KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

MATRIC INTERVENTION PROGRAMME

GRADE 12

2022

TABLE OF CONTENTS

SUMMARY OF THE CONTENT TAUGHT IN TRIGONOMETRY FROM GRADE 10 TO GRADE 12

GRADE 10

- Definitions of trig ratios and their reciprocals using right angled triangle.
- Special angles
- Simple trig equations.
- Numerical values of ratios for angles 0^0 to 360^0 (using a diagram).
- Trigonometric functions (sketch, find equations and interpret).
- Effects of parameters *a* and *q* on trig functions.
- Solve 2 – dimensional problems (*right angled* Δ)

GRADE 11

- Derivation and use of fundamental identities.
- Derivation and use of reduction formulae.
- Proving trigonometric identities.
- Determining for which values of a variable an identity holds
- Determining the general solution and / or specific solutions of trigonometric equations.
- Sketching of basic trig graphs for $\theta \epsilon[-360°; 360°]$
- Effects of parameters k and p on trig functions.
- Sketch trig graph with at most 2 parameters at a time.
- Prove and apply the sine, cosine and area rules.
- Solve problems in two dimensions using the sine, cosine and area rules.

GRADE 12

- Compound and double angle identities.
- Solve problems in two and three dimensions.

THE TRIGONOMETRIC RATIOS

The three basic trigonometric ratios are:

sine	$\sin\theta$	$\sin A$	$\sin 37°$
cosine	$\cos\beta$	$\cos x$	$\cos 115°$
tangent	$\tan\alpha$	$\tan S\hat{P}R$	$\tan 360°$

QUESTION 1 **WCS20**

1.1 If $\sin 42° . \cos 14° = a$ and $\cos 42° . \sin 14° = b$, determine the following in terms of a and b.

1.1.1 $\sin 56°$ (2)

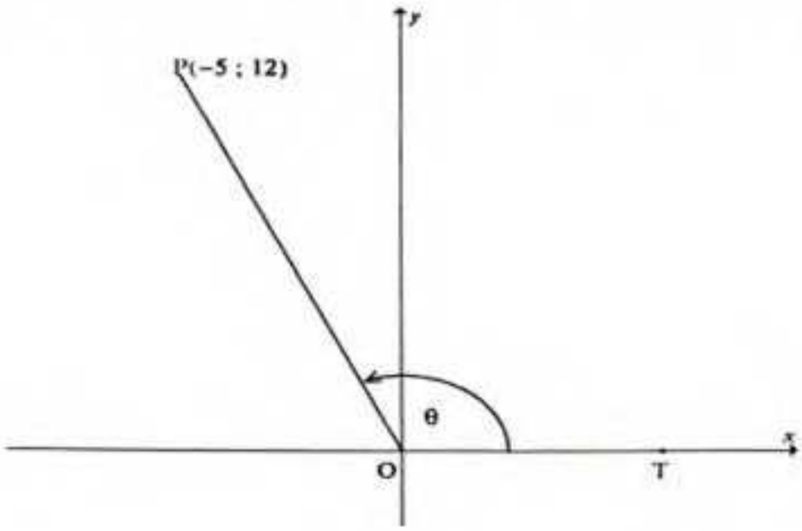
1.1.2 $\sin 28°$ (2)

1.1.3 $\cos 56°$ (3)

1.2 Determine the value of $\tan\theta$, if the distance between the point $(\cos\theta ; \sin\theta)$ and (-2; 1) is $\sqrt{6}$. (3)

QUESTION2 **NW20**

2.1 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $P\hat{O}T = \theta$



Answer the following **without using a calculator**:

2.1.1 Write down the value of $\tan\theta$ (1)

2.1.2 Calculate the value of $\cos\theta$ (3)

2.1.3 S(a ; b) is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and OS = 6,5 units. Calculate the value of b. (4)

2.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

2.3 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

Determine the value of $\cos 2A$ **without using a calculator.** (5)

QUESTION 3 LPS16

THIS QUESTION HAS TO BE ANSWERED WITHOUT THE USE OF A CALCULATOR:

3.1 Simplify fully: 3.1.1 $\frac{\sin 140°.\tan(-315°)}{\cos 230°.\sin 420°}$ (5)

3.1.2 $\frac{\sin 15°.\cos 15°}{\cos(45° - x)\cos x - \sin(45° - x)\sin x}$ (5)

3.2.1 Express $\cos^2 A$ in terms of $\cos 2A$ (2)

3.2.2 Hence show that $\cos 15° = \frac{\sqrt{\sqrt{3}+2}}{2}$ (4)

3.3 Calculate x when $\sin 2x = \cos(-3x)$ for $x \in [-90°; 90°]$ (6)

QUESTION 4 NWS16

4.1.1 Prove the identity: $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$.

4.1.2 Hence calculate, without using a calculator, the value of $\cos 15° - \cos 75°$.

4.2 Determine the value of $\tan\theta$, if the distance between $A(\cos\theta; \sin\theta)$ and $B(6;7)$ is $\sqrt{86}$.

QUESTION 5 KZNJ17

5.1 If sin 161° = t, express the following in terms of t:

5.1.1 cos 19° (3)

5.1.2 tan 71° (3)

5.1.3 $\frac{1}{\cos(-341^\circ)}$ (2)

5.2 If A + B = 90° and tan A = 0,2 then determine without the use of the calculator:

5.2.1 sin A (2)

5.2.2 cos (–180° – B) (3)

5.3 Determine the maximum value of:

$8 - 10 \sin x \cos x$ (3)

QUESTION 6 KZNM16

6.1 If $4 \tan \alpha - 3 = 0$ and $90^\circ \leq \alpha \leq 360^\circ$, determine without the use of a calculator the value of $\cos^2 \alpha - \sin \alpha$. (5)

6.2 Simplify, without using a calculator:

6.2.1 $\frac{\sin 61^\circ \cdot \sin(90^\circ - \theta)}{\cos 29^\circ \cdot \sin(180^\circ - \theta)}$ (4)

6.2.2 sin 15° cos 15° (3)

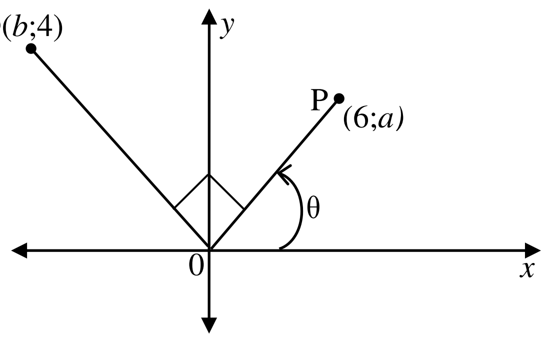
6.3 Prove the following identity:

$$\frac{\sin A - \cos A}{\sin A + \cos A} + \frac{\sin A + \cos A}{\sin A - \cos A} = \frac{-2}{\cos 2A}$$ (6)

[18]

QUESTION 7 KZNJ16

In the diagram alongside, $D\hat{O}P = 90^\circ$. The angle which OP makes with the x – axis is θ. Given D $(b;4)$; P $(6;a)$ and $\sqrt{5}\cos\theta - 2 = 0$



7.1 Determine, without the use of a calculator, the numerical value of:

7.1.1 a (3)

7.1.2 b (4)

7.2 Simplify without the use of the calculator.

7.2.1 $\sin^2 20^\circ + \sin^2 70^\circ$ (3)

7.2.2 $\dfrac{\cos 330^\circ . \sin 140^\circ}{\sin(-160^\circ) . \tan 405^\circ . \sin 290^\circ}$ (10)

QUESTION8 GPS16

8.1 If $5\tan\theta + 2\sqrt{6} = 0$ and $0^\circ < \theta < 270^\circ$, determine with the aid of a sketch and without the use of a calculator, the value of:

8.1.1 $\sin\theta$ (2)

8.1.2 $\cos\theta$ (1)

8.1.3 $\dfrac{14\cos\theta + 7\sqrt{6}\sin\theta}{\cos(-240^\circ).\tan 225^\circ}$ (4)

QUESTION 9 NWS18

9.1 Prove that:

$$\frac{\sqrt{4(1-\cos\theta)(1+\cos\theta)}}{\sin 2\theta} = \frac{1}{\cos\theta}$$ (4)

9.2 It is given that $\sin p + \sqrt{3}\cos p = 1$.

9.2.1 Show that the equation can be written as $\sin(60^\circ + p) = \frac{1}{2}$ (3)

9.2.2 Hence, determine the general solution of $\sin p + \sqrt{3}\cos p = 1$. (4)

9.3 **Without using a calculator**, determine the value of:

$\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \ldots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 2$ (3)

QUESTION10 WCS18

10.1 Prove that: $\dfrac{\sin 2\theta}{\sin\theta} = 4\cos\theta - \dfrac{\cos 2\theta + 1}{\cos\theta}$ (4)

QUESTION 11 **FSS18**

11.1 Prove the identity: $\dfrac{2\sin^2 x}{2\tan x - \sin 2x} = \dfrac{\cos x}{\sin x}$ (5)

11.2 Determine the general solution of $2 + 2\cos 2x = 0$ (4)

[9]

QUESTION 12 **KZNJ19**

12.1 Prove the identity:

$$\frac{\cos^2(90° + \theta)}{\cos(-\theta) + \sin(90° - \theta)\cos\theta}) = \frac{1}{\cos\theta} - 1$$ (5)

12.2 It is given that

$$p = \cos\alpha + \sin\alpha$$
$$q = \cos\alpha - \sin\alpha$$

deduce the following trigonometric ratios in terms of p and q.

12.2.1 $\cos 2\alpha$ (2)

12.2.2 $\dfrac{1 + \sin 2\alpha}{\cos 2\alpha}$ (5)

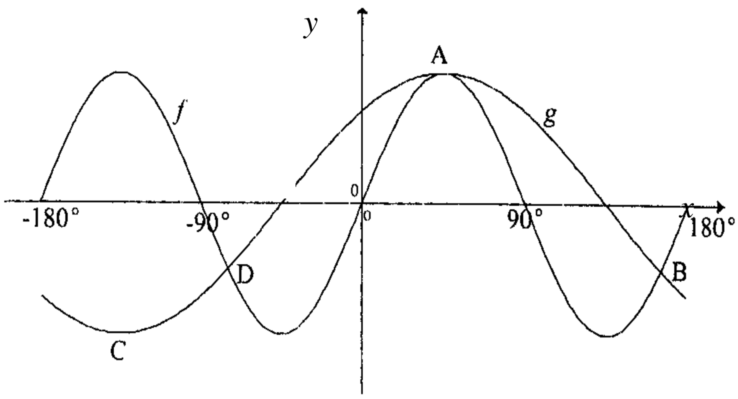
12.3 Determine the general solution of $6\cos^2 x + \sin x - 5 = 0$. (6)

QUESTION 13 **KZNM16**

13.1 The sketch below, shows the graphs of :

$f = \{(x\,;\,y\,/\,y = \sin px\}$ and
$g = \{(x\,;\,y)\,/\,y = \cos(x + q)\,;\,x \in [-180°\,;\,180°]$

A (45° ; 1) and B $\left(165°\,;\,-\dfrac{1}{2}\right)$ are two points of intersection of f and g.



13.1.1 Determine the value(s) of p and q. (4)

13.1.2 What is the period of g? (1)

13.1.3 Write down the co-ordinates of C, the turning point of the curve g (1)

13.1.4 Write down the co-ordinates of D, a point of intersection of f and g. (1)

QUESTION 14 ECS16

Given: $f(x) = \cos 2x$ and $g(x) = \sin(x + 60°)$ for $x \in [-90°; 180°]$.

14.1 Solve for x if $f(x) = g(x)$ and $x \in [-90°; 180°]$. (5)

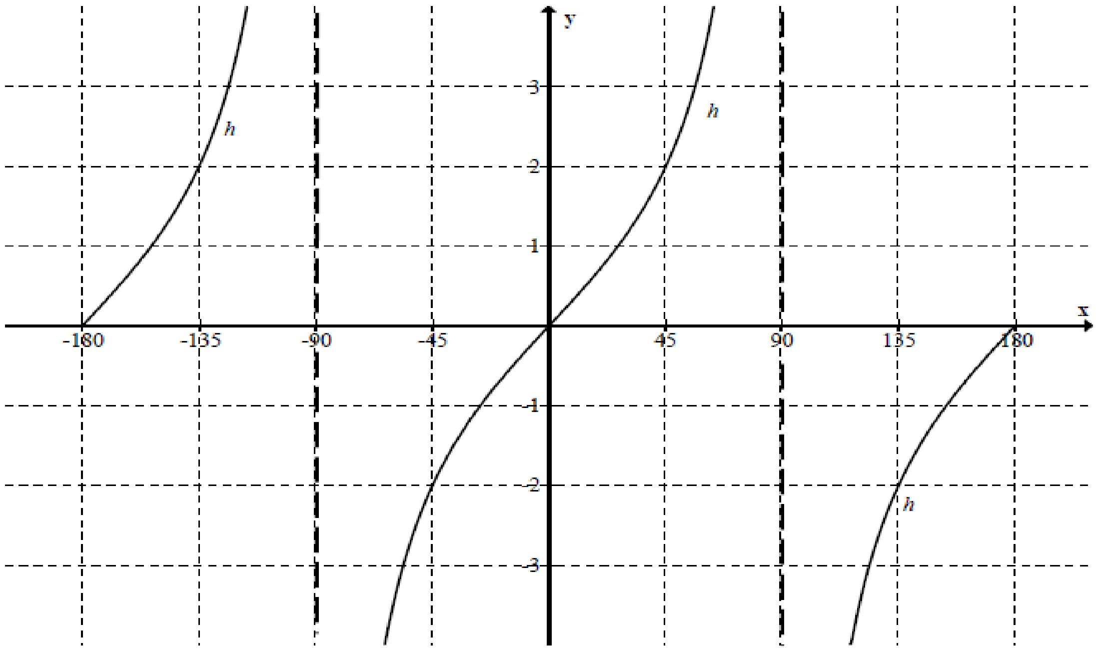
14.2 Sketch the graph of f and g on the same set of axes for $x \in [-90°; 180°]$. Clearly show ALL intercepts with the axes, points of intersection as well as turning points. (6)

14.3 Write down the period of $g\left(\frac{3}{2}x\right)$. (1)

14.4 Determine the equation of h if $h(x) = f(x - 45°) - 1$. (2)

QUESTION 15 FSS16

The graph of $h(x) = a \tan x$; for $x \in [-180°; 180°]$, $x \neq -90°$, is sketched below.



