GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2023

MATHEMATICS

(PAPER 1)

TIME: **3 hours**

MARKS: **120**

7 pages and an information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Given: $12x = x^2$

1.1.1 Solve for x. (3)

1.1.2 Hence, or otherwise, determine the value(s) of p if $\left(p^2-1\right)^2 = 12\left(p^2-1\right)$. (Leave your answer in surd form, where necessary). (4)

1.2 Solve for x if $5x^2 + 7x - 2 = 0$. (Round-off the answer to TWO decimal places.) (4)

1.3 Solve for x if $\sqrt{x+6} = x$. (5)

1.4 Use the solution for x in QUESTION 1.3 to determine the value of y for which $\sqrt{y+1} = y-5$. (2)

1.5 A race requires an athlete to run 10 km and cycle 50 km. Tendani runs at a speed of x km/h and cycles at $(x+31)$ km/h. He takes $\frac{10}{x}$ hours for the 10 km run.

1.5.1 Express the time he takes for the 50 km cycle in terms of x. (1)

1.5.2 Calculate the speed (correct to TWO decimal places) at which he must run to complete the entire race in 2 hours. (6)

[25]

QUESTION 2

In a geometric series, the sum of the first n terms is given by $S_n = k\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

2.1 Calculate the value(s) of k. (4)

2.2 Calculate the second term of the series. (4)

[8]

QUESTION 3

3.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$. (4)

3.2 Calculate the value of $\sum_{p=1}^{50}(100-3p)$. (4)

3.3 A quadratic sequence is defined with the following properties:

$T_2 - T_1 = 7$

$T_3 - T_2 = 13$

$T_4 - T_3 = 19$

3.3.1 Write down the values of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

3.3.2 Calculate the value of T_{69} if T_{69} = 23 594. (5)

[17]

QUESTION 4

4.1 Given: $f(x) = x^2 - 2x - 3$ and $g(x) = x - 5$

4.1.1 Show that the turning point of f is $(1;-4)$. (3)

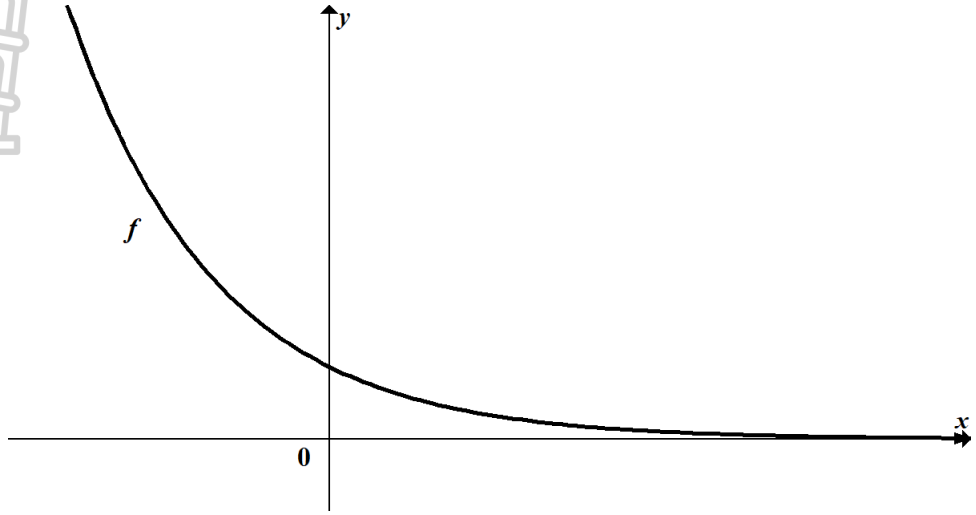
4.1.2 Determine the coordinates of the x- and y-intercepts of the graph of f. (3)

4.1.3 Determine the points of intersection of the graphs of f and g. (4)

4.1.4 Sketch neat graphs of f and g on the same system of axes.
Clearly label the turning point and where the graphs of f and g intersect each other as well as the x- and y-intercepts of both graphs. (6)

