}{\sin\alpha} = 2$ (4)

10.6 Consider $\cos 6x + \cos 2x = 2\cos 4x\cos 2x$

a) Show that $\cos 6x + \cos 2x = 2\cos 4x\cos 2x$ (4)

b) Hence otherwise, write down the general solution of the equation (4)
$\cos 6x + \cos 2x + \cos 4x = 0$

10.7 If $\theta \in [-180°; 180°]$, determine the value(s) of θ:

(a) $\sin 5\theta\cos 20° - \cos 5\theta\sin 20° = 1$ (4)

(b) $2\cos 3\theta\cos 30° - 2\sin 3\theta\sin 30° = 1$ (4)

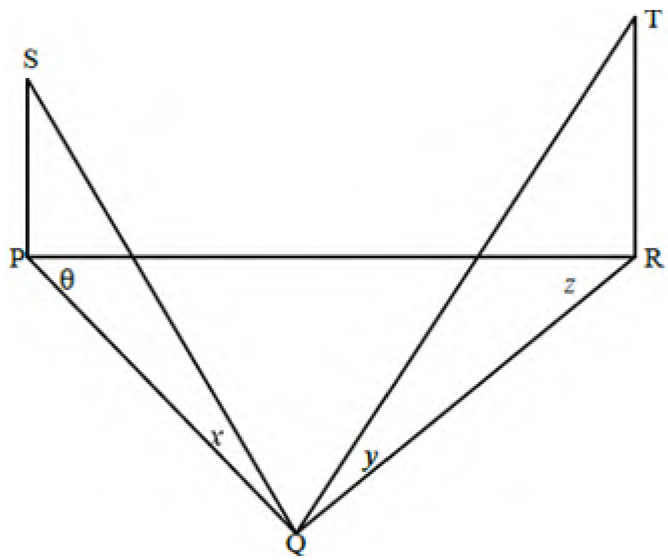
10.8 Solve for x if $\cos(x - 30°) = 2\sin x$ (4)

10.9 Solve for θ if $\cos 2\theta = \sin(30° + \theta)$ (4)

D and 3D TRIGONOMETRY

QUESTION 11

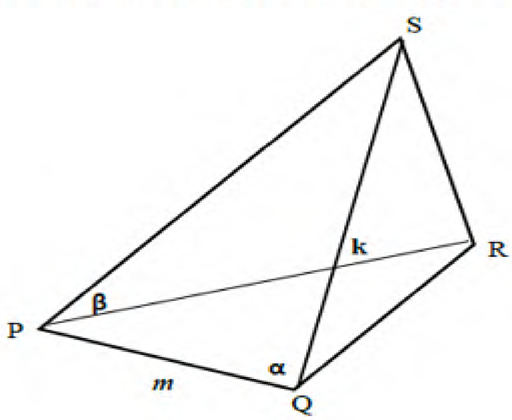
P, Q and R are three points on the same horizontal plane. PS and RT are the two vertical poles. Wires are strung from Q to the tops of the poles. The wire from Q to S forms an angle of x^0 with the ground. The other wire forms an angle of y^0 with the horizontal plane and is t metres long.



11.1 Show that $QR = t.\cos y$ (3)

11.2 Show that $PQ = \dfrac{t.\cos y.\sin z}{\sin\theta}$ (3)

11.3 Prove that $PS = \dfrac{t.\sin z.\tan x.\cos y}{\sin\theta}$ (3)

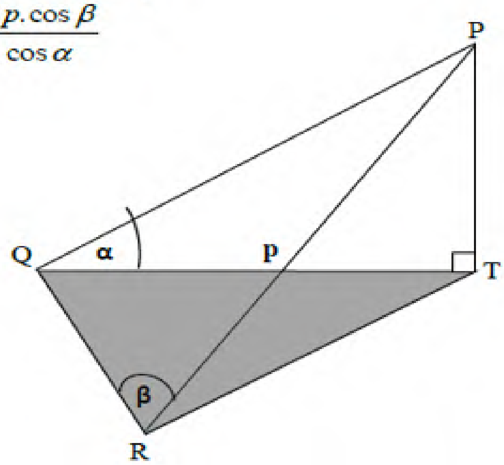
QUESTION 12 In the figure SR is a vertical mast. P, Q and R are 3 points in the same horizontal plane. PS and QS are stay ropes. $PQ = m$; $QS = k$; $P\hat{Q}S = \alpha$. The angle of elevation of S from P is β. If $k = 2m$, show that: $PS = m\sqrt{5 - 4\cos\alpha}$  	(5)

QUESTION 13

In the figure, Q, T and R are points in the horizontal plane and TP represents a vertical pole positioned at T. The angle of elevation of P from Q is α.

