KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 10

MATHEMATICS PAPER 1

NOVEMBER 2023

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 7 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   Write neatly and legibly.

QUESTION 1

1.1   Given: $T = \dfrac{\sqrt{x+5}}{x-1}$, where $x \in \{-6; -2; 1; 4\}$.

Choose from the given set of numbers a value of x for which T will be:

    1.1.1   rational   (1)

    1.1.2   non-real   (1)

1.2   Factorise the following expressions fully:

    1.2.1 $3x - x^2$ (2)

    1.2.2 $y - xy + x - 1$ (3)

1.3   Simplify the following fully:

    1.3.1 $(x+3)(x^2 - 3x + 9)$ (2)

    1.3.2 $\dfrac{x^3 + x^2 + x}{x^3 - 1} + \dfrac{2x+2}{1-x^2} + \dfrac{x+1}{x^2 - 1}$ (4)

    1.3.3 $\dfrac{10^{2x+3} \cdot 4^{1-x}}{25^{2+x}}$ (4)

1.4   Determine the numerical value of $(3x + y)^2$ if it is given that $9x^2 + y^2 = 12$ and $xy = -3$.   (3)

[20]

QUESTION 2

2.1   Solve for x **without using a calculator**:

    2.1.1 $2x^2 - 5x = 3$ (3)

    2.1.2 $\left(\dfrac{1}{2}\right)^{3x-1} = 128$ (3)

2.2   Solve for x, $x \in \mathbb{R}$

$-5 \leq 3x - 2 < 7$ (2)

2.3   Solve for x and y simultaneously if:

$2x - y = 17$

$4y + 3x = 9$ (4)

[12]

QUESTION 3

3.1   Consider the following linear number pattern:

37 ; 33 ; 29 ; p ; 21 ; .....

   3.1.1   Write down the value of p .   (1)

   3.1.2   Determine the nth term of the number pattern.   (2)

   3.1.3   Determine the 15th term of the sequence.   (2)

   3.1.4   Which term is the first to have a negative value?   (3)

3.2   Consider the number pattern below:

$$\frac{-3}{5}; \frac{-6}{9}; \frac{-9}{13}; ......; ....$$

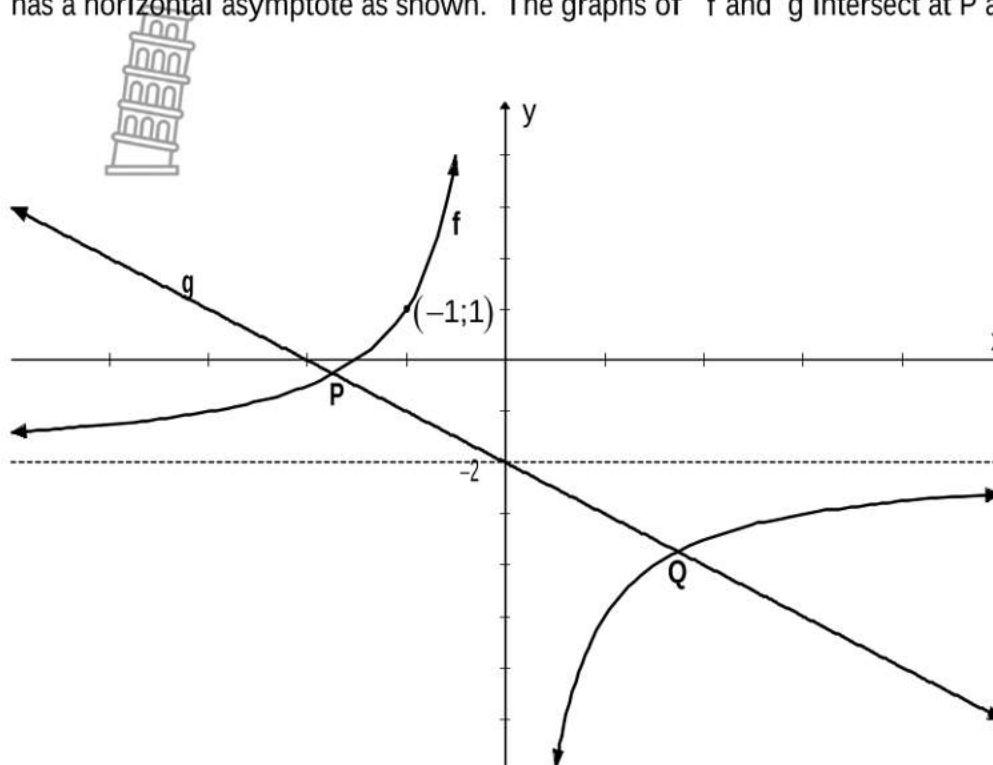   3.2.1   Determine the next two terms of the number pattern.   (2)