4.1.5 Use the graph to determine the values of x where $f(x) \geq 0$. (2)

4.2 The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



4.2.1 Write down the equation of the asymptote of f. (1)

4.2.2 Write down the equation of f^{-1} in the form $y = \ldots$ (2)

4.2.3 Sketch the graph of f^{-1} in your ANSWER BOOK.
Indicate the intercept and ONE other point on the graph. (3)

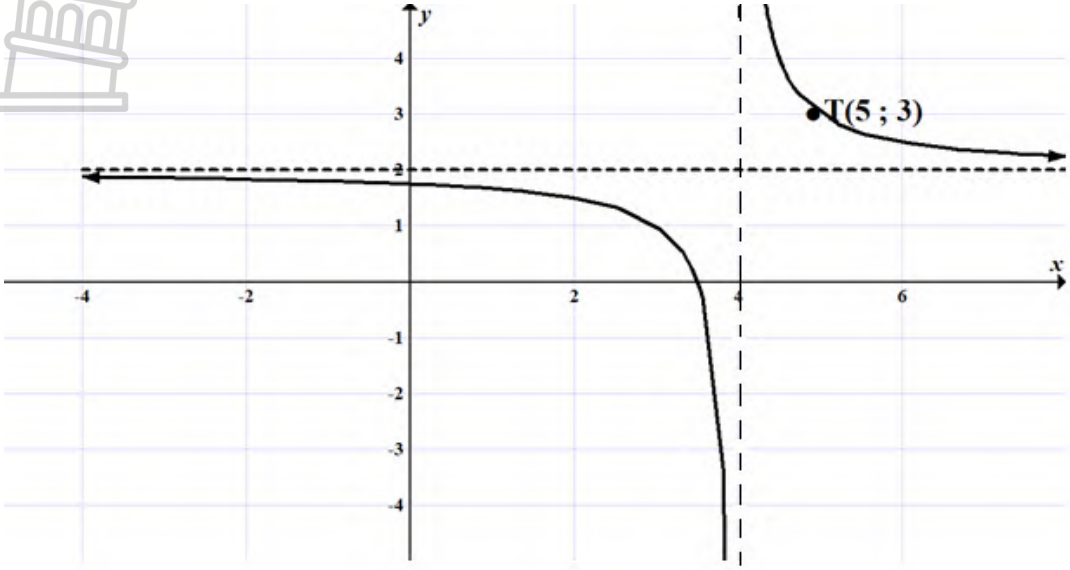
4.2.4 Write down the equation of the asymptote of $f^{-1}(x + 2)$. (2)

4.2.5 Prove that: $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$ for all values of x. (3)

[29]

QUESTION 5

The diagram below represents the graph of $f(x) = \frac{a}{x-p} + q$. $T(5;3)$ is a point on f.



5.1 Determine the values of a, p and q. (4)

5.2 If the graph of f is reflected across the line having equation $y=-x+c$ and the new graph coincides with the graph of $y=f(x)$, determine the value of c. (2)

[6]

QUESTION 6

6.1 Given: $f(x) = 3x - x^2$.

Use the definition (from first principles) of the derivative to calculate $f'(x)$. (4)

6.2 Determine $\frac{dy}{dx}$ if:

6.2.1 $y=\frac{x-3\sqrt{x}}{x^2}$ (4)

6.2.2 $\frac{y}{3x}=(1+x)^2$ (4)

6.3 A function h is given by $h(x)=ax^2+\frac{b}{x}$ and has a minimum value of 12 if $x=2$.

Calculate the values of a and b. (7)

[19]

QUESTION 7

The graph of the cubic function f has a turning point at $A(-1 ; p)$ and $B(2 ; q)$. The function f has the following properties:

$f'(x) > 0$ for $x < -1$ and $x > 2$

$f'(x) < 0$ for $-1 < x < 2$

$f(2) > 2$

7.1 Draw a neat sketch of f. Clearly label points A and B on the sketch. (It is NOT necessary to show x- and y-intercepts.) (4)

7.2 If $f(x) = x^3 + bx^2 + cx + d$, calculate the values of b and c. (6)

[10]

QUESTION 8

During an experiment, the temperature T (in degrees Celsius), varies with time t (in hours), according to the formula $T(t) = 30 + 4t - \frac{1}{2}t^2$, $t \in [0;10]$.