15.1 Determine the value of a. (2)

15.2 If $f(x) = \cos(x + 45°)$, sketch the graph of f for $x \in [-180°; 180°]$, on the diagram provided in your ANSWER BOOK. (4)

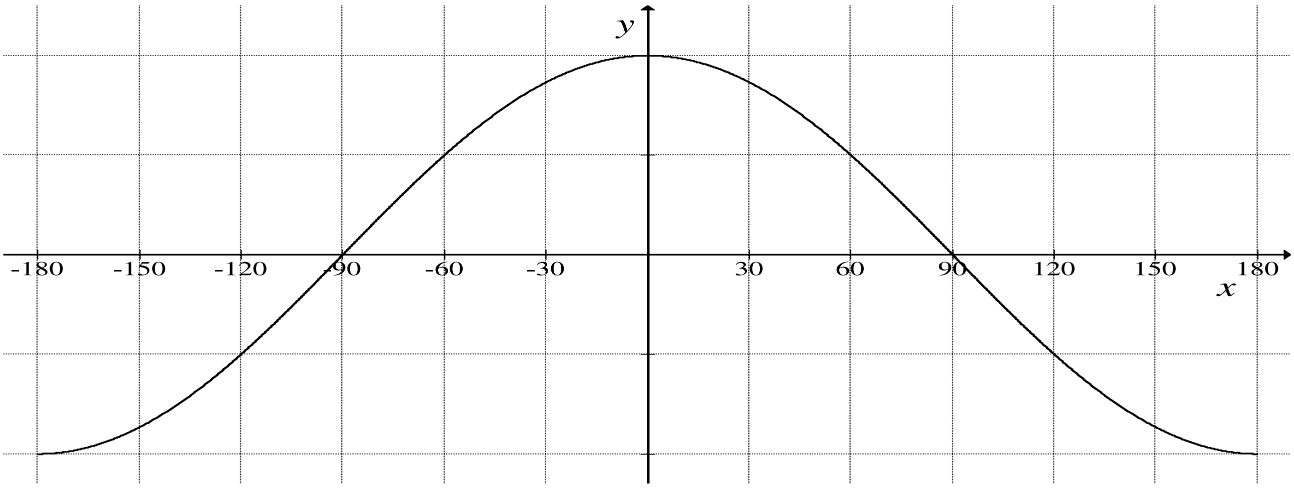
15.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°; 180°]$? (1)

QUESTION 16 GPS16

16.1 Show that the equation $2\cos x = \sin(x + 30°)$ is equivalent to $\sqrt{3}\sin x = 3\cos x$. (3)

16.2 Hence or otherwise, calculate the value of x for $x \in [-180°; 180°]$ if $2\cos x = \sin(x + 30°)$. (4)

16.3 In the diagram below, the graph of $f(x) = 2\cos x$ is drawn for $x \in [-180°; 180°]$



QUESTION17 NWS16

Consider $f(x) = \cos(x - 45^\circ)$ and $g(x) = \tan\frac{1}{2}x$ for $x \in [-180^\circ; 180^\circ]$.

17.1 Use the grid provided to draw sketch graphs of f and g on the same set of axes for $x \in [-180° ; 180°]$. Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)

17.2 Use your graphs to answer the following questions for $x \in [-180° ; 180°]$

17.2.1 Write down the solutions of $\cos(x - 45°) = 0$ (2)

17.2.2 Write down the equations of asymptote(s) of g. (2)

17.2.3 Write down the range of f. (1)

17.2.4 How many solutions exist for the equation $\cos(x - 45°) = \tan\frac{1}{2}x$? (1)

17.2.5 For what value(s) of x is $f(x).g(x) > 0$ (3)

QUESTION 18 WCS16

In the diagram, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \in [-90°; 180°]$.

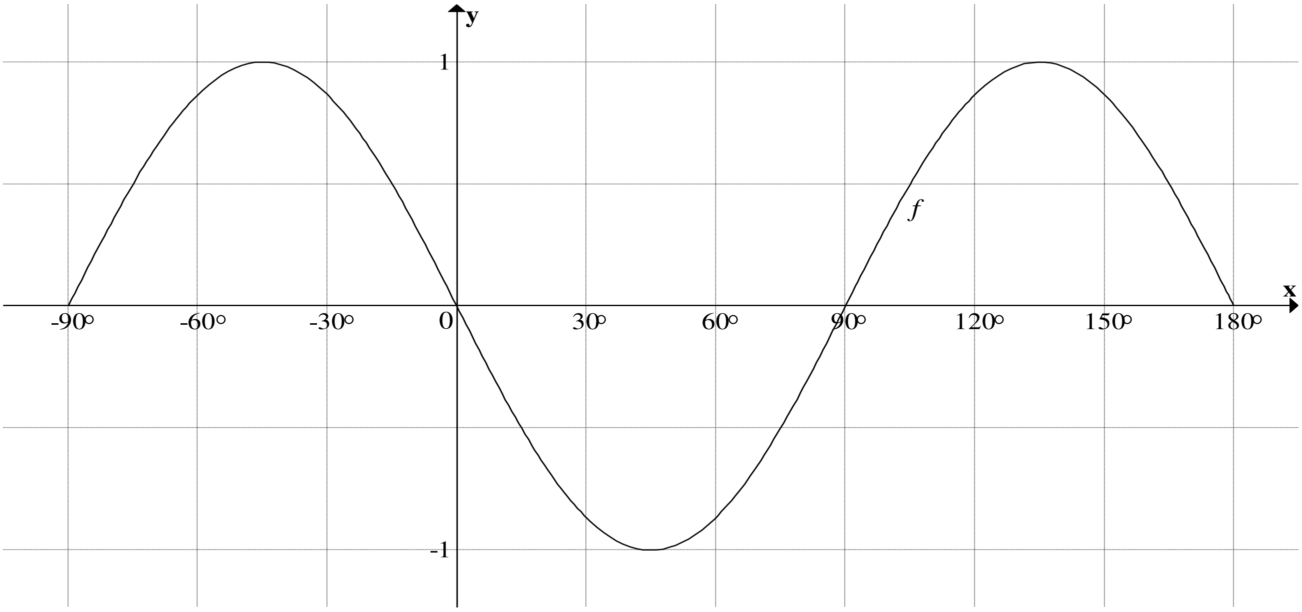
18.1 Draw the graph of g, where $g(x) = \cos(x - 60°)$, on the same system of axes for the interval $x \in [-90°; 180°]$ in the ANSWER BOOK. (3)

18.2 Determine the general solution of $f(x) = g(x)$. (5)

18.3 Use your graphs to solve x if $f(x) \leq g(x)$ for $x \in [-90°; 180°]$ (3)

18.4 If the graph of f is shifted 30° left, give the equation of the new graph which is formed. (2)

18.5 What transformation must the graph of g undergo to form the graph of h, where $h(x) = \sin x$? (2)



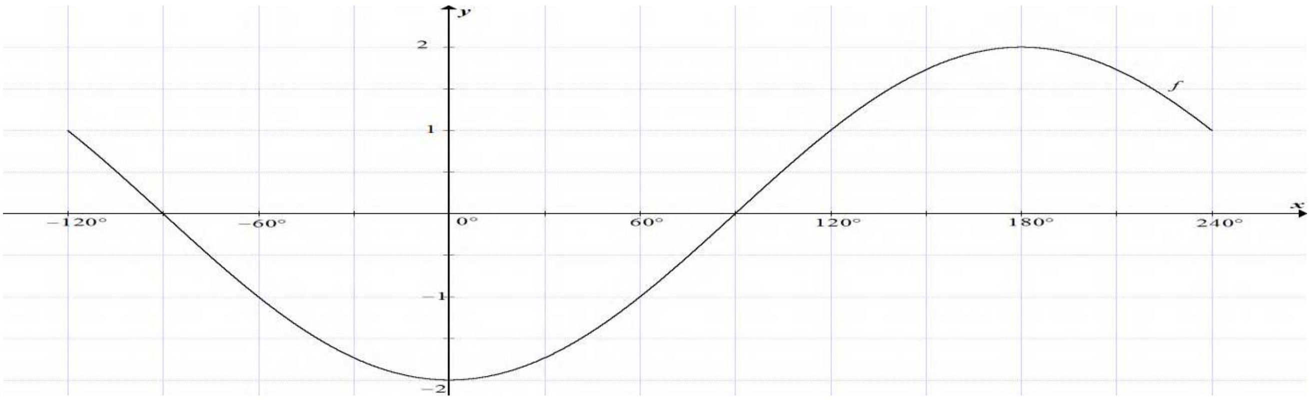
QUESTION 19 NM16

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

19.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

19.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

19.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



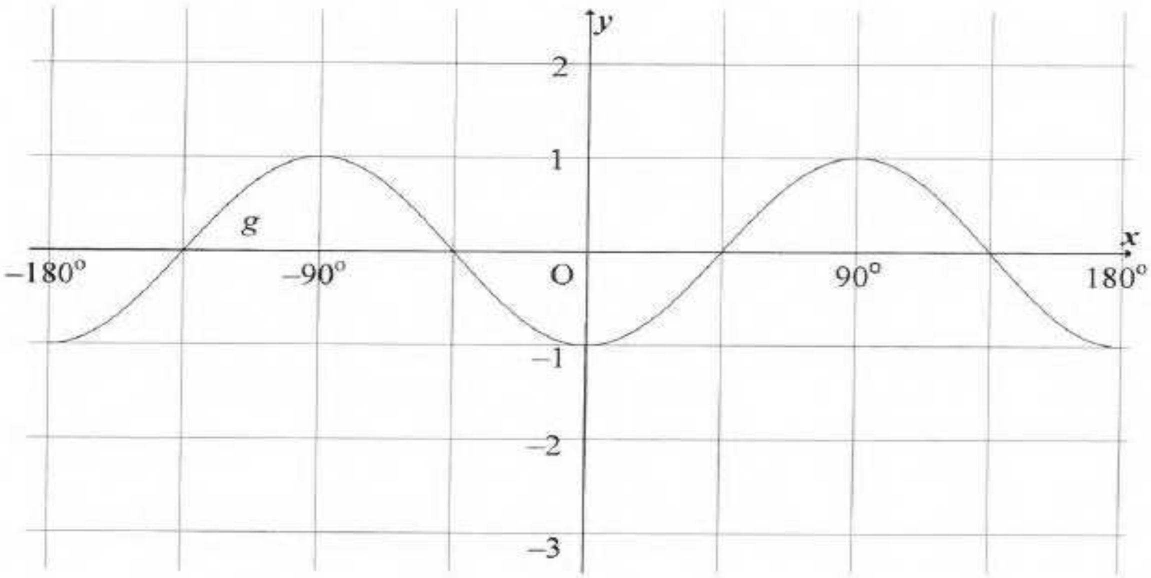
19.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

19.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

QUESTION 20 NJ16

20.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$

20.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$ is drawn below.



20.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180° ; 180°]$ on the set of axes provided in the ANSWER BOOK. (3)

20.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180° ; 0°]$ (2)

20.2.3 Write down the value(s) of x for which $f(x + 30°) - g(x + 30°) = 0$ for $x \in [-180° ; 180°]$ (2)

QUESTION 21 IEB 2019 NOV

Given $f(x) = \cos 2x + 1$ and $g(x) = \tan x$ for $x \epsilon [0°; 270]$.

21.1 Write down the period of f.

21.2 Draw the graphs of f and g on the same system of axes.

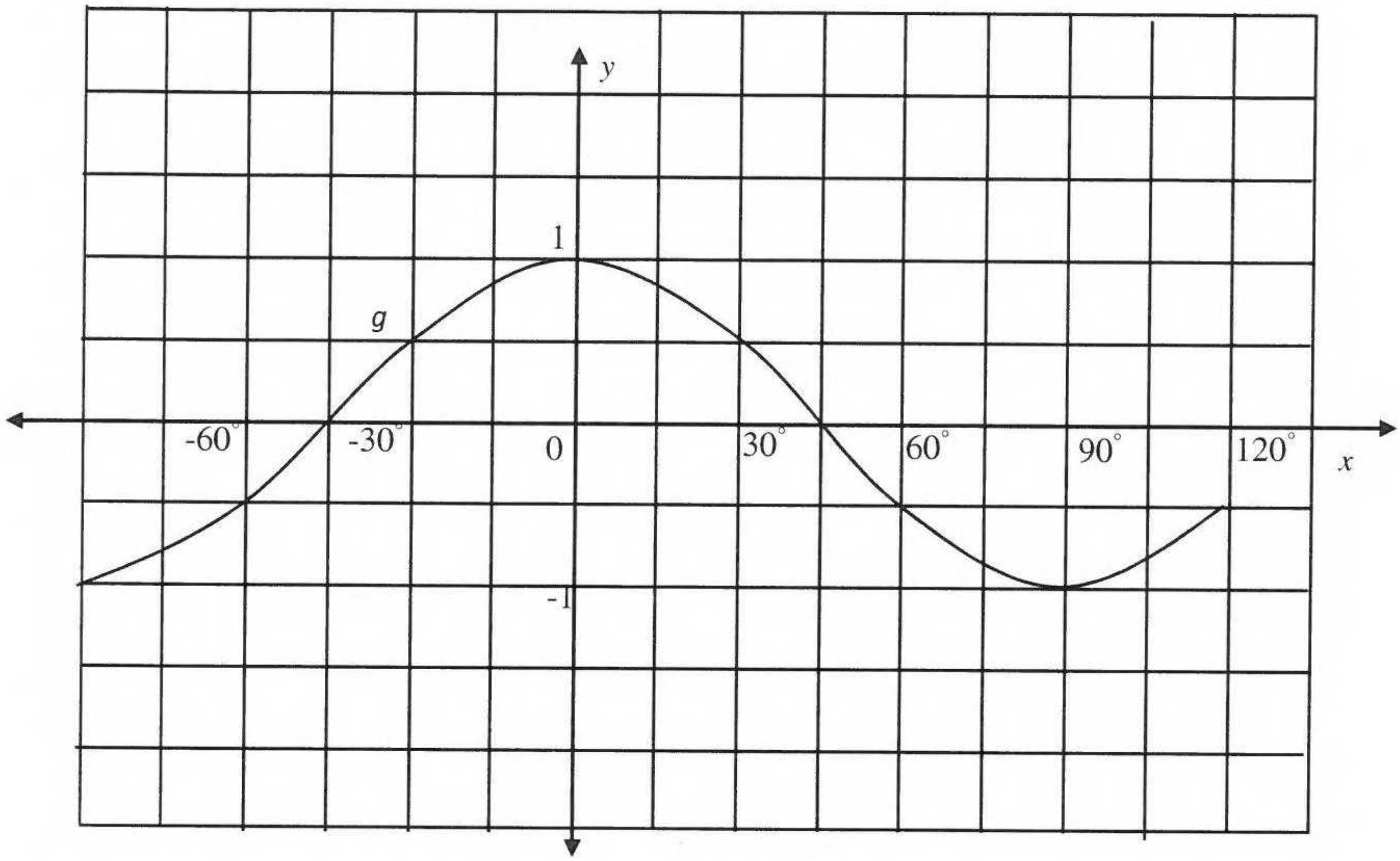
21.3 Use the graphs to write down the point(s) of intersection.

21.4 Write the values of x if $f(x).g(x) < 0$

21.5 Write the values of x if $f(x) > g(x)$

QUESTION 22 FSS17

In the diagram below, the graph of $g(x) = \cos 2x$, for $x \in [-90^\circ; 120^\circ]$ is drawn.