   3.2.2   Determine the nth term of the number pattern.   (2)

[12]

QUESTION 4

4.1   The sketch below shows the graphs $f(x) = \dfrac{a}{x} + q$ and $g(x) = -x - 2$. The graph of f has a horizontal asymptote as shown. The graphs of f and g intersect at P and Q.

4.1.1	Write down the value of q.	(1)
4.1.2	The point $(-1; 1)$ lies on f. Calculate the value of a.	(2)
4.1.3	Determine the x-coordinates of P and Q.	(4)
4.1.4	The graph of h is obtained by reflecting f in the y-axis. Write down the equation of h.	(2)

4.2   The function $p(x) = b^x + q$ is described by the following properties:

- $b > 0;\ b \neq 1$
- x-intercept at $(2; 0)$
- The horizontal asymptote is $y = -9$

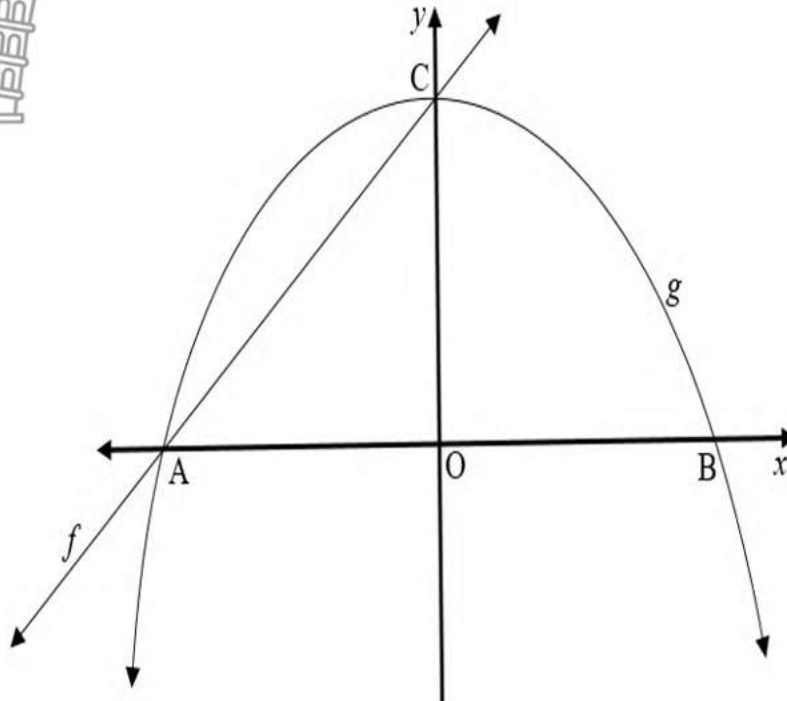|		
4.2.1	Write down the range of p.	(1)
4.2.2	Determine the value of b, hence write down the equation of p.	(3)
4.2.3	Sketch the graph of p. Clearly indicate the intercepts with the axes and the asymptote.	(3)

[16]

QUESTION 5

5.1   The diagram shows the graphs of $g(x) = -x^2 + 4$ and $f(x) = mx + c$. A and B are
       the x-intercepts of g and C is the y-intercept of g. Graph f passes through A and C.

5.1   Write down the coordinates of C.   (1)

5.2   Determine the coordinates of A and B.   (3)

5.3   Determine the values of m and c.   (2)

5.4   How should g be transformed so that the new graph will have a y-intercept of -2
       and still have a maximum turning point?   (2)

5.5   For which values of x is:

       5.5.1 $f(x) < g(x)$?   (2)

       5.5.2 $f(x).g(x) \leq 0$?   (3)

                                                                                          [13]

QUESTION 6

6.1   Gugu buys a double bed which costs R12 000 on hire purchase. She is charged a
        simple interest of 12%p.a. over six years.

