GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2023

GRADE 11

MATHEMATICS
PAPER 2

TIME: **2 hours**

MARKS: **100**

11 pages and 2 answer sheets

INSTRUCTIONS AND INFORMATION

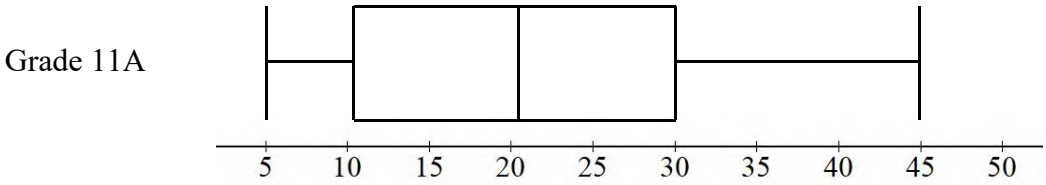
1. This question paper consists of 8 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. Answer sheets for QUESTION 1.2 and QUESTION 8.1 are provided at the end of the question paper. Write you name in the spaces provided on each answer sheet and submit them together with your ANSWER BOOK.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

The following box and whisker plot and accompanying 5-number summary, shows the marks obtained by Grade 11A for a Mathematics test out of 50.

The 5-number summary for Grade 11A:

Min. = 5 ; $Q_1 = 11$; $Q_2 = 21$; $Q_3 = 30$; Max. = 45



The following data shows the marks obtained by the learners in another class (Grade 11B) for the same Mathematics test out of 50.

10 ; 13 ; 17 ; 21 ; 21 ; 23 ; 27 ; 29 ; 31 ; 34 ; 37 ; 38 ; 42 ; 43 ; 46 ; 48

1.1 Determine the 5-number summary for Grade 11B. (3)

1.2 On the diagram sheet provided in ANSWER SHEET A, draw a box and whisker plot of the marks for Grade 11B. (3)

1.3 Calculate the mean mark of Grade 11B. (2)

1.4 Calculate the standard deviation of Grade 11B. (2)

1.5 How many learners in Grade 11B obtained a mark that is higher than one standard deviation above the mean? (2)

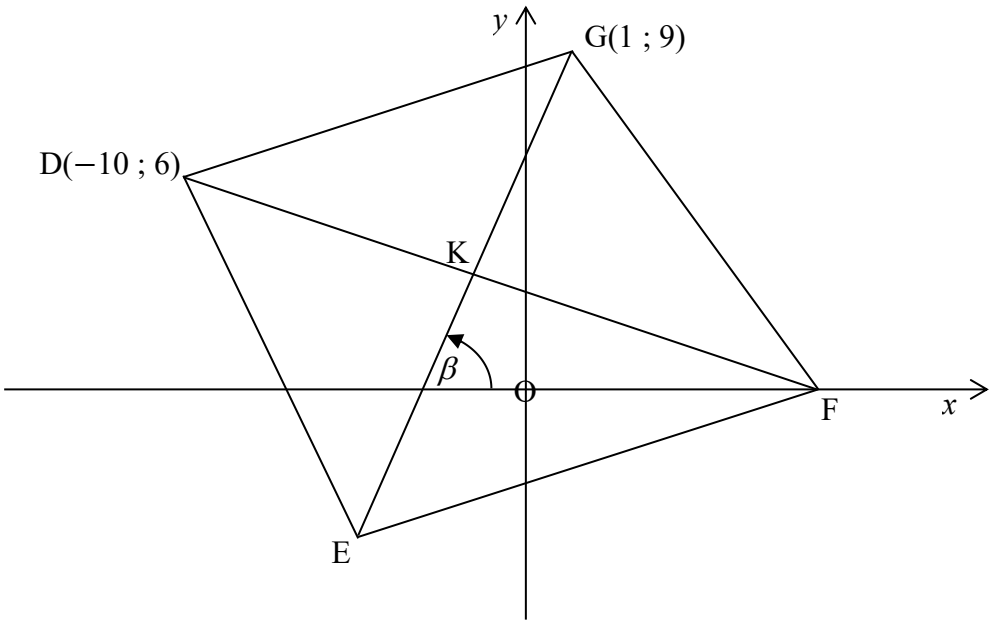
1.6 Taking the interquartile range of the two grades into account, comment on the performance of Grade 11A and Grade 11B. (2)

1.7 Determine the percentage of learners in Grade 11A who achieved less than 30 for the test. (1)

[15]

QUESTION 2

In the following diagram D(–10 ; 6), E, F and G(1 ; 9) are the vertices of a quadrilateral. The equation of EG is $3x - y + 6 = 0$. The diagonals of the quadrilateral bisect each other at point K. Point F is on the x-axis and β is the angle of inclination of EG.



