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GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2023

GRADE 11

MATHEMATICS

PAPER 1

TIME: 2 hours

MARKS: 100

7 pages

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. Answer ALL the questions.
2. This question paper consists of 6 questions.
3. Present your answers according to the instructions of each question.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which were used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in the question paper.
10. Write neatly and legibly.

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QUESTION 1

1.1 If $x \in \{0; 1; 2; 3; 4; 5\}$, determine the values of x for which:

$$\sqrt{\frac{16}{4-x}}$$

1.1.1 is not defined. (2)

1.1.2 is a natural number. (2)

1.1.3 is an irrational number. (2)

1.2 Solve for x:

1.2.1 $3x^2 - 4x = 0$ (3)

1.2.2 $3x - 14 = -6x^2$ (4)

1.2.3 $(x+1)(x-3) > 12$ (4)

1.2.4 $\sqrt{2-x} + 2 = x$ (4)

1.2.5 If $x - 6 = 0$ is one of the solutions of the equation $x + \frac{40}{x} = 16$, determine ONE value of y for which $2y + 3 + \frac{40}{2y+3} = 16$. (3)

1.3 Solve for x and y simultaneously:

$$y - 1 = 2x$$
$$x^2 + xy - 3x - y + 2 = 0$$ (5)

1.4 Show that the equation $x^2 - px - p^2 = 2$ has TWO real and unequal roots for all real values of p. (3)

1.5 A farmer constructs a rectangular enclosure using 100 m of fencing. He uses one existing boundary wall as one side of the rectangular enclosure.
Calculate the dimensions of the rectangle to obtain the maximum enclosed area. (7)

[39]

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QUESTION 2

2.1 Simplify WITHOUT the use of a calculator:

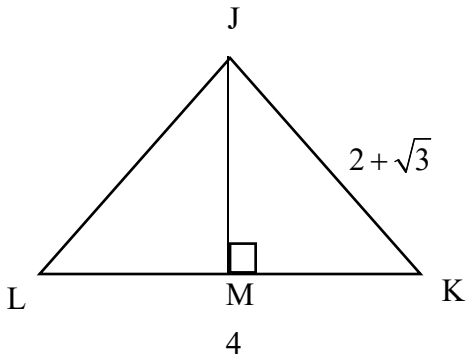
2.1.1 $\dfrac{3^{n+2}.9^{n+1}}{27^{n-1}}$ (3)

2.1.2 $\dfrac{x^2}{1+x}$ if $x = 1 + \sqrt{3}$ (4)

2.1.3 $\dfrac{\sqrt{a^2-b^2}\times(a+b)^{\frac{5}{2}}}{(a-b)^{\frac{1}{2}}}$ if $a \neq b$ (4)

2.2 Prove that: $\dfrac{2}{1+\sqrt{2}} - \dfrac{8}{\sqrt{8}} = -2$ (4)

2.3 Given: Isosceles Δ JKL with JK = $2 + \sqrt{3}$ and LK = 4.



Calculate the area of Δ JKL correct to ONE decimal place. (6)

[21]

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QUESTION 3

3.1 The FIRST three terms of a linear sequence are:

$x;\ 4x+5;\ 10x-5;\ \ldots$

Determine the numerical value of x. (2)

3.2 Consider the linear sequence:

17; 14; 11; …; –106

3.2.1 Determine n if the n^{th} term is given as $T_n = -3n + 20$. (2)

3.2.2 Which term is the FIRST negative term in the sequence? (2)

3.2.3 Determine the value of the 20^{th} ODD term in the sequence. (3)

3.3 Consider the pattern:

3; a; 10; b; 21

The pattern has a second difference of 1.
Determine the values of a and b. (4)
[13]

