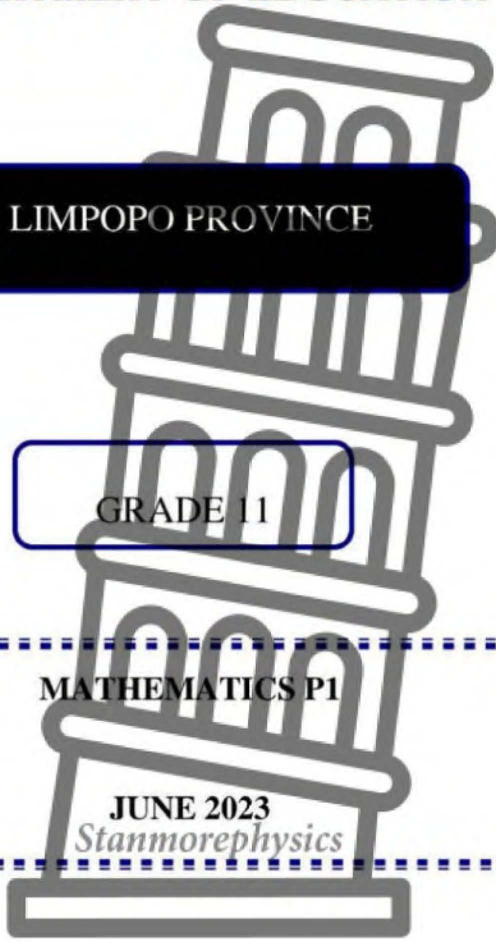
LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION



LIMPOPO PROVINCE

GRADE 11

MATHEMATICS P1

JUNE 2023

MARKS : 100

DURATION : 2 hours

This question paper consists of 7 pages including 1 diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions and 1 diagram sheet.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $2x(x-3)=0$ (2)

1.1.2 $3x^2-2x=4$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{5-x}=x+1$ (5)

1.1.4 $2x^2+5x \leq 3$ (4)

1.1.5 $5x^{\frac{2}{5}}=20$ (3)

1.1.6 $2^x-\frac{12}{2^x}=-4$ (5)

1.2 Solve for x and y simultaneously:

$5y-x=2$ and $x^2-3xy+4y=4$ (6)

[29]

QUESTION 2

2.1 Simplify the following, without using a calculator:

2.1.1 $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ (3)

2.1.2 $(\sqrt{12}+2)(\sqrt{3}-1)$ (3)

2.1.3 $\frac{3^{2x+1}\,.\,15^{2x-3}}{27^{x-1}\,.\,3^{x}\,.\,5^{2x-4}}$ (4)

2.2 WITHOUT using a calculator, show that $\frac{\sqrt{2}}{\sqrt{2}+1}+\frac{4}{\sqrt{2}}$ simplifies to $2+\sqrt{2}$. (4)

2.3 Determine the value of m if $x^2-mx+(m+3)=0$ has equal roots. (5)

[19]

QUESTION 3:

Given: $f(x) = \frac{1}{x-3} - 2$

3.1 Write down the equation(s) of the asymptote(s) of f. (1)

3.2 Determine the x-intercept of f. (3)

3.3 Sketch the graph of f on the attached diagram sheet. Clearly show ALL the intercepts with the axes and the asymptote(s). (3)

3.4 Determine the equation of the axis of symmetry of f having a positive gradient. (2)

3.5 The graph of f is transformed to obtain the graph of $h(x) = \frac{1}{x}$.
Describe the transformation from f to h. (2)

3.6 Write down the domain of h. (1)

[12]

QUESTION 4

Given: $f(x) = -x^2 + 6x + 7$

4.1 Determine the coordinates of the turning point of f. (3)

4.2 Write down the equation of the axis of symmetry of f. (1)

4.3 The graph of f is shifted 4 units to the left and reflected in the x-axis to form h.
Write down the equation of h in the form $h(x) = a(x+p)^2 + q$. (2)

