Education

KwaZulu-Natal Department of Education

UMLAZI DISTRICT

MATHEMATICS

INVESTIGATION – THE COMPOUND ANGLE FORMULA

11 FEBRUARY 2019

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 50 | **DURATION : 2 PERIODS**

NAME :________________________________ **GRADE :** _________________

The aim of this this task is to investigate if cos $(\alpha - \beta)$ is distributive and to then derive the compound angle identity

INSTRUCTIONS

1. This task consists of 3 parts and 3 Questions and is spread over three pages,Answer all questions
2. Answers may be left in surd form where necessary
3. Calculators may be used only in questions which allow it.
4. Answers must be done in pages and submitted on due date
5. Write nealty and legibly

PART 1

In the table below two methods are used by different learners to arrive at answers for given questions. Study the table and answer the questions that follow:

1.1 Follow the trend used by Learner A and Learner B and complete all blank spaces on the table. You should redraw the table in your answer book and may use a calculator to get your answers. (4)

Learner A			**Learner B**		
Question	Working	Answer	Question	Working	Answer
$\cos(60° - 30°)$	$\cos 30°$	$\frac{\sqrt{3}}{2}$	$\cos(60° - 30°)$	$\cos 60° - \text{Cos } 30°$	$\frac{1-\sqrt{3}}{2}$
$\cos(150° - 90°)$			$\cos(150° - 90°)$		
$\cos(225° - 180°)$			$\cos(225° - 180°)$		
$\cos(360° - 210°)$			$\cos(360° - 210°)$		
$\cos(60° - 300°)$			$\cos(60° - 300°)$		
$\cos(210° - 180°)$			$\cos(210° - 180°)$		

1.2 Are the answers obtained by learner A the same as that of learner B for the same question? (1)

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1.3 Give a reason why you think there is a difference in the answers? (2)

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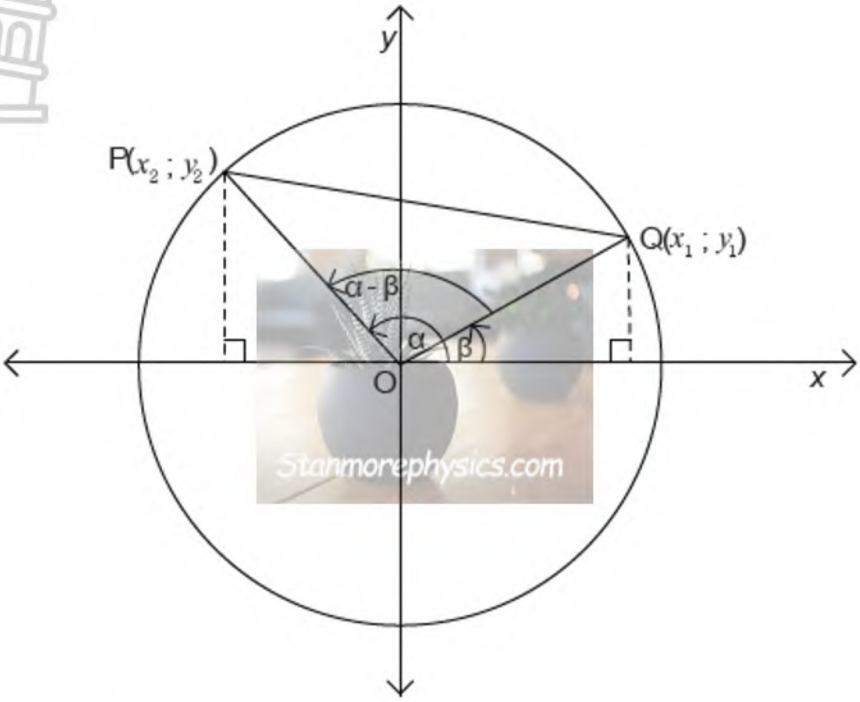
PART 2

Now consider the diagram below which is a unit circle with α and β the angles formed with the positive *x*-axis. The expression (α – β) represents the difference between angles.



2.1 Define the unit circle (2)

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2.2 Write the coordinates of Q in terms of β (1)

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2.3 Write the coordinates of P in terms of α (1)

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2.4 Use the distance formula to calculate the length of PQ in terms of α and β (4)

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2.5 Use the cosine rule in Δ POQ to calculate the length of PQ in terms of α and β (4)

2.6 Equate the value of PQ in 2.4 and 2.5 and make $\cos(\alpha - \beta)$ the subject of the formula (3)

2.7 Use the formula above and to evaluate the following: (4)

(a.) $\cos(150^0 - 90^0)$

(b) $\cos(225^0 - 180^0)$

2.8 If $\cos(\alpha+\beta)=\cos[\alpha-(-\beta)]$, use the results obtained in 2.6 to derive an expression for $\cos(\propto+\beta)$ **(3)**

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2.9 Use co-function and expansion of $\cos[(90^0-\alpha)-\beta]$ to show that:

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$ (3)

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2.10 Use 2.9 to show that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$ (2)

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2.11 Use 2.8 to expand $\cos 2\alpha$ (2)

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2.12 Use 2.11 to write down two other expressions representing $\cos 2\alpha$ (2)

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2.13 Use 2.9 to expand $\sin 2\alpha$ (3)

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PART 3

Use the formulae you derived in Part 2, identities and special angles to evaluate the following without using a calculator:

3.1 $\sin(60^0 - 45^0)$ (3)

3.2 $\cos(15^0)$ (3)

3.3 $2\sin 15^0 \cos 15^0$ (3)

3.4 $\cos 70^0 \cos 10^0 + \cos 20^0 \cos 80^0$ (5)

TOTAL: 55 MARKS