22.1 Draw the graph of $f(x) = \sin(x + 30^\circ)$ for $x \in [-90^\circ; 120^\circ]$ on the set of axes provided in the ANSWER BOOK. (3)

22.2 Determine the value(s) of x, $x \in [-90^\circ; 120^\circ]$ for which both graphs are decreasing. (2)

22.3 Consider $h(x) = f(x + 60^\circ)$.

22.3.1 Describe the transformation the graph of f has to undergo to form the graph of h: (2)

22.3.2 Determine the equation of h in its simplest form. (2)

[9]

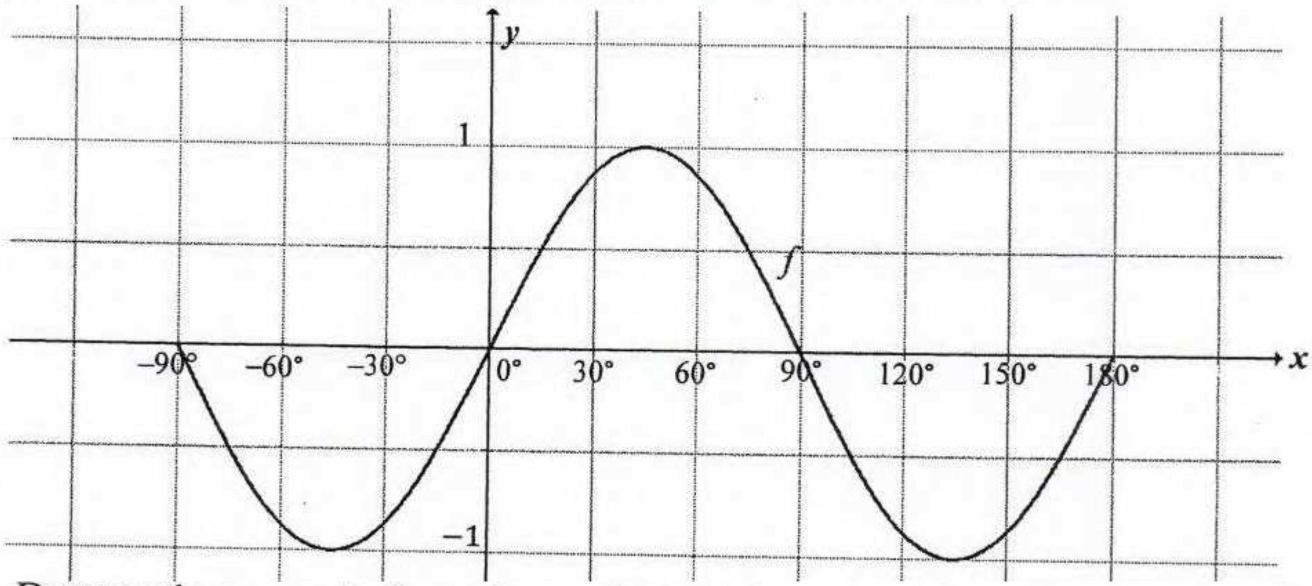
QUESTION 23 MPS17

23.1 Given: $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30^\circ)$.

Use the given set of axis in the ANSWER BOOK and sketch the graphs of f and g for $x \in [-120^\circ;\ 60^\circ]$. Show all intercepts with the axis, coordinates of turning points and coordinates of endpoints. (6)

QUESTION 24 **LPS17**

The graph of $f(x) = \sin 2x$ is drawn in the diagram for the interval $x \in [-90°;180°]$



24.1 Draw on the same set of axes the graph of $g(x) = \cos(x + 60°)$ for $x \in [-90°\,;180°]$. Clearly show all intercepts with the axes, coordinates of the turning points and the endpoints of the graphs. (3)

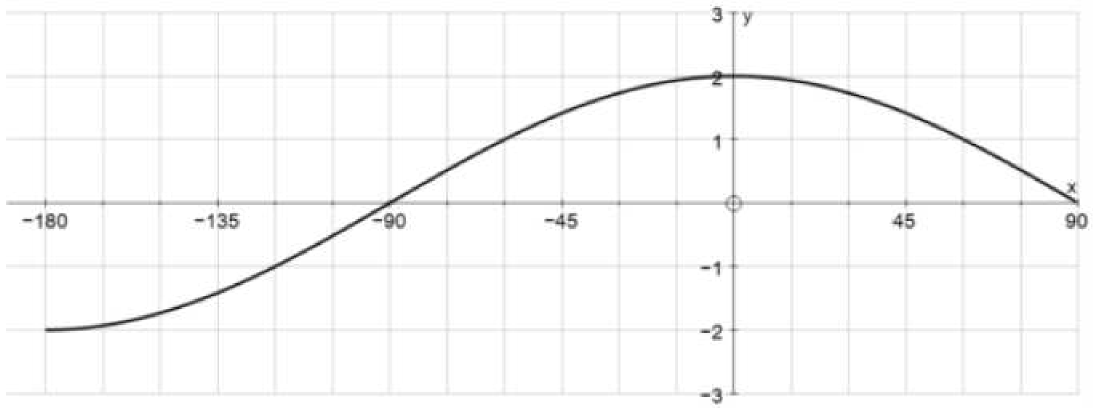
24.2 Calculate the general solution for $\sin 2x = \cos(x + 60°)$. (6)

24.3 Write down the solution for $\sin 2x = \cos(x + 60°)$, $x \in [-90°\,;180°]$. (2)

24.4 Determine for which values of x is $f(x).g(x) < 0$, for $x \in [-90°\,;180°]$. (3)

QUESTION 25 WCS17

Below is the sketch of the graph of $f(x) = a\cos x$ for $x \in [-180°; 90°]$.



25.1 Write down the value of a. (1)

25.2 Sketch the graph of $g(x) = \sin 2x$ on the given axes for $x \in [-180°; 90°]$ in the ANSWERBOOK (3)

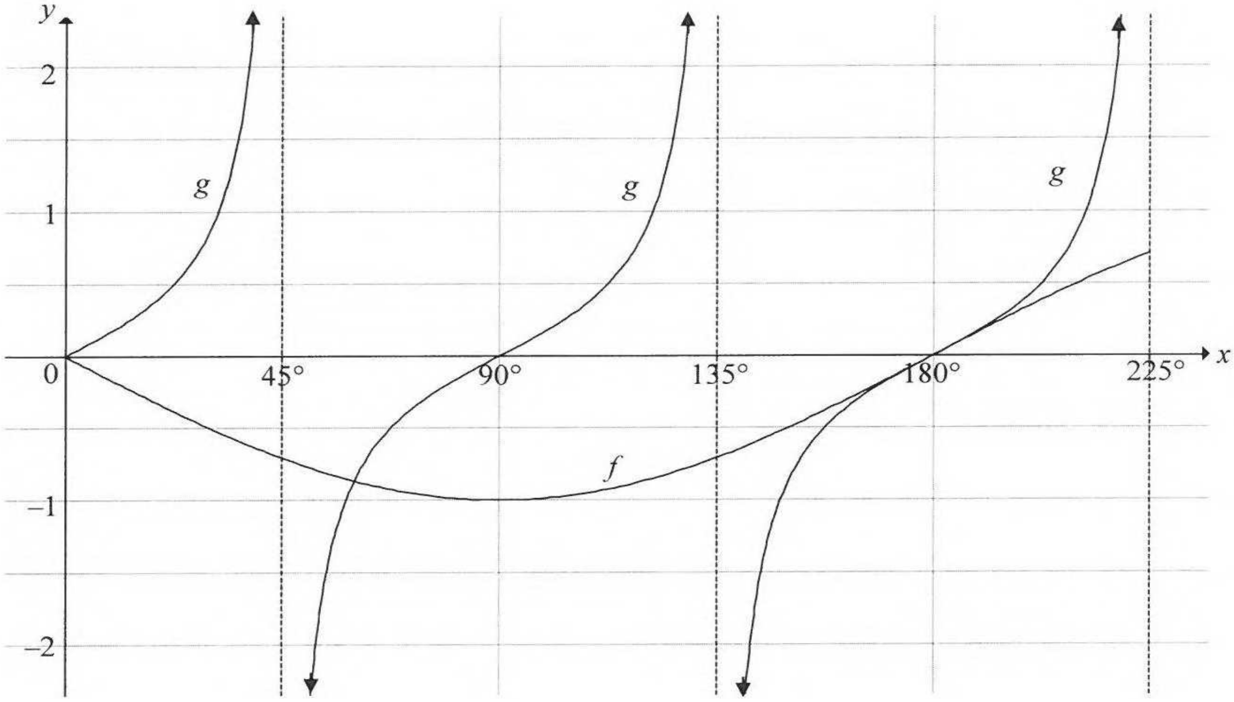
25.3 What is the period of g? (1)

25.4 Use your sketch and determine the value(s) of x for which $f(x) - g(x) = 0$ (2)

25.5 Calculate the maximum value of $3 - 10\sin x \cos x$ (3)

QUESTION 26 NM17

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \leq x \leq 225°$.



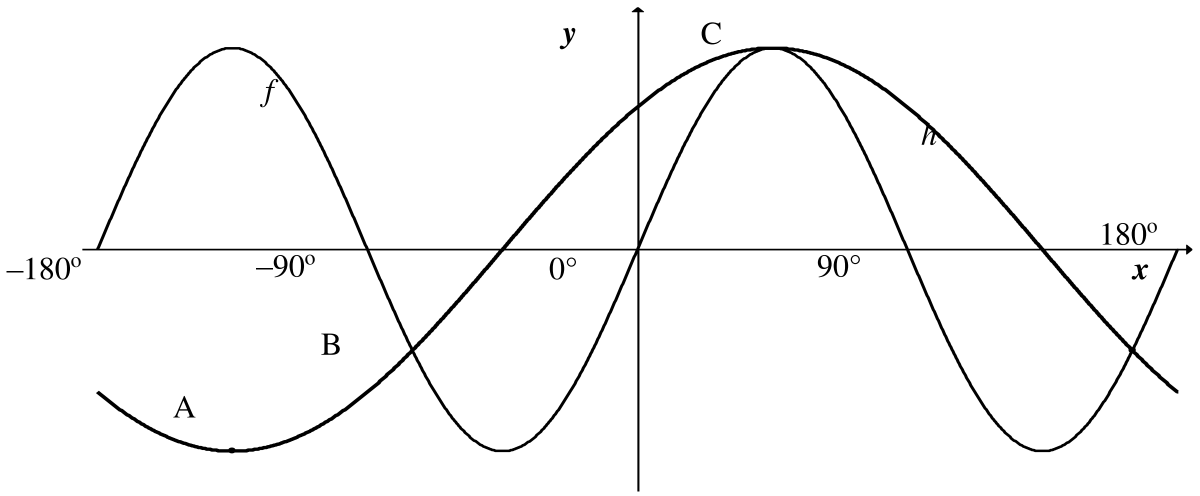
26.1 Write down the values of a and b. (2)

26.2 Write down the period of $f(3x)$. (2)

26.3 Determine the values of x in the interval $90^\circ \leq x \leq 225^\circ$ for which $f(x).g(x) \leq 0$. (3)

QUESTION 27 NJ17

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45°)$ for the interval $x \in [-180° ; 180°]$. $A(-135° ; -1)$ is a minimum point on graph h and $C(45° ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and $D\left(165°; -\frac{1}{2}\right)$.



27.1 Write down the period of f. (1)

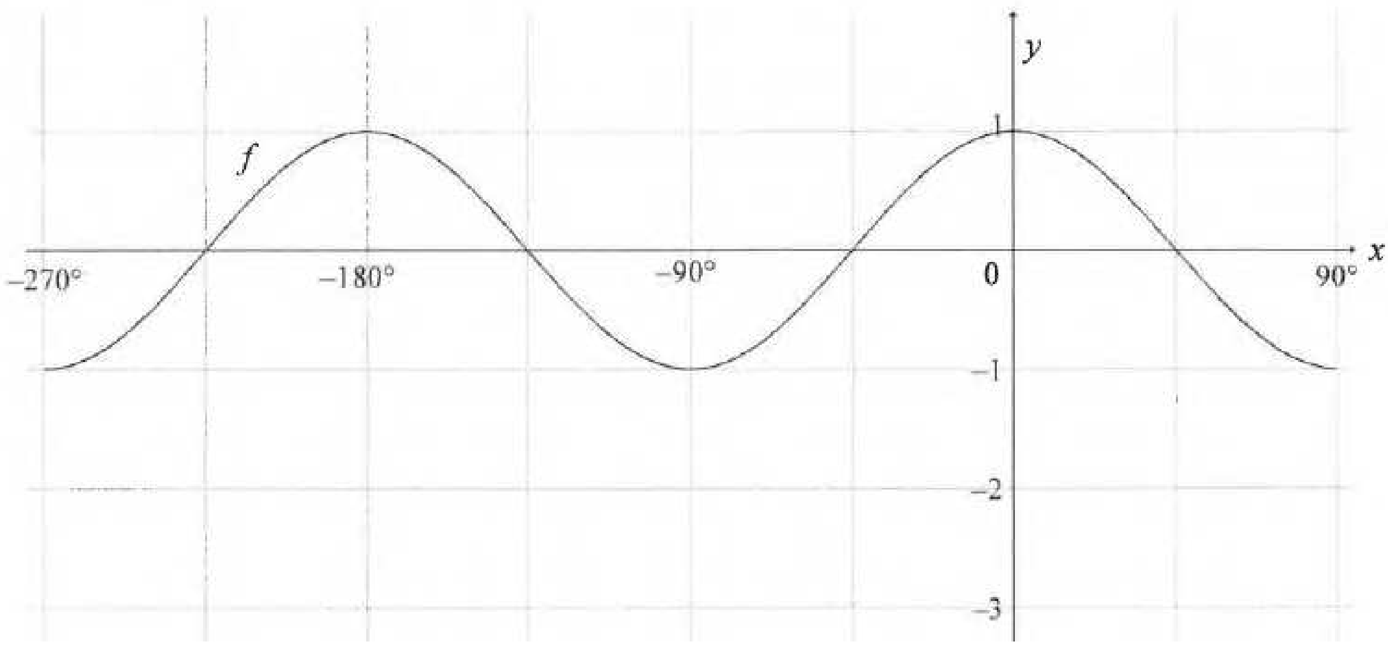
27.2 Determine the x-coordinate of B. (1)

27.3 Use the graphs to solve $2\sin x.\cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180° ; 180°]$. Show ALL working. (4)

[6]

QUESTION 28 NN17

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270° ; 90°]$.



28.1 Draw the graph of $g(x)=2\sin x-1$ for the interval $x\in[-270°;90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

28.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x=\dfrac{-1+\sqrt{5}}{2}$. (4)

28.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x\in[-270°;90°]$. (4)

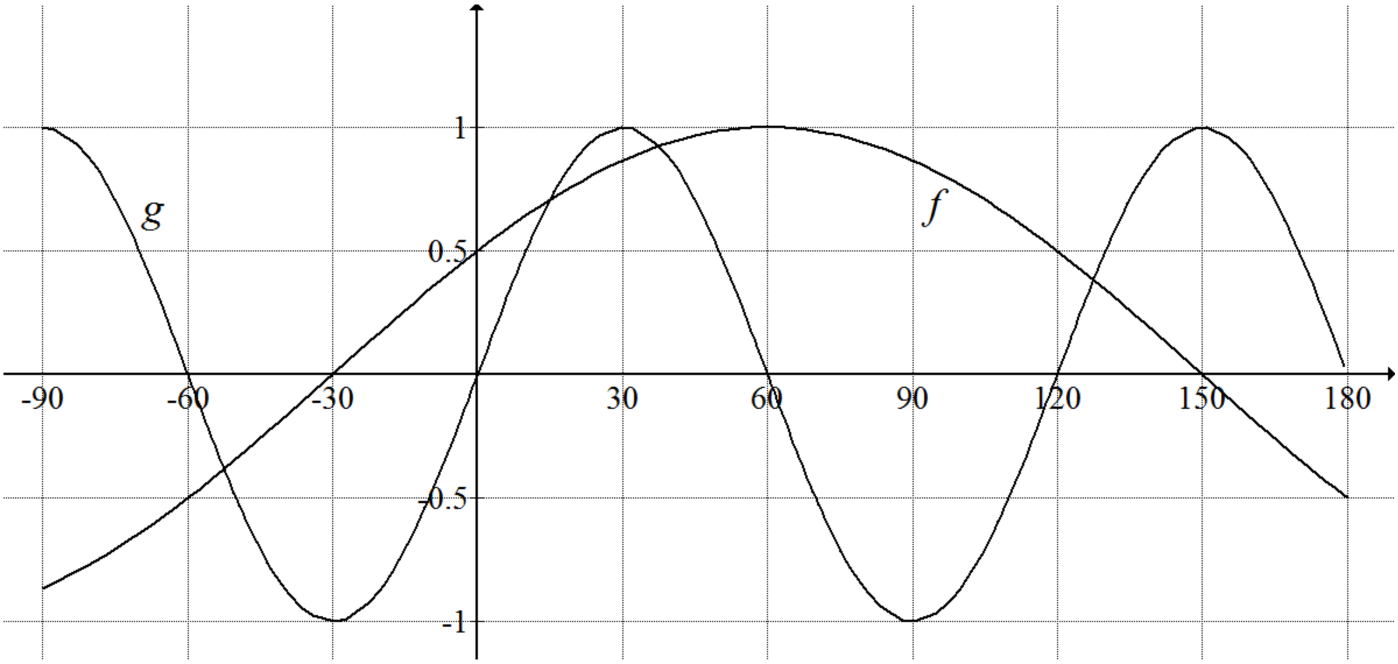
QUESTION 29 **ECS18**

29.1 A function is defined as $f(x)=a\cos(x-p)+1$.
The function satisfies the following conditions:

- The period is $360°$
- The range is $y\in[-1;3]$
- The co-ordinates of a maximum point are $[210°;3]$

