

School Name

Badge

GRADE 12

MATHEMATICS INVESTIGATION

DATE: 15 FEBRUARY 2021

MARKS: 70

Duration: 2 Weeks

This investigation consists of 6 pages including the cover sheet.

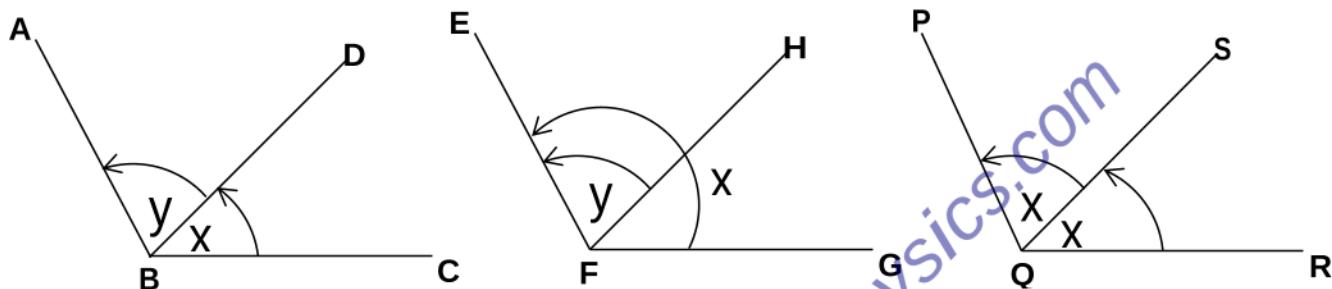
### INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This task paper consists of 4 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

**INVESTIGATING COMPOUND ANGLES AND THEREFORE, DOUBLE ANGLES.****QUESTION 1**

- 1.1. In the following diagrams,  $\hat{ABD} = y$ ,  $\hat{DBC} = x$ ,  $\hat{EFH} = y$ ,  $\hat{EFG} = x$  and  $\hat{PQS} = \hat{SQR} = x$



Write each of the following in terms of  $x$  and/or  $y$

1.1.1  $\hat{ABC}$  (1)

1.1.2  $\hat{HFG}$  (1)

1.1.3  $\hat{PQR}$  (1)

- 1.2 In the table below, different methods are used by learner X and learner Y to answer given questions. Study the table and answer the questions that follow:

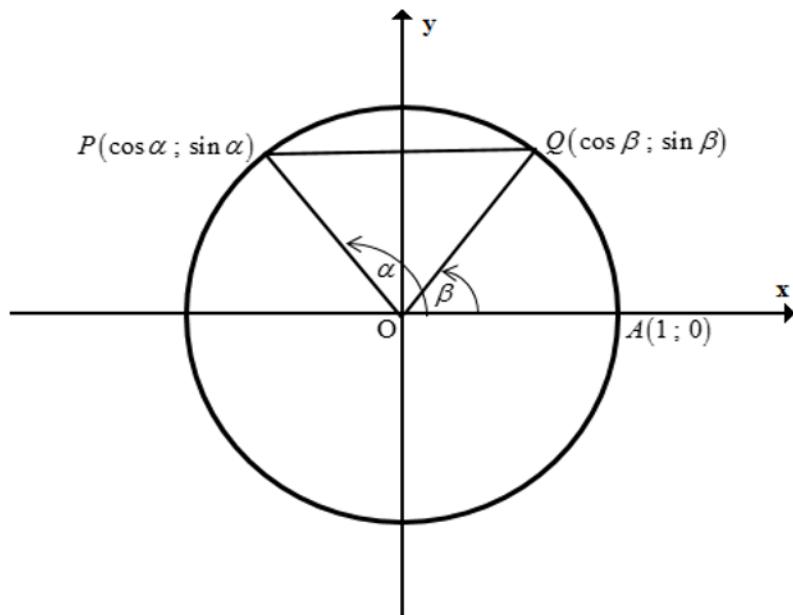
- 1.2.1 Redraw the table in your answer book and use a calculator to complete it. (4)

Learner X			Learner Y		
Question	Working	Answer	Question	Working	Answer
$\cos(45^\circ - 15^\circ)$	$\cos 30^\circ$	$\frac{1}{2}$	$\cos(45^\circ - 15^\circ)$	$\cos 45^\circ - \cos 15^\circ$	$-\sqrt{6} + \sqrt{2}$ 4
$\cos(150^\circ - 30^\circ)$	$\cos 120^\circ$		$\cos(150^\circ - 30^\circ)$	$\cos 150^\circ - \cos 30^\circ$	
$\sin(150^\circ + 60^\circ)$			$\sin(150^\circ + 60^\circ)$		
$\cos(90^\circ - 150^\circ)$			$\cos(90^\circ - 150^\circ)$		
$\sin(60^\circ - 300^\circ)$			$\sin(60^\circ - 300^\circ)$		

- 1.2.2 Are the answers obtained by learner X the same as that of learner Y for the same question? (1)

- 1.2.3 Whose method do you think is mathematically correct? Give a reason. (3)

- 1.3 Consider the sketched unit circle. By the definition of trigonometric functions, the points P and Q on the terminal sides of angles  $\alpha$  and  $\beta$ , and are labelled as shown in the figure below.

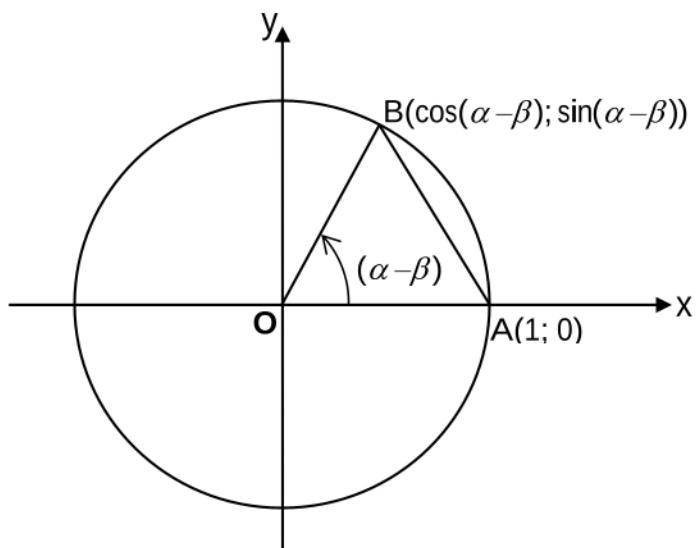


Make use the distance formula to prove that:

$$PQ^2 = 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \quad (4)$$

- 1.4 Consider the following sketch.  
Make use of the distance formula and prove that:

$$AB^2 = 2 - 2\cos(\alpha - \beta) \quad (4)$$



- 1.5 Use 1.2 and 1.3 above to prove that  $\triangle POQ \equiv \triangle BOA$  (4)
- 1.6 Hence, compare sides PQ and AB and write a conclusion about  $\cos(\alpha - \beta)$ . (4)
- 1.7 Use  $\cos(\alpha - \beta)$  to derive a formula for  $\sin(\alpha - \beta)$ . (Hint: use co-function) (3)
- 1.8 By applying the compound angles and without using a calculator. Determine the value of  $\cos 15^\circ$  (3)

**[33]****QUESTION 2**

Using your CALCULATOR, go to TABLE mode.