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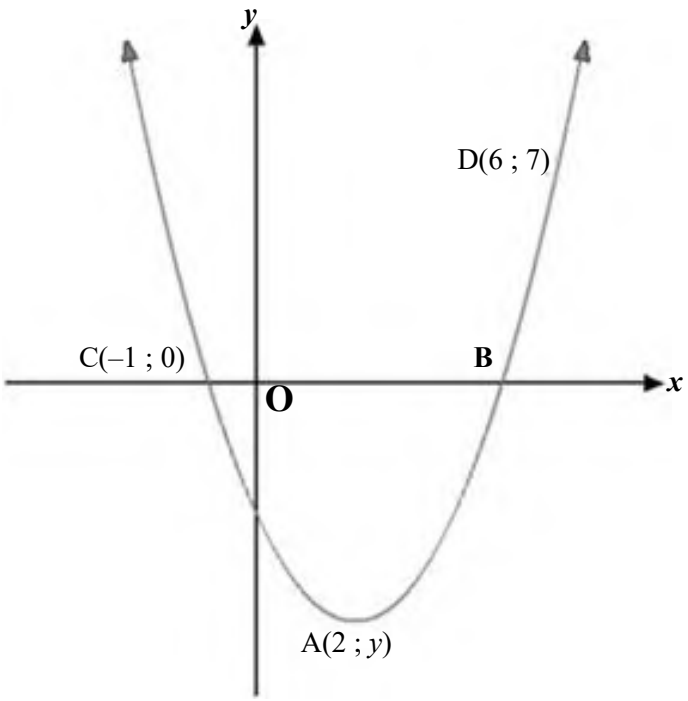
QUESTION 4

The diagram shows the graph of $f(x) = ax^2 + bx + c$ with the following essential properties:

- A(2 ; y) is the turning point of f.
- B and C(–1 ; 0) are the x-intercepts of f.
- D(6 ; 7) is a point on f.



4.1 Write down the coordinates of B. (2)

4.2 Show that the equation of f can be written as:

$f(x) = x^2 - 4x - 5$ (3)

4.3 Determine the equation of a line $h(x)$ passing through point C which is perpendicular to the line passing through point B and the y-intercept of f. (3)

4.4 For which values of x will $x.f(x) \geq 0$? (2)

[10]

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QUESTION 5

Given: $g(x)=\frac{6}{x+2}-1$ and $p(x)=\frac{6}{x-3}+2$.

5.1 Sketch the graph of g showing clearly the asymptotes and the intercepts with the axes. (3)

5.2 Determine the equation h, the line of symmetry of g, that has an angle of inclination of 135° in the form $y = \ldots$ (3)

5.3 Determine value(s) of x for which $g(x) < h(x)$. (1)

5.4 If the graph of g is shifted so that it coincides with the graph of p,

5.4.1 by how many units must the graph be shifted horizontally? (1)

5.4.2 by how many units must the graph be shifted vertically? (1)

[9]

QUESTION 6

6.1 Given: $h(x)=3.2^x-6$

6.1.1 Write down the equation of the asymptote of h. (1)

6.1.2 Determine the x-intercept of h. (2)

6.1.3 Determine the y-intercept of h. (1)

6.1.4 Determine for which values of x will $h(x) > 0$. (1)

6.2 Draw a neat sketch of the graph of $g(x) = a.b^x + q$ if:

- $a = 1$
- $0 < b < 1$
- $q = -1$

(3)

[8]

TOTAL: 100

END

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PROVINCIAL EXAMINATION

JUNE 2023

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 1)

10 pages

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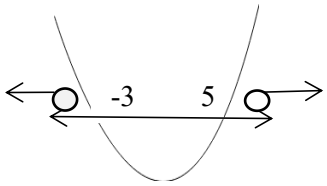
QUESTION 1

