GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2023

MATHEMATICS

(PAPER 2)

TIME: **3 hours**

MARKS: **130**

11 pages, 1 information sheet and an answer book of 23 pages

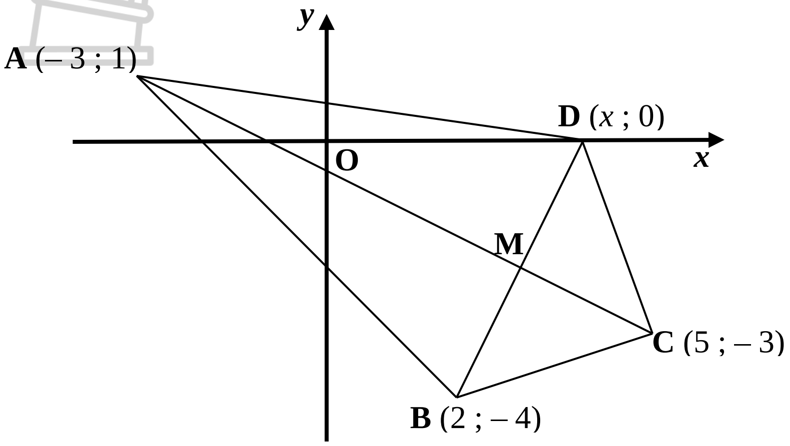
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

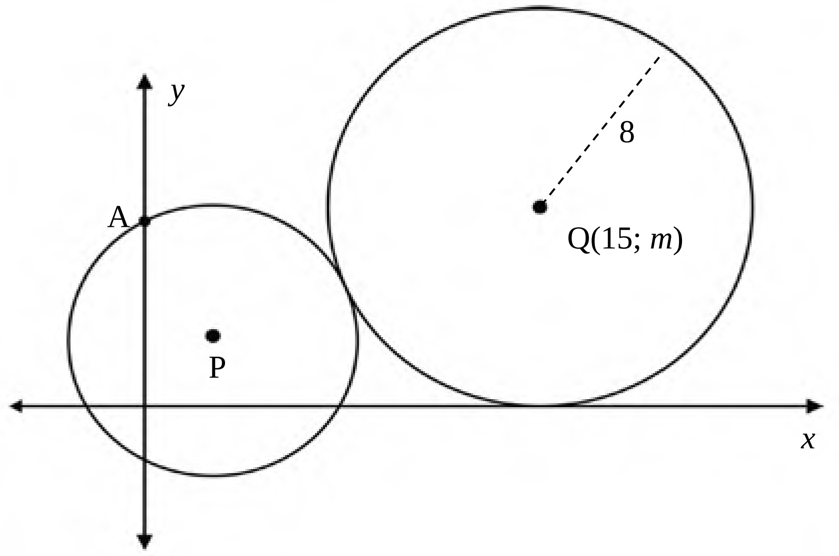
ABCD is a quadrilateral on the Cartesian plane with vertices, A(–3; 1), B(2 ; –4), C(5 ; –3) and D(x ; 0).



1.1	Determine the gradient of AC.	(2)
1.2	Calculate the equation of AC.	(3)
1.3	Calculate the angle of inclination of AC.	(3)
1.4	Determine the coordinates of D if AC is the perpendicular bisector of DB.	(5)
1.5	Hence, deduce that ABCD is a kite.	(3)
1.6	Calculate the area of ΔABD.	(5)
		[21]

QUESTION 2

2.1 In the diagram below, Q(15; m) is the centre of the larger circle which touches both the x-axis and the circle centred at P. The circle with centre P, has the y-intercept at A and has equation $(x-3)^2 + (y-3)^2 = 25$. The radius of the circle, centre Q, is 8 units.



2.1.1 Determine the equation of the circle with centre Q in terms of x; y and m. (1)

2.1.2 Determine the value of m. (2)

2.1.3 Determine the length of PQ. (2)

2.1.4 Calculate the coordinates of A. (3)

2.1.5 Determine the equation of the tangent to the circle with centre P, at A. (4)

2.2 Two other circles are given:

- One has centre K, and equation $x^2-6x+y^2+4y-12=0$.
- The other has centre T, and equation $(x-12)^2+(y-10)^2=100$

2.2.1 Determine the centre and radius of the circle with centre K. (4)

2.2.2 Calculate the distance between the centres, K and T. (2)

2.2.3 At how many points do the two circles intersect? Motivate your answer. (2)

[20]

QUESTION 3

3.1 Simplify **without the use of a calculator**:

$$\sin^2(360^\circ - x) - \frac{\cos(x - 180^0).\tan(-x).\sin(90^o - x).\cos(360^o + x)}{\sin(180^o + x)}$$ (8)

