KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12
DIRECTORATE

NCS (CAPS)

LEARNER SUPPORT DOCUMENT

GRADE 10

MATHEMATICS
STEP AHEAD PROGRAMME

2023

This document has been compiled by the KZN FET Mathematics Subject Advisors.

PREFACE

This support document serves to assist Mathematics leaners on how to deal with curriculum gaps, it also captures the challenging topics in the Grade 10 – 12 work. It will cover the following:

TABLE OF CONTENTS

TOPIC 2: EUCLIDEAN GEOMETRY

Weighting: (30/100 marks from Paper 2)

Overview

EUCLIDEAN GEOMETRY

EUCLIDEAN GEOMETRY
2 BRANCHES
(Grade 10)

1. LINES
- *Parallel lines
- *Perpendicular lines
- * Neither Parallel nor Perpendicular but Crossing each other

2. TRIANGLES and QUADRILATERALS

Triangles
- *Name by their properties (Angles or sides)
- *Prove Congruency and Similarity
- *Examinable Proof (Midpoint Theorem)

Quadrilaterals
- *Name by their properties
- *Examinable Proofs

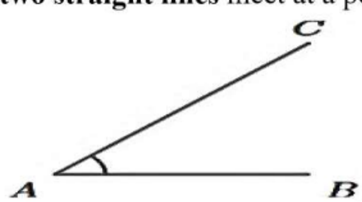
SUMMARY OF EUCLIDEAN GEOMETRY FROM EARLY GRADES

- Revision results about the lines (alt ∠s , corr. ∠s , co-int ∠s , vert opp ∠s , ∠s on str line or ∠s around point), triangles(sum of ∠s in Δ ,ext angle, isosceles, similar and congruence triangles) etc.
- Properties of quadrilaterals (parallelogram, rectangle, rhombus, square, kite and trapezium, know areas, diagonals, and sides
- 6 proofs for those quadrilaterals to be known for exam purpose

LESSON 1: REVISION OF LINES AND ANGLES FROM GR. 9 EUCLIDEAN GEOMETRY

NOTES

An angle is formed where **two straight lines** meet at a point, also known as a **vertex.**

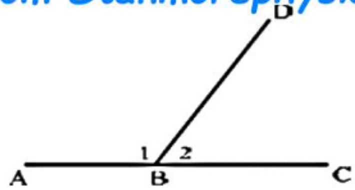


Angles are labelled with a caret on a letter, for example, Â.
Angles can also be labelled according to the **line segments** that make up the angle, for example $C\hat{A}B$ *or* $B\hat{A}C$.
The "∠" symbol is a short method of writing angle in geometry and is often used in phrases such as "sum of ∠*s* in Δ".
Angles are measured in degrees which is denoted by °, a small circle raised above the text, similar to an exponent.

Types of Angles

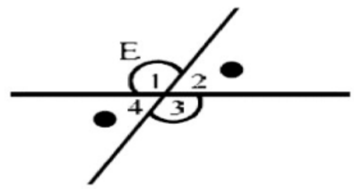
1. **Adjacent angles** on a straight line are supplementary. **Supplementary angles** add up to 180°

$\hat{B}_1$ and $\hat{B}_2$ **share a vertex** and a common side. Hence $B_1 + B_2 = 180°$.



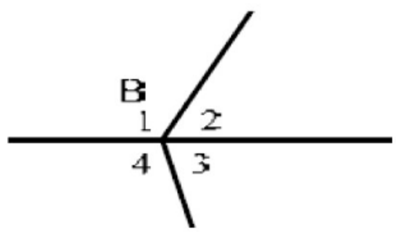
2. If two lines intersect, **vertically opposite angles** are equal. Two lines intersect if they **cross each** other at a point.

   Vertically opposite angles are **angles opposite each other** when two lines intersect. They share a vertex and are equal.



$\hat{E}_1 = \hat{E}_3$ and $\hat{E}_2 = \hat{E}_4$

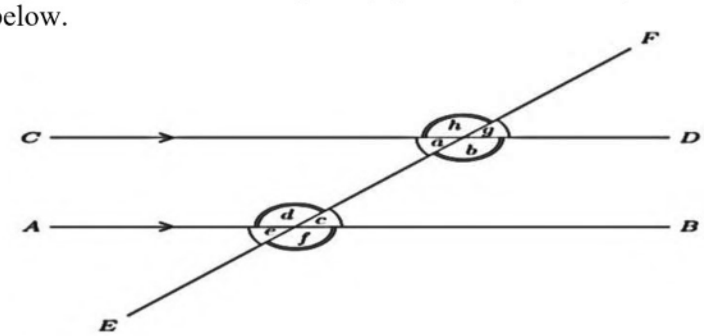
3. The **angles around a point** add up to 360° (A revolution).



$B_1 + B_2 + B_3 + B_4 = 360°$

PARALLEL LINES

Parallel lines are always the **same distance apart (equidistant)** and they are **denoted by arrow symbols** as shown below.



$CD \parallel AB$. EF is a **transversal line**. A transversal line **intersects** two or more parallel lines. Below are the **properties of the angles** formed by the above intersecting lines.

1. Corresponding Angles

Corresponding angles lie either both above or both below the lines and on the same sideof the transversal. If the lines are parallel, the corresponding angles will be equal.

2. Alternate Angles

Alternate angles lie on opposite sides of the transversal and between the lines. If the lines are parallel, the alternate angles will be equal.

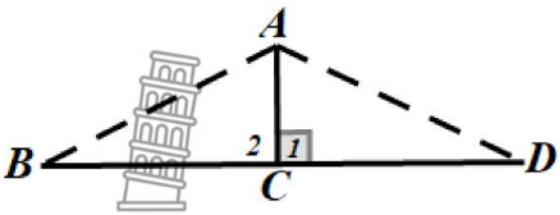
3. Co-interior Angles

Co-interior angles lie on the same side of the transversal between the lines. If the lines are parallel, the co-interior angles are supplementary

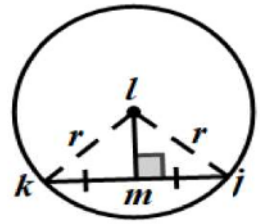
NB: If two lines are intersected by a transversal such that **corresponding angles are equal**; or **alternate angles are equal**; or co-interior angles are supplementary, then the **two lines are parallel**.

1. Examples on Straight Lines, Parallel Lines and Triangles

1.1. PERPENDICULAR LINES

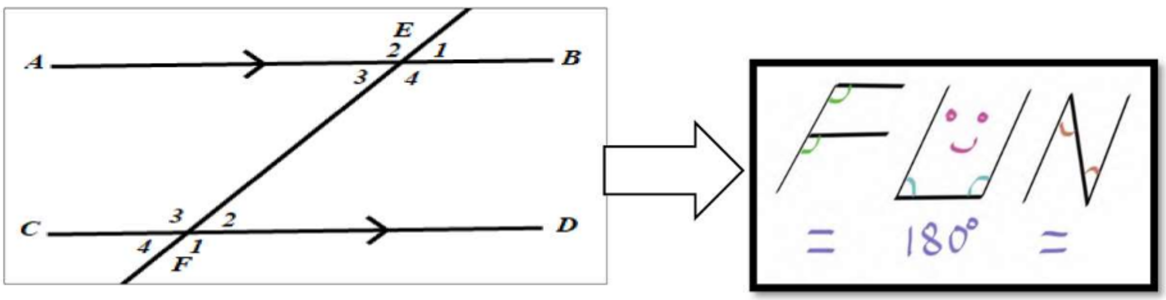


OR



- $\hat{C}_1 + \hat{C}_2 = 180°$ **($\angle$s on str line)**
- **BUT** $(AD)^2 = (AC)^2 + (CD)^2$ **(Pythagoras)**
- $\Delta ABC \equiv \Delta ADC$ **(RHS)**

1.2. PARALLEL LINES



- **Alternating angles**

  $\hat{E}_3 = \hat{F}_2$ **(Alt $\angle$s AB // CD)**

  $\hat{E}_4 = \hat{F}_3$ **(Alt $\angle$s AB // CD)**
- **Corresponding angles**

  $\hat{E}_1 = \hat{F}_2$ **(Corresp $\angle$s AB // CD)**

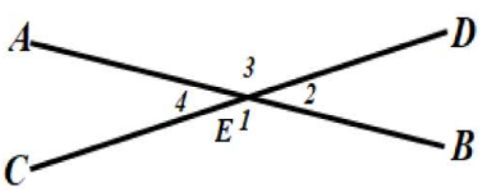
  $\hat{F}_4 = \hat{E}_3$ **(Corresp $\angle$s AB // CD)**

  $\hat{E}_4 = \hat{F}_1$ **(Corresp $\angle$s AB // CD)**
- **Co-interior angles**

  $\hat{E}_4 + \hat{F}_2 = 180°$ **(Co-int $\angle$s AB // CD)**

  $\hat{F}_3 + \hat{E}_2 = 180°$ **(Co-int $\angle$s AB // CD)**

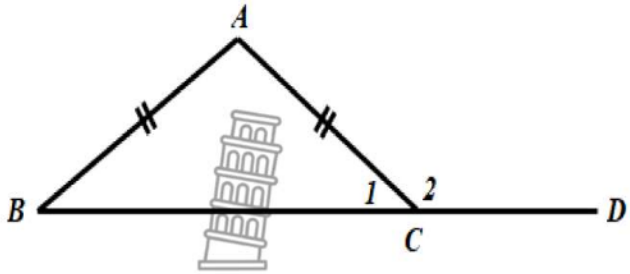
1.3. CROSSING LINES



- $\hat{E}_1 = \hat{E}_3$ **(Vert opp $\angle$s)**
- $\hat{E}_2 = \hat{E}_4$ **(Vert opp $\angle$s)**
- $\hat{E}_1 + \hat{E}_2 + \hat{E}_3 + \hat{E}_4 = 360°$ **($\angle$s around pt)**

2. FACTS ABOUT THE TRIANGLE

2.1 PROPERTIES OF TRIANGLES must be known **(isosceles, equilateral, right angled triangle etc.)**

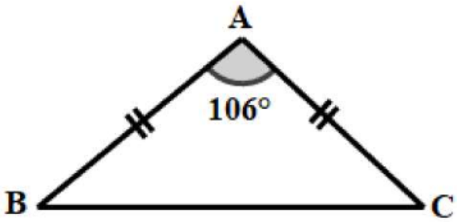


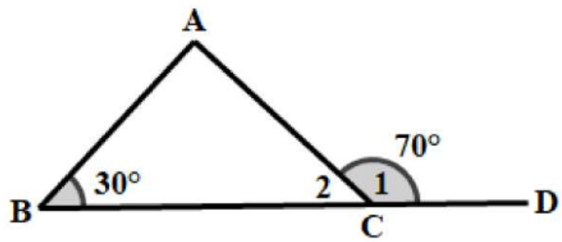
(a) $\hat{A}+\hat{B}+\hat{C}_1=180^\circ$ **(Sum of $\angle$s in Δ)**

(b) $\hat{A}+\hat{B}=\hat{C}_2$ **(ext $\angle$ of Δ)**

(c) $\hat{B}=\hat{C}_1$ **(opp $\angle$s = sides)**

2.2 Solve for the unknown angles in each of the following triangles.





1.$\hat{B}=\hat{C}=\dfrac{180^\circ-106^\circ}{2}=37^\circ$ **(opp $\angle$s = sides)**

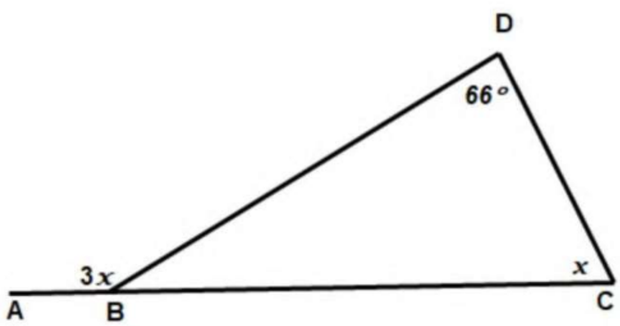
2. $\hat{A}=40^\circ$ **(ext $\angle$ of Δ,** $\hat{C}_2=30^\circ$

LESSON 2: REVISION OF LINES AND ANGLES

NOTES

Examples on how to calculate the value of x in the following sums

a)

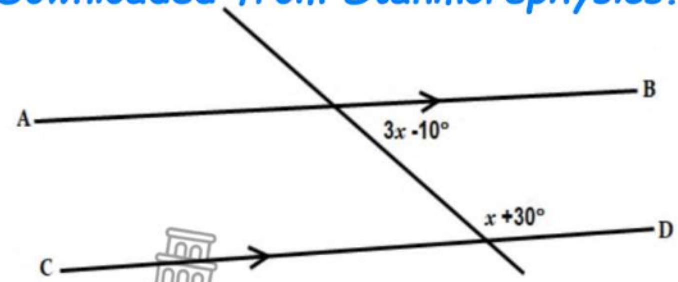


$\hat{ABC}=\hat{D}+\hat{C}$ $\angle$s of a Δ

$3x=66^\circ+x$

$x=33^\circ$

b)



$(3x-10°)+(x+30°)=180°$ Co-int $\angle$ s, AB//CD

$4x+20°=180°$

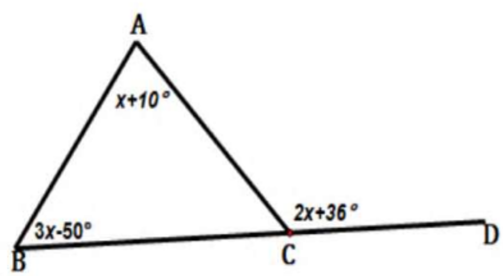
$4x=160°$

$x=40°$

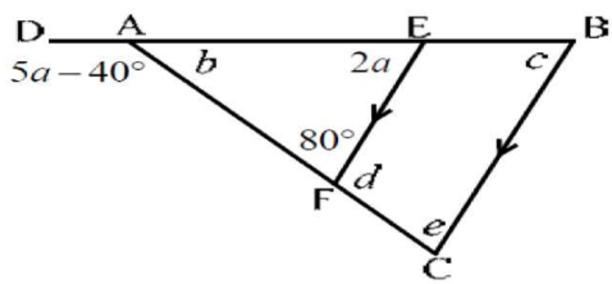
ACTIVITIES 1

Exercise 1:

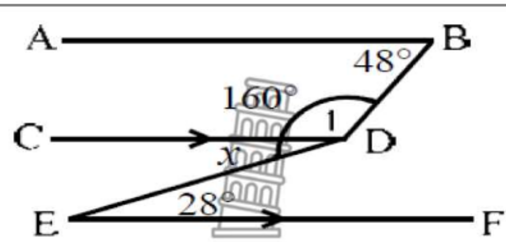
1. Calculate the value of x



2. In ΔABC, EF∥BC. BA is produced to D.
   a) Calculate, **with reasons,** the value of $\boldsymbol{a}$ and hence show that AE = AF.
   b) Calculate, **with reasons**, the value of b, c, d, $and\ e$.



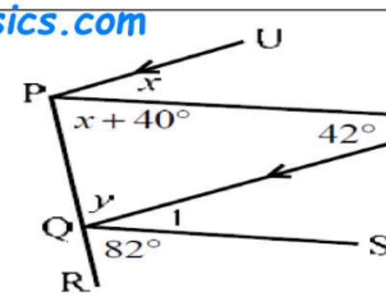
3. In the diagram below, CD∥EF, $D\hat{E}F = 28°$, $\hat{B} = 48°$ and $B\hat{D}E = 160°$. Prove that AB∥CD.



4. In the diagram below, PU $\parallel$ QT, $\hat{T} = 42°$, $R\hat{Q}S = 82°$, $P\hat{Q}T = y$, UPT $= x$ and QPT $= x + 40°$.

a) Prove that PT$\parallel$QS

b) Calculate y



LESSON 3: REVISION OF TRIANGLES FROM GR. 9 EUCLIDEAN GEOMETRY

NOTES

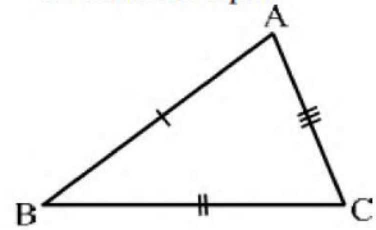
PROPERTIES OF TRIANGLES

A triangle is a three-sided polygon. Triangles can be classified **according to sides** and also be classified **according to angles.**

1. TYPES OF TRIANGLES according to **sides**

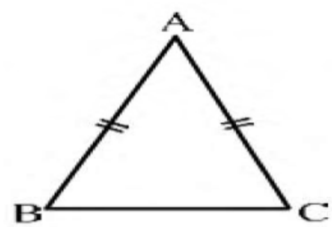
a) Scalene Triangle:

No sides are equal.



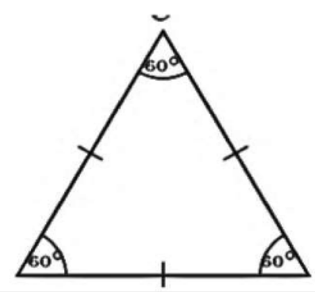
b) Isosceles Triangle:

- Two sides are equal.
- Angles opposite equal sides are equal.



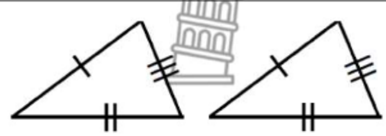
c) Equilateral Triangle

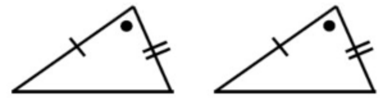
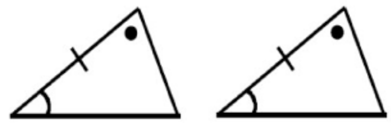
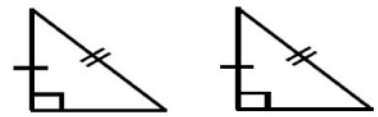
- All three sides are equal**.**
- All three interior angles are equal



2. **CONGRUENT TRIANGLES** – 4 Conditions

a) If three sides of a triangle are equal in length to the corresponding sides of another triangle, then the two triangles are congruent.
Side, Side, Side (S, S, S)



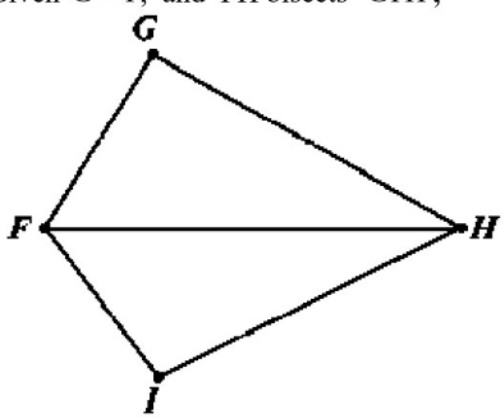
b) If two sides and the **included** angle of a triangle are equal to the corresponding two sides and **included** angle of another triangle, then the two triangles are congruent. Side, Angle, Side (S, A, S)	
c) If one side and two angles of a triangle are equal to the corresponding one side and two angles of another triangle, then the two triangles are congruent. Angle, Angle, Side (A, A, S)	
d) If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle, then the two triangles are congruent. 90°, Hypotenuse, Side (R, H, S).	

We use ≡ to indicate that triangles are congruent.

NOTE: The order of letters when labelling congruent triangles is very important.

Worked out example

Given $G = \hat{I}$, and FH bisects GFH, **Prove that** $\Delta GFH \equiv \Delta IFH$



In ΔGFH and ΔIFH

i) $G = \hat{I}$ **given**

ii) $G\hat{F}H = H\hat{F}I$ **given**

iii) $FH = FH$ **common side**

$\therefore \Delta GFH \equiv \Delta IFH$ **(AAS)**

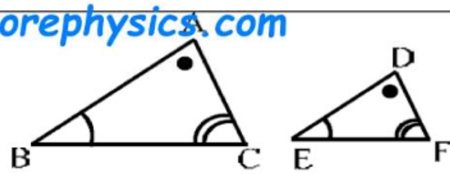
3. SIMILAR TRIANGLES

Two triangles are similar if **one triangle is a scaled version of the other**. This means that their **corresponding angles are equal** in measure and the **ratio of their corresponding sides are in proportion**. The two triangles have **the same shape**, but different scales.

Congruent triangles are similar triangles, but **not all similar triangles are congruent**.

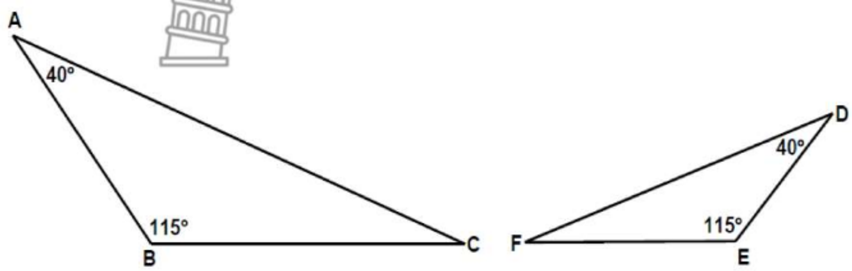
We use /// to indicate that two triangles are similar.

a) If all three pairs of corresponding angles of two triangles are equal, then the triangles are similar.
Angle, Angle, Angle (AAA)



b) If all three pairs of corresponding sides of two triangles are in proportion, then the triangles are similar.

Worked Example
Prove that $\Delta GFH /// \Delta IFH$



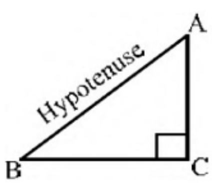
In ΔGFH *and* ΔIFH

i) $\hat{A}=\hat{D}=40^{\circ}$ given
ii) $\hat{A}=\hat{E}=115^{\circ}$ given
iii) $\hat{C}=\hat{F}$ 3^{rd} $\angle$ of a Δ
$\therefore \Delta GFH /// \Delta IFH$ (AAA)

NOTE: The order of letters for similar triangles is very important. Always label similar triangles in corresponding order.

4. THEOREM OF PYTHAGORAS

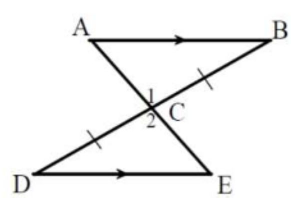
$AB^2 = AC^2 + BC^2$ if $\hat{C} = 90°$



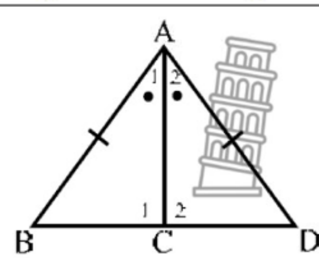
ACTIVITIES 2

Exercise 1: Mind Action Series (Pg. 166)

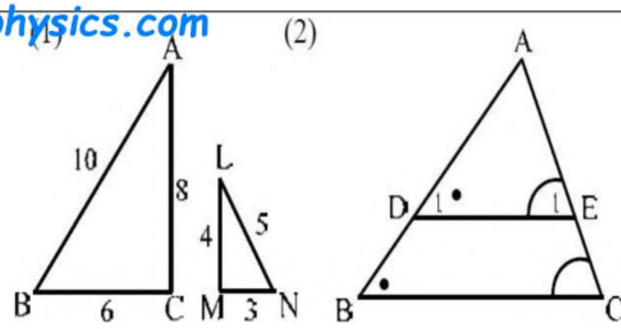
1. AB∥DE and DC = CB
   a) Prove that AC = CE and AB = DE



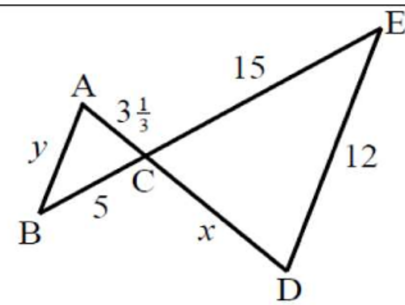
2. Prove that $C_1 = 90°$ using congruency.



3. Show that the following triangles are similar.

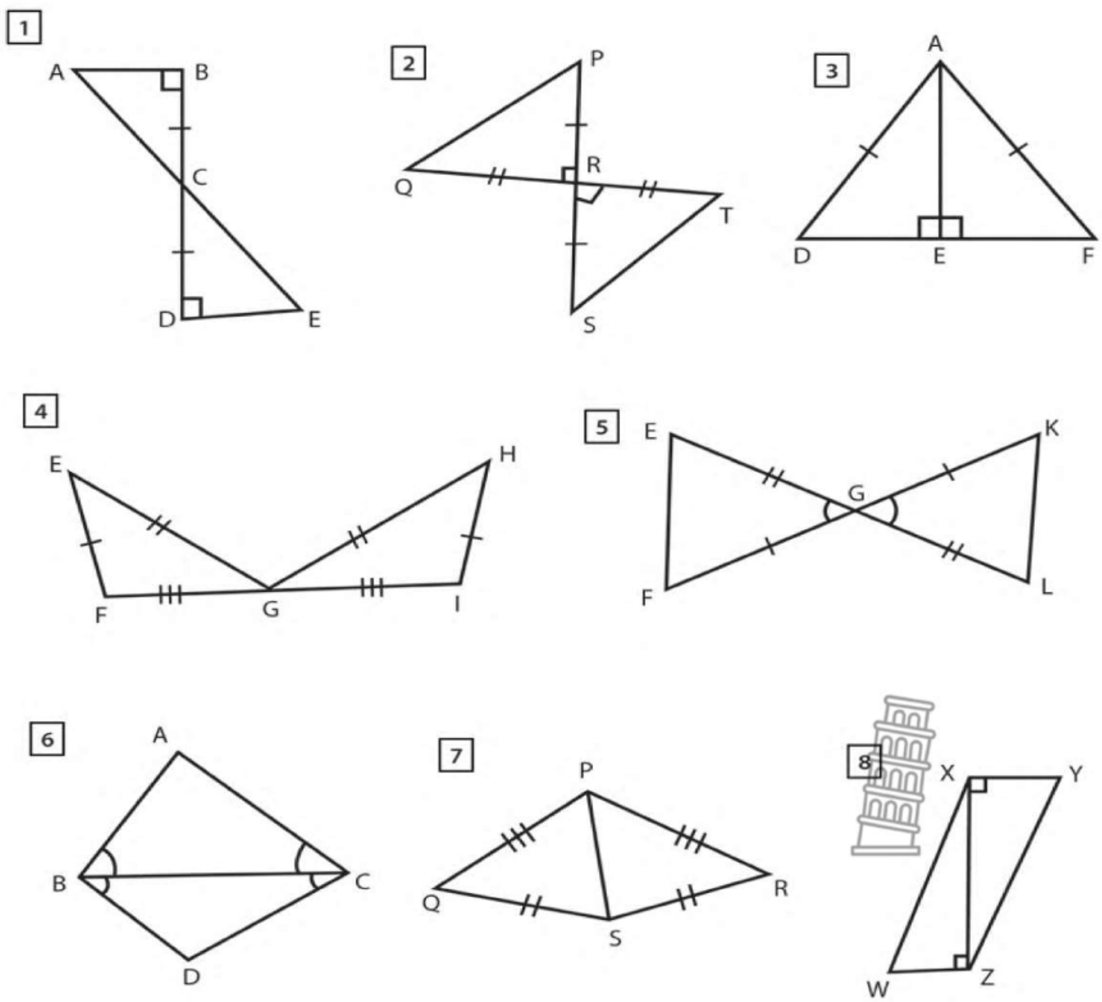


4. If $\Delta ABC /// \Delta DEC$, calculate x and y.



ACTIVITY 3

Proving Triangle Congruency Worksheet. Prove whether the given triangles are congruent or not.



LESSON 4 : REVISION OF TRIANGLES FROM GR. 9 EUCLIDEAN GEOMETRY

NOTES

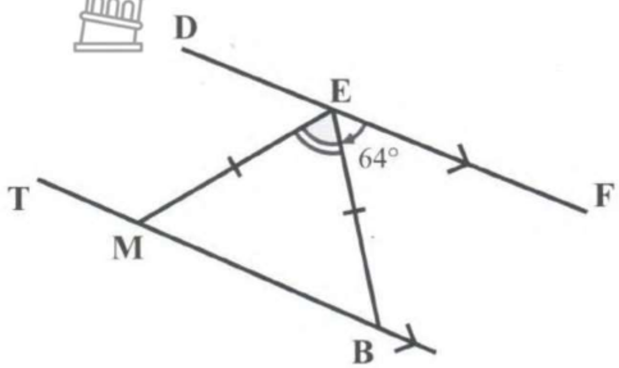
Activity 4

INDEPENDENT CLASS ACTIVITY 4: LINES, ANGLES AND TRIANGLES

QUESTIONS

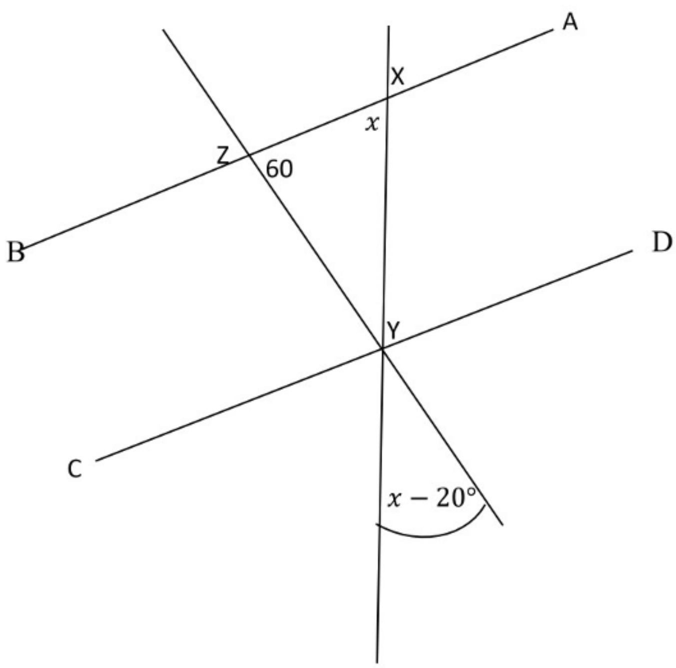
1. In the diagram below, straight lines DEF and TMB are parallel to each other.

