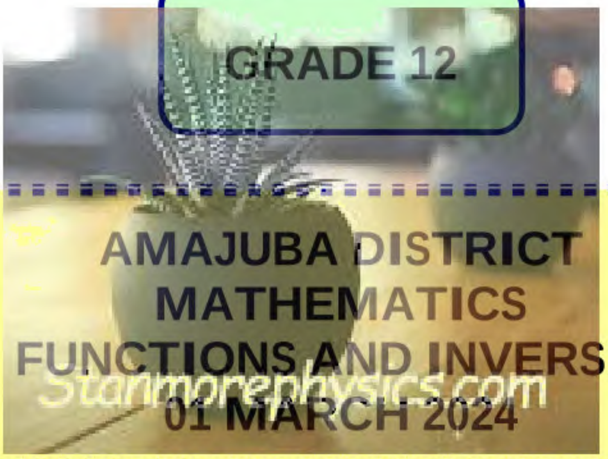
education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12



AMAJUBA DISTRICT
MATHEMATICS
FUNCTIONS AND INVERSES

01 MARCH 2024

MARKS: 35 **TIME:** 45 MINUTES

This question paper consists of 6 pages.

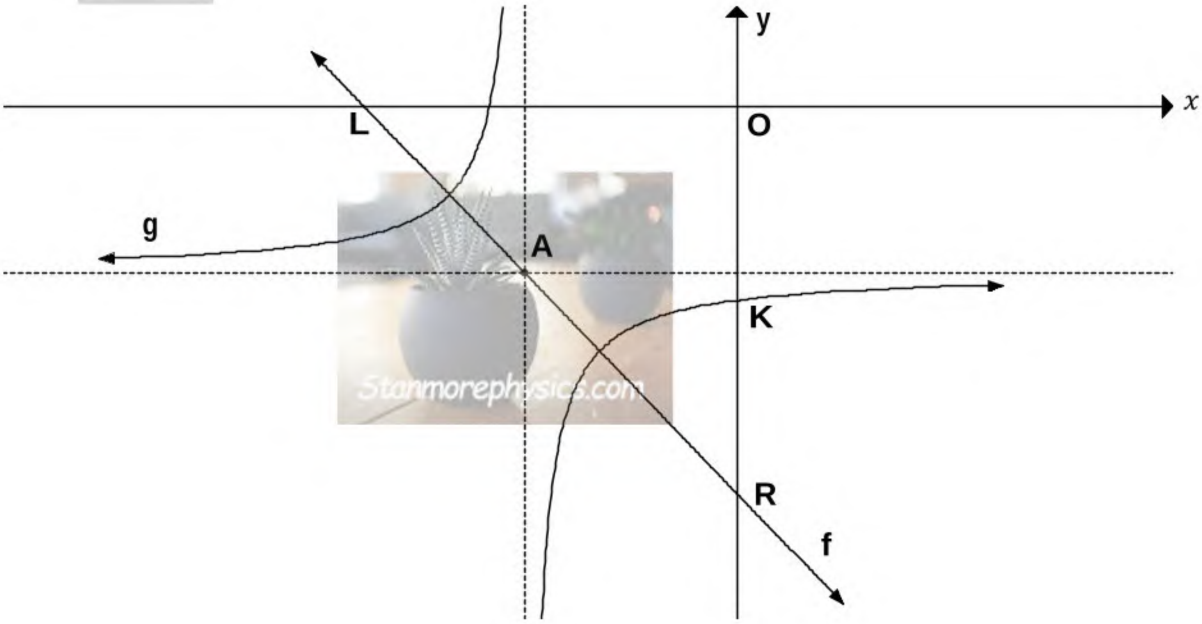
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **3** questions.
2. Answer **ALL** the questions.
3. Number the answers **correctly** according to the numbering system used in this question paper.
4. Clearly show **ALL** calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will **NOT** necessarily be awarded full marks.
6. You may use an **approved** scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers correct to **TWO** decimal places, unless stated otherwise.
8. Diagrams are **NOT** necessarily drawn to scale
9. Write **neatly** and legibly.

QUESTION 1

In the diagram below, the graph of $g(x) = \frac{-2}{x+4} - 3$ **is drawn. The graph of** f **passes through** A**, the point of intersection of the asymptotes of** g**, and cuts the x-axis and the y-axis at** L **and** R **respectively.** K **is the y-intercept of g.**



1.1 Determine the equation of **f** in the form $y = mx + c$, given that **g** is symmetrical about **f**. (3)

1.2 Write down the equation of the vertical asymptote of $g(x-2)$ (1)

1.3 Calculate the length of **KR**. (3)

1.4 The graph of **h**, where **h** is the reflection of **f** in the line **y = –7**, passes through the point **S(– 4 ; p)**.

