KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12
DIRECTORATE

NCS (CAPS)

LEARNER SUPPORT DOCUMENT

GRADE 10

MATHEMATICS
STEP AHEAD PROGRAMME

BOOK 2

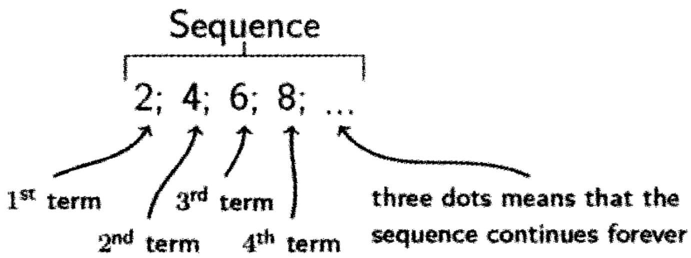
2022

This document has been compiled by the KZN FET Mathematics Subject Advisors.

TOPIC: NUMBER PATTERNS					
LESSON 1					
Term	2	**Week**		**Grade**	10
Duration	2 hours	**Weighting**	15±3	**Date**	
Sub-topics	Investigate number patterns leading to those where there is a constant difference, and the general term is therefore linear.				
RESOURCES					
Mind Action Series Grade10 Answer Series Grade 10					
NOTES					

INTRODUCTION:

A sequence is an ordered list of items, usually numbers. Each item which makes up a sequence is a called a term.



Number pattern is a pattern in which a series / list of numbers follows a certain sequence. This pattern generally establishes a common relationship between all numbers.

Number patterns could be ascending, descending, multiples of a certain number, or series of even numbers, odd numbers, etc.

Learners should now do Activity 1.1.1

LINEAR NUMBER PATTERNS:

Linear pattern is a sequence of numbers in which there is a common difference (a) between any term and the term before.

The common difference is the difference between any term and the term before it.
Consider the number pattern 3;5;7;9;11; …

The pattern is formed by adding 2 to each new term. We say that the constant difference between the terms is 2
NOTE: A number pattern with a constant difference is called a linear number pattern.
To solve the problem of number patterns, the general term /rule being followed in the pattern must be understood.

To determine the general term of linear number patterns, find the difference between consecutive terms/numbers.

Example 5

Given the sequence 2; 9; 16; 23; …, determine the general term T_n

General term : $T_n = an + c$

Common difference : a

$a = 9 - 2 = 7$

$a = 16 - 9 = 7$

$a = 23 - 16 = 7$

$T_n = 7n + c$

Substitute T_1 (term1) :

$2 = 7(1) + c$

$c = -5$

$T_n = 7n - 5$

ACTIVITIES/ASSESSMENTS

ACTIVITY 1.1.1: DAY 1

1. Consider the following sequences :
   1.1. 1; 3; 5; 7; …
   1.2. 2; 5; 8; 11; …
   1.3. 2;4; 8; …
   1.4. 1; 8; 27; 64; …
   1.5. 2 ; 9; 28; 65; …
   1.6. 1 ; 1 ; 2 ; 3 ; 5 ;…

In each case,

a) Write down the next three terms.

Explain in words what you notice, i.e. make a rule.

ACTIVITY 1.1.2 DAY 2

2. For each of the following sequences , determine the general term (nth term) and hence calculate the 25th term

a) 6; 9; 12; 15; …

b) 3; 8; 13; 18; …

c) 5; 0; -5; -10; …

d) 10; 16; 22; 28; …

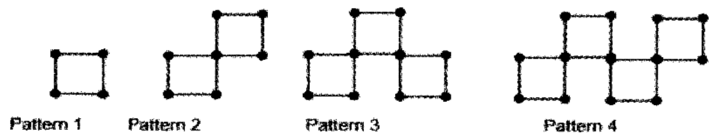
e) – 6; – 11; – 16; …

f) – 8; – 3; 2; …

g) 30; 27; 24; …

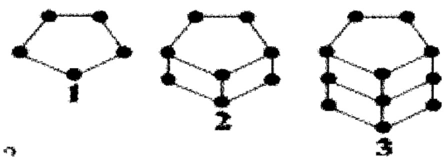
h) 1; 10; 19; …

TOPIC : NUMBER PATTERNS					
LESSON 2					
Term	2	**Week**		**Grade**	10
Duration	I hour	**Weighting**	15±3	**Date**	
Sub-topics	Investigate number patterns leading to those where there is a constant difference and the general term is therefore linear. Linear Number Pattern : Shapes				
RESOURCES					
Mind Action Series Grade10 Answer Series Grade 10					



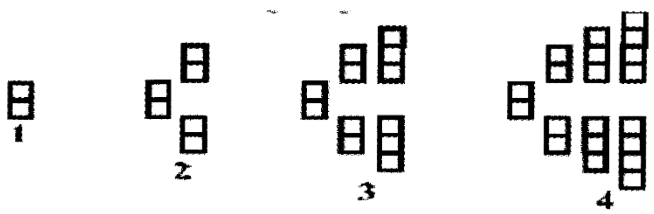
a) Write down the sequence
b) Determine the common difference
c) Determine the general term of the pattern.
d) How many sticks will be in the 50^{th} pattern?

2. Consider the diagram made up of black dots joined by thin black lines.



a) How many dots are there in figure 4?
b) How many lines are there in figure 4?
c) How many dots are there in figure 8?
d) How many lines are there in figure 8?
e) Determine the general rule to find the number of dots in the nth figure.
f) How many dots are there in the 186^{th} figure?
g) Which figure will contain 272 dots?
h) Determine the general rule to find the number of lines in the nth figure
i) How many lines are there in the 900^{th} figure?
j) Which figure will contain 650 lines?

3. Consider the following designs:



a) Write down the number of squares in design 1; 2; 3; 4; and 5.
b) Determine the number of squares in design n .
c) How many squares are there in design 20?

4. Consider the following sequence of Es:

$\therefore T_n = 6n+c$

Substitute (1;4)

$4 = 6(1) + c$

$4 = 6 + c$

$-2 = c$

$\therefore T_n = 6n - 2$

c) Calculate $T_{73} - T_{37}$ (L2)

$T_{73} = 6(73) - 2 = 436$

$T_{37} = 6(37) - 2 = 220$

$\therefore T_{73} - T_{37} = 436 - 220 = 216$

d) Prove that all the terms of this sequence are POSITIVE, EVEN integers (L4)

$T_n = 6n - 2$

$= 2(3n - 1)$

$T_n = 6n - 2$

$= 2(3n - 1)$

$T_5 = a + 4d$

b) State the common difference (L2)

$d = T_3 - T_2$

$= (a+2d) - (a+d)$

$= a + 2d - a - d$

$= d$

c) Determine the sum of the first 5 terms (L3)

$\text{Sum} = T_1 + T_2 + T_3 + T_4 + T_5$

$= a + a+d + a+2d + a+3d + a+4d$

$= 5a + 10d$

ACTIVITIES/ASSESSMENTS

Activity 1.3.1

1. The height of water in a tank is recorded whilst the tank is being filled. The results have been recorded at five minute intervals:

	First reading	**After 5 min**	**After 10 min**	**After 15 min**	**After 20 min**
Level in *cm*	3	11	19	27	35

1.1. What will be the height of the water after 25 minutes?

1.2. What will be the height after an hour?

1.3. At what rate is the level rising? Give your answer in $cm / minute$.

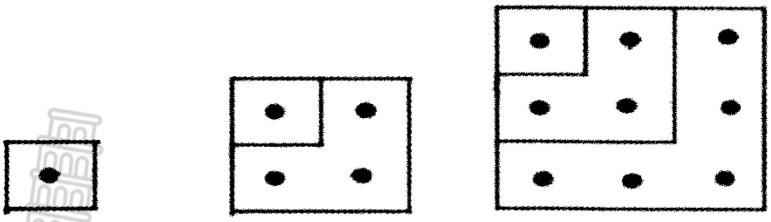
1.4 What will be the water level after $5n$ minutes?

1.5. After how many minutes will the water level be 403 cm?

Given the linear pattern: $2x+1$; $3x+3$; $4x+5$;

2.1. Write down the next term in the pattern

Samantha is investigating a pattern of dots represented in the diagram below :



Pattern number : 1   2   3

Number of dots : $1^2 = 1$ $1+3 = 2^2 = 4$ $1+3+5 = 3^2 = 9$

6.1 Write down :

6.1.1 The number of dots in the 4^{th} pattern.

6.1.2 The number of dots in the 13^{th} pattern.

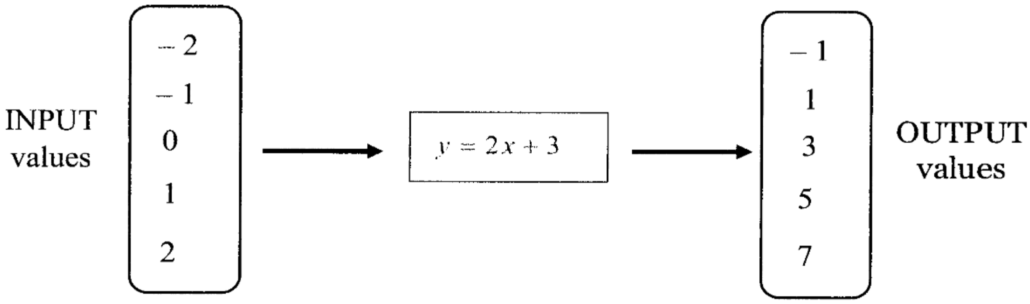
6.1.3 A formula for the number of dots in the n^{th} pattern.

6.2 Hence , or otherwise , calculate the value of : $1+3+5+\ldots+43$

III. Each form of an equation has a graph with a distinct shape.
(As they proceed with the lessons, learners will discover which shape goes with which type of equation.)

- **Functions**:
    - A graph is a set of points $(x \,;\, y)$ and indicates a relationship between the two variables (x and y).
    - A function is a graph, or a set of points, where every x-value has only one y-value.
    - Consider the equation $y = 2x + 3$. If we PUT a value of x IN, e.g. 5, we get a value of y OUT: y will be 13.
      So: the x-values are the INPUT values and the y-values are the OUTPUT values.

- **Different ways of representing functions**:
    - A mapping:



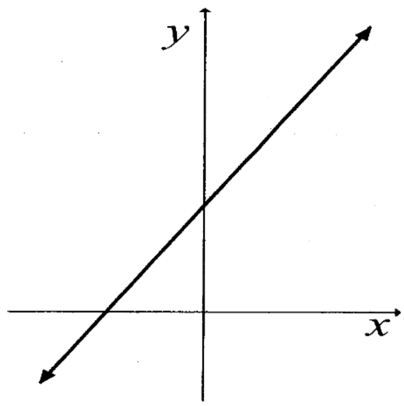
The set of all the input values is called the **domain**.

The set of all the output values is called the **range**.

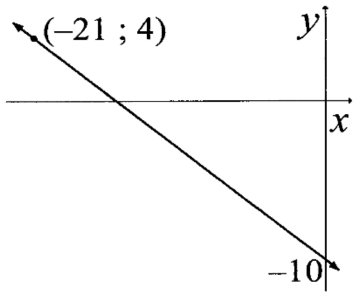
- A table of values:

x	-2	-1	0	1	2
$y = 2x + 3$	-1	1	3	5	7

- Ordered pairs:
  $(-2\,;\,-1)$, $(-1\,;\,1)$, $(0\,;\,3)$, $(1\,;\,5)$, $(2\,;\,7)$
- A graph:



- Example 2:
  Determine the equation of the line in the sketch in the form $y = mx + c$:

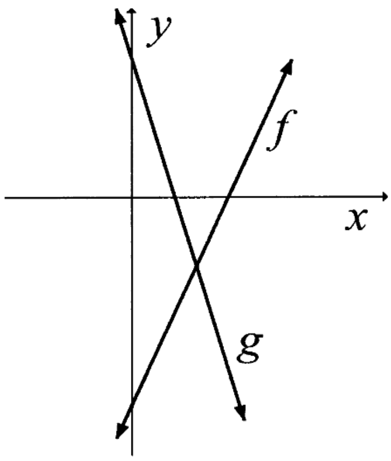


  Solution:
  The y-intercept is -10. Therefore $c = -10$.
  $\therefore y = mx - 10$
  Substitute the coordinates of the point $(-21\,;4)$, into the equation:
  $$4 = m(-21) + 10$$
  $$21m = -14$$
  $$m = -\frac{2}{3}$$
  Therefore the equation is $y = -\frac{2}{3}x - 10$.
- Learners should now do Activity 3.