   It is also given that EM = EB and BEF=64° .

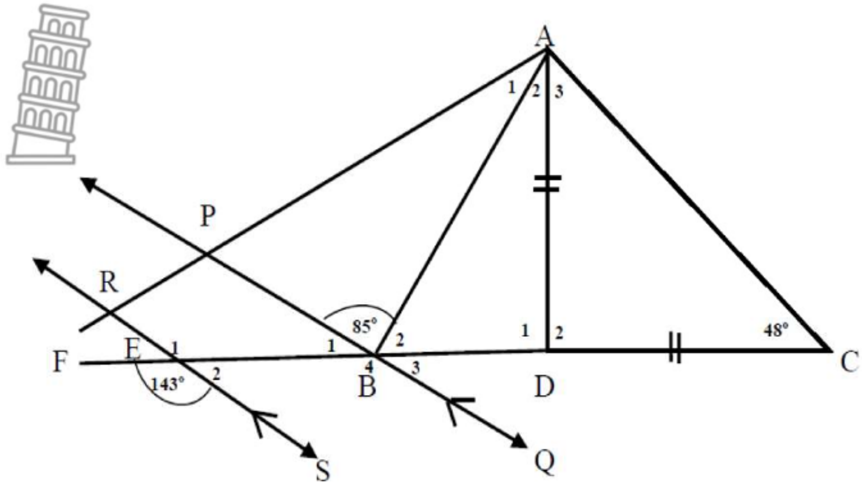


Calculate the size of MEB (4)

2. In the diagram below, AB and DC are two parallel lines cut by two transversal lines at X, Y and Z respectively.



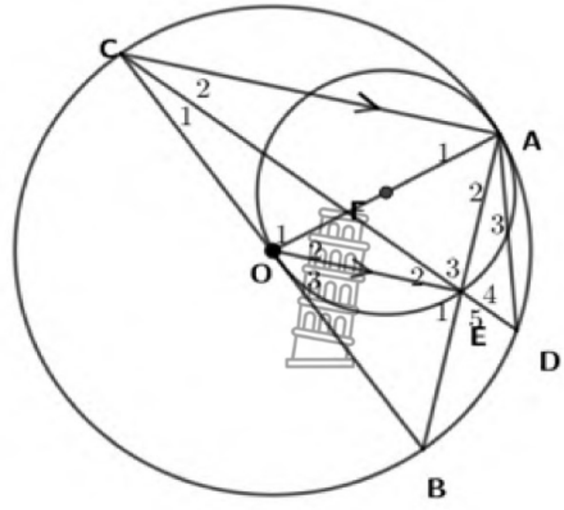
2.1	Determine giving reasons, the value of x in the diagram:	(6)
2.2	Name one pair of co-interior angles	(1)
2.3	Name one pair of alternate angles	(1)

2.4	Complete: If two parallel lines are cut by a transversal, then the co-interior angles are ……………..	(1)
2.5	Complete: The size of angle XYD = ………………. Reason	(2)
3.	In the diagram below, AD = CD and PQ∥RS. AR and FC are straight lines. RS and FC intersect at E. PQ also intersects FC at B. 	

3.1	Determine the sizes of the following angles, giving appropriate reasons:		
	a	D_1	(2)
	b	B_1	(2)
	c	A_2	(2)
3.2	Show that $\hat{REF}=\hat{B}_3$		(3)
			[9]

4.Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle such that CB is a diameter and $O_1 = 90°$. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle.

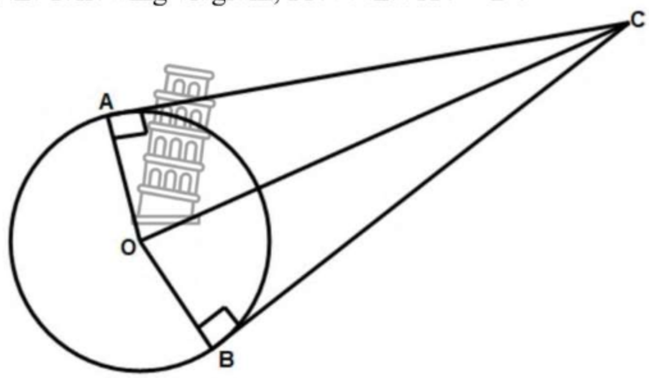
OE ∥ CA.



4.1 Give TWO lines each equals to OC (2)

4.2 Prove that ΔOCA ≡ ΔOBA. (3)

5. In the following diagram, Prove that AC = BC



(5)

LESSON 5: PROPERTIES OF QUADRILATERALS (1)

NOTES

A quadrilateral is a closed shape (polygon) consisting of four straight line segments.A **polygon** is a two-dimensional figure with three or more straight sides. The **interior angles** of a quadrilateral add up to 360°.

1. PARALLELOGRAM
   - A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

2. RECTANGLE
   - A rectangle is a parallelogram that has all four angles equal to 90°.

3. RHOMBUS
   - A rhombus is a parallelogram with all four sides of equal length.

4. SQUARE
   - A square is a rhombus with all four interior angles equal to 90°.
     A square has all the properties of a rhombus.

     OR
   - A square is a rectangle with all four sides equal in length.

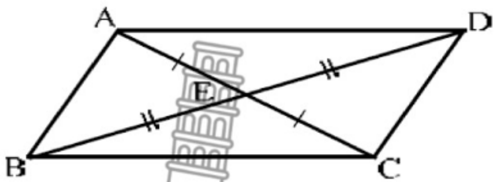
5. TRAPEZIUM
   - A trapezium is a quadrilateral with at least one pair of opposite sides parallel.
     NOTE: A trapezium is sometimes called a trapezoid.

6. KITE
   - A kite is a quadrilateral with two pairs of adjacent sides equal.

PROPERTIES OF QUADRILATERALS

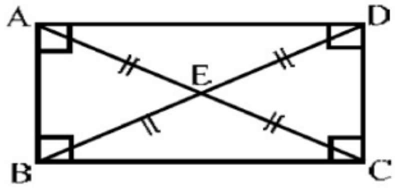
QUATRILATERAL	**PROPERTIES**

PARALLELOGRAM



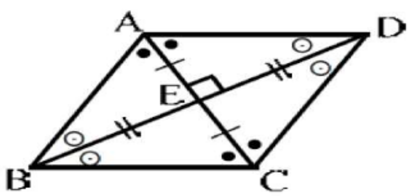
- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- Both diagonals bisect each other.

RECTANGLE



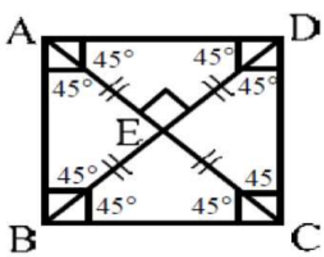
- **Diagonals are equal in length.**
- **All interior angles are equal to 90°**

RHOMBUS



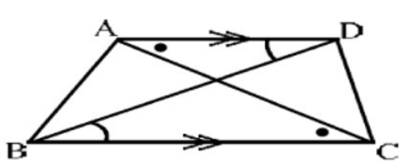
- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- Both diagonals bisect each other.
- **All sides are equal in length.**
- **The diagonals bisect each other at 90°**
- **The diagonals bisect both pairs of opposite angles.**

RECTANGLE

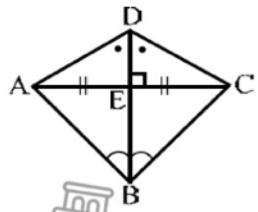


- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- Both diagonals bisect each other.
- All sides are equal in length.
- The diagonals bisect each other at 90°
- The diagonals bisect both pairs of opposite angles.
- **All interior angles equal 90°.**
- **Diagonals are equal in length.**

TRAPEZIUM



- One pair of opposite side are parallel.
- The diagonals of a trapezium intersect but don't bisect each other.
- Diagonals lie between parallel lines and therefore, the alternate angles are equal.

KITE 	• Diagonal between equal sides bisects the other diagonal. • One pair of opposite angles are equal (the angles between unequal sides). • Diagonal between equal sides bisects the interior angles and is an axis of symmetry. • Diagonals intersect at 90°

ACTIVITY 5

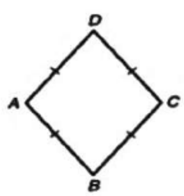
1. The following are properties of some quadrilaterals.
   a) Having a pair of parallel sides.
   b) Having two pairs of parallel sides.
   c) Having four right angles.
   d) Having four equal sides.
   e) Having equal diagonals.

   In the table below, mark a "✓" in the box if the quadrilateral has the property referred to in a) to e) above:

	PROPERTIES				
QUADRILATERALS	a)	b)	c)	d)	e)
Kite					
Trapezium					
Parallelogram					
Rectangle					
Square					
Rhombus					

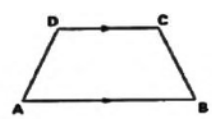
2. Referring to the figure below, use the names of the quadrilaterals to complete the sentences.

1.



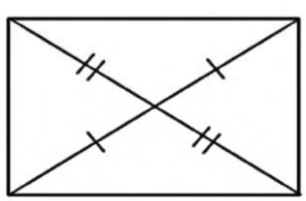
This is a _________________.

2.



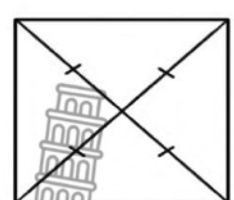
This is a _________________.

3.



This is a _________________.

4.



This is a _________________.

LESSON 6: PROPERTIES OF QUADRILATERALS (2)

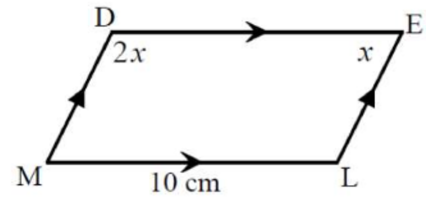
NOTES

EXAMPLES:

1. DELM is a parallelogram.

   a) Calculate the value of x and hence the sizes of the interior angles.

   b) If DE = 2DM and ML = 10 cm, determine the length of the other sides of DELM.



SOLUTION:

a) $2x + x = 180°$ [co-int $\angle s$; DM∥EL]

$3x = 180°$

$x = 60°$

$\hat{E} = \hat{M} = 60°$ [opp $\angle s$ of parm equal]

$\hat{D} = 120°$

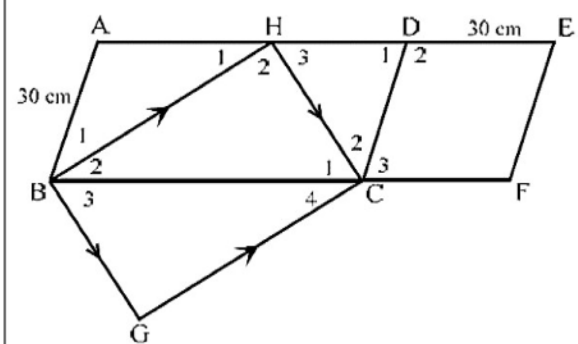
$\hat{L} = 120°$ [opp $\angle s$ of parm equal]

b) DE = 10 cm [opp sides of parm]

DM = 5 cm [DE = 2DM]

EL = 5 cm [opp sides of a parm]

2. ABCD is a parallelogram. BH bisects $A\hat{B}C$ and HC bisects $B\hat{C}D$. $A\hat{B}C = 60°$. $\hat{F} = 120°$. BH∥GC and BG∥HC.
   AD is produced to E such that AB = DE = 30 cm.
   BC is produced to F.
   Prove that
   a) BGCH is a rectangle.
   b) DCFE is a rhombus.



SOLUTION:

a) BCGH is a parallelogram [both pairs opp. sides are parallel]

$A\hat{B}C = 60°$ [given]

$\hat{B}_1 = \hat{B}_2 = 30°$ [BH bisects $A\hat{B}C$]

$B\hat{C}D = 120°$ [co-int $\angle s$; AB∥DC]

$\hat{C}_1 = \hat{C}_2 = 60°$ [HC bisects $B\hat{C}D$]

$\hat{H}_2 = 90°$ [sum of $\angle s$ *of* Δ]

$\therefore$ BGCH is a rectangle [BGCH is a parm with an interior $\angle$ = 90°]

b) F = 120°

$\hat{C}_1 + \hat{C}_2 = 120°$

$\therefore \hat{F} = \hat{C}_1 + \hat{C}_2$

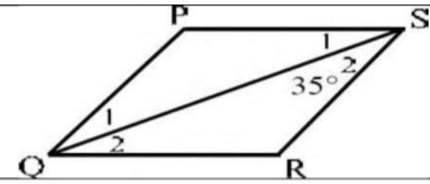
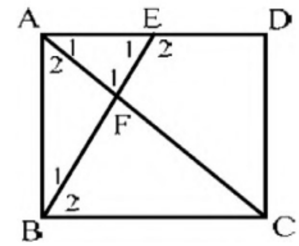
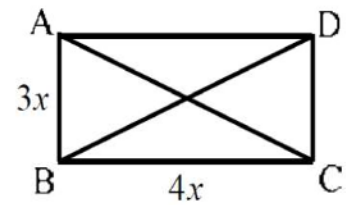
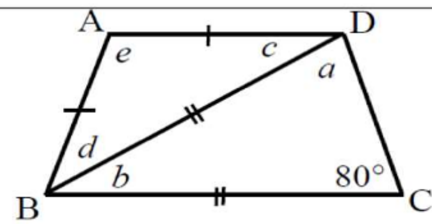
$\therefore$ DC∥EF [corresponding angels are equal]

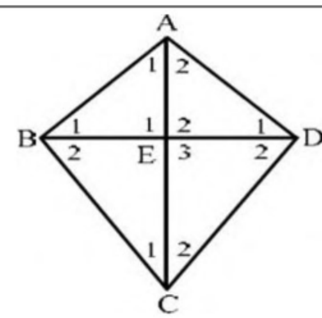
AD∥BC [opp. sides of parallelogram ABCD]

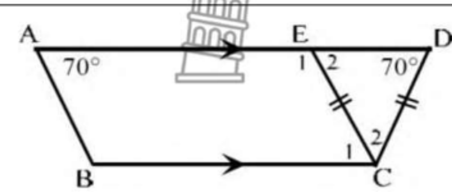
ADE and BCE are straight lines [given]

$\therefore$ DE‖CF	
DCFE is a parallelogram	[both pairs opp sides are ‖
DC = AB = 30 cm	[opp sides of a parm are equal]
$\therefore$ DC = DE = 30 cm	
$\therefore$ DCEF is a rhombus	[DCEF is a parm with adjacent sides equal]

ACTIVITY 6

1.	PQRS is a rhombus with $\hat{S}_2 = 35°$. Calculate the sizes of all the interior angles	
2.	ABCD is a square. $A\hat{E}B = 55°$ Calculate F_1	
3.	In rectangle ABCD, AB = $3x$ and BC = $4x$ Determine the length of AC and BD in terms of x.	
4.	ABCD is a trapezium with AD‖BC. AB = AD and BD = BC. $\hat{C} = 80°$ Determine the unknown angles.	

5.	ABCD is a kite. The diagonals intersect at E. BD = 14 cm, AD = $\sqrt{85}$ cm and DC = 25 cm. Determine: a) AE b) AC c) $\hat{B}_1$ if $\hat{A}_1 = 20°$.	

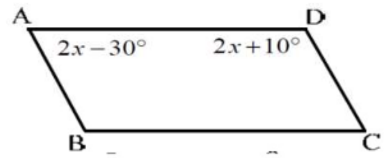
LESSON 7: PROPERTIES OF QUADRILATERALS (3)	
NOTES	
Example: In trapezium ABCD, AD‖BC with $\hat{A} = \hat{D} = 70°$ and EC = DC. Prove that ABCE is a parallelogram.	

Solution:

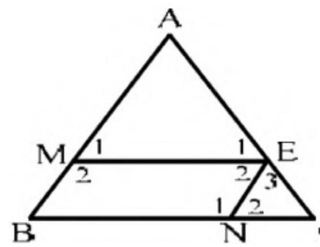
$\hat{E}_2 = 70°$	[∠s opp = sides]
AB ∥ EC	[corresponding ∠s are equal]
ABCE is a parallelogram	[both pairs opposite sides are equal]

ACTIVITY 7

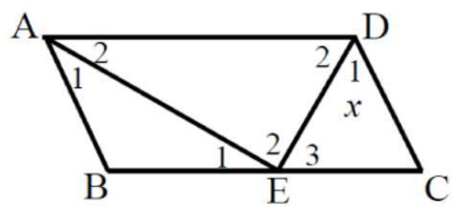
1. Determine the sizes of the interior angles of parallelogram ABCD.



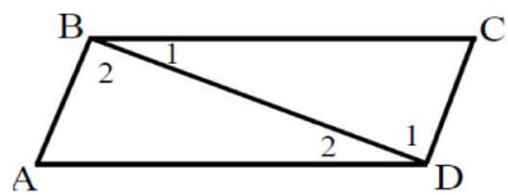
2. In ΔABC, $\hat{A} = 80°$ and $\hat{C} = 35°$ and parallelogram MENB
Calculate the size of angle MEN.



3. In parallelogram ABCD, AB = BE = DE.
$\hat{D}_1 = x$ and $\hat{A}_1 = 28°$.
Calculate x.



4. AD = BD = BC.
$\hat{C} = 75°$ and $A\hat{D}B = 30°$.
Prove that ABCD is a parallelogram.



LESSON 8: PROPERTIES OF QUADRILATERALS (4)

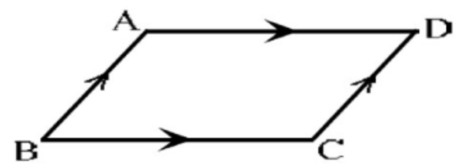
NOTES

The proofs of the following theorems are examinable in Gr. 10 Euclidean Geometry:

1. The opposite sides and angles of a parallelogram are equal.
2. The diagonals of a parallelogram bisect each other.
3. The diagonals of a rectangle are equal.
4. The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

As an example the proof of the first of these theorems is given below:

Prove that the opposite sides and angles of a parallelogram are equal.



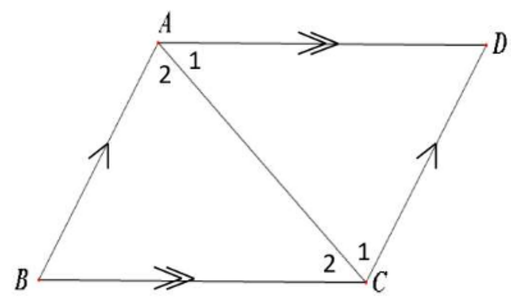
PROOF:

Given: Parallelogram ABCD.

R.T.P.: AB = CD and AD = BC

$B\hat{A}D = B\hat{C}D$ and $\hat{B} = \hat{D}$

Construction: Draw diagonal AC.



Proof: In ΔABC and ΔCDA:

1. $\hat{A}_2 = \hat{C}_1$ [alt. $\angle$'s; AB || CD]
2. $\hat{A}_1 = \hat{C}_2$ [alt. $\angle$'s; AD || BC]
3. $AC = AC$ [common]

$\therefore \Delta ABC \equiv \Delta CDA$ [$\angle;\angle;s$]

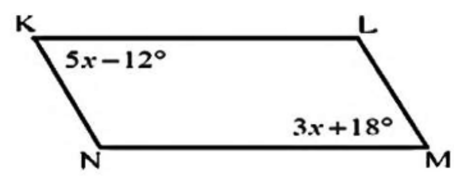
$\therefore AB = CD$ and $BC = AD$ and $\hat{B} = \hat{D}$ [$\equiv \Delta' s$]

Also: $\hat{A}_1 + \hat{A}_2 = \hat{C}_1 + \hat{C}_2$ [$\hat{A}_2 = \hat{C}_1$; $\hat{A}_1 = \hat{C}_2$]

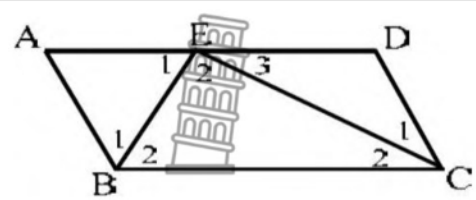
$\therefore B\hat{A}D = B\hat{C}D$

ACTIVITY 8

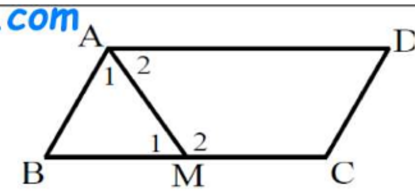
1. KLMN is a parallelogram
Calculate the size of the interior angles



2. In parallelogram ABCD, AB = 50 cm and E is a point on AD such that AB = AE and CD = DE.
Determine:
   a) DE
   b) the perimeter of ABCD.



3. ABCD is a parallelogram. AM bisects $\hat{A}$. AB = AM. $\hat{C} = 120°$.
Calculate the size of the interior angles.



LESSON 9: PROPERTIES OF QUADRILATERALS (5)

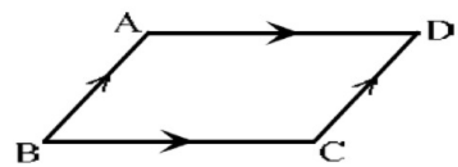
NOTES

The proofs of the following theorems are examinable in Gr. 10 Euclidean Geometry:

1. The opposite sides and angles of a parallelogram are equal.
2. The diagonals of a parallelogram bisect each other.
3. The diagonals of a rectangle are equal.
4. The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

As a further example the proof of the second of these theorems is given below:

Prove that the diagonals of a parallelogram bisect each other:



Given: Parallelogram ABCD with diagonals AC and BC intersecting in E.

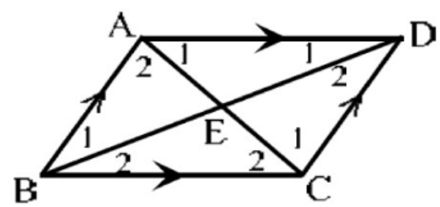
R.T.P.: AE = EC and BE = ED

Proof: In ΔABE and ΔCDE:

1. $\hat{A}_2 = \hat{C}_1$ [alt. ∠'s; AB || CD]
2. $\hat{B}_1 = \hat{D}_2$ [alt. ∠'s; AB || CD]
3. AB = CD [opp. sides of parm]

$\therefore \Delta ABE \equiv \Delta CDE$ [AAS]

$\therefore$ AE = EC and BE = ED [$\equiv \Delta$s]



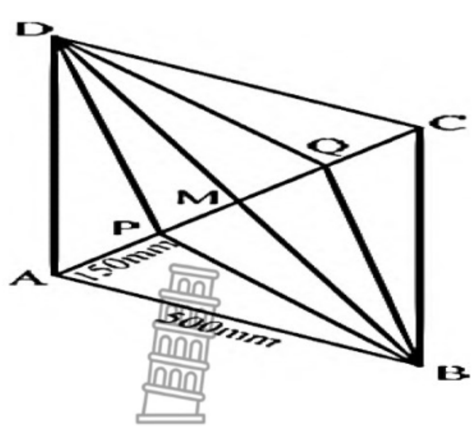
EXAMPLE:

Diagonals AC and BD of parallelogram ABCD intersect at M. AP = QC and AC =600 mm, AB = 500 mm and AP =150 mm.

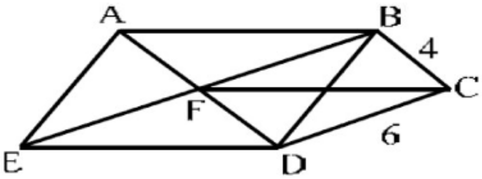
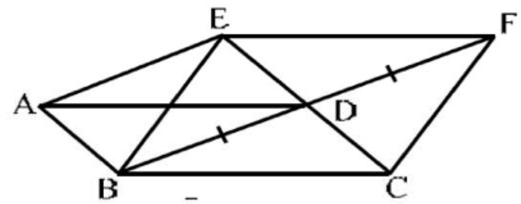
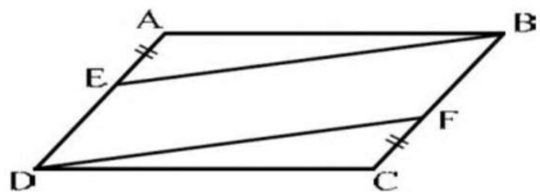
Prove that PBQD is a parallelogram

SOLUTION:

AM = MC [diagonals of a parm]
But AC = 600 mm [given]
AM = MC = 300 mm
AP = QC = 150 mm [given]
PM = MQ = 150 mm
Also, BM = MD [diagonals of parm]
∴ PM = MQ and BM = MD
∴ PBQD is a parallelogram [diagonal of quad bisect]



ACTIVITY 9

1.	In the diagram, BCDF, EDCF and ABCF are parallelograms. BC = 4 units and CD = 6 units. Prove that ABDE is a parallelogram.	
2.	Parallelograms ABCD and ABDE are given with DF = DB. Prove that BCFE is a parallelogram.	
3.	ABCD is a parallelogram with AE = FC. Prove that BEDF is a parallelogram.	

LESSON 10: PROPERTIES OF QUADRILATERALS (6)

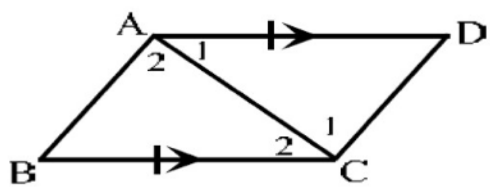
NOTES

TAKE NOTE: Learners need to be able to apply the theorem below, but the **proof** of the theorem is not examinable. It is given here as enrichment.

Theorem: If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram

Required to prove: ABCD is a parallelogram:

Construction: Draw diagonal AC.



Proof:

In ΔABC and ΔCDA:

(a) $\hat{C}_2 = \hat{A}_1$ [alt. $\angle$s; AD $\parallel$ BC]

(b) AC = AC [common]

(c) BC = AD [given]

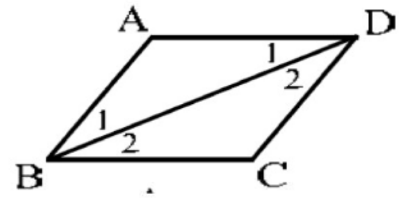
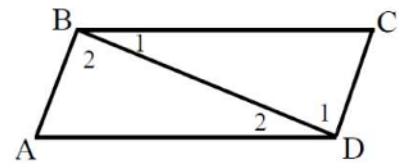
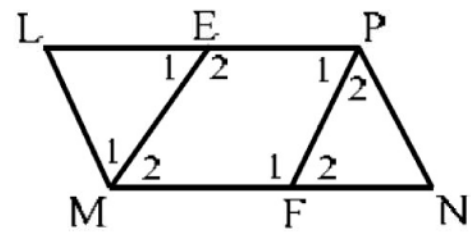
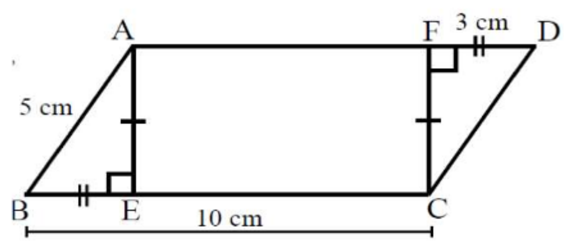
$\therefore \Delta ABC \equiv \Delta CDA$ [SAS]

$\hat{A}_2 = \hat{C}_1$ [$\equiv \Delta$s]

$\therefore$ AB $\parallel$ CD [alt. $\angle$s are equal]

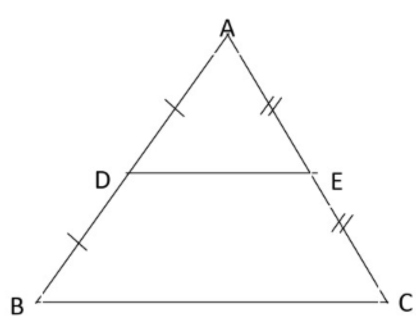
$\therefore$ ABCD is a parallelogram [both pairs opp. sides are $\parallel$]

ACTIVITY 10

1.	In parallelogram ABCD, AB =AD and $\hat{C} = 100°$. Calculate the sizes of all the interior angles.	
2.	ΔABD and ΔBCD are two isosceles triangles. $\hat{C} = 75°$ and $A\hat{D}B = 30°$. Prove that ABCD is a parallelogram.	
3.	In quadrilateral LMNP, $\hat{E}_1 = 62°$, $\hat{P}_1 = 68°$, FP = FN and LE = LM. Prove that: a) LP‖MN b) LMNP is a parallelogram	
4.	In quadrilateral ABCD, AB = 5 cm, BC = 10 cm, FD = 3 cm, BE = FD and AE = FC. AE ⊥ BC and CF ⊥ AD. Prove that ABCD is a parallelogram.	