Write down the values of a and p. (2)

29.2 In the diagram below, the functions $f(x)=\cos(x-60°)$ and $g(x)=\sin 3x$ are drawn for $x\in[-90°;180°]$.



For which values of x is

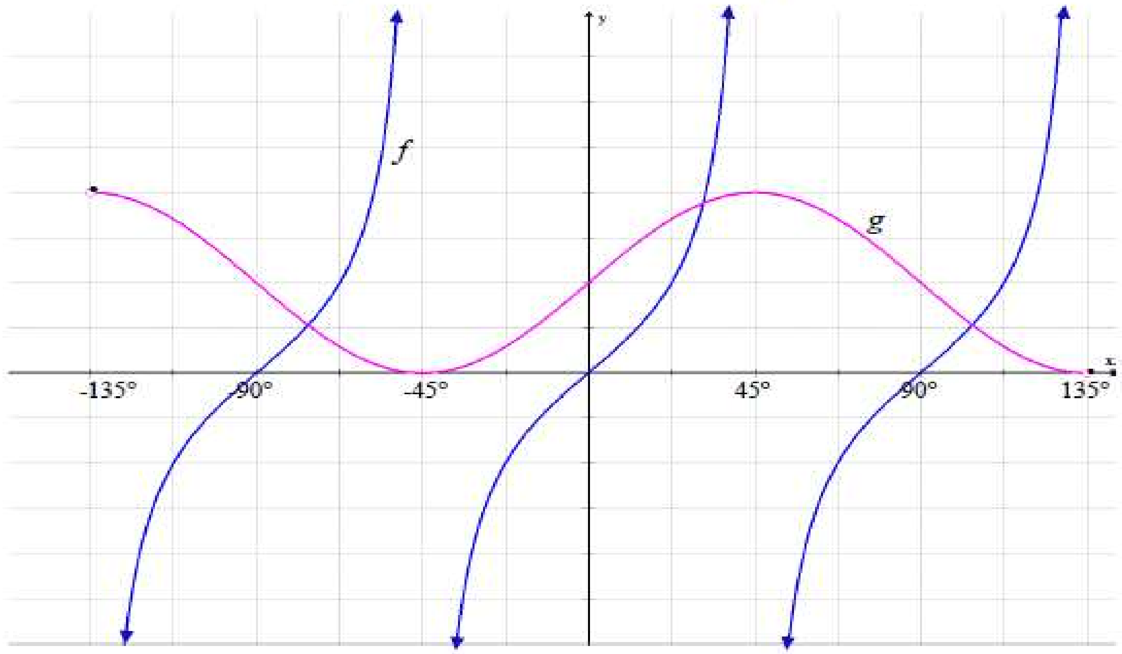
29.2.1 $f'(x)=0$ where $x\in[-90^\circ;180^\circ]$? (1)

29.2.2 $f(x)=g(x)$, where $x\in[-90^\circ;30^\circ]$? Show all relevant calculations. (6)

29.2.3 $f(x)>g(x)$, where $x\in[-90^\circ;30^\circ]$? (2)

QUESTION 30 **GPS18**

The functions $f(x)=\tan 2x$ and $g(x)=1+\sin 2x$ are sketched for $x\in[-135^\circ;135^\circ]$.



30.1 Write down the equation of the asymptote in the interval $x\in[-135^\circ;0^\circ]$. (1)

30.2 If $h(x)=\dfrac{\sin x-2\sin^3 x}{2\sin^2 x.\cos x}$, determine h in terms of f. (4)

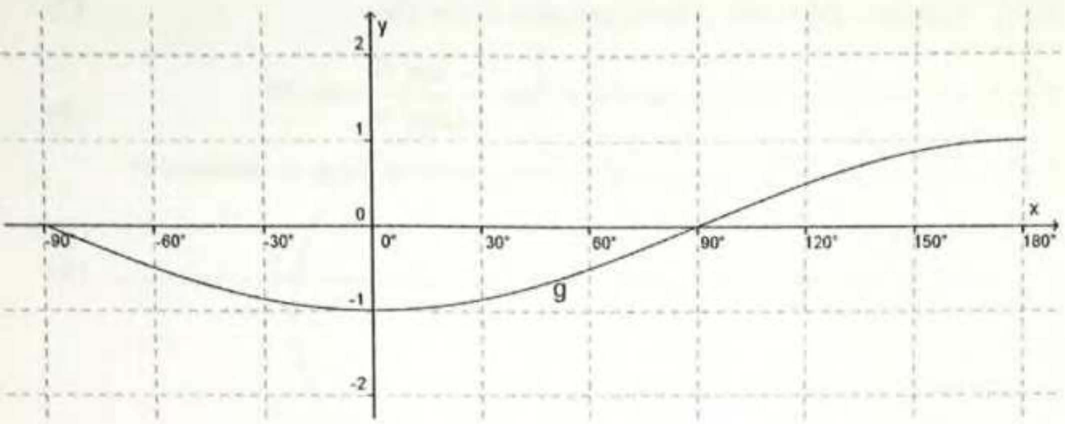
30.3 Determine the equation of p in its simplest form, if graph g is translated by moving the y-axis 45° to the right. (3)

30.4 Determine the values of x for which $(\tan 2x)(-1-\sin 2x)\le 0$ for $x\in[-135^\circ;0^\circ)$. (3)

[11]

QUESTION 31 MPS18

In the diagram the graph of $g(x) = -\cos x$ is drawn for the interval $x \in [-90^\circ; 180^\circ]$.



31.1 On the same set of axis in the answer book draw the graph of $f(x) = \sin(x + 30^\circ)$ for $x \in [-90^\circ; 180^\circ]$. Show clearly all the intercepts with the axes, as well as the turning points. (4)

31.2 Write down the period of $g(2x)$. (2)

31.3 Determine for which values of x; $x \in [-90^\circ; 180^\circ]$, the graphs of f and g are both increasing. (2)

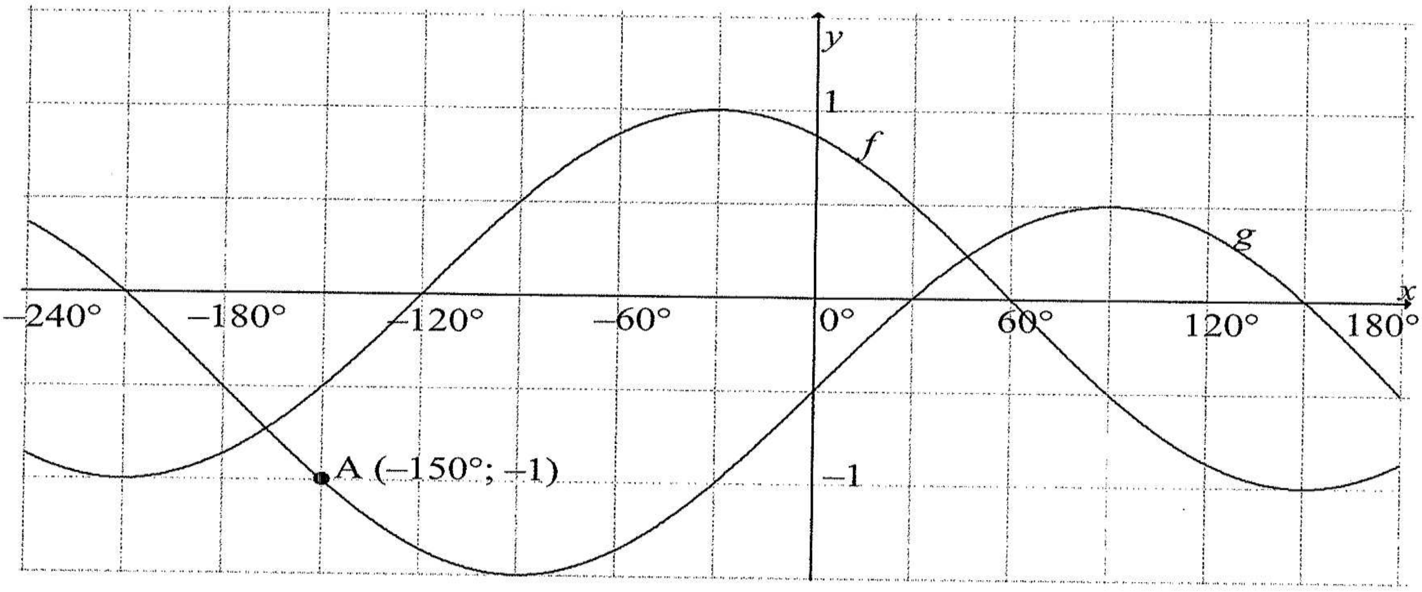
QUESTION 32 NWS18

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = \sin x + q$ are drawn on the same set of axes for $-240^\circ \leq x \leq 180^\circ$. A$(-150^\circ ; -1)$ is a point on g.

32.1 Determine the values of p and q. (4)

32.2 Determine graphically the values of x, for $-240^\circ \leq x \leq 180^\circ$, where $f(x) = g(x) + \frac{1}{2}$ (2)

32.3 Describe the transformation that the graph of f has to undergo to form the graph of h, where $h(x) = -\sin x$. (2)



QUESTION 33 NM18

33.1 Consider: $g(x) = -4\cos(x + 30°)$

33.1.2 Write down the maximum value of $g(x)$. (1)

33.1.3 Determine the range of $g(x) + 1$. (2)

33.1.4 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)

QUESTION 34 NWN18

Consider: $f(x) = -2\tan\frac{3}{2}x$

34.1 Write down the period of f. (1)

34.2 The point $A(t\,;2)$ lies on the graph. Determine the general solution of t. (3)

34.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120°\,;180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

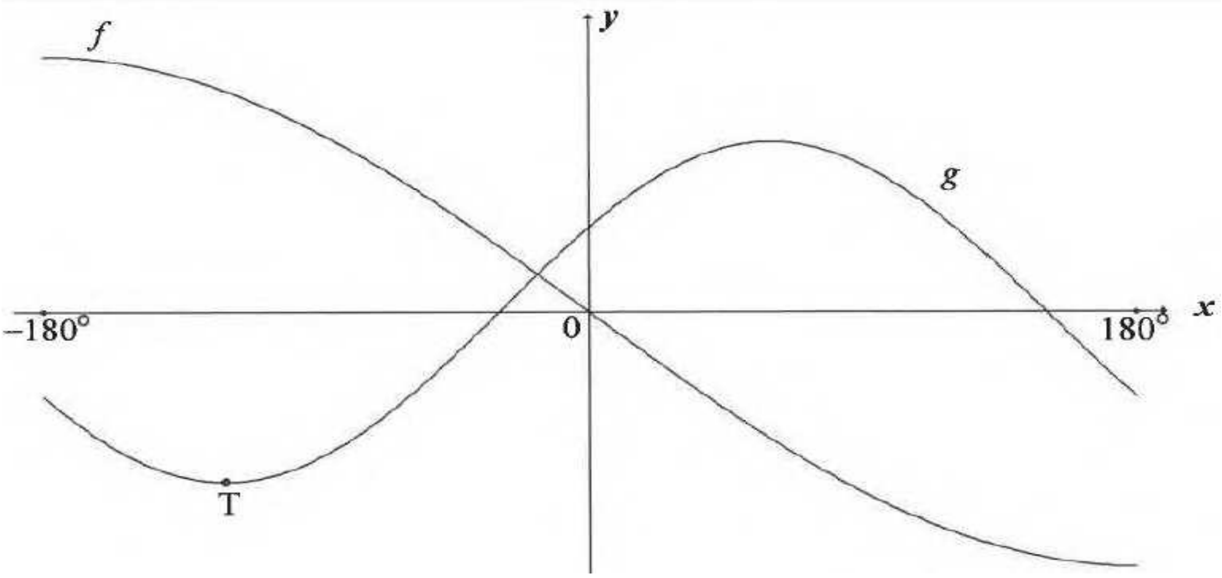
34.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120°\,;180°]$. (3)

34.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

QUESTION 35 **NWJ18**

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x - 60°)$ are drawn in the interval $x \in [-180°; 180°]$. T$(p\,;q)$ is a turning point of g with $p < 0$.



35.1 Write down the period of f. (1)

35.2 Write down the range of g. (2)

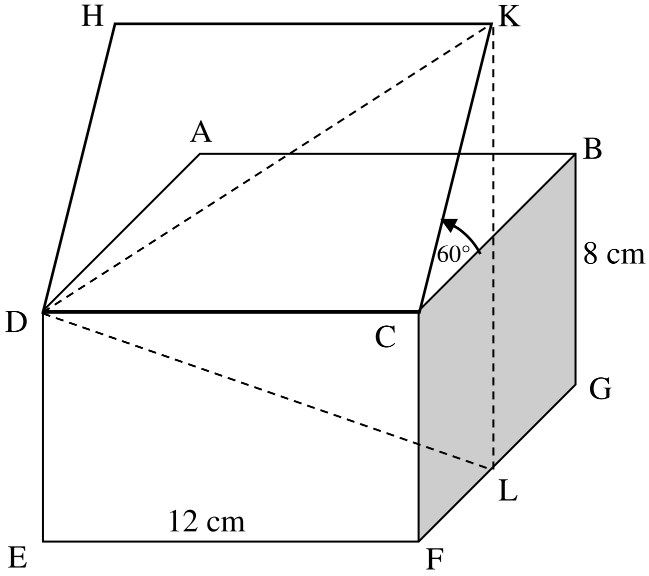
35.3 Calculate $f(p) - g(p)$. (3)

35.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°; 180°]$ for which:

35.4.1 $g(x) > 0$ (3)

35.4.2 $g(x).g'(x) > 0$ (4)

QUESTION 36 NWS 2O



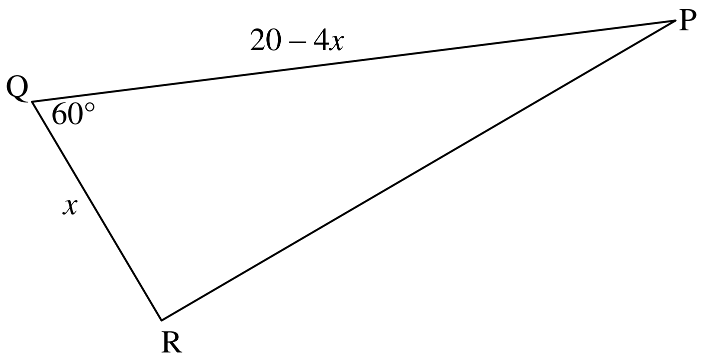
36.1 Write down the length of KC. (1)

36.2 Determine KL, the perpendicular height of K, above the base of the box. (3)

36.3 Hence, determine the value of $\frac{\sin K\hat{D}L}{\sin D\hat{L}K}$. (4)

QUESTION 37 NM16

37.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

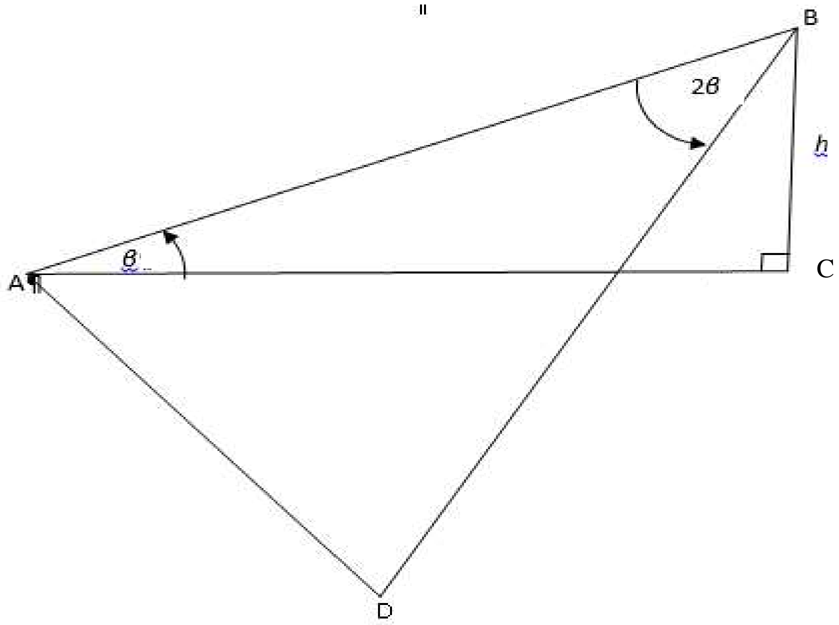


37.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

37.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

37.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

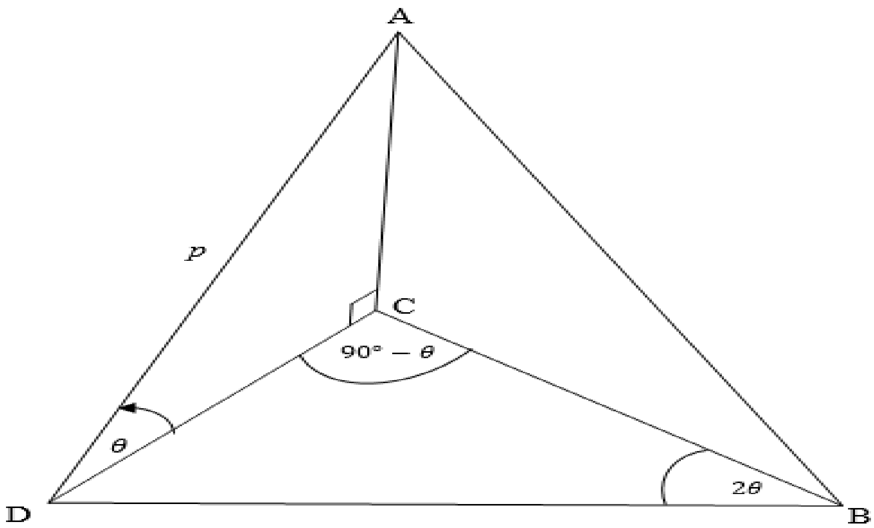
37.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and $BA = BD$.