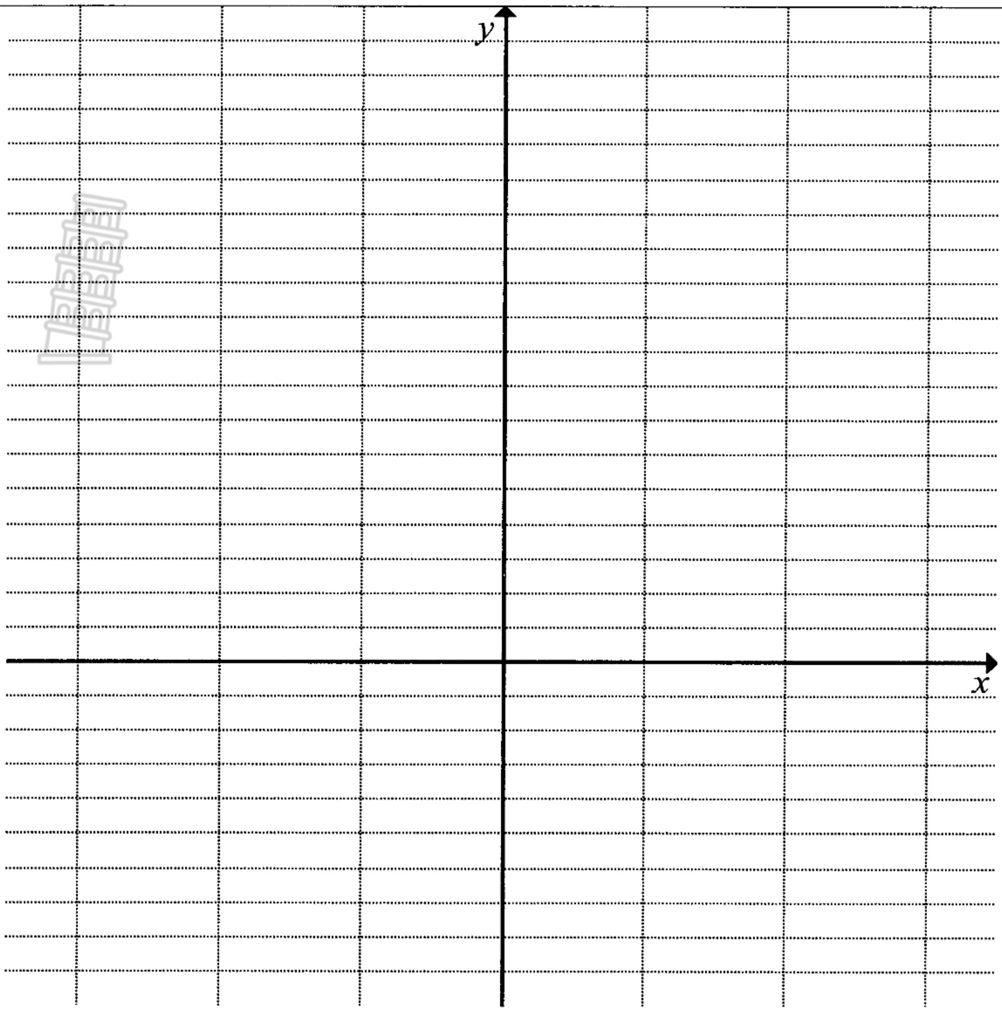
Determining the point of intersection of two lines algebraically:

- If a graph showing two lines has been drawn accurately, (estimated) coordinates for the point of intersection can be read off.
- However, if the equations of the two graphs are known, the precise coordinates of the point of intersection can be calculated.
- Any graph is a set of points $(x\ ;\ y)$ all of which satisfy the equation of the graph. Two different straight lines have different equations and therefore different points that satisfy their equations – except for one point $(x\ ;\ y)$, namely the point on intersection that satisfies both equations. The coordinates of that point is the solution to solving the two equations simultaneously.

- **Example**:
  Determine algebraically the point of intersection of the graphs of $f(x) = 2x - 6$ and $g(x) = -3x + 4$.



  Solution:
  Solve simultaneously:
  $y = 2x - 6$ and $y = -3x + 4$.
  By equating:
  $$2x - 6 = -3x + 4$$
  $$5x = 10$$
  $$x = 2$$
  Substitute into $y = 2x - 6$:
  $$y = 2(2) - 6$$
  $$y = -2$$
  Point of intersection: $(2\ ;\ -2)$.
  Take note:
  Any other suitable method could also have been used to solve the two equations simultaneously, e.g. substitution or elimination.



1.2 How does changing the value of a influence the graph of the parabola $y = ax^2 + q$?

....

....

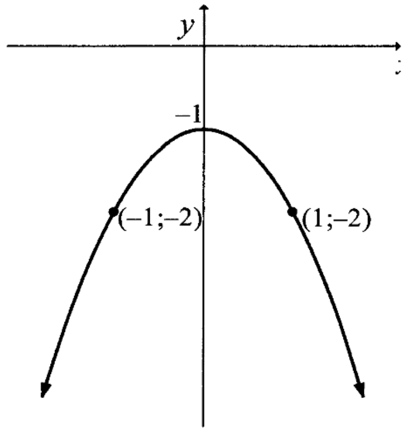
2. The quadratic functions $y = x^2$, $y = x^2 + 1$ and $y = x^2 - 2$.

2.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.
Also: compare the equation for each of the graphs to the general equation $y = ax^2 + q$ and then complete the last two columns of the table.

x	-3	-2	-1	0	1	2	3
$y = x^2$							
$y = 2x^2$							
$y = \frac{1}{2}x^2$							

a	q

Example 2:
Draw a sketch graph of $y = -x^2 - 1$.
Solution:
Because $a > 0$ we know it is a "sad face" parabola.
Because $q = -1$, we know the y-intercept, and the maximum value is -1.
The graph is therefore not cutting the x-axis.
To draw the graph more precisely, one can substitute two x-values, e.g. -1 and 1, and so obtain two more points to plot, namely $(-1\,;-2)$ and $(1\,;-2)$.



Learners should now do Activity 2.

Activity 2:
Draw sketch graphs of the following functions:

1. $y = -x^2 + 9$
2. $y = x^2 + 3$
3. $y = \frac{1}{3}x^2$
4. $y = -4x^2$
5. $f(x) = -\frac{1}{2}x^2 + 8$
6. $g(x) = 3x^2 - 6$

TOPIC: FUNCTIONS
Weighting: (30/100 marks from Paper 1)

LESSON 4: THE PARABOLA (PART 2)

Term	2	**Duration**	1 hour	**Grade**	10	**Date**	
Sub-topics	Determining the equations of parabolas with general equation $y = ax^2 + q$.						

RESOURCES
Gr. 10 textbooks :
Siyavula; Platinum, Survival Series, Classroom Maths and Mind Action Series
The Answer Series 3 in 1 Study Guide for Gr. 10

NOTES

How to determine the equation of a parabola:

- **Example 1**:Determine the equation of the parabola shown in the sketch alongside.
  Solution:

Substitute $q = 4$ into $y = ax^2 + q$

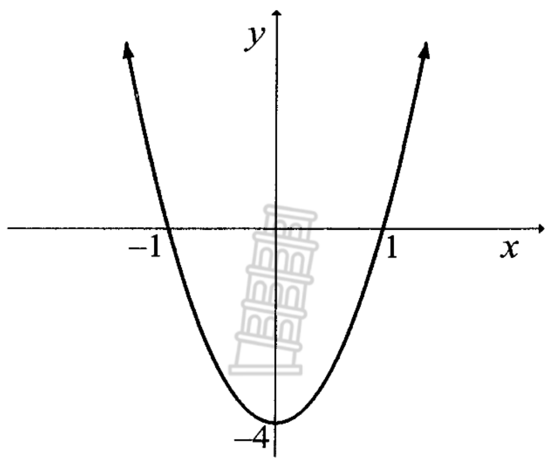
$y = ax^2 - 4$

Substitute any of the two x- intercepts into the equation

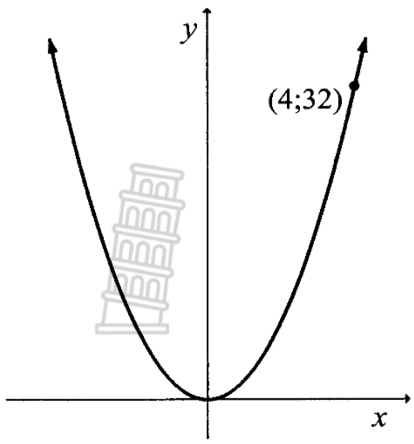
E.g. $(1\,;0)$: $0 = a(1)^2 - 4$

$a = 4$

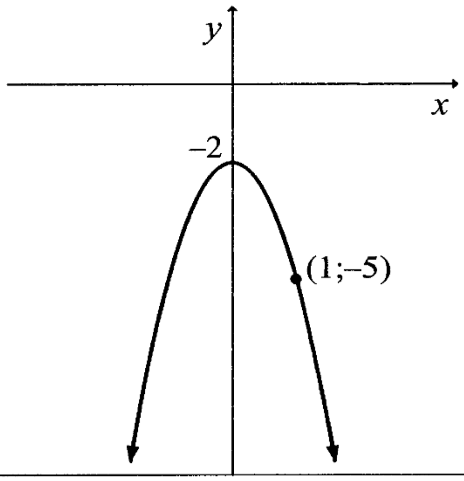
Therefore the equation of the parabola is $y = 4x^2 - 4$



3.



4.



TOPIC: FUNCTIONS Weighting: (30/100 marks from Paper 1)							
LESSON 5: THE HYPERBOLA (PART 1)							
Term	2	**Duration**	2 hours	**Grade**	10	**Date**	

RESOURCES

Textbooks: Mind Action Series; The Answer Series and Previous question papers.

NOTES

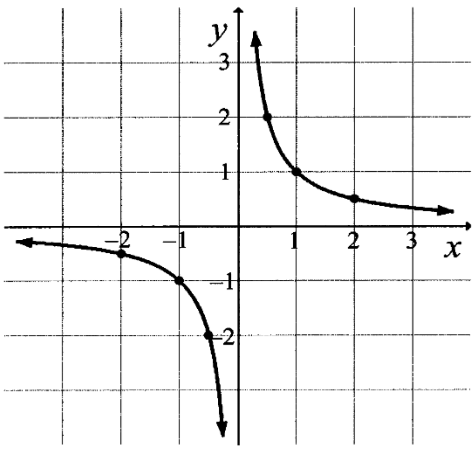
Hyperbola : $y=\frac{a}{x}+q$

Effect of a when $q=0$ ($y=\frac{a}{x}$)

1. For the graph of $y=\frac{1}{x}$
2. Using the table method

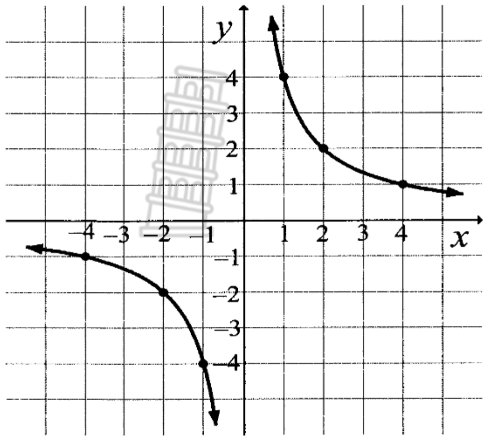
x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	$\frac{1}{0}$	2	1	$\frac{1}{2}$

3. Note: if $x=0$ then $y=\frac{1}{0}$ which is undefined, i.e. there is no y-intercept and the graph is not touching the y axes.
4. If $y=0$ and solve for x: $0=\frac{1}{x}$ $\Rightarrow$ there is no value of x, hence no x-intercept.