LESSON 11: MIDPOINT THEOREM							
Term	2	**Duration**	2 hours	**Grade**	10	**Date**	
NOTES							
The **midpoint** is the centre of a line segment (it bisects the line segment).							

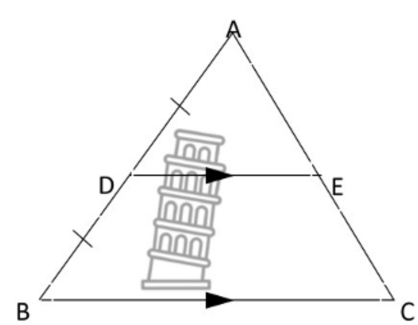
MIDPOINT THEOREM:



If AD = DB and AE = EC,

then DE‖BC and $DE = \frac{1}{2}BC$

CONVERSE OF MIDPOINT THEOREM:



If AD = BD and DE‖BC

then AE = EC and $DE = \frac{1}{2}BC$

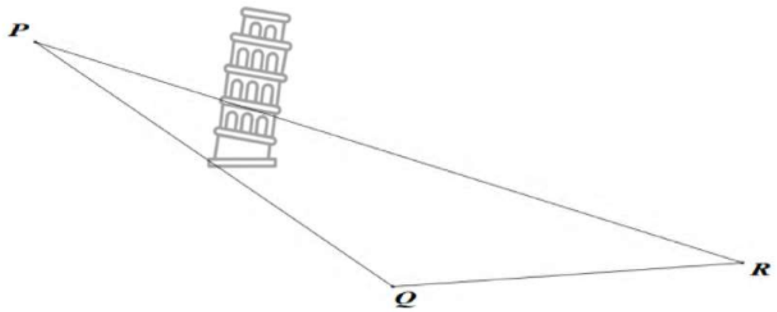
ACTIVITY 11

PART A

INVESTIGATING THE MIDPOINT THEOREM

Use a ruler and determine midpoint S of side PQ and midpoint T of side PR of triangle PQR.

Indicate on the sketch that PS = SQ and PT = TR. Draw ST.



1. Measure each of the following angles, using a protractor:

1.1 $\hat{Q}$ = ... degrees　　1.3 $P\hat{S}T$ = ... degrees

1.2 $\hat{R}$ = ... degrees　　1.4 $P\hat{T}S$ = ... degrees　　(4)

2. What do you notice concerning your answers in question 1?

…………………………………………………………………………………………(2)

3. Using your answer to question 2, what can you conclude concerning ST and QR?

…………………………………………………………………………………………(1)

4. Give a reason for your answer in question 3.

…………………………………………………………………………………………(1)

5. Measure the lengths of the following sides:

5.1 ST = ………………. mm　　5.2 QR = ………………. mm　　(2)

6. What do you notice concerning the lengths of ST and QR?

…………………………………………………………………………………………(1)

7. Hence, make a conjecture regarding the line joining the midpoints of two sides of any triangle:

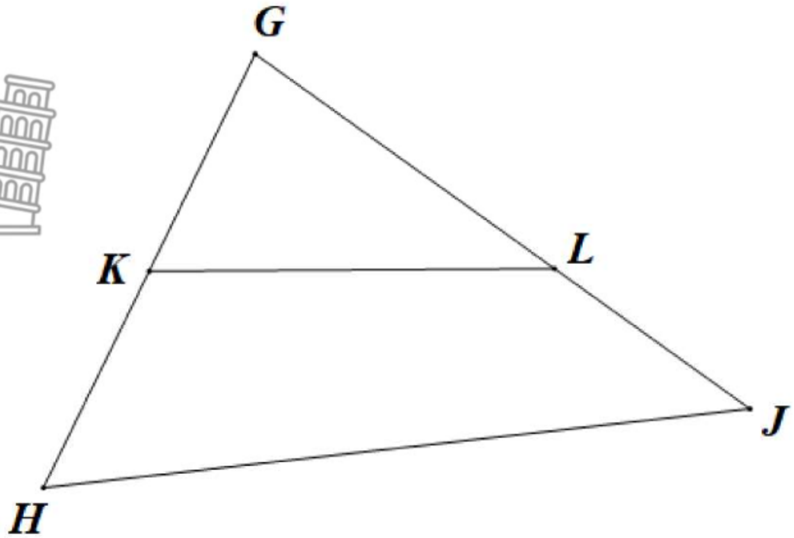
…………………………………………………………………………………………

…………………………………………………………………………………………(2)

PART B

In this part all answers have to be justified, using accurate measurements of distances and/or angles.

Given: ΔGHJ with K on GH, and L on GJ.



1. Are K and L the midpoints of GH and GJ respectively?

   …………………………………………………………………………………..…………… (2)

2. Is KL || HJ?

   ……………………………………………………………………………………………….. (3)

3. Is HJ = 2KL?

   ………………………………………………………………………………………………….(1)

ACTIVITY 12

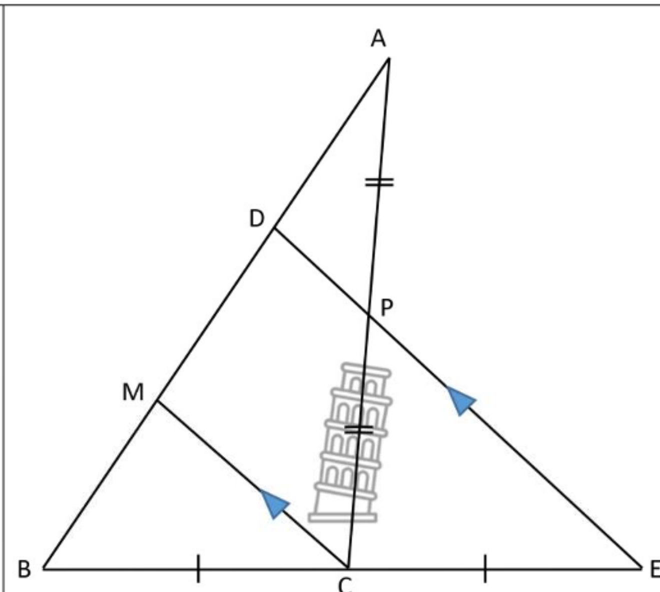
1. Given: AD = 5 cm and MC = 6 cm.

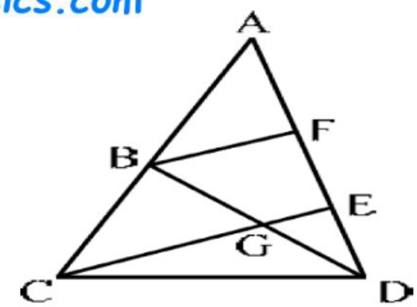
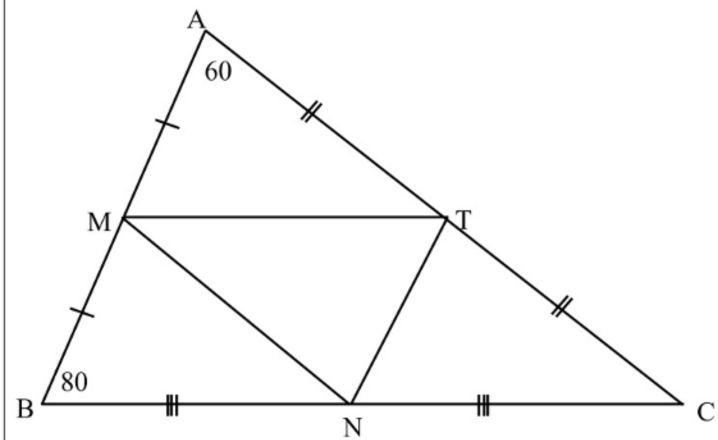
   Calculate, with reasons:

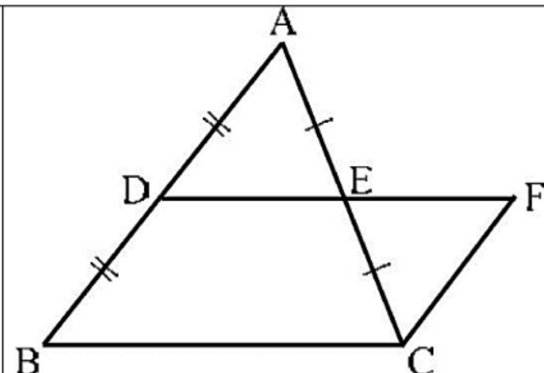
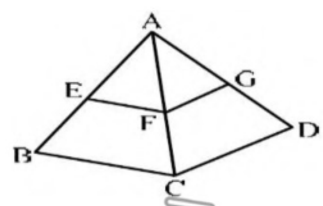
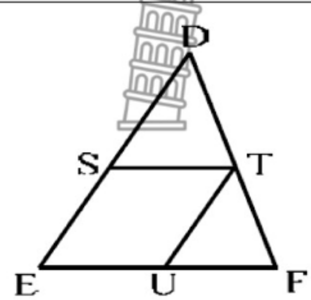
   1.1 The length of BM

   1.2 The length of DP

   1.3 The length of DE



2.	In ΔACD, AB = BC, GE = 15 cm, AF = FE = ED. Calculate the length of CE.	
3.	M, N and T are the midpoints of AB, BC and AC of ΔABC. $\hat{A}=60°$ and $\hat{B}=80°$. Calculate the interior angles of ΔMNT.	

ACTIVITY 13		
1.	In ΔABC, AD = DB and AE = EC. DE is produced to F. DB||FC and BC = 32 mm. a) Prove that DBCF is a parallelogram. b) Calculate the length of DE.	
2.	In ΔABC, AE = EB and EF||BC. In Δ ACD, FG||CD. Prove that AG = GD.	
3.	In ΔDEF, DS = SE, EU = EF and ST||EF. Prove that SEUT is a parallelogram.	

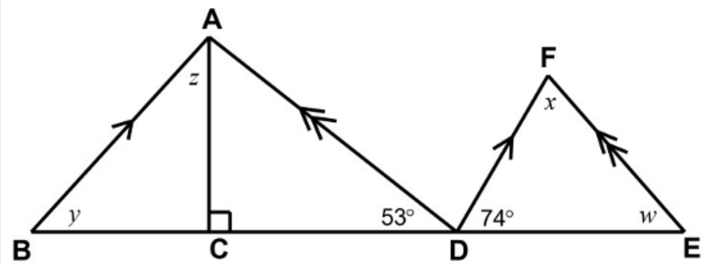
LESSON 12: CONSOLIDATION OF GR. 10 EUCLIDEAN GEOMETRY

ACTIVITIES/ASSESSMENTS

Activity 14

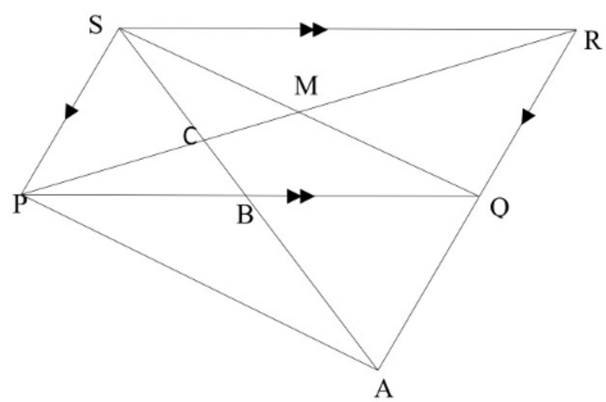
1. Study the diagram below and calculate the unknown angles w, x, y and z.

   Give reasons for your statements.



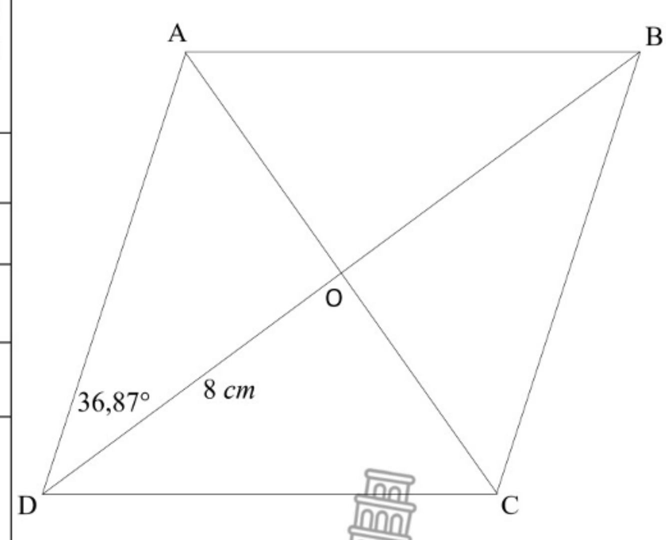
2. **DBE NOV. 2015 GRADE 10**

   In the diagram below, PQRS is a parallelogram having diagonals PR and QS intersecting in M. B is a point on PQ such that SBA and RQA are straight lines and SB = BA. SA cuts PR in C and PA is drawn.

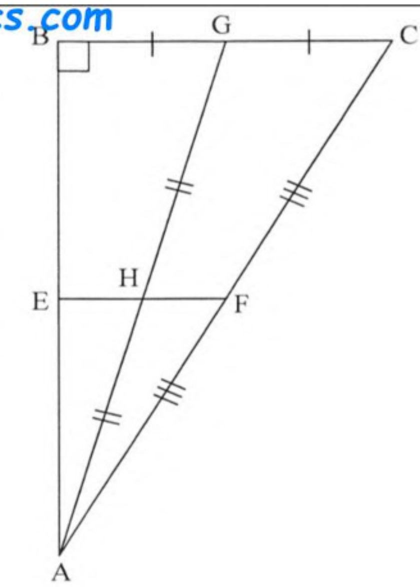


2.1	Prove that SP = QA.
2.2	Prove that SPAQ is a parallelogram
2.3	Prove that AR = 4MB.

3. In the diagram, ABCD is a rhombus having diagonals AC and BD intersecting in O. $A\hat{D}O = 36{,}87°$ and DO = 8 cm.



3.1	Write down the size of the following angles:	
	3.1.1	$C\hat{D}O$
	3.1.2	$A\hat{O}D$
3.2	Calculate the length of AO.	
3.3	If E is a point on AB such that OE | | AD, calculate the length of OE.	

4.	ΔABC is right angled at B. F and G are midpoints of AC and BC respectively. H is the midpoint of AG. E lies on AB such that FHE is a straight line.		
	4.1	Prove that E is the midpoint of AB.	
	4.2	If EH = 3,5cm and the area of $\Delta AEH = 9{,}5\text{cm}^2$, calculate the length of AB.	
	4.3	Hence, calculate the area of ΔABC.	

LESSON 1: DERIVATION AND APPLICATION OF A DISTANCE FORMULA

NOTES

Calculating the length of a line passing through the given two points

- Write down the correct formula $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ OR $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ where d_{AB} represents the distance of line passing through the points A and B
- Substitute the points into the formula
- Simplify the answer
- Correct your answer to two decimal places unless stated otherwise

Examples

1. Determine the length of a line passing through the points $A(-3;4)$ and $B(4;4)$

$$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3-4)^2 + (4-4)^2}$$
$$= \sqrt{(-7)^2 + 0^2}$$
$$= \sqrt{49}$$
$$= 7$$

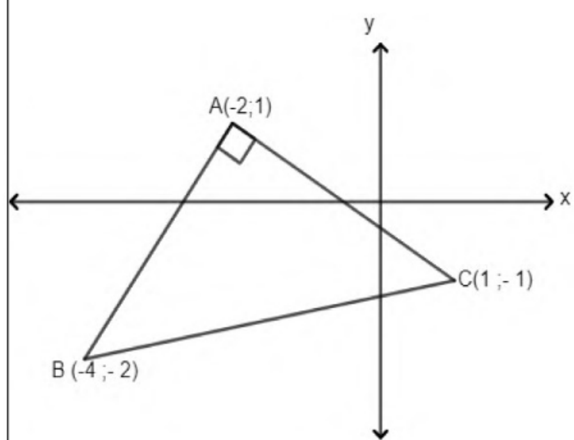
2. Given the points $P(2;-1)$ and $Q(-3;5)$;determine the length of line passing through the points

$$d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2-(-3))^2 + (-1-5)^2}$$
$$= \sqrt{5^2 + (-6)^2}$$
$$= \sqrt{25+36}$$
$$= \sqrt{61}$$
$$= 7,81$$

3. Determine the length of XY if it passes through the points $X(4;-5)$ and $Y(4;3)$

$$d_{XY} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-4)^2 + (5-(-3))^2}$$
$$= \sqrt{(0)^2 + (8)^2}$$
$$= \sqrt{0+64}$$
$$= 8$$

4. Given triangle ABC. Determine the following:
   4.1 The length of AB.
   4.2 The length of AC.
   4.3 Based on 4.1 and 4.2. write down the type triangle ABC.



Solution

4.1. $$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2+4)^2 + (1+2)^2}$$
$$= \sqrt{(2)^2 + (3)^2}$$
$$= \sqrt{(4+9}$$
$$= \sqrt{13}$$
$$= 3,61$$

4.2. $$d_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-1)^2 + (1+1)^2}$$
$$= \sqrt{(-3)^2 + (2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$
$$= 3,61$$

4.3. Right-angled isosceles triangle

ACTIVITIES/ ASSESSMENT: LESSON 1

1. Determine the distance between the two points. Correct your answer to TWO decimal places where necessary

1.1. $(2;-4)$ and $(1;1)$
1.2. $(3;-1)$ and $(-3;5)$
1.3. $(2;7)$ and $(6;4)$

2. Given the diagram below

2.1. Determine the length of

2.1.1 PS

2.1.2. PQ

2.1.3. SR

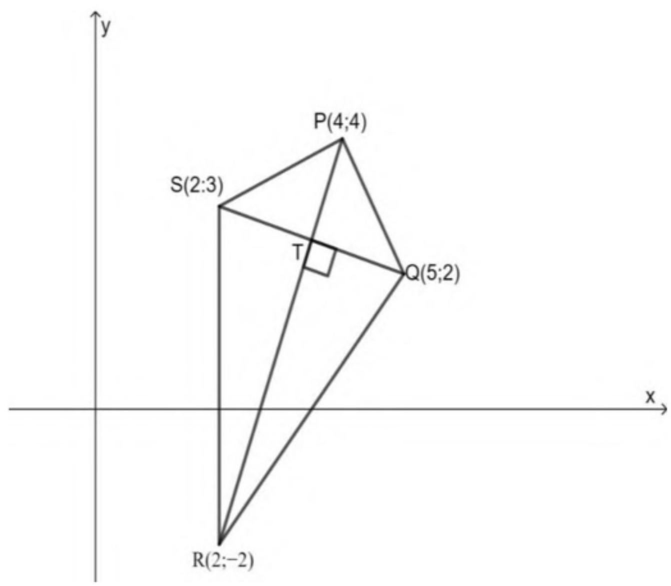
2.1.4. QR

2.1.5. PR

2.2. Given $T\left(\frac{7}{2};\frac{5}{2}\right)$ is a point where PR and SQ meet, Calculate the following lengths

2.2.1. TR

2.2.2. TQ



LESSON 2 : APPLICATION OF A GRADIENT FORMULA

NOTES

DEFINITION

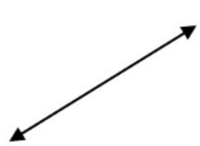
The symbol used for the gradient is m

$$m=\frac{\text{vertical change}}{\text{horizontal change}}\text{ or }\frac{\text{change in }y}{\text{change in }x}=\frac{\Delta y}{\Delta x}=\frac{\text{how much you go up/ down}}{\text{how much you go forward/backwards}}$$

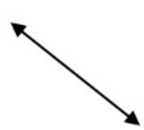
A summary of important facts concerning gradient:

- Types of lines:

A rising line (**FROM LEFT TO RIGHT)** has a **POSITIVE** gradient



A declining line **(FROM LEFT TO RIGHT)** has a **NEGATIVE** gradient

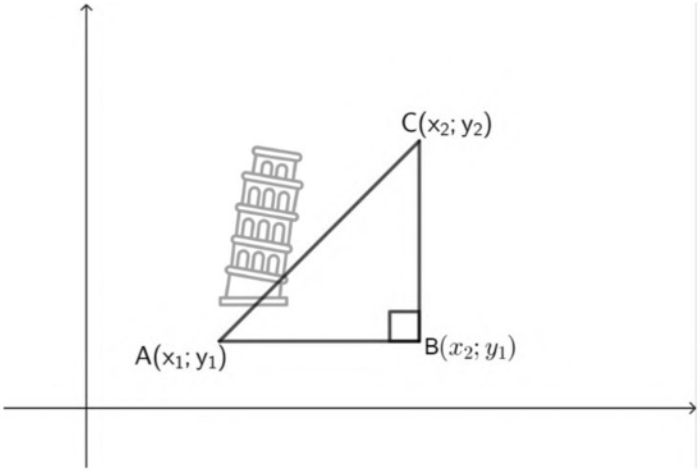


A horizontal line has a gradient of **ZERO**

The gradient of a vertical line is **UNDEFINED**.

- If two lines are parallel, then $m_1 = m_2$.

- If two lines are perpendicular then $m_1 \times m_2 = -1$.



horizontal change = AB = $x_2 - x_1$

vertical change = BC = $y_2 - y_1$

$$m = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In order to calculate the gradient of a line passing through the given two points :

- Write down the formula
- Substitute the points into the formula
- Simplify the answer
- Correct your answer to two decimal places unless stated otherwise

EXAMPLES

Determine the gradient of a line passing through the given points:

1.1 $P(1;-1)$ and $Q(-1;5)$

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-5}{1-(-1)}$$

$$= 3$$

1.2 A(5 ;4) and B(2 ;1)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-1}{5-2}$$

$$= 3$$

1.3 Comment about answers on 1.2 and 1.1.

It means $AB // PQ$, $m_{AB} = m_{PQ}$.

1.4 $X\left(-\frac{2}{3};\frac{1}{2}\right)$ and $Y\left(-\frac{1}{3};-\frac{3}{2}\right)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{1}{2}-\left(-\frac{3}{2}\right)}{-\frac{2}{3}-\left(-\frac{1}{3}\right)}$$

$$= -6$$

1.5 A(5 ;0) and B(2 ;0)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-0}{5-2}$$

$$= 0$$

1.6 Comment about your answer in 1.5.

AB is a horizontal line, $m_{AB} = 0$.

1.7 A(0 ;4) and B(0 ;1)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-1}{0}$$

AB is a vertical line, gradient is undefined.

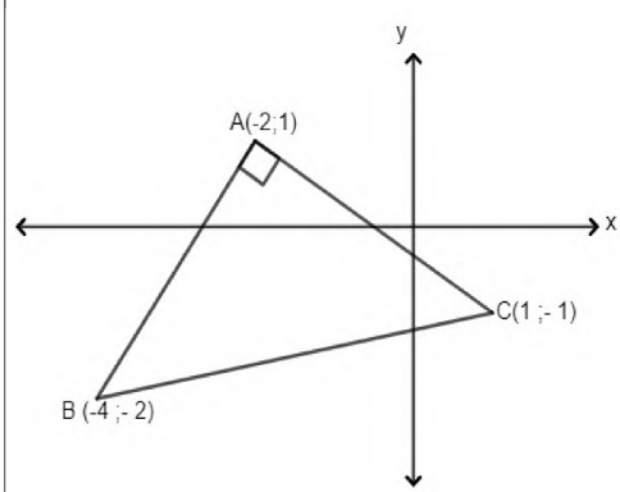
1.8 Given the triangle ABC with points. Determine the gradient of:

1.8.1 AB

1.8.2 AC

1.8.3 BC

1.8.4 Comment about your answer to 1.8.1 and 1.8.2 .



Solutions

1.6.1 $m_{AB} = \frac{y_B - y_A}{x_B - x_A}$

$$= \frac{-2-1}{-4+2}$$

$$= \frac{-3}{-2}$$

$$= \frac{3}{2}$$

1.6.2 $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-1-1}{1+2}$$

$$= -\frac{2}{3}$$

1.6.3 $m_{BC} = \frac{y_C - y_B}{x_C - x_B}$

$$= \frac{-2+1}{-4-1}$$

$$= \frac{-1}{-5}$$

$$= \frac{1}{5}$$

1.6.4 They confirmed that $AB \perp AC$,

$m_{AB} \times m_{AC} = -1$.

ACTIVITIES/ASSESSMENTS: LESSON 2

1 Determine the gradient of a line passing through the points

1.1 (2 ;–4) and (1 ;1)

1.2 (3;–1) and (–3 ;5)

1.3 $\left(\frac{1}{2};-\frac{1}{6}\right)$ and $\left(\frac{2}{3};-\frac{5}{2}\right)$

1.4 The gradient of a line that passes through the points $(-2;4)$ and $(x;3)$ is 2. Determine the value of x.

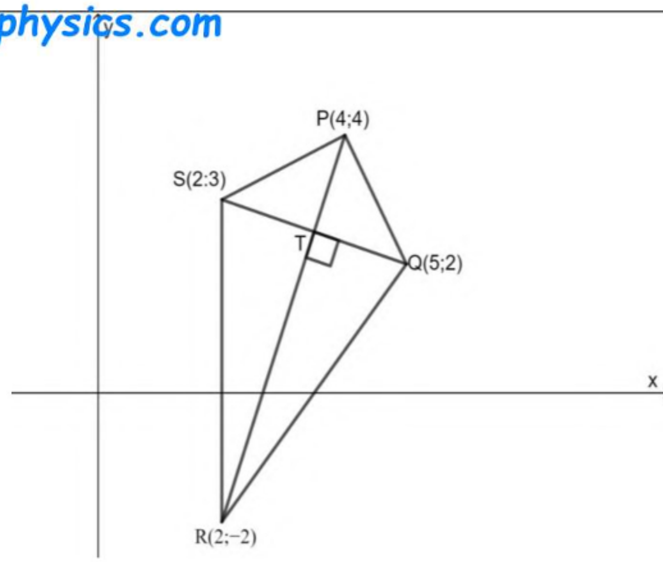
2 In the diagram alongside, determine the gradient of:

2.1 PS

2.2 PQ

2.3 If $T\left(\frac{7}{2};\frac{5}{2}\right)$ is the point where SQ and PR meet, show that PQRS is a kite.

2.4 Comment about your answers in 2.1 and 2.2.



LESSON 3: THE GRADIENT OF A LINE
NOTES

Definition of terms

- **Parallel** lines are lines that are the same distance apart. Parallel lines never intersect (meet). They have equal gradients.
- **Perpendicular** lines are lines that intersect at $90°$. The product of their gradients result to -1.
- A **vertical** line is upright, perpendicular to a plane or any line that is regarded as a base. In a Cartesian plane this line is parallel to the *y* axis and perpendicular to the *x* axis.
- A **horizontal** line is parallel to the horizon. This line is parallel to the *x* axis and perpendicular to the *y* axis
- **Collinear** points are points that lie on the same line. The gradients between the points are the same.

ACTIVITIES/ASSESSMENTS: LESSON 3

Activity 1				
1.	Refer to worksheet 1 and answer the following			
	1.1.	Show by calculation that the following lines are parallel		
	.	1.1.1.	JG and LN	
		1.1.2.	PQ and EF	
	1.2.	Will the line passing through SK and CH be parallel? Justify your answer. With calculations.		
	1.3.	The line parallel to GT passes through the points $(-1\ ;\ 3)$ and $(1\ ;\ y)$. Determine the value of *y*.		
	1.4.	Calculate the gradients of the lines passing through AB; CD; and SN.		
	1.5.	Hence conclude about the gradient of a horizontal line		
	1.6.	Comment on the gradient of a vertical line based on calculations.		

1.7	Explain why is $m_{JR} = m_{KR}$.	

Activity 2

Refer to worksheet 1		
2.1	Show that $RM \perp LN$ given that R is the midpoint.	
2.2	Calculate the gradient of a line perpendicular to line passing through QJ.	
2.3	The line drawn perpendicular to QG passes through (x ;-2) and the origin . Determine the value of x.	

Activity 3

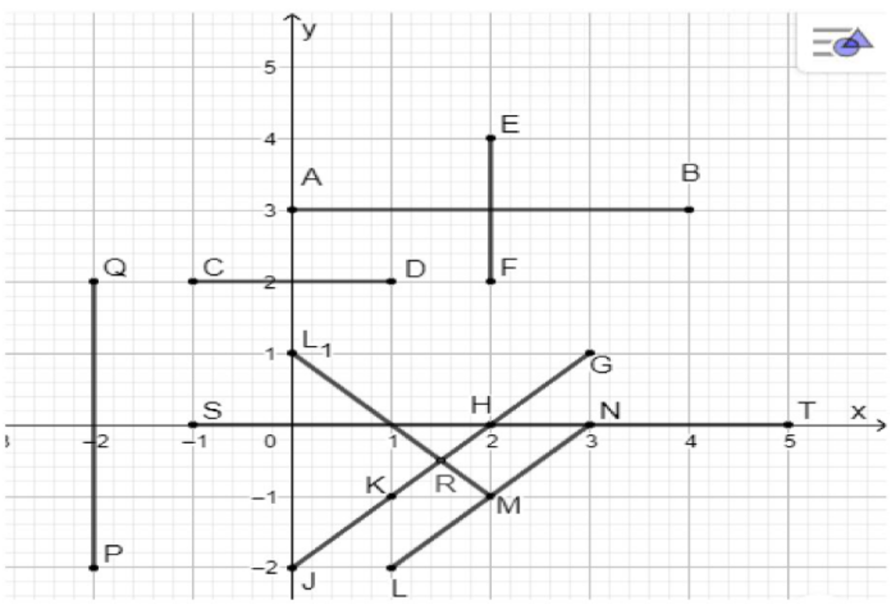
3.1 Given : A(-1;-3), B(1 ; -2) and C(x ;-1).