2.1 Determine the size of β. (2)

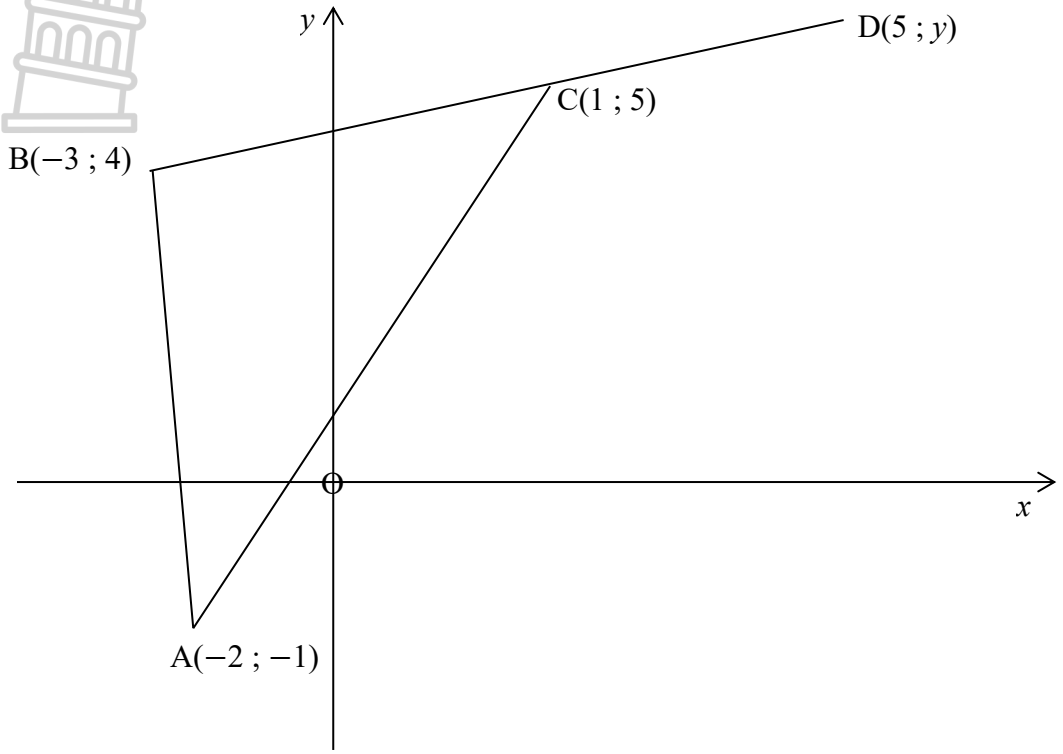
2.2 Calculate the coordinates of F given the equation of DF is $x + 3y = 8$. (2)

2.3 Determine the coordinates of E. (4)

2.4 Prove that DGFE is a rhombus. (3)
[11]

QUESTION 3

In the diagram below, A is the point (–2;–1), B(–3 ; 4), C(1 ; 5) and D(5 ; y).



3.1 Find the length of AC in simplified surd form. (2)

3.2 Determine the gradient of BC. (2)

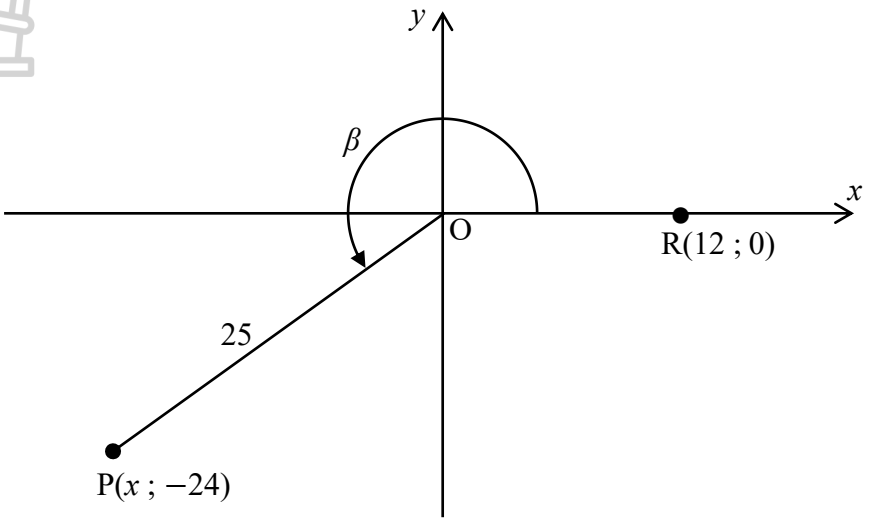
3.3 Determine the value of y if B, C and D are collinear. (3)

3.4 If H is a new point such that AH ⊥ BC, determine the equation of AH. (3)

[10]

QUESTION 4

4.1 In the diagram below, P(x; –24) is a point such that OP = 25, and R(12 ; 0) with $\hat{ROP} = \beta$, where $180° < \beta < 270°$.



4.1.1 Calculate the value of x. (2)

4.1.2 Determine the value of each of the following WITHOUT the use of a calculator.

(a) $\sin\beta$ (1)

(b) $\cos(180° - \beta)$ (2)

(c) $\tan^2(-\beta)$ (2)

4.1.3 T is a point on OP such that OT = 15.

(a) Show that $T\left(-\frac{21}{5}; -\frac{72}{5}\right)$ WITHOUT the use of a calculator. (4)

(b) Determine the area of ΔROT. (4)