QT = p and PQ = PR. $P\hat{R}Q = \beta$

Prove that $PQ = \dfrac{2p.\cos\beta}{\cos\alpha}$



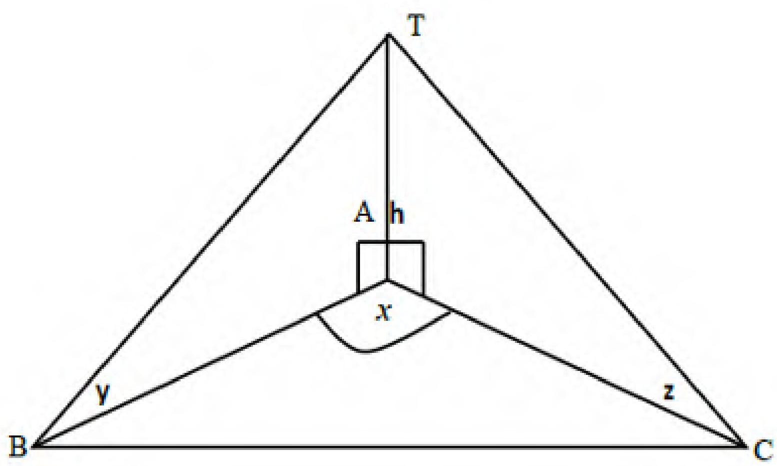
QUESTION 14

In the diagram below TA represents the vertical pole of height h erected in the horizontal plane ABC.

$A\hat{B}T = y$

$B\hat{A}C = x$

$A\hat{C}T = z$



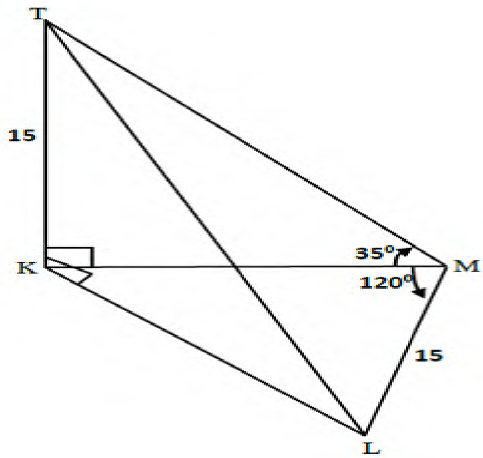
14.1 Prove that:

Area $\Delta ABC = \dfrac{h^2 \sin x}{2\tan y.\tan z}$ (4)

14.2 Calculate the value of h if the area of $\Delta ABC = 51{,}8m^2$, $x = 123{,}7^0$, $y = 37{,}2^0$ and $z = 61{,}6^0$ (3)

QUESTION 15

In the sketch below, K, L and M are three points in the same horizontal plane such that $K\hat{M}L = 120^0$. T represents a point vertically above K such that TK=LM=15cm and $T\hat{K}L = 90^0$.



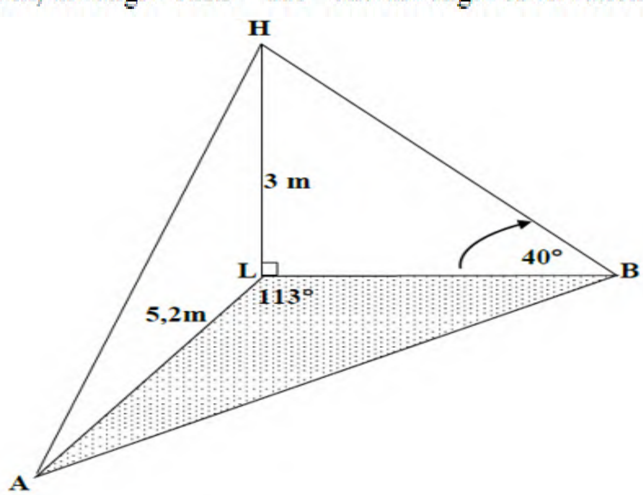
15.1 Determine the length of KM (2)

15.2 Show that the length of KL= 31; 7m (Show all your calculations) (3)

15.3 Determine the size of $K\hat{T}L$ (3)

QUESTION 16

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.



16.1 Calculate the length of LB. (2)

16.2 Hence, or otherwise, calculate the length of AB. (3)

16.3 Determine the area of ΔABL. (3)

TRIGONOMETRIC GRAPHS

QUESTION 17

Consider: $g(x) = -4\cos\,(x + 30°)$

17.1 Write down the maximum value of $g(x)$. (1)

17.2 Determine the range of $g(x) + 1$. (2)

17.3 The graph of g is shifted 60° to the left and then reflected about th x-axis to form a new graph$h$. Determine the equation of h in its simplest form. (3)