Calculate the value of x if the points A, B and C are collinear.

3.2 Points A(2;8), B(8;-1) and C(4;a) lie on the Cartesian plane. Find the value of a if A, B and C are collinear points.

WORKSHEET 1

Consider the following lines and answer the questions that follow.



LESSON 4 : THE MIDPOINT OF A LINE SEGMENT
NOTES
• **To calculate the distance between the two points or the length of a horizontal line segment:**

Examples

1. In the sketch below: M lies on AB

   1.1. Write down the coordinates of M by reading from the sketch.

   1.2. Confirm by using the distance formula that M is the midpoint of AB.

Solutions

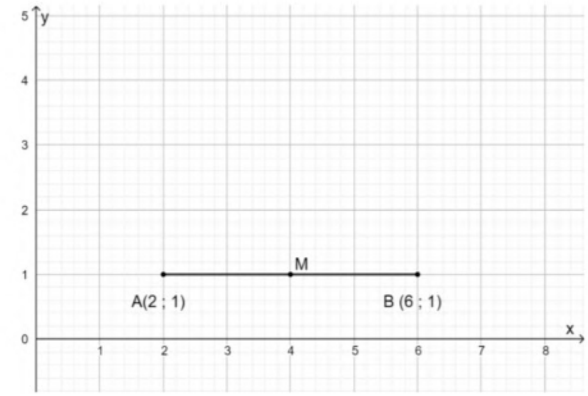
1.1 $M(4;1)$

1.2 $M\left(\dfrac{x_A+x_B}{2};\dfrac{y_A+y_B}{2}\right)$

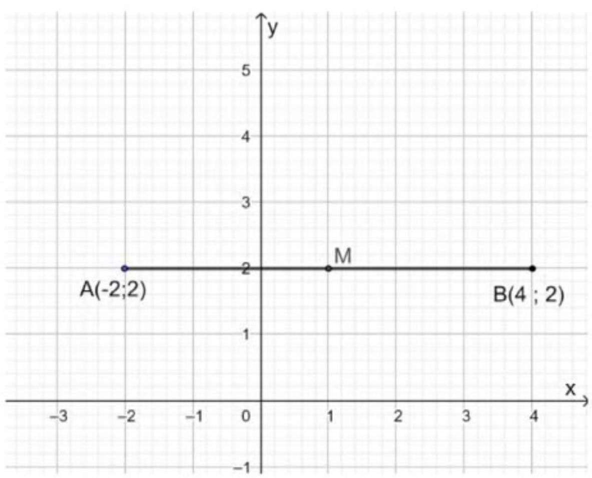
$M\left(\dfrac{2+6}{2};\dfrac{1+1}{2}\right)$

$M\left(\dfrac{8}{2};\dfrac{2}{2}\right)$

$M(4;1)$



2. Repeat the same procedures for the sketches that follow:



Solutions

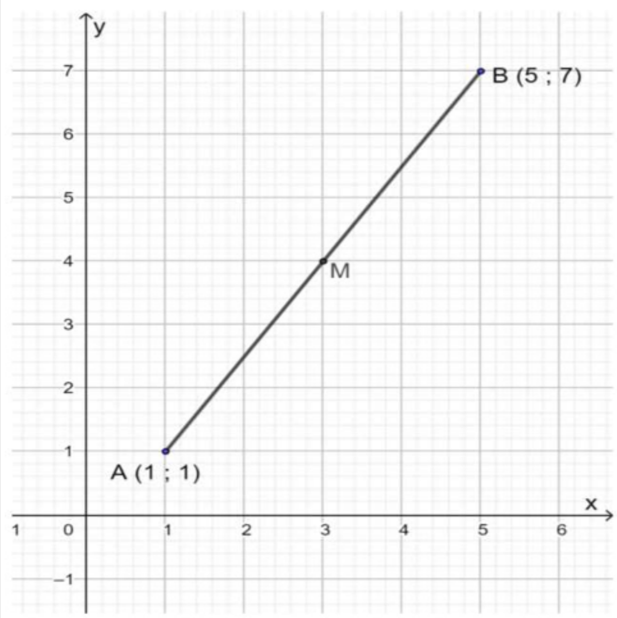
By reading from diagram: $M(1;2)$

By calculation: $M\left(\dfrac{x_A+x_B}{2};\dfrac{y_A+y_B}{2}\right)$

$M\left(\dfrac{-2+4}{2};\dfrac{2+2}{2}\right)$

$M\left(\dfrac{2}{2};\dfrac{4}{2}\right)$

3.



Solutions

By reading from diagram: $M(3;4)$

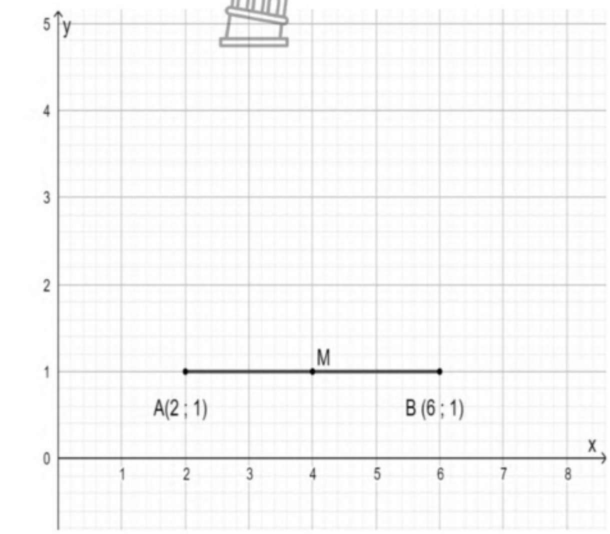
By calculation: $M\left(\dfrac{x_A+x_B}{2};\dfrac{y_A+y_B}{2}\right)$

$M\left(\dfrac{1+5}{2};\dfrac{1+7}{2}\right)$

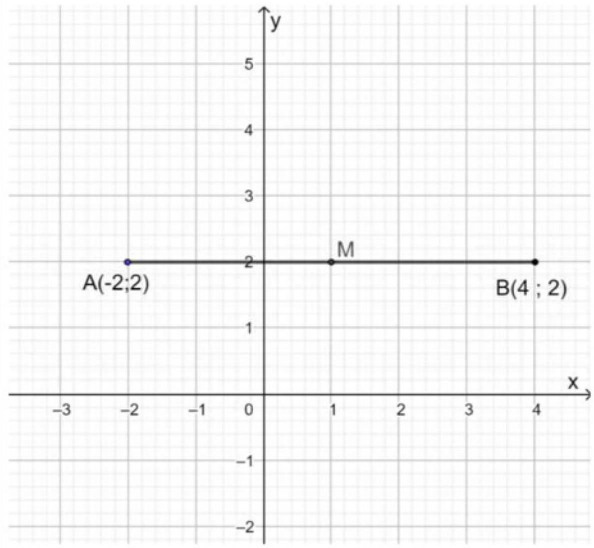
$M(1;2)$	$M\left(\frac{6}{2};\frac{8}{2}\right)$ $M(3;4)$

Worksheet 2 (This must be printed for learners and must be attached to the learners working)

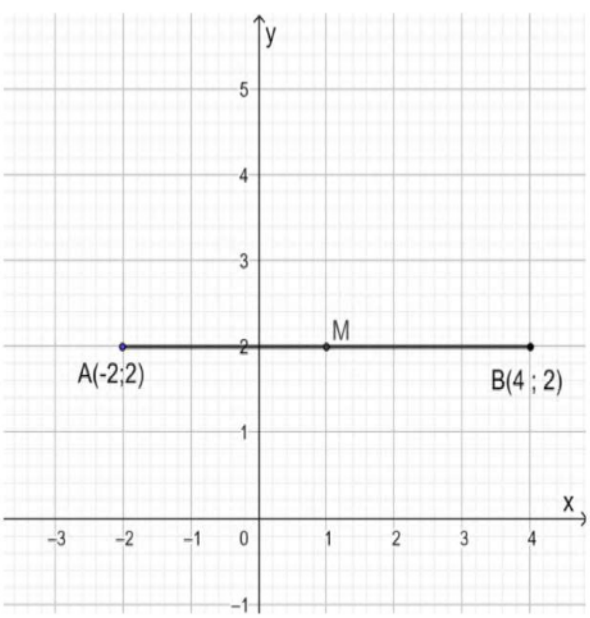
1.



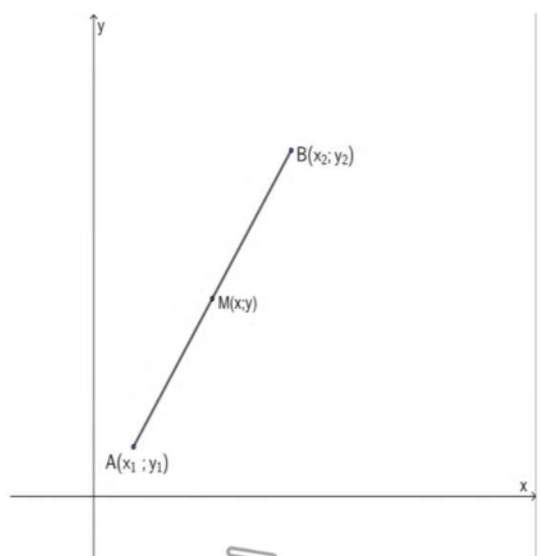
2.



3.



4.



ACTIVITIES/ASSESSMENTS: LESSON 4

1.1 Calculate the coordinates of M the midpoint of the line segment joining points A (1;6) and B (5;8)

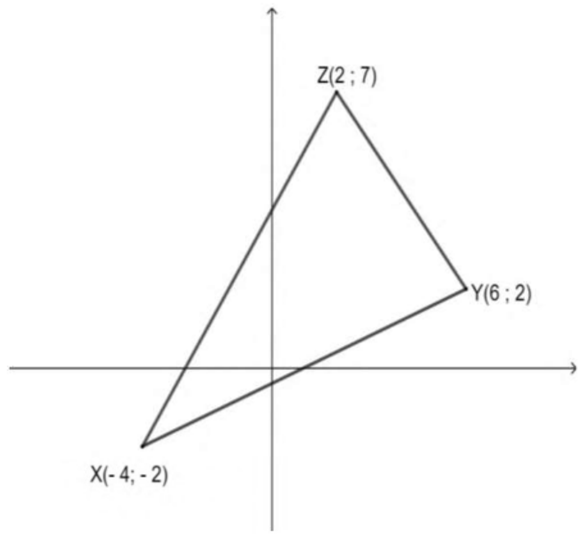
1.2 If M (-3;2) is the midpoint of the line joining the points A (x;1) and B (-1;y), calculate the value of x and y.

2 In the diagram $X(-4;\ -2)$ $Y(6\ ;\ 2)$ and $Z(2\ ;\ 7)$ and Z are the vertices of Δ XYZ. Calculate the following:

2.1. The coordinates of K, the midpoint of XY.

2.2. The length of ZK

2.3 Sipho claims that $YZ \perp XY$. Is he correct? Motivate your answer by calculations.



3 Given is a quadrilateral with AB// CD where A (-1; 4), B (0; *p*), C (2; 3) and D (3; 0).

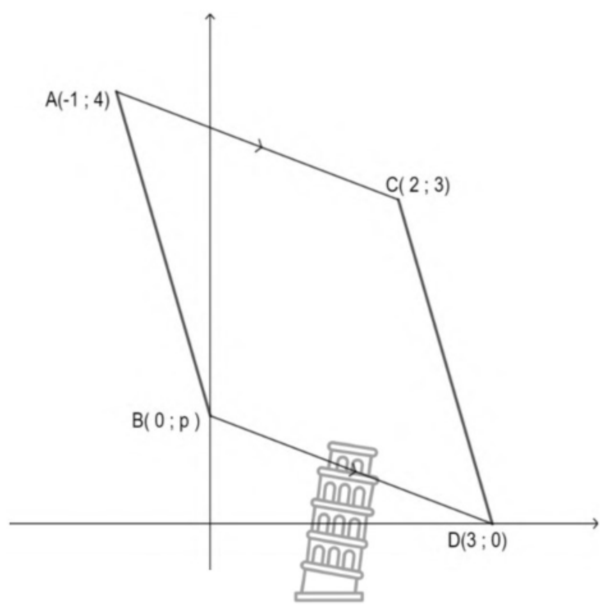
3.1 Calculate the length of AD

3.2. Calculate the value of *p*

3.3. Show that $AD \perp BC$.

3.4. Determine M the midpoint of AB.

3.5. What type of a quadrilateral is ABCD? Motivate your answer.



LESSON 5: DERIVATION AND APPLICATION OF A DISTANCE FORMULA

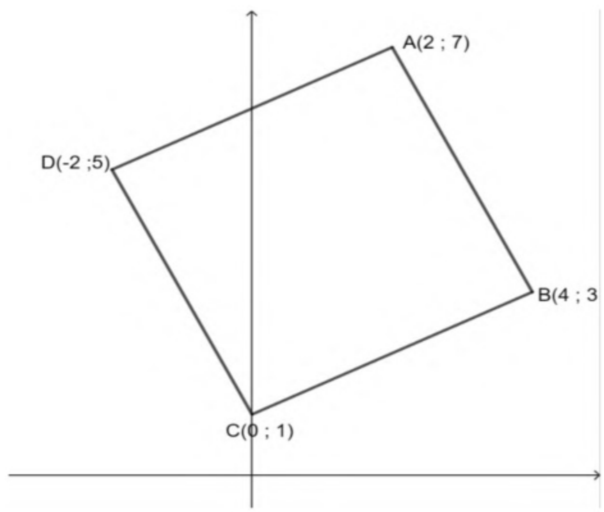
NOTES

To prove that a quadrilateral is a parallelogram, one the following must be true

- AB // CD; AD // BC.
- $AB = DC; AD = BC.$
- AD = BC; AD // BC.
- AB = CD; AB // CD.
- $M\left(\frac{x_A+x_C}{2};\frac{y_A+y_C}{2}\right)$ is the same as $M\left(\frac{x_B+x_D}{2};\frac{y_B+y_D}{2}\right)$

Examples

1. In the diagram, A(4 ; 3), B(2 ; 7), C(-2 ; 5) and D(0 ; 1) are the vertices of a quadrilateral. Use the diagonals (midpoint) and inspection method to prove that quadrilateral ABCD is a parallelogram.



Solution

Consider the midpoint of DB to be P.

$P\left(\frac{x_B+x_D}{2};\frac{y_B+y_D}{2}\right)$

$P\left(\frac{4-2}{2};\frac{3+5}{2}\right)$

$P(1;4)$

Consider the midpoint to be Q.

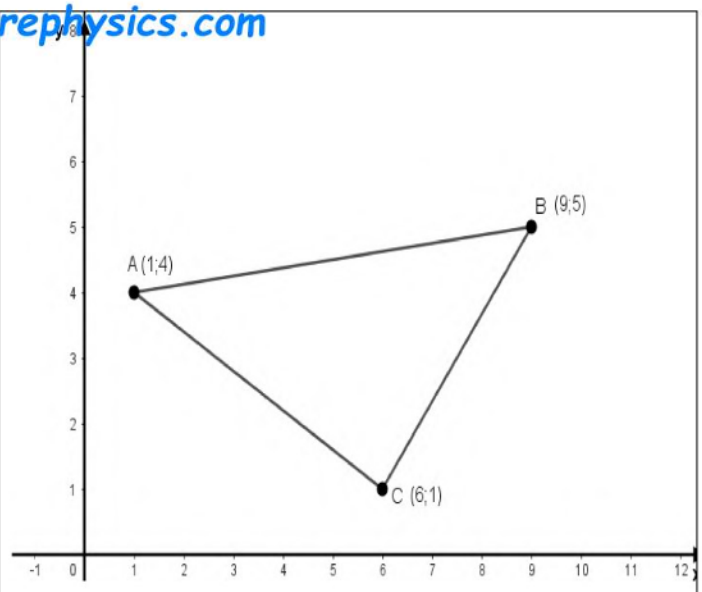
$Q\left(\frac{x_C+x_A}{2};\frac{y_C+y_A}{2}\right)$

$Q\left(\frac{0+2}{2};\frac{1+7}{2}\right)$

$Q(1;4)$

Since the coordinates of P and Q are the same, it proves that ABCD is parallelogram.

2. $A(1;4)$, $B(9;5)$ and $C(6;1)$ are the coordinates of a triangle.
   2.1 If ADBC is a parallelogram, determine the coordinates of D.
   2.2 Determine the equation of a height passing through point C.



Solutions

2.1 Learners need to remember that diagonals of a parallelogram bisect each other. Let M be the midpoint of AB.

$$\mathrm{M}\left(\frac{x_B+x_A}{2};\frac{y_B+y_A}{2}\right)$$

$$M\left(\frac{9+1}{2};\frac{5+4}{2}\right)$$

$$\mathrm{M}\left(5;\frac{9}{2}\right)$$

$$x_M=\frac{x_D+x_C}{2}$$

$$5=\frac{x_D+6}{2}$$

$$10=x_D+6$$

$$x_D=4$$

$$y_M=\frac{y_D+y_C}{2}$$

$$\frac{9}{2}=\frac{y_D+1}{2}$$

$$9=y_D+1$$

$$y_D=8$$

$$\therefore D(4;8)$$

Alternatively, by inspection:

Learners may check horizontal shift and vertical shift. Moving from point C to be B gives the same move as A to D.

$x_C+k\rightarrow x_B$ i.e. moving from C you add/ remove something to reach B.

Solutions

2.2 Learners have to recall that the height is perpendicular to the base (AB in this case).

$$m_{AB}=\frac{y_B-y_A}{x_B-x_A}$$

$$=\frac{5-4}{9-1}$$

$$=\frac{1}{8}$$

$$m_{\perp}=-8 \quad \text{(Remember } m_1\times m_2=-1\text{)}$$

$$y-y_1=m(x-x_1)$$

$$y-1=-8(x-6)$$

$$y-1=-8x+48$$

$$y=-8x+49$$

$6+3 \to 9$ $x_A + k \to x_D$ $1+3 \to 4$ $y_C + k \to y_B$ $1+4 \to 5$, then for D, $4+4 \to 8$ $\therefore D(4;8)$	

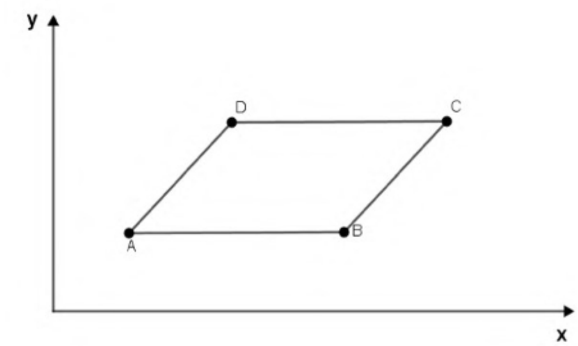
ACTIVITIES/ ASSESSMENT: LESSON 5

1. The diagram represents a quadrilateral with points $A(2;2)$, $B(5;3)$, $C(6;6)$ and $D(3;5)$.

1.1 Calculate the length of AB.

1.2 Determine the coordinates of M, the midpoint of AC.

1.3 Prove that quadrilateral ABCD is a parallelogram. (Use the diagonals).



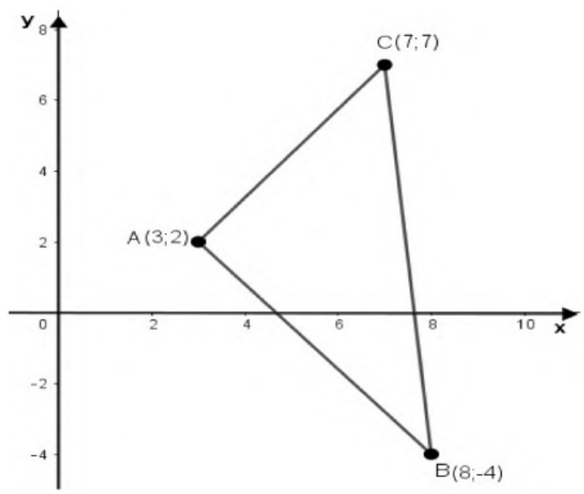
2. Given is triangle ABC with $A(3;2)$, $B(8;-4)$ and $C(7;7)$.

2.1 Determine the midpoint of BC.

2.2 Find the coordinates of P if ABPC is a parallelogram.

2.3 Determine the gradient of AC.

2.4 Calculate the equation of a line perpendicular to AC passing through A.

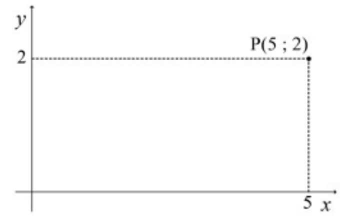


LESSON 1: INTRODUCTION TO THE CARTESIAN PLANE AND FUNCTIONS

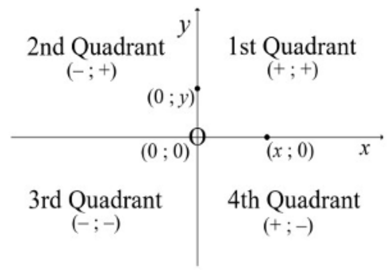
NOTES

- **Revision**:

I. Coordinates of a point:
If a point P has coordinates (5 ; 2),
we can say that:
the x-coordinate of P = 5
and the y-coordinate of P = 2.



II. The Cartesian plane:



- A point can be in any one of the four quadrants or on the x- or y-axis.
  - Every point on the y-axis has $x = 0$.
  - Every point on the x-axis has $y = 0$.
  - Points with the same x-coordinate lie on the same vertical line.
  - Points with the same y-coordinate lie on the same horizontal line.

I. Learners now do Activity 1.

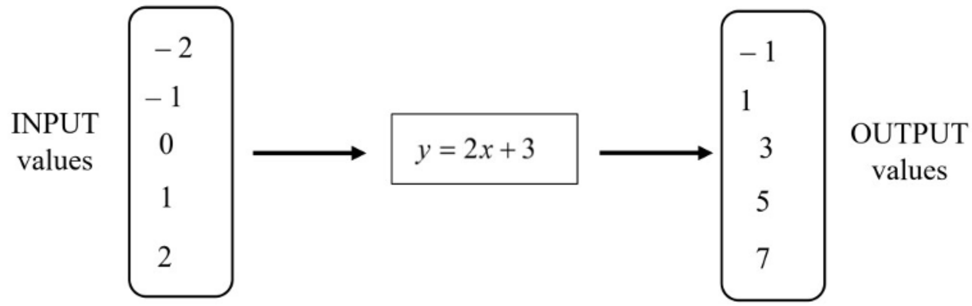
- **Three basic facts about graphs**:
DRAW A CARTESIAN PLANE
  I. Every point on the y-axis has an x-coordinate = 0.
  Every point on the x-axis has a y-coordinate = 0.

  II. The equation of a graph is true for all points on the graph.
  E.g. $(5\,;5)$ is a point on the graph of $y = x$.
  (…;…) is a point on the graph of $y = x$
  $(-7\ ;\ 7)$ is a point on the graph of $y = -x$.
  (…;…) is a point on the graph of $y = -x$
  $(4\,;11)$ is a point on the graph of $y = 2x + 3$.
  (…;…) is a point on the graph of y =$y = 3x + 1$
  $(4\,;16)$ is a point on the graph of $y = x^2$
  (…;…) is a point on the graph of y = $y = x^2$

  III. Each form of an equation has a graph with a distinct shape.
  (As they proceed with the lessons, learners will discover which shape goes with which type of equation.)

- **Functions**:
  - A graph is a set of points $(x\ ;\ y)$ and indicates a relationship between the two variables (x and y).
  - A function is a graph, or a set of points, where every x-value has only one y-value.
  - Consider the equation $y = 2x + 3$. If we PUT a value of x IN, e.g. 5, we get a value of y OUT: y will be 13.
    So: the x-values are the INPUT values and the y-values are the OUTPUT values.

- **Different ways of representing functions**:
  - A mapping:



The set of all the input values is called the **domain**.

The set of all the output values is called the **range**.

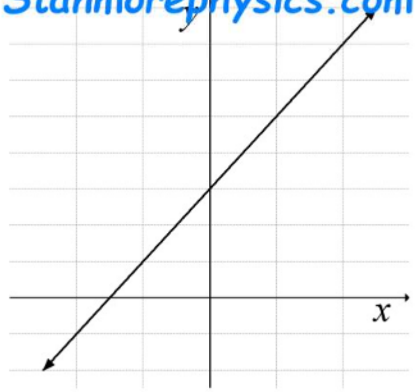
  - A table of values:

x	-2	-1	0	1	2
$y = 2x$	-4		0		

  -

x	-2	-1	0	1	2
$y = 2x + 3$	-1		3		7

  - Ordered pairs:
    Learners must plot the following points on the cartesian plane and join them.
    $(-2\ ;-1)$, $(-1\ ;1)$, $(0\ ;3)$, $(1\ ;5)$, $(2\ ;7)$
  - A graph:



- **Function notation:**

  We can write $y = 2x + 3$ as $f(x) = 2x + 3$ [y = *f*(*x*)] We read: the function value of *x* is $2x + 3$.
  E.g. (x; y) = (x; *f*(*x*))
  $f(4) = 2(4) + 3 = 11$.
  In other words: if $x = 4$, then $f(x)$ or $y = 11$.
  This means that $(4\ ;11)$ will be a point on the graph of *f*.

- **Independent and dependent variables**:

  We choose whichever value of x that we wish and then the value of y depends on that.
  Therefore *x* is called the independent variable, and *y* the dependent variable.

LESSON 1: ACTIVITIES/ASSESSMENTS

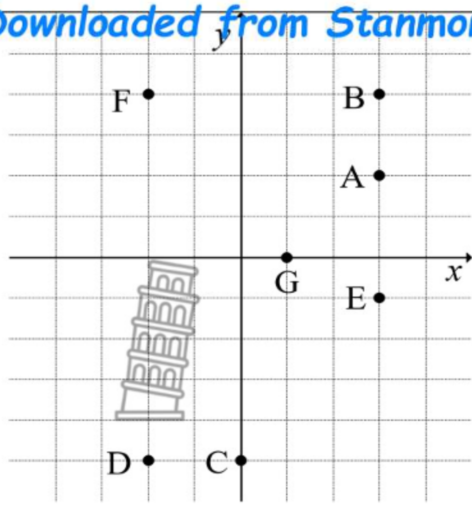
1. In which quadrant does each of the following points lie? Or on which axis?

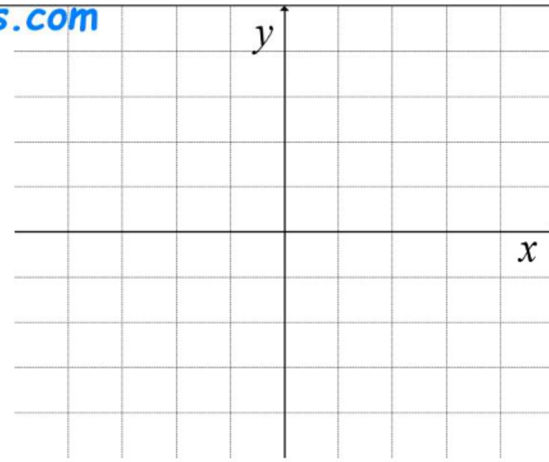
   1.1. $(-4\ ;\ 3)$
   1.2. $(\ 5\ ;-3)$
   1.3. $(\ 5\ ;0)$
   1.4. $(\ 6\ ;\ 1)$
   1.5. $(-7\ ;-2)$
   1.6. $(\ 0\ ;-4)$

2. Write down the coordinates of points A to G:

3. Draw the lines represented by the following equations:

   3.1 $x = 3$; 3.2 $x = 0$; 3.3 $x = -3$
   3.4 $y = 2$; 3.5 $y = 0$; 3.6 $y = -2$

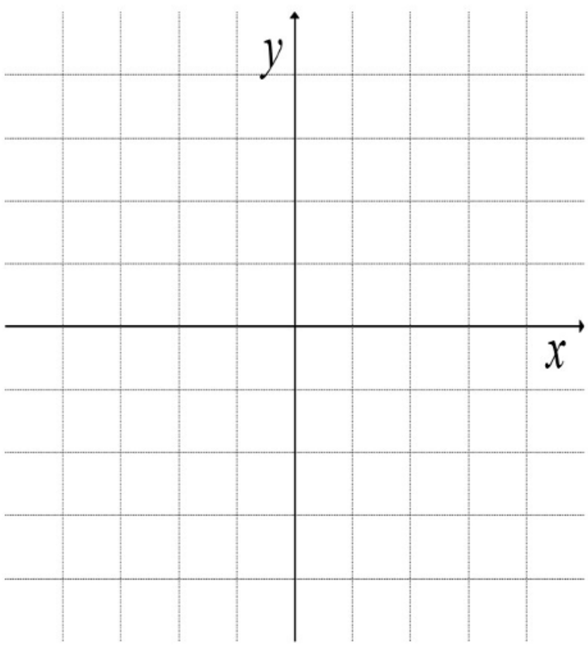




4. Draw the graphs of the points where ;

4.1 $y = x$

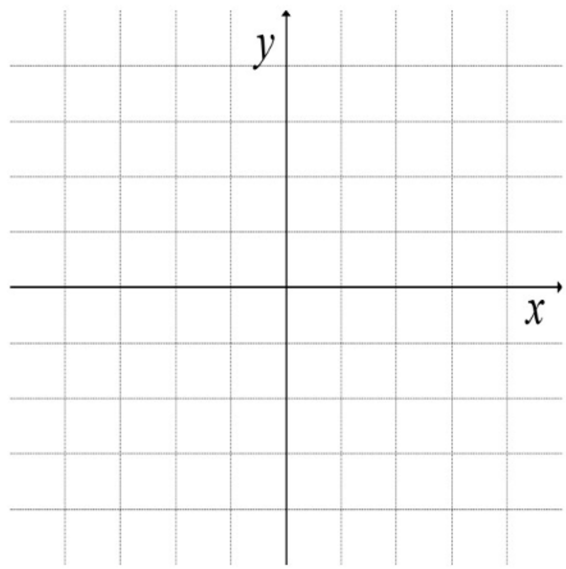
4.2 $y = -x$



5. In which quadrants are the points with/:

5.1 positive y-coordinates (i.e. $y > 0$) found?