1.1	1.1.1	$x \in \{4\ ;\ 5\}$	✓ answer ✓ answer	(2)
	1.1.2	$x \in \{0\ ;\ 3\}$	✓ answer ✓ answer	(2)
	1.1.3	$x \in \{1\ ;\ 2\}$	✓ answer ✓ answer	(2)
1.2	1.2.1	$3x^2 - 4x = 0$ $\therefore x(3x-4) = 0$ $x = 0 \cdots \textbf{or} \cdots x = \frac{4}{3}$ **NOTE**: Any other valid method.	✓ factors ✓✓ answers	(3)
	1.2.2	$3x - 14 = -6x^2$ $\therefore\ 6x^2 + 3x - 14 = 0$ $\therefore\ x = \frac{-(3) \pm \sqrt{(3)^2 - 4(6)(-14)}}{2(6)}$ $\therefore\ x = 1{,}29$ ***or*** $x = -1{,}79$	✓ standard form ✓ substitution ✓✓ answers	(4)
	1.2.3	$(x+1)(x-3) > 12$ $x^2 - 2x - 3 > 12$ $x^2 - 2x - 15 > 0$ $(x-5)(x+3) > 0$ $\therefore x > 5 \cdots \textbf{or} \cdots x < -3$ 	✓ standard form ✓ factors ✓✓ answers	(4)
	1.2.4	$\sqrt{2-x} + 2 = x$ $\sqrt{2-x} = x - 2$ $(\sqrt{2-x})^2 = (x-2)^2$ $2 - x = x^2 - 4x + 4$ $0 = x^2 - 3x + 2$ $0 = (x-2)(x-1)$ $\therefore\ x = 2$ ... ***or*** ... $x = 1 (NA)$	✓ squaring both sides ✓ standard form ✓ factors ✓ answers with rejection	(4)

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	1.2.5 $x-6=0$ $\therefore x=6$ $\because x=2y+3$ $6=2y+3$ $2y=3$ $\therefore y=\frac{3}{2}$	✓ value of x ✓ substitution ✓ value of y	(3)
1.3	$y-1=2x$ $\therefore y=2x+1$ ……(1) $x^2+xy-3x-y+2=0$ ……(2) $\therefore x^2+x(2x+1)-3x-(2x+1)+2=0$ $x^2+2x^2+x-3x-2x-1+2=0$ $\therefore 3x^2-4x+1=0$ $(3x-1)(x-1)=0$ $\therefore x=\frac{1}{3}$ ... ***or*** ... $x=1$ $\therefore y=\frac{5}{3}$ ... ***or*** ... $y=3$	✓ expression for y ✓ substitution ✓ standard form ✓ x-values ✓ y-values	(5)
1.4	$x^2-px-p^2=2$ $\therefore x^2-px-p^2-2=0$ $\Delta=b^2-4ac$ $\Delta=(-p)^2-4(1)(-p^2-2)$ $\Delta=p^2+4p^2+8$ $\Delta=5p^2+8$ $\therefore p^2\geq 0\cdots\cdots p\in\Re$ $\therefore 5p^2\geq 0\cdots\cdots p\in\Re$ $\therefore 5p^2+8>0\cdots\cdots p\in\Re$ $\therefore$ roots are real and unequal.	✓ substitute into Δ ✓ $\Delta=5p^2+8$ ✓ $p^2\geq 0$, $5p^2\geq 0$ and $5p^2+8>0$	(3)

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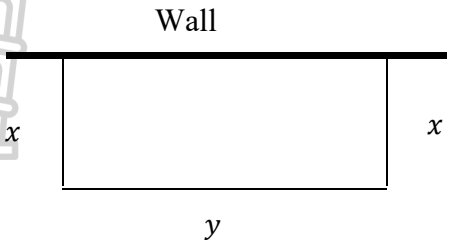
1.5	The 100 m fencing will make up three sides because the existing wall will make one side.  $fencing = 2x + y = 100$ $\therefore y = -2x + 100$ $\because A = xy$ $A = x(-2x + 100)$ $A = -2x^2 + 100x$ $A = -2(x^2 - 50x)$ $\therefore A = -2(x^2 - 50x - 625 - 625)$ $A = -2(x - 25)^2 - 625)$ Maximum area: $x = 25$ $A = -2(25)^2 + 100(25)$ $\therefore A = 1\,250$ $\because 1\,250 = 25y$ $\therefore y = 50$	✓ expression for y ✓ expression for A ✓ complete square ✓ value of x ✓ value of A ✓ value of y	(7)
			[39]