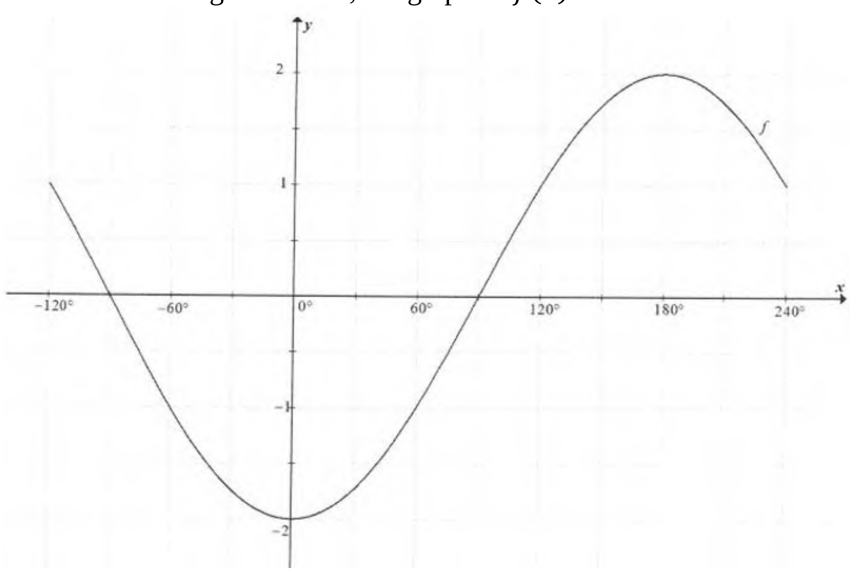
QUESTION 18

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

18.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

18.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the Interval $-180° \leq x \leq 180°$. (3)

18.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.

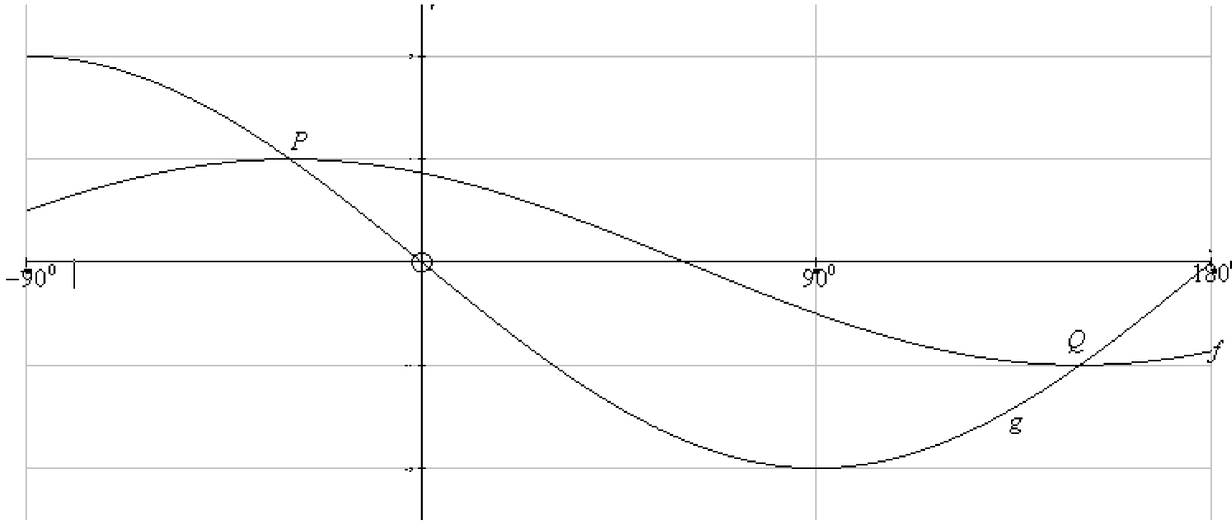


18.3.1 Draw the graph of $g(x) = \sin\,(x + 60°)$ for $-120° \leq x \leq 240°$. (3)

18.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

QUESTION 19

The graphs of $f(x) = \cos\left(x + 30^0\right)$ and $g(x) = -2\sin x$ for $-90^0 \leq x \leq 180^0$ are given below. The graphs intersect at point P and Q.