2.1 Insert the function  $f(x) = 2\cos^2 x - 1$

- Start:  $-180^\circ$ ; End:  $180^\circ$
- Step  $45^\circ$

2.1.1 Complete the following table: (1)

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$2\cos^2 x - 1$									

2.1.2 Use the table to sketch the graph of  $f(x) = 2\cos^2 x - 1$ ,  $x \in [-180^\circ; 180^\circ]$ . (3)

2.2 Using your calculator insert the following function  $g(x) = 1 - 2\sin^2 x$

2.2.1 Complete the following table: (1)

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$1 - 2\sin^2 x$									

2.2.2 Use the table to sketch the graph of  $g(x) = 1 - 2\sin^2 x$ ,  $x \in [-180^\circ; 180^\circ]$ . (3)

2.3 Using your calculator insert the following function  $h(x) = \cos^2 x - \sin^2 x$

2.3.1 Complete the following table: (1)

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$\cos^2 x - \sin^2 x$									

2.3.2 Use the table to sketch the graph of  $h(x) = \cos^2 x - \sin^2 x$  where

$x \in [-180^\circ; 180^\circ]$ . (3)

2.4 Using your calculator insert the following function  $j(x) = \cos 2x$

2.4.1 Complete the following table: (1)

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$\cos 2x$									

2.4.2 Use the table to sketch the graph of  $j(x) = \cos 2x$ ,  $x \in [-180^\circ; 180^\circ]$ . (3)

2.5 If the graphs f, g and h were drawn on the set of axes as j what will your observation be? (1)

2.6 Make a comparison of the four expressions. (3)

**[20]**

**QUESTION 3**

Using your CALCULATOR, go to TABLE mode.

3.1 Using your calculator insert the following function  $u(x) = 2\sin x \cos x$

3.1.1 Complete the following table: (1)

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$2\sin x \cos x$									

3.1.2 Use the table to sketch the graph of  $u(x) = 2\sin x \cos x$ ,  $x \in [-180^\circ; 180^\circ]$ . (3)

3.2 Using your calculator insert the following function  $v(x) = \sin 2x$

3.2.1 Complete the following table: (1)

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\sin 2x$									

3.2.2 Use the table to sketch the graph of  $v(x) = \sin 2x$  where

$x \in [-180^\circ; 180^\circ]$ . (3)

3.3 If the graph of  $u$  was drawn on the same set of axes as  $v$ , what will you observe? (1)

3.4 Make a comparison of the two expressions. (1)

[10]

**QUESTION 4****APPLICATION**

4.1 Without the use of tables or a calculator prove that  $\frac{1 - \cos 2A}{\sin 2A} = \tan A$  (3)

4.2 Hence, calculate the value of  $\tan 15^\circ$  (4)

[7]

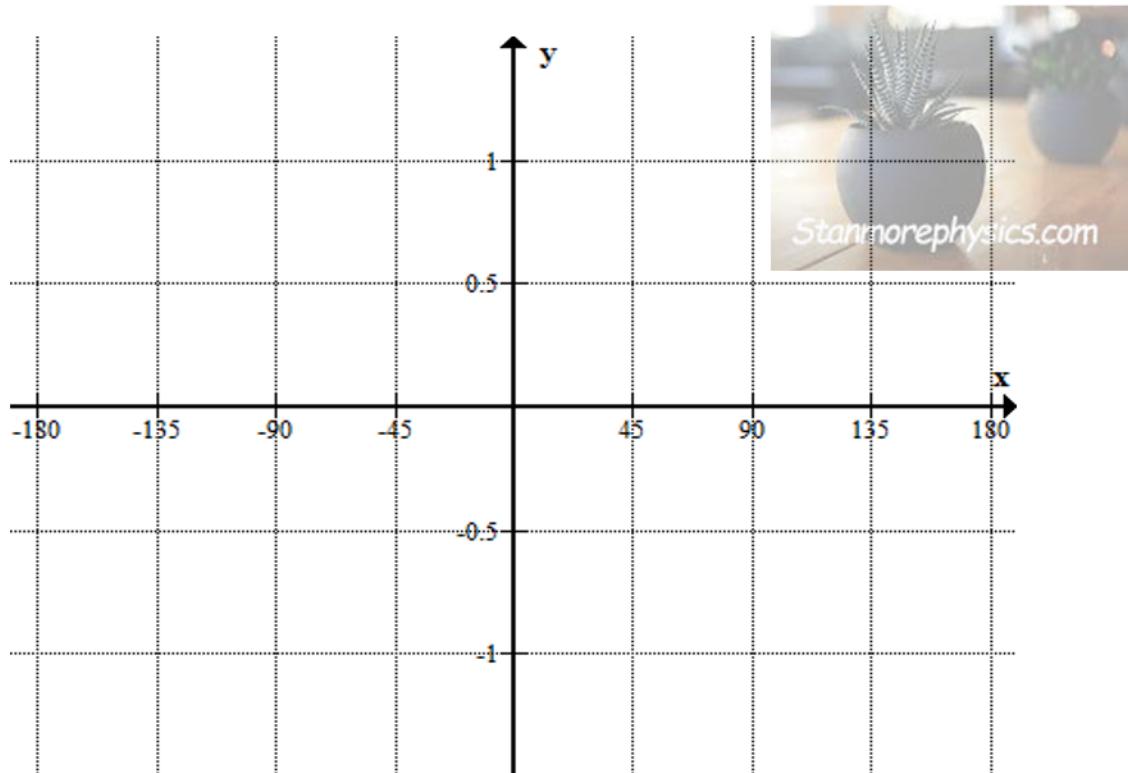
**TOTAL:** 70

NAME OF LEARNER: ..... CLASS: .....