Determine the distance AD between the two anchors in terms of h. (7)

QUESTION 38 WCS16

In the diagram below, D, B and C are points in the same horizontal plane. AC is a vertical pole and the length of the cable from D to the top of the pole, A, is p meters. $AC \perp CD$. $A\hat{D}C = \theta$; $D\hat{C}B = (90° - \theta)$ and $C\hat{B}D = 2\theta$.



38.1 Prove that:

$$BD = \frac{p\cos\theta}{2\sin\theta}$$ (5)

38.2 Calculate the height of the flagpole AC if $\theta = 30°$ and $p = 3$ meters. (2)

38.3 Calculate the length of the cable AB if it is further given that $A\hat{D}B = 70°$ (5)

[12]

MORE RE-ENFORCING TRIG CHALLENGES

QUESTION 1

Simplify the following without the use of a calculator:

1.1 $\sin 10^\circ \cos 20^\circ + \cos 10^\circ \sin 20^\circ$

1.2 $\cos 55^\circ \cos 10^\circ - \sin 55^\circ \sin 10^\circ$

1.3 $2\sin 15^\circ . \cos 15^\circ$

1.4 $\sin 22,5^\circ . \cos 22,5^\circ$

1.5 $\cos^2 15^\circ - \sin^2 15^\circ$

1.6 $\cos^2 15^\circ + \sin^2 15^\circ$

1.7 $(\cos 15^\circ - \sin 15^\circ)^2$

1.8 $1 - 2\sin^2 15^\circ$

1.9 $2\sin^2 22,5^\circ - 1$

QUESTION 2

Identities

2.1 Prove that $\dfrac{\sin^2 \beta}{1-\cos \beta} = 1 + \dfrac{\sin \beta}{\tan \beta}$. (4)

2.2 Prove: $\dfrac{1}{1-\cos \theta} + \dfrac{1}{1+\cos \theta} = \dfrac{2}{\sin^2 \theta}$

2.3 Prove that $\dfrac{1}{\tan x} + \cos x = \dfrac{\cos^2 x}{\tan x - \tan x \sin x}$. (6)

2.4 Prove the identity:

$$\frac{(\sin x - \cos x)^2}{1 - \sin^2 x} = \frac{1}{\cos^2 x} - 2\tan x$$

QUESTION 3 – REDUCTION

3.1 Simplify the following:

3.1.1 $\cos(-\theta)\cdot\cos\theta+\cos(90°-\theta)\cdot\sin(180°+\theta)$

3.1.2 $\sin(90°+x)\cos(-x)+\sin(180°+x)\cos(90°+x)$.

3.1.3 $\cos(90°+x)\cdot\sin(180°+x)-\sin(90°+x)\cdot\cos(180°-x)-\dfrac{\sin^2(-x)}{\sin 150°}$

3.2 Determine the value of the following without the use of a calculator:

3.2.1 $\sin 280°.\cos 160°-\cos 100°.\sin 200°$

3.2.2 $\cos 265°.\sin 355°-\sin 85°.\cos 175°$

3.2.3 $\cos 65°.\cos 295°-\sin 115°.\cos 205°$

3.3 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

3.3.1 $\sin 191°$

3.3.2 $\cos 22°$

3.4 Simplify $\cos(x-180°)+\sqrt{2}\sin(x+45°)$ to a single trigonometric ratio.

3.5 Given: $\sin P+\sin Q=\dfrac{7}{5}$ and $\hat{P}+\hat{Q}=90°$

Without using a calculator, determine the value of $\sin 2P$.

3.6 **Show that** $\tan 89°\times\tan 88°\times\tan 87°\times......\times\tan 1°=1$ without the use of a calculator. Show all steps.

3.7 **Calculate the value of** $\sin^2 89°+\sin^2 88°+\sin^2 87°+......+\sin^2 2°+\sin^2 1°$ without the use of a calculator. Show all steps.

QUESTION 4 - IDENTITIES

4.1 Prove the following identities:

4.1.1 $\dfrac{1-\cos 2\theta}{1-\cos\theta}=\dfrac{\sin 2\theta+2\sin\theta}{\sin\theta}$

4.1.2 $\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$

4.1.3 $\dfrac{\sin 2x-2\sin x}{\cos 2x-1}=\dfrac{1}{\sin x}-\dfrac{1}{\tan x}$

4.1.4 $\dfrac{\cos 2\theta+\cos\theta}{\sin^2\theta}=\dfrac{\sin 2\theta-\sin\theta}{\sin\theta-0{,}5\sin 2\theta}$

4.1.5 $\dfrac{1+\sin 2A}{\cos 2A}=\dfrac{\cos A+\sin A}{\cos A-\sin A}$

4.1.6 $\dfrac{\sin 3x+\sin 7x}{\cos 3x+\cos 7x}=\tan 5x$

4.1.7 $\sin 3x+\cos 5x=2\cos 4x\sin x$

4.2

4.2.1 Prove that $\cos(60°+\theta)-\cos(60°-\theta)=-\sqrt{3}\sin\theta$

4.2.2 Hence evaluate $\cos 105°-\cos 15°$ without using a calculator.

4.1 Prove that $\dfrac{\sin^2\beta}{1-\cos\beta}=1+\dfrac{\sin\beta}{\tan\beta}$. (4)

4.2 Prove: $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}=\dfrac{2}{\sin^2\theta}$

4.3 Prove that $\dfrac{1}{\tan x}+\cos x=\dfrac{\cos^2 x}{\tan x-\tan x\sin x}$. (6)

4.4 Prove the identity:

$$\frac{(\sin x-\cos x)^2}{1-\sin^2 x}=\frac{1}{\cos^2 x}-2\tan x$$

QUESTION 5

"Write in terms of …"

5.1 If $\cos 36° = k$, write the following in terms of k:

5.1.1 $\cos 144°$

5.1.2 $\sin(-54°)$

5.1.3 $\tan 36°$

5.2 If $\sin 70° = p$, express the following in terms of p:

5.2.1 $\sin 290°$

5.2.2 $\cos 70°$

5.2.3 $\tan 70°$

5.3 If $\cos 20^o = \frac{1}{p}$, find the value of $\tan 160^o$ in terms of p:

QUESTION 4

Identities

4.1 Prove that $\dfrac{\sin^2\beta}{1-\cos\beta} = 1 + \dfrac{\sin\beta}{\tan\beta}$. (4)

4.2 Prove: $\dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta} = \dfrac{2}{\sin^2\theta}$

4.3 Prove that $\dfrac{1}{\tan x} + \cos x = \dfrac{\cos^2 x}{\tan x - \tan x \sin x}$. (6)

4.4 Prove the identity:

$$\frac{(\sin x - \cos x)^2}{1-\sin^2 x} = \frac{1}{\cos^2 x} - 2\tan x$$

QUESTION 6

Equations

6.1 Solve for A if $3\sin 2A = -1$ and $A \in (-180°;360°)$

6.2 Find the general solution of $2\sin x - 3\cos x = 0$

6.3 Determine the general solution of $3\sin x + 2\cos x = 0$

6.4 Find the general solution of $\sin\alpha = \cos(\alpha + 30°)$

QUESTION 1 – BASIC COMPOUND AND DOUBLE ANGLES

1.1 Simplify the following:

1.1.1 $\sin(\theta + 30°) - \sin(\theta - 30°)$

1.1.2 $\sin(A - 60°) + \cos(A - 30°)$

1.1.3 $\sqrt{3}\sin(\theta + 60°) - \sin(\theta + 30°)$

1.2 Determine the value of $\sin 75°$ without the use of a calculator.

1.3 Write the following as a single trigonometric ratio:

1.3.1 $\cos 3\theta.\cos\theta + \sin 3\theta.\sin\theta$

1.3.2 $\cos 2x.\cos 3x - \sin 2x.\sin 3x$

1.3.3 $\sin 4P.\cos 2P - \cos 4P.\sin 2P$

1.3.4 $\sin 10°.\cos 4\theta + \cos 10°.\sin 4\theta$

SOME OTHER INTERESTING QUESTIONS

1. Determine the value of $\dfrac{\sin^2 10° + \sin^2 80° - \cos^2 11°}{\cos^2 259°}$ without a calculator.

2. Simplify the following without the use of a calculator:

$$\frac{\cos 70°}{\sin 10°} + \frac{\sin 70°}{\cos 10°}$$

3. If $\sin 11° = t$, determine $\sin 33°$ in terms of t

4. Simplify: $\cos(x + 60°)\cos x + \sin(x + 60°)\sin x$

**5. Determine the maximumvalue of $2\sin x\cos 10° + 2\cos x\sin 10°$

QUESTION 5

5.1 Simplify the expression to a **single trigonometric term:**

$\tan(-x).\cos x.\sin(x - 180°) - 1$ (5)

5.2 Given: $\cos 35° = m$

Without using a calculator, determine the value of EACH of the following in terms of m:

5.2.1 $\cos 215°$ (2)

5.2.2 $\sin 20°$ (3)

5.3 Determine the general solution of:

$\cos 4x.\cos x + \sin x.\sin 4x = -0{,}7$ (4)

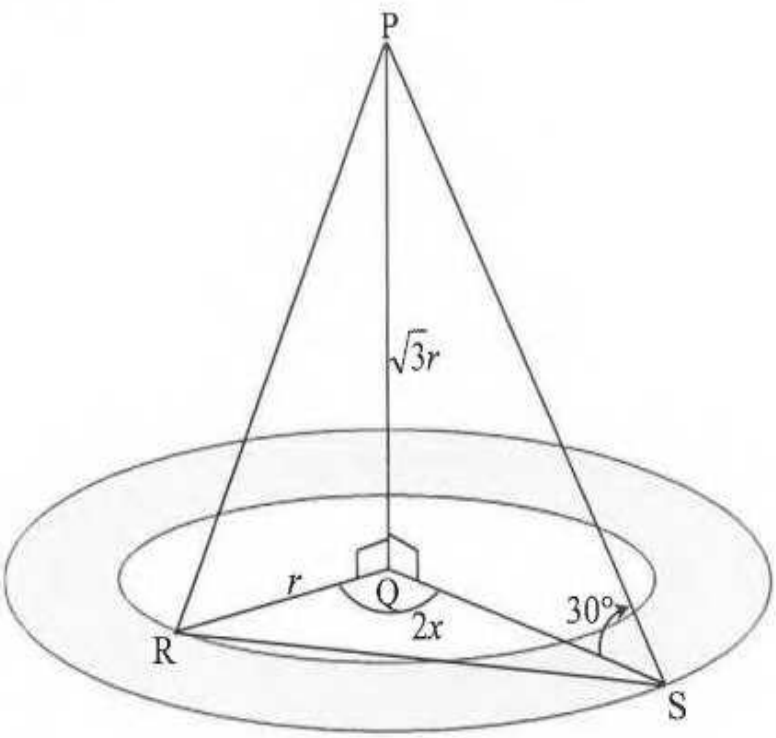
5.4 Prove the identity: $\dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x} = \cos^2 x - \sin^2 x$ (4)

[18]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is 30°.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1 Show that $QS = 3r$ (3)

7.2 Determine, in terms of r, the area of the flower garden. (2)

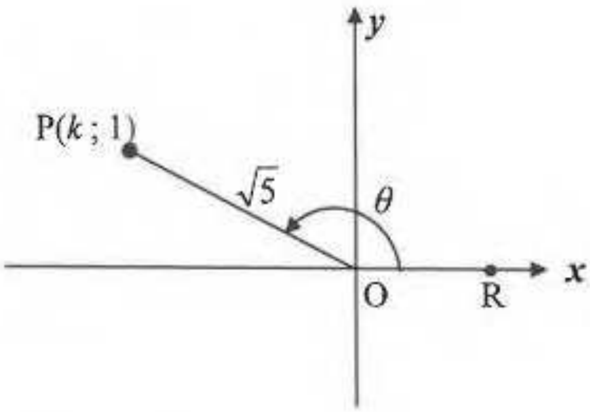
7.3 Show that $RS = r\sqrt{10 - 6\cos 2x}$ (3)

7.4 If $r = 10$ metres and $x = 56°$, calculate RS. (2)

[10]

QUESTION 5

5.1 In the diagram, $P(k\ ;\ 1)$ is a point in the 2[nd] quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $R\hat{O}P = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\dfrac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

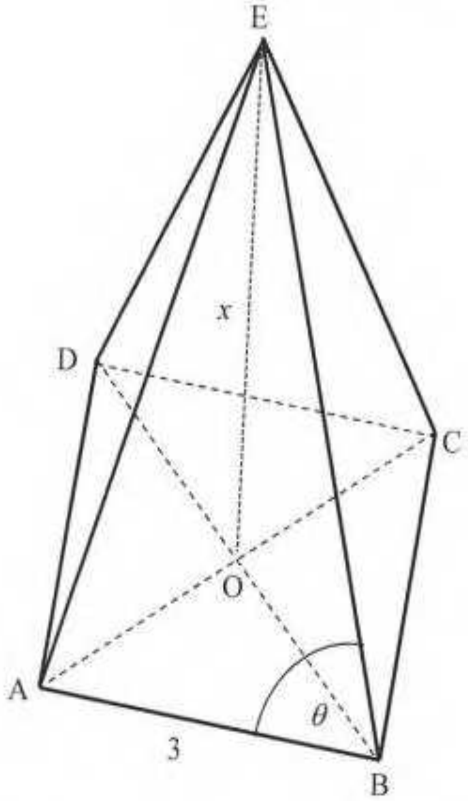
5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)

[23]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height)



7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \dfrac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is 15 m^3, calculate the value of θ. (4)

[12]