4.2 Simplify to a single trigonometric ratio:

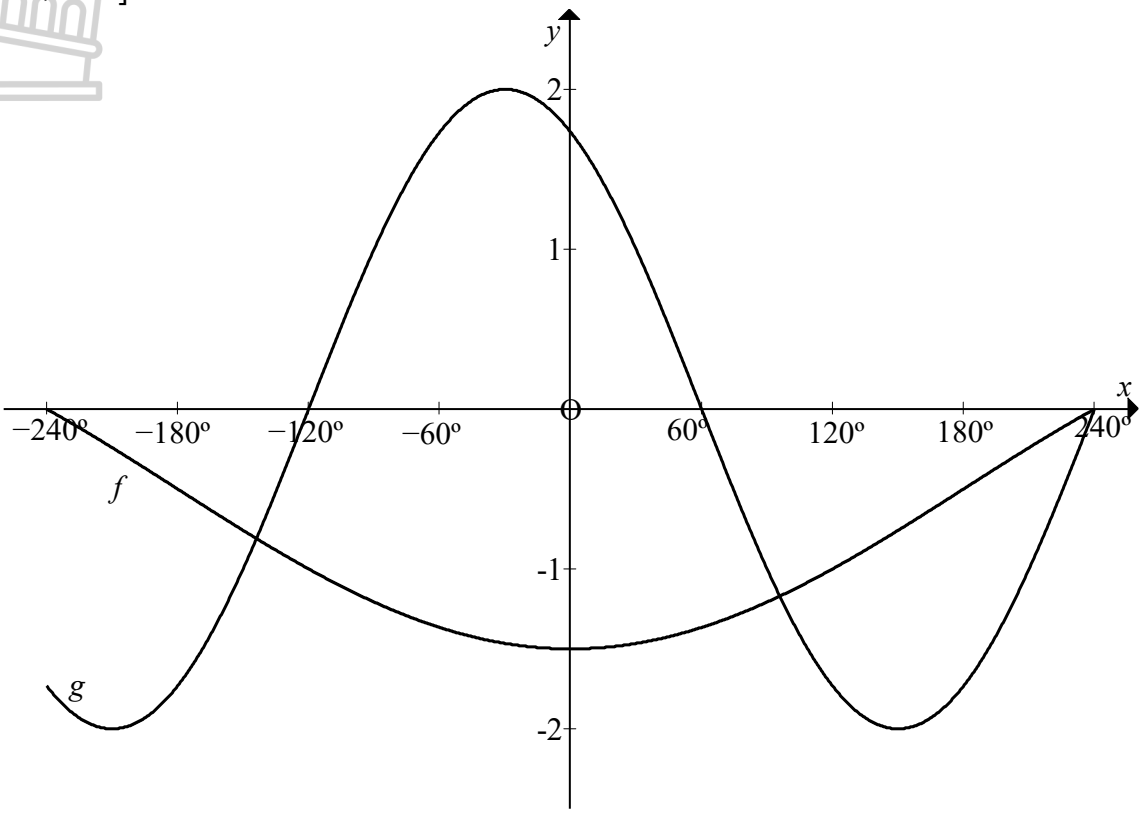
$$\frac{\tan 225° + \sin(180° - \theta)\cos(90° + \theta)}{\cos(90° - \theta)\sin(-\theta - 540°)}$$

(6)

[21]

QUESTION 5

In the diagram below, $f(x) = a\cos\left(\frac{1}{2}x\right) - \frac{1}{2}$ and $g(x) = -2\sin(x + p)$ are drawn for $x \in [-240°; 240°]$.



5.1 Write down the amplitude of g. (1)

5.2 Determine the period of f. (1)

5.3 Determine the values of a and p. (2)

5.4 For which value(s) of x is $f(x).g(x) \leq 0$? (2)

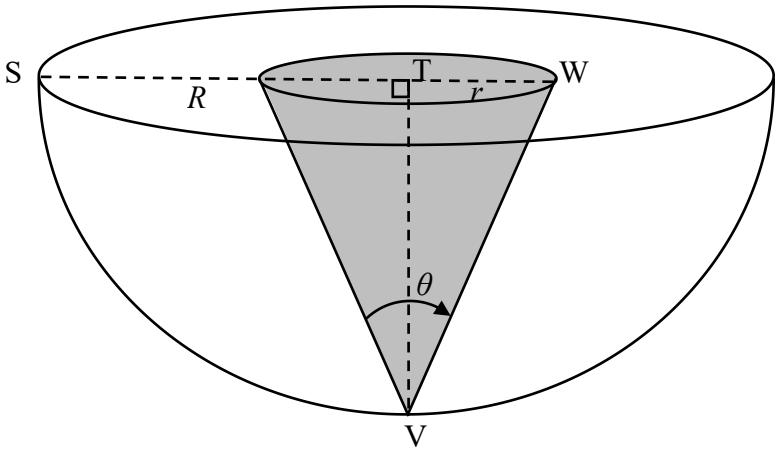
5.5 If $h(x) = 2\cos(x + k)$, and $h(x) = g(x)$, write down a possible value for k. (2)
[8]

QUESTION 6

A candle holder is made in the shape of a hemisphere where a conical section is drilled out for the candle wax.