5.2 positive x-coordinates (i.e. $x > 0$) found?



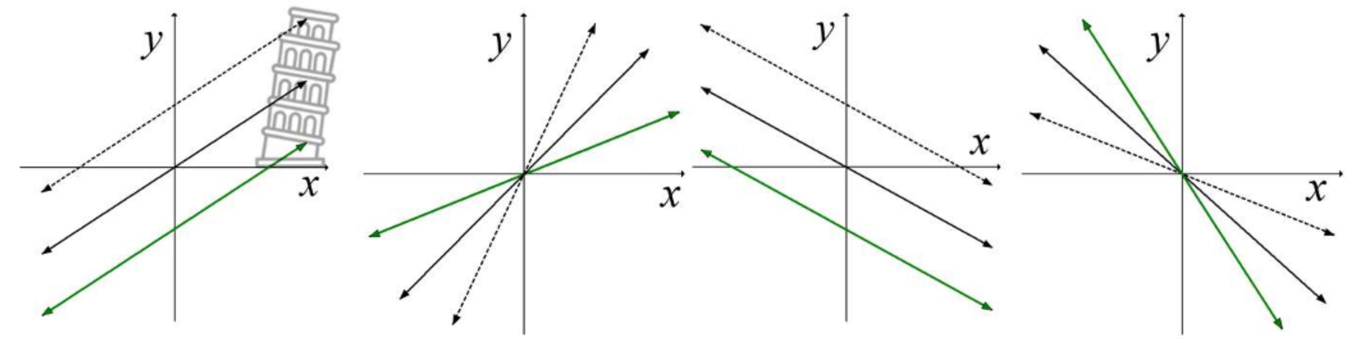
LESSON 2: STRAIGHT LINE GRAPHS

NOTES

Introduction:

Below some examples of straight-line graphs have been drawn. Look at each system of axes and explain in your own words the difference between the graphs.

A B C D



- That the steepness of all three graphs in A and C are the same.
- In A and C the different graphs are formed by translating one graph up and down.
- The graphs in A are all rising or increasing when viewed from left to right, which is the standard way to view a graph.
- The graphs in C are all declining or decreasing when viewed from left to right, which is the standard way to view a graph.
- In A and C the learners should note that in each case one graph goes through the origin and the other two graphs have specific x- and y-intercepts, some positive and others negative.
- In B all three graphs are increasing, but at different rates, i.e. their steepness differs.
- In D all three graphs are decreasing, but at different rates, i.e. their steepness differs.
- All the graphs in B and D passes through the origin, which means that their x- an y-intercepts are zero.

The equation of a straight line graph:

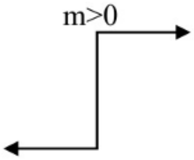
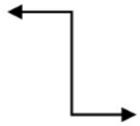
Activity 1

1. Draw straight line graphs on one system of axes, by means of point by point plotting:
(Use the following x-values $-2; -1; 0; 1; 2$)

1.1.	(a) $y = x$	(b) $y = x + 1$	(c) $y = x - 2$	
1.2.	(a) $y = x$;	(b) $y = 2x$;	(c) $y = \frac{1}{2}x$	
1.3.	(a) $y = -x$	(b) $y = -2x$;	(c) $y = -\frac{1}{2}x$	
1.4.	(a) $y = 2x + 2$;	(b) $y = -\frac{3}{2}x + 3$		
1.5.	(a) $y = 2$;	(b) $y = -3$;	(c) $x = 2$;	(d) $x = -4$

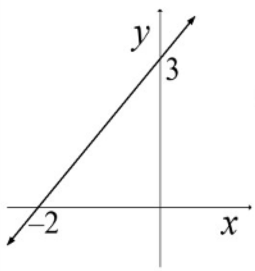
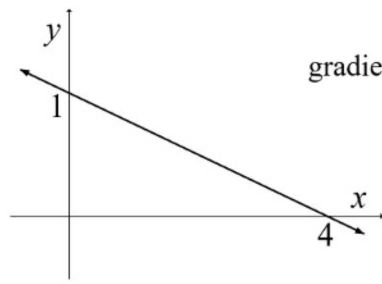
- The standard form of a straight line graph:
$y = mx + c$ or $y = ax + q$, with m *or* a = the gradient and c or q = the y-intercept.

- A summary of important facts concerning gradient:

A rising line has a **POSITIVE** gradient	A declining line has a **NEGATIVE** gradient	A horizontal line has a gradient of **ZERO**	The gradient of a vertical line is **UNDEFINED**.
m>0 	m<0 	m=0 	

- Value of the gradient:

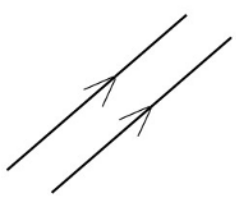
  Examples:

 gradient $= +\frac{3}{2}$	 gradient $= -\frac{1}{4}$

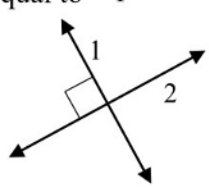
The gradient of a line $= \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{how much you go up}}{\text{how much you go forward/backwards}}$

- Parallel and perpendicular lines:

Parallel lines have equal gradients



The product of the gradients of non-vertical perpendicular lines is equal to -1



In the figure line 1 is perpendicular to line 2.
If the gradient of line 1 $(m_1) = -\frac{3}{2}$,

then the gradient of line 2 $(m_2) = +\frac{2}{3}$.

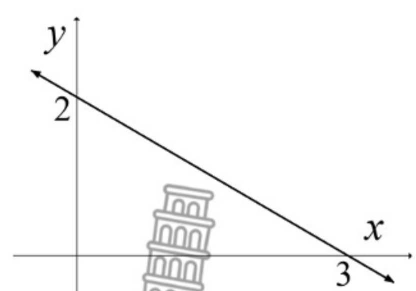
And $m_1 \times m_2 = \left(-\frac{3}{2}\right)\left(+\frac{2}{3}\right) = -1$

There are three ways to draw a straight line graph:

1. Table method
2. Dual- intercept method
3. Gradient-y-intercept method

- The Table Method

  Example: Sketch the graph of $2x + 3y = 6$ using table method.



- Rewrite the equation in the form $y = mx + c$
  (i.e. make y the subject of the formula):

X	-2	-1	0	1	2
$y=-\frac{2}{3}x+2$	3,3	2,7	2	1,3	0,7

- Dual-intercept method:
  Example: Sketch the graph of $2x+3y=6$.
  - Determine the y-intercept by substituting $x=0$: $(0\ ;\ 2)$
  - Determine the x-intercept by substituting $y=0$. $(3\ ;\ 0)$
  - Then mark off those two points on the system of axes, join the points and extend the line.

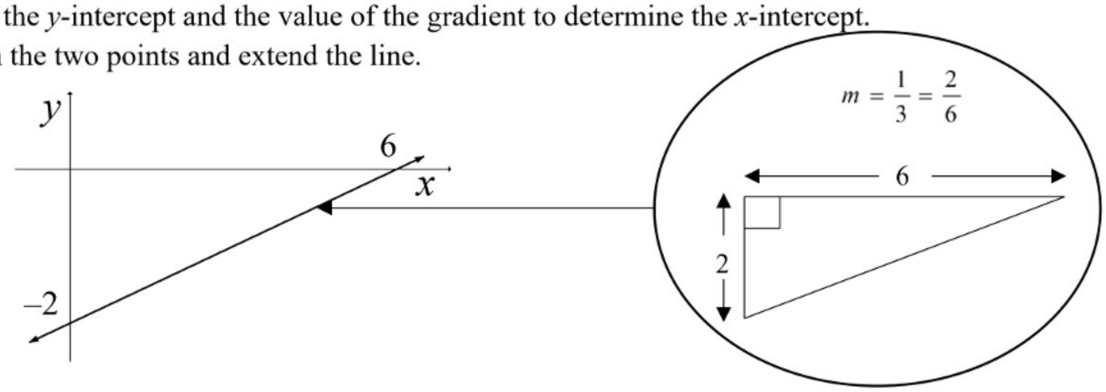
  Take note: to draw the correct straight line, only two points on the line are needed.
- Gradient-y-intercept method:
  Example: Sketch the graph of $2x-6y=12$.
  - Rewrite the equation in the form $y=mx+c$ (i.e. make y the subject of the formula):

    $2x-6y=12$

    $-6y=-2x+12$

    $y=\frac{1}{3}x-2$

    The gradient, m, is equal to $\frac{1}{3}$ ($m=\frac{1}{3}$), and the y-intercept, c, is equal to -2.
  - Mark off the y-intercept on the system of axes: $(0\ ;\ -2)$.
  - Now use the y-intercept and the value of the gradient to determine the x-intercept.
  - Now join the two points and extend the line.



- Take Note:
  - In most cases, the dual-intercept method and table method are much easier.

However, in the case of a straight-line graph passing through the origin, e.g. $y=3x$, the dual-intercept method cannot be used. Then, either point-by-point plotting or the gradient-y-intercept method can be used. **Domain and range**:

- There are no limits to the values of x and y in a straight-line graph; therefore, both x and y can be any real number.
- Domain: $x\in R$. It can also be written as $x\in(-\infty\ ;\ \infty)$.
  Range: $y\in R$. It can also be written as $y\in(-\infty\ ;\ \infty)$.
  - The only exceptions occur in the case of the range of a horizontal line and the domain of a vertical line, as well as both the domain and the range of a line segment

Activity 2.

Draw graphs of the following functions and determine their range and domain. Do not use point by point plotting.

2.1. $f(x) = 2x - 4$

2.2. $g(x) = \frac{3}{4}x + 6$

2.3. $2x + 6y = 18$

2.4. $-3x + 2y = 9$

2.5. $-2x + 3y - 9 = 0$

2.6. $h(x) = -4x + 4$

2.7. $p(x) = \frac{1}{4}x + 2$

2.8. $q(x) = -\frac{3}{4}x$

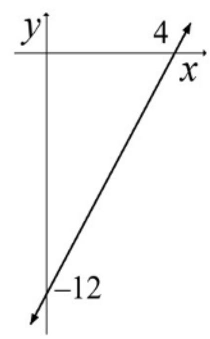
2.9. $x + 1 = 0$

LESSON 3: DETERMINING THE EQUATION OF A STRAIGHT LINE

NOTES

How to determine the equation of a straight line graph:

- Example 1:



Determine the equation of the line in the sketch in the form $y = mx + c$:

Solution:

The y-intercept is -12. Therefore $c = -12$.

$\therefore y = mx - 12$

Substitute the coordinates of the x-intercept, $(4 ; 0)$, into the equation:

$\therefore 0 = m(4) - 12$

$4m = 12$

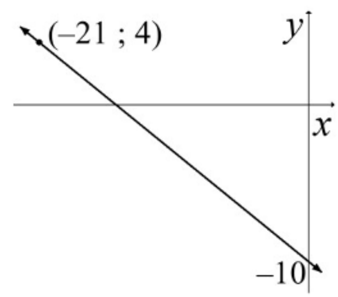
$m = 3$

Therefore the equation is $y = 3x - 12$.

[Alternatively: After substituting $c = -12$, the gradient can also be obtained from $m = \frac{\Delta y}{\Delta x} = \frac{12}{4} = 3$.]

- Example 2:

Determine the equation of the line in the sketch in the form $y = mx + c$:



Solution:

The y-intercept is -10. Therefore $c = -10$.

$\therefore y = mx - 10$

Substitute the coordinates of the point $(-21 ; 4)$, into the equation:

$\therefore 4 = m(-21) - 10$

$21m = -14$

$m = -\frac{2}{3}$

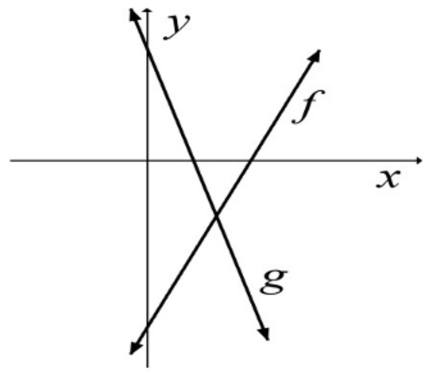
Therefore the equation is $y = -\frac{2}{3}x - 10$.

- Learners should now do Activity 3.

Determining the point of intersection of two lines algebraically:

- If a graph showing two lines has been drawn accurately, (estimated) coordinates for the point of intersection can be read off.
- However, if the equations of the two graphs are known, the precise coordinates of the point of intersection can be calculated.
- Any graph is a set of points $(x\ ;\ y)$ all of which satisfy the equation of the graph. Two different straight lines have different equations and therefore different points that satisfy their equations – except for one point $(x\ ;\ y)$, namely the point on intersection that satisfies both equations. The coordinates of that point is the solution to solving the two equations simultaneously.

- **Example**:
  Determine algebraically the point of intersection of the graphs of $f(x)=2x-6$ and $g(x)=-3x+4$.



Solution:

Solve simultaneously:

$y=2x-6$ and $y=-3x+4$.

By equating:

$2x-6=-3x+4$

$5x=10$

$x=2$

Substitute into $y=2x-6$:

$y=2(2)-6$

$y=-2$

Point of intersection: $(2\ ;\ -2)$.

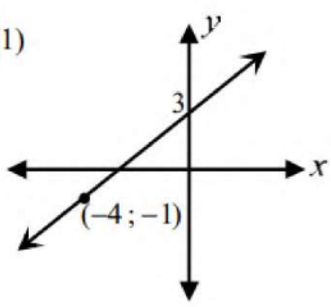
Take note:

Any other suitable method could also have been used to solve the two equations simultaneously, e.g. substitution or elimination.

LESSON 3 : ACTIVITY

Determine the equations of the following lines in the form $f(x) = ax + q$:

(1)



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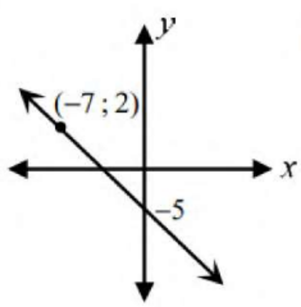

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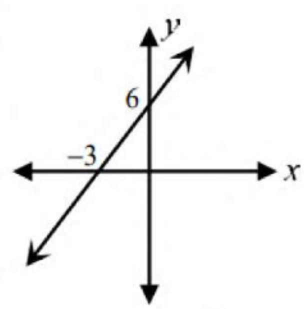

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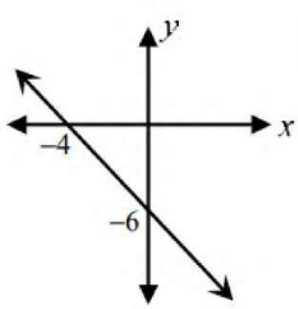

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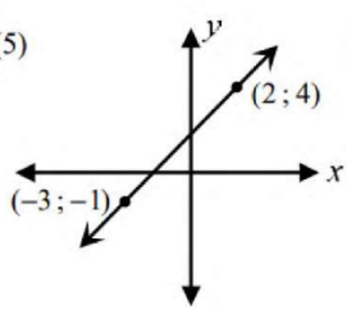

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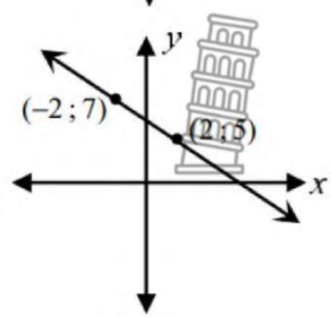


LESSON 4: THE PARABOLA (PART 1)

NOTES

Introduction:

- Plotting linear functions always give straight line graphs, as seen in the previous lesson.
- The next step is to explore the shape of the graph obtained by plotting a quadratic function.
- Complete Activity 1: Worksheet on Sketching Parabolas of the form $y = ax^2 + q$.
    - The gradient does not stay constant as is the case with the straight line graph, but the graph become steeper as one goes further away from $x = 0$;
    - The range is not all the real numbers any more, but the graph has a turning point, and therefore a minimum (or maximum) value;
    - The axis of symmetry; and
    - The graph is not just increasing or just decreasing, but first decreasing and then increasing (or vice versa if $a < 0$.)

- If $a > 0$, shape is a smiley face  ; If $a < 0$, shape is a sad face 
- The value of a affects the shape: influences the amount of vertical stretching.
  For $a > 0$: The bigger the value of a, the more the vertical stretch (narrower graph).
  For $a < 0$: The smaller the value of a, the more the vertical stretch (narrower graph).
- q is the y-intercept; and the turning point is $(0\,;\,q)$.
  q also indicates vertical shifting: the number of units and the direction.
- q is the minimum value of the parabola if $a > 0$; and the maximum value if $a < 0$.
- The range of the parabola will be $[q\,;\,\infty)$ if $a > 0$; and $(-\infty\,;\,q]$ if $a < 0$.
- The domain of the parabola is $x \in R$.
- The axis of symmetry is always the y-axis, i.e. the line $x = 0$.

Drawing a sketch graph of a parabola (without plotting points):

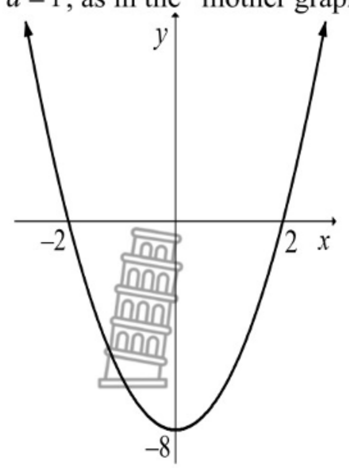
Example 1:

Draw a sketch graph of $y = 2x^2 - 8$.

Solution:

Because $a > 0$ we know it is a "smiley face" parabola.

Because $a = 2$ we know that there is more vertical stretch than when $a = 1$, as in the "mother graph" $y = x^2$.



Determining the intercepts with the axes:
Because $q = -8$, we know the y-intercept and the minimum value is -8.
(Could also substitute $x = 0$ into the equation.)
For the x-intercepts, substitute $y = 0$ and solve for x:

$2x^2 - 8 = 0$
$2(x^2 - 4) = 0$
$x^2 - 4 = 0$
$(x - 2)(x + 2) = 0$
$x = 2$ or $x = -2$

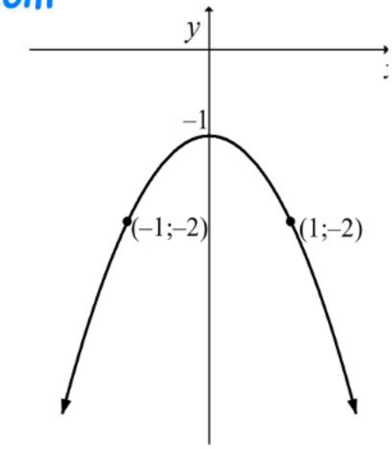
We now have enough information to draw the graph

Example 2:

Draw a sketch graph of $y=-x^2-1$.

Solution:

Because $a>0$ we know it is a "sad face" parabola.
Because $q=-1$, we know the y-intercept, and the maximum value is -1.
The graph is therefore not cutting the x-axis.
To draw the graph more precisely, one can substitute two x-values, e.g. -1 and 1, and so obtain two more points to plot, namely $(-1\,;-2)$ and $(1\,;-2)$.



LESSON 4: ACTIVITIES/ASSESSMENTS

Activity 1 Worksheet on Parabolas of the form $y=ax^2+q$.

The quadratic functions $y=x^2$, $y=2x^2$, $y=\frac{1}{2}x^2$ and $y=-x^2$.

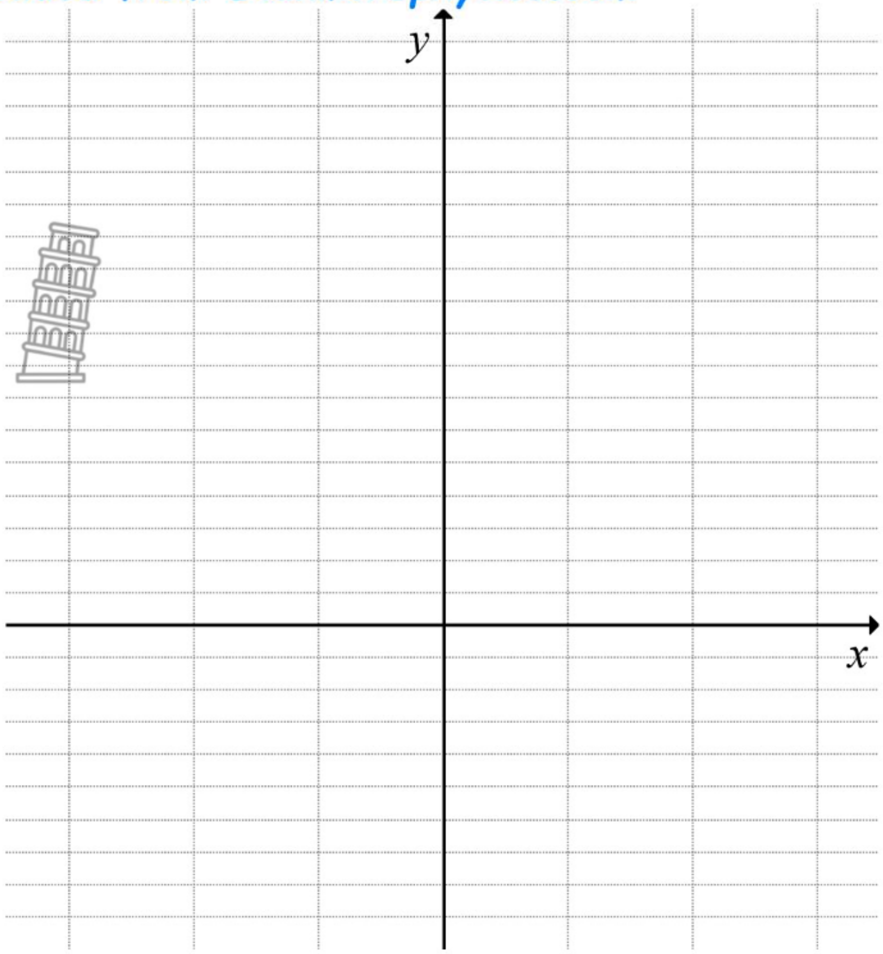
1.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation of each of the graphs to the general equation $y=ax^2+q$ and then complete the last two columns of the table.

x	-3	-2	-1	0	1	2	3
$y=x^2$							
$y=2x^2$							
$y=\frac{1}{2}x^2$							
$y=-x^2$							

a	q

1.



1.2 How does changing the value of a influence the graph of the parabola $y = ax^2 + q$?

……

……

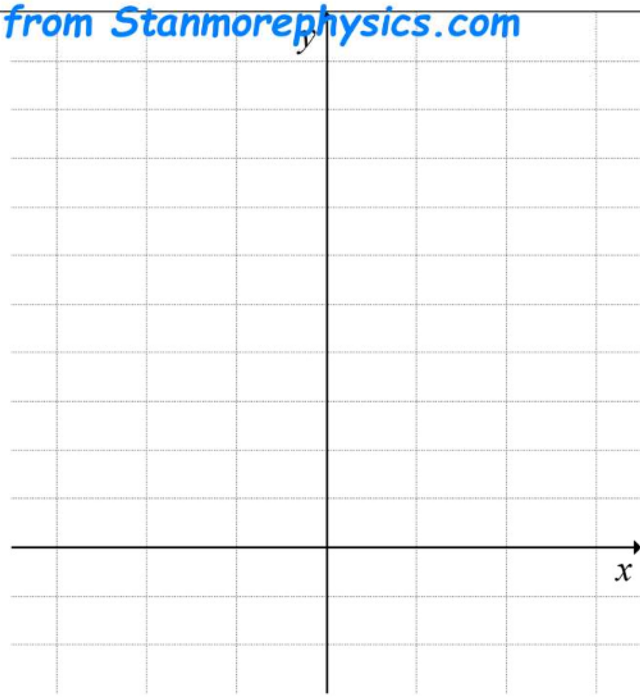
2. The quadratic functions $y = x^2$, $y = x^2 + 1$ and $y = x^2 - 2$.

2.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation for each of the graphs to the general equation $y = ax^2 + q$ and then complete the last two columns of the table.

x	-3	-2	-1	0	1	2	3
$y = x^2$							
$y = 2x^2$							
$y = \frac{1}{2}x^2$							

a	q



2.2 How does changing the value of q influence the shape of the graph of the parabola $y = ax^2 + q$?

……

……

Activity 2:

Draw sketch graphs of the following functions:

1. $y = x^2 - 4$
2. $y = x^2 + 3$
3. $y = \frac{1}{3}x^2$
4. $y = -4x^2$
5. $f(x) = -2x^2 + 8$
6. $g(x) = 3x^2 - 6$

LESSON 5: THE PARABOLA (PART 2)

NOTES

To determine the equation of a parabola:

- **Example 1**: Determine the equation of the parabola shown in the sketch alongside.

  Solution:

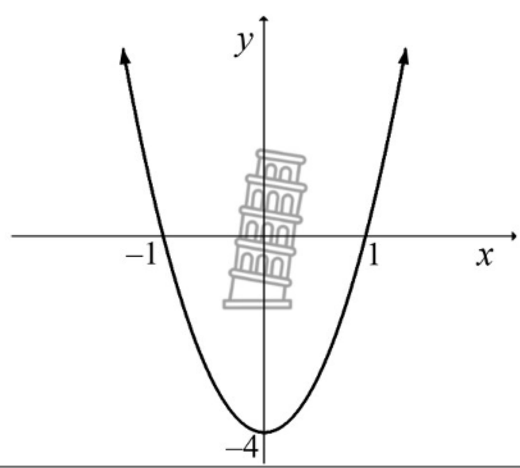
Substitute $q = 4$ into $y = ax^2 + q$ $\quad y = ax^2 - 4$

Substitute any of the two x- intercepts into the equation

E.g. $(1\,;0)$: $0 = a(1)^2 - 4$

$a = 4$

Therefore the equation of the parabola is $y = 4x^2 - 4$



- Example 2:

Determine the equation of the parabola shown in the sketch alongside.

Solution: Substitute $q = 5$ into $y = ax^2 + q$:

$y = ax^2 + 5$.

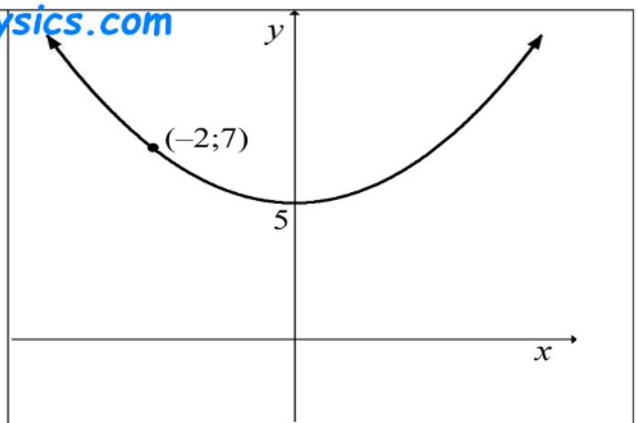
Substitute $(-2\ ;\ 7)$ into the equation to determine the value of a:

$7 = a(-2)^2 + 5$

$4a = 2$

$a = \frac{1}{2}$

Therefore the equation of the parabola is $y = \frac{1}{2}x^2 + 5$.