19.1 Determine $f(0)-g(0)$ without using a calculator. (2)

19.2 Calculate the x-coordinates of the points P and Q. (6)

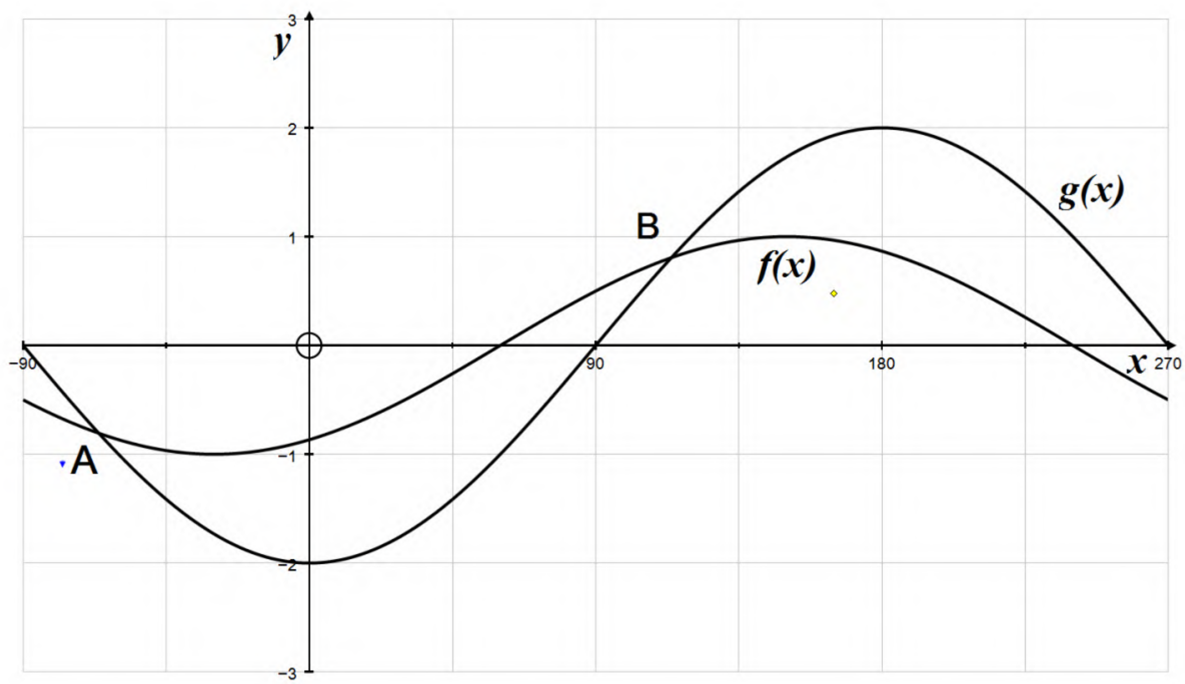
19.3 For which values of x will $f(x)\geq g(x)$? (2)

19.4 Graph h is obtained by the following transformation of f:
$h(x)=3f(x+60^0)$.

19.5 Write down the simplified equation of h after the transformation. (2)

QUESTION 20

The diagram below shows the graphs of $f(x) = sin\ (x - 60°)$ and $g(x) = -2\ cos\ x$ for $x \in [-90°; 270°]$.



20.1 Give the period of f. (1)

20.2 If the point of intersection at A has an x-value of $-66{,}2°$, find the corresponding y-value and the co-ordinates of point B. (Write answer correct to 1 dec. place) (3)

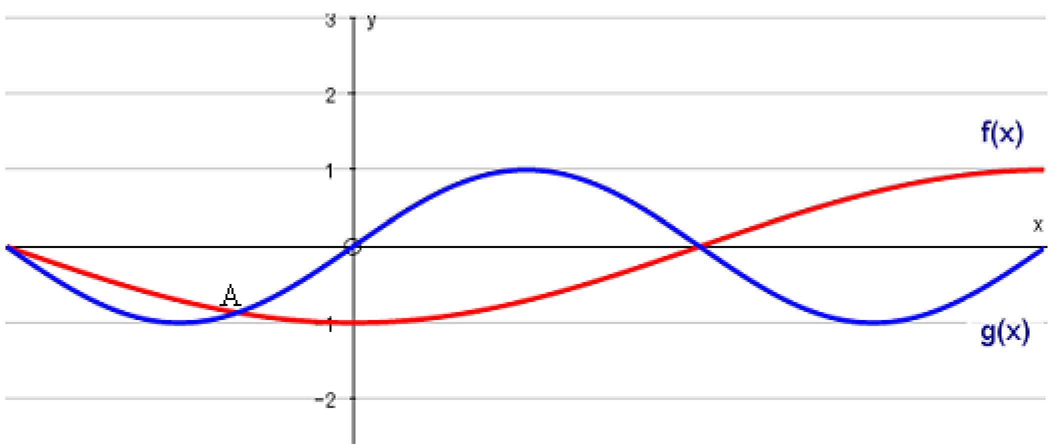
20.3 Use the graph to find the values of x for which $f(x) > g(x)$. (2)

20.4 If $g(x)$ is shifted up two units, give the range of the new graph. (2)

QUESTION 21

The graphs of $f(x) = -\cos x$ and $g(x) = \sin 2x$ for $x \in [-90°;180°]$ are drawn in the diagram below.

The coordinates of $A\left(-30°;-\frac{\sqrt{3}}{2}\right)$ is given.