8.1 Determine an expression for the rate of change of temperature with time. (2)

8.2 During which time interval was the temperature decreasing? (4)

[6]

TOTAL: 120

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta\text{ABC} = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})}$$

P(A or B) = P(A) + P(B) – P(A and B)

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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EDUCATION
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JUNE EXAMINATION
JUNIE EKSAMEN
GRADE/*GRAAD* 12

2023

MARKING GUIDELINES/
NASIENRIGLYNE

MATHEMATICS/
WISKUNDE

(PAPER/*VRAESTEL* 1)

16 pages/*bladsye*

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and has not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aannames van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

QUESTION/*VRAAG* 1

1.1	$12x = x^2$		
	1.1.1 $12x = x^2$ $x^2 - 12x = 0$ $x(x - 12) = 0$ $x = 0$ or/*of* $x = 12$	✓ std form/*standaard vorm* ✓ factors/*faktore* ✓ both answers correct/ *beide antwoorde korrek*	(3)
	1.1.2 Let $p^2 - 1 = x$ $\therefore p^2 - 1 = 0$ or/*of* $p^2 - 1 = 12$ $p^2 = 1$ $p^2 = 13$ $p \pm 1$ $p^2 \pm \sqrt{13}$ **OR/*OF*** $p^4 - 2p^2 + 1 - 12p^2 + 12 = 0$ $p^4 - 14p^2 + 13 = 0$ $(p^2 - 1)(p^2 - 1) = 0$ $\therefore p = \pm 1$ or/*of* $p = \pm\sqrt{13}$	✓ subst./*vervang* $p^2 - 1 = x$ ✓ values for/*waardes van* p^2 ✓✓ roots/*wortels* ✓ expansion/*uitbreiding* ✓ factors/*faktore* ✓✓ roots/*wortels*	(4)
1.2	$5x^2 + 7x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(7) \pm \sqrt{(7)^2 - 4(5)(-2)}}{2(5)}$ $= \frac{-(7) \pm \sqrt{89}}{10}$ $x = 0{,}24$ or/*of* $x = 1{,}64$ **Minus 1 mark for incorrect rounding-off. If wrong formula used, Max. 2/4 for *x*-values (CA)/*Minus 1 punt vir verkeerde afronding. As verkeerde formule gebruik is, Maks 2/4 vir x-waardes (CA)***	✓subst. into correct formula/ *subt in die korrekte formule* ✓ simplification *vereenvoudiging* ✓✓ values of *x* *waardes van x*	(4)

1.3		$\sqrt{x+6}=x$ $\therefore x+6=x^2$ $x^2-x-6=0$ $(x+2)(x-3)=0$ $x=-2$ or/*of* $x=3$ *check* : $\sqrt{4}\neq -2$ or/*of* $\sqrt{3+6}=3$ (*No solution/geen oplossing nie*) $\therefore x=3$	✓ squaring both sides ✓ std form ✓ factors ✓ x-values ✓ answer **Answer only: Full marks** ***Antwoord alleen: Volpunte***	(5)
1.4		$x=y-5$ $3=y-5$ $\therefore y=8$	✓ relationship between x and y *verwantskap tussen x en y* ✓ resultant y-value/ *resulterende y-waarde*	(2)
1.5	1.5.1	$\frac{50}{x+31}$	✓ answer/*antwoord*	(1)
	1.5.2	$\frac{10}{x}+\frac{50}{x+31}=2$ $\therefore \frac{10}{x}(x)(x+31)+\frac{50}{x+31}(x)(x+31)=2(x)(x+31)$ $\therefore 10x+310+50x=2x^2+62x$ $\therefore 2x^2+2x-310=0$ $\therefore x^2+x-155=0$ $x=\frac{-1\pm\sqrt{1-4(1)(-155)}}{2(1)}$ $=\frac{-1\pm\sqrt{621}}{2}$ $x=11{,}96$ or/*of* $x=-12{,}96$ $\therefore x=11{,}96\ km/h$ **Minus 1 mark for negative speed** ***Minus 1 punt vir negatiewe spoed***	✓ setting up equation/*opstel van vergelyking* ✓✓ multiplying: LHS : RHS/ *vermenigvuldig LK : RK* ✓ std form/*standardvorm* ✓ subst. into correct formula/ *subst. in korrekte formule* ✓ answer/*antwoord*	(6)
				[25]