3.2 If $\beta \in [-90^0; 270^0]$, determine β **without the use of a calculator**,

$$\cos(\beta + 80^0) = \frac{\sin(-300^0)\cos 45^0}{\cos 405^0}$$ (7)

3.3 Determine the value of the following **without the use of a calculator**:

3.3.1 $\dfrac{\sin 49^\circ}{\cos 41^\circ}$ (2)

3.3.2 $\sin 85^\circ \cos 65^\circ + \cos 85^\circ \sin 65^\circ$ (3)

3.3.3 $\dfrac{1}{2}(\cos 15^0 + \sqrt{3}\sin 15^0)$ (4)

[24]

QUESTION 4

4.1 Determine the general solution of the following equation:
$\cos(54^\circ - x) = \sin 2x$ (6)

4.2 If $13\sin x + 5 = 0$ and $x \in [0^\circ; 270^\circ]$, determine **without the use of a calculator**, the value of $\sin 2x$. (5)

4.3 Prove the following identity: $\dfrac{1+\sin 2x}{\cos 2x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$ (7)

4.4 If $\sin 39^\circ = p$, determine the following in terms of p:

4.4.1 $\sin 129^\circ$ (3)

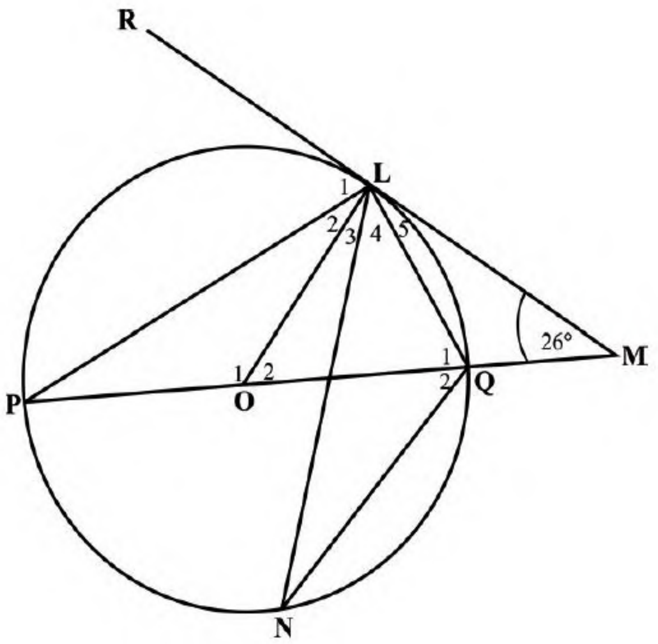
4.4.2 $\tan 321^\circ$ (2)

4.4.3 $\sin 78^\circ$ (2)

[25]

QUESTION 5

5.1 In the diagram below, O is the centre of the circle and L is a point on the circumference. The diameter, PQ makes an angle of 26° with the tangent RLM. N is a point on the lower part of the circle.



Determine, with reasons, the size of:

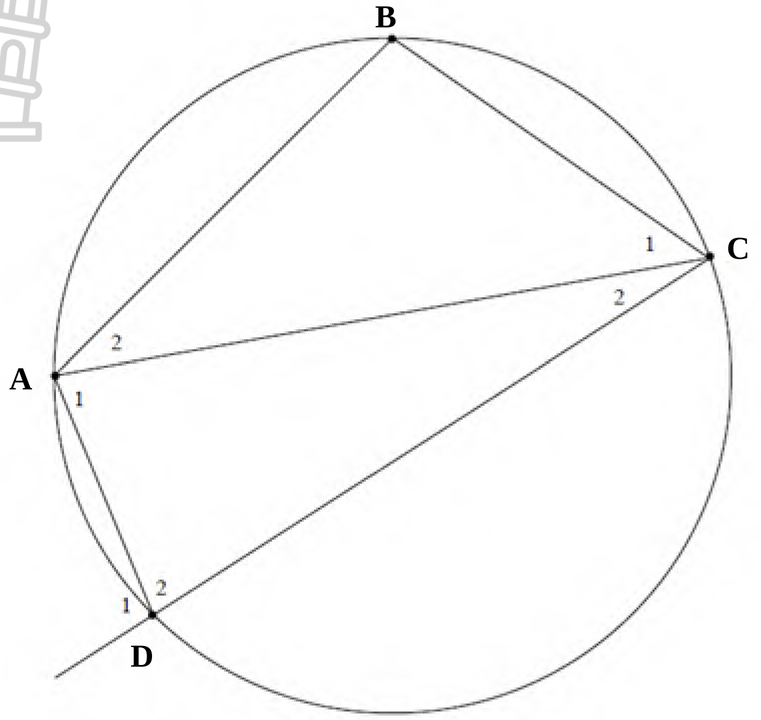
5.1.1 $O\hat{L}M$ (2)