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QUESTION 2

2.1	2.1.1	$\frac{3^{n+2}.9^{n+1}}{27^{n-1}}$ $\frac{3^{n+2}.3^{2n+2}}{3^{3n-3}}$ $\frac{3^{3n+4}}{3^{3n-3}}$ $3^{3n+4-3n+3}$ 3^7	✓ 3^{2n+2} and 3^{3n-3} ✓ $3^{3n+4-3n+3}$ ✓ answer	(3)
	2.1.2	$\frac{x^2}{1+x}$ $=\frac{(1+\sqrt{3})^2}{1+1+\sqrt{3}}$ $=\frac{1+2\sqrt{3}+3}{2+\sqrt{3}}$ $=\frac{4+2\sqrt{3}}{2+\sqrt{3}}$ $=\frac{2(2+\sqrt{3})^2}{2+\sqrt{3}}$ $=2$	✓ substitution ✓ simplification ✓ factorisation ✓ answer	(4)
	2.1.3	$\frac{\sqrt{(a^2-b^2)}\times(a+b)^{\frac{5}{2}}}{(a-b)^{\frac{1}{2}}}$ $=\frac{\sqrt{(a-b)(a+b)}\times(a+b)^{\frac{5}{2}}}{(a-b)^{\frac{1}{2}}}$ $=\frac{(a-b)^{\frac{1}{2}}(a+b)^{\frac{1}{2}}\times(a+b)^{\frac{5}{2}}}{(a-b)^{\frac{1}{2}}}$ $(a+b)^{\frac{1}{2}}\times(a+b)^{\frac{5}{2}}$ $(a+b)^3$ $a^3+3a^2b+3ab^2+b^3$	✓ factorisation ✓ simplification ✓ simplification ✓ answer	(4)

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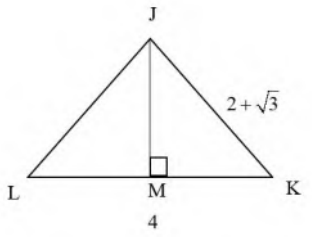
2.2	*RTP*: $\frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}} = -2$ $\frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}}$ $= \frac{2}{1+\sqrt{2}} - \frac{8}{2\sqrt{2}}$ $= \frac{2(2\sqrt{2}) - 8(1+\sqrt{2})}{2\sqrt{2}(1+\sqrt{2})}$ $= \frac{4\sqrt{2} - 8 - 8\sqrt{2}}{2\sqrt{2} + 2\sqrt{4}}$ $= \frac{4\sqrt{2} - 8 - 8\sqrt{2}}{2\sqrt{2} + 4}$ $= \frac{-4\sqrt{2} - 8}{2\sqrt{2} + 4}$ $= \frac{-4\sqrt{2} + 2}{2\sqrt{2} + 2}$ $= -2$	✓ $2\sqrt{2}$ ✓ $2\sqrt{2}(1+\sqrt{2})$ ✓ simplification ✓ factorisation	(4)
2.3	 $MK = LM = 2$ $JM^2 = (2+\sqrt{3})^2 - 2^2$ pythag $JM^2 = 4 + 4\sqrt{3} + 3 - 4$ $JM^2 = 4\sqrt{3} + 3$ $JM = \sqrt{4\sqrt{3}+3}$ $A = \frac{1}{2}(4)\sqrt{4\sqrt{3}+3}$ $A = 6{,}3\ units^2$	✓ MK= 2 ✓ substitution ✓ simplification ✓ JM ✓ substitution into area formula ✓ answer	(6)
			[21]