QUESTION/*VRAAG* 2

2.1	$S_n = k\left[1-\left(\frac{1}{2}\right)^n\right]$ $a = k\left[1-\left(\frac{1}{2}\right)^1\right]$ $= \frac{k}{2}$ $r = \frac{1}{2}$ $\therefore 10 = \frac{\frac{k}{2}}{1-\frac{1}{2}}$ $5 = \frac{k}{2}$ $\therefore k = 10$ **OR/*OF*** $S_n = \frac{a\left(1-r^n\right)}{1-r}$ $= \frac{a}{1-r}\left(1-r^n\right)$ $but\ \ S_\infty = \frac{a}{1-r} = 10$ $\Rightarrow S_n = 10\left(1-r^n\right)$ $and\ \ S_n = k\left[1-\left(\frac{1}{2}\right)^n\right]$ $\therefore k = 10$ and/*en* $r = \frac{1}{2}$ **OR/*OF***	$\left(\frac{1}{2}\right)^n \to 0\ \ as\ \ n \to \infty$ $\therefore S_n = k$ $\therefore k = 10$	✓ $a = \frac{k}{2}$ ✓ $r = \frac{1}{2}$ ✓ subst. into correct formula/ *subst in die korrekte formule* ✓ $k = 10$ **OR/*OF*** ✓ $\left(\frac{1}{2}\right)^n \to 0$ ✓ $n \to \infty$ ✓ $S_\infty = k$ ✓ $k = 10$ **OR/*OF*** ✓ $S_\infty = \frac{a}{1-r} = 10$ ✓ $S_n = k\left[1-\left(\frac{1}{2}\right)^n\right]$ ✓ $k = 10$ ✓ $r = \frac{1}{2}$ **OR/*OF***	(4) (4) (4)

	$S_1=\frac{k}{2};\ S_2=\frac{k}{4};\ S_3=\frac{k}{2}$ $S_\infty=\frac{a}{1-r}=10$ $a=\frac{k}{2}$ and/*en* $k=\frac{1}{2}$ $1=\frac{\frac{k}{2}}{\left(1-\frac{1}{2}\right)}$ $\therefore k=10$	✓ $S_\infty=\frac{a}{1-r}=10$ ✓ $a=\frac{k}{2}$ ✓ $r=\frac{1}{2}$ ✓ $k=10$	(4)
2.2	$r=\frac{1}{2}$ $\frac{a}{1-\frac{1}{2}}=10$ $a=5$ $T_2=ar=\frac{5}{2}$	✓ $r=\frac{1}{2}$ ✓ substitution/*substitusie* ✓ $a=5$ ✓ answer/*antwoord*	(4)
			[8]