5.1.2 $\hat{O}_2$ (1)

5.1.3 $\hat{P}$ (2)

5.1.4 Q_1 (3)

5.2 In the diagram below, points A, B, C and D lie on the circumference of a circle.

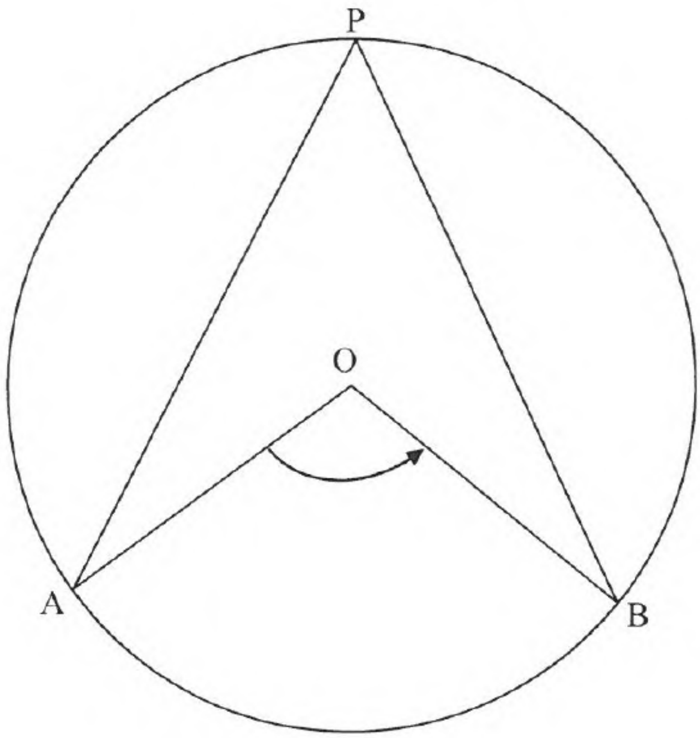


Prove that $\hat{C}_2 = \hat{B} - \hat{A}_1$ (5)

[13]

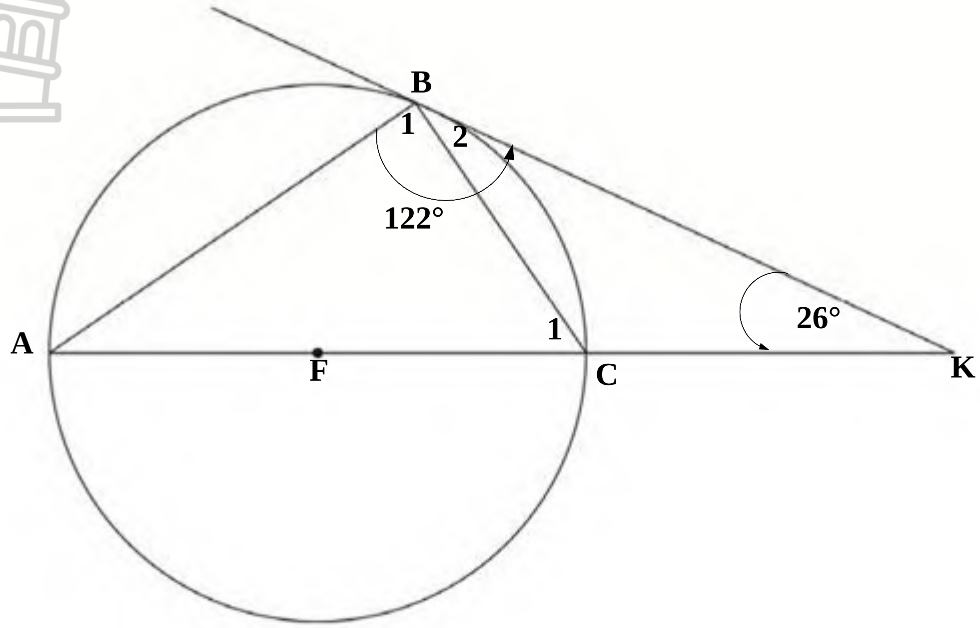
QUESTION 6

6.1 In the diagram below, O is the centre of the circle and A, P and B are points on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem which states that $A\hat{O}B = 2A\hat{P}B$. (5)

6.2 In the diagram below, F is the centre of the circle and AC is a diameter. AC is produced to K. $\hat{B}_1 + \hat{B}_2 = 122°$, and $\hat{K} = 26°$.