        6.1.1   Calculate the total amount she will pay for the double bed.   (3)

        6.1.2   How much interest will she pay over this period?   (1)

        6.1.3   Calculate her monthly instalment.   (2)

6.2   The population of a city in KwaZulu-Natal is 2 500 000 in the year 2020.
        Assuming that the population will continue to increase at a constant rate of 5,25% each
        year, estimate the population of the city at the beginning of 2024.
        (Give your answer correct to the nearest whole number.)   (3)

6.3   If the current exchange rate is $1 =R19,08 and £1 =R23,31, determine the exchange
        rate between dollar and pound.   (2)

6.4   Brent crude oil costs $93,78 a barrel. Calculate the cost in rands, of importing a barrel
        when the exchange rate is R19,08 to the dollar.   (2)
                                                                                                [13]

QUESTION 7

7.1   A bag contains 3 green balls and 5 red balls. The balls are placed in a bag and one is chosen at random. Determine the probability that the ball selected is red.   (1)

7.2   Two events A and B are complementary and also P(not A) = 0,25

    7.2.1   Complete the statement: P(A) + P(B) = …….   (1)

    7.2.2   Write down the value of P(A).   (2)

7.3   For two events V and W which are not mutually exclusive, you are given the following information:
- P(V) = 0,35
- P(W) = 0,55
- P(V or W) = 0,8

Let the value of P(V and W) = x.

    7.3.1   Draw a Venn diagram based on the information given above.   (4)

    7.3.2   Determine the value of P(V and W).   (2)

    7.3.3   Write down the value of P(not V or W).   (2)

    7.3.4   Write down the value of P(V and not W).   (2)
    [14]

                                TOTAL: **100**

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \qquad\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} ; \ r \neq 1 \qquad S_\infty = \frac{a}{1 - r} ; \ -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \qquad\qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc.\cos A$$
$$\text{area } \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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MATHEMATICS P1

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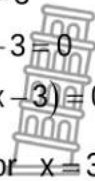**MARKING GUIDELINE**

**NATIONAL
SENIOR CERTIFICATE**

GRADE 10

MARKS: 100

QUESTION 1

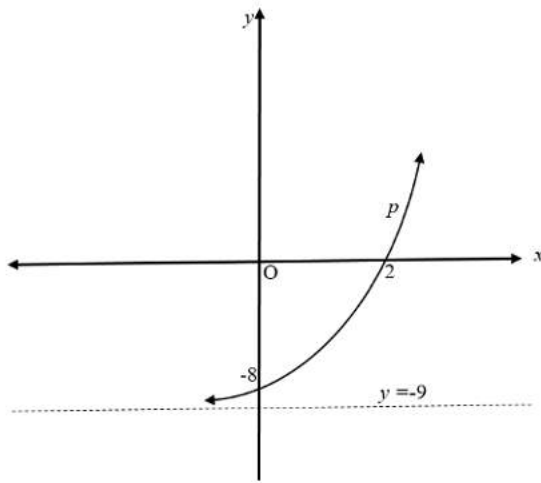| 1.1.1	$x = 4$	✓answer (1)
1.1.2	$x = -6$	✓answer (1)
1.2.1	$3x - x^2$ $= x(3-x)$	✓✓ $x(3-x)$ (2)
1.2.2	$y - xy + x - 1$ $= y(1-x) - 1(1-x)$ $= (1-x)(y-1)$	✓✓ each term $y(1-x) - 1(1-x)$ ✓$(1-x)(y-1)$ (3)
1.3.1	$(x+3)(x^2 - 3x + 9)$ $= x^3 - 3x + 9x + 3x^2 - 9x - 27$ $= x^3 - 27$	✓ $x^3 - 3x + 9x + 3x^2 - 9x - 27$ ✓ $x^3 - 27$ **Answer only: Full marks** (2)
1.3.2	$\dfrac{x^3 + x^2 + x}{x^3 - 1} + \dfrac{2x+2}{1 - x^2} + \dfrac{x+1}{x^2 - 1}$ $= \dfrac{x(x^2 + x + 1)}{(x-1)(x^2 + x + 1)} - \dfrac{2(x+1)}{(x-1)(x+1)} + \dfrac{x+1}{(x-1)(x+1)}$ $= \dfrac{x}{(x-1)} - \dfrac{2}{(x-1)} + \dfrac{1}{(x-1)}$ $= \dfrac{x-1}{(x-1)}$ $= 1$	✓ $(x-1)(x^2 + x + 1)$ ✓ $-\dfrac{2(x+1)}{(x-1)(x+1)}$ ✓simplification ✓answer (4)
1.3.3	$\dfrac{10^{2x+3}.4^{1-x}}{25^{2+x}}$ $= \dfrac{2^{2x+3}.5^{2x+3}.2^{2-2x}}{5^{4+2x}}$ $= 2^{2x+3+2-2x}.5^{2x+3-4-2x}$ $= 2^5.5^{-1}$ $= \dfrac{32}{5}$ or $6\dfrac{2}{5}$	✓ $\dfrac{2^{2x+3}.5^{2x+3}.2^{2-2x}}{5^{4+2x}}$ ✓ $2^{2x+3+2-2x}.5^{2x+3-4-2x}$ ✓ $2^5.5^{-1}$ ✓answer (4)
1.4	$(3x + y)^2$ $= 9x^2 + 6xy + y^2$ But $9x^2 + y^2 = 12$ and $xy = -3$ $\therefore (3x + y)^2 = 12 + 6(-3)$ $\qquad\qquad = -6$	✓ $9x^2 + 6xy + y^2$ ✓ $12 + 6(-3)$ ✓ -6 (4)
		[20]