21.1 Determine the range of k if $k(x) = 2f(x) - 3$ (2)

21.2 How do you need to shift $h(x) = \sin(2x + 60°)$ to obtain $g(x)$? (1)

21.3 Determine the x-values, $x \in [-90°;180°]$, for which

21.3.1 $g(x) < f(x)$ (3)

21.3.2 $f'(x) \times g(x) > 0$ (3)

LGEBRA, EQUATIONS AND INEQUALITIES ANSWERS

1.1.1)	$x = 5$ or $x = -1$
1.1.2)	$x = 4{,}95$ or $x = 0{,}05$
1.1.3)	$x = -1$ or $x = 3$ $y = -4$ or $y = 0$
3.1.1)	$x = 0{,}4$ or $x = 4{,}6$
3.1.2)	$x \leq -3$ *or* $x \geq 12$
3.1.3)	$x = 4$
3.2.1)	$n = -5$
3.2.2)	$m = \frac{3}{2}$
3.2.3)	$m = \frac{3}{2}$
4.1.1)	$x = 4$ or $x = -2$
4.1.2)	$x = \frac{1}{3}$
4.1.3)	$x = -1$
4.2)	$x = 0$ or $x = -k$
4.3.1)	$x = \frac{-b\pm\sqrt{n^2-1764}}{18}$
4.3.2)	$n = \pm 42$
6.1.1)	$x = \frac{5}{2}$ *or* $x = -2$
6.1.2)	$x = 0$ or $x = 0.43$ *or* $x = -0.77$
6.1.3)	$x = \frac{p}{2-p}$; $p \neq 2$
6.1.4)	$x = \frac{1}{16}$ or $x = \frac{1}{25}$
6.2)	$x = \frac{1}{2}$ or $x = 1$
	$y = 3$ or $y = 2$
9.1.1.	$x = -\frac{1}{2}$ *or* $x = 3$
9.1.2.	$x \leq 0$ or $x \geq 7$
9.2.1.	$x = -0{,}68$ *or* $x = 0{,}88$
9.2.2.	$y = -0{,}06$ **OR** $y = -0{,}84$
9.3.	$x = -3$ *or* $x = 2$ $y = 1$ or $y = -4$
9.4.	$m = \frac{49}{36}$
11.1.1.	$x = \pm\frac{9}{2}$
11.1.2(a)	$x = 5{,}37$ or $-0{,}37$
11.1.2.(b)	$x = \pm 2{,}71$ *or* $x = \pm 1{,}28$
11.1.3	$-4 \leq x \leq 2$
11.1.4	$x = -1$ *or* $x = 0$
11.2.	$x = 1$ *or* $x = 2$ $y = 2$ *or* $y = 1$
11.1.1	$x = \pm\frac{9}{2}$
13.1.1.	$x = -7$ or $x = -2$
13.1.2.	$x = 0{,}29$ or $x = -2{,}54$
13.1.3.	$x = 5$ or $x = -1 (n/a)$
13.2.	$x = 1$ or $x = 7$ and $y = -1$ or $y = 17$

2.1.1)	$x = 2$ or $x = 3$
2.1.2)	$-\frac{1}{3} < x < 4$
2.1.3)	$x = 0$ or $x = \frac{4}{5}$ (n/a)
2.1.4)	$x = -\frac{5}{3}$
2.1.5)	$x = 5$ or $x = 3$ $y = 2$ or $y = -2$
5.1.1)	$x = -\frac{2}{3}$ or $x = 4$
5.1.2)	$x = 3$ or $x = 6$ (n/a)
5.1.3)	$x \leq -\frac{5}{2}$ *or* $x \geq 3$
5.2.1)	$P = \frac{1}{2^x}$
5.2.2)	$x = -1{,}58$
5.3.1)	irrational
5.3.2)	irrational
5.3.2)	non-real
7.1.1)	$x = -\frac{5}{2}$ or $x = \pm\sqrt{2}$
7.1.2)	$x \leq -1$ *or* $x \geq 5$
7.2)	$x = 1{,}12$ or $x = -3{,}12$
7.3)	$y = -3$ and $x = 5$
8.1.1)	$x = 0$ *or* $x = -2$
8.1.2)	$x \geq 0$
8.2)	$x = 2{,}77$ *or* $x = -1{,}27$
8.3.1)	$k = -7$ *or* $k = 2$
8.3.2)	$x = -1$ *or* $x = 44$ (n/a)
8.4)	$y = -\frac{12}{7}$ or $y = -3$ $x = -\frac{3}{7}$ *or* $x = -3$
8.5)	$k \leq \frac{1}{5}$
10.1.1.	$x = 0$ *or* $x = \frac{5}{2}$
10.1.2.	$x = 2{,}77$ or $x = -1{,}77$
10.1.3.	$x = 9$ or $x = 6$ (n/a)
10.1.4.	$-\frac{1}{3} < x < 0$
10.2.	$x = \frac{2}{3}$ *or* $x = 3$ and $y = \frac{5}{3} = 1$ or $y = -3$
12.1.1	$x = -2{,}55$ or $x = -0{,}78$
12.1.2	$x = -\frac{8}{5}$ or $x = 2$ $\therefore x = 2$
12.1.3	$x \leq -6$ or $x \geq 4$
12.2	$x = -2$ or $x = 4$ $y = 13$ or $y = -17$
12.3.1	$p = -1$ or $p = 49$
12.3.2	$x = 2$ or no solution
14.1.1	$x = -9$ or $x = 2$