2.1.1 Complete the following table:

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$2 \cos^2 x - 1$									

2.1.2

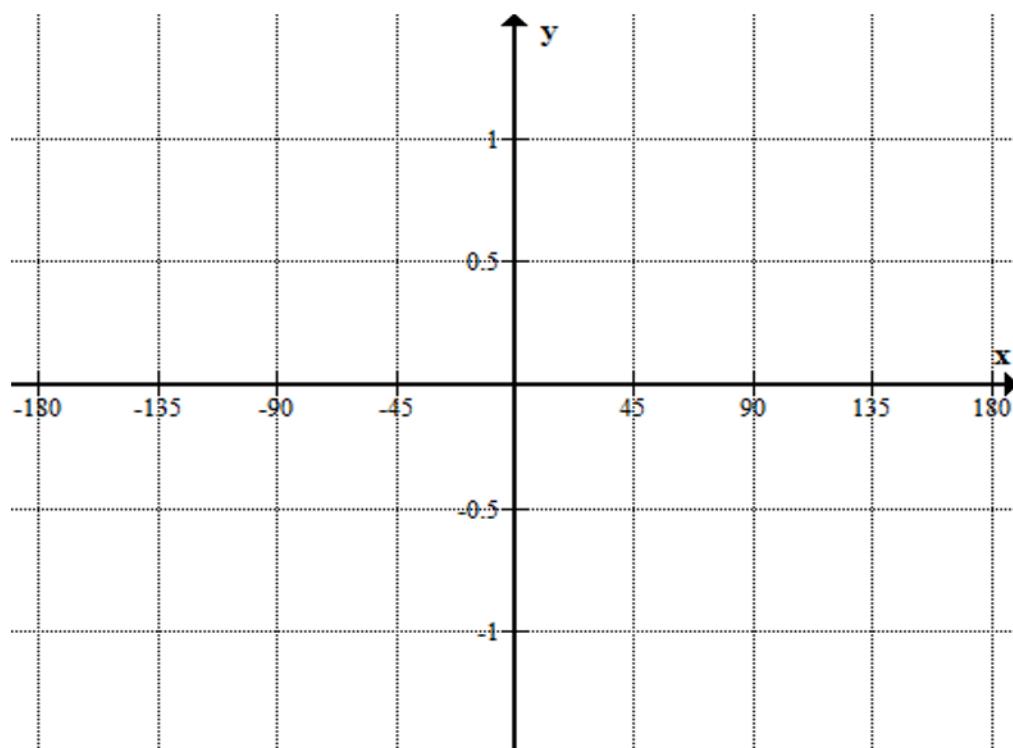


NAME OF LEARNER: ..... CLASS: .....

2.2.1 Complete the following table:

X	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$1 - 2 \sin^2 x$									

2.2.2

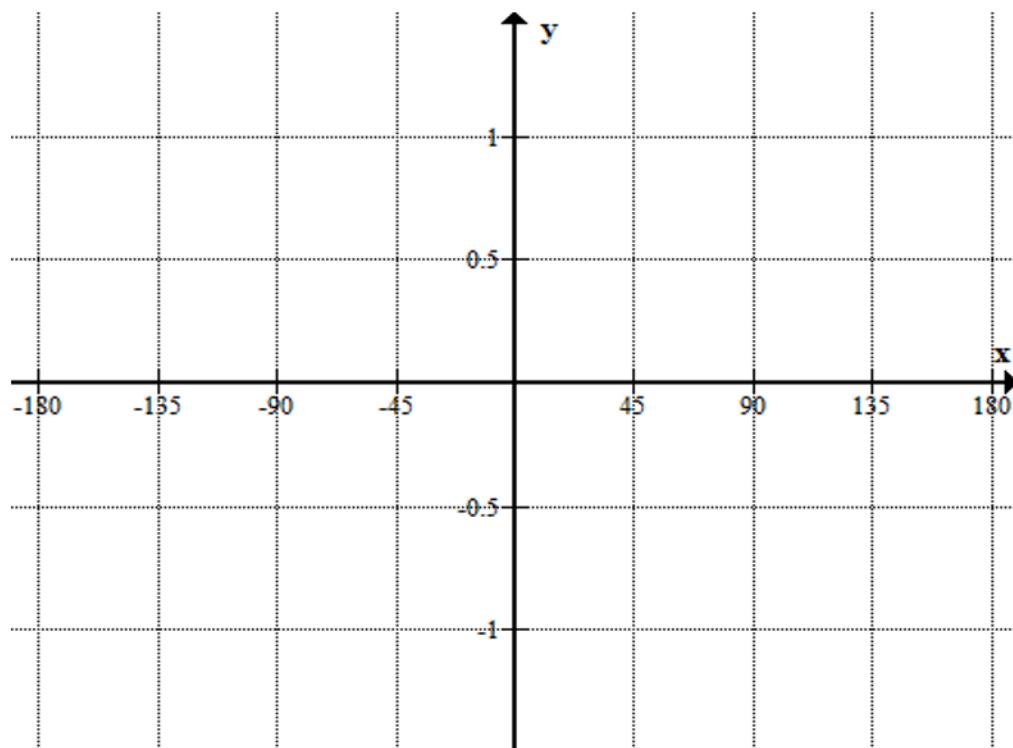


NAME OF LEARNER: ..... CLASS: .....

2.3.1 Complete the following table:

X	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$\cos^2 x - \sin^2 x$									

2.3.2

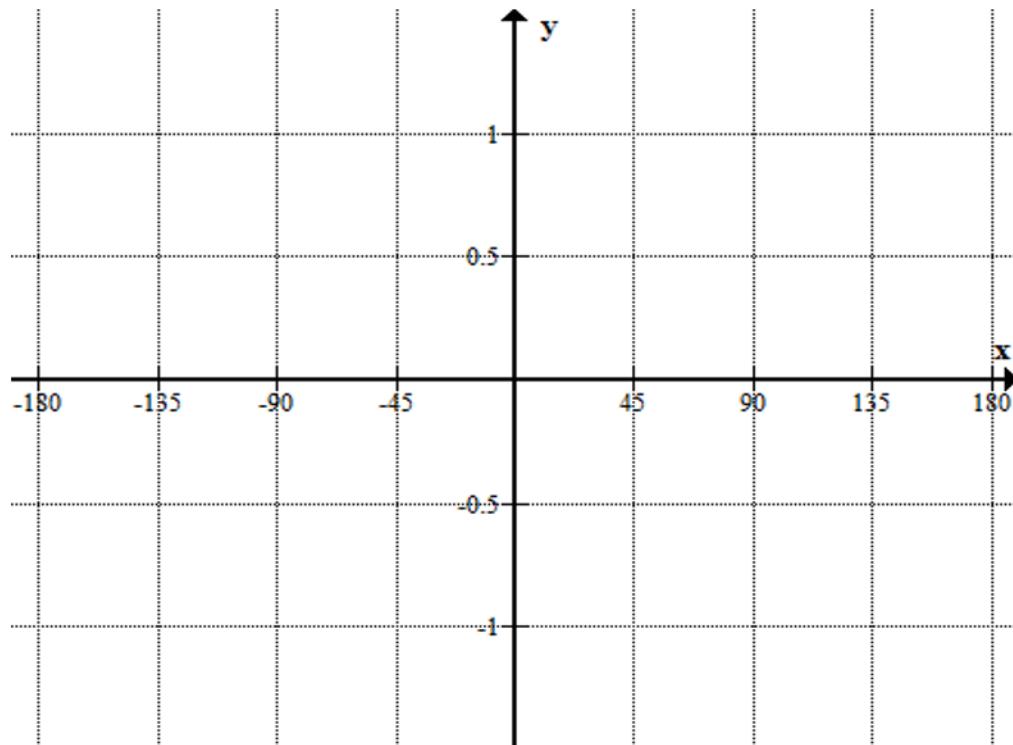


NAME OF LEARNER: ..... CLASS: .....