GRADE 10
Marking Guideline

QUESTION 2

2.1.1	$2x^2 - 5x = 3$ $2x^2 - 5x - 3 = 0$ $(2x+1)(x-3) = 0$ $x = -\dfrac{1}{2}$ or $x = 3$	✓ $2x^2 - 5x - 3 = 0$ ✓ $(2x+1)(x-3) = 0$ ✓both answers (3)
2.1.2	$\left(\dfrac{1}{2}\right)^{3x-1} = 128$ $2^{-3x+1} = 2^7$ $-3x = 6$ $x = -2$	✓ $2^{-3x+1} = 2^7$ ✓ $-3x = 6$ ✓ $x = -2$ (3)
2.2	$-3 \le 3x < 9$ $-1 \le x < 3$	✓ $-3 \le 3x < 9$ ✓ $-1 \le x < 3$ (2)
2.3	$2x - y = 17 \dotfill (1)$ $4y + 3x = 9 \dotfill (2)$ $(1) \times 4 : -4y + 8x = 68 \dotfill (3)$ $(2) + (3) : 11x = 77$ $x = 7$ Substitute $x = 7$ into (1) $2(7) - y = 17$ $y = -3$ **OR** $2x - y = 17 \dotfill (1)$ $4y + 3x = 9 \dotfill (2)$ $y = 2x - 17 \dotfill (3)$ Substitute (3) into (2) $4(2x - 17) + 3x = 9$ $11x = 77$ $x = 7$ $y = 2(7) - 17$ $y = -3$	✓multiplying (1) by 4 ✓adding (2) + (3) ✓x-value ✓y-value (4) **OR** ✓equation (3) ✓substitution ✓x-value ✓y-value (4)
		[12]

QUESTION 3

3.1.1	$p = 25$	✓answer	(1)
3.1.2	$T_n = -4n + b = 37$ $b = 41$ $T_n = -4n + 41$	✓ $-4n$ ✓41	(2)
3.1.3	$T_{15} = -4(15) + 41$ $T_{15} = -19$	✓substitution ✓ $T_{15} = -19$ **Answer only: Full marks**	(2)
3.1.4	$-4n + 41 = -3$ $n = 11$ 11th term is the first term which has a negative value. **OR** 37 ; 33 ; 29 ; 25 ; 21 ;17 ;13 ; 9 ; 5 ; 1 ; -3 $\therefore 11^{th}$ term is the first term which has a negative value.	✓ equating by -3 ✓ $4n + 41 = -3$ ✓ $n = 11$ **Answer only: Full marks** **OR** ✓✓37 ; 33 ; 29 ; 25 ; 21 ;17 ;13 ; 9 ; 5 ; 1 ; -3 ✓ $n = 11$	(3)
3.2.1	$\dfrac{-12}{17} ; \dfrac{-15}{21}$	✓ $\dfrac{-12}{17}$ ✓ $\dfrac{-15}{21}$	(2)
3.2.2	Numerator: $-3; -6; -9;.....$ $T_n = -3n$ Denominator: $5; 9; 13;.....$ $T_n = 4n + 1$ Combined: $T_n = \dfrac{-3n}{4n + 1}$	✓ $T_n = -3n$ ✓ $T_n = 4n + 1$ $T_n = \dfrac{-3n}{4n + 1}$ **Answer only: Full marks**	(2)
			[12]