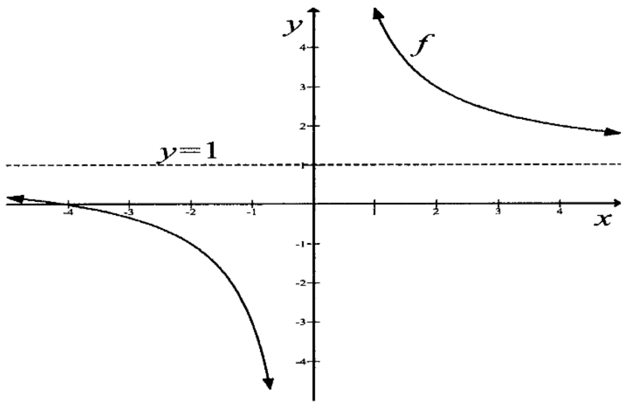


Effect of q

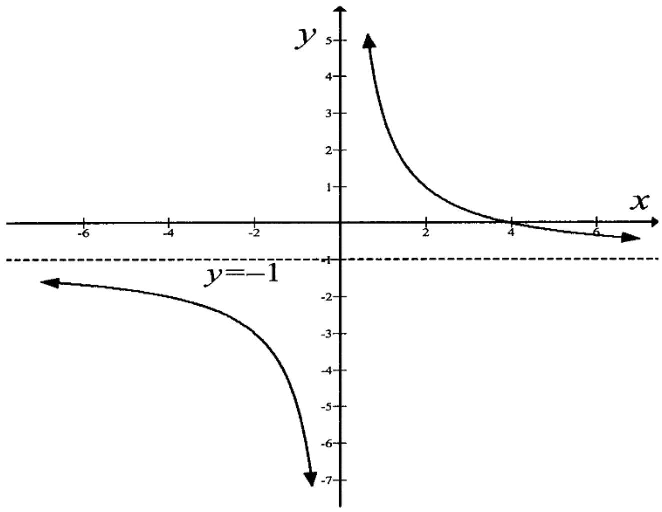
1. $y = \frac{4}{x}$ $q = 0$



2. $y = \frac{4}{x} + 1$ q > 0



3. $y = \frac{4}{x} - 1$ $q < 0$



LESSON 5: ACTIVITIES/ASSESSMENTS

Classwork

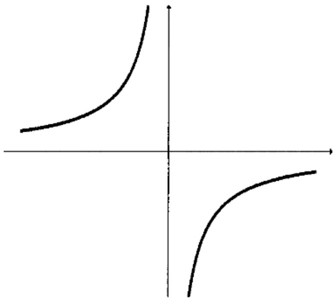
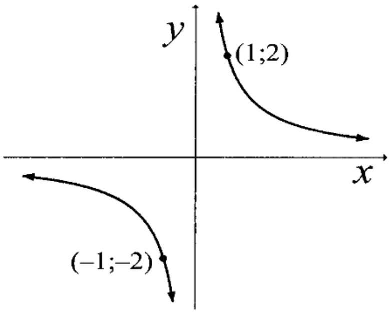
Draw the following graphs on the same set of axis using table method:

1. ***f***: $y = \frac{6}{x}$ 2. ***g***: $y = -\frac{10}{x}$ 3. ***h***: $y = \frac{12}{x}$ 4. ***k***: $y = \frac{16}{x}$

Homework

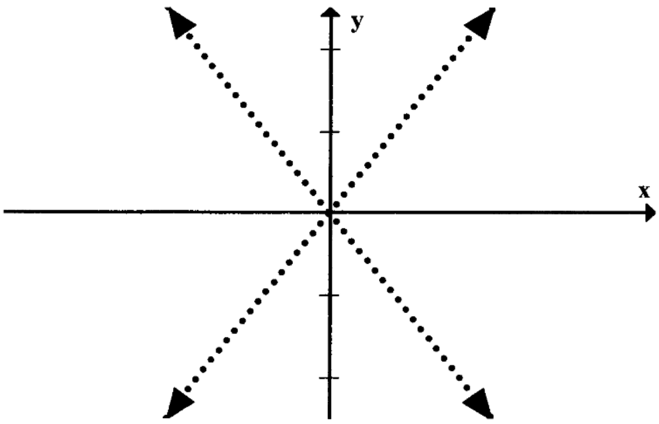
Draw the following graphs on the same set of axis using table method:

1 . $f(x) = \frac{8}{x}$ 2. $g(x) = -\frac{8}{x}$ 3. $h(x) = \frac{8}{x} + 1$ 4. $k(x) = \frac{8}{x} - 1$

Example 3:	$f(x) = -\frac{2}{x}$			
Step 1 :	Asymptotes $x = 0$ and $y = 0$			
Step 2 :	No x-intercept and no y-intercept			
Step 3 :	Shape $a > 0$ 			
Step 4 :	Drawing the graph 			

Lines of symmetry

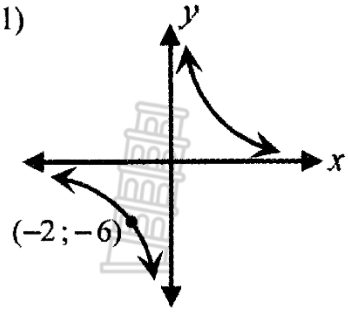
Lines of axes of symmetry are lines $y = x$ and $y = -x$



Asymptotes

An asymptote is a line, which the graph approaches/tends towards but not touches or cuts. Each hyperbola has a vertical and horizontal asymptote.

(b) Determine the equation of each of the following hyperbolas in the form $f(x)=\frac{a}{x}+q$.

(1)


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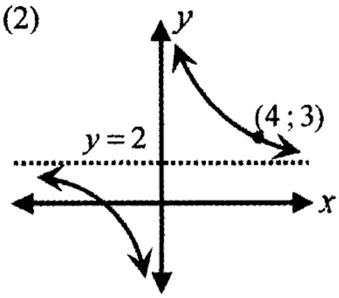

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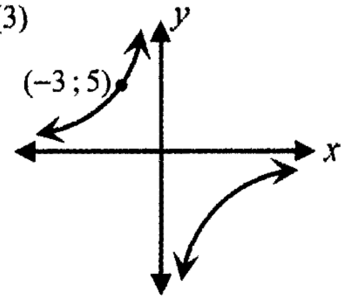

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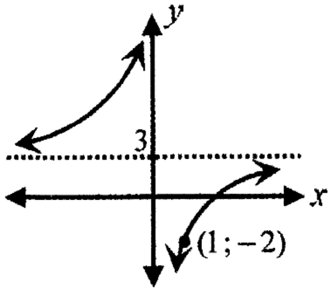

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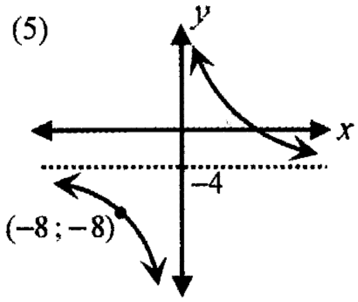

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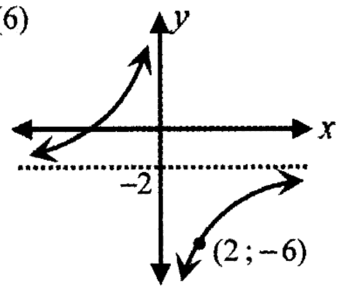

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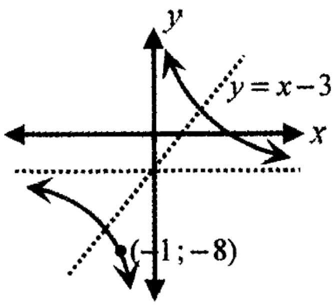

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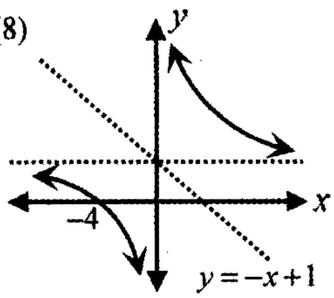

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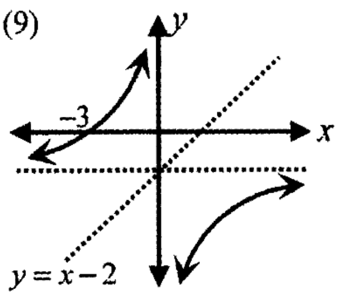

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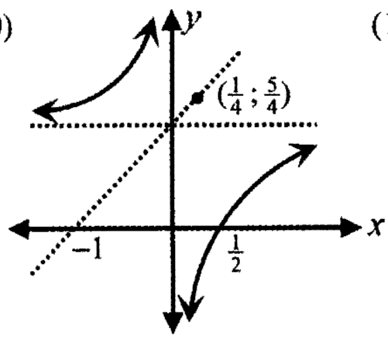

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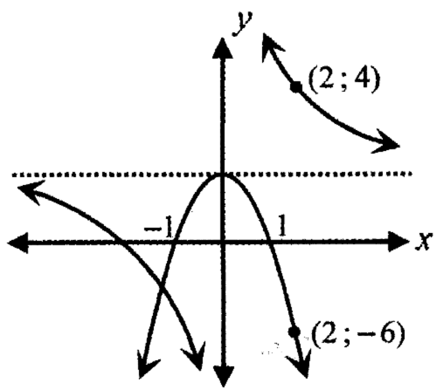
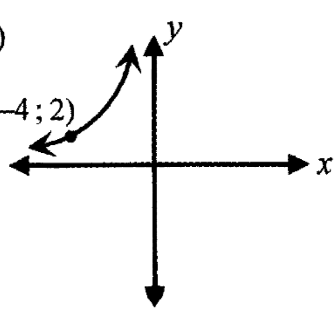


(c) For each function below, state the domain and range and determine the equation:

(1)


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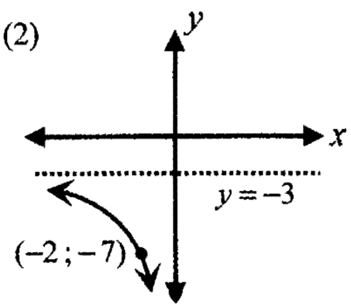

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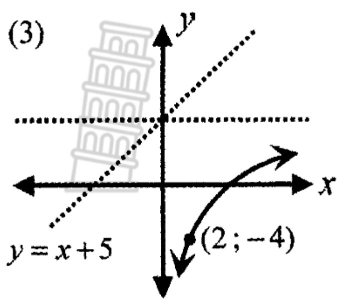


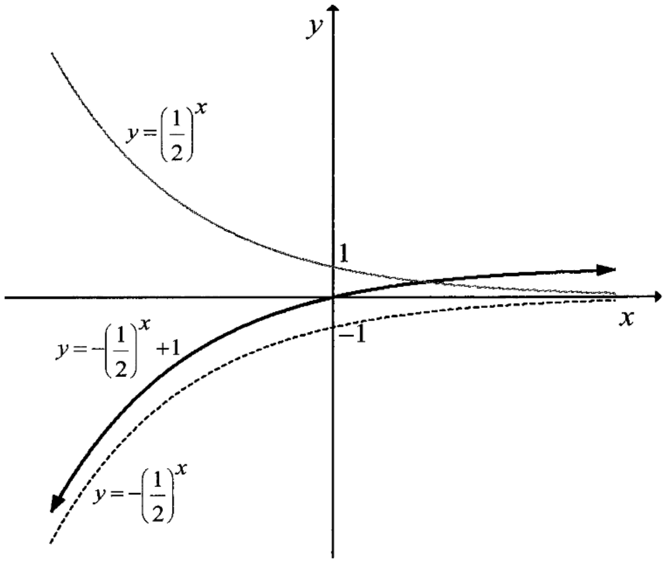
- The graph of $y = 2.3^x$ will be the same as $y = 3^x$, except with a vertical stretch by a factor of 2, causing e.g. the y-intercept now to be 2.
- Finally: the graph of $y = 2.3^x + 3$ will be the same as $y = 2.3^x$, just translated (shifted) up by 3 units.
- Take note:
  It is not necessary to draw all three graphs; only the final one. However, it is a very good habit and method to do it in this way.

Example 2:

Draw a sketch graph of $y = -\left(\frac{1}{2}\right)^x + 1$.

Solution:

- The "mother graph" in this case is $y = 2^x$, which is increasing, and has a y-intercept of 1.
- $y = \left(\frac{1}{2}\right)^x$ or $y = 2^{-x}$ is a reflection of $y = 2^x$ in the y-axis.
- $y = -\left(\frac{1}{2}\right)^x$ is a reflection of $y = \left(\frac{1}{2}\right)^x$ in the y-axis.
- $y = -\left(\frac{1}{2}\right)^x + 1$ is $y = -\left(\frac{1}{2}\right)^x$ translated up by one unit.



- Learners should now do Activity 2.

LESSON 7: ACTIVITIES/ASSESSMENTS

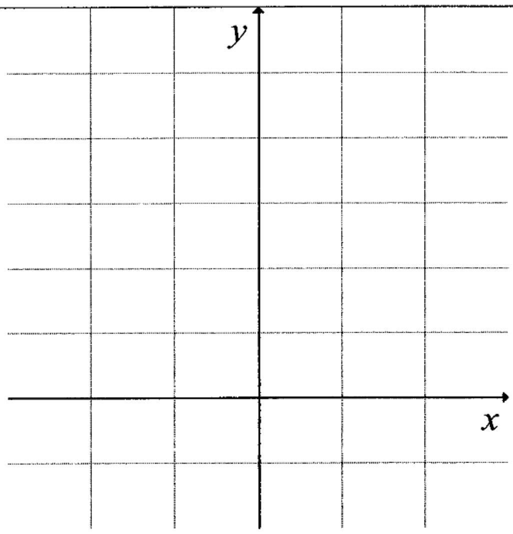
Activity 1 Worksheet on Sketching Exponential graphs of the form $y = a.b^x + q$.