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QUESTION 3

3.1	$T_2 - T_1 = T_3 - T_2$ $4x + 5 - x = 10x - 5 - (4x + 5)$ $3x + 5 = 10x - 5 - 4x - 5$ $3x + 5 = 6x - 10$ $\therefore 3x = 15$ $\therefore x = 5$		✓ method ✓ answer	(2)
3.2	3.2.1	$-3n + 20 = -106$ $3n = 126$ $\therefore n = 42$	✓ equating ✓ answer	(2)
	3.2.2	$-3n + 20 < 0$ $20 < 3n$ $\therefore \frac{20}{3} < n$ $\therefore n = 7$	✓ $T_n < 0$ ✓ answer	(2)
	3.2.3	Odd valued terms: 17; 11; 5; …….. General term: $T_n = -6n + 23$ $T_n = -6(20) + 23$ $\therefore T_n = -97$	✓ sequence ✓ T_n ✓ answer	(3)
3.3	3; a; 10; b; 21 $a - 3$; $10 - a$; $b - 10$; $21 - b$ 1^{st} difference 2^{nd} differences: $10 - a - a + 3 = 1$ $-2a + 13 = 1$ $-2a = -12$ $\therefore a = 6$ and $21 - b - b + 10 = 1$ $-2b + 31 = 1$ $-2b = -30$ $\therefore b = 15$		✓ 1^{st} differences ✓ equating 2^{nd} difference in terms of a, then b. ✓ value of a ✓ value of b	(4)
				[13]

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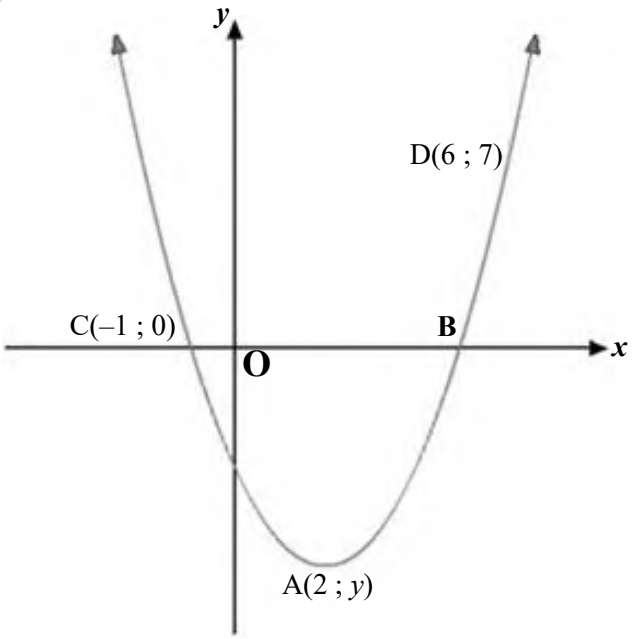
QUESTION 4



4.1	B(5; 0) **NOTE**: Must be in coordinate form.	✓✓ answer	(2)
4.2	$y = a(x - x_1)(x - x_2)$ $\therefore 7 = a(6-(-1))(6-5)$ $7 = 7a$ $\therefore a = 1$ $\therefore y = 1(x+1)(x-5)$ $y = x^2 - 5x + x - 5$ $\therefore y = x^2 - 4x - 5$	✓ substitute roots and point D(6;7) ✓ value for a ✓ $y = x^2 - 5x + x - 5$	(3)
4.3	B(5 ; 0) C(0 ; –5) $m_{BC} = \frac{-5-0}{0-5}$ $m_{BC} = \frac{-5}{-5}$ $\therefore m_{BC} = 1$ $\therefore m_h = -1$ $\therefore y - y_1 = m(x - x_1)$ $y - (-1) = -1(x - 0) \cdots\cdots pt(-1;0)$ $\therefore y = -x - 1$	✓ m_{BC} ✓ m_h ✓ answer	(3)