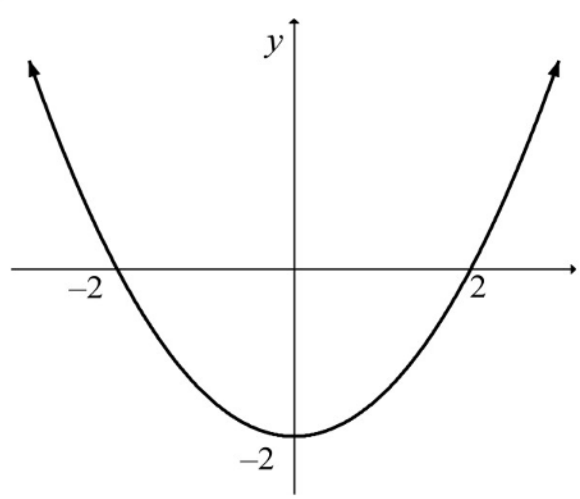
LESSON 5: ACTIVITIES/ASSESSMENTS

Activity 1:

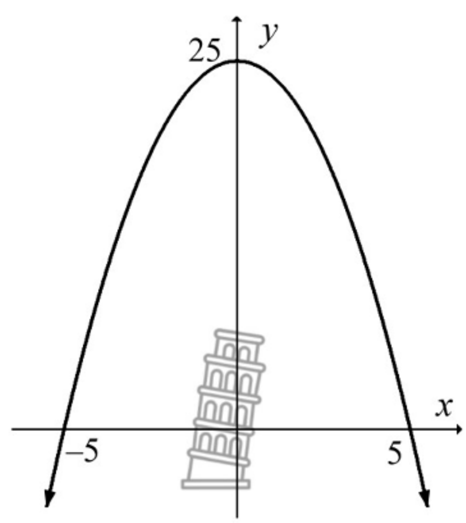
In each case:

1.1 Determine the equation of the parabola.
1.2 Write down the domain of the graph.
1.3 Write down the range of the graph.
1.4 Write down the coordinates of the turning point.
1.5 Write down the equation of the axis of symmetry.
1.6 Write down the values of x for which
  1.6.1 $y > 0$
  1.6.2 $y < 0$
  1.6.3 the graph is increasing
  1.6.4 the graph is decreasing.

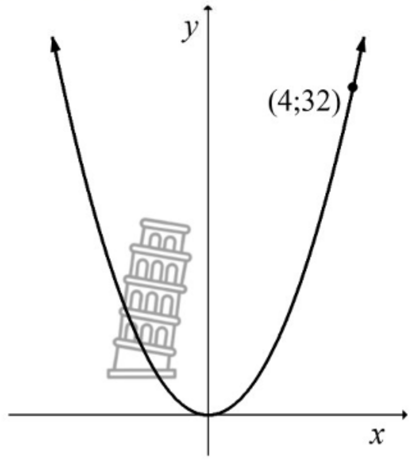
A.



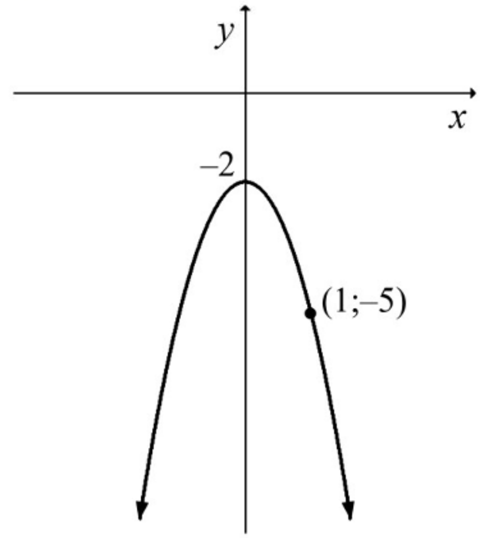
B.



C.



D.



LESSON 6: THE HYPERBOLA (PART 1)

NOTES

Demonstrate to learners how to sketch the graph of a Hyperbolic function using a table.

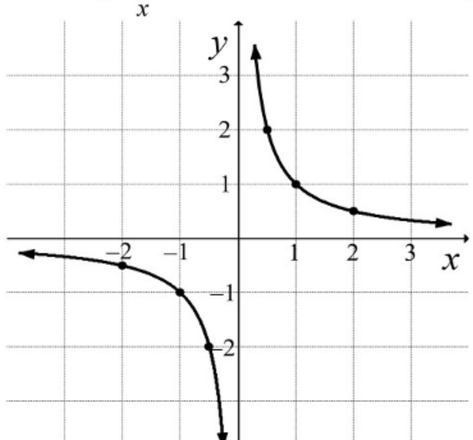
Hyperbola : $y = \frac{a}{x}$

Effect of a when $q = 0$; $y = \frac{a}{x}$

1. For the graph of $y = \frac{1}{x}$
2. Using the table method

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	$\frac{1}{0}$	2	1	$\frac{1}{2}$

3. Note: if $x = 0$ then $y = \frac{1}{0}$ which is undefined, i.e. there is no y-intercept and the graph is not touching the y axes.
4. If $y = 0$ and solve for x: $0 = \frac{1}{x} \Rightarrow$ there is no value of x, hence no x-intercept.



5. The graph will keep on approaching the vertical and horizontal lines but never touches. The vertical and horizontal lines will be called asymptotes viz. vertical and horizontal asymptotes.

6. Demonstrate using the table method draw the following graphs on the same set of axes:

A. $f(x) = \frac{2}{x}$ B. $g(x) = \frac{4}{x}$ C. $h(x) = \frac{8}{x}$

A.

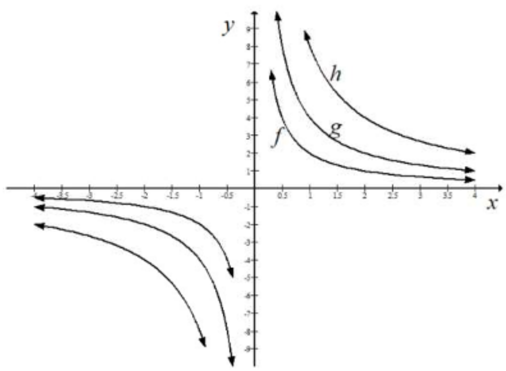
x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$f(x)$	-1	-2	-4	$\frac{2}{0}$	4	2	1

B.

x	-4	-2	-1	0	1	2	4
$g(x)$	-1	-2	-4	$\frac{4}{0}$	4	2	1

C.

x	-8	-4	-2	0	2	4	8
$h(x)$	-1	-2	-4	$\frac{1}{0}$	4	2	1



7. As the number in the numerator gets larger, the branches of the hyperbola are stretched vertically away from the x-axis.

8. All the graphs don't touch the x and y-axis because at $x = 0$ the graph is undefined hence x-axis and y – axis are called asymptotes.

An asymptote is a line, which the graph approaches/tends towards but not touches or cuts. Each hyperbola has a vertical and horizontal asymptote.

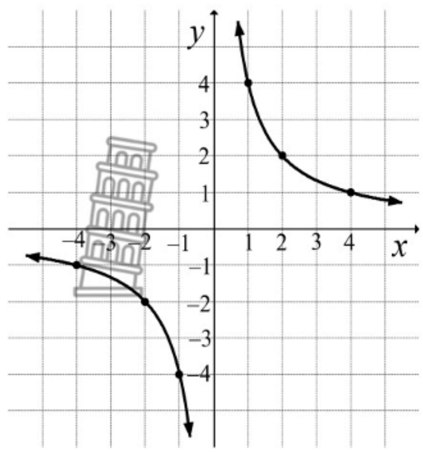
9. Put negative in all of the above graphs and let the learners do the same investigation in groups or individually:

A. $y = -\frac{1}{x}$ B. $y = -\frac{2}{x}$

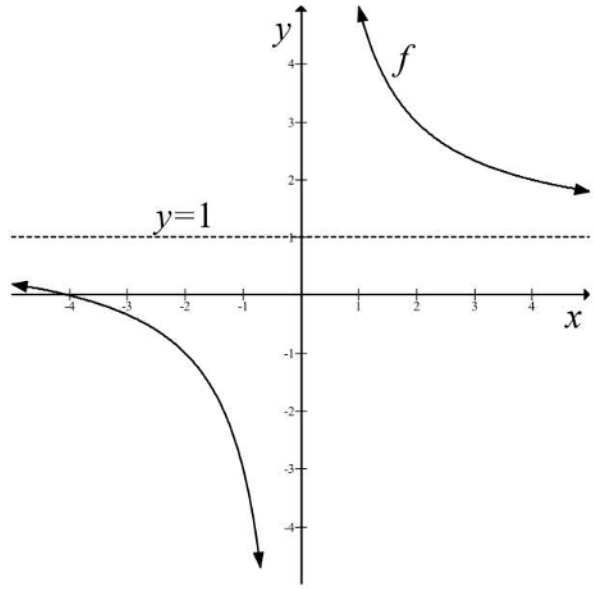
C. $y = -\frac{4}{x}$ D. $y = -\frac{8}{x}$

Effects of q

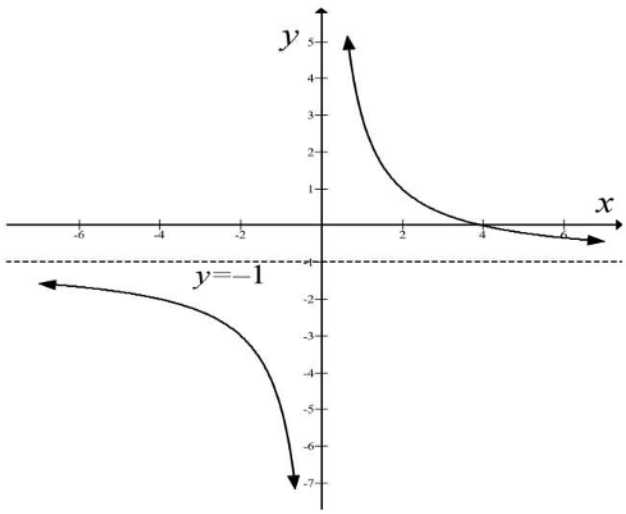
1. $y=\frac{4}{x}$ $q=0$



2. $y=\frac{4}{x}+1$ $q>0$



3. $y=\frac{4}{x}-1$ $q<0$



LESSON 6: ACTIVITIES/ASSESSMENTS

Activity 1

Draw the following graphs on the same set of axis using table method:

1.1. ***f*****:** $y=\frac{6}{x}$

1.2. ***g***: $y=-\frac{10}{x}$

1.3. ***h***: $y=\frac{12}{x}$

1.4. k: $y = \frac{16}{x}$

Activity 2

Draw the following graphs on the same set of axis using table method:

2.1. $f(x) = \frac{8}{x}$

2.2. $g(x) = -\frac{8}{x} + 1$

2.3. $h(x) = \frac{8}{x} + 1$

2.4. $k(x) = \frac{8}{x} - 1$

LESSON 7: THE HYPERBOLA (PART 2)

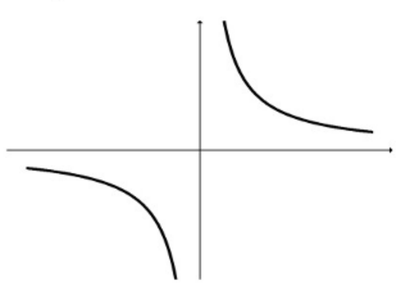
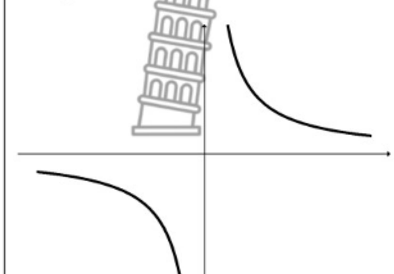
NOTES

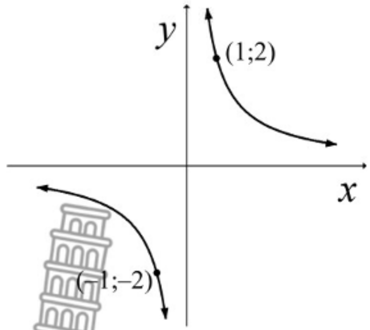
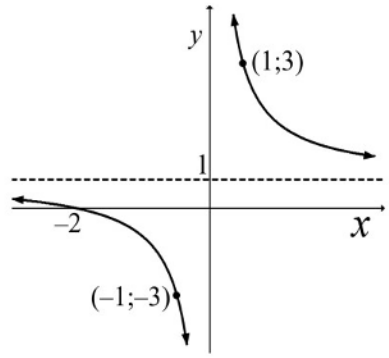
Sketching the graphs

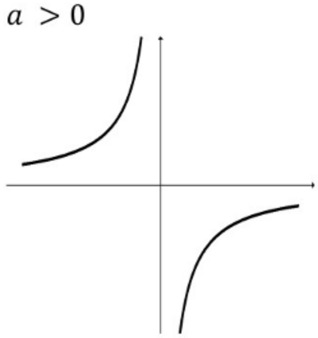
Standard form : $y = \frac{a}{x} + q$

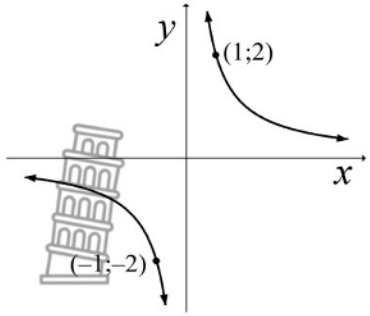
Steps:

1. Determine the asymptotes ($x = 0$ *and* $y = q$)
2. Determine the x – intercept.
3. Determine the shape.
4. Plot the graph

Example 1:	$f(x) = \frac{2}{x}$		Example 2:	$f(x) = \frac{2}{x} + 1$
Step 1 :	Asymptotes $x = 0$ and $y = 0$		Step 1 :	Asymptotes $x = 0$ and $y = 1$
Step 2 :	x – intercept $x = 0$		Step 2 :	x – intercept : $0 = \frac{2}{x} + 1$ $0 = 2 + x$ $x = -2$
Step 3 :	Shape $a > 0$ 		Step 3 :	Shape $a > 0$ 

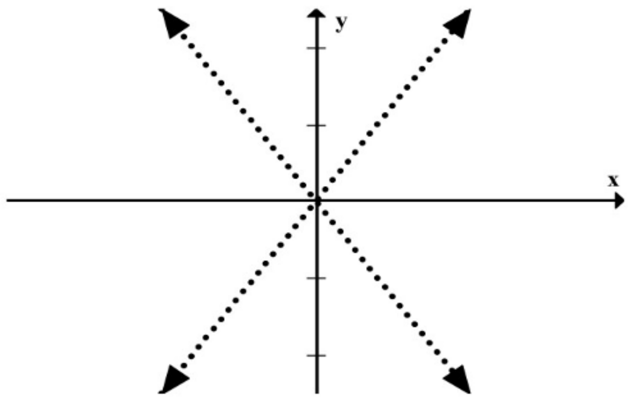
Step 4 :	Plotting the graph 		Step 4:	Plotting the graph 

Example 3:	$f(x) = -\frac{2}{x}$		Example 4	$f(x) = -\frac{2}{x} + 1$
Step 1 :	Asymptotes $x = 0$ and $y = 0$			Asymptotes $x = 0$ and $y = 1$
Step 2 :	No x-intercept and no y-intercept		Step 2 :	x – intercept : $0 = -\frac{2}{x} + 1$ $-1 = -\frac{2}{x}$ $x = 2$
Step 3 :	Shape $a > 0$ 			

Step 4 :	Drawing the graph 		

Lines of symmetry

Lines of axes of symmetry are lines $y = x$ and $y = -x$



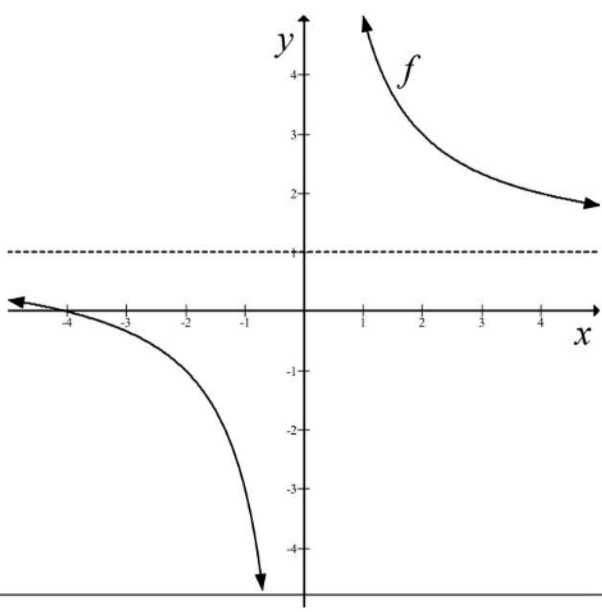
Domain and Range

Domain: $x \in R$, but $x \neq 0$

Range : $y \in R$, but $y \neq 0$ [Or $y \neq a$ if $y = a$ is the equation of the asymptote.

B. Finding the equation of the hyperbola

To find the equation of the hyperbola, you need to determine the values of a and q.



Solution:

Asymptote is $y = 1$, then $q = 1$

Substitute point (-4:0) into the hyperbola equation:

$$0 = \frac{a}{-4} + 1$$

$$-1 = \frac{a}{-4}$$

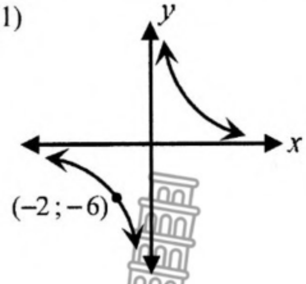
$a = 4$

$$\therefore y = \frac{4}{x} + 1$$

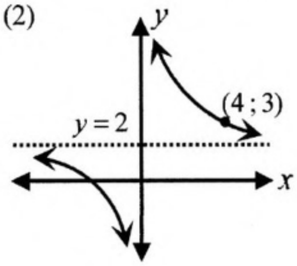
LESSON 7: ACTIVITIES/ASSESSMENTS
<u>**Activity 1**(See attached appendix in case you don't have a textbook)</u> Mind Action Series: Exercise 10 (b) No. 1 - 6 <u>**Activity 2**(See attached appendix in case you don't have a textbook)</u> Mind Action Series: Exercise 10 (b) No 7 – 10 Exercise 10 (c) No 1 - 3

(b) Determine the equation of each of the following hyperbolas in the form $f(x)=\frac{a}{x}+q$.

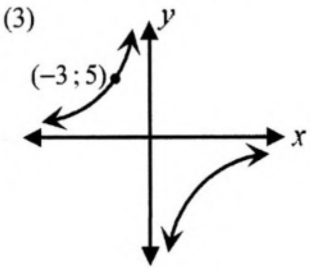
(1)



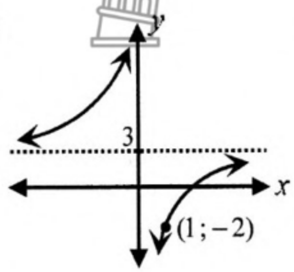
(2)



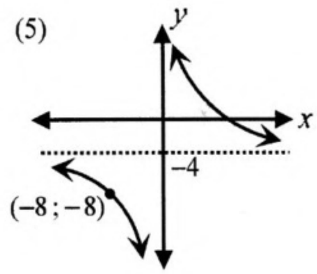
(3)



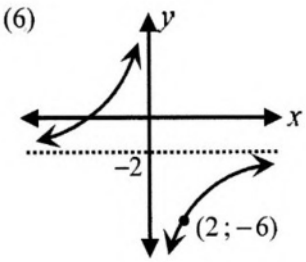
(4)



(5)



(6)



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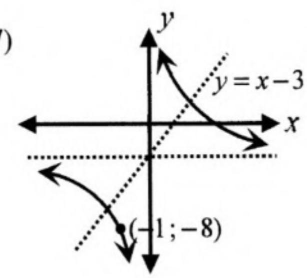

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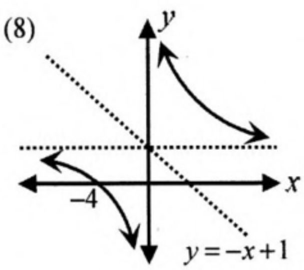
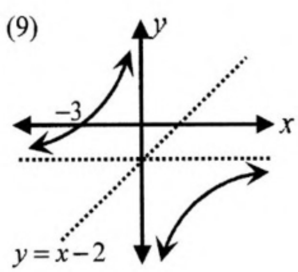


(9)



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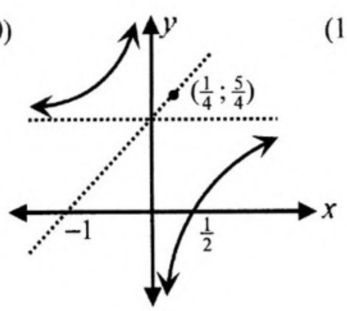

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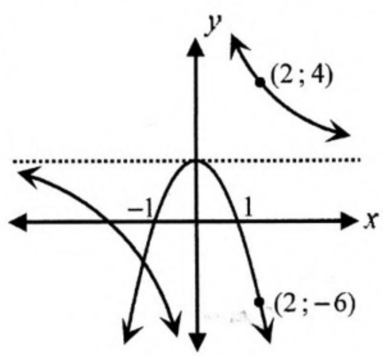
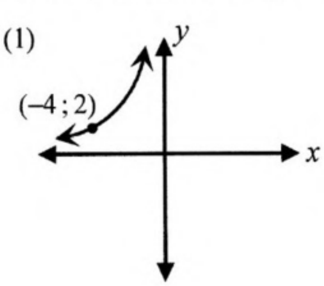


(c) For each function below, state the domain and range and determine the equation:

(1)



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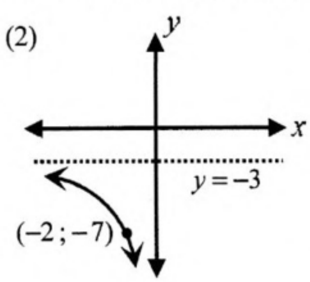

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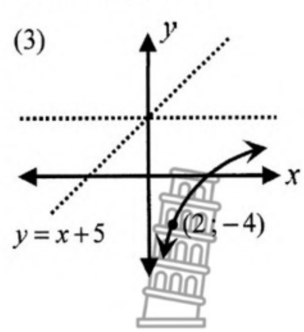


LESSON 8: THE EXPONENTIAL GRAPH (PART 1)

NOTES

- Complete Activity 1: Worksheet on sketching Exponential graphs of the form $y = a.b^x$ .using table method.
  - The graph is increasing all the time, but very slowly at first, and then faster and faster.
  - For very small values of x the graph approaches the *x*-axis, but will never touch or cut it, because for any value of x, $2^x > 0$. This means that the *x*-axis is an asymptote to the graph.
  - The range of the graph is $y > 0$; but the domain is $x \in R$.
  - The graph does not have an axis of symmetry.
- Properties of the exponential graph with general equation $y = a.b^x + q$, $b > 0$ and $b \neq 1$:
  - If $b > 1$, the graph is increasing; if $0 < b < 1$ the graph is decreasing.
  - For $b > 1$, the bigger the value of *b*, the steeper graph , e.g. the graph of $y = 3^x$ is steeper than the graph of $y = 2^x$.

    For $b < 1$, the smaller the value of *b*, the faster the graph decreases, e.g. $y = \left(\frac{1}{3}\right)^x$ decreases faster than $y = \left(\frac{1}{2}\right)^x$.
  - Take note that e.g. $y = \left(\frac{1}{2}\right)^x$ can also be written as $y = 2^{-x}$.
  - For any graph $y = b^x$ the *y*-intercept will always be 1, because $(\text{any number})^0 = 1$.
  - The value of *a* affects the shape: influences the amount of vertical stretching.

    For $a > 0$: The bigger the value of a, the more the vertical stretch.

    For $a < 0$: The graph of e.g. $y = -2^x$ is a reflection of $y = 2^x$ in the *x*-axis.
  - *q* indicates vertical shifting: the number of units and the direction.

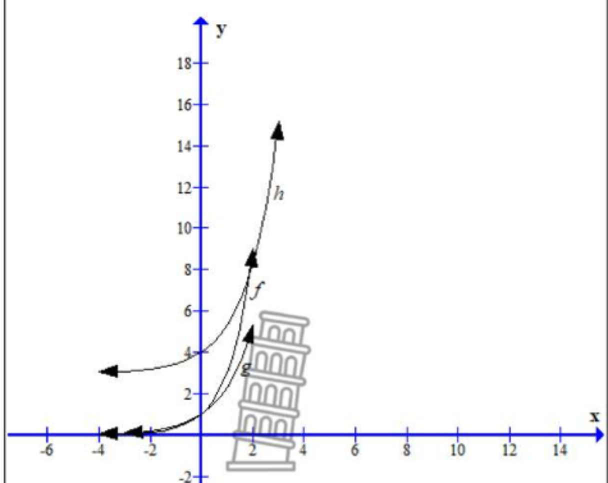
Sketching an exponential graph (without plotting points):

Example 1:

Draw a sketch graph of $h(x) = 2.3^x + 3$.

Solution:

- The "mother graph" in this case is $f(x) = 3^x$, which is increasing, and has a y-intercept of 1, because $3^0 = 1$.
- The graph of $g(x) = 2.3^x$ will be the same as $f(x) = 3^x$, except with a vertical stretch by a factor of 2, causing e.g. the *y*-intercept now to be 2.
- Finally: the graph of $h(x) = 2.3^x + 3$ will be the same as $g(x) = 2.3^x$, just translated (shifted) up by 3 units.
- Take note: It is not necessary to draw all three graphs; only the final one.



However, it is a very good habit and method to do it in this way.

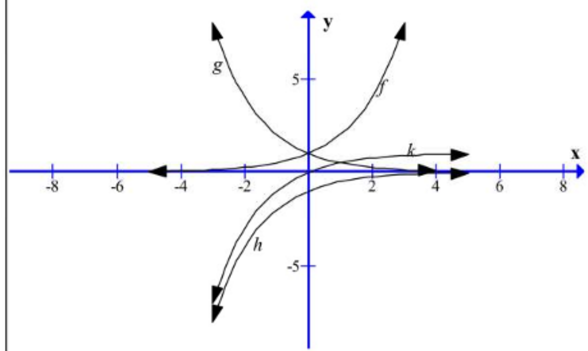
Example 2:

Draw a sketch graph of $y=-\left(\frac{1}{2}\right)^{x}+1$.

Solution:

- The "mother graph" in this case is $f(x)=2^{x}$, which is increasing, and has a y-intercept of 1.
- $g(x)=\left(\frac{1}{2}\right)^{x}$ or $g(x)=2^{-x}$ is a reflection of $f(x)=2^{x}$ in the y-axis.
- $h(x)=-\left(\frac{1}{2}\right)^{x}$ is a reflection of $g(x)=\left(\frac{1}{2}\right)^{x}$ in the y-axis.
- $k(x)=-\left(\frac{1}{2}\right)^{x}+1$ is $h(x)=-\left(\frac{1}{2}\right)^{x}$ translated up by one unit.

Learners should now do Activity 2.



LESSON 8: ACTIVITIES/ASSESSMENTS

Activity 1 Worksheet on Sketching Exponential graphs of the form $y=a.b^{x}+q$.

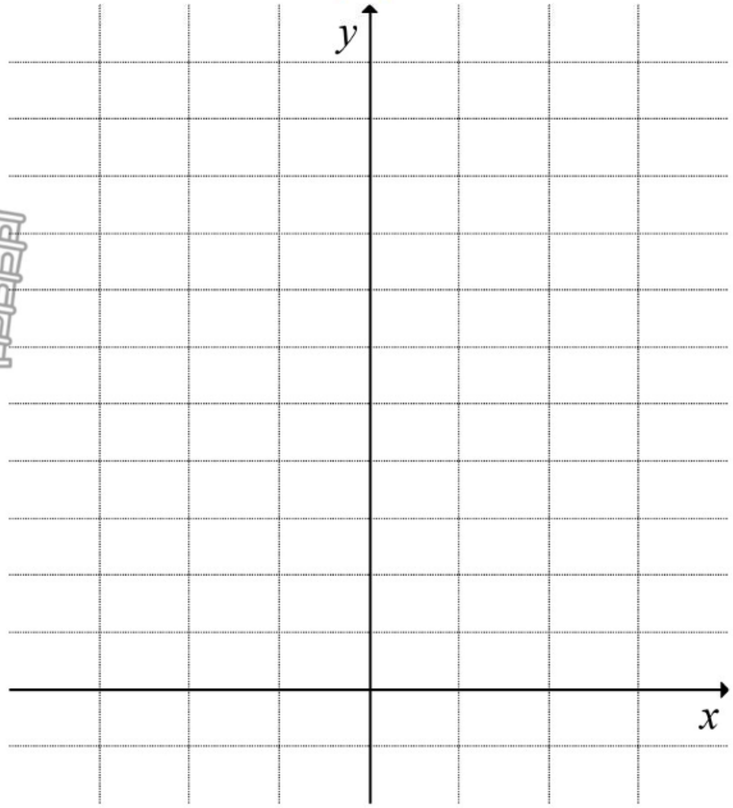
Drawing the graphs of $y=2^{x}$, $y=3^{x}$ and $y=\left(\frac{1}{2}\right)^{x}$.

1.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.
Also: compare the equation of each of the graphs to the general equation $y=a.b^{x}+q$ and then complete the last three columns of the table.

x	-2	-1	0	1	2
$y=2^{x}$					
$y=3^{x}$					
$y=\left(\frac{1}{2}\right)^{x}$					

a	b	q

1.



1.2 How does changing the value of b influence the shape of the exponential graph $y = a.b^x + q$?

……………………………………………………………………………………………………

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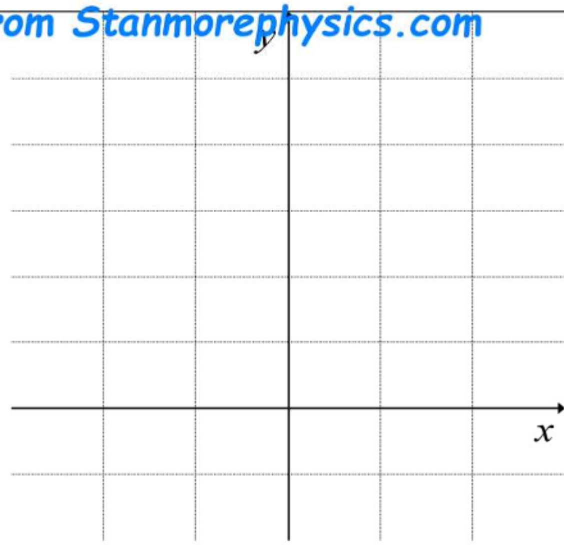
2. Drawing the graphs of $y = 2^x$, $y = 2^x + 1$ and $y = 2^x - 1$.