[6]

QUESTION 5

5.1 Given: $g(x) = \left(\frac{1}{2}\right)^x - 4$

5.1.1 Write down the equation of asymptote of g. (1)

5.1.2 Write down the range of g. (1)

5.1.3 Determine the coordinates of the x- intercept of g. (3)

5.1.4 Hence, write down the values of x for which $g(x) < 0$. (1)

5.2 The point A(3 ; 54) lies on the graph of $f(x) = 3^{x+p} - 27$.

5.2.1 Determine the value of p. (3)

5.2.2 Determine the range of f. (1)

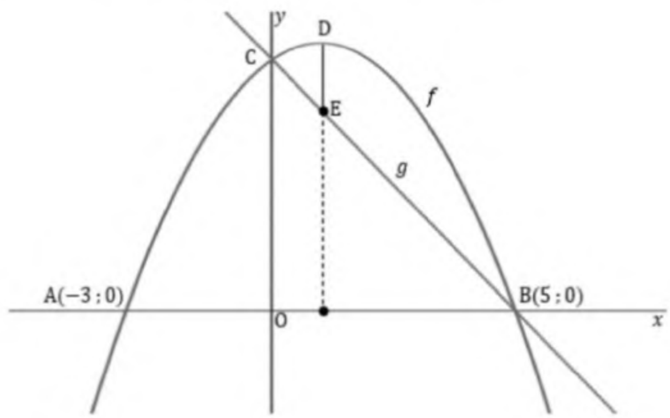
5.2.3 Graph g is obtained be reflecting graph f about the x-axis.
Determine the coordinates of the y-intercept of g. (3)

[13]

QUESTION 6:

The diagram shows the graphs of $f(x) = -x^2 + 2x + 15$ and $g(x) = -3x + k$.

- Graph f cuts the x-axis at A(−3 ; 0) and B(5 ; 0), the y-axis at C and has a turning point at D.
- Graph g cuts the x-axis at B and the y-axis at C.
- E is a point on g such that DE is parallel to the y-axis.



6.1 Show that $k = 15$. (1)

6.2 Determine the coordinates of D, the turning point f. (3)

6.3 Determine the values of x for which f is increasing. (1)

6.4 Calculate the average gradient between points A and D. (2)

6.5 Calculate the length of DE. (2)

6.6 If $h(x) = f(x-1) - 2$, determine the equation of h in the form:
$h(x) = a(x+p)^2 + q$. (3)

6.7 For which values of x is:

6.7.1 $f(x) \geq 0$ (1)

6.7.2 $f(x) < g(x)$. (2)