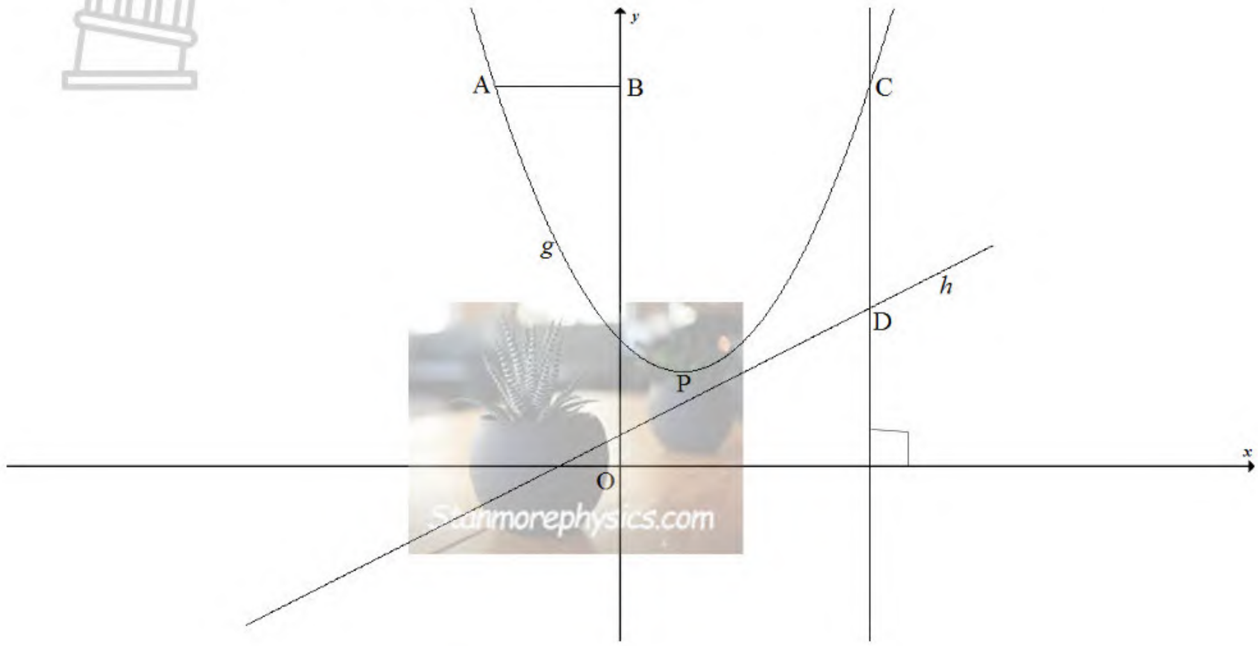
1.4.1 Write down the value of **p.** (1)

1.4.2 Calculate the area of **ΔARS.** (3)

[11]

QUESTION 2

The graphs of $g(x) = x^2 - 2x + 4$ **and** $h(x) = x + 1$ **are sketched below.** AB **is perpendicular to the y – axis and** CD **is perpendicular to the x – axis.**



2.1 Calculate the coordinates of **P**, the turning point of g. (2)

2.2 **If OB = 12** units, determine the coordinates of **A**. (3)

2.3 **If C (x; y)** is on **g**, **D** is on **h**, such that **CD** is always perpendicular to the x-axis, determine an expression in terms of **x** for the length of **CD**. (2)