2.4.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
cos 2x									

2.4.2

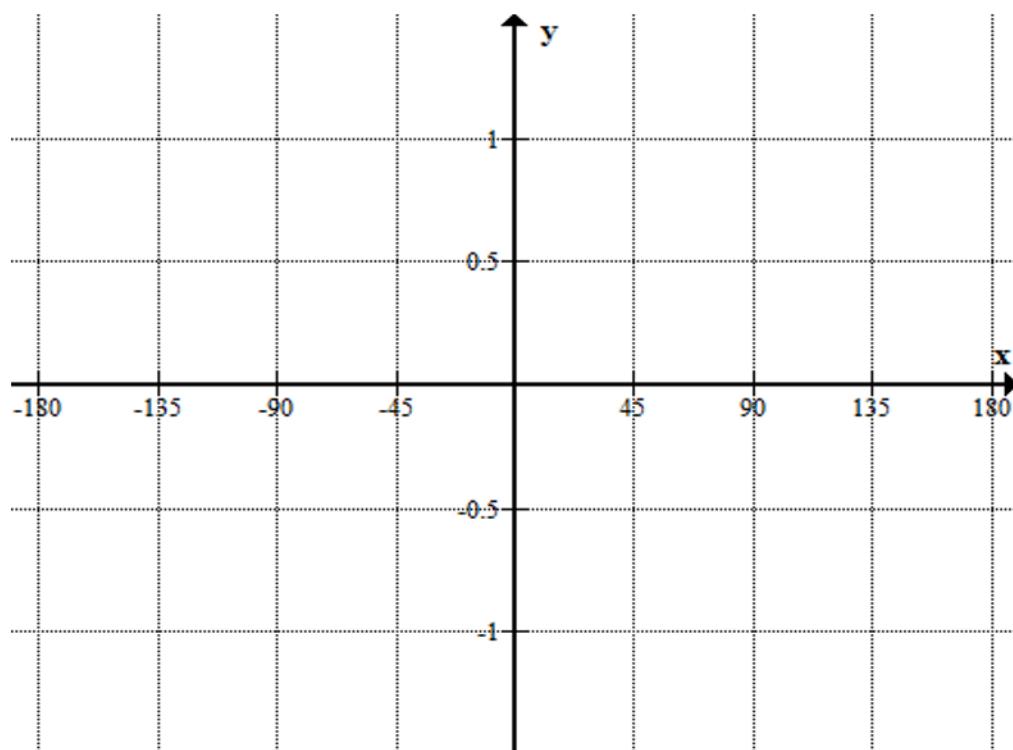


NAME OF LEARNER: ..... CLASS: .....

3.1.1 Complete the following table:

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$2\sin x \cos x$									

3.1.2

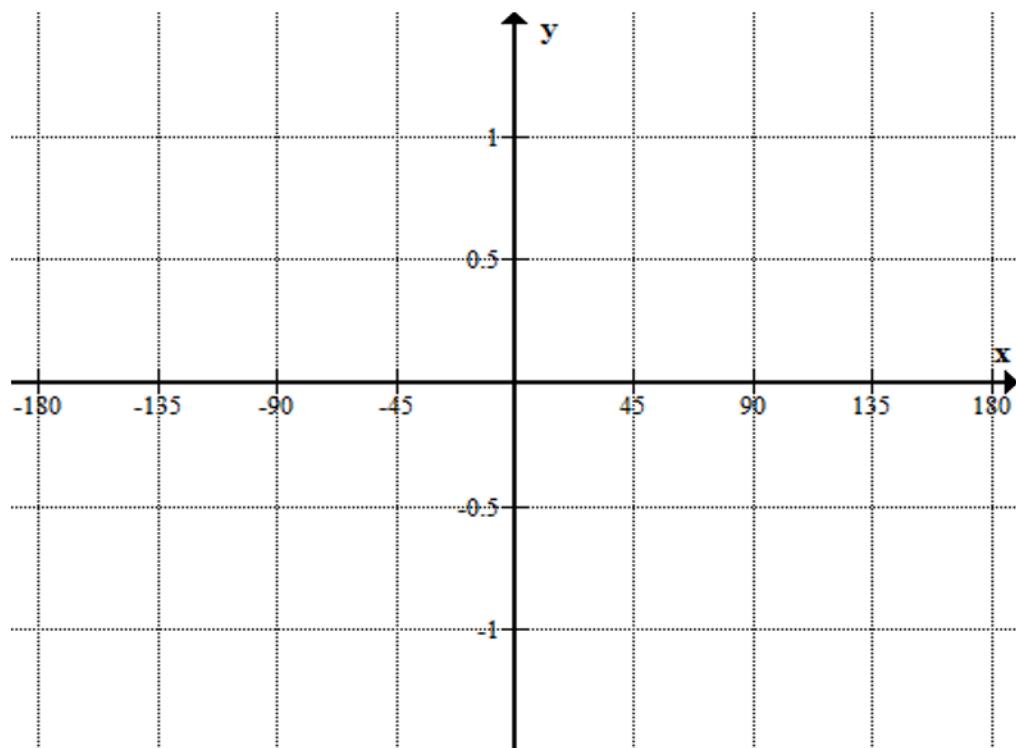


NAME OF LEARNER: ..... CLASS: .....

3.2.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
sin 2x									

3.2.2



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GRADE 12

**MATHEMATICS INVESTIGATION:**

**MARKING GUIDELINE**

**DATE: 15 FEBRUARY 2021**

**MARKS: 70**

This marking guideline consists of 13 pages.