QUESTION/*VRAAG* 3

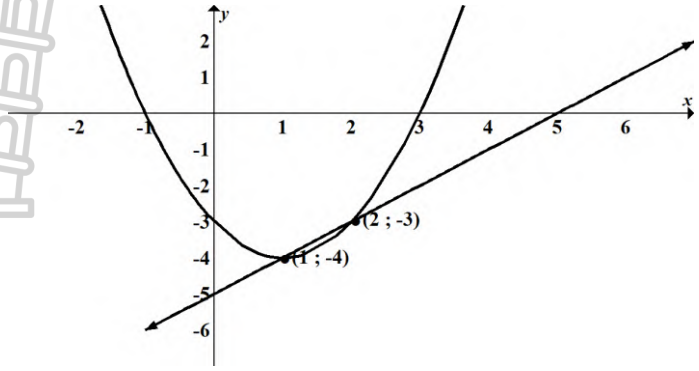
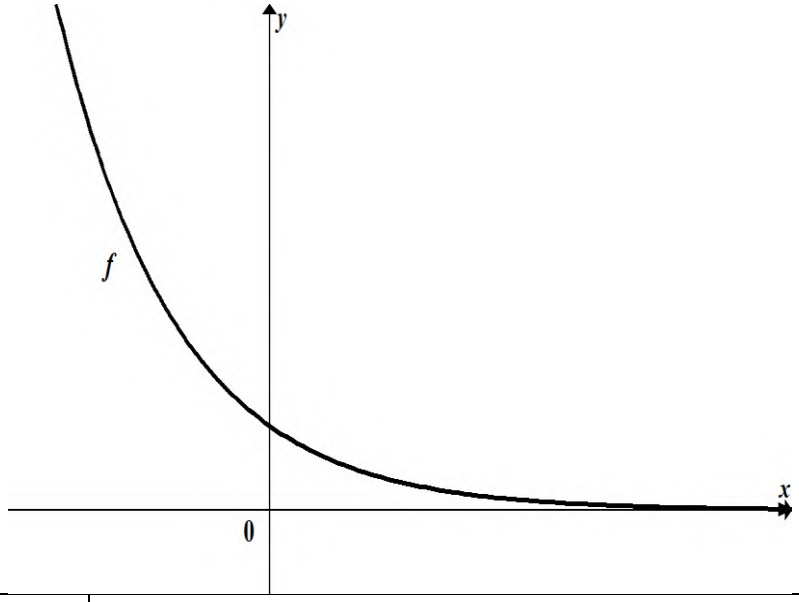
3.1	$S_n = a + (a+d) + (a+2d) + \ldots + a + (n-1)d$ (1) $S_n = a + (n-1)d + (a+(n-2)d) + (a+(n-3)d) + \ldots + a$ (2) $2S_n = n(2a + (n-1)d)$ $S_n = \frac{n}{2}(2a + (n-1)d)$	✓ first series/ *eerste reeks* ✓ series reversed/ *omgekeerde reeks* ✓ sum of series/ *som van die reeks* ✓ dividing by 2/ *deel deur 2*	(4)
3.2	$\sum_{p=1}^{50}(100-3p)$ $\sum_{p=1}^{50}(100-3p) = 97 + 94 + 91 + \ldots$ $T_1 = a = 97$ $d = -3$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{50}{2}[2(97) + (50-1)(-3)]$ $= 1175$ **OR/*OF*** $\sum_{p=1}^{50}(100-3p) = 97 + 94 + 91 + \ldots$ $T_1 = a = 97$ $l = 100 - 3(50) = -50$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[a + l]$ $S_n = \frac{50}{2}[97 - 50]$ $= 1175$	✓ a = 97 ✓ d= -3 ✓ n=50 ✓ answer/*antwoord* **OR/*OF*** ✓ a = 97 ✓ l = –50 ✓ n = 50 ✓ answer/*antwoord*	(4) (4)

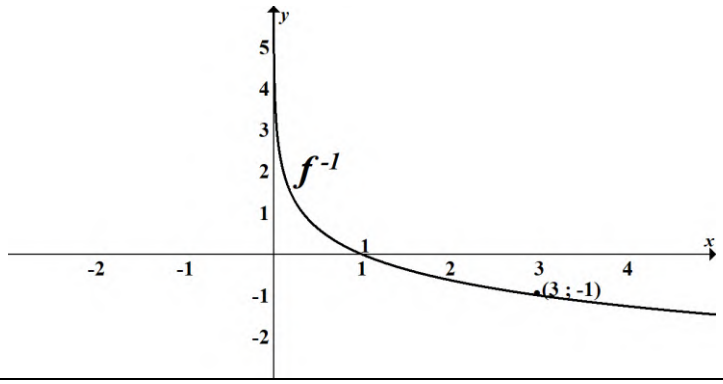
3.3	3.3.1	(a)	$T_5 - T_4 = 25$	✓ answer (1)
		(b)	$T_{70} - T_{69} = 7 + (69-1)6$ $= 415$	✓ subst. into correct formula/*subt. in die korrekte formule* ✓ $n = 69$ ✓ answer/*antwoord* (3)
	3.3.2		$T_{89} - T_{69} = (T_{70} - T_{69}) + (T_{71} - T_{70}) + \ldots + (T_{89} - T_{88})$ = 415 + 421 + … to 20 terms/*tot 20 terme* $= \frac{20}{2}[2(415) + 19(6)]$ $= 9440$ ∴ T_{69} = T_{89} – (sum of the differences from/*som van die verskille van* T_{69} to T_{89}) $\therefore T_{69} = 23594 - 9440$ $= 14154$ **OR/*OF*** $T_{n+1} - T_n = 7 + 6(n-1)$ $\therefore T_{89} - T_1 = \sum_{n=1}^{88} (T_{n+1} - T_n)$ $= \frac{n}{2}[2a + (n-1)d]$ $= \frac{88}{2}[14 + (87)6]$ $= 23584$ $\therefore T_1 = 23594 - 23584 = 10$ $\therefore T_{69} - 10 = \sum_{n=1}^{68} (T_{n+1} - T_n)$ $= 34(15 + (67)6)$ $= 14144$ $\therefore T_{69} = 14154$ **OR/*OF***	✓ expansion/ *uitbreiding* ✓ $n = 20$ ✓ $a = 415$ ✓ method/*metode* ✓ answer/*antwoord* (5) **OR/*OF*** ✓ formula/*formule* ✓ value of S_{88} / *waarde van* S_{88} ✓ $T_1 = 23594 - 23584 = 10$ **OR/*OF***