1. Drawing the graphs of $y = 2^x$, $y = 3^x$ and $y = \left(\frac{1}{2}\right)^x$.

1.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation of each of the graphs to the general equation $y = a.b^x + q$ and then complete the last three columns of the table.

x	-2	-1	0	1	2		a	b	q
$y = 2^x$									
$y = 3^x$									
$y = \left(\frac{1}{2}\right)^x$									



2.2 How does changing the value of q influence the exponential graph $y = a.b^x + q$?

....

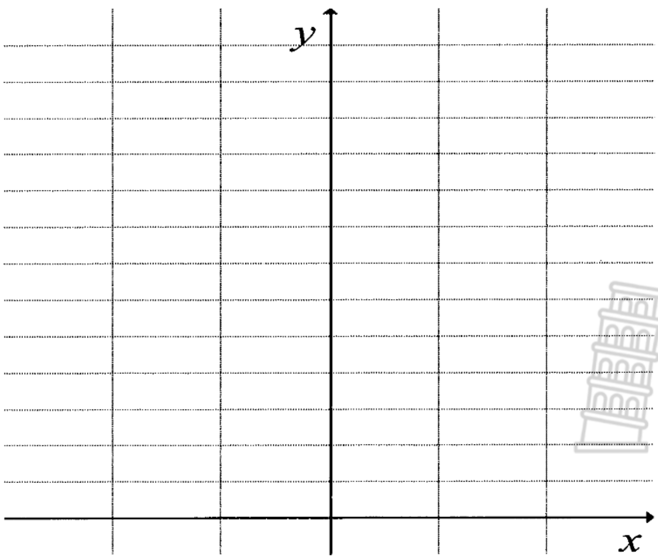
....

3. Drawing the graphs of $y = 2^x$, $y = 3.2^x$ and $y = -2^x$.

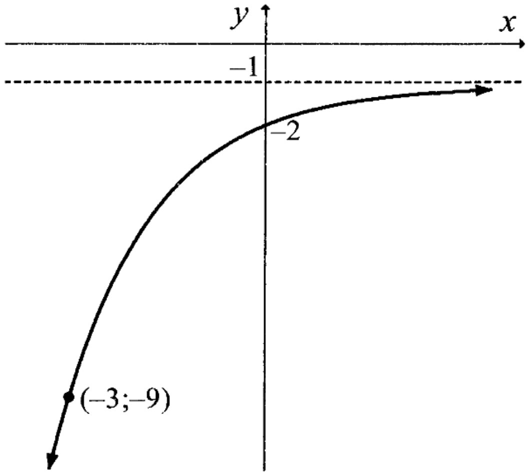
3.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation of each of the graphs to the general equation $y = a.b^x + q$ and then complete the last three columns of the table.

x	-2	-1	0	1	2		a	b	q
$y = 2^x$									
$y = 3.2^x$									
$y = -2^x$									



Example 2:
Determine the equation of the exponential graph shown in the sketch alongside.



Solution:

$q = -1$, because the asymptote is the line $y = -1$.

$\therefore y = a.b^x - 1$.

Substitute $(0\ ;\ -2)$ into $y = a.b^x - 1$:

$-2 = a.b^0 - 1$

$a = -1$

Substitute $(-3\ ;\ -9)$ into $y = -b^x - 1$:

$-9 = -b^3 - 1$

$b = 2$

Therefore the equation of the exponential graph is $y = -2^{-x} - 1$.

- Learners can now do Activities 1 and 2.

LESSON 8: ACTIVITIES/ASSESSMENTS

Activity 1:

In each case:

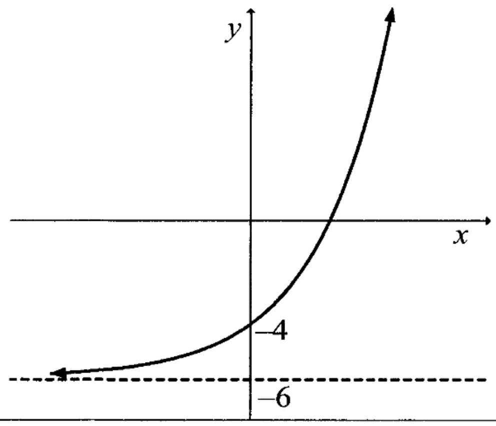
(a) Determine the equation of the exponential graph.

(b) Write down the domain of the graph.

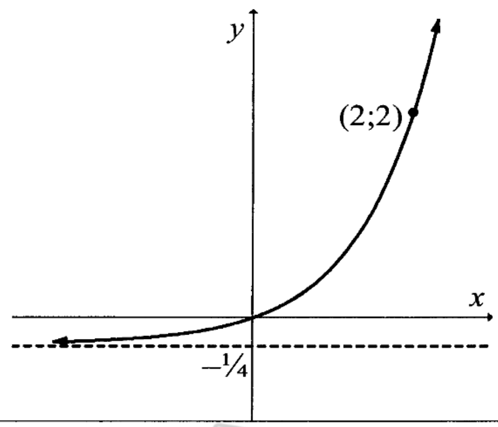
(c) Write down the range of the graph.

(d) Write down the equation of the asymptote to the graph.

1. $f(x) = a.3^x - 6$



2. $f(x) = a.b^x + q$



FUNCTIONS – MIXED QUESTIONS – GRADE 10

QUESTION 1

Given: $f(x)=\frac{3}{x}+1$ and $g(x)=-2x-4$

1.1 Sketch the graphs of f and g on the same set of axes. (4)

1.2 Write down the equations of the asymptotes of f. (2)

1.3 Write down the domain of f. (2)

1.4 Solve for x if $f(x)=g(x)$. (5)

1.5 Determine the values of x for which $-1\leq g(x)<3$. (3)

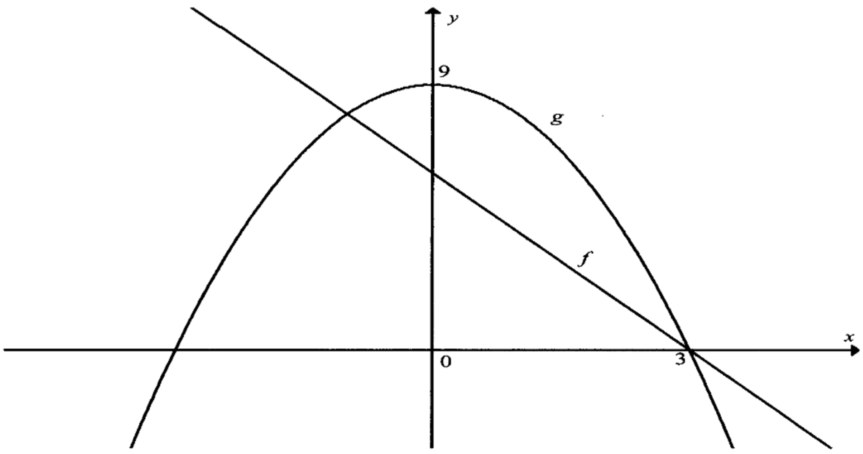
1.6 Determine the y-intercept of k if $k(x) = 2g(x)$. (2)

1.7 Write down the coordinates of the x- and y-intercepts of h if h is the graph of g reflected about the y-axis. (2)

[20]

QUESTION 2

Sketched below are the graphs of $f(x)=-2x+6$ and $g(x)=ax^2+q$.



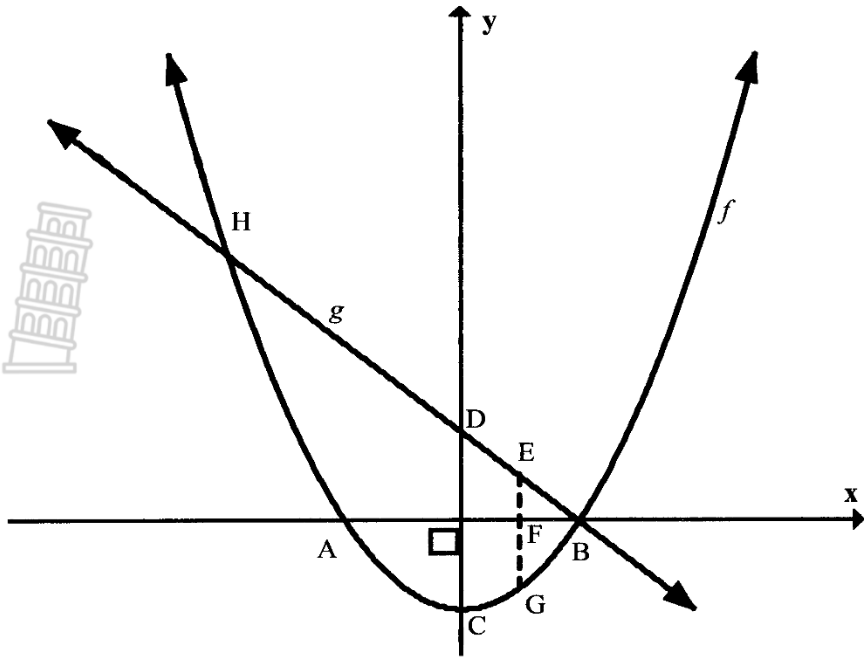
2.1 Determine the values of a and q. (3)

2.2 Calculate the values of x for which $f(x) = g(x)$. (5)

2.3 Hence or otherwise, write down the values of x for which $g(x) > f(x)$. (2)

2.4 Write down the coordinates of the turning point of h if $h(x)=g(x)-4$. (2)

[12]



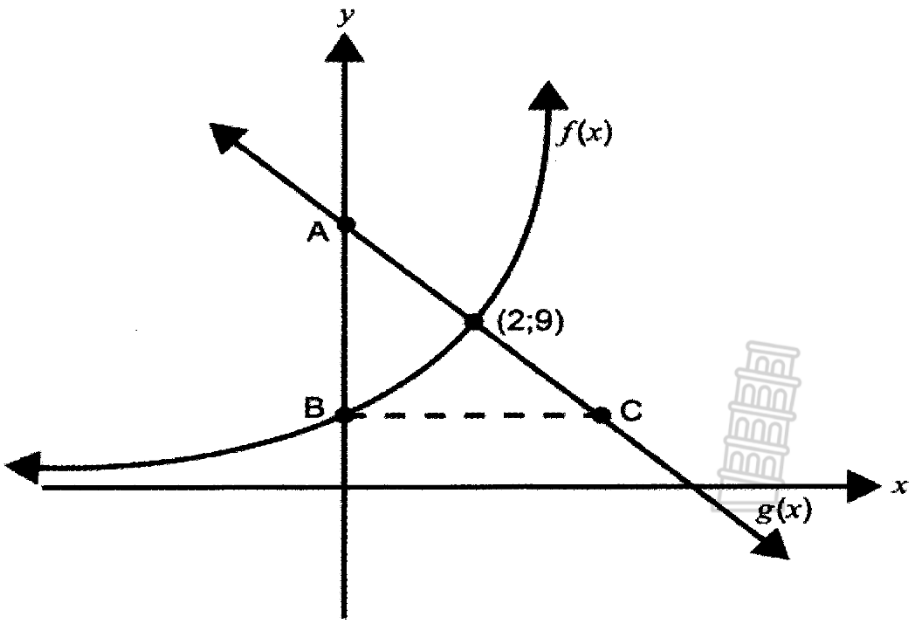
Use the graphs and the information above to determine the following:

4.1	The coordinates of A and B.	(4)
4.2	The coordinates of C.	(1)
4.3	The coordinates of D.	(1)
4.4	The length of EG if OF = $\frac{1}{2}$ unit and E lies on g and G lies on f.	(5)

[11]

QUESTION 5

Below are the graphs of $f(x) = a^x$, $a > 0$ and $g(x) = -x + 11$. (2; 9) is the point of intersection of $f(x)$ and $g(x)$.