2.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation of each of the graphs to the general equation $y = a.b^x + q$ and then complete the last three columns of the table.

x	-2	-1	0	1	2
$y = 2^x$					
$y = 2^x + 1$					
$y = 2^x - 1$					

a	b	q



2.2 How does changing the value of q influence the exponential graph $y = a.b^x + q$?

……

……

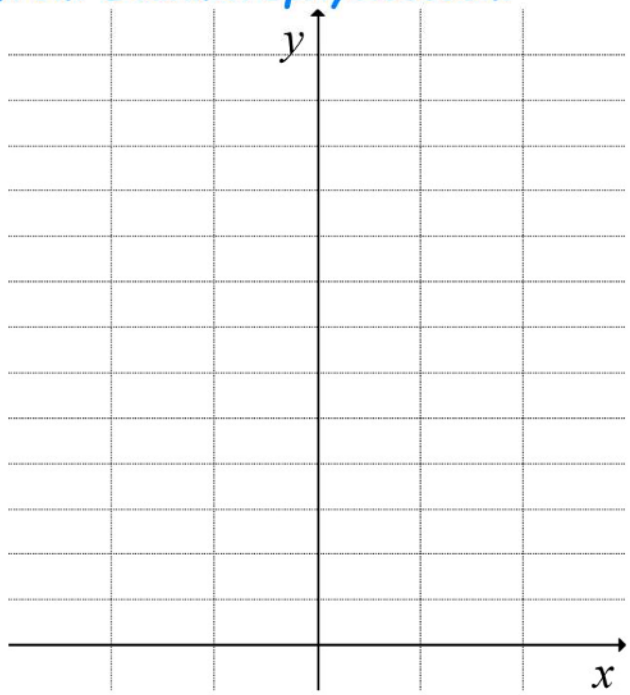
3. Drawing the graphs of $y = 2^x$, $y = 3.2^x$ and $y = -2^x$.

3.1 Complete the table below and then sketch the graphs on the given system of axes. Label each graph, using its equation.

Also: compare the equation of each of the graphs to the general equation $y = a.b^x + q$ and then complete the last three columns of the table.

x	-2	-1	0	1	2
$y = 2^x$					
$y = 3.2^x$					
$y = -2^x$					

a	b	q



3.2 How does changing the value of a influence the exponential graph $y = a.b^x + q$?

……………………………………………………………………………………………………

……………………………………………………………………………………………………

Activity 2

Draw sketch graphs of the following functions. Indicate the asymptote, the *y*-intercept and the *x*-intercept (where applicable):

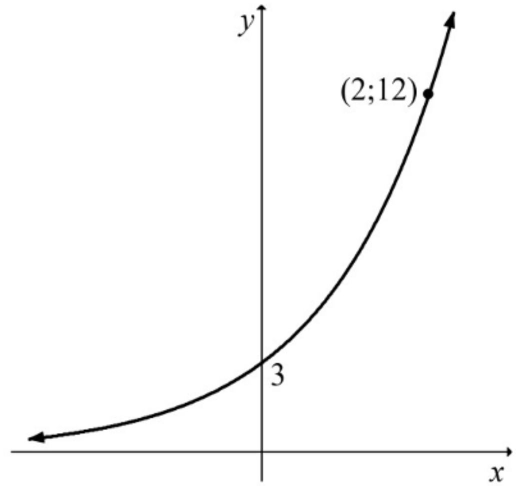
1. $y = 3^x$
2. $y = -3^x$
3. $y = 2.3^x$
4. $y = 3^x + 2$
5. $y = 3^x - 1$
6. $y = 3^{-x}$
7. $y = -\left(\frac{1}{3}\right)^x$
8. $y = 6\left(\frac{1}{3}\right)^x - 2$

LESSON 9: THE EXPONENTIAL GRAPH (PART 2)

NOTES

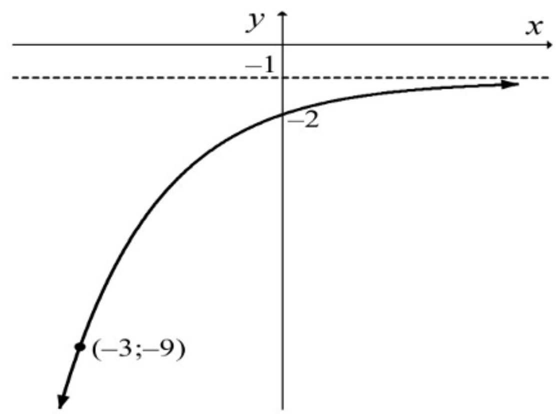
How to determine the equation of an exponential graph:

- Example 1:
  Determine the equation of the exponential graph shown in the sketch alongside.



  Solution:
  $q = 0$, because the asymptote is the x-axis.
  Substitute $(0\ ;\ 3)$ into $y = a.b^x$:
  $3 = a.b^0$
  $a = 3$
  $\therefore y = 3.b^x$
  Substitute $(2\ ;\ 12)$ into $y = 3.b^x$:
  $12 = 3.b^2$
  $b = 2$.
  Therefore the equation of the exponential graph is $y = 3.2^x$.

- Example 2:
  Determine the equation of the exponential graph shown in the sketch alongside.



  Solution:
  $q = -1$, because the asymptote is the line $y = -1$.
  $\therefore y = a.b^x - 1$.
  Substitute $(0\ ;\ -2)$ into $y = a.b^x - 1$:
  $-2 = a.b^0 - 1$
  $a = -1$
  Substitute $(-3\ ;\ -9)$ into $y = -b^x - 1$:
  $-9 = -b^3 - 1$
  $b = 2$
  Therefore the equation of the exponential graph is $y = -2^{-x} - 1$.
- Learners can now do Activities 1 and 2.

LESSON 9: ACTIVITIES/ASSESSMENTS

Activity 1:

In each case:

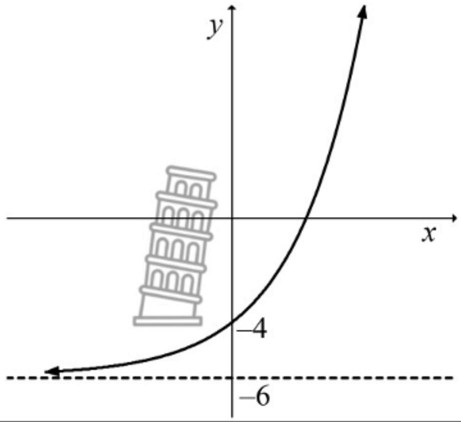
(a) Determine the equation of the exponential graph.

(b) Write down the domain of the graph.

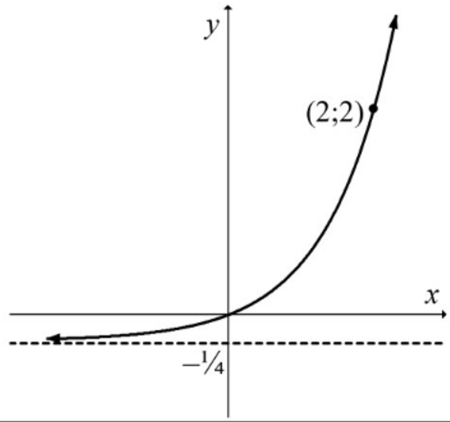
(c) Write down the range of the graph.

(d) Write down the equation of the asymptote to the graph.

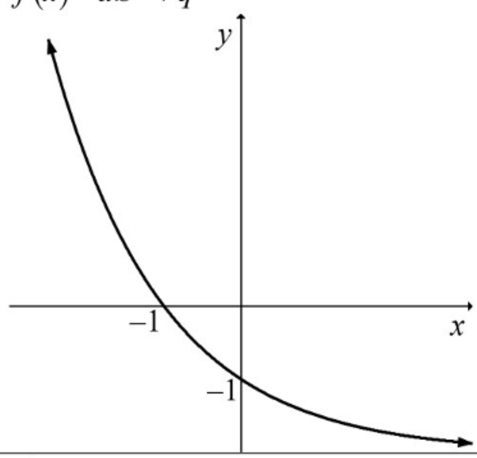
1. $f(x)=a.3^x-6$



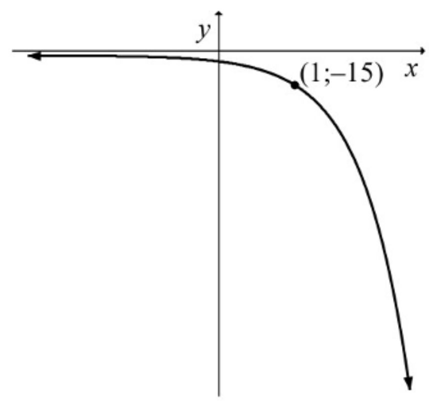
2. $f(x)=a.b^x+q$



3. $f(x)=a.b^x+q$

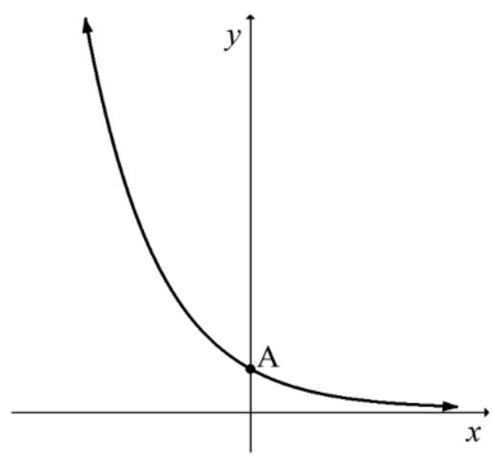


4. $f(x)=a.b^x+q$



Activity 2:

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is drawn alongside.



Answer the following questions by using the graph:

1. Write down the coordinates of A.
2. If B$(-2\,;\,m)$ is a point on the graph, calculate the value of m.
3. Write down the equation of the reflection of f in the y-axis.
4. Graph h is obtained by reflecting f in the x-axis and translating it down by 2 units. Write down the equation of h.
5. Write down the equation of the asymptote of h.

LESSON 10: MIXED QUESTIONS ON FUNCTIONS

NOTES

Explaining the functional notation where you have to find the x- values for which:

- $f(x) < 0$
- $f(x) > 0$
- $x.f(x) > 0$
- $x.f(x) < 0$
- $f(x) = g(x)$

ACTIVITIES/ASSESSMENTS

Classwork Activity:

Two selected problems per day from the 15 questions given.

Homework Activity:

Two selected problems per day from the 15 questions given.

FUNCTIONS – MIXED QUESTIONS – GRADE 10

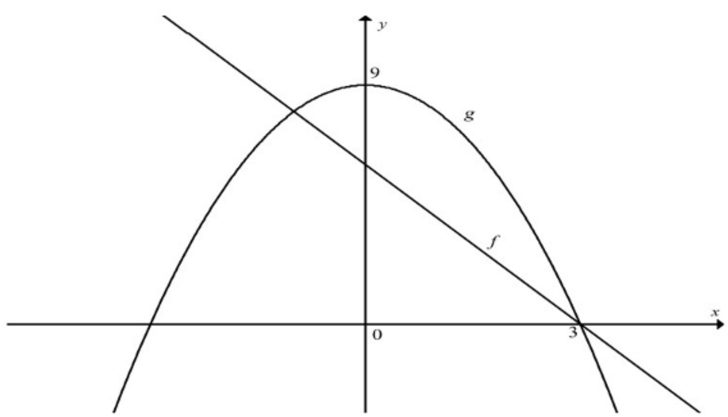
QUESTION 1

Given: $f(x)=\frac{3}{x}+1$ and $g(x)=-2x-4$

1.1	Sketch the graphs of f and g on the same set of axes.	(4)
1.2	Write down the equations of the asymptotes of f.	(2)
1.3	Write down the domain of f.	(2)
1.4	Solve for x if $f(x)=g(x)$.	(5)
1.5	Determine the values of x for which $-1\leq g(x)<3$.	(3)
1.6	Determine the y-intercept of k if $k(x)=2g(x)$.	(2)
1.7	Write down the coordinates of the x- and y-intercepts of h if h is the graph of g reflected about the y-axis.	(2)
		[20]

QUESTION 2

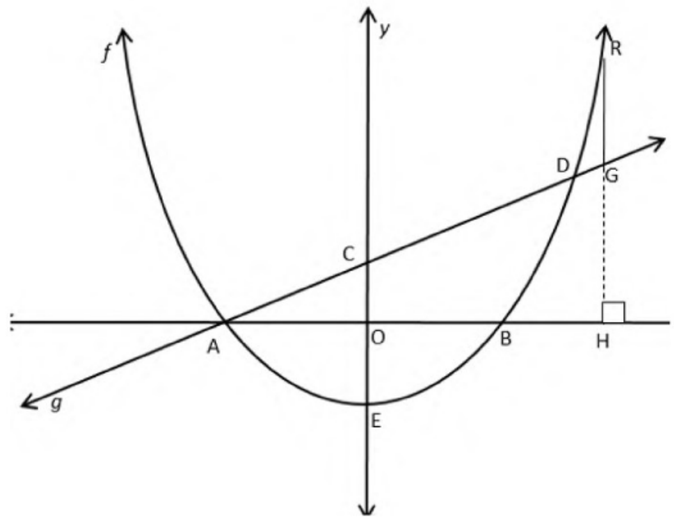
Sketched below are the graphs of $f(x)=-2x+6$ and $g(x)=ax^2+q$.



2.1	Determine the values of a and q.	(3)
2.2	Calculate the values of x for which $f(x)=g(x)$.	(5)
2.3	Hence or otherwise, write down the values of x for which $g(x)>f(x)$.	(2)
2.4	Write down the coordinates of the turning point of h if $h(x)=g(x)-4$.	(2)
		[12]

QUESTION 3

Sketched belo are the graphs of $f(x)=x^2-1$ and $g(x)=x+1$ The graph of f intersects the x-axis at A and B and the y-axis at E. The graph of g intersects the x-axis at A and the y-axis at C. f and g intersect at D.



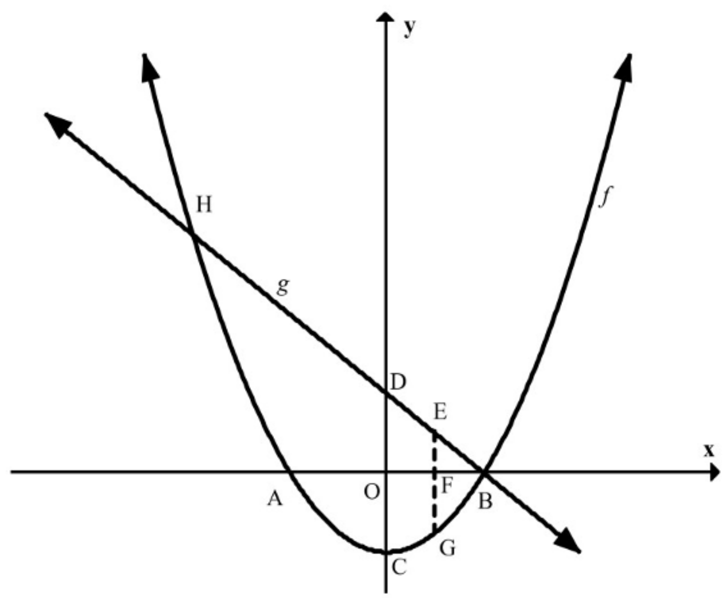
Use the graphs and the information above to determine the following

3.1	The coordinates of A and B.	(3)
3.2	The coordinates of C.	(1)
3.3	The coordinates of D.	(5)
3.4	The range of f.	(2)
3.5	The length of RG if OH is 6 units and F lies on f and G lies on g.	(3)
3.6	The value(s) of x for which $g(x)>f(x)$.	(2)

[16]

QUESTION 4

Sketched below are the graphs of $f(x)=2x^2-2$ and $g(x)=-2x+2$.



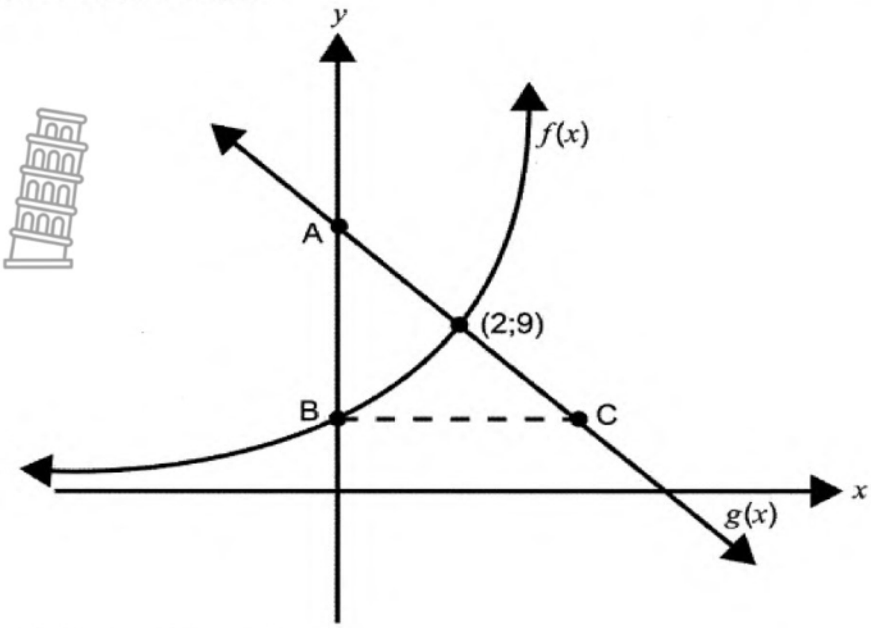
The graph of f intersects the x–axis at A and B and the y–axis at C. The graph of g intersects the x–axis at B and the y–axis at D. f and g intersect at H and B.

Use the graphs and the information above to determine the following:

4.1	The coordinates of A and B.	(4)
4.2	The coordinates of C.	(1)
4.3	The coordinates of D.	(1)
4.4	The length of EG if OF = $\frac{1}{2}$ unit and E lies on g and G lies on f.	(5)

QUESTION 5

Below are the graphs of $f(x) = a^x$, $a > 0$ and $g(x) = -x + 11$. (2; 9) is the point of intersection of $f(x)$ and $g(x)$.



5.1 Show that the value of $a = 3$. (2)

5.2 Determine the equation of $f(x)$. (1)

5.3 Write down the equation of the asymptote of $f(x)$. (1)

5.4 Write down the domain of g. (1)

5.5 Find the coordinates of:

5.5.1 point A (1)

5.5.2 point B (1)

5.5.3 point C (2)

5.6 The graph of $f(x)$ is shifted up to point **A.** Give the new equation of $f(x)$ in the form $h(x)=$... (2)

5.7 The graph of $g(x)$ is reflected in the y-axis. Give the new equation of $g(x)$ in the form $k(x) =$ ... (2)

[13]

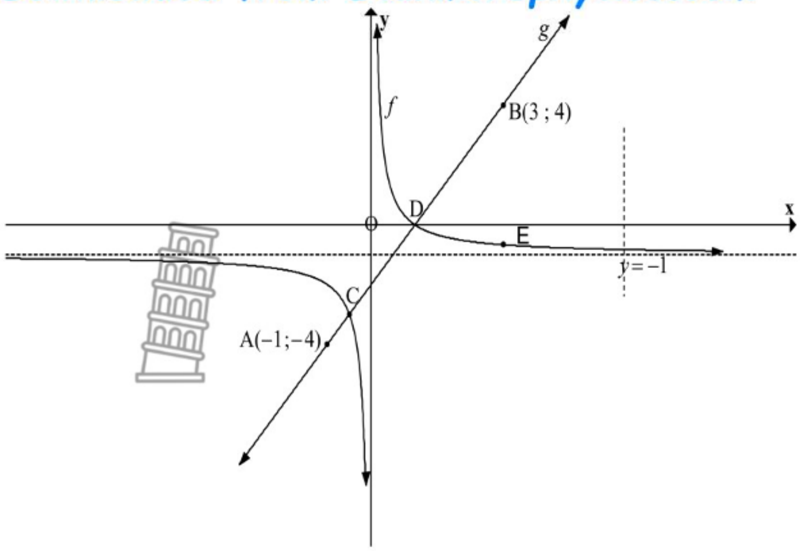
QUESTION 6

The sketch below shows f and g, the graphs of $f(x) = \frac{1}{x} - 1$ and $g(x) = ax + q$ respectively.

Points A(–1 ; –4) and B(3 ; 4) lie on the graph g.

The two graphs intersect at points C and D.

Line BE is drawn parallel to the y-axis, with E on f



6.1 Show that $a = 2$ and $q = -2$. (2)

6.2 Determine the values of x for which $f(x) = g(x)$. (4)

6.3 For what values of x is $g(x) \geq f(x)$? (3)

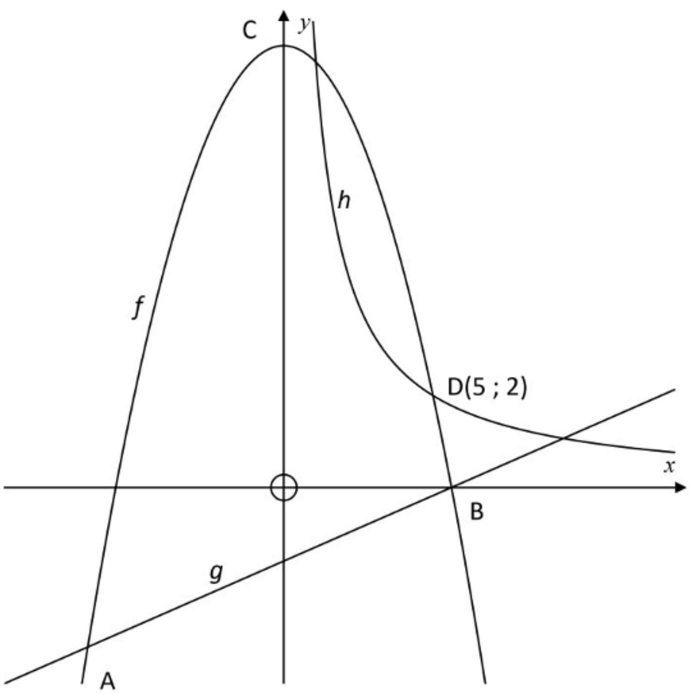
6.4 Calculate the length of BE. (3)

6.5 Write down an equation of h if $h(x) = f(x) + 3$. (1)

[13]

QUESTION 7

Sketched below are the graphs of $f(x) = -2x^2 + 18$, $g(x) = mx + c$ and $h(x) = \frac{k}{x}$ where $x > 0$. The graph of f intersects the x–axis at A and B and the y–axis at C, which is also the turning point of f. The graph of g also intersects the x–axis at B. One of the points of intersection of g and h is D(5 ; 2).



Use the graphs and the information given above to answer the following :

7.1 The coordinates of A and B. (4)

7.2 The length of OC. (1)

7.3 The values of m and c. (3)

7.4 The equation of h. (2)

7.5 The range of f. (2)

7.6 The equation of the straight line through B perpendicular to g. (3)

[15]

QUESTION 9

9.1 Given: $f(x) = x^2 - 2$ and $g(x) = 3^x$

9.1.1 Sketch the graph of $f(x)$ and $g(x)$ on the same system of axes. Clearly indicate all the intercepts on the graph. (4)

9.1.2 For which value(s) of x is $g(x) > 1$? (1)

9.1.3 Write down the range of $f(x)$. (1)

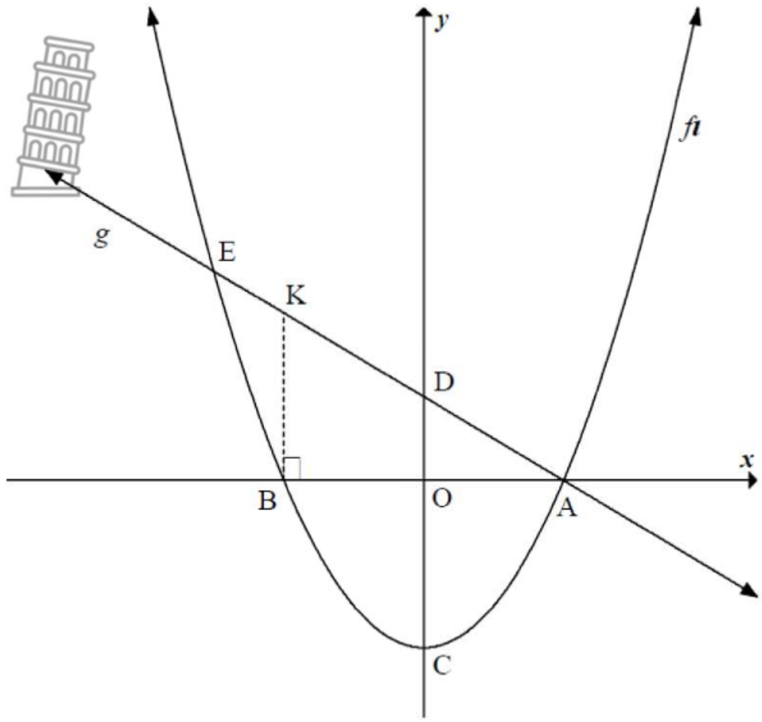
9.1.4 Describe in words the transformation of $f(x)$ to $h(x) = (x - 2)(x + 2)$. (2)

9.2 Determine the equation of the function $g(x) = \frac{a}{x} + q$, with the asymptote $y = -2$. The straight line $f(x) = -x$ intersects the graph of $g(x)$ at the point $(5; -5)$. (3)

[11]

QUESTION 10

The graphs of $f(x) = x^2 - 4$ and $g(x) = -x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK $\parallel$ x-axis. f and g intersect at A and E.



10.1 Write down the coordinates of C. (1)

10.2 Write down the coordinates of D. (1)

10.3 Determine the length of CD. (1)

10.4 Calculate the coordinates of B. (3)

10.5 Determine the coordinates of E, a point of intersection of f and g. (4)

10.6 For which values of x will:

10.6.1 $f(x) < g(x)$ (2)

10.6.2 $f(x).g(x) \geq 0$ (2)

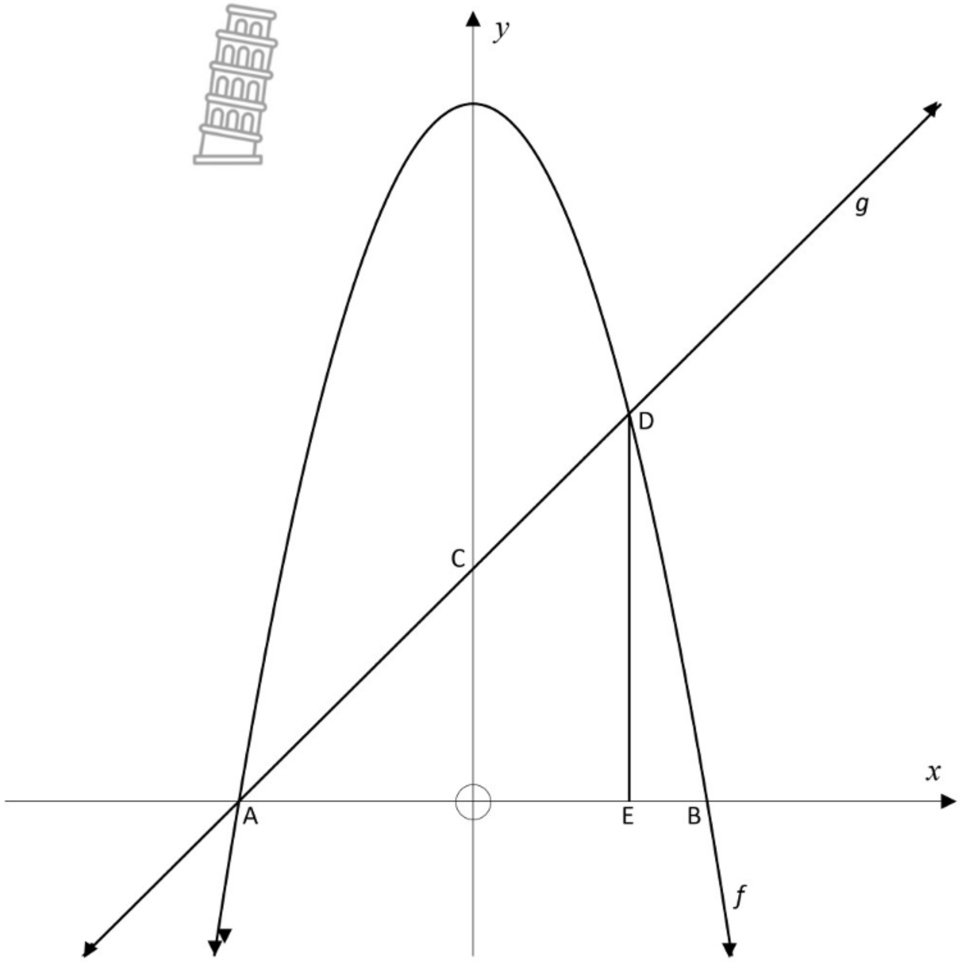
10.7 Calculate the length of AK. (4)

[18]

QUESTION 11

Sketched below are the graphs of $f(x) = -4x^2 + 9$ and $g(x) = 2x + 3$.