## INVESTIGATING COMPOUND ANGLES AND THEREFORE, DOUBLE ANGLES.

### QUESTION 1

1.1.1	$\angle ABC = x + y$	answer
1.1.2	$\angle HFG = x - y$	answer
1.1.3	$\angle PQR = 2x$	answer

1.2.1

Learner X			Learner Y			
Question	Working	A	Question	Working	Answer	
<b>NOTE: 1 mark for each correct row</b>						
$\cos(150^\circ - 30^\circ)$	$\cos 120^\circ$	$-\frac{1}{2}$	$\cos(150^\circ - 30^\circ)$	$\cos 150^\circ - \cos 30^\circ$	$-\sqrt{3}$	
$\sin(150^\circ + 60^\circ)$	$\sin 210^\circ$	$-\frac{1}{2}$	$\sin(150^\circ + 60^\circ)$	$\sin 150^\circ + \sin 60^\circ$	$\frac{1 + \sqrt{3}}{2}$	
$\cos(90^\circ - 150^\circ)$	$\cos(-60^\circ)$	$\frac{1}{2}$	$\cos(90^\circ - 150^\circ)$	$\cos 90^\circ - \cos 150^\circ$	$\frac{\sqrt{3}}{2}$	
$\sin(60^\circ - 300^\circ)$	$\sin(-240^\circ)$	$-\frac{\sqrt{3}}{2}$	$\sin(60^\circ - 300^\circ)$	$\sin 60^\circ - \sin 300^\circ$	$\sqrt{3}$	
1.2.2	No, the answers are not the same				answer	
1.2.3	Learner X Learner Y distributed the trig ratio into the bracket (consider any mathematically acceptable reason)				answer	$\checkmark \checkmark$ reason
1.3					substitution expansion use of $\cos^2 \theta + \sin^2 \theta = 1$ simplification	

	$\begin{aligned} d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ PQ^2 &= (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 \\ &= \cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta \\ &\quad + \sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta \\ &= \cos^2 \alpha + \sin^2 \alpha - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta \\ &\quad + \cos^2 \beta + \sin^2 \beta \\ &= 1 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) + 1 \\ &= 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \end{aligned}$	
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1.4	$\begin{aligned} AB^2 &= (\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2 \\ &= \cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta) \\ &= 1 - 2\cos(\alpha - \beta) + 1 \\ &= 2 - 2\cos(\alpha - \beta) \end{aligned}$	✓ correct substitution ✓ correct expansion ✓ use of identities ✓ simplification										
1.5	<p>In <math>\Delta POQ \equiv \Delta BOA</math></p> <table border="1"> <thead> <tr> <th>Statement</th> <th>Reason</th> </tr> </thead> <tbody> <tr> <td><math>OP = OB</math></td> <td>both radii = 1</td> </tr> <tr> <td><math>OQ = OA</math></td> <td>both radii = 1</td> </tr> <tr> <td><math>\hat{POQ} = \hat{BOA}</math></td> <td>both = <math>\alpha - \beta</math></td> </tr> <tr> <td><math>\therefore \Delta POQ \equiv \Delta BOA</math></td> <td>SAS</td> </tr> </tbody> </table>	Statement	Reason	$OP = OB$	both radii = 1	$OQ = OA$	both radii = 1	$\hat{POQ} = \hat{BOA}$	both = $\alpha - \beta$	$\therefore \Delta POQ \equiv \Delta BOA$	SAS	✓ S/R ✓ S/R ✓ S/R ✓ conclusion
Statement	Reason											
$OP = OB$	both radii = 1											
$OQ = OA$	both radii = 1											
$\hat{POQ} = \hat{BOA}$	both = $\alpha - \beta$											
$\therefore \Delta POQ \equiv \Delta BOA$	SAS											
1.6	<p><math>PQ = AB</math> from congruency</p> <p>Therefore</p> $\begin{aligned} AB^2 &= PQ^2 \\ 2 - 2\cos(\alpha - \beta) &= 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ -2\cos(\alpha - \beta) &= -2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ \therefore \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{aligned}$	✓ S/R ✓ $PQ^2 = AB^2$ ✓ substitution ✓ simplification										

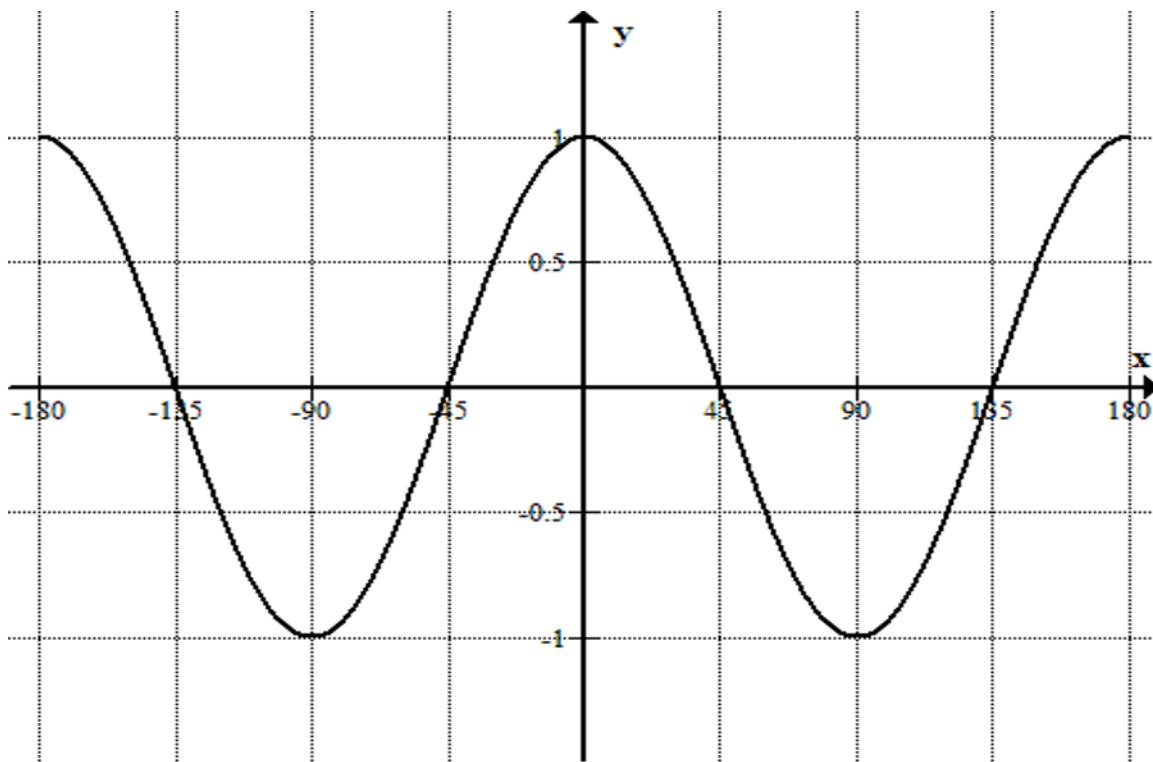
1.7	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ $\begin{aligned} \sin(\alpha + \beta) &= \cos[(90^\circ - (\alpha + \beta))] \\ &= \cos[(90^\circ - \alpha) - \beta] \\ &= \cos(90^\circ - \alpha) \cdot \cos \beta + \sin(90^\circ - \alpha) \cdot \sin \beta \\ &= \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \end{aligned}$	✓ $\cos(\alpha + \beta) = \cos[\alpha - (-\beta)]$ ✓ expansion ✓ answer
1.8	$\begin{aligned} \sin 105^\circ &= \sin(60^\circ + 45^\circ) \\ &= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$	✓ correct expansion ✓ correct substitution ✓ answer
		[33]