2.4 Hence, calculate the **minimum** length of **CD**. (3)

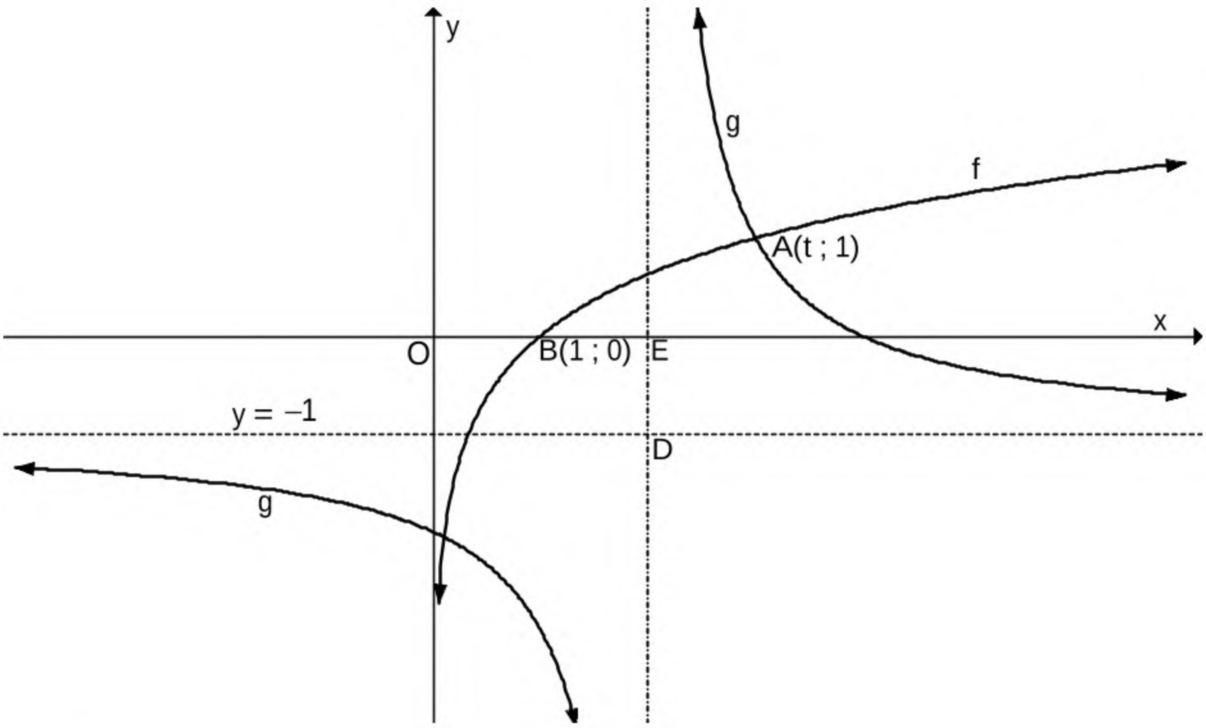
2.5 If $y = 2x + c$ is a **tangent** to the graph of **g**, calculate the value(s) of **c**. (3)

[13]

QUESTION 3

The diagram below shows the graphs of $g(x)=\frac{2}{x+p}+q$ **and** $f(x)=\log_3 x$.

- **y = –1** is the horizontal asymptote of **g**.
- **B(1 ; 0)** is the x-intercept of **f**.
- **A(t ; 1)** is a point of intersection between **f** and **g.**
- The vertical asymptote of **g** intersects the x-axis at **E** and the horizontal asymptote at D.
- OB = BE.



3.1 Write down the range of **g**. (1)

3.2 Determine the equation of **g**. (1)

3.3 Calculate the value of **t.** (2)

3.4 Write down the equation of f^{-1}, the inverse of **f**, in the form **y =…** (2)

3.5 For which values of **x** will $f^{-1}(x)<3$? (2)

3.6 Determine the point of intersection of the graphs of **f** and the axis of symmetry of **g** that has a **negative** gradient. (3)

[11]

TOTAL MARK: 35

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a(r^n-1)}{r-1}$; $r\neq 1$ $\quad S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $\quad P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

area $\Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y}=a+bx$

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

AMAJUBA DISTRICT
MATHEMATICS
FUNCTIONS AND INVERSES
01 MARCH 2024

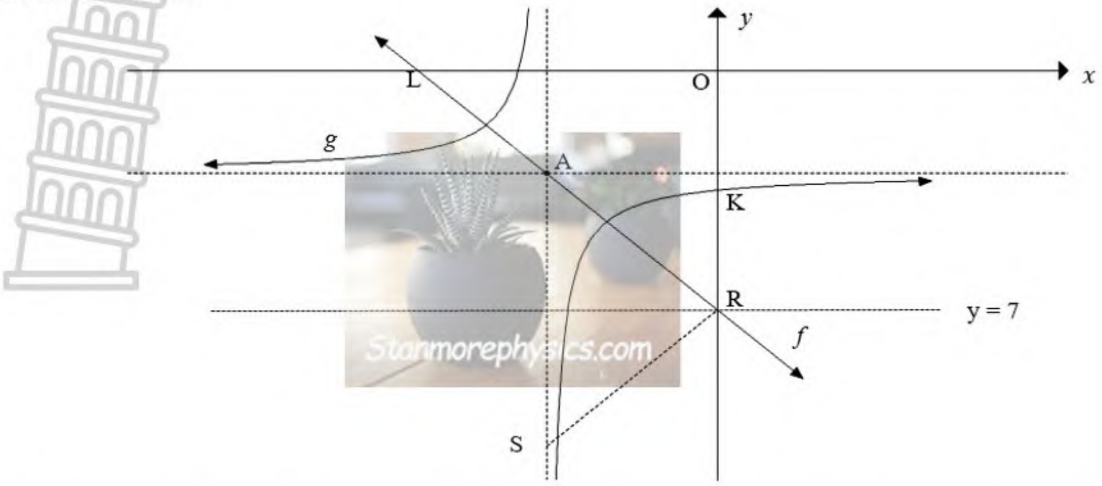
MARKING GUIDELINES

MARKS: 35

TIME: 45 MINUTES

This question paper consists of 4 pages.