The graph of f intersects the x–axis at A and B. The graph of g intersects f at A and D and the y–axis at C. DE // y–axis.



11.1 Write down the coordinates of C. (1)

11.2 Calculate the length of AB. (3)

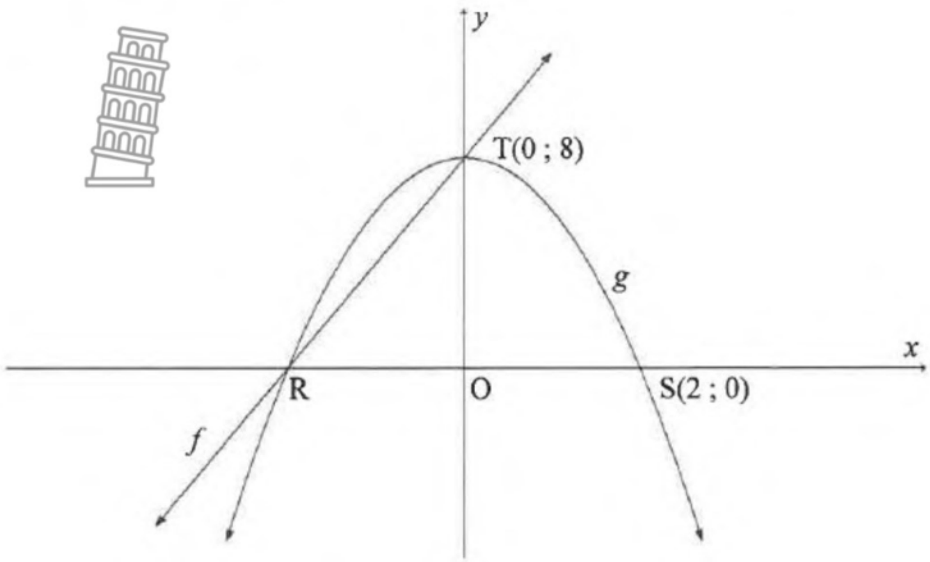
11.3 Calculate the length of DE if $AE = 2\frac{1}{2}$ units. (2)

11.4 For which values of x will $f(x) > g(x)$? (2)

11.5 Calculate the length of AD. (2)

QUESTION 12

The diagram shows the graphs of $g(x) = ax^2 + q$ and $f(x) = mx + c$.
R and S(2 ; 0) are the x-intercepts of g and T(0 ; 8) is the y-intercept of g.
Graph f passes through R and T.



12.1 Write down the range of g. (1)

12.2 Write down the x-coordinate of R. (1)

12.3 Calculate the values of a and q. (3)

12.4 Determine the equation of f. (3)

12.5 Use the graphs to determine the value(s) of x for which:

12.5.1 $f(x) = g(x)$ (2)

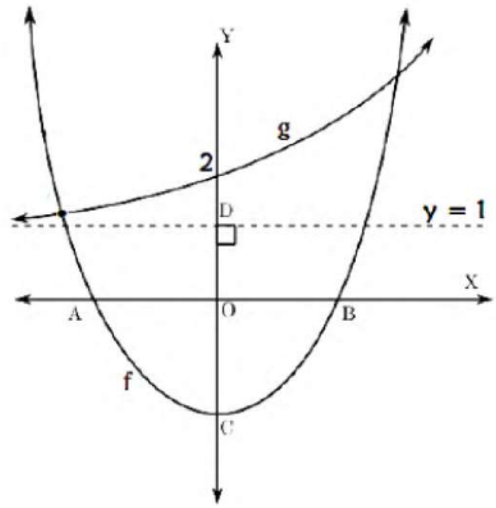
12.5.2 $x \,.\, g(x) \leq 0$ (3)

12.6 The graph h is obtained when g is reflected along the line $y = 0$.
Write down the equation of h in the form $h(x) = px^2 + k$. (2)

[15]

QUESTION 13

The graph represents $f(x) = 2x^2 - 2$ and $g(x) = a^x + q$.
$(-2; 1\frac{1}{4})$ is a point of intersection of f and g.



13.1 Calculate the co-ordinates of A, B and C. (5)

13.2 Calculate the value of a and q. (2)

13.3 Write down the length of CD. (1)

13.4 State the range of f. (1)

13.5 For which value(s) of x is:

13.5.1 $f(x) \leq 0$ (1)

13.5.2 $f(x)$ increasing (1)

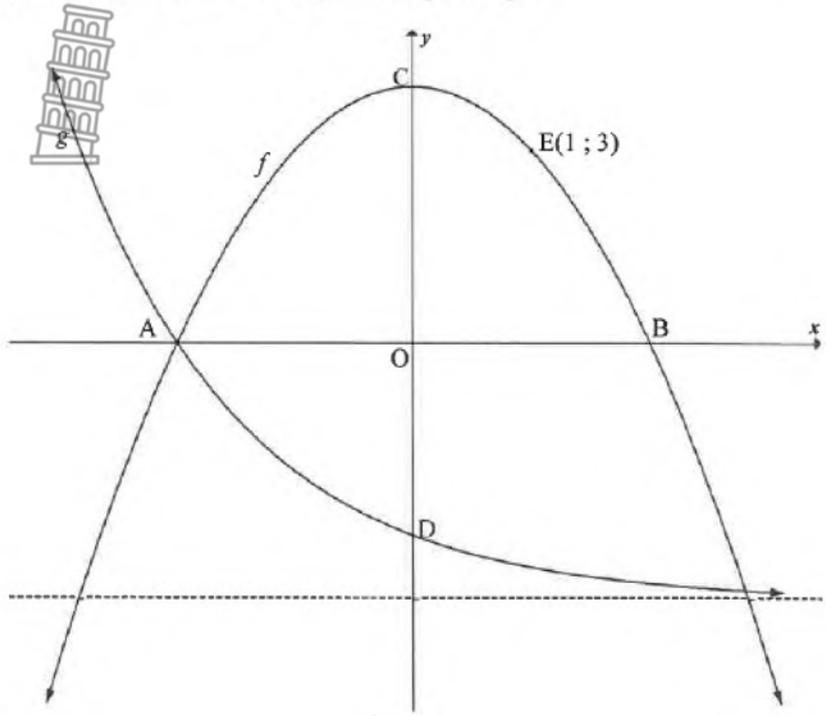
13.5.3 $g(x) < f(x)$, if $x < 0$ (1)

[12]

QUESTION 14

Sketched below are the graphs of $f(x) = ax^2 + q$ and $g(x) = \left(\frac{1}{2}\right)^x - 4$.

A and B are the x-intercepts of f. The graphs intersect at A and point E $(1 ; 3)$ lies on f. C is the turning point of f and D is the y-intercept of g.



14.1 Write down the:

QUESTION 15

14.1.1	Coordinates of D	(2)
14.1.2	Range of g	(1)

14.2 Calculate the:

14.2.1	Coordinates of A	(2)
14.2.2	Values of a and q	(4)

14.3 Determine the:

14.3.1	Length of CD	(2)
14.3.2	Equation of a straight line through A and D	(3)

14.4 For which values of x is:

14.4.1	$f(x) > 0$?	(2)
14.4.2	f decreasing?	(1)

[17]

QUESTION 4

Given: $f(x)=x^2-16$ and $g(x)=3^x-3$

15.1	Write down the coordinates of the y–intercept of f.	(1)
15.2	Calculate the x–intercept of g.	(2)
15.3	Sketch the graphs of f and g on the same system of axes. Clearly indicate ALL the intercepts and the asymptote on the graph.	(5)
15.4	For which value(s) of x is $g(x) > 6$?	(1)
15.5	Describe in words the transformation of f to h if $h(x)=-(x-4)(x+4)$	(1)
		[10]

LESSON 1: TRIGONOMETRIC GRAPHS (1)

NOTES:

Point by point plotting of graphs defined by $y = \sin\theta$ **and** $y = \cos\theta$ **for** $\theta \in [0°;360°]$**:**

Explanation of terminology:

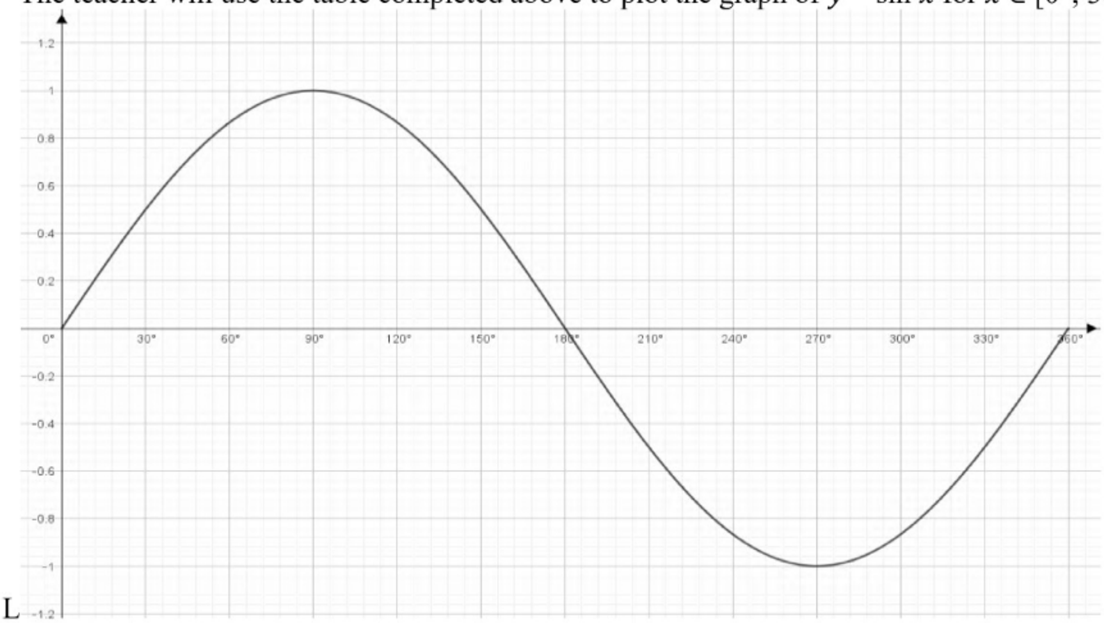
- **Domain:** the x values (input) that are represented on the $x-axis$
- **Range:** the y values (output) that are represented on the $y-axis$ (definitions of these would have been covered in the previous topic of functions).
- **Period:** Number of degrees it takes a function to complete a circle.
- The interval notation and the meaning thereof:
  - ✓ **Square brackets** mean that the end values are included.
  - ✓ **Round brackets** mean that the end values are not included
- From algebra the concept of substitution and calculator skills.

➢ Teacher will ask learners to complete the table using the calculator.

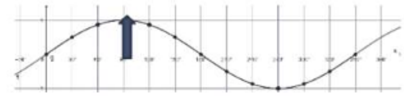
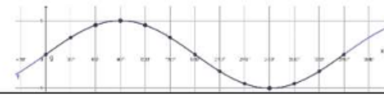
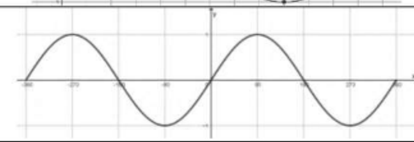
x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin x$	0	0,5	0,87			0,5				-1			0

EXAMPLE

➢ The teacher will use the table completed above to plot the graph of $y = \sin x$ for $x \in [0°; 360°]$



➢ The teacher will explain the main features of this graph to learners.

	Solution	Meaning	Example
Amplitude	1	half the distance between maximum and minimum value (must always be positive	
Period	360°	One complete cycle Starts at 0° and ends at 360°	
Range	$y \in$ [-1;1]	All possible y- values	
Domain	$x \in [-360°; 360°]$	All x-values for which the graph was sketched.	

ACTIVITY 1

1.1 Using point-by-point plotting, sketch the graph of $f(x) = \cos x$ in, for θ∈[0°;360°].using the tables below.

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$y = \cos x$									

1.2. Use the graph drawn in 1.1 to complete the following table below

	solution
Amplitude	
Period	
Range	
Domain	

1.3. Using point-by-point plotting, sketch the graph of $f(x) = -\sin x$ in, for θ∈[0°;360°].using the tables below.

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y = -sin$x$									

1.4. Use the graph drawn in 1.3 to complete the following table below.

	solution
Amplitude	
Period	
Range	
Domain	

LESSON 2: TRIGONOMETRIC GRAPHS (1)

NOTES:

Point by point plotting of graphs defined by $y = \sin\theta$ and $y = \cos\theta$ for $\theta \in [0°;360°]$:

Explanation of terminology:

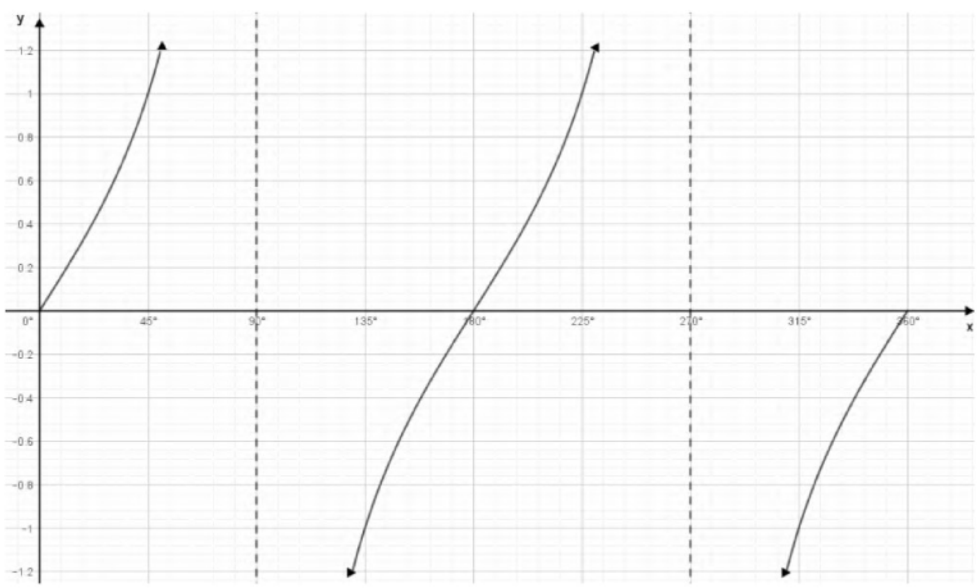
- Domain: the x values (input) that are represented on the $x - axis$
- Range: the y values (output) that are represented on the $y - axis$ (definitions of these would have been covered in the previous topic of functions).
- Period: Number of degrees it takes a function to complete a circle.
- The interval notation and the meaning thereof:
  - ✓ Square brackets mean that the end values are included.
  - ✓ Round brackets mean that the end values are not included
- From algebra the concept of substitution and calculator skills.
- Asymptote: Special facts regarding the $y = \tan x$
- Asymptotes at $x = 90° + k.\ 180°$ for k an integer Remember the tan graph repeats every 180°(hint, when using calculator asymptote will appear as MATHS ERROR)

Example

Complete the table using the calculator

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$y = \tan x$	0		E						

Use the table completed above to plot the graph of $y = \sin x$ for $x \in [0°;360°]$



	Solution	Meaning
Asymptote	x= 90° and x =270°	
Period	180°	One complete cycle Starts at 0° and ends at 360°
Range	$y \in [-\infty;\infty]$	All possible y- values

Domain	$x \in [0°; 360°]$	All x-values for which the graph was sketched.

Activity 1

1.1. Using point-by-point plotting, sketch the graphs of the following trigonometric functions using the tables below.

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y = -tan x	0°		E						

1.2. Use the graph drawn in 1.1 to complete the following table below.

	Solution
Asymptote	
Period	
Range	
Domain	

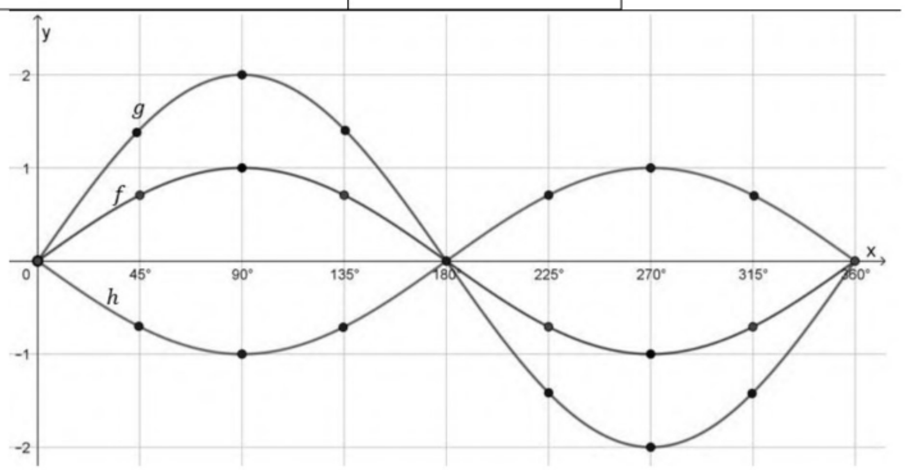
LESSON 3: TRIGONOMETRIC GRAPHS (2)

NOTES:

Explanation of the characteristics of the graph sketched

- **Learners identify highest and lowest points on the graph. Link the highest to term maximum and lowest to term minimum.**
- **Amplitude: half the distance between maximum and minimum value (must always be positive** $\frac{\max value - \min value}{2}$
- **Range: [min value; max value]**

$y = a\sin\theta$ **and** $y = a\cos\theta$	$y = a\tan\theta$	
• **For** $a > 1$ **Vertical stretch from min to max value and the amplitude increases** • a **is the amplitude of the graph (ignore the sign)?** • **for** $0 < a < 1$**,the amplitude decreases** • **if** a **is negative (** a **<0), there is a reflection in the x- axis.** • **For** $-1 < a < 0$**, there is a reflection in the x- axis and the amplitude decreases** • **For** $a < -1$**, there is a reflection in the x- axis and the amplitude increases.**	• **vertical stretch from the x-axis (ignore the signs)** • **if** a **is negative, the there is a reflection in the x-axis**	**The effect of** $\boldsymbol{a}$ **in the function** $\boldsymbol{y = a \sin x}$**.** **The teacher will draw the following graphs so that learners can see the effect of** $\boldsymbol{a}$ $\boldsymbol{f(x) = \sin x}$**;** $\boldsymbol{g(x) = 2\sin x}$ **and** $\boldsymbol{h(x) = -\sin x}$**.**



➢ How does the value of $\boldsymbol{a}$ affect the shape of the graph?

Possible answers: for $g(x)$ the max and the min changes and for $h(x)$ there is reflection along the x-axis.

Activity 1.

1.1 Sketch the following graphs in the same set of axes for $x \in [0°; 360°]$ and complete the table below

$f(x) = \cos x$, $g(x) = 2\cos x$ and $h(x) = -2\cos x$

	$f(x)$	$g(x)$	$h(x)$
Amplitude			
Period			
Range			
Domain			

1.2. Sketch the following graphs in the same set of axes for $x \in [0°; 360°]$ and complete the table below

$f(x) = \tan x$, $g(x) = 2\tan x$ and $h(x) = -2\tan x$

	$f(x)$	$g(x)$	$h(x)$
Asymptotes			
Period			
Range			
Domain			

LESSON 4: TRIGONOMETRIC GRAPHS (2)

NOTES:

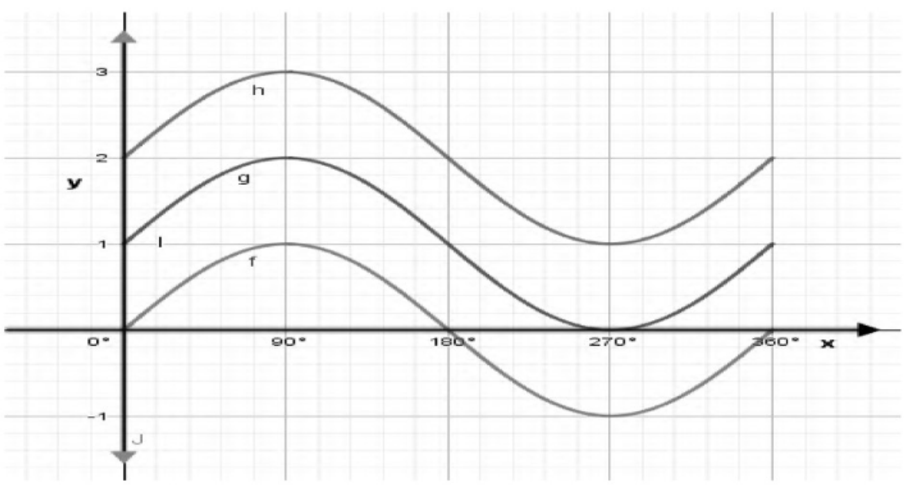
Explanation of the characteristics of the graph sketched

- Learners identify highest and lowest points on the graph. Link the highest to term maximum and lowest to term minimum.
- **Amplitude:** half the distance between maximum and minimum value (must always be positive $\frac{\max value - \min value}{2}$
- **Range:** [min value; max value]
- **Effect of 'q':** it is the vertical shift of the graph. (q>0, the graph shift vertically upward by q units, q<0, the graph shift vertically downward by q units)

Examples: The effect of q in $y = \sin x + q$

Study the three graphs below:

$f(x) = \sin x$
$g(x) = \sin x + 1$
$h(x) = \sin x + 2$



What transformation do you notice?

$\mathbf{y = \sin x}$ **moved 1 unit up to give the graph of** $\mathbf{y = \sin x + 1}$

$\mathbf{y = \sin x}$ **moved 2 units up to give the graph of** $\mathbf{y = \sin x + 2}$

ACTIVITY 1

1.1. Complete the following table by use of a calculator, rounding off answers to 2 decimal places.

X	0°	45°	90°	135°	180°	225°	270°	315°	360°
f(x) = cos x									
g(x) = cos x+2									
h(x) = cos x-2									
p(x) = -cos x+1									

1.1.1 Sketch the graph of f and g , (use different coloured pencils for each graph to highlight the differences and similarities.

1.1.2 Explain the effect of 2 in the graph of $y = \cos x + 2$ (q-value).

1.1.3 Similarly explain the effect of -2 on the mother graph of $y = \cos x$

1.1.4 Comparing yours answers to questions 1.1.2 and 1.1.3, explain the effects of q on the trigonometric functions. (hint: use the words, stretch and shift to differentiate between the effects)

1.2. Complete the following table by use of a calculator, rounding off answers to 2 decimal places and answer the question that follows

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
f(x) = tan x									
g(x) = tan x+2									
h(x) = tan x-2									

1.2.1 Sketch the graph of f and g , (use different coloured pencils for each graph to highlight the differences and similarities.

1.2.1 Explain the effect of the 2 in the graph of $y = \tan x + 2$ (q-value).

1.2.2 Similarly explain the effect of -2 on the mother graph of $y = \tan x$

1.2.3 Comparing yours answers to questions1.2. 2 and1.2 3, explain the effects of q on the trigonometric functions. (hint: use the words, stretch and shift to differentiate between the effects)

LESSON 5: TRIGONOMETRIC GRAPHS (2)

NOTES:

Explanation of the characteristics of the graph sketched

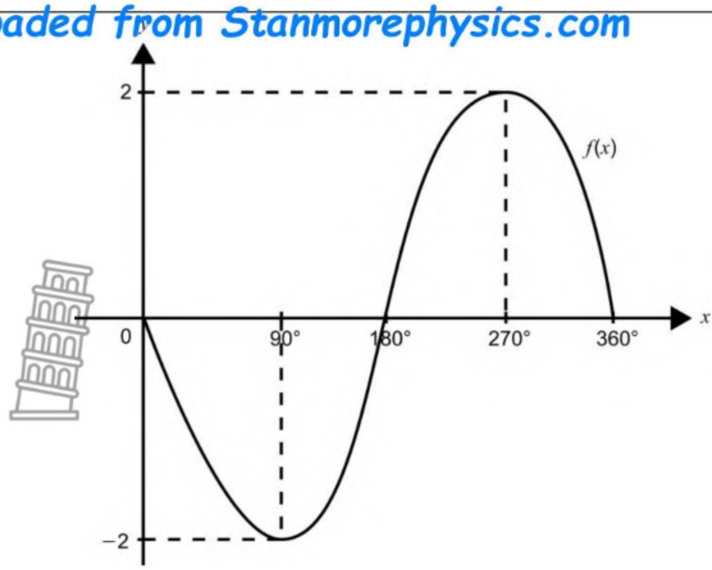
- Learners identify highest and lowest points on the graph. Link the highest to term maximum and lowest to term minimum.
- **Amplitude:** half the distance between maximum and minimum value (must always be positive

$$\frac{\max value - \min value}{2}$$

- **Range:** [min value; max value]
- **Effect of 'q':** it is the vertical shift of the graph.

Example

Below is the graph of $f(x) = a.\sin x + q$



1.1 Determine the values of a and q.
1.2 Write down the range of $f(x)$.
1.3 What is the amplitude of $f(x)$?

Solution:

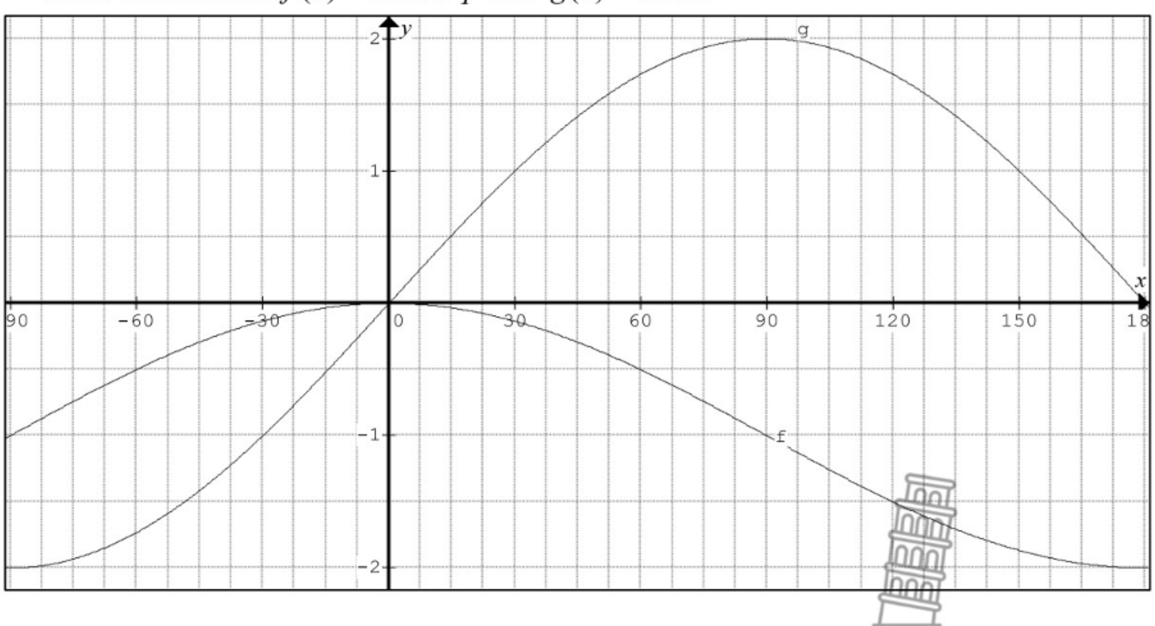
1.1 $a = -2$
$b = 0$

1.2 $-2 \le y \le 2$, $y \in R$

1.3 2

Activity 1

1.1 Below is a sketch of $f(x) = \cos x + q$ and $g(x) = a\sin x$



1.1.1 Write down the amplitude of *f*.

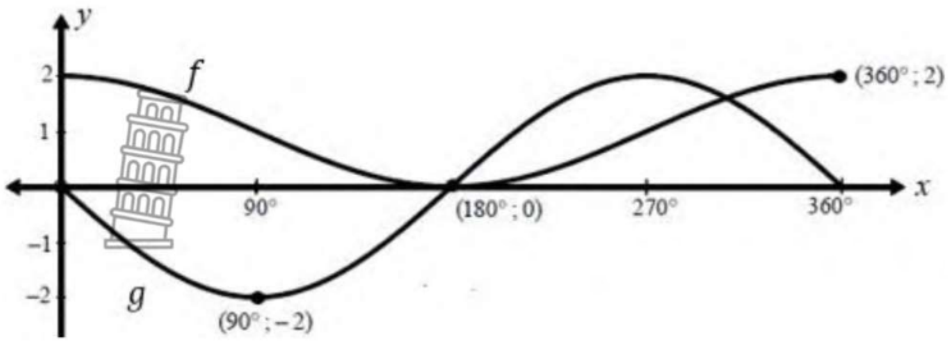
1.1.2 For which value(s) of *x* is g(*x*) = 2 ?

1.1.3 Determine the values of *a* and *q*.

1.1.4 For which values of $x \in [-90°; 180°]$ is $f(x) \geq g(x)$?

1. In the diagram below, the graphs of $f(x) = a \cos x + q$ and

   $g(x) = m \sin x + n$ are shown the domain $x \in [0°; 360°]$.



2.1 Write down the amplitude and range of f.

2.2 Write down the amplitude and range of g

2.3 Determine the values of a and q.

2.4 Determine the values of m and n.