6.2 Determine the values of x for which $f(x) = g(x)$. (4)

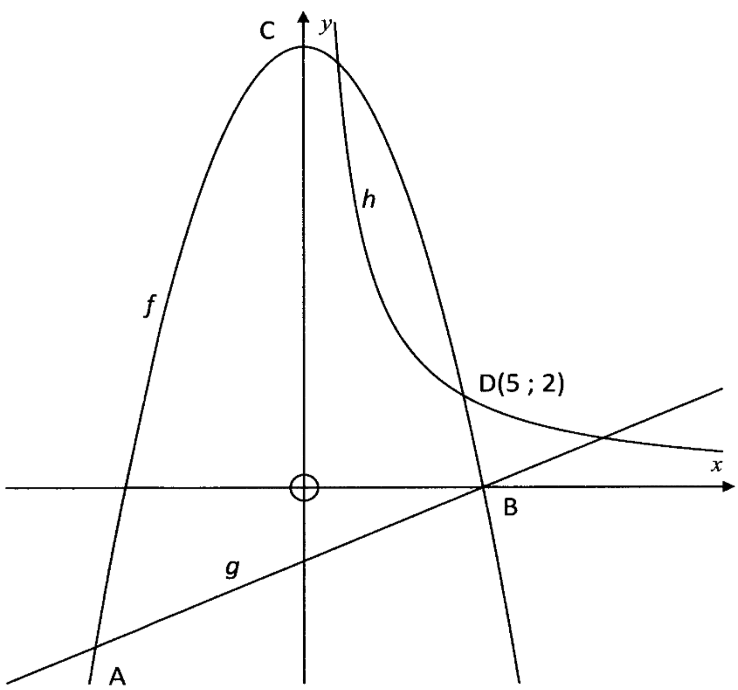
6.3 For what values of x is $g(x) \geq f(x)$? (3)

6.4 Calculate the length of BE. (3)

6.5 Write down an equation of h if $h(x) = f(x) + 3$. (1)
[13]

QUESTION 7

Sketched below are the graphs of $f(x) = -2x^2 + 18$, $g(x) = mx + c$ and $h(x) = \frac{k}{x}$ where $x > 0$. The graph of f intersects the x–axis at A and B and the y–axis at C, which is also the turning point of f. The graph of g also intersects the x–axis at B. One of the points of intersection of g and h is D(5 ; 2).



Use the graphs and the information given above to answer the following :

7.1 The coordinates of A and B. (4)

7.2 The length of OC. (1)

7.3 The values of m and c. (3)

7.4 The equation of h. (2)

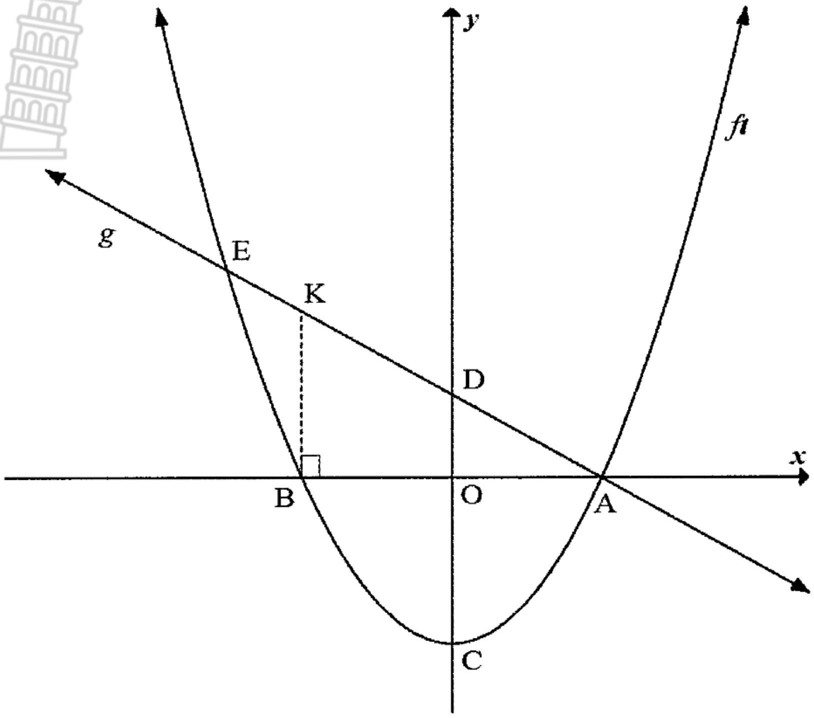
7.5 The range of f. (2)

7.6 The equation of the straight line through B perpendicular to g. (3)

[15]

QUESTION 10

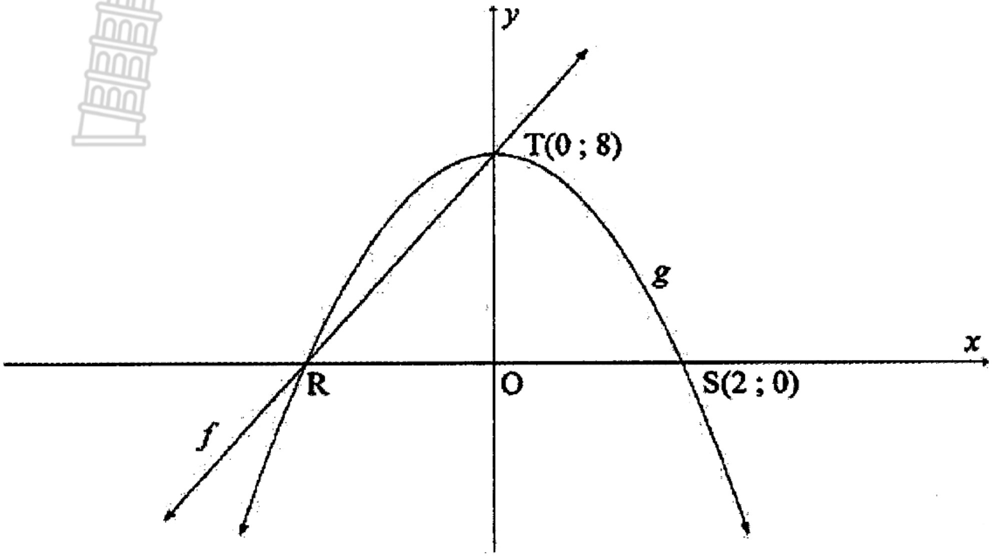
The graphs of $f(x)=x^2-4$ and $g(x)=-x+2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK || x-axis. f and g intersect at A and E.



10.1	Write down the coordinates of C.	(1)
10.2	Write down the coordinates of D.	(1)
10.3	Determine the length of CD.	(1)
10.4	Calculate the coordinates of B.	(3)
10.5	Determine the coordinates of E, a point of intersection of f and g.	(4)
10.6	For which values of x will:	
	10.6.1 $f(x)<g(x)$	(2)
	10.6.2 $f(x).g(x)\geq 0$	(2)
10.7	Calculate the length of AK.	(4)
		[18]

QUESTION 12

The diagram shows the graphs of $g(x) = ax^2 + q$ and $f(x) = mx + c$.
R and S(2 ; 0) are the x-intercepts of g and T(0 ; 8) is the y-intercept of g.
Graph f passes through R and T.

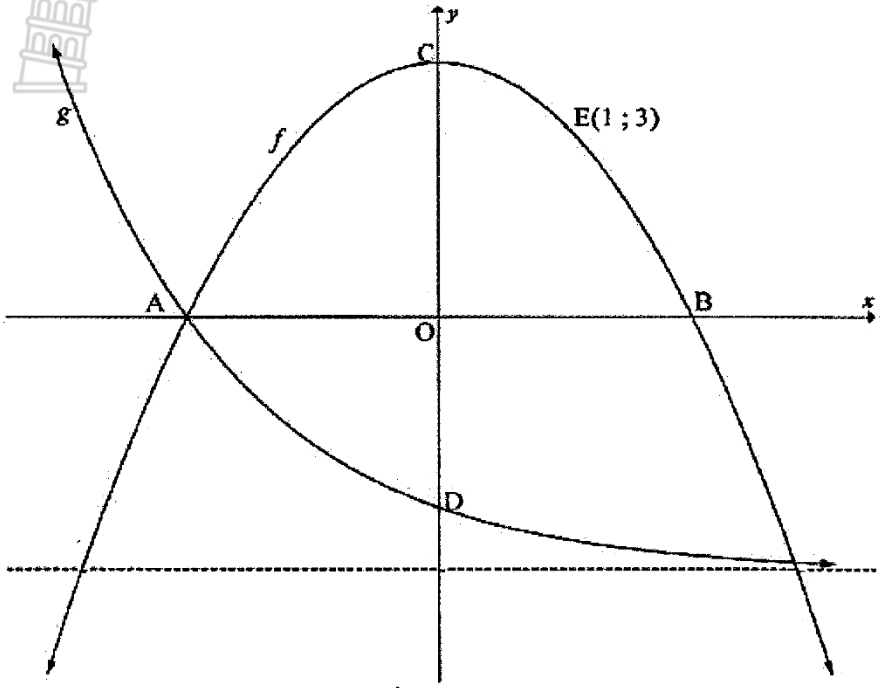


12.1	Write down the range of g.	(1)
12.2	Write down the x-coordinate of R.	(1)
12.3	Calculate the values of a and q.	(3)
12.4	Determine the equation of f.	(3)
12.5	Use the graphs to determine the value(s) of x for which:	
	12.5.1 $f(x) = g(x)$	(2)
	12.5.2 $x \cdot g(x) \leq 0$	(3)
12.6	The graph h is obtained when g is reflected along the line $y = 0$. Write down the equation of h in the form $h(x) = px^2 + k$.	(2)
		[15]

QUESTION 14

Sketched below are the graphs of $f(x) = ax^2 + q$ and $g(x) = \left(\frac{1}{2}\right)^x - 4$.

A and B are the x-intercepts of f. The graphs intersect at A and point E (1 ; 3) lies on f. C is the turning point of f and D is the y-intercept of g.



14.1	Write down the:		
	14.1.1	Coordinates of D	(2)
	14.1.2	Range of g	(1)
14.2	Calculate the:		
	14.2.1	Coordinates of A	(2)
	14.2.2	Values of a and q	(4)
14.3	Determine the:		
	14.3.1	Length of CD	(2)
	14.3.2	Equation of a straight line through A and D	(3)
14.4	For which values of x is:		
	14.4.1	$f(x) > 0$?	(2)
	14.4.2	f decreasing?	(1)
			[17]

TOPIC: MEASUREMENT Weighting: (8/100 marks from Paper 2)							
LESSON 1: SURFACE AREA AND VOLUME OF PRISMS							
Term	2	**Duration**	2 hours	**Grade**	10	**Date**	
Sub-topic	Grade 9. Revision Surface area and Volume of the cylinder and prism						

RESOURCES

Mind Action Series Gr 10

Sasol Inzalo Gr 9

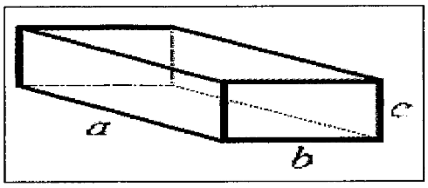
Past papers Gr 10

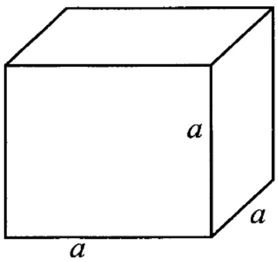
Other material

NOTES

- A **prism** is a three- dimensional shape with two congruent parallel polygons faces at opposite ends. The faces are referred to as the bases (or ends) of the prism.
- The **lateral surface area** of a prism is the sum of the area of the lateral faces.
- **Volume** is defined as the amount of space occupied by any three-dimensional solid
- The **surface area** of a prism is the sum of the prism's lateral area and the areas of the two bases.

Below are formulas for total surface area and volume

PRISM	**SURFACE AREA**	**VOLUME**
Cuboid Rectangular Prism 	Sum of the areas of the six rectangles: Surface Area $= 2ab + 2ac + 2bc$	Area of the base multiplied by the distance moved by the base Volume $= (ab) \times c$ $= abc$

PRISM	**SURFACE AREA**	**VOLUME**
Cube 	Sum of the areas of the six sides: Surface Area $= 2(a)(a) + 2(a)(a) + 2(a)(a)$ Surface Area $= 6a^2$	Area of the base multiplied by the distance moved by the base Volume $= a \times a \times a$ $= a^3$

EXAMPLE 2

1. Calculate the volume of a cylinder with a radius of 3 and a height of 12

solution :

$volume = \pi r^2 h$

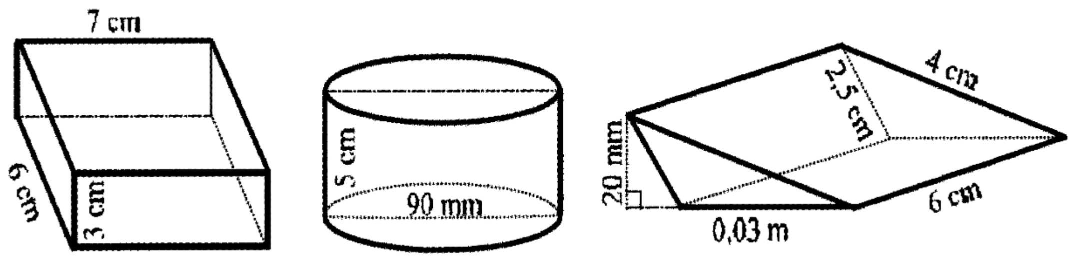
$= \pi(3)^2(12)$

$= \pi(9)(12)$

$= 108\pi$

ACTIVITIES/ASSESSMENTS

1. Consider the following three closed hollow prisms:



(a) Calculate the volume of each of these prisms.