**QUESTION 2**

2.1.1										
x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$	
$2\cos^2 x - 1$	1	0	-1	0	1	0	-1	0	1	✓ answer (1)

2.1.2 Use the table to sketch the graph of  $f(x) = 2\cos^2 x - 1$  where

$$x \in [-180^\circ ; 180^\circ].$$

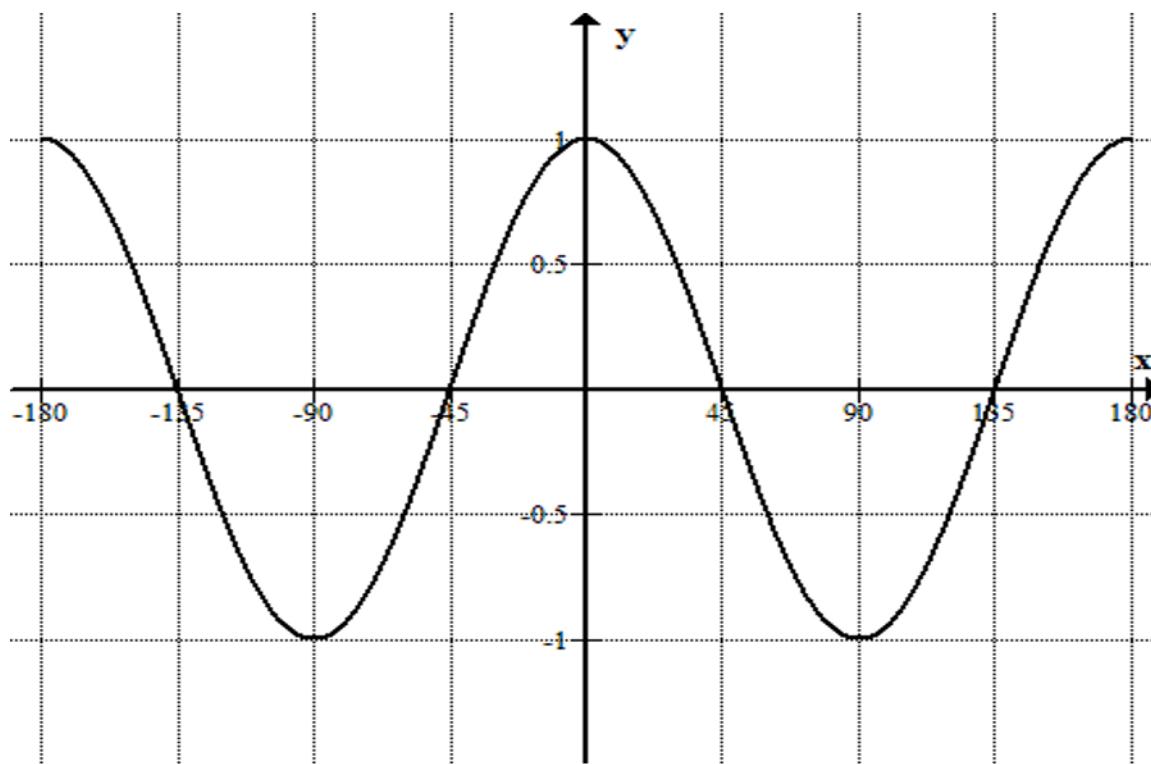


2.1.2	Refer to diagram	✓ shape ✓ all x- intercepts ✓ all turning points
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2.2.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
1 - 2sin² x	1	0	-1	0	1	0	-1	0	1	✓ answer (1)