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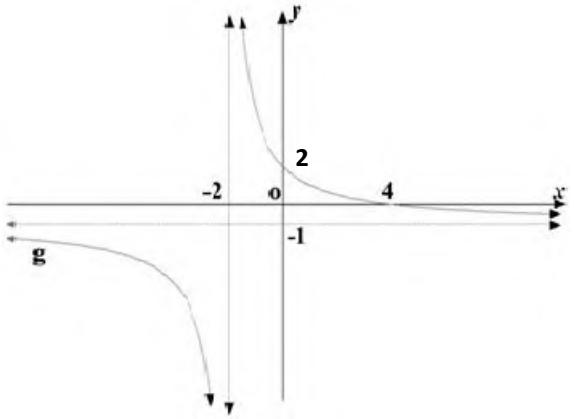
4.4	$-1 \le x \le 0$ $x \ge 5$ **OR** $x \in [-1\ ;\ 0]$ **or** $[5\ ;\ \infty]$ **NOTE**: Deduct 1 mark if brackets are incorrect in the alternative solution.	✓ all critical values (independent) ✓ answers ✓ all critical values (independent) ✓ answers	(2)
			[10]

QUESTION 5

5.1	 **NOTE**: If the candidate calculates the intercepts and lists the asymptotes but does not sketch the graph, award 2 marks.	✓ asymptotes ✓ intercepts ✓ shape	(3)
5.2	tan 135° = –1 $y - y_1 = m(x - x_1)$ $y - (-1) = -1(x - (-2))$ $y + 1 = -x - 2$ $y = -x - 3$	✓ $m = -1$ ✓ subs. point (2;–1) ✓ answer	(3)
5.3	$x < -2$	✓ answer	(1)
5.4	5.4.1 5 units right **NOTE**: Accept an answer of 5 units.	✓ answer	(1)
	5.4.2 3 units up **NOTE**: Accept an answer of 3 units.	✓ answer	(1)
			[9]

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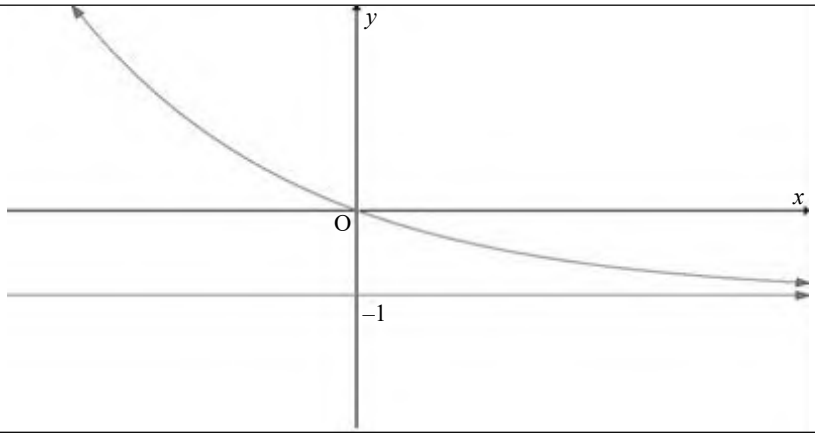
QUESTION 6

6.1	6.1.1	$y = -6$	✓ answer	(1)
	6.1.2	$h(x) = 3.2^x - 6$ $0 = 3.2^x - 6$ $6 = 3.2^x$ $2 = 2^x$ $x = 1$	✓ equate to 0 ✓ answer	(2)
	6.1.3	$h(x) = 3.2^x - 6$ $y = 3.2° - 6$ $y = -3$	✓ answer	(1)
	6.1.4	$x > 1$	✓ answer	(1)
6.2			✓ x-intercept ✓ asymptote ✓ shape	(3)
				[8]
			TOTAL:	**100**