13.3.1. $x \in R,\ x \neq -4$
15.1.1 $x = 1$ or $x = -2$
15.1.2. $x = 1,43$ or $x = 0.23$
15.1.3. $x = \frac{5}{2}$
15.1.4 $x = 8$
15.1.5 $x = 1$ or $x \geq 2$
15.2. $y = 1$ or $y = -1$ $x = 2$ or $x = 1$
15.3. $b \leq \frac{4}{5}$

14.1.2 $x = -0,43$ or $x = -2,32$
14.1.3 $x < 0$ or $x > 2$
14.2. $x = -2$ or $x = -1$ $y = \frac{1}{2}$ or $y = 1$

NUMBER PATTERNS, SEQUENCES AND SERIES ANSWERS	
LIMP 2013 1.1.1 $p = 6$ $q = 12$ 1.1.2 $p = 10$ $q = 17$ 1.2.1 630 1.2.2 $T_1 = 36$ $T_2 = 42$ $T_3 = 48$ 1.3 $n = 6$ 1.4.1. $-3 < x < 3$ 1.4.2 $K = \frac{-2}{5}$ 2.1.1. $T_n = 2n^2 - 3n + 4$ 2.1.2 So n = 17 or term 17	**MPUM 2013** 3.1 Tn $= \frac{5}{2}n^2 + \frac{5}{2}n - 4$ 3.2 $T_{14} = 521$ 4.1 d = 4, a = -10 4.2 k = 16 4.3.1 $r =$ t - ½ 4.3.2 $-\frac{1}{2} < t < \frac{3}{2}$ 4.3.3 $\frac{-4}{5}$
NW 2014 5.1 – 558 5.2.1 180l 5.2.2 $\approx 72,34\ l$ 5.3.1 $0 < x < \frac{2}{3}$ 5.3.2 $S_\infty = 2$ 5.4 $x = 4$ $y = 26$	**WC METRO 2014** 6.1.1 n = 28 6.1.2 868 6.2.1 $T_n = x.(\frac{x}{3})^{n-1}$ 6.2.2 $-3 < x < 3$ 6.3 - 45 7.1 $T_n = n^2 + 3n - 1$ 7.2 303
KZN 2015 8.1.1 12; 2 8.1.2 3925 8.2 Bookwork 9.1 196602 9.2 $-2 < x < 2$ 9.3 $x > 2$ 10.1 $T_n = 2n^2 - 4n + 7$ 10.2 Between th e 505th and the 506th	**M. COLLEGE 2015** 11.1.1 193 11.1.2 $3n - 2$ 11.1.3 $k = 20$ 12.1.1 16 12.1.2 14 12.2.1 $\therefore T_n = -n^2 + 7$ 12.2.2 Terms 20 and 21 12.3 $\frac{1}{2} < x < \frac{3}{2}$ $x \neq 1$
FS 2016 13.1 $\sum_{4}^{13} 3 = 10(3) = 30$ 13.2.1 $b = 11$ 13.2.2 $T_n = 3 + (n-1)8 = 8n - 5$ 13.2.3 235 13.2.4 3570 13.3.1 $T_4 = 34$	**KZN 2016** 14.1 $T_n = 2n^2 - 6n + 8$ 14.2 Between the 7023rd and 7024th terms. 15.1.1 18 ; 2 15.1.2 100 15.1.3. 5300 15.2 Bookwork 16.1 3 16.2 20