QUESTION 4

4.1.1	$q = -2$	✓answer	(1)
4.1.2	$f(x) = \dfrac{a}{x} - 2$ through $(-1;1)$ $1 = \dfrac{a}{-1} - 2$ $a = -3$	✓substitution by $(-1;1)$ ✓answer	 (2)
4.1.3	$f(x) = g(x)$ $\dfrac{-3}{x} - 2 = -x - 2$ $x^2 - 3 = 0$ $x_P = -\sqrt{3}$ $x_Q = +\sqrt{3}$	✓ equating ✓ $x^2 - 3 = 0$ ✓ $x_P = -\sqrt{3}$ ✓ $x_Q = +\sqrt{3}$	 (4)
4.1.4	$h(x) = f(-x) = \dfrac{-3}{-x} - 2$ $h(x) = \dfrac{3}{x} - 2$	✓ $h(x) = f(-x) = \dfrac{-3}{-x} - 2$ ✓ $h(x) = \dfrac{3}{x} - 2$ **Answer only: Full marks**	 (2)
4.2.1	Range: $y > -9$ **OR** Range: $y \in (-9; \infty)$	✓answer	 (1)
4.2.2	$p(x) = b^x - 9$ through $(2;0)$ $b^2 - 9 = 0$ $b = 3$ $p(x) = 3^x - 9$	✓ horizontal asymptote ✓substitution by $(2;0)$ ✓ $p(x) = 3^x - 9$	 (3)
.2.3		✓y-intercept ✓shape and x-intercept ✓asymptote	 (3)
			[16]

QUESTION 5

5.1	$C(0;4)$	✓answer (1)
5.2	$-x^2 + 4 = 0$ $(x-2)(x+2) = 0$ $x = 2$ or $x = -2$ $A(-2;0)$ $B(2;0)$	✓equating by 0 ✓factorising ✓ both coordinates (3)
5.3	$m = \dfrac{4-0}{0+2} = 2$ $c = 4$	✓value of m ✓value of c (2)
5.4	It must be shifted 6 units downwards.	✓✓answer (2)
5.5.1	$-2 < x < 0$ **OR** $x \in (-2;0)$	✓values ✓notation (2)
5.5.2	$x = -2$ and $x \geq 2$ **OR** $x = -2$ and $x \in [2;\infty)$	✓ $x = -2$ ✓✓ $x \geq 2$ **OR** ✓✓ $x \in [2;\infty)$ each end (3)
		[13]

QUESTION 6

6.1.1	$A = P(1 + in)$ $A = 12000(1 + 0,12 \times 6)$ $A = R20640$	✓ $A = P(1 + in)$ ✓substitution ✓answer (3)
6.1.2	Interest $= 20600 - 12000$ $= R8640$	✓answer (1)
6.1.3	Monthly instalment $= \dfrac{20640}{72}$ $= R286,67$	✓$\dfrac{20640}{72}$ ✓answer (2)
6.2	$A = P(1 + i)^n$ $A = 2500000(1 + 0,0525)^4$ $A = 3067810$	✓ $A = P(1 + i)^n$ ✓substitution ✓answer (3)

6.3	£1 =$1,22 **OR** $1 =£0,82	✓✓answer (2)
6.4	$1 = R19,08 $93,78 = x x = R1789,32	✓✓answer (2)
		[13]

QUESTION 7

7.1	$P(R) = \dfrac{5}{8}$	✓answer (1)
7.2.1	P(A) + P(B) =1	✓ 1 (1)
7.2.2	P(A) =1−0,25 = 0,75	✓P(A) =1−0,25 ✓answer **Answer only: Full marks** (2)
7.3.1		✓ 0,35 − x ✓ 0,55 − x ✓x ✓0,2 (4)
7.3.2	0,35 − x + x + 0,55 − x + 0,2 =1 x = 0,1	✓ 0,35 − x + 0,55 − x + 0,2 =1 ✓answer (2)
7.3.3	P(not V or W) =0,1 + 0,45 + 0,2 =0,75	✓0,1 + 0,45 + 0,2 ✓answer **Answer only: Full marks** (2)
7.3.4	P(V and not W) =0,25	✓✓answer (2)
		[14]

TOTAL: 100