FUNCTIONS

Straight Line	Parabola	Hyperbola	Exponential
$y = mx + c$ m … **gradient** and c … **y-intercept**	$y = a(x+p)^2 + q$ $-p$ … **axis of symmetry with equation** $x = -p$ q … **maximum** or minimum value $(-p;q)$ Turning point	$y = \frac{a}{x+p} + q$ $-p$ **… vertical asymptote with equation** $x = -p$ q **… horizontal asymptote with equation** $y = q$	$y = a.b^{x+p} + q$ $b > 0$ **and** $b \neq 1$ q **… horizontal asymptote with equation** $y = q$
$m < 0$ **… graph slant to the left** $m > 0$ **… graph slant to the right**	$a < 0$ **… graph faces downwards and has a minimum turning point** $a > 0$ **… graph faces upwards and has a maximum turning point**	$a < 0$ **… graph is on the second and the fourth quadrant** $a > 0$ **… graph is on the first and the third quadrant**	$a < 0$ **… graph is below the asymptote** $a > 0$ **… graph is above the asymptote**
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$	**Domain:** $x \in R$, $x \neq -p$ **Range:** $y \in R$, $y \neq q$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$
$y - y_1 = m(x - x_1)$	$y = ax^2 + bx + c$ **Axis of symmetry:** $x = \frac{-b}{2a}$ $y = a(x - x_1)(x - x_2)$ x_1 **and** x_2 **are x-intercepts**	**Axis of symmetry/lines of symmetry:** $\left.\begin{cases} y = x + c \\ y = -x + c \end{cases}\right\}$ **substitute point of intersection of asymptotes** **OR** $\begin{cases} y = (x-p)+q \\ y = -(x-p)+q \end{cases}$	

INVERSE FUNCTION

Indicated as f^{-1}

- **Swop x and y in the given function**
- **Make y subject of the formula in the new function**
- **The graph of the given function and the graph of its inverse are reflected about the line** $y = x$

Straight line	Parabola	Exponential
$y = mx + c$	$y = ax^2$	$y = b^x$
Inverse is a function $x = my + c$ $y = \frac{x}{m} - \frac{c}{m}$	**Inverse is not a function** $x = ay^2$ $y = \pm\sqrt{\frac{x}{a}}$	**Inverse is a function** $x = b^y$ $y = \log_b x$

	Restrict domain of $y = ax^2$ **so that the inverse is a function** **Restrictions:** $\begin{Bmatrix} x \geq 0 \\ x \leq 0 \end{Bmatrix}$	
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \geq 0$ **or** $x \leq 0$ **Range:** $y > 0$ **if** $a > 0$ $y < 0$ **if** $a < 0$	**Domain:** $x > 0$ **Range:** $y \in R$

QUESTION 1 **DBE/November 2018**

Given: $f(x) = \dfrac{-1}{x-1}$

1.1 Write down the domain of f. (1)

1.2 Write down the asymptotes of f. (2)

1.3 Sketch the graph of f, clearly showing all intercepts with the axes and any asymptotes. (3)

1.4 For which values of x will $x.f'(x) \geq 0$? (2)

[8]

QUESTION 2 **DBE/November 2021**

Given: $f(x) = \dfrac{-1}{x-3} + 2$

2.1 Write down the equation of the asymptotes of f. (2)

2.2 Write down the domain of f. (1)

2.3 Determine the coordinates of the x-intercept of f. (2)

2.4 Write down the coordinates of the y-intercept of f. (2)

2.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)

[10]

QUESTION 3 **DBE/Feb.-Mar. 2018**

The function f, defined by $f(x) = \dfrac{a}{x+p} + q$, has the following properties:

- The range of f is $y \in R, y \neq 1$.
- The graph f passes through the origin.
- $P\left(\sqrt{2}+2; \sqrt{2}+1\right)$ lies on the graph of f.

3.1 Write down the values of q. (1)

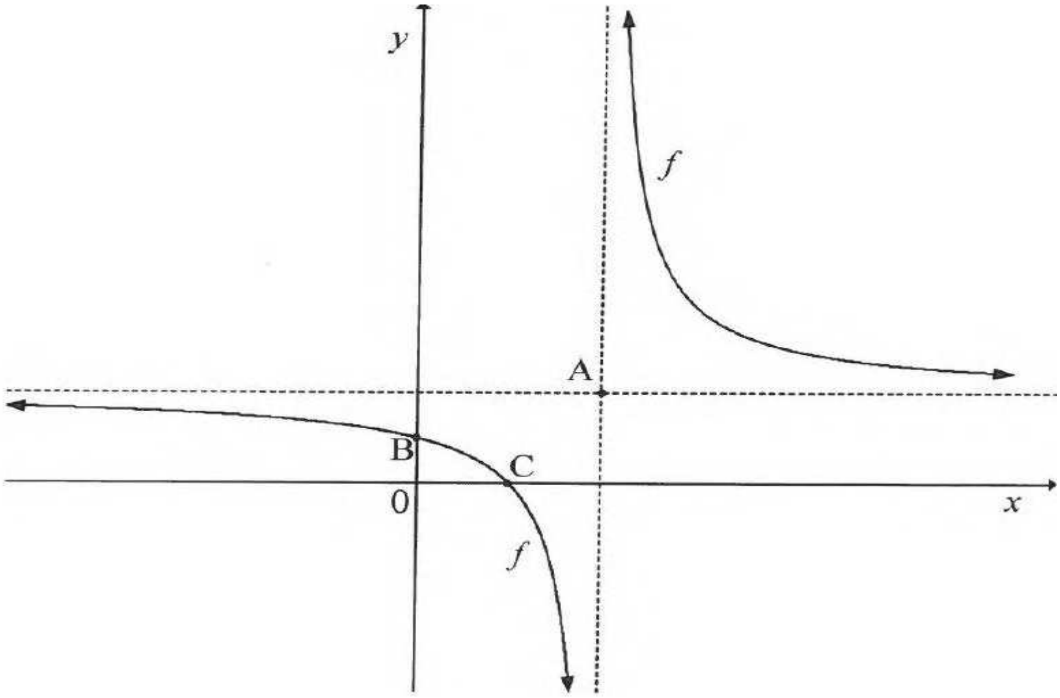
3.2 Calculate the values of a and p. (5)

3.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)

[10]

The sketch below shows the graph of $f(x)=\dfrac{6}{x-4}+3$. The asymptotes of f intersect at A.

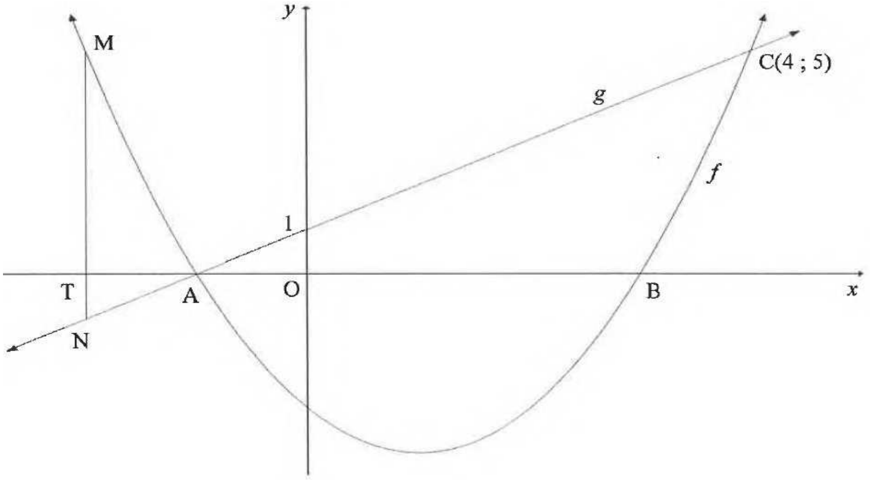
The graph f intersects the x-axis and y- axis at C and B respectively.



4.1	Write down the coordinates of A.	(1)
4.2	Calculate the coordinates of B.	(2)
4.3	Calculate the coordinates of C.	(2)
4.4	Calculate the average gradient of f between B and C	(2)
4.5	Determine the equation of a line of symmetry of f which has a positive y- intercept.	(2)
		[9]

QUESTION 5 **DBE/November 2018**

In the diagram below, A and B are the x-intercepts of the graph of $f(x)=x^2-2x-3$. A straight line, g, through A cuts f at $C(4;5)$ and the y-axis at $(0;1)$. M is a point on f and N is a point on g such that MN is parallel to the y-axis. MN cuts the x-axis at T.



5.1 Show that $g(x)=x+1$. (2)

5.2 Calculate the coordinates of A and B. (3)

5.3 Determine the range of f. (3)

5.4 If MN=6:

5.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working. (4)

5.4.2 Hence, write down the coordinates of N. (2)

5.5 Determine the equation of the tangent to f drawn parallel to g. (5)

5.6 For which value(s) of k will $f(x)=x^2-2x-3$ and $h(x)=x+k$ NOT intersect? (1)

[20]

QUESTION 6 **DBE/November 2017**

Given: $f(x)=-ax^2+bx+6$

6.1 The gradient of the tangent to the graph of f at the point $\left(-1,\frac{7}{2}\right)$ is

Show that $a=\frac{1}{2}$ and $b=2$. (5)

6.2 Calculate the x-intercepts of f. (3)

6.3 Calculate the coordinates of the turning point of f. (3)

6.4 Sketch the graph of f. Clearly indicate All intercepts with the axes and the turning point of f. (4)

6.5 Use the graph to determine the values of x for which $f(x)>6$ (3)

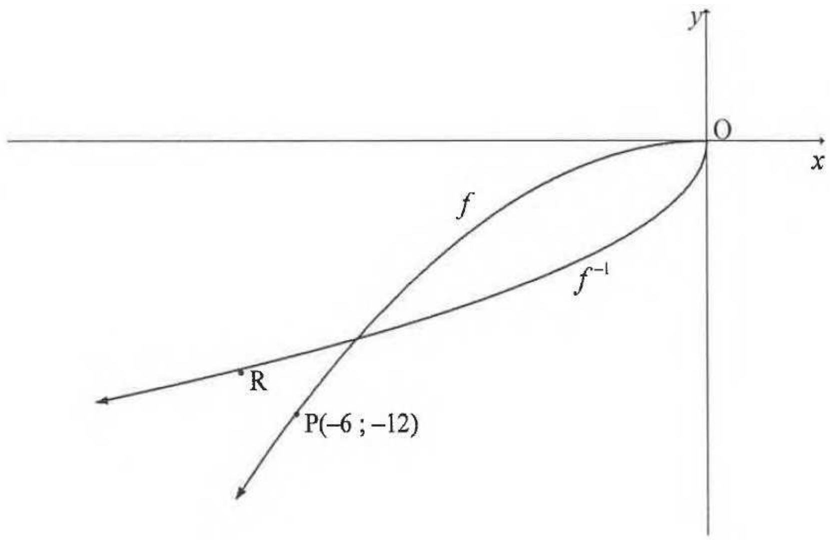
6.6 Sketch the graph of $g(x)=-x-1$ on the same set of axes as f. Clearly indicate ALL intercept with the axes. (2)

6.7 Write down the values of x for which $f(x).g(x)\leq 0$. (3)

[23]

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.

The graph of f^{-1} is also drawn. $P(-6, -12)$ is a point on f and R is a point on f^{-1}.



7.1 Is f^{-1} a function? Motivate your answer. (2)

7.2 If R is the reflection of P in the line $y = x$, write down the coordinate of R. (1)

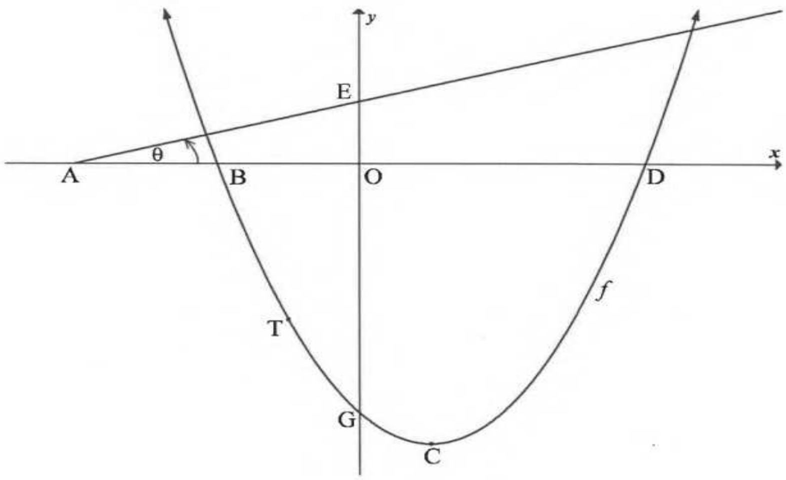
7.3 Calculate the value of . (2)

7.4 Write down the equation of f^{-1} in the form $y = ...$ (3)

[8]

QUESTION 8 **DBE/November 2021**

The graph of $f(x) = (x-4)(x-6)$ is drawn below. The parabola cuts the x-axis at B and D and the y-axis at G. C is the turning point of f. Line AE has an angle of inclination of θ and cuts the x-axis and y-axis ate A and E respectively. T is a point on f between B and G.



8.1 Write down the coordinates of B and D. (2)

8.2 Calculate the coordinates of C. (2)

8.3 Write down the range of f. (1)

8.4 Given that $\theta = 14,04°$ and the tangent to f at T is perpendicular to AE.

8.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

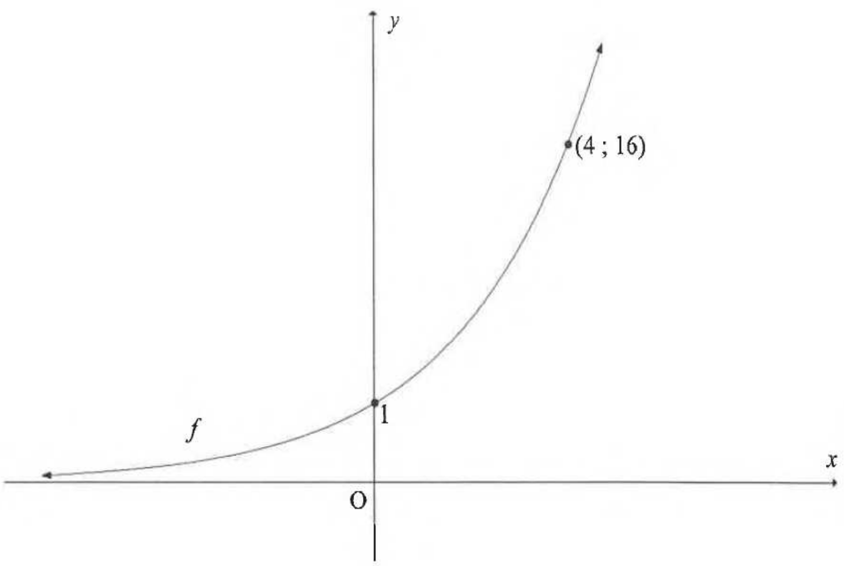
8.4.2 Calculate the coordinates of T. (5)

8.5 A straight line, g, parallel to AE, cuts f at $K(-3;-9)$ and R. Calculate the x coordinate of R. (6)

[17]

QUESTION 9 **DBE/ November 2019**

Sketched below is the graph of $f(x) = k^x; k > 0$. The point (4;16) lies on f.