In the diagram below, the radius of the hemisphere is represented by R, where ST = VT = R.
The radius of the cone is represented by r, where TW = r.
The angle at the vertex of the cone is given by θ.

$$V_{SPHERE} = \frac{4}{3}\pi R^3 \qquad V_{CONE} = \frac{1}{3}\pi r^2 h$$



6.1 Express r in terms of R and θ. (2)

6.2 Show that the remaining volume of the hemisphere can be represented by:

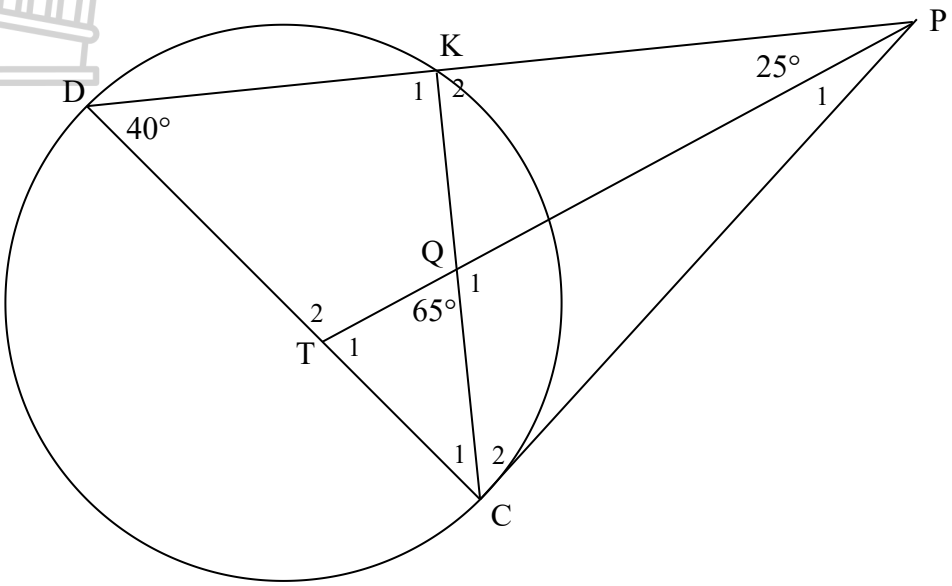
$$V = \frac{\pi R^3}{3}\left(2 - \tan^2\left(\frac{\theta}{2}\right)\right)$$ (4)

6.3 The cone is filled with wax and a wick. The wick is always 1 *cm* below the level of the wax but the wick must not protrude over the level of the flat surface of the hemisphere. The radius of the hemisphere (R) is 12 *cm* and the angle of the cone (θ) is 36°.

Determine the volume of wax that must be put into each candle holder. (3)
[9]

QUESTION 7

CD is a diameter and PC a tangent to the circle. Chord DK is produced to P.
PT intersects KC at Q. $C\hat{D}P = 40°$, $D\hat{P}T = 25°$ and $T\hat{Q}C = 65°$.



7.1 Determine, with reasons, the size of the following:

7.1.1 $\hat{C}_2$ (2)

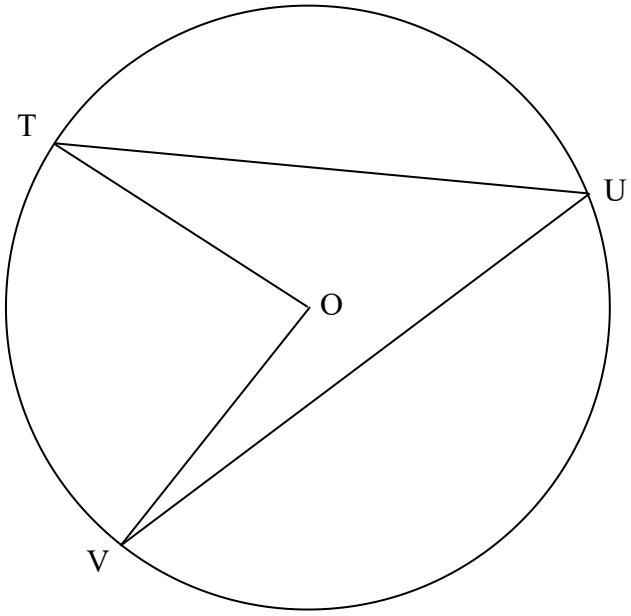
7.1.2 $\hat{K}_1$ (2)

7.1.3 $\hat{P}_1$ (2)

7.2 Prove, with reasons, that TC = QC. (2)
[8]

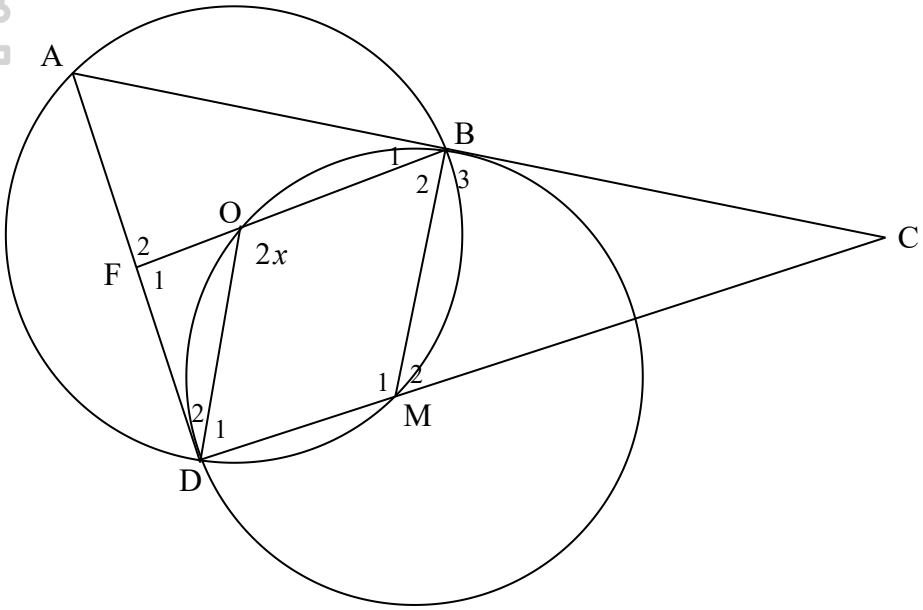
QUESTION 8

In the diagram below, the points T, U and V lie on the circumference of the circle with centre O.



