education

Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

GRADE 12

INFORMAL TEST 6

23 FEBRUARY 2024

MARKS: 50

DURATION: 60 MINUTES

This question paper consists of 4 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of **5** questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless otherwise stated.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise

QUESTION 1

1. Given: $(x+3)(3x-1)=p$

    1.1 Solve for x if $p=0$ (2)

    1.2 Solve for x, rounded to TWO decimal places, if $p=0$ (4)

    1.3 $3^x\left(x+\dfrac{1}{3}\right)<0$ (3)

    [09]

QUESTION 2

Given the geometric series: $x+90+81+\ldots\ldots\ldots$

2.1 Calculate the value of x. (2)

2.2 Show that the sum of first n terms is $S_n=1000\left[1-(0,9)^n\right]$ (2)

2.3 Hence, or otherwise, calculate the sum to infinity. (2)

[06]

QUESTION 3

3 An arithmetic and a geometric sequence are combined to form the pattern,

   which is given by: $P_n=x;\dfrac{1}{3};2x;\dfrac{1}{9};3x;\dfrac{1}{27};$

   3.1 Write down the next TWO terms of the pattern. (2)

   3.2 Determine the general term $\left(T_n\right)$ for the odd terms of this pattern.

        Write down your answer in terms of x. (2)

   3.3 Calculate the value of P_{26}. (3)

   3.4 If $\displaystyle\sum_{n=1}^{21}P_n=33,5$, determine the value of x. (6)
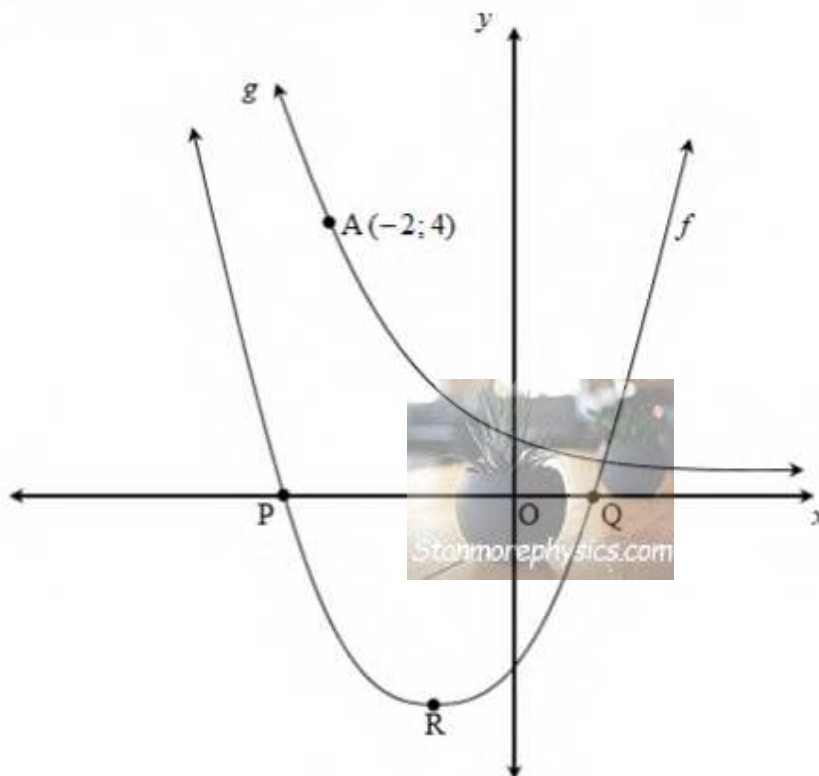
   [13]

QUESTION 4

The graphs of $f(x) = 2(x+1)^2 - 8$ and $g(x) = \left(\dfrac{1}{2}\right)^x$ are represented in the sketch below.

P and Q are the x-intercepts of f and R is the turning point of f.
A$(-2; 4)$ is a point on the graph of g.

4.1 Write down the equation of the axis of symmetry of f. (1)

4.2 Write down the coordinates of R, the turning point of f. (1)

4.3 Determine the equation of the g^{-1}, the inverse of g, in the form $y = ...$ (2)

4.4 Sketch the graph of g^{-1}. Clearly indicate the intercept with the axis. (2)

4.5 For which value(s) of x, is :

    4.5.1 $g^{-1}(x) \geq -2$ (2)

    4.5.2 $x.f(x) < 0$ (3)

[11]

4

QUESTION 5

5.1 Given that: $\cos 26° = p$

Express each of the **following** in **terms** of p, **without using a calculator.**

5.1.1 $\tan 154° = p$ (2)

5.1.2 $\sin 13° \cos 13°$ (2)

5.2 Consider: $\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$

5.2.1 Prove the identity. (4)

5.2.2 For which value(s) of x in the interval $x \in [-180°; 180°]$ is the identity not valid? (3)

[11]

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FREE STATE PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