	$7 \quad 13 \quad 19 \quad 25$ $\quad 6 \quad 6 \quad 6$ $\therefore 2a = 6$ $\therefore a = 3$	✓ substitution/*substitusie* ✓ answer/*antwoord* (5)
	$7 - 6 = 1$ $T_1 - T_0 = 1$ $a + b + c - c = 1$ $3 + b = 1$ $b = -2$	OR/***OF*** ✓ a and/*en* b
	$T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $T_{69} = 3(69)^2 - 2(69) + 9$ $= 14154$	✓ T_{89} (subst. n = 89) ✓ $T_n = 3n^2 - 2n + 9$ ✓ T_{89} (subst./*vervang* n = 69) ✓ answer/*antwoord* (5)
	OR/*OF*	**OR/*OF***
	$7 \quad 13 \quad 19 \quad 25$ $\quad 6 \quad 6 \quad 6$ $\therefore 2a = 6$ $\therefore a = 3$	✓ substitution/*substitusie* ✓ answer/*antwoord*
	$3a + b = 7$ $b = -2$	✓ a and/*en* b
	$T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $T_{69} = 3(69)^2 - 2(69) + 9$ $= 14154$	✓ T_{89} (subst./*vervang* n = 89) ✓ $T_n = 3n^2 - 2n + 9$ ✓ T_{69} (subst./*vervang* n = 69) ✓ answer/*antwoord* (5)
		[17]

QUESTION/*VRAAG* 4

4.1	Given/*Gegee*: $f(x)=x^2-2x-3$ and/*en* $g(x)=x-5$		
	4.1.1 $f(x)=x^2-2x-3$ $x=\frac{-b}{2a}$ and/*en* $y=\frac{4ac-b^2}{4a}$ $x=\frac{-(-2)}{2(1)}$ $y=\frac{4(1)(-3)-(-2)^2}{4(1)}$ $x=1$ $y=-4$ $\therefore TP\ is\ (1;-4)$ **OR/*OF*** $f'(x)=0$ $f(1)=(1)^2-2(1)-3$ $f'(x)=2x-2$ $=1-2-3$ $\therefore 2x-2=0$ $f(1)=-4$ $x=1$ $\therefore TP\ is\ (1;-4)$	✓ formula (axis of symmetry)/*Formule (simmetrie-as)* ✓ substitution/*substitusie* ✓ subst. into *y*-formula/ *substitusie in y-formule* **OR/*OF*** ✓ $f'(x)=0$ ✓ derivative/*afgeleide* ✓ subst. *x*-values	(3)
	4.1.2 $f(x)=x^2-2x-3$ $(x+1)(x-3)=0$ $y=(0)^2-2(0)-3=-3$ $x=-1$ and/*en* $x=3$ $\therefore (0);-3) \rightarrow$ coordinate of the *y*-intercept/ *koördinate van die y-afsnit* are the coordinates of the *x*-intercept/*is die koördinate van die x-afsnit* $\therefore$ (–1 : 0) and/*en* (3 ; 0)	✓ factors/*faktore* ✓ both *x*-values/*beide x waardes* ✓ *y*-value/*y-waarde*	(3)
	4.1.3 $x-5=x^2-2x-3$ $x^2-3x+2=0$ $(x-1)(x-2)=0$ $x=1$ or/*of* $x=2$ $y=-4$ or/*of* $y=-3$ $\therefore (1;-4)$ or/*of* $(2;-3)$ are the points of intersection/*is die snypunte*	✓ std form/*standaard vorm* ✓ factors/*faktore* ✓ both *x*-values/*beide x-waardes* ✓ both *y*-values/*beide y-waardes*	(4)