8.1 On the ANSWER SHEET B provided, prove the theorem which states that $T\hat{O}V = 2T\hat{U}V$. (5)

8.2 In the diagram below, the two EQUAL circles with centres O and M are drawn. Chords AB and DM are produced to C. Chord OB is produced to meet AD at F. AB is a tangent to the circle with centre M at B, and AD is a tangent at D. $D\hat{O}B = 2x$.



8.2.1 Provide a reason why DOBM is a rhombus. (1)

8.2.2 Provide the geometric reason why each of the following angles are equal to x.

(a) $\hat{A}$ (1)

(b) $\hat{M}_2$ (1)

(c) $\hat{D}_1$ (1)

(d) $\hat{B}_2$ (1)

8.2.3 Prove, with reasons, that $\hat{D}_2 = \hat{C}$. (4)

8.2.4 Prove, with reasons, that AB = BC. (4)

[18]

TOTAL: 100

END

Name and Surname: _____ Grade: _____

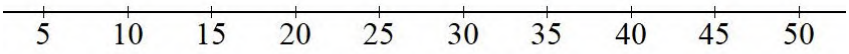
ANSWER SHEET A

QUESTION 1

1.2

Grade 11A

Grade 11B



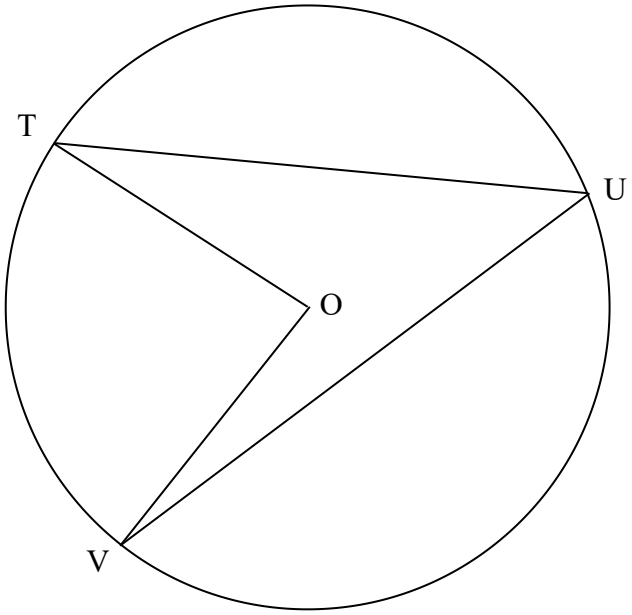
(3)

Name and Surname: _____ Grade: _____

ANSWER SHEET B

QUESTION 8

8.1



(5)

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2023

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 2)

10 pages

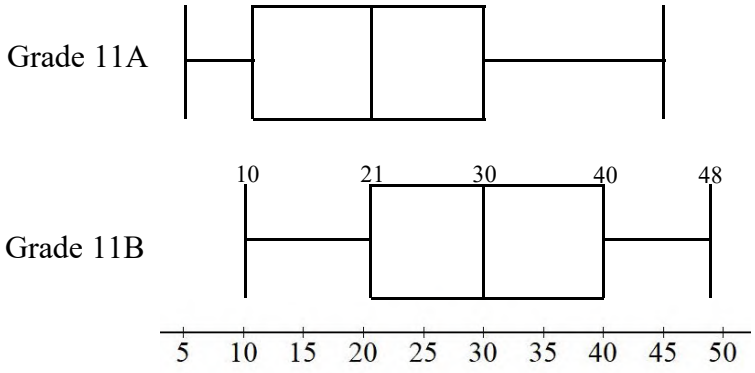
INSTRUCTIONS AND INFORMATION:

- A – ACCURACY
- CA – CONSISTENT ACCURACY
- S – STATEMENT
- R – REASON
- S & R – STATEMENT with REASON

NOTES:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is unacceptable to adopt values/answers in order to solve a problem.

QUESTION 1

1.1	10; 13; 17; 21; 21; 23; 27; 29; 31; 34; 37; 38; 42; 43; 46; 48 Min. =10 ; $Q_1 = 21$; Max. = 48 $Q_2 = \frac{29+31}{2} = 30$; $Q_3 = \frac{38+42}{2} = 40$	✓ Correct Min. Q_1 and Max. ✓ Correct Q_2 ✓ Correct Q_3	(3)
1.2		✓ Correct Min. plot/lower whisker ✓ Correct Max. plot/upper whisker ✓ Correct box	(3)
1.3	$\bar{x} = \frac{480}{16}$ $\bar{x} = 30$	✓ Fraction ✓ Answer **NB**: Answer only full marks	(2)
1.4	σ = 11,46	✓✓ Answer	(2)
1.5	1σ interval: (30 – 11,46 ; 30 + 11,46) (18,54 ; 41,46) 4 learners achieved over 41,46 marks.	✓ Interval ✓ Answer	(2)
1.6	There is no difference in the IQR. IQR = 19 for both classes. However, the middle 50% for Grade 11B was between 21 and 40 compared to only 11 and 30 for Grade 11A. The box for 11B is shifted to the right. Thus, Grade 11B generally performed better.	✓ Comment ✓ IQR value	(2)
1.7	75%	✓ Answer	(1)
			[15]