QUESTION 1



1.1	A(–4 ; –3) $m_f = -1$ $y - y_1 = m(x - x_1)$ $y - (-3) = -(x - (-4))$ $y = -x - 7$	✓ coordinates of A and m_f ✓ subst. m and A ✓ equation (3)
1.2	x = –2	✓ x = –2 (1)
1.3	$g(x) = \frac{-2}{x+4} - 3$ $y = \frac{-2}{0+4} - 3 = \frac{-7}{2} = -3\frac{1}{2}$ $K\left[0; \frac{-7}{2}\right]$ R(0 ; –7) **KR** $= -\frac{7}{2} - (-7) = \frac{3}{2}$ units	✓ K ✓R ✓ answer (3)
1.4.1	$p = -11$	✓ answer (1)
1.4.2	A(–4 ; –3) and S(–4 ; –11) $m_{SR} = \frac{-11-(-7)}{-4-0} = 1$ and $m_{AR} = -1$ $\therefore m_{SR} \times m_{AR} = -1$ $\therefore SR \perp AR$ $SR = \sqrt{[0-(-4)]^2 + [-7-(-11)]^2} = 4\sqrt{2}$ $AR = \sqrt{[(-4)-0]^2 + [(-3)-(-7)]^2} = 4\sqrt{2}$ $\therefore Area\ \Delta ARS = \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2}$ $= 16\ units^2$	✓ Gradient of SR ✓ base = $4\sqrt{2}$ and $\perp h = 4\sqrt{2}$ ✓ answer (3)
		[11]

QUESTION 2

2.1	$g(x) = x^2 - 2x + 4$ $x = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$ $y = (1)^2 - 2(1) + 4 = 3$ $P(1;3)$	✓x = 1 ✓y = 3 (coordinate form) (2)
2.2	$g(x) = x^2 - 2x + 4 = 12$ $x^2 - 2x - 8 = 0$ $(x+2)(x-4) = 0$ $x = -2$ or $x = 4$ $y = 12$ $A(-2\,;12)$	✓equating to 12 ✓ x values ✓ coordinates of A (3)
2.3	$CD = x^2 - 2x + 4 - (x+1)$ $= x^2 - 3x + 3$	✓for subtraction of both graphs ✓answer (2)
2.4	$CD = x^2 - 3x + 3$ $x = -\frac{b}{2(a)} = \frac{-(-3)}{2(1)} = \frac{3}{2}$ $CD = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 3 = \frac{3}{4}\ units$	✓ method ✓x value ✓CD value (3)
2.5	$x^2 - 2x + 4 = 2x + c$ $x^2 - 4x + 4 - c = 0$ $\Delta = b^2 - 4ac = 0$ $(-4)^2 - 4(1)(4-c) = 0$ $16 - 16 + 4c = 0$ $c = 0$	✓equating graphs ✓for discriminant =0 ✓answer (3)
		[13]

QUESTION 3

3.1	$y \in R$; $y \neq -1$	✓Answer (1)
3.2	$g(x)=\dfrac{2}{x-2}-1$	✓ $\dfrac{2}{x-2}-1$ (1)
3.3	$f(x)=\log_3 x.$ $\log_3 t=1$ $t=3$	✓correct substitution of A ✓ $t=3$ (2)
3.4	$x=\log_3 y$ $y=3^x$	✓interchange x and y ✓ $y=3^x$ (2)
3.5	$3^x<3^1$ $x<1$	✓ $3^x<3^1$ ✓ $x<1$ (2)
3.6	Equation of the axis of symmetry: $y=-x+1$ x-intercept of the axis of symmetry is at x = 1 f has an x-intercept at B(1 ; 0) Point of intersection: $(1;0)$ **OR/OF** Since BE = ED = 1 and D lies on the axis of symmetry and the gradient of the axis of symmetry is –1, B will also lie on the axis of symmetry. But B also lies on f. Therefore B(1 ; 0) is the point of intersection between f and the axis of symmetry with a negative gradient.	✓ equation of axis of symmetry ✓ x-intercept ✓ $(1;0)$ **OR** ✓ BE = ED = 1 ✓ reasoning ✓ $(1;0)$ (3) **[11]**

TOTAL MARK: 35