2.2.2 Use the table to sketch the graph of  $g(x) = 1 - 2\sin^2 x$  where

$$x \in [-180^\circ ; 180^\circ].$$

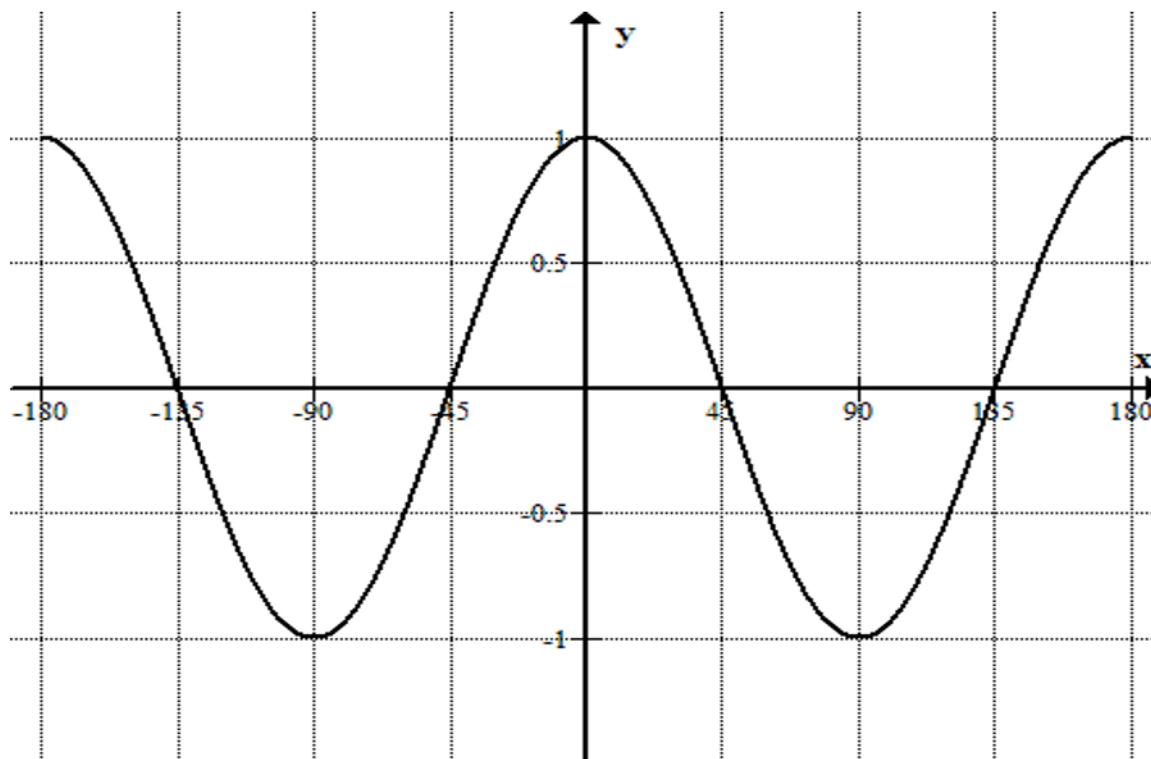


2.2.2	Refer to the diagram	✓ shape ✓ all x- intercepts ✓ all turning points
-------	----------------------	--

2.3.1									
x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$
$\cos^2 x - \sin^2 x$	1	0	-1	0	1	0	-1	0	1

2.3.2 Use the table to sketch the graph of  $h(x) = \cos^2 x - \sin^2 x$  where

$$x \in [-180^\circ ; 180^\circ].$$

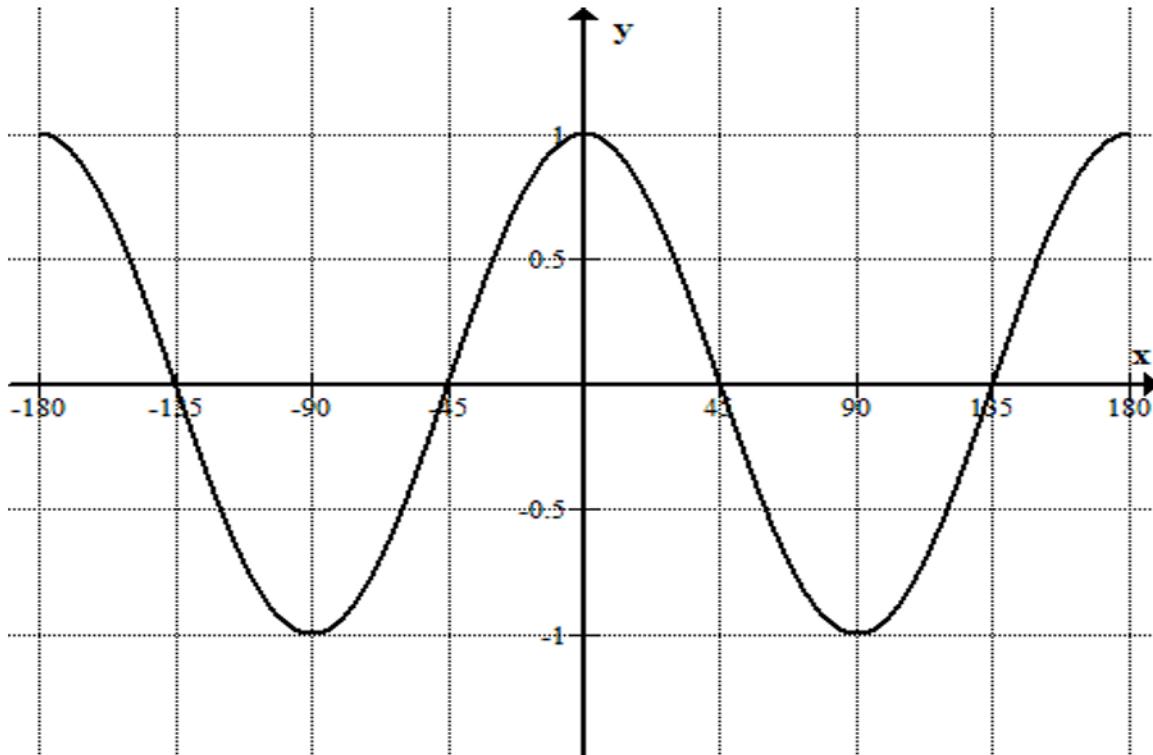


2.3.2	Refer to the diagram	<input checked="" type="checkbox"/> shape <input checked="" type="checkbox"/> all x- intercepts <input checked="" type="checkbox"/> all turning points
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2.4.1										
x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$	
$\cos 2x$	1	0	-1	0	1	0	-1	0	1	<input checked="" type="checkbox"/> answer

									(1)
--	--	--	--	--	--	--	--	--	-----

2.4.2 Use the table to sketch the graph of  $j(x) = \cos 2x$  where  $x \in [-180^\circ ; 180^\circ]$ .



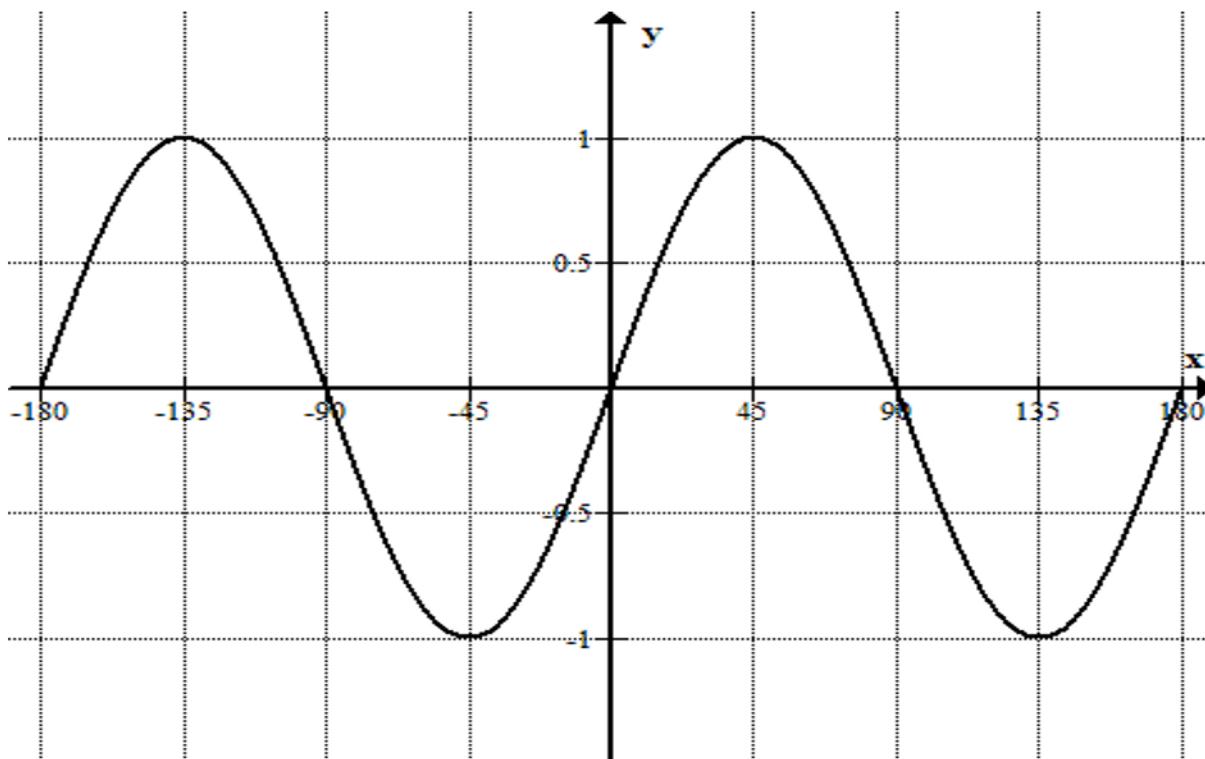
2.4.2	Refer to the diagram	✓ shape ✓ all x- intercepts ✓ all turning points
2.5	$f$ , $g$ and $h$ will coincide with $j$	✓ answer
2.6	$\cos 2x = 2\cos^2 x - 1$ $\cos 2x = 1 - 2\sin^2 x$ $\cos 2x = \cos^2 x - \sin^2 x$	✓ answer ✓ answer ✓ answer
		[20]