9.1 Determine the value of k. (2)

9.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = ...$ (2)

9.3 Sketch the graph of g, indicate on your graph the coordinate of two points on g. (4)

9.4 Use the graph to determine the value(s) of x for which:

9.4.1 $f(x) \times g(x) > 0$ (2)

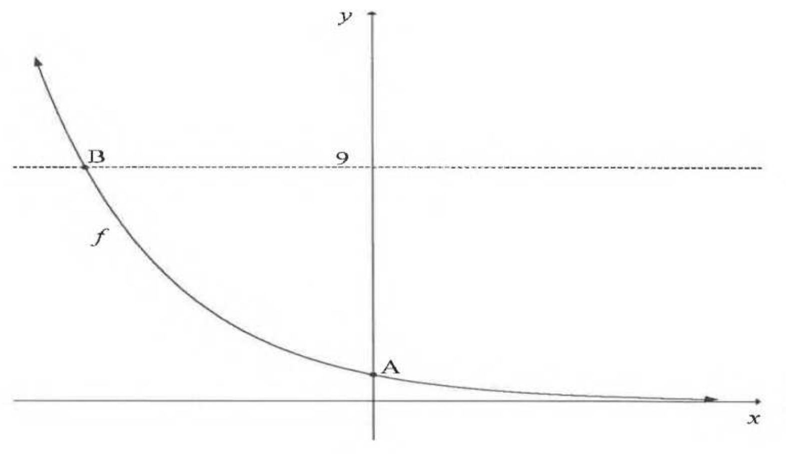
9.4.2 $g(x) \leq -1$ (2)

9.5 If $h(x) = f(-x)$, calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$ (4)

[16]

QUESTION 10 **DBE/ November 2020**

The graph of $f(x) = 3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y = 9$.



10.1 Write down the coordinates of A. (1)

10.2 Determine the coordinates of B. (3)

10.3 Write down the domain of f^{-1}. (2)

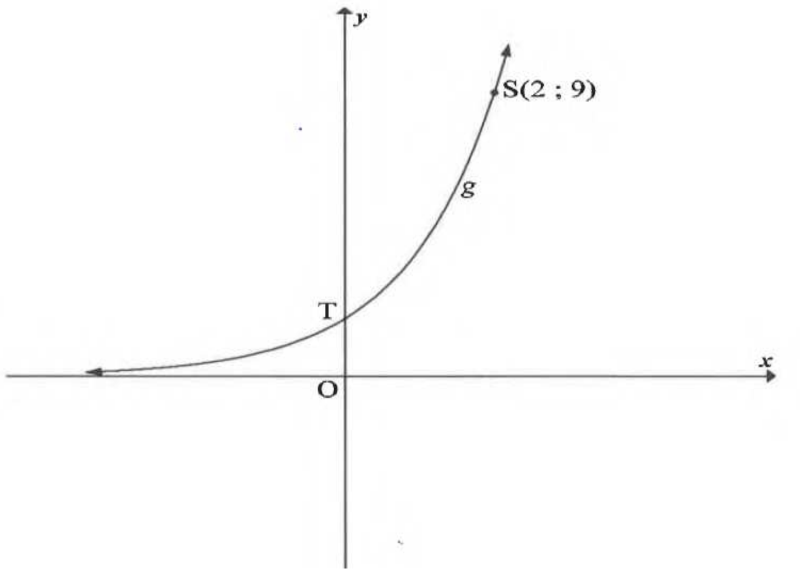
10.4 Describe the translation from f to $h(x) = \frac{27}{3^x}$. (3)

10.5 Determine the values of x for which $h(x) < 1$. (3)

[12]

QUESTION 11 **DBE/Feb.-Mar. 2018**

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2;9) lies on g. T is the y-intercept of g.



11.1 Write down the coordinates of T. (2)

11.2 Calculate the value of a. (2)

11.3 The graph h is obtained by reflecting g in the y-axis. Write down the equation of h. (2)

11.4 Write down the values of x for which $0 < \log_3 x < 1$. (2)

[8]

QUESTION 12 ***DBE/2019***

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

12.1 Write down the range of g. (1)

12.2 Determine the equation of g^{-1} in the form $y = ...$ (2)

12.3 Is g^{-1} a function? Justify your answer. (2)

12.4 The point $M(a;2)$ lies on g^{-1}.

12.4.1 Calculate the value of a (2)

12.4.2 M', the image of M,lies on g. Write down the coordinates of M' (1)

12.5 If $h(x) = g(x+3)+2$, write down the coordinates of the image of M' on h. (3)

[11]

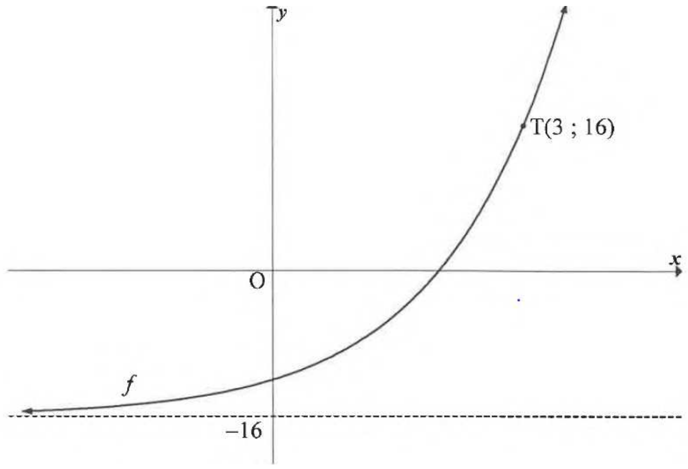
QUESTION 13 ***DBE/2021***

13.1 Given : $g(x) = 3^x$

13.1.1 Write down the equation g^{-1} in the form $y = ...$ (2)

13.1.2 Point P(6;11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to from g. Write down the coordinate of the image of P on g. (2)

13.2 Sketched is the graph of $f(x)=2^{x+p}+q$. T(3;16) is a point on f and the asymptote of f is $y=-16$.



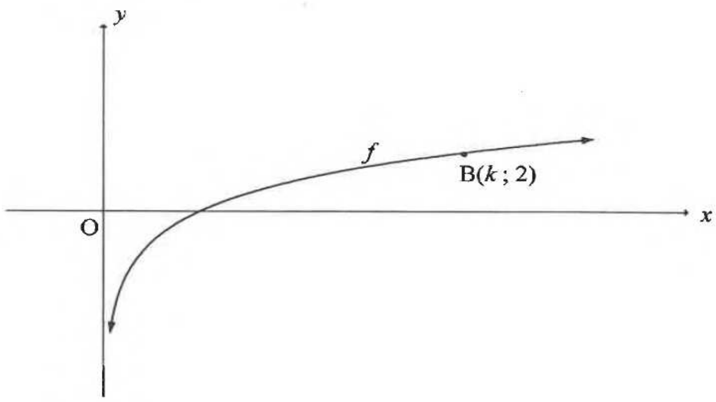
Determine the values of p and q (4)

[8]

QUESTION 14 **DBE/November 2021**

The graph of $f(x)=\log_4 x$ is drawn below.

B$(k;2)$ is a point on f.



14.1 Calculate the value of k. (2)

14.2 Determine the values of x for which $-1\leq f(x)\leq 2$. (2)

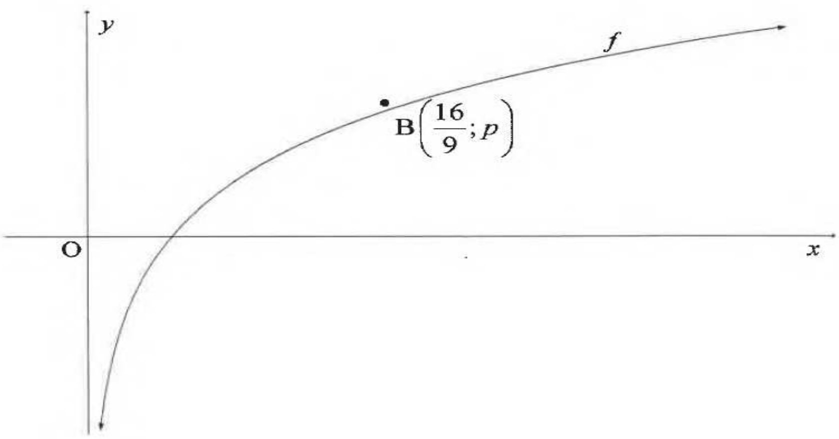
14.3 Write down the equation of f^{-1}, the inverse of f, in the form $y=...$ (2)

14.4 For which values of x will $x.f(x)<0$? (2)

[8]

QUESTION 15 ***DBE/2018***

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f.



15.1 For which value(s) of x is $\log_{\frac{4}{3}} x < 0$? (2)

15.2 Determine the value of p ,without using a caluclator. (3)

15.3 Write down the equation of the inverse of f in the form $y = ...$ (2)

15.4 Write down the range of $y = f^{-1}(x)$ (2)

15.5 The function $h(x) = \left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f. Write down the coordinates of B'', the image of B an h. (2)

[11]

QUESTION 16 ***DBE/2019***

16.1 Given: $f(x) = \frac{1}{x+2} + 3$

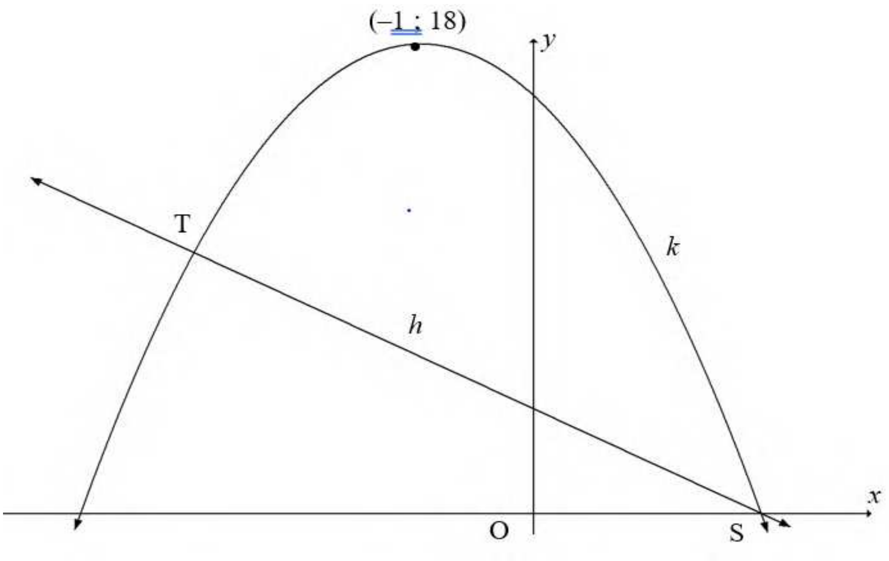
16.1.1 Determine the equation of the asymptotes of f. (2)

16.1.2 Write down the y =intercept of f. (1)

16.1.3 Calculate the x -intercept of f. (2)

16.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes (3)

16.2 Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$.
Graph k has a turning point at (-1;18). S is the x -intercept of h and k.
Graphs h and k also intersect at T.



16.2.1 Calculate the coordinates of S. (2)

16.2.2 Determine the equation of k in the form $y = a(x + p)^2 + q$ (3)

16.2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T. (5)

16.2.4 Determine the value(s) of x for which $k(x) < h(x)$. (2)

16.2.5 It is further given that k is the graph of $g'(x)$. (2)

(a) For which values of x will the graph of g be concave up? (2)

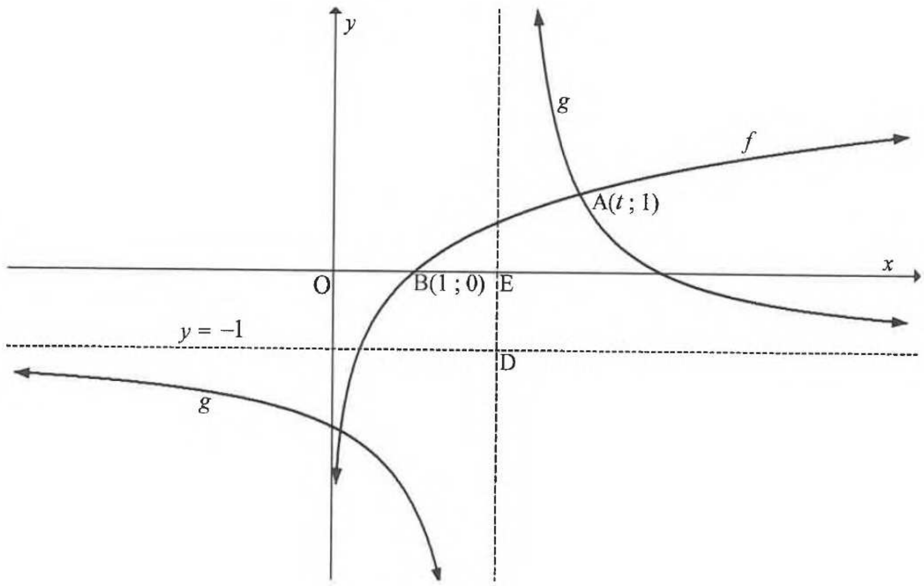
(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3)

[25]

QUESTION 17 **DBE/November 2017**

The diagram below shows the graph of $g(x) = \frac{2}{x + p} + q$ and $f(x) = \log_3 x$.

- $y = -1$ is the horizontal asymptote at g.
- $B(1;0)$ is the x-intercept of f.
- $A(t;1)$ is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB=BE.



17.1 Write down the range of g. (2)

17.2 Determine the equation of g. (2)

17.3 Calculate the value of t. (3)

17.4 Write down the equation of f^{-1}, the inverse of f, in the form $y = ...$ (2)

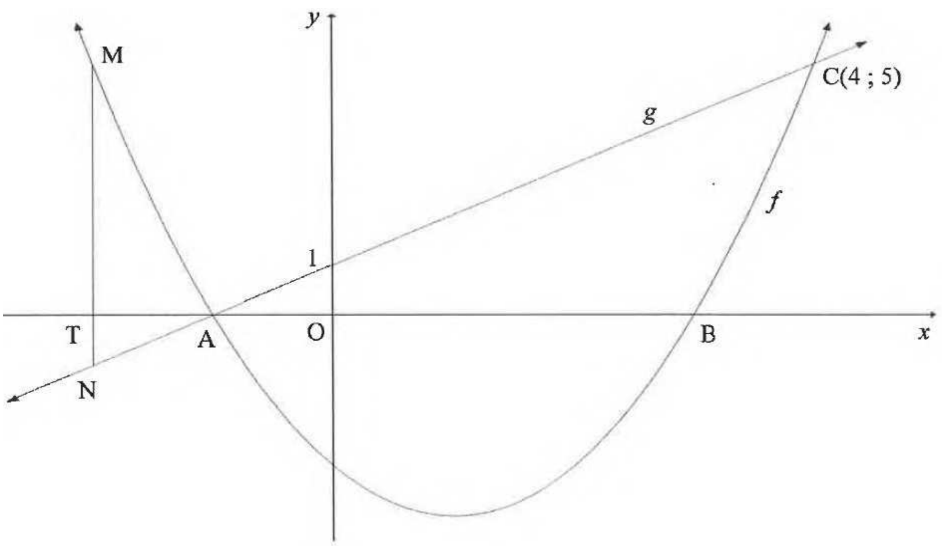
17.5 For which values of x will $f^{-1}(x) < 3$? (2)

17.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (2) (3)

[14]

QUESTION 18 **DBE/November 2018**

In the diagram below, A and B are the x-intercepts of the graph of $f(x) = x^2 - 2x - 3$. A straight line, g, through A cuts f at $C(4;5)$ and the y-axis at $(0;1)$. M is a point on f and N is a point on g such that MN is parallel to the y-axis. MN cuts the x-axis at T.