13.3.2 $T_n = 4n^2 - 9n + 6$

13.3.3 $n = 45$

WC 2016

19.1.1 $3\frac{1}{2}; 3; 2\frac{1}{2}$

19.1.2 $d = -\frac{1}{2}$

19.1.3 $n = 97$

19.2.1 No negative term. Converges to + number.

19.2.2 $\frac{2}{2187}$

19.2.3 $4{,}57 \times 10^{-4}$

LIMP 2016

17.1 $a = \frac{1}{2}$

17.2 Yes $-1 < r < 1$

17.3 series converges to $\frac{2}{3}$

18.1 Proof

18.2 $T_n = 3n^2 + n - 2$

18.3 4838

18.4 70

WC WINELANDS 2017

22.1 $n = 9$

22.2.1 2 ; 8 ; 20 ; 38

22.2.2 4628

EDEN & KAROO 2018

23.1 Bookwork

23.2.1 $n = 11$

23.2.2 165

23.3.1 $T_1 = 1$

23.3.2 - 75

23.4 59

24.1.1 $19; 7; -1; -5$

24.1.2 $a = 2,\ b = -18,\ c = 35$

24.2.1 $-4 < p < 4$

24.2.2 $p = \frac{-4}{3}$

FS 2019

28.1 4

28.2 $T_n = 2n^2 - n$

28.3 $n = 37$

29.1 $n = 36$

29.2 13650

30.1 48; 63

30.2 $T_n = n^2 + 4n + 3$

31.1 $p = 3$

31.2 36060

31.3.1 $x = k + 1$ and $y = k + 2$

31.3.2 $T_x + T_y = 11 + 10k$

KZN 2017

20.1.1 11; 3

20.1.2 128

20.1.3 2921

20.2.1 $-3 < x < -1$

20.2.2 $S_\infty = \frac{x-2}{1-(x+2)} = \frac{x-2}{-x-1}$

$S_\infty \neq 0$ since $x \neq 2$

21.1 35

21.2 4421

21.3 $T_n = 2n^2 - 4n + 5$

$= 2(n^2 - 2n + 2) + 1$

Since $2(n^2 - 2n + 2)$ is even

$\therefore 2(n^2 - 2n + 2) + 1$ is odd

21.4 $T_n = 2n^2 - 4n + 105$

KZN 2018

25.1 73, 99

25.2 $T_n = 2n^2 + 4n + 3$

25.3 For the first difference

$T_n = 4n + 6 = 2(2n + 3)$

An even number of the first difference is always added to first term of the quadratic sequence to get an odd number. This process continues to produce all odd numbers of the sequence.

26.1 $t = -8$

26.2 $\therefore$ 4 terms are positive.

27.1.1 $r = (x - 3)$

27.1.2 $2 < x < 4$

27.2 $p = \frac{2}{3}$

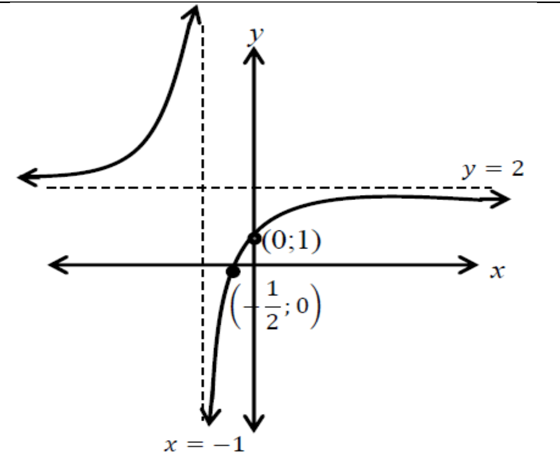
KZN 2019

33.1.1 15 ; 5

	33.1.2 22, 5 33.2 $n = 10$

FUNCTIONS ANSWERS

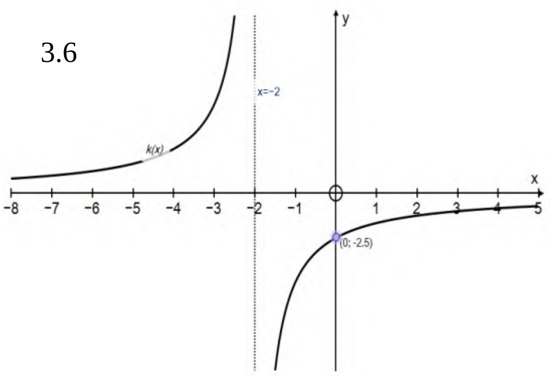
Question 1 (Metro East Sept 2020)



Question 2 (Feb/March 2018)

2.1	$E(4;-9)$
2.2	$A(1;0)$
2.3	$M(8;7)$
2.4	$y=-\frac{7}{4}x+7$
2.5	$y=-\frac{4}{7}x+4$
2.6	$x\leq 0$ or $1\leq x\leq 7$

Question 3 Sept 2018

3.1	$f(x)=1-\frac{5}{x+2}$
3.2	$x=-2$ and $y=1$
3.3	y-intercept: $\left(0;-\frac{3}{2}\right)$ x-intercept: $(3;0)$
3.4	$c=3$
3.5	$k(x)=-\frac{5}{x+2}$
3.6	
3.7	$x\in R; x\neq -2$

ANALYTICAL GEOMETRY SPRING CLASSES MATERIAL ANSWERS:

1.1	$y=5$
1.2.1	2
1.2.2	$63{,}43^\circ$
1.2.3	$D(0;-1)$
1.3	-3
1.4	$N(2;-5)$
1.5	$53{,}13^\circ$
2.1	$x=3$