QUESTION 2

2.1	Equation of EG: $3x - y + 6 = 0$ $y = 3x + 6$ then $m_{EG} = 3$ $\tan \beta = 3$ $\beta = 71{,}57°$	✓ Gradient ✓ Answer	(2)
2.2	$x + 3(0) = 8$ $x = 8$ $\therefore (8\,;0)$	✓ $x = 0$ ✓ Coordinate form	(2)
2.3	$M_{DF} = M_{GE}$ diagonals bisect $K\left(\frac{-10+8)}{2}\,;\frac{6+0}{2}\right)$ $K(-1\,;3)$ $-1 = \frac{x+1}{2}$ $\quad 3 = \frac{y+9}{2}$ $x = -3$ $\quad y = -3$ $\therefore E(-3\,;-3)$	✓ $M_{DE} = M_{GE}$ ✓ Coordinates of K ✓ x-value ✓ y-value	(4)
2.4	$m_{DF} \times m_{GE}$ $= -\frac{1}{3} \times 3$ $= -1$ $\therefore DF \perp GE$ DGFE is a rhombus. (diagonals bisect perpendicularly)	✓ m_{DF} ✓ DF ⊥ GE ✓ Reason	(3)
			[11]

QUESTION 3

3.1	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(1-(-2))^2 + (5-(-1))^2}$ $AC = 3\sqrt{5}$	✓ Substitution ✓ Answer	(2)
3.2	$m_{BC} = \frac{5-4}{1-(-3)}$ $m_{BC} = \frac{1}{4}$	✓ Substitution ✓ Answer	(2)
3.3	$m_{BC} = m_{BD} = m_{CD}$ $\frac{1}{4} = \frac{y-5}{5-1}$ **or** $\frac{1}{4} = \frac{y-4}{5-(-3)}$ $y = 6$	✓ Equal gradients ✓ Substitution ✓ $y = 6$	(3)
3.4	$m_{AH} = -4 \quad (\text{AH} \perp \text{BC})$ $y-(-1) = -4(x-(-2))$ $y = -4x - 9$	✓ m_{AH} ✓ Substitute A ✓ Equation	(3)
			[10]

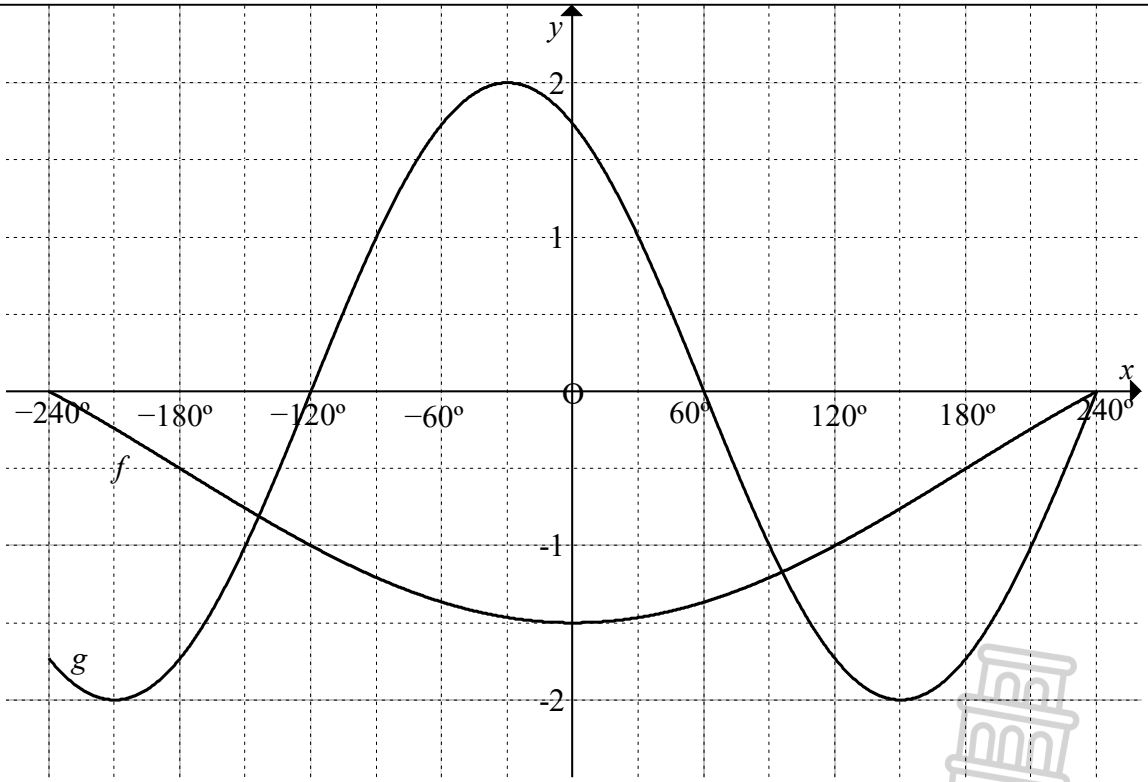
QUESTION 4

4.1	4.1.1		$x^2 + (-24)^2 = 25^2$ $x = -7$	✓ Substitute into Pythagoras ✓ Answer	(2)
	4.1.2	(a)	$\sin\beta$ $= -\frac{24}{25}$	✓ Answer	(1)
		(b)	$\cos(180° - \beta)$ $= -\cos\beta$ $= -\left(\frac{-7}{25}\right)$ $= \frac{7}{25}$	✓ Reduction ✓ Simplified answer	(2)