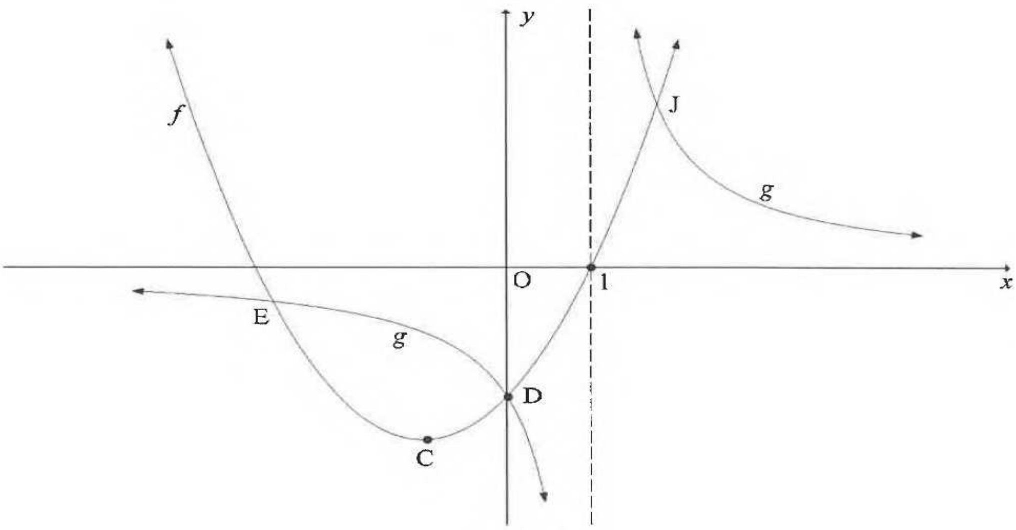
18.1	Show that $g(x) = x + 1$.	(2)
18.2	Calculate the coordinates of A and B.	(3)
18.3	Determine the range of f.	(3)
18.4	If MN=6:	
	18.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working.	(4)
	18.4.2 Hence, write down the coordinates of N.	(2)
18.5	Determine the equation of the tangent to f drawn parallel to g.	(5)
18.6	For which value(s) of k will $f(x) = x^2 - 2x - 3$ and $h(x) = x + k$ NOT intersect?	(1)

[20]

QUESTION 19 **DBE/ November 2019**

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at (1;0).
- D is the y-intercept of both f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



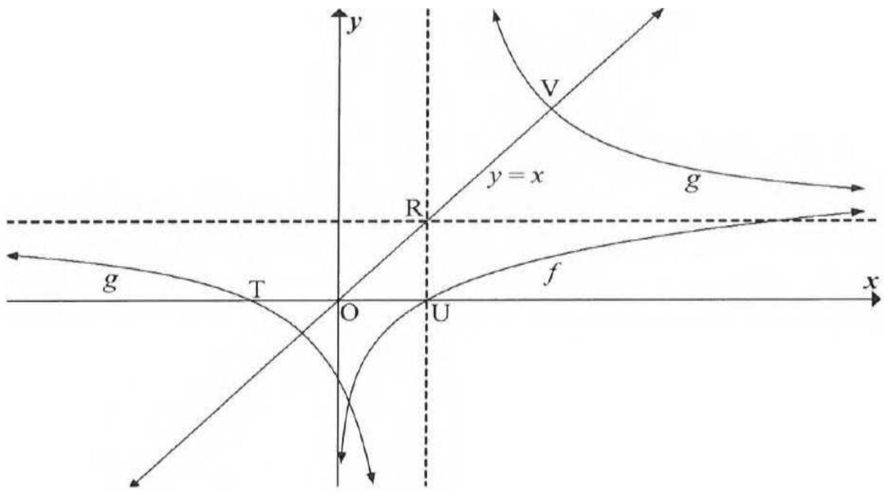
19.1	Write down the value of p.	(1)
19.2	Show that $a = 3$ and $b = 2$.	(3)
19.3	Calculate the coordinate of C.	(4)
19.4	Write down the range of f.	(2)
19.5	Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = ...$	(3)
19.6	Is the straight line determine in QUESTION 19.5, a tangent to f? Explain your answer.	(2)
19.7	The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q.	(4)

[19]

QUESTION 20 **DBE/Feb-March 2017**

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



20.1 Write down the coordinates of U. (1)

20.2 Write down the equation of the asymptotes of g. (2)

20.3 Determine the coordinates of T. (2)

20.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = ...$ (2)

20.5 Write down the equation of the asymptotes of $h(x-3)$. (1)

20.6 Calculate the coordinates of V. (4)

20.7 Determine the coordinates of T' the point which is symmetrical to T about the point R. (2)

[14]

QUESTION 21 ***DBE/2017***

Given: $f(x) = x^2 - 5x - 14$ and $g(x) = 2x - 14$

21.1 On the same set , sketch the graph of f and g .Clearly indicatie all intercepts with the axes and turning points. (6)

21.2 Determine the equation of the tangent to f at $x = 2\frac{1}{2}$. (2)

21.3 Determine the values of k for which $f(x) = k$ will have two unequal positive real roots. (2)

21.4 A new graph h is obtained by first reflecting g in the x-axis and the translating it 7 units to the left. Write down the equation of h in the form $h(x) = mx + c$. (2)

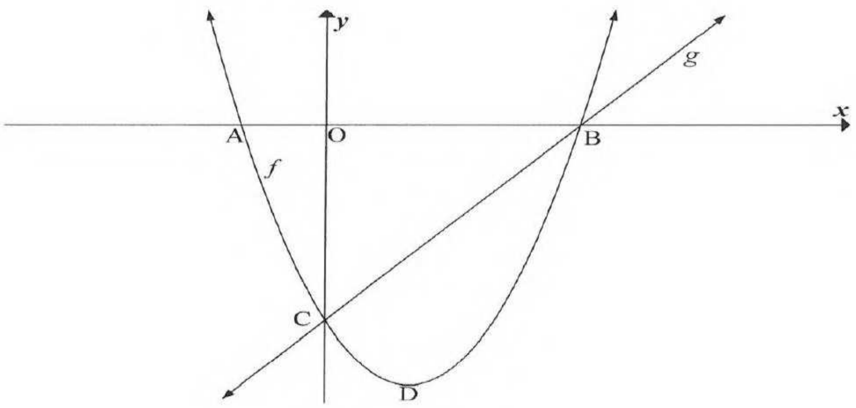
[12]

QUESTION 22 **DBE/Feb.-Mar. 2017**

22.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



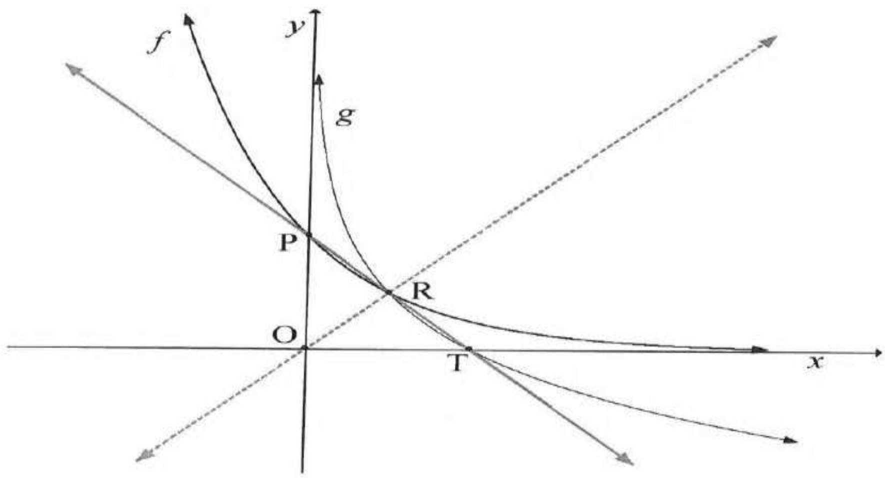
22.1.1	Determine the coordinates of C.	(1)
22.1.2	Calculate the lengthof AB.	(4)
22.1.3	Determine the coordinates of D.	(2)
22.1.4	Calculate the average gradient of f between C and D.	(2)
22.1.5	Calculate the size of $O\hat{C}B$.	(2)
22.1.6	Determine the values of k for which $f(x) = k$ will have two unequal positive real roots.	(3)
22.1.7	For which values of x will $f'(x).f''(x) > 0$?	(3)

22.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

QUESTION 23 ***DBE/*2017**

In the sketch below, P is the y-intercpt of the graph of $f(x) = b^x$. T is the x-intercept of graph g, the inverse of f. R is the point of intersection of f and g. Straight lines are drawn through O and R and through P and T.



23.1 Determine the equation of g (in terms of b) in the form $y = ...$ (2)

23.2 Write down the equation of the line passing through O and R. (1)

23.3 Write down the coordinates of point P. (1)

23.4 Determine the equation of the line passing through P and T. (2)

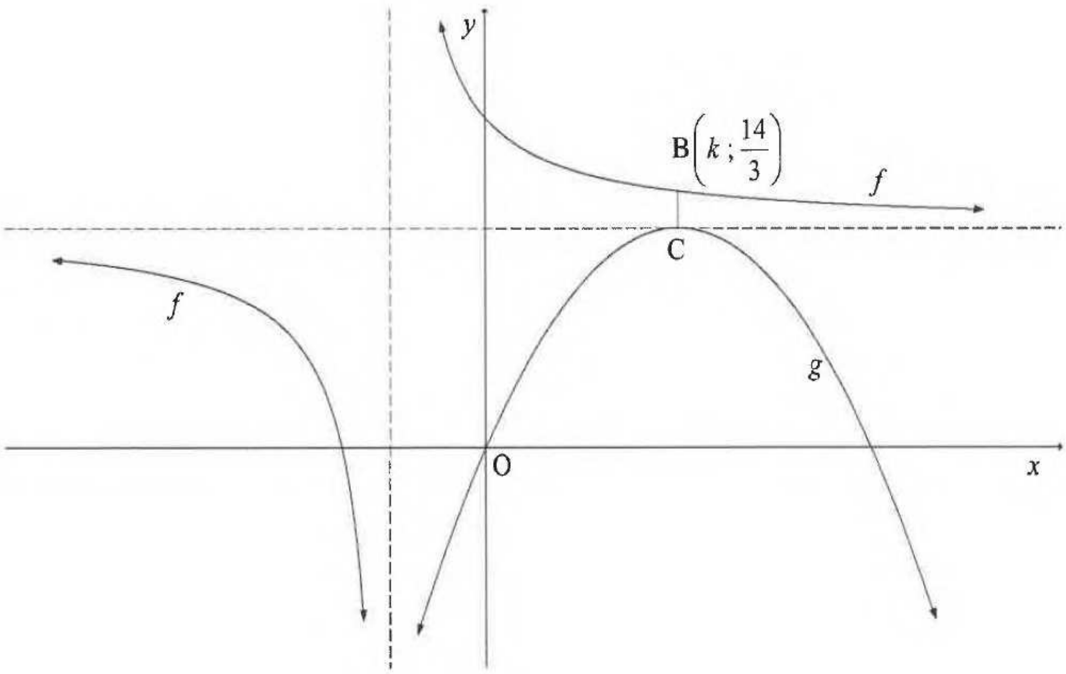
23.5 Calculate the value of b. (5)

[11]

QUESTION 24 ***DBE*/2018**

The graphs of $f(x) = \frac{2}{x+1} + 4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptotes of f.
- The graph of g passes through the origin.
- $B\left(k; \frac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.



24.1 Write down the domain of f. (2)

24.2 Determine the x-intercept of f. (2)

24.3 Calculate the value of k. (3)

24.4 Write down the coordinates of C. (2)

24.5 Determine the equation of g in the form $y = a(x+p)^2 + q$. (3)

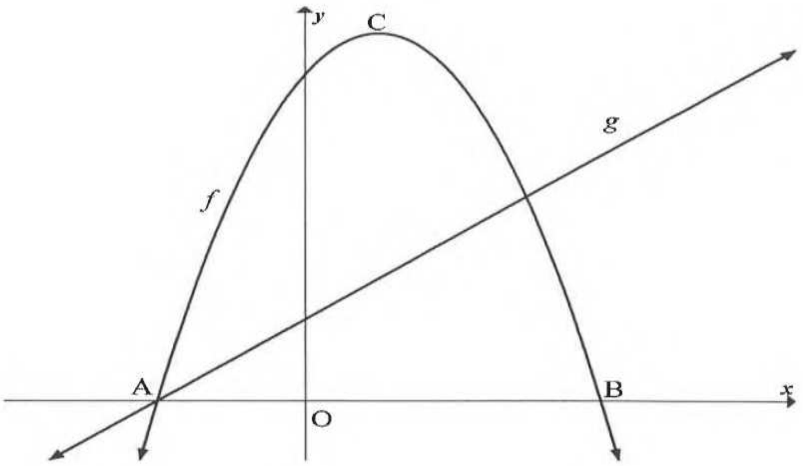
24.6 For which value(s) of x will $\frac{f(x)}{g(x)} \leq 0$? (4)

24.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x} - 5 = -(x-3)^2 - 5$. Give reasons for your answer. (4)

[20]

QUESTION 25 *DBE/2021*

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$. A and B are the x-intercepts of f .C is the point on f .



25.1	Calculate the coordinates of A and B.	(3)
25.2	Determine the coordinates of C the turning point of f .	(2)
25.3	Write down the range of f .	(1)
25.4	The graph of $h(x) = f(x+p)+q$ has a maximum value of 15 at $x = 2$. Determine the values of p and q .	(3)
25.5	Determine the eqauation of g^{-1}, the inverse of g, in the form $y = ...$	(2)
25.6	For which value(s) of x will $g^{-1}(x).g(x) = 0$?	(2)
25.7	If $p(x) = f(x) + k$, determine the values of k for which p and g will not intersect.	(5)

[18]

QUESTION 26 **DBE/ November 2020**

26.1 Given: $h(x) = \dfrac{-3}{x-1} + 2$

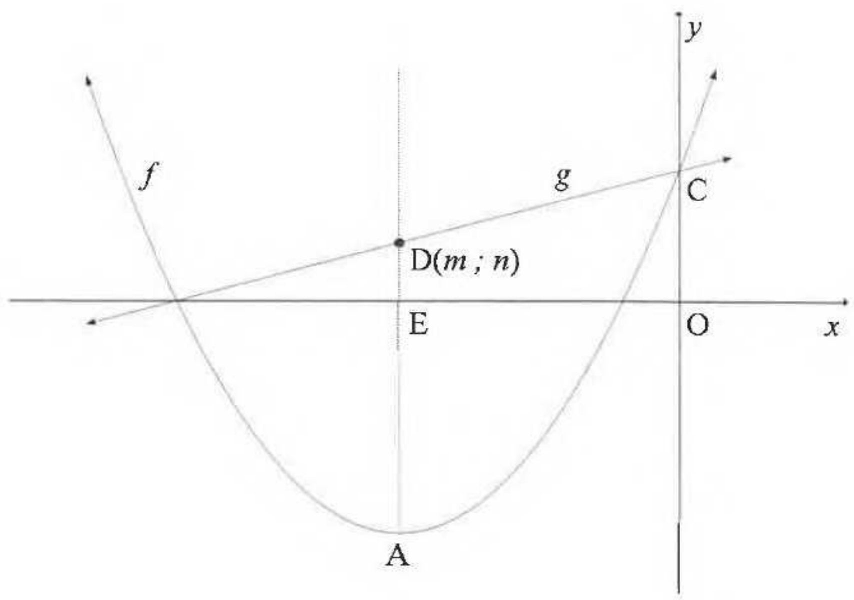
26.1.1 Write down the equation of the asymptotes of h . (2)

26.1.2 Determine the equation of the axis of symmetry of h that has a negative gradient (2)

26.1.3 Sketch the graph of h , showing the asymptotes and the intercepts with the axes. (4)

26.2 The graphs of $h(x) = \dfrac{1}{2}(x+5)^2 - 8$ and $g(x) = \dfrac{1}{2}x + \dfrac{9}{2}$ are sketched below, (2)

- A is the turning point of f .
- The axis of symmetry of f intersects the x-axis at E and line g at $D(m;n)$.
- C is the y-intercept of f and g .



26.2.1	Write down the coordinates of A.	(2)
26.2.2	Write down the range of f .	(1)
26.2.3	Calculate the values of m and n .	(3)
26.2.4	Calculate the area of OCDE.	(3)
26.2.5	Determine the equation of g^{-1}, the inverse of g, in the form $y = ...$	(2)
26.2.6	If $h(x) = g^{-1}(x) + k$ is a tangent to f, determine the coordinates of the point of contact between h and f	(4)

[23]