3.1.1	$-\frac{3}{2}$
3.1.2	$123{,}69^\circ$
3.3	$y=x+3$
3.4	$\sqrt{13}$
3.5.1	$(-5;-2)$ or $(-6;-3)$
3.5.2	$L(-5;-2)$
3.6	12,5 square units

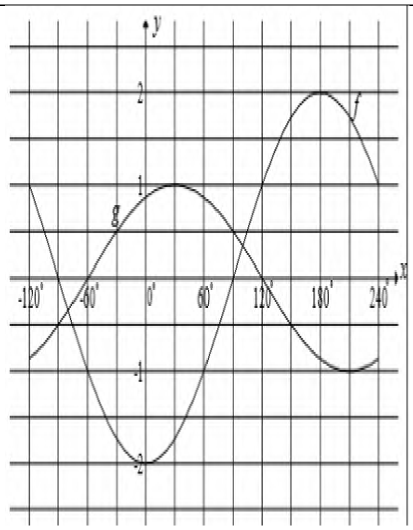
2.2	3
2.3	$y = 3x + 19$
2.4	$2\sqrt{26}$
2.5	$\frac{1}{5}$
2.6	15 units^2
5.1	$M(6;-4)$
5.2	$m_{ns} = 2$
5.3	$y = -\frac{1}{2}x - 6$
5.4	$x^2 + y^2 = 256$
5.5	$T(0;-1)$
5.6	$\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ or $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$
5.7	25 square units
6.1.1	$\frac{1}{5}$
6.1.2	$y = \frac{1}{2}x + 7$
6.1.3	$(x+2)^2 + (y-1)^2 = 20$
6.1.4	$26{,}57^\circ$
5.1.5	$y = \frac{1}{2}x - 3$
5.2	$-3 < t < 7$
5.3	25 square units

4.1.1	$y = -\frac{3}{2}x + 8$
4.1.2	$G(4;2)$
4.3	show that midpoint theorem applies
4.4	$D(16;6)$
7.1	$k = -2$
7.2	BC=2 units
7.3.1	$\alpha = 45°$
7.3.2	VWB=45°
7.4.1	$Q(-3;-2)$
7.4.2	$(x+3)^2 + (y+2)^2 = 10$
7.4.3	$x = -2$ or $x = -4$
8.1	$F(3;1)$
8.2	5
8.3	10
8.4	two tangents from the same
8.5.1	$5\sqrt{17} = 20{,}62$
8.5.2	$(x-m)^2 + (y-n)^2 = 100$
8.5.3	$G(12;-11)$

TRIGONOMETRY ANSWERS

1.1	$\tan\theta = \frac{k}{6}$
1.2	$k = 3$
2.1	$\frac{-5}{\sqrt{39}}$
2.2	$\frac{5\sqrt{39}}{32}$
3.1.1	$-q$
3.1.2	$= 2q\sqrt{1-q^2}$
3.1.3	$\pm\sqrt{\frac{q+1}{2}}$
3.2	$-\frac{1}{2}$
4.	$-\frac{1}{\cos x}$
5.1.1	$\frac{2}{\sqrt{3}}$
5.1.2	$\frac{1}{2\sqrt{2}}$
6.1	Proof
6.2	$\cos^2 x$
7.1	$-\tan 86^\circ$
7.2	2

8.1	Proof
8.2	Proof
8.3	Proof
8.4	Proof
8.5	Proof
8.6	Proof
9.1	$\theta = n.360^\circ \pm 48,19^\circ$ or $\theta = n.360^\circ \pm 120^\circ$, $n \in \mathbb{Z}$
9.2	$x = 9,22^\circ + k.90^\circ$ or / of $x = -31,72^\circ + k.90^\circ$ $(k \in Z)$
11.3	Proof
12	Proof
13	Proof
14.1	Proof
14.2	13.22
15.1	21.42m
15.2	Proof
15.3	$64,68^0$
16.1	3.58m
16.2	7,38m
16.3	$8,57m^2$

17.3.1	
17.3.2	$\sin(x + 60°) > -2\cos x$ $x \in (-80{,}10°; 99{,}90°)$ or $-80{,}10° < x < 99{,}90°$
19.1	$\frac{\sqrt{3}}{2}$
19.2	$x = 30^0$ $x = -30^0$ OR $x = 150^0$
19.3	$x \in [-30^0; 150^0]$
19.4	$-3\sin x$
20.1	360°
20.2	$y = -2\cos(-66{,}2°)$ $= -0{,}8$ B(113,8° ; 0,8)
20.3	$-66{,}2° < x < 113{,}8°$
20.4	$0 \leq y \leq 4,\ \ y \in R$
21.1	$y \in [-5; -1]$

21.2	30^0 to the right
21.3.1	$x \in (-90^0; -30^0)$ *or* $x \in (90^0; 180^0)$
21.3.2	$x \in [-90^0; 90^0)$