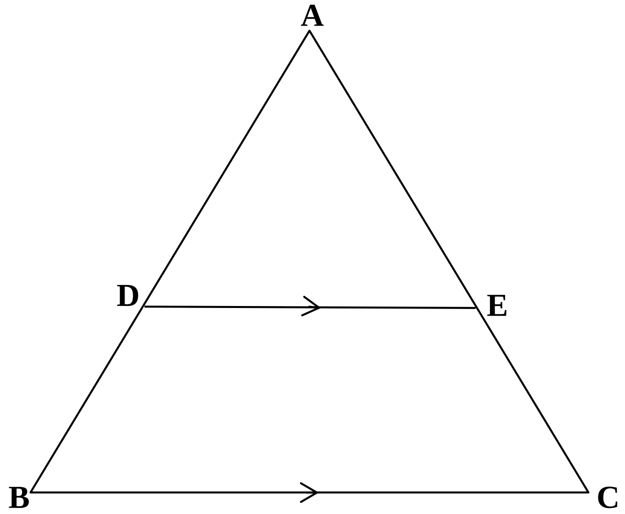
Show that BK is a tangent to the circle at B. (5)

[10]

QUESTION 7

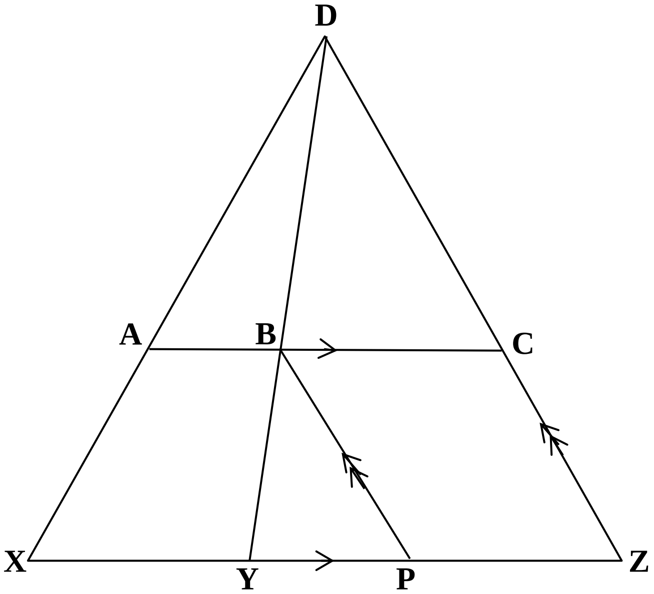
7.1 In ΔABC below, D and E are points on AB and AC respectively such that DE || BC.

Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

7.2 In ΔDXZ below, AC || XZ and BP || DZ. DY is drawn to intersect AC at B.

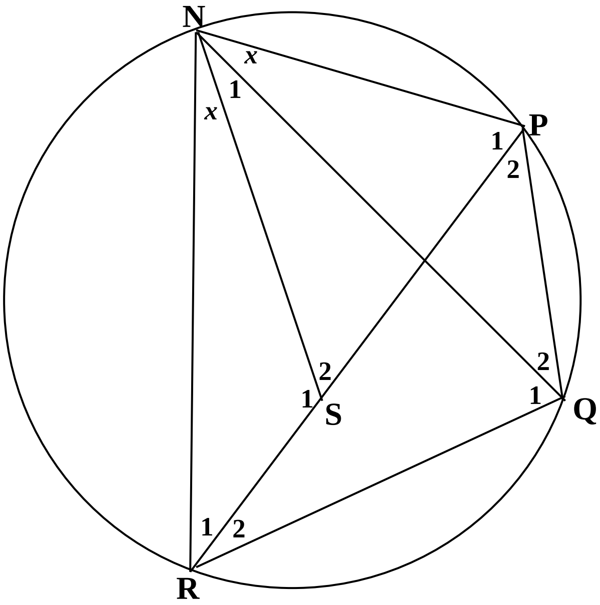


Prove that: $\frac{BC}{YZ} = \frac{DA}{DX}$

(5)
[11]

QUESTION 8

In the diagram below, NPQR is a cyclic quadrilateral with S a point on chord PR. N and S are joined and $R\hat{N}S = P\hat{N}Q = x$.