(b) Calculate the surface area of each of these prisms.

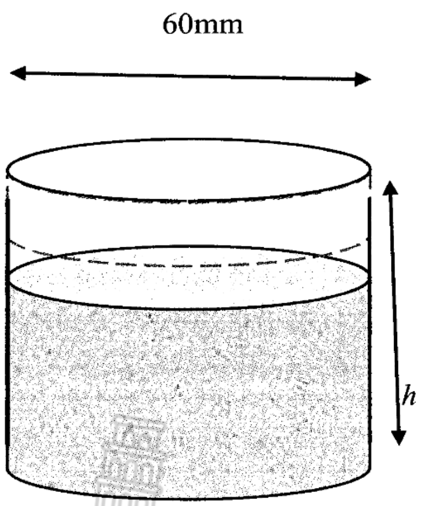
(c) If the cylinder and cuboid area open on top, calculate the surface area of these prisms.

2. The diagram below shows a cup with a volume of $117\pi cm^3$ and an inner diameter of 60 mm. ignore the thickness of the cup.

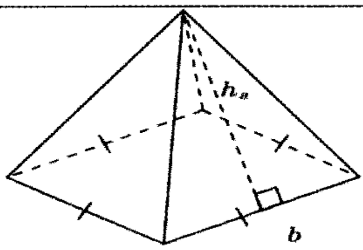
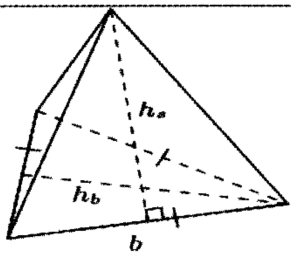
Calculate

(a)The height of the cup

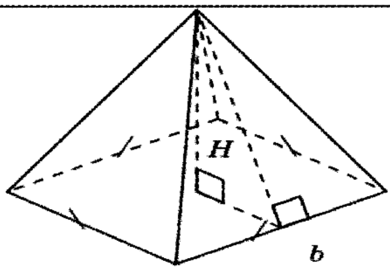
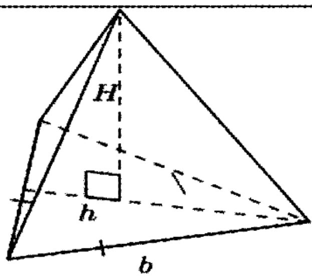
(b)Total surface areas touched by the water if it touches the cup if it is the cup is 80% full of water.



SURFACE AREA

Square pyramid		Surface area $= $ area of base + area of triangular sides $= b^2 + 4\left(\frac{1}{2}bh_s\right)$ $= b\left(b + 2h_s\right)$
Triangular pyramid		Surface area $=$ area of base + area of triangular sides $= \left(\frac{1}{2}b \times h_b\right) + 3\left(\frac{1}{2}b \times h_s\right)$ $= \frac{1}{2}b\left(h_b + 3h_s\right)$

VOLUME

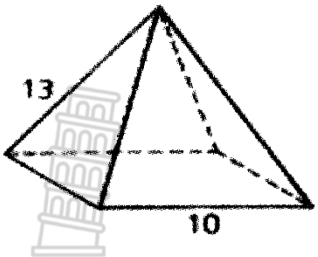
Square pyramid		Volume $= \frac{1}{3} \times$ area of base $\times$ height of pyramid $= \frac{1}{3} \times b^2 \times H$
Triangular pyramid		Volume $= \frac{1}{3} \times$ area of base $\times$ height of pyramid $= \frac{1}{3} \times \frac{1}{2}bh \times H$

Example.

The Louvre Pyramid located in Paris is a large right square pyramid made of metal and glass. It serves as the main entrance to the Louvre Museum, which houses the famous paintings such as the Mona Lisa. The length of one side of the base is 35,4 m and the heights 21,6 m. the base of the pyramid is open

ACTIVITIES/ASSESSMENTS

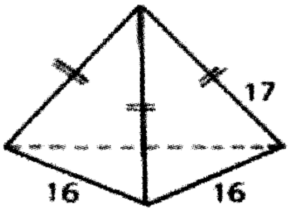
1. The pyramid shown is regular and has a square base.



(a) Calculate the pyramid's total surface area.

(b) Calculate the volume of the pyramid

2. The pyramid shown is a regular base and has a triangular base

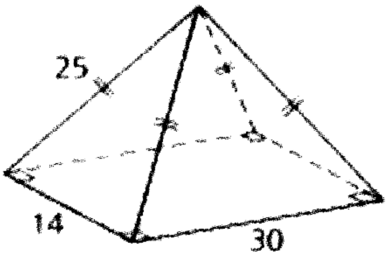


Determine:

(a) The total surface area

(b) The volume of the pyramid.

3. The pyramid shown has a rectangular base, its lateral edges are congruent

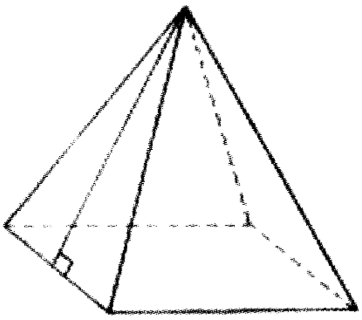


(a) Why is this pyramid not regular?

(b) What is the lateral face area

(c) Calulate the total surface area.

4. A regular pyramid has a slant height of 12 and a lateral edge of 15.



Determine :

(a) The pyramid's lateral area?

(b) The area of the base

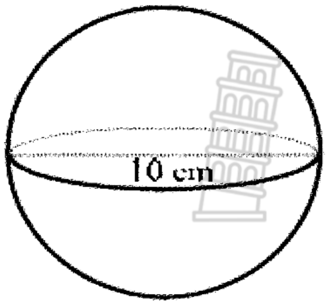
(c) The pyramid total area

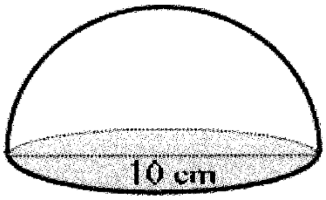
5. PABCD is a regular square pyramid.

(a) If each side of the base has a length of 14 and the altitude (PQ) is 24, find the pyramid's lateral area and total area.

Example:1

Calculate the surface area and volume of the following closed solids;





Surf. area $= 4\pi(5)^2$

$= (100\pi)\text{cm}^2$

$= 314{,}16 \text{ cm}^2$

vol. of sphere $= \frac{4}{3}\pi(5)^3$

$= \left(\frac{500\pi}{3}\right)cm^3$

$=523{,}60 \text{ cm}^3$

Surf. Area of hemisphere(half of sphere)

= curved surface + circular base

$= \frac{1}{2}\left[4\pi(5)^2\right] + \pi 5^2$

$= (75\pi)cm^2$

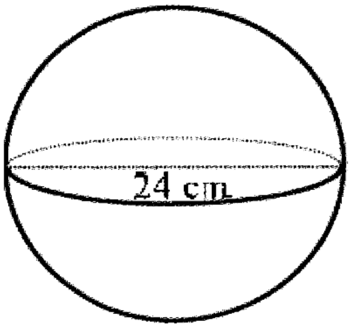
$= 235{,}62cm^2$

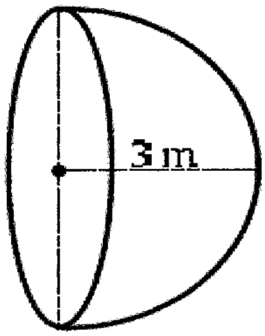
Vol. of herm $= \frac{1}{2} \times \frac{4}{3}\pi(5)^3$

$= 261{,}80cm^3$

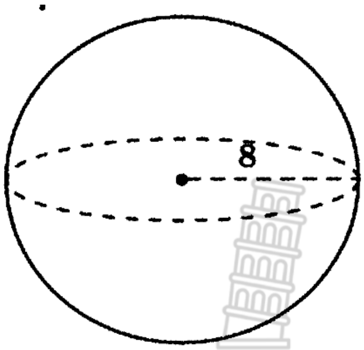
ACTIVITIES/ASSESSMENTS

1. Calculate the surface area and volume of the following closed solids;

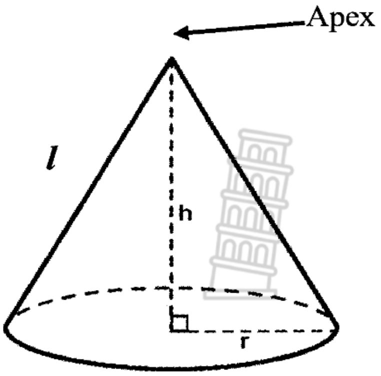




2 The figure shows a sphere. The radius of the sphere is $r = 8$ units. Determine the volume of the figure. round your answer to two decimal places



CONE



$S = \pi rl + \pi r^2$ where l is the slant height,

$V = \frac{1}{3}\pi r^2 h$

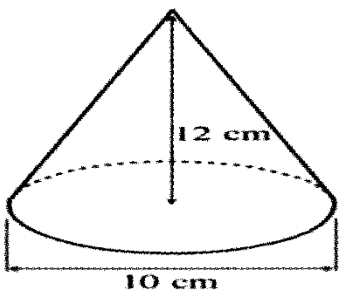
πrl is the area of the curved surface.

πr^2 is the area of a circular base.

NOTES

Example : 1

The following sketch shows a cone with a base diameter of 10 cm and a height of 12 cm:



Calculate , correct to two decimal places:

(a) Volume

(b) Surface area of area of a cone

(c)

Solution

(a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5^2 \times 12$

$= 314,16$

(b) $l = \sqrt{5^2 + 12^2} = 13$

Surface Area $= \pi r^2 + \pi rl$

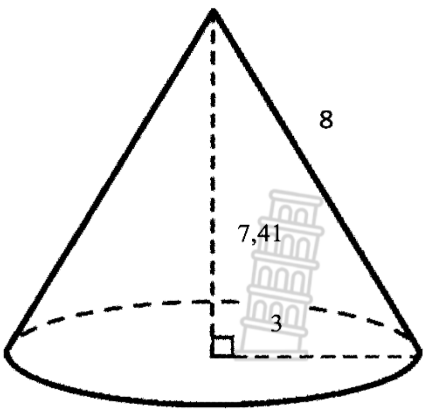
$= \pi(5)^2 + \pi(5)(13)$

$= 282,74 \text{ cm}^2$

ACTIVITIES/ASSESSMENTS

1. The figure here is a cone. The vertical height of the cone is $h = 7,41$ units and the slant height of the cone is $h = 8$ units ; the radius of the cone is shown, $r = 3$ units. Find the surface area and

wo decimal places.



TOPIC: MEASUREMENT
Weighting: (8/100 marks from Paper 2)

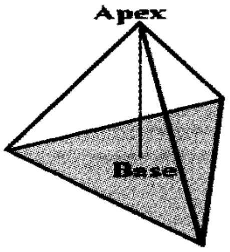
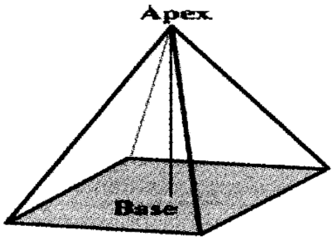
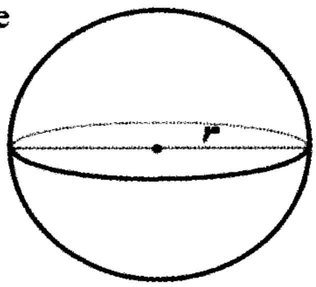
LESSON 6: COMPOSITE SHAPES

Term	2	**Duration**	2 hours	**Grade**	10	**Date**	
Sub-topics	Composite shapes: Pyramid , cone ,cylinder and sphere						

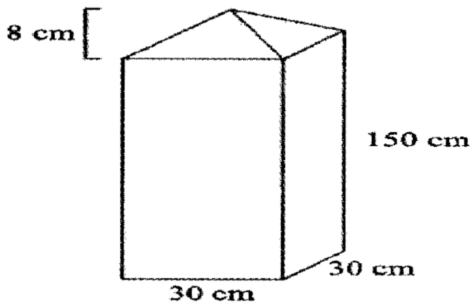
RESOURCES

Gr. 10 textbooks :
Siyavula; Past Exam papers, mind action series, Sasol Inzalo textbook, Classroom Mathematics and resources from the internet.