		(c)	$\tan^2(-\beta)$ $= \tan^2\beta$ $= \left(\frac{-24}{-7}\right)^2$ $= \frac{576}{49}$	✓ Reduction ✓ Simplified answer	(2)
	4.1.3	(a)	$\sin\beta = \frac{-24}{25}$ $\frac{y}{15} = \frac{-24}{25}$ $y = \frac{-72}{5}$ $\cos\beta = \frac{-7}{25}$ $\frac{x}{15} = \frac{-7}{25}$ $x = \frac{-21}{5}$	✓ $\sin\beta = \frac{y}{15}$ ✓ Equate sine ratios ✓ $\cos\beta = \frac{x}{15}$ ✓ Equate cosine ratios	(4)
		(b)	$\text{Area}\Delta\text{ROT} = \frac{1}{2}(12)\left(\frac{72}{5}\right)$ $\text{Area}\Delta\text{ROT} = 86{,}4 \text{ units}^2$ **ALTERNATIVE** $OT = 15 \quad OR = 12$ $\beta = 253{,}7397...°$ $\text{Area}\Delta\text{ROT} = \frac{1}{2}(15)(12)\sin(106{,}26°)$ $\text{Area}\Delta\text{ROT} = 86{,}4 \text{ units}^2$	✓ Formula for area Δ ✓ Base = 12 ✓ Height = $\frac{72}{5}$ ✓ Area ✓ OT length ✓ 106,26° ✓ Substitute sine area formula ✓ Area	(4)

4.2	$\frac{\tan 225° + \sin(180° - \theta)\cos(90° + \theta)}{\cos(90° - \theta)\sin(-\theta - 540°)}$ $= \frac{1 + \sin\theta . - \sin\theta}{\sin\theta . \sin\theta}$ $= \frac{1 - \sin^2\theta}{\sin^2\theta}$ $= \frac{\cos^2\theta}{\sin^2\theta}$ $= \tan^2\theta$	✓ $\tan 225° = 1$ ✓ $\sin(180° - \theta) = \sin\theta$ ✓ $\cos(90° + \theta) = -\sin\theta$ ✓ $\sin(-\theta - 540°) = \sin\theta$ ✓ $1 - \sin^2\theta = \cos^2\theta$ ✓ $\frac{\cos^2\theta}{\sin^2\theta} = \tan^2\theta$	(6)
			[21]

QUESTION 5



5.1	$A = 2$	✓ Answer	(1)
5.2	Period = 720°	✓ Answer	(1)
5.3	$a = -1$ $p = -60°$	✓ Value of a ✓ Value of p	(2)
5.4	$-120° \leq x \leq 60°$ **or** $x = -240°$ **or** $x = 240°$	✓ Interval ✓ $-240°$; $240°$	(2)
5.5	$p = 30°$	✓✓ Value of p	(2)
			[8]