Prove that:

8.1 $\Delta NSR \,|||\, \Delta NPQ$ (3)

8.2 $\Delta NQR \,|||\, \Delta NPS$ (3)

[6]

TOTAL: 130

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab \sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ *P(A or B) = P(A) + P(B) – P(A and B)*

$\hat{y} = a + bx$

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION/ *JUNIE EKSAMEN*
GRADE/*GRAAD* 12

2023

MATHEMATICS/*WISKUNDE*

(PAPER/*VRAESTEL* 2)

ANSWER BOOK/*ANTWOORDBOEK*

23 pages/*bladsye*

LEARNER'S NAME / *LEERDER SE NAAM*	

CLASS/*KLAS*	

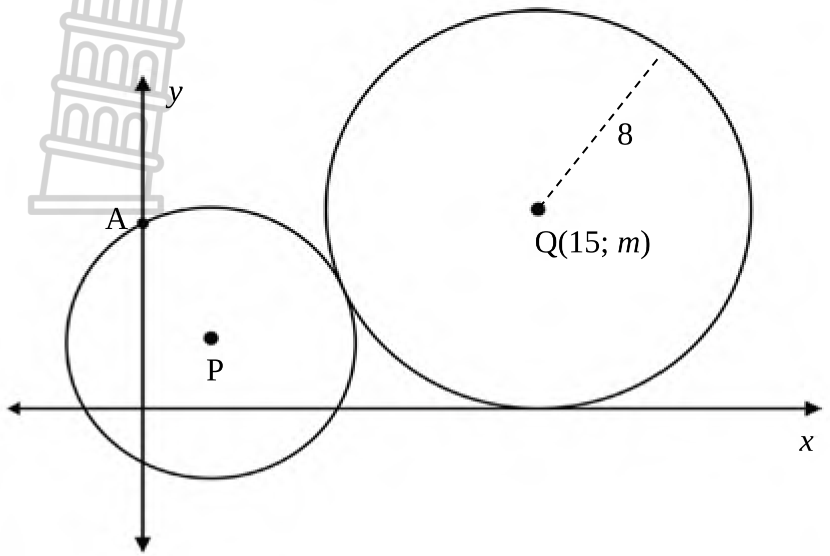
QUESTION/*VRAAG*	MARKS/*PUNTE*
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL/*TOTAAL*: 130	

QUESTION/*VRAAG* 1

	Solution/*Oplossing*	**Marks/ *Punte***
1.1		(2)
1.2		(3)
1.3		(3)
1.4		(5)

	Solution/*Oplossing*	**Marks/ *Punte***
1.5		(3)
1.6		(5)
		[21]

QUESTION/*VRAAG* 2



	Solution/*Oplossing*	**Marks/ *Punte***
2.1.1		
		(1)
2.1.2		
		(2)
2.1.3		
		(2)

	Solution/*Oplossing*	**Marks/ *Punte***
2.1.4		(3)
2.1.5		(4)
2.2.1		(4)
2.2.2		(2)

2.2.3		
		(2)
		[20]

QUESTION/*VRAAG* 3

	Solution/*Oplossing*	**Marks/ *Punte***
3.1		
		(8)

	Solution/*Oplossing*	**Marks/ *Punte***
3.2		
		(7)
3.3.1		
		(2)

3.3.2		
		(3)
3.3.3		
		(4)
		[24]

QUESTION/*VRAAG* 4

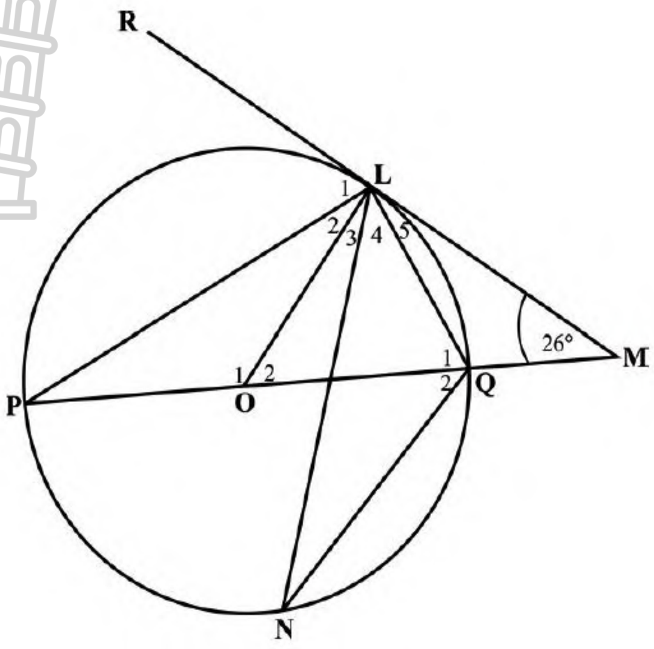
	Solution/