NOTES

Solid	Surface area	Volume
Right triangular pyramid 	Sum of the area of the base (equilateral triangle) and three congruent triangles.	$V = \frac{1}{3}(A \times H)$ where: A = area of base H = height
Right square pyramid 	Sum of the area of the base (square) and four congruent triangles.	$V = \frac{1}{3}(A \times H)$ where: A = area of base H = height
Sphere 	$S = 4\pi r^2$ where r is the radius of the sphere	$V = \frac{4}{3}\pi r^3$

5. A concrete gate comprises a right rectangular prism having a square base and a pyramid at the top, as shown in the diagram below. The length of the sides of the base is 30cm and the height of the rectangular section is 150 cm. the perpendicular height of the pyramid section is 8 cm. Calculate the ff:

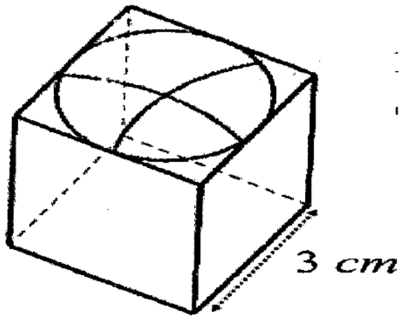


(a) The amount of the concrete required to make one post

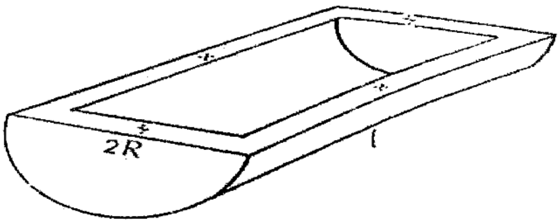
(b) The surface area of the pyramid section of a post.

(c) If the length of the sides of the base is halved, how many posts with the same design can be made using the same amount of concrete as the original post?

6 A hemisphere (half sphere) is placed on top of a cube side 3 cm.Calculate the total surface area of the composite shape.



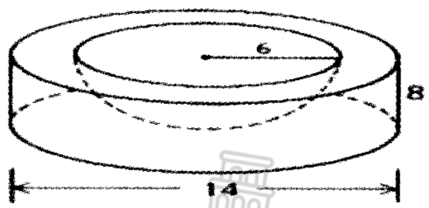
7.



(a) Calculate the amount of concrete required to make the trough.

(b) Calculate the volume of water that can ce held by a trough.

8 A plastic bowl on the right is in the shape of a cylinder with a hemisphere cut out. The dimensions are shown. Calculate:

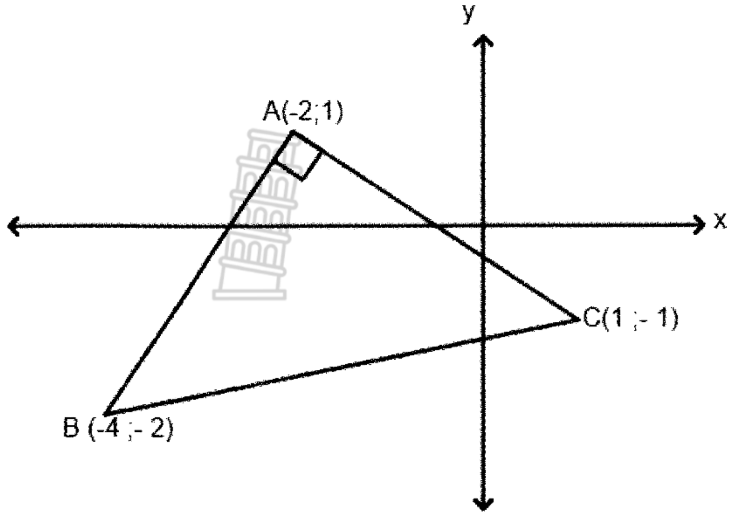


(a) the volume of a cylinder.

(b) the volume of a hemisphere.

(c) The amount of plastic used to make a solid bowl.

Given triangle ABC. Determine the following:



3.1. The length of AB.

3.2. The length of AC.

3.3. Based on 4.1 and 4.2. write down the type triangle ABC.

Solution

3.1. $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2+4)^2 + (1+2)^2}$

$= \sqrt{(2)^2 + (3)^2}$

$= \sqrt{(4+9}$

$= \sqrt{13}$

$= 3,6$

3.2. $d_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-1)^2 + (1+1)^2}$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

$= \sqrt{13}$

$= 3,6$

3.3. Isosceles triangle

ACTIVITIES/ ASSESSMENT: LESSON 1

1. Determine the distance between the two points. Correct your answer to two decimal places where necessary

   1.1. $(2;-4)$ and $(1;1)$

   1.2. $(3;-1)$ and $(-3;5)$

   1.3. $\left(\frac{1}{2};-\frac{1}{6}\right)$ and $\left(-\frac{2}{3};-\frac{5}{2}\right)$

2. Given the diagram below

2.1. Determine the length of

2.1.1 PS

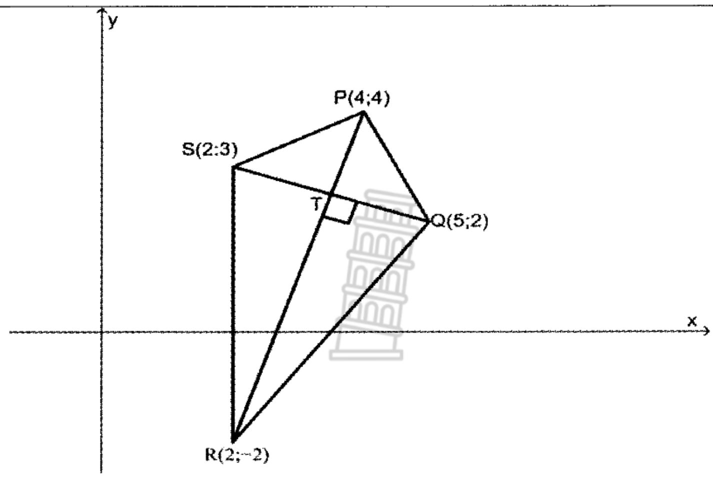
2.1.2. PQ

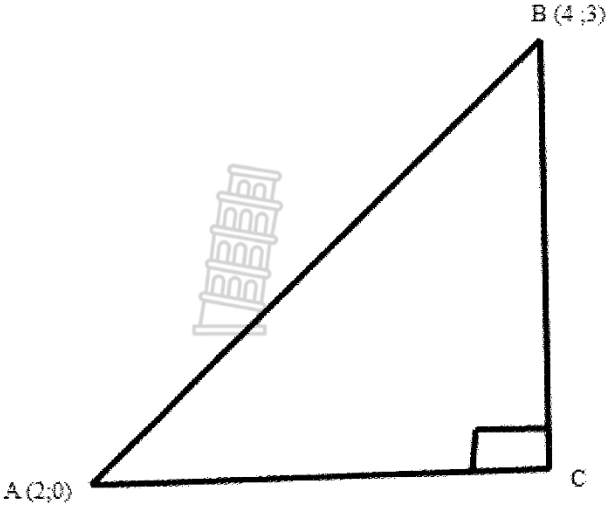
2.1.3. SR

2.1.4. QR

2.1.5. PR

2.1.6. SQ





3.2. The length of AC

....

3.3. The length of BC

....

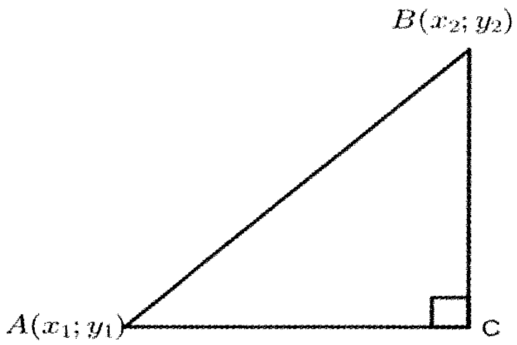
3.4. **HENCE** determine the length of AB

....

....

....

4. Given the triangle ABC with the points $A(x_1;y_1)$ and $B(x_2;y_2)$.



Express the following in terms of $(x_1;y_1;x_2;y_2)$:

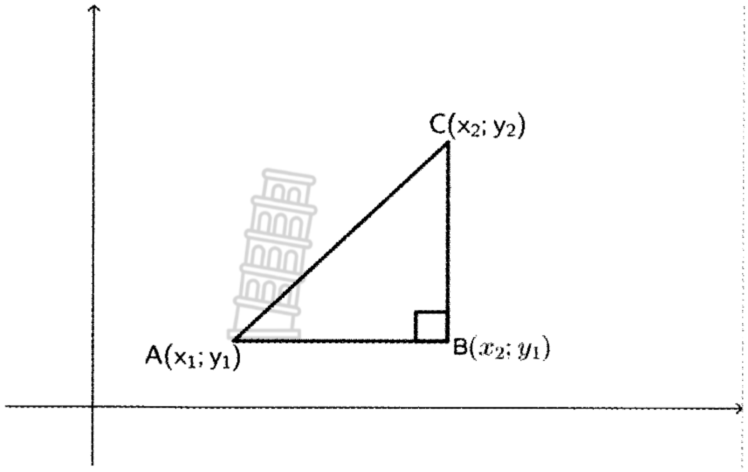
4.1. Coordinates of C...

4.2. the length of AC ....

4.3. the length of BC ....

4.4. the length of AB in terms of using the Pythagoras theorem (SHOW ALL STEPS)

....



horizontal change = AB = $x_2 - x_1$

vertical change = BC = $y_2 - y_1$

$$m = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating the gradient of a line passing through the given two points :

- Write down the formula
- Substitute the points into the formula
- Simplify the answer
- Correct your answer to two decimal places unless stated otherwise

The following examples must be done

EXAMPLES

Determine the gradient of a line passing through :

1.1 $P(1;-1)$ and $Q(-1;5)$

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{1-(-1)} = \frac{-6}{2} = 3$$

1.2 A(5 ;4) and B(2 ;1)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{5-2} = \frac{3}{1}$$

1.3 $X\left(-\frac{2}{3};\frac{1}{2}\right)$ and $Y\left(-\frac{1}{3};-\frac{3}{2}\right)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{1}{2}-\left(-\frac{3}{2}\right)}{-\frac{2}{3}-\left(-\frac{1}{3}\right)} = \frac{\frac{4}{2}}{-\frac{1}{3}} = -6$$

1.4 A(5 ;0) and B(2 ;0)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{5-2} = 0$$

ACTIVITIES/ASSESSMENTS: LESSON 2

1. Determine the gradient of a line passing through the points

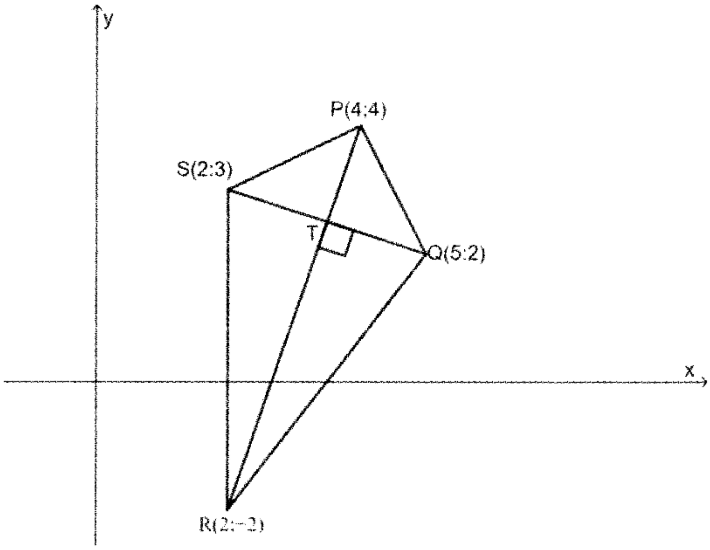
   1.1. $(2\,;-4)$ and $(1\,;1)$

   1.2. $(3;-1)$ and $(-3\,;5)$

   1.3. $(\frac{1}{2};-\frac{1}{6})$ and $(-\frac{2}{3};-\frac{5}{2})$

   1.4. The gradient of a line that passes through the points $(-2;4)$ and $(x;3)$ is 2.

   Determine the value of x.