6.8 Determine the maximum value of $p(x) = 3^{f(x)-12}$. (3)

6.9 Determine the values of k for which $f(x) + k = 0$ will have two distinct real roots. (3)

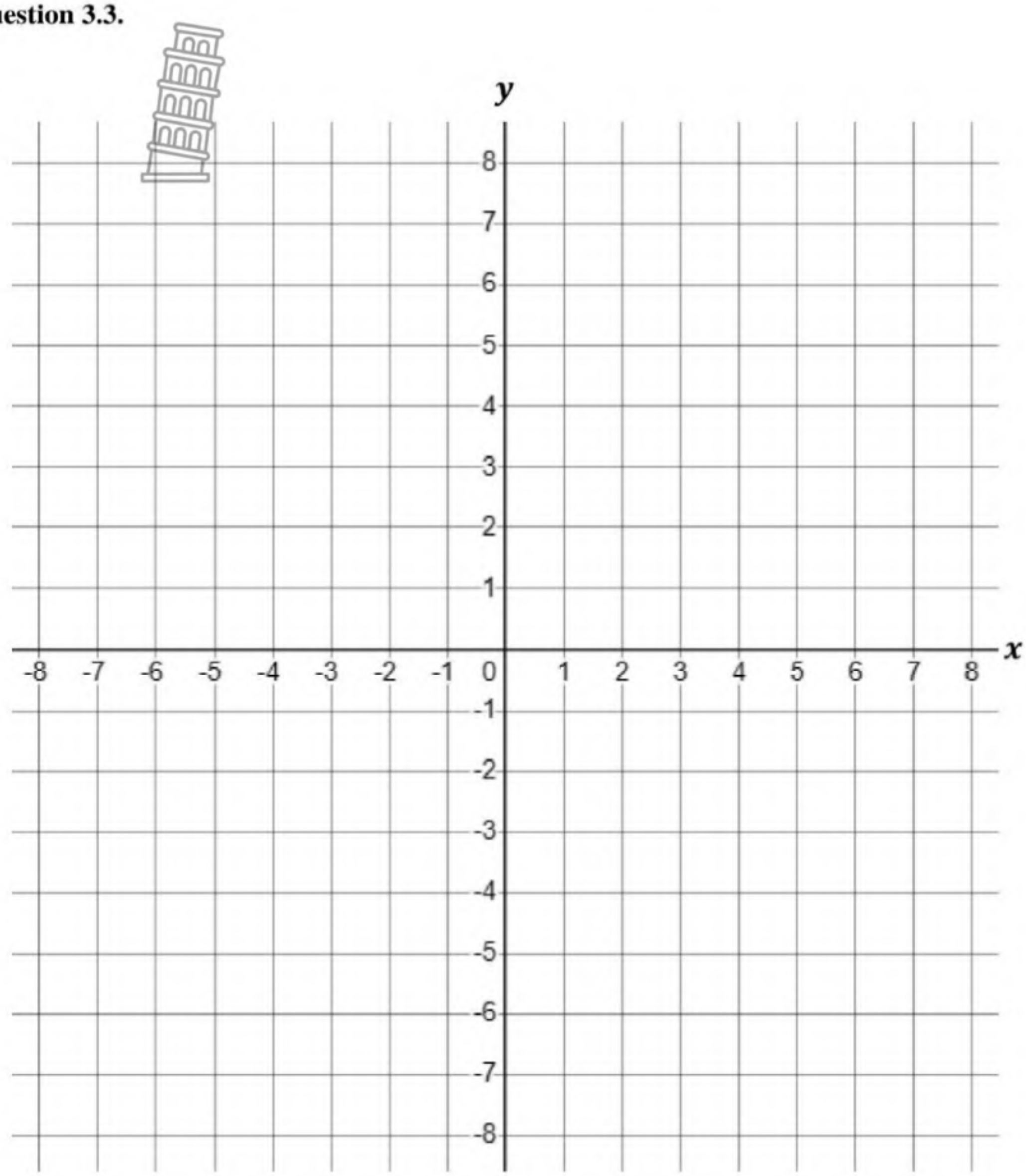
[21]

TOTAL: 100

DIAGRAM SHEET

Name of learner: ____

Question 3.3.



(4)

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MATHEMATICS

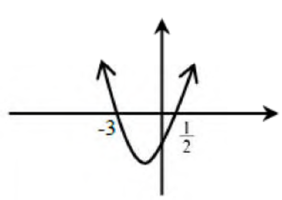
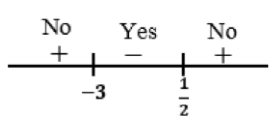
JUNE P1 2023