	4.1.4		$f(x)=x^2-2x-3$: ✓ *y*-intercept/*afsnit* ✓ *x*-intercept/*afsnit* ✓ turning point/*draaipunt* ✓ $(2;-3)$ on the graph/ *op die grafiek* $g(x)=x-5$: ✓ *y*-intercept/*afsnit* ✓ *x*-intercept/*afsnit*	(6)
	4.1.5	$x \leq -1$ or $x \geq 3$	✓ roots/*wortel* ✓ inequality signs/*ongelykheidsteken*	(2)
4.2				
	4.2.1	$y = 0$	✓ answer/*antwoord*	(1)

	4.2.2	$x=\left(\frac{1}{3}\right)^y$ $y=\log_{\frac{1}{3}} x$ **OR/*OF*** $x=\left(\frac{1}{3}\right)^y \qquad -y=\log_3 x$ $x=3^{-y} \qquad y=-\log_3 x$	✓ swopping *x*- and *y*-values *ruil die x- en y-waardes* ✓ answer/*antwoord* **OR/*OF*** ✓ swopping *x*- and *y*-values *ruil die x- en y-waardes* ✓ answer/*antwoord*	(2) (2)
	4.2.3		✓ shape/*vorm* ✓ *x*-intercept/*x-afsnit* ✓ any other point on the graph/*enige ander punt op die grafiek*	(3)
	4.2.4	$x=-2$	✓✓ answer/*antwoord*	(2)
	4.2.5	$LHS=\left[f(x)\right]^2-\left[f(-x)\right]^2$ $=\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$ $=3^{-2x}-3^{2x}$ $RHS=f(2x)-f(-2x)$ $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ $=3^{-2x}-3^{2x}$ $\therefore LHS=RHS$ $\left[f(x)\right]^2-\left[f(-x)\right]^2=f(2x)-f(-2x)$	✓ subst. into LHS/ *subt. in die LK* ✓ subst. into RHS/ *subt. in die RK* ✓ $3^{-2x}-3^{2x}$	(3)
				[29]

QUESTION/*VRAAG* 5

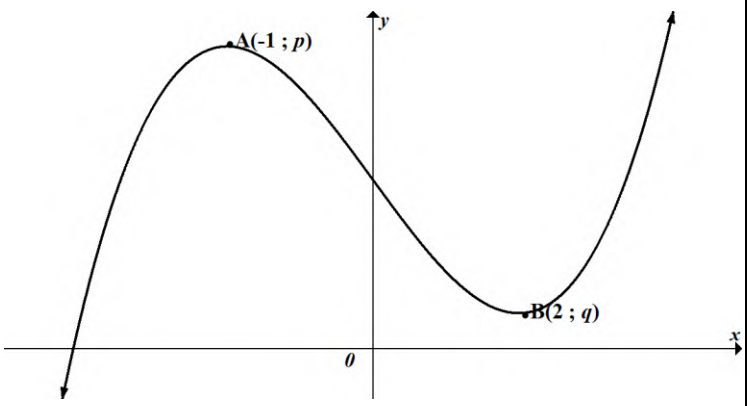
	$f(x)=\frac{a}{x-p}+q$		
5.1	$p=4$ $q=2$ $3=\frac{a}{5-4}+2$ $1=\frac{a}{1}$ $\therefore a=1$	✓ $p=4$ ✓ $q=2$ ✓ subst. of/*van* $T(5;3)$ ✓ $a=1$	(4)
5.2	$y=-x+2$ substitute/*vervang* (4 ; 2) $2=-4+c$ $c=6$ **OR/*OF*** Translation of the line $y=-x$, 2 units up and 4 units to the right./*Vertaling van die lyn y = –x, 2 eenhede op en 4 eenhede na regs* $y=-(x-4)+2$ $y=-x+6$ $\therefore c=6$	✓ subst. of/*van* $T(4;2)$ ✓ $c=6$ **OR/*OF*** ✓ subst. of/*van* $x-4$ ✓ answer/*antwoord*	(2) (2)
			[6]