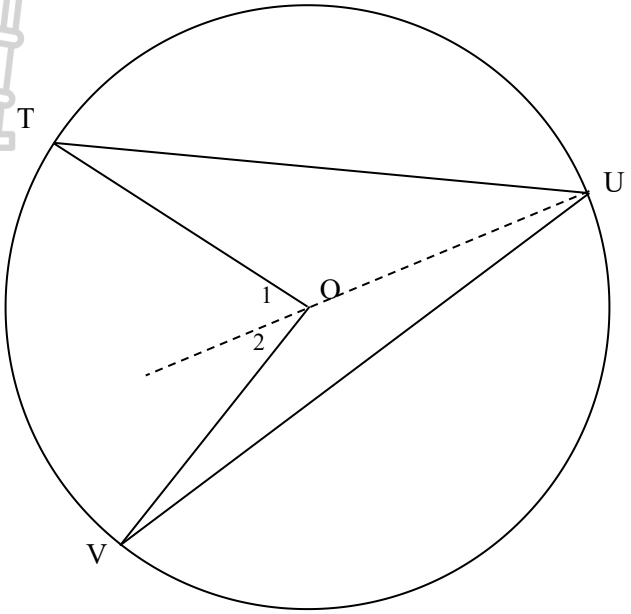
QUESTION 6

6.1	$\tan\left(\frac{\theta}{2}\right)=\frac{r}{R}$ $R.\tan\left(\frac{\theta}{2}\right)=r$	✓ Substitution ✓ Manipulation	(2)
6.2	$V_{hemisphere}=\frac{1}{2}.\frac{4}{3}\pi R^3$ $V_{hemisphere}=\frac{2}{3}\pi R^3$ $V_{cone}=\frac{1}{3}\pi r^2 R$ $V_{cone}=\frac{1}{3}\pi\left(R.\tan\left(\frac{\theta}{2}\right)\right)^2 R$ $V_{cone}=\frac{2}{3}\pi.R^3\tan^2\left(\frac{\theta}{2}\right)$ $V_{remaining}=\frac{2}{3}\pi R^3-\frac{1}{3}\pi.R^3\tan^2\left(\frac{\theta}{2}\right)$ $V_{remaining}=\frac{\pi R^3}{3}\left(2-\tan^2\left(\frac{\theta}{2}\right)\right)$	✓ $\frac{1}{2}$ Volume Sphere ✓ Substitute r from 6.1 ✓ Subtract volumes (Method) ✓ Factors	(4)
6.3	$V_{cone}=\frac{1}{3}\pi.R^3\tan^2\left(\frac{\theta}{2}\right)$ $V_{cone}=\frac{1}{3}\pi(11)^3\tan^2\left(\frac{36°}{2}\right)$ $V_{cone}=147{,}15\ cm^3$	✓ $V_{cone}=\frac{1}{3}\pi.R^3\tan^2\left(\frac{\theta}{2}\right)$ ✓ Substitute $R=11$ and $\theta=36°$ ✓ Volume of wax	(3)
			[9]

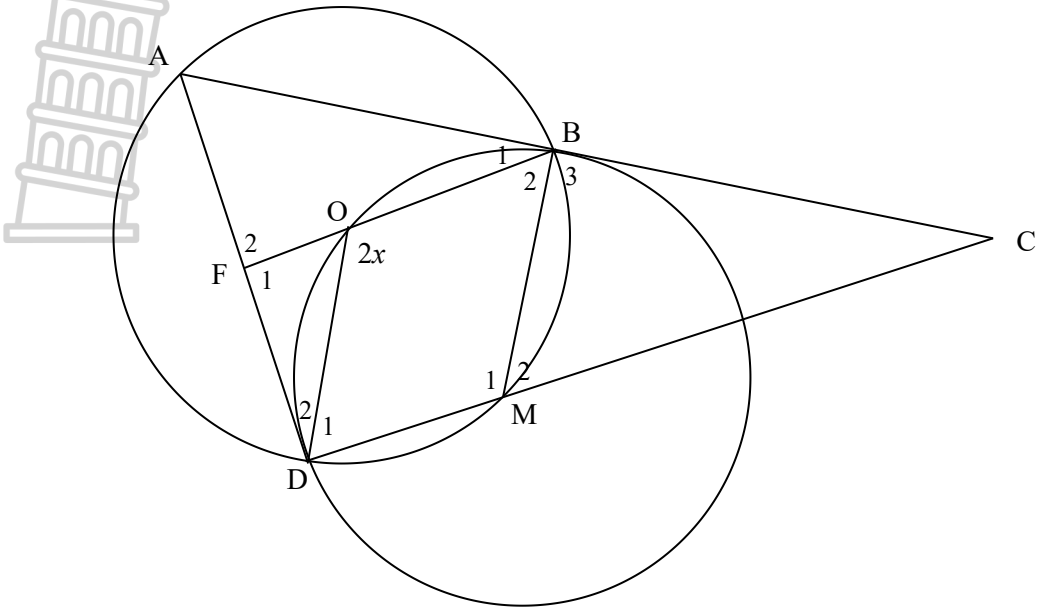
QUESTION 7

7.1	7.1.1	$\hat{C}_2=40°$	tan chord theorem	✓ S ✓ R	(2)
	7.1.2	$\hat{K}_1=90°$	∠'s in a semi circle	✓ S ✓ R	(2)
	7.1.3	$\hat{P}_1=25°$	ext ∠ of a Δ	✓ S ✓ R	(2)
7.2	$\hat{T}_1=65°$ $TC=QC$		ext ∠ of Δ sides opp equal ∠'s	✓ R ✓ R	(2)
					[8]

QUESTION 8



8.1	*Construct UO and extend.* In ΔTOU $\hat{O}_1 = \hat{T} + \hat{U}_1$ exterior $\angle$ of Δ $\hat{T} = \hat{U}_1$ $\angle$'s opp equal radii $\therefore \hat{O}_1 = 2\hat{U}_1$ Similarly in ΔTOU $\therefore \hat{O}_2 = 2\hat{U}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{U}_1 + 2\hat{U}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{U}_1 + \hat{U}_2)$ $T\hat{O}V = 2T\hat{U}V$	✓ Construction ✓ SR ✓ SR ✓ S ✓ Conclusion	(5)



8.2	8.2.1		Two pairs of adjacent sides equal.	✓ R	(1)
	8.2.2	(a)	∠ at centre = 2 × ∠ at circumference	✓ R	(1)
		(b)	ext ∠ of cyclic quad	✓ R	(1)
		(c)	corr ∠'s, OD ∥ MB	✓ R	(1)
		(d)	opp ∠'s, rhombus **ALTERNATIVE** alt ∠'s, OB ∥ DM	✓ R	(1)
	8.2.3		$\hat{D}_2 + x = 90°$ tan ⊥ rad $\hat{D}_2 = 90° - x$ $\hat{B}_3 = 90°$ tan ⊥ rad $\hat{C} = 90° - x$ int ∠s of Δ $\therefore \hat{D}_2 = \hat{C}$ both $90° - x$	✓ R ✓ S ✓ SR ✓ Conclusion	(4)
	8.2.4		$\hat{F}_2 = 90°$ corr ∠s, FB ∥ DM AF = FD line from centre ⊥ chord AB = BC line through midpt ∥ to 2nd side	✓ SR ✓ S ✓ R ✓ R	(4)
					[18]
				TOTAL:	**100**