MARKING GUIDELINES

MARKS : 100

DURATION : 2 hours

These marking guidelines consist of 10 pages including the cover page

QUESTION 1			
1.1.1	$2x(x-3)=0$ $\therefore x=0$ or $x=3$	✓ $x = 0$ ✓ $x = 3$	(2)
1.1.2	$3x^2-2x=4$ $3x^2-2x-4=0$ $x=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(-4)}}{2(3)}$ $x=-0{,}87$ or $x=1{,}54$	✓ standard form ✓ substitution into correct formula ✓✓answers	(4)
1.1.3	$\sqrt{5-x}=x+1$ $\left(\sqrt{5-x}\right)^2=(x+1)^2$ $5-x=x^2+2x+1$ $x^2+3x-4=0$ $(x+4)(x-1)=0$ $x\neq-4$ or $x=1$ $\therefore x=1$	✓ squaring both sides ✓ standard form ✓ factors ✓ both solutions to x ✓ rejecting $x=4$	(5)
1.1.4	$2x^2+5x\leq 3$ $2x^2+5x-3\leq 0$ $(x+3)(2x-1)\leq 0$ CV: $x=-3$ or $x=\frac{1}{2}$  ALTERNATIVE METHOD:  $\therefore -3\leq x\leq\frac{1}{2}$	✓ standard form ✓ critical values ✓✓ answer	(4)

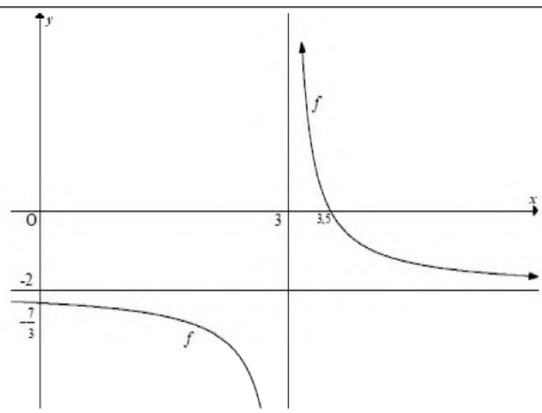
1.1.5	$5x^{\frac{2}{5}} = 20$ $x^{\frac{2}{5}} = 4$ $\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = (2^2)^{\frac{5}{2}}$ $x = 2^5$ $x = 32$ ALTERNATIVE METHOD: $x^{\frac{2}{5}} = 4$ $\left(x^{\frac{2}{5}}\right)^5 = (2^2)^5$ $x^2 = 2^{10}$ $x = \sqrt{2^{10}}$ $x = 2^5$ $x = 32$	✓ divide both sides by 5 ✓ apply exponent law ✓ $x = 32$ ALTERNATIVE: ✓ raise both sides to power 5 ✓ apply exponent law ✓ $x = 32$	(3)
1.1.6	$2^x - \frac{12}{2^x} = -4$ $(2^x)^2 - 12 = -4.2^x$ $(2^x)^2 + 4.2^x - 12 = 0$ $(2^x + 6)(2^x - 2) = 0$ $2^x \neq -6$ or $2^x = 2$ $\therefore x = 1$ ALTERNATIVE METHOD: Let $k = 2^x$ $k - \frac{12}{k} = -4$ $k^2 - 12 = -4k$ $k^2 + 4k - 12 = 0$ $(k+6)(k-2) = 0$ $\therefore 2^x \neq -6$ or $2^x = 2$ $\therefore x = 1$	✓ LCD ✓ $(2^x)^2 - 4.2^x - 12 = 0$ ✓ factors ✓ $2^x \neq -6$ ✓ $x = 1$ ALTERNATIVE: ✓ k-method ✓ $k^2 - 4k - 12 = 0$ ✓ factors ✓ $2^x \neq -6$ ✓ $x = 1$	(5)

1.2	$5y - x = 2$ (1) $x^2 - 3xy + 4y = 4$ (2) $x = 5y - 2$ (3) Sub (3) into (2): $x^2 - 3xy + 4y = 4$ $(5y-2)^2 - 3y(5y-2) + 4y - 4 = 0$ $25y^2 - 20y + 4 - 15y^2 + 6y + 4y - 4 = 0$ $10y^2 - 10y = 0$ $10y(y-1) = 0$ $y = 0$ or $y = 1$ $x = 5(0) - 2 = -2$ or $x = 5(1) - 2 = 3$	✓ generate 3rd equation ✓ substitution ✓ standard form ✓ factors ✓ both solutions to y ✓ both solutions to x	(6)
			[29]