QUESTION/*VRAAG* 6

6.1	$3x-x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h},\ h\neq 0$ $=\lim_{h\to 0}\frac{3(x+h)-(x+h)^2-(3x-x^2)}{h}$ $=\lim_{h\to 0}\frac{3x+3h-x^2-2xh-h^2-3x+x^2}{h}$ $=\lim_{h\to 0}\frac{3h-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(3-2x-h)}{h}$ $=\lim_{h\to 0}(3-2x-h)$ $f'(x)=3-2x$ **Minus 1 mark for incorrect notation/*Minus 1 punt vir verkeerde notasie***	✓ substitution into correct formula/*substitusie in die korrekte formule* ✓ simplification/*vereenvoudiging* ✓ factorization/*faktorisering* ✓ answer/*antwoord*	(4)
6.2	6.2.1 $y=\frac{x-3\sqrt{x}}{x^2}$ $y=\frac{x-3x^{\frac{1}{2}}}{x^2}$ $y=x^{-1}-3x^{-\frac{3}{2}}$ $\frac{dy}{dx}=-x^{-2}+\frac{9}{2}x^{-\frac{5}{2}}$	✓ x^{-1} ✓ $-3x^{-\frac{3}{2}}$ ✓ $-x^{-2}$ ✓ $\frac{9}{2}x^{-\frac{5}{2}}$	(4)
	6.2.2 $\frac{y}{3x}=(1+x)^2$ $\frac{y}{3x}=1+2x+x^2$ $y=3x+6x^2+3x^3$ $\frac{dy}{dx}=3+12x+9x^2$	✓ simplification/*vereenvoudig* ✓ 3 ✓ $12x$ ✓ $9x^2$	(4)

6.3	$h(x) = ax^2 + \frac{b}{x}$ subst./*vervang* $(2\,;12) : 12 = a(2)^2 + \frac{b}{2}$ $\therefore\ 24 = 8a + b$ (1) Minimum if/*as: h'(x)* = 0 $\therefore\ 2ax - \frac{b}{x^2} = 0$ but/*maar* $x = 2$ $\quad\therefore\ 4a - \frac{b}{4} = 0$ $\therefore$ 16a – b = 0 (2) (1) + (2) : 24*a* = 24 Subst. into/*vervang in* (1) : 8 + b = 24 *b* = 16	✓ setting up equation/*opstel van vergelyking* ✓ eqn (1) ✓ $h'(x) = 0$ ✓ $2ax - \frac{b}{x^2} = 0$ ✓ $4a - \frac{b}{2} = 0$ ✓ $a = 1$ ✓ $b = 16$	(7)
			[19]

QUESTION/*VRAAG* 7

7.1		✓ Shape (increasing to the left of A and the right of B./*Vorm (neem toe links van A en regs van B)* ✓ A as Max. Tp and B as Min.Tp/*A as Maks DP en B Min DP* ✓ Decreasing between A and B/*Neem af tussen A en B* ✓ B above *x*-axis/*B bo x-as* **Minus 1 mark if turning points A and B not labelled/*Minus 1 punt as draaipunte A en B nie benoem is nie***	(4)

7.2	$f(x) = x^3 + bx^2 + cx + d$ $f'(x) = 3x^2 + 2bx + c$ $f'(-1) = 3(-1)^2 + 2b(-1) + c = 0$ $\therefore\ 3 - 2b + c = 0$ (1) $f'(2) = 3(2)^2 + 2b(2) + c = 0$ $\therefore\ 12 + 4b + c = 0$ (2) $(1) - (2) : -9 - 6b = 0$ $\therefore\ b = -\frac{3}{2}$ $\therefore\ 3 + 3 + c = 0$ $\therefore\ c = -6$	✓ derivative method and accuracy/*afgeleide metode en akkuraatheid* ✓ $f'(x) = 0$ at/*by* -1 and/*en* 2 ✓✓ deriving each equation/*afgeleide van elke vergelyking* ✓ value of *b*/*waarde van b* ✓ value of *c*/*waarde van c*	(6)
			[10]

QUESTION/*VRAAG* 8

8.1	$T(t) = 30 + 4t - \frac{1}{2}t^2,\ t \in [0;10]$. $T'(t) = 4 - t$	✓ 4 ✓ $-t$	(2)
8.2	$T'(t) \leq 0$ $4 - t \leq 0$ $t \geq 4$ $\therefore 4 \leq t \leq 10$	✓ $T'(t) \leq 0$ ✓ $4 - t \leq 0$ ✓ *t*-subject of formula/*t die onderwerp van formule* ✓ answer/*antwoord* **accept/*aanvaar*** $4 < t \leq 10$	(4)
			[6]
		TOTAL/*TOTAAL*:	**120**