### QUESTION 3

3.1.1								
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x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$	
$2\sin x \cos x$	0	1	0	-1	0	1	0	-1	0	✓ answer (1)

3.1.2 Use the table to sketch the graph of  $u(x) = 2\sin x \cos x$  where  $x \in [-180^\circ ; 180^\circ]$ .

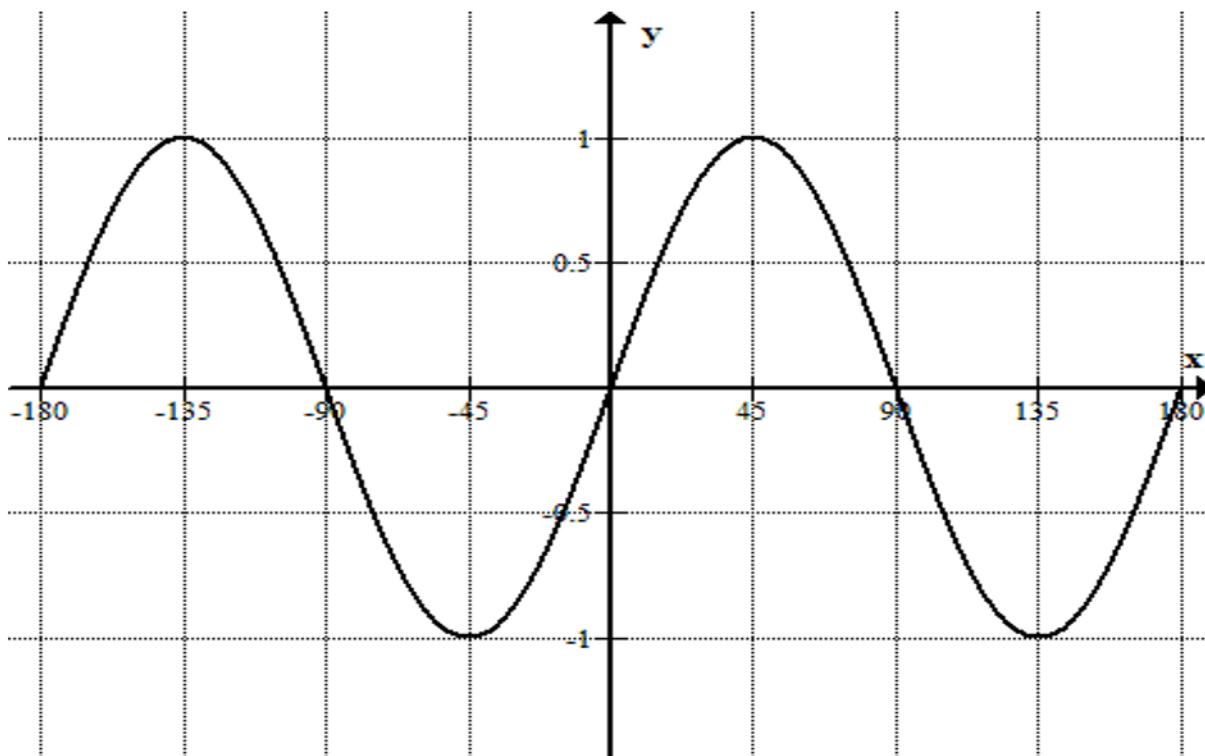


3.1.2	Refer to the diagram	✓ shape ✓ all x-intercepts ✓ all turning points
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3.2.1								
-------	--	--	--	--	--	--	--	--

x	$-180^\circ$	$-135^\circ$	$-90^\circ$	$-45^\circ$	$0^\circ$	$45^\circ$	$90^\circ$	$135^\circ$	$180^\circ$	
$\sin 2x$	0	1	0	-1	0	1	0	-1	0	✓ answer (1)

3.2.2 Use the table to sketch the graph of  $v(x) = \sin 2x$  where  $x \in [-180^\circ ; 180^\circ]$ .



3.2.2	Refer to the diagram	✓ shape ✓ all x- intercepts ✓ all turning points
3.3	u will coincide with v	✓ answer
3.4	$\sin 2x = 2\sin x \cos x$	✓ answer
		[10]

**QUESTION 4****APPLICATION**

4.1	$\begin{aligned} \text{LHS} &= \frac{1-\cos 2A}{\sin 2A} \\ &= \frac{1-(1-2\sin^2 A)}{2\sin A \cos A} \\ &= \frac{2\sin^2 A}{2\sin A \cos A} \\ &= \frac{\sin A}{\cos A} \\ &= \tan A \\ \therefore \text{LHS} &= \text{RHS} \end{aligned}$	✓ $1-2\sin^2 A$ ✓ $2\sin A \cos A$ ✓ $\frac{\sin A}{\cos A}$
4.2	$\begin{aligned} \tan 15^\circ &= \frac{1-\cos 2(15^\circ)}{\sin 2(15^\circ)} \\ &= \frac{1-\cos 30^\circ}{\cos 30^\circ} \\ &= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ &= 2 - \sqrt{3} \end{aligned}$	✓ $\frac{1-\cos 2(15^\circ)}{\sin 2(15^\circ)}$ ✓ $\frac{1-\cos 30^\circ}{\sin 30^\circ}$ ✓ correct substitution ✓ answer
		<b>[7]</b>

**TOTAL: 70**