QUESTION 2			
2.1.1	$\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left[\left(\frac{8}{27}\right)^{\frac{1}{3}}\right]^2$ $= \left(\sqrt[3]{\frac{8}{27}}\right)^2$ $= \left(\frac{2}{3}\right)^2$ $= \frac{4}{9}$ ALTERNATIVE METHOD: $\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{2^3}{3^3}\right)^{\frac{2}{3}}$ $= \left(\frac{2}{3}\right)^2$ $= \frac{4}{9}$	✓ apply exponent law ✓ method ✓ answer ALTERNATIVE: ✓ prime factor base ✓ method ✓ answer	(3)
2.1.2	$(\sqrt{12} + 2)(\sqrt{3} - 1) = (2\sqrt{3} + 2)(\sqrt{3} - 1)$ $= 2.3 - 2\sqrt{3} + 2\sqrt{3} - 2$ $= 6 - 2$ $= 4$	✓ $2\sqrt{3}$ ✓ $2.3 - 2\sqrt{3} + 2\sqrt{3} - 2$ ✓ answer	

	ALTERNATIVE METHOD: $(\sqrt{12}+2)(\sqrt{3}-1)=\sqrt{36}-2\sqrt{3}+2\sqrt{3}-2$ $=6-2\sqrt{3}+2\sqrt{3}-2$ $=4$	ALTERNATIVE: ✓ $\sqrt{36}$ ✓ $6-2\sqrt{3}+2\sqrt{3}-2$ ✓ answer	(3)
2.1.3	$\frac{3^{2x+1}.15^{2x-3}}{27^{x-1}.3^{x}.5^{2x-4}}$ $=\frac{3^{2x+1}.(5\times3)^{2x-3}}{(3^3)^{x-1}.3^x.5^{2x-4}}$ $=\frac{3^{2x+1}.5^{2x-3}.3^{2x-3}}{3^{3x-3}.3^x.5^{2x-4}}$ $=3^{2x+1+2x-3-3x+3-x}.5^{2x-3-2x+4}$ $=3.5$ $=15$	✓ prime bases ✓ apply exponent law ✓ adding and subtracting exponents ✓ answer	(4)
2.2	$\frac{\sqrt{2}}{\sqrt{2}+1}+\frac{4}{\sqrt{2}}$ $=\left(\frac{\sqrt{2}}{\sqrt{2}+1}\times\frac{\sqrt{2}}{\sqrt{2}}\right)+\left(\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$ $=\frac{\sqrt{2}.\sqrt{2}}{2+\sqrt{2}}+\frac{4\sqrt{2}+4}{2+\sqrt{2}}$ $=\frac{(\sqrt{2})^2+4\sqrt{2}+2^2}{2+\sqrt{2}}$ $=\frac{(\sqrt{2}+2)^2}{2+\sqrt{2}}$ $=2+\sqrt{2}$ ALTERNATIVE METHOD: $\frac{\sqrt{2}}{\sqrt{2}+1}+\frac{4}{\sqrt{2}}$ $=\left(\frac{\sqrt{2}}{\sqrt{2}+1}\times\frac{\sqrt{2}}{\sqrt{2}}\right)+\left(\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$ $=\frac{2}{2+\sqrt{2}}+\frac{4\sqrt{2}+4}{2+\sqrt{2}}$ $=\frac{6+4\sqrt{2}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$ $=\frac{12+2\sqrt{2}-8}{4-2}$ $=\frac{4+2\sqrt{2}}{2}$ $=\frac{2(2+\sqrt{2})}{2}$ $=2+\sqrt{2}$	✓ Multiply by LCD: $\sqrt{2}(\sqrt{2}+1)$ ✓ simplify ✓ factors ✓ answer ALTERNATIVE: ✓ Multiply by LCD: $\sqrt{2}(\sqrt{2}+1)$ ✓ simplify ✓ multiply with the conjugate $\frac{2-\sqrt{2}}{2-\sqrt{2}}$ ✓ factorise (HCF:2) ✓ answer	(4)