2. In the diagram alongside, determine the gradient of

   2.1. PS

   2.2. PQ

   2.3. If $T\left(\frac{7}{2};\frac{5}{2}\right)$ is the point where SQ and PR meet, show that PQRS is a kite.

1.3. The line parallel to GT passes through the points $(-1\ ;\ 3)$ and $(1\ ;\ y)$. Determine the value of y.

1.4. Calculate the gradients of the lines passing through AB; CD; and SN.

1.5. Hence conclude about the gradient of a horizontal line

1.6. Comment on the gradient of a vertical line based on calculations.

1.7. Explain why is $m_{JR} = m_{KR}$.

DAY 2

Refer to worksheet 2

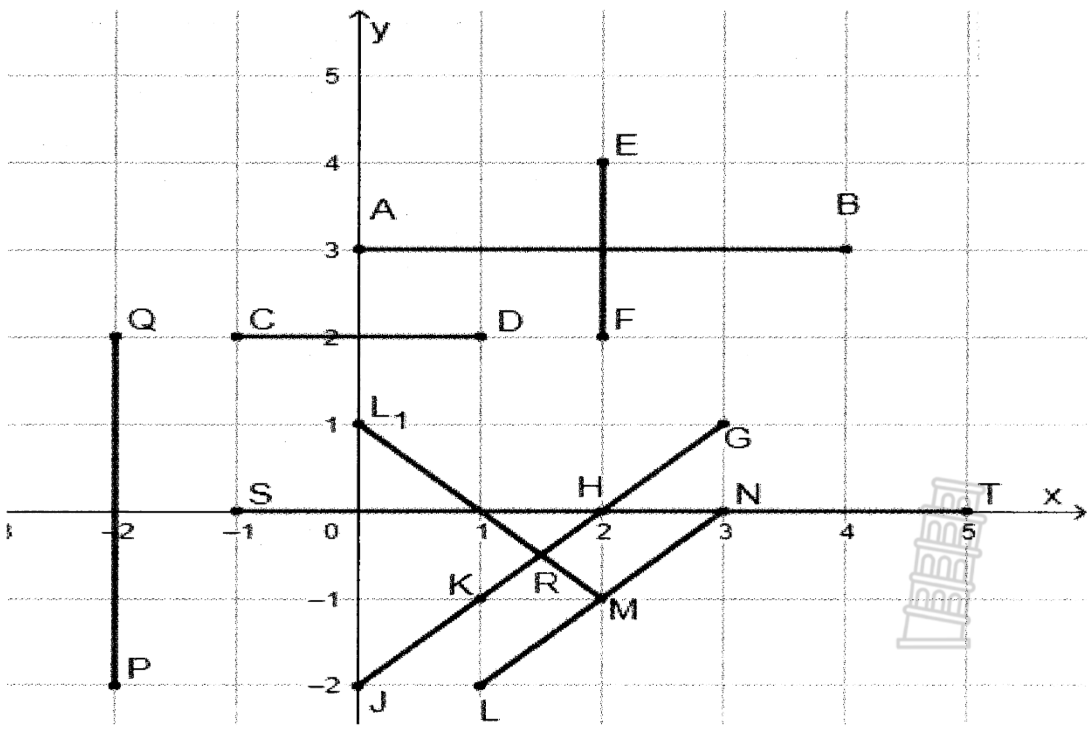
1.1. Show that $RM \perp LN$ given that R is the midpoint.

1.2. Calculate the gradient of a line perpendicular to line passing through QJ.

1.3. The line drawn perpendicular to QG passes through (x ;-2) and the origin . Determine the value of x.

WORKSHEET 2

1. Consider the following lines and answer the questions that follow



TOPIC: ANALYTICAL GEOMETRY Weighting: (15/ 100 marks from Paper 2)							
LESSON 4 : THE MIDPOINT OF A LINE SEGMENT							
Term	3	**Duration**	1 hour	**Grade**	10	**Date**	
Sub-topics	Midpoint of a line segment						

RESOURCES

Gr. 10 textbooks:
Mind Action Series
Grade 11 textbook: Clever Mathematics

NOTES

- **To calculate the distance between the two points or the length of a horizontal line segment:**

Start by allowing learners to draw a horizontal line segment on a Cartesian plane. The Cartesian plane must ensure equal intervals.

Example

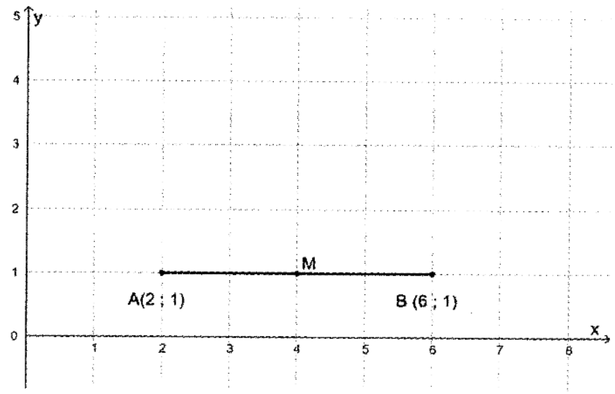
1. In the sketch below: M lies on AB

 1.1. Write down the coordinates of M by reading from the sketch.

 1.2. Confirm by using the distance formula that M is the midpoint of AB

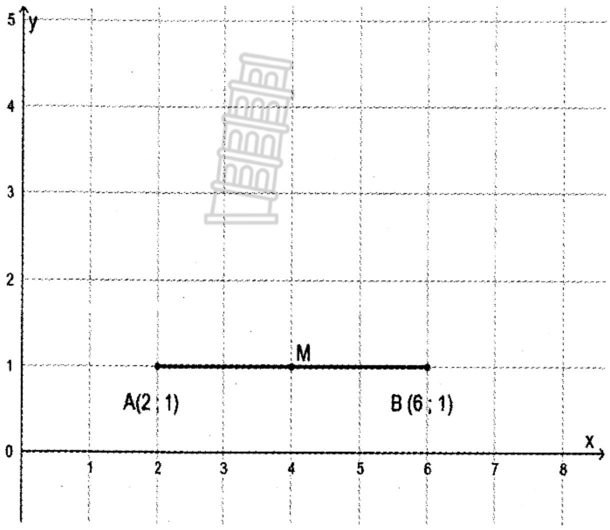
 1.3. Calculate the value of $\frac{x_1 + x_2}{2}$ and $\frac{y_1 + y_2}{2}$

 1.4. Compare the coordinates of M with the values of $\frac{x_1 + x_2}{2}$ and $\frac{y_1 + y_2}{2}$

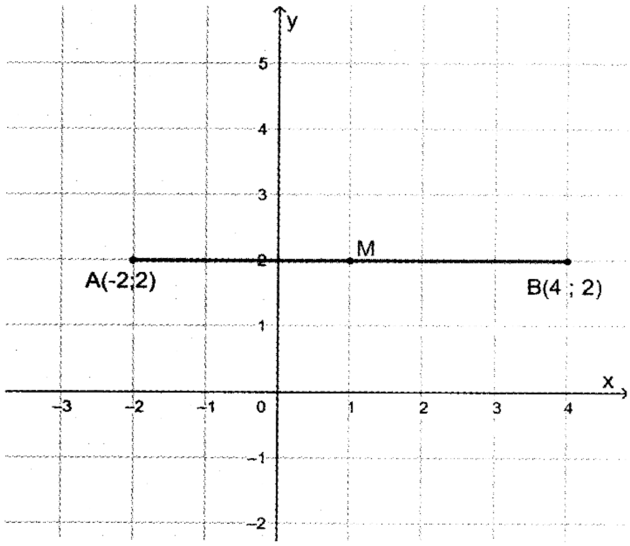


Worksheet 3 (This must be printed for learners and must be attached to the learners working)

1.



2.



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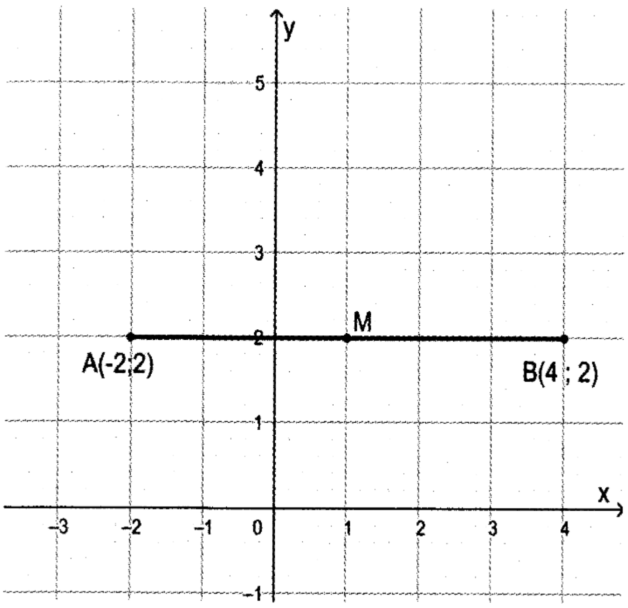

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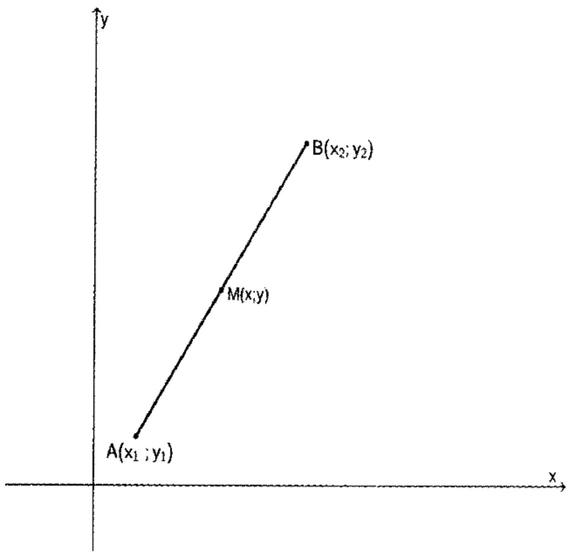


ACTIVITIES/ASSESSMENTS: LESSON 4

1. 1.1. Calculate the coordinates of M the midpoint of the line segment joining points A (1;6) and B (5;8)

   1.2. If M (-3;2) is the midpoint of the line joining the points A (x;1) and B (-1;y), calculate the value of x and y.

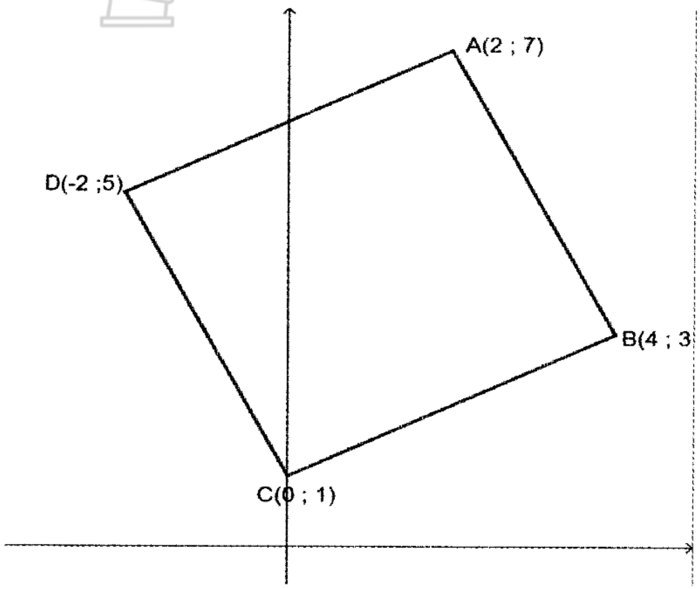
SHORT TEST

TOTAL MARK : 23 MARKS **DURATION: 30 min**

All the answers must be correct to two decimal places if necessary.

QUESTION 1

1. In the diagram, A(4 ; 3), B(2 ; 7), C(-2 ; 5) and D(0 ; 1) are the vertices of a quadrilateral.



1.1. Determine the length of DC (3)

1.2. Calculate the gradient of CB (3)

1.3. If M is the midpoint of AC, determine the coordinates of M (3)

1.4. Show that $AD \perp DC$ (5)

[14]

QUESTION 2

2. Given the quadrilateral PQRS with P(3;2), Q(-2;0) , (7;k) and AD || BC