TERM 1

2024 INFORMAL TEST 6

MARKING GUIDELINE

This marking guideline consists of 5 pages

QUESTION 1

1.1	$(x+3)(3x-1)=0$ $$x=-3 \text{ or } x=\frac{1}{3}$$	✓✓ answers (2)
1.2	$(x+3)(3x-1)=4$ $(x+3)(3x-1)=6$ $3x^2-x+9x-3-6=0$ $3x^2+8x-9=0$ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ $$x=\frac{-(8)\pm\sqrt{(8)^2-4(3)(-9)}}{2(3)}$$ $$x=\frac{-8\pm\sqrt{172}}{6}$$ $x=0,85 \text{ or } x=-3,52$	✓ standard form ✓ substitution ✓✓ answers (4)
1.3	$$3^x\left(x+\frac{1}{3}\right)<0$$ $3^x>0$ for all real values of x $$x+\frac{1}{3}<0$$ $$x<-\frac{1}{3}$$	✓ $3^x>0$ ✓ $x+\frac{1}{3}<0$ ✓ $x<-\frac{1}{3}$ (3)
		[09]

QUESTION 2

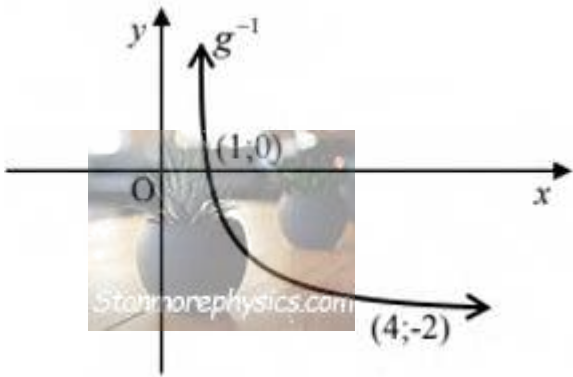| 2.1	$x+90+81+\ldots\ldots\ldots$ $$r=\frac{81}{90}=\frac{9}{10}$$ $x=100$	✓✓ answer (2)
2.2	$$S_n=\frac{a(r^n-1)}{r-1}$$ $$S_n=\frac{100\left(\left(\frac{9}{10}\right)^n-1\right)}{\frac{9}{10}-1}$$ $$S_n=-1000\left((0,9)^n-1\right)$$	✓ substitution ✓ simplification (2)

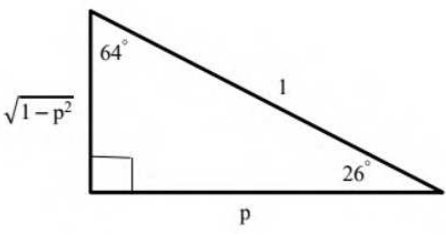|	$S_n = 1000\left((1-0,9)^n\right)$	
2.3	$S_\infty = \dfrac{a}{1-r}$ $S_\infty = \dfrac{100}{1-\left(\dfrac{9}{10}\right)}$ $S_\infty = 1000$	✓ substitution ✓ answer (2)
		[06]

QUESTION 3

3.1	$4x; \dfrac{1}{81}$	✓✓ answer (2)
3.2	$T_n = x + (n-1)x$ $T_n = x + xn - x$ $T_n = xn$	✓ substitution ✓ answer (2)
3.3	$T_n = ar^{n-1}$ $T_{13} = \dfrac{1}{3}\left(\dfrac{1}{3}\right)^{13-1}$ $T_{13} = \left(\dfrac{1}{3}\right)^{13}$	✓ n ✓ r ✓ answer (3)
3.4	$\displaystyle\sum_{n=1}^{21} P_n = S_{11} + S_{10}$ $= \dfrac{11}{2}[2x+10x] + \dfrac{\dfrac{1}{3}\left[1-\left(\dfrac{1}{3}\right)^{10}\right]}{1-\dfrac{1}{3}}$ $= 66x + 0,5$ $66x + 0,5- = 33,5$ $x = \dfrac{1}{2}$	✓ S_{11} ✓ S_{11} ✓ geometric ✓ arithmetic ✓ $= 66x + 0,5$ ✓ answer (6)

QUESTION 4

4.1	$x = -1$	✓ answer (1)
4.2	$R(-1; -8)$	✓ answer (1)
4.3	$g: y = \left(\dfrac{1}{2}\right)^x$ $g^{-1}: x = \left(\dfrac{1}{2}\right)^y$ $\therefore g^{-1}: y = \log_{\frac{1}{2}} x$	✓ swap x and y ✓ answer (2)
4.4		✓ x-intercept ✓ shape (2)
4.5.1	$0 < x \le 4$ or $x \in (0; 4]$	✓ ✓ answer(2)
4.5.2	$x < -3$ or $0 < x < 1$	✓ $x < -3$ ✓ or ✓ $0 < x < 1$ (3)
		[11]

QUESTION 5

5.1.1	$\tan 154° = -\tan 26°$ $= \dfrac{-\sqrt{1-p^2}}{p}$	✓ reduction ✓ answer (2)

4

5.1.2	sin 13° cos 13° $\sin 26° = 2\sin 13° \cos 13°$ $\sin 13° \cos 13° = \dfrac{\sin 26°}{2}$ $\sin 13° \cos 13° = \dfrac{\sqrt{1-p^2}}{2}$	✓ reduction ✓ answer (2)
5.2.1	$\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ LHS: $\dfrac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x \cos x - \cos x}$ $= \dfrac{2\sin^2 x - \sin x}{2\sin x \cos x - \cos x}$ $= \dfrac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$ $= \tan x = \text{RHS}$	✓ expansion $\cos 2x$ ✓ expansion $\sin 2x$ ✓ simplification ✓ answer (4)
5.2.2	$\sin 2x = \sin x$ $x = -90°; x = 30°; x = 90° \text{ and } x = 150°$	✓ $\sin 2x = \sin x$ ✓ ✓ any two (3)
		[11]