2.3	$x^2 - mx + (m+3) = 0$ $\Delta = b^2 - 4ac$ For equal roots: $\Delta = 0$ $\therefore (-m)^2 - 4(m+3)(1) = 0$ $m^2 - 4m - 12 = 0$ $(m+2)(m-6) = 0$ $\therefore m = -2$ or $m = 6$	✓ $\Delta = 0$ ✓ substitution ✓ simplify ✓ factorise ✓ both answers	(5)
			[19]

QUESTION 3			
3.1	$x = 3$ and $y = -2$	✓ $x = 3$ and $y = -2$	(1)
3.2	x-intercept: $y = 0$ $0 = \frac{1}{x-3} - 2$ $2(x-3) = 1$ $2x - 6 = 1$ $2x = 7$ $x = \frac{7}{2}$	✓ $y = 0$ ✓ simplify ✓ $x = \frac{7}{2}$	(3)
3.2		✓ asymptotes ✓ shape ✓ x-and y- intercepts	(3)
3.3	$y = x + c$ $-2 = 3 + c$ $c = -5$ $\therefore y = x - 5$	✓ substitution of $(3\,;\,-2)$ ✓ $y = x - 5$	(2)
3.4	Translate f 3 units to the left and 2 units up.	✓ 3 units to the left ✓ 2 units up	(2)

3.5	$x \in (-\infty\ ;\ +\infty)\ ; x \neq 0$ ALTERNATIVE ANSWER: $x \in \mathbb{R}\ ; x \neq 0$	✓ answer	(1)
			[12]

QUESTION 4			
4.1	At TP: $x = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3$ $\therefore y = -(3)^2 + 6(3) + 7 = 16$ $\therefore (3\ ;16)$ ALTERNATIVE METHOD: $f(x) = -x^2 + 6x + 7$ $= -(x^2 - 6x - 7)$ $= -[(x^2 - 6x + 9) - 9 - 7]$ $= -[(x-3)^2 - 16]$ $= -(x-3)^2 + 16$ $\therefore (3\ ;16)$ ALTERNATIVE METHOD: x- intercepts at 7 and -1: $\therefore x_{TP} = \frac{7+(-1)}{2} = 3$ $\therefore y = -(3)^2 + 6(3) + 7 = 16$ $\therefore (3\ ;16)$	✓ method ✓ x-coordinate ✓ y-coordinate ALTERNATIVE: ✓ method ✓ x-coordinate ✓ y-coordinate ALTERNATIVE: ✓ method ✓ x-coordinate ✓ y-coordinate	(3)
4.2	$x = 3$	✓ answer	(1)
4.3	$f(x) = -x^2 + 6x + 7$ $= -(x-3)^2 + 16$ $\therefore h(x) = (x - 3 + 4)^2 + 16$ $= (x+1)^2 + 16$	✓ $a = 1$ and $q = 16$ ✓ $p = 1$	(2)
			[6]

QUESTION 5			
5.1.1	$y = -4$	✓ answer	(1)
5.1.2	$y > -4$	✓ answer	(1)
5.1.3	$g(x) = \left(\frac{1}{2}\right)^x - 4$ x-intercept: $y = 0$ $0 = \left(\frac{1}{2}\right)^x - 4$ $4 = \left(\frac{1}{2}\right)^x$ $\therefore 2^2 = 2^{-x}$ $\therefore x = -2$ $\therefore (-2\,;0)$	✓ substitute $y = 0$ ✓ mathematical procedure ✓ coordinate of x-intercept	(3)
5.1.4	$x > -2$	✓ answer	(1)
5.2.1	$y = 3^{x+p} - 27$ $54 = 3^{3+p} - 27$ $81 = 3^{3+p}$ $3^4 = 3^{3+p}$ $\therefore 4 = 3 + p$ $\therefore p = 1$	✓ substitution (3; 54) ✓ equating indices ✓ answer	(3)
5.2.2	$y > -27$ or $y \in (-27\,;\,\infty)$	✓ answer	(1)
5.2.3	$g(x) = -f(x)$ $g(x) = -(3^{x+1} - 27)$ $= -3^{x+1} + 27$ $y = -3^{0+1} + 27 = 24$ $\therefore y$-intercept at $(0\,;24)$	✓ $g(x) = -f(x)$ ✓ new equation ✓ answer	(3)
			[13]

QUESTION 6			
6.1	$y = -3x + k$ $0 = -3(5) + k$ $\therefore k = 15$	✓ substitute with $(5\,;\,0)$	(1)

6.2	$x = -\frac{b}{2a} = \frac{-2}{2(-1)} = 1$ $\therefore y = -(1)^2 + 2(1) + 15 = 16$ $\therefore D(1\,;16)$ ALTERNATIVE METHOD: $x = \frac{-3+5}{2} = 1$ $\therefore y = -(1)^2 + 2(1) + 15 = 16$ $\therefore D(1\,;16)$	✓ $x = 1$ ✓ substitution ✓ $y = 16$ ALTERNATIVE: ✓ $x = 1$ ✓ substitution ✓ $y = 16$	(3)
6.3	$x < 1$ ALTERNATIVE ANSWER: $x \in (-\infty\,;1)$	✓ answer	(1)
6.4	$A(-3\,;0)$ and $D(1\,;16)$ $m = \frac{y_A - y_D}{x_A - x_D} = \frac{0-16}{-3-1} = 4$	✓ subst. into gradient formula ✓ answer	(2)
6.5	$D(1\,;16)$ and $E(1\,;12)$ $\therefore DE = 4$ units	✓ $E(1\,;\,12)$ ✓ answer	(2)
6.6	$h(x) = f(x-1) - 2$ $= -(x-1)^2 + 2(x-1) + 15 - 2$ $= -x^2 + 2x - 1 + 2x - 2 + 13$ $= -x^2 + 4x + 10$ $= -(x^2 - 4x - 10)$ $= -(x^2 - 4x + 4 - 4 - 10)$ $= -(x-2)^2 + 14$ ALTERNATIVE METHOD: $D(1\,;16)$ and $a = -1$ $\therefore f(x) = -(x-1)^2 + 16$ $h(x) = f(x-1) - 2$ $= -(x-1-1)^2 + 16 - 2$ $= -(x-2)^2 + 14$	✓ $-(x-1)^2 + 2(x-1) + 15 - 2$ ✓ $-x^2 + 4x + 10$ ✓ $h(x) = -(x-2)^2 + 14$ ALTERNATIVE: ✓ substitute D (1 ; 16) and $a = -1$ ✓ $h(x) = -(x-1-1)^2 + 16 - 2$ ✓ $h(x) = -(x-2)^2 + 14$	(3)
6.7.1	$-3 \leq x \leq 5$	✓ answer	(1)

6.7.2	$x < 0$ and $x > 5$	✓ $x < 0$ ✓ $x > 5$	(2)
6.8	Max value of $f(x)$ is 16 $\therefore f(x) - 12 = 16 - 12 = 4$ $\therefore$ max value of $p(x) = 3^4$ $= 81$	✓ Max value of $f(x)$ is 16 ✓ $f(x) - 12 = 4$ ✓ answer	(3)
6.9	$-x^2 + 2x + 15 + k = 0$ $b^2 - 4ac > 0$ $(2)^2 - 4(-1)(15 + k) > 0$ $4k > -64$ $k > -16$	✓ $\Delta > 0$ ✓ correct substitution ✓ answer	(3)
			[21]

TOTAL: 100