Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

CALCULUS TEST

19 MAY 2023

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 25

TIME: 30 minutes

N.B. This question paper consists of 5 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

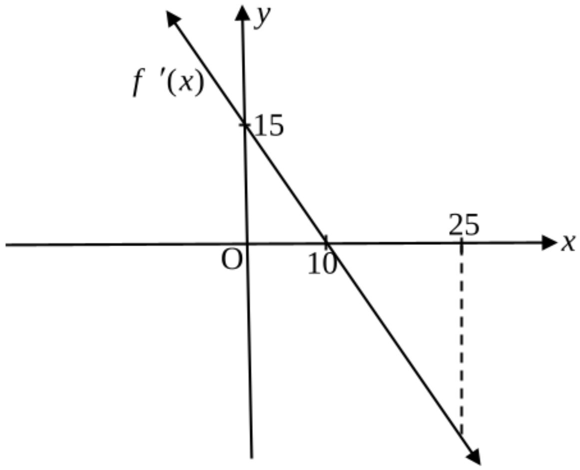
1. This question paper consists of 2 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The graphs of $g(x)=x^3-ax^2+6$ and $h(x)=2x^2+bx+3$ touch when $x=1$.
The two graphs also have a common tangent at $x=1$.
Determine the coordinates of the point of contact of the two graphs. (7)

1.2 Given: A function defined by $f(x)=3x^3-4k^2x+5$, where *k* is a positive number.
Determine the value(s) of *x* in terms of *k* for which the function is decreasing. (3)

1.3 In the diagram below, the graph of $f\ '(x)$ is sketched for $x \in R$.



1.3.1 Write down the gradient of *f* at $x=0$. (1)

1.3.2 Write down the *x* – coordinate of the turning point of *f*. (1)

1.3.3 What is the gradient of *f* at $x=1$? (3)

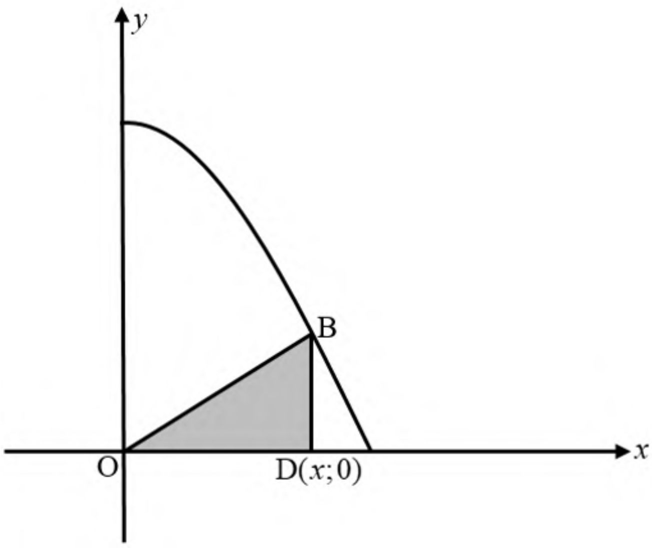
1.3.4 Calculate the point of inflection of *f*. (1)

[16]

QUESTION 2

The figure below represents the parabola given by $f(x) = 4 - \frac{x^2}{4}$ with $0 \leq x \leq 4$.

D(x; 0) is a point on the x – axis and B on the graph of *f*. DB is parallel to the y – axis.



2.1 Show that A, the area of ΔOBD, is given by $A = 2x - \frac{x^3}{8}$. (3)

2.2 Determine how far D should be from O in order that the area of ΔOBD is as large as possible. (4)

2.3 Hence determine the area of ΔOBD when D is on the point calculated in 2.2 above. (2)

[09]

TOTAL: 25 MARKS

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

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MARKING GUIDELINE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 25

TIME: 30 minutes

N.B. This guideline consists of 3 pages.

QUESTION 1

1.	$g(1)=h(1)$ $(1)^3-a(1)^2+6=2(1)^2+b(1)+3$ $1-a+6=2+b+3$ $a=2-b$.......................(1) $g'(x)=3x^2-2ax$ $h'(x)=4x+b$ $g'(1)=h'(1)$ $3(1)^2-2a(1)=4(1)+b$ $3-2a=4+b$......................(2) (1) in (2): $3-2(2-b)=4+4$ $b=5$ $a=-3$ $\therefore h(x)=2x^2+5x+3$ $\therefore h(1)=10$ $\therefore$ Point of contact is: (1 ; 10)	✓ $g(1)$ and $h(1)$ ✓ $g'(x)$ ✓ $g'(1)=h'(1)$ ✓Value of b ✓Value of a ✓10 ✓Answer (7)
1.2	A function decreases when $g'(x)<0$ $g'(x)=9x^2-4k^2$ $\therefore \quad 9x^2-4k^2<0$ $(3x-2k)(3x+2k)<0$ CV's: $\pm\frac{2k}{3}$ $\therefore \quad -\frac{2k}{3}<x<\frac{2k}{3}$	✓Setting up the inequality ✓Factors ✓Solution (3)
1.3.1	$f'(0)=15$	✓ Answer (1)
1.3.2	$x=10$	✓ Answer (1)
1.3.3	$m=\frac{15-0}{0-10}=-\frac{3}{2}$ $\therefore f'(x)=-\frac{3}{2}x+15$ $\therefore f'(1)=-\frac{3}{2}(1)+15$ $=\frac{27}{2}$	✓ $m=-\frac{3}{2}$ ✓ $f'(x)=-\frac{3}{2}x+15$ ✓ Answer (3)
1.3.4	There is no point of inflection, $f(x)$ is a parabola.	✓ Answer/ Any logical reasoning (1)
		[16]

QUESTION 2

2.1	Coordinates of B: $B\left(x;4-\frac{x^2}{4}\right)$ $\therefore DB = 4-\frac{x^2}{4}$ *units* and $OD = x$ *units* $\therefore$ Area $\Delta OBD = \frac{1}{2}\times OD\times DB$ $=\frac{1}{2}\times x\times\left(4-\frac{x^2}{4}\right)$ $=2x-\frac{x^3}{8}$	✓ $x-$co-ord of B ✓ $y-$co-ord of B ✓ Substitution in correct formula (3)
2.2	For maximum: $\frac{dA}{dx}=0$ $\therefore\ 2-\frac{3x^2}{8}=0$ $\frac{3x^2}{8}=2$ $3x^2=16$ $x^2=\frac{16}{3}$ $x=\pm 2.31$ $\therefore$ D must be 2.31 *units* away from O.	✓Equating derivative to 0 ✓Standard form ✓x – values ✓ Answer/Reasoning (4)
2.3	$A=2x-\frac{x^3}{8}$ $A(2.31)=2(2.31)-\frac{(2,31)^3}{8}$ $=3,08$ *units*^{2}	✓ Substitution ✓Answer (2)
		[09]

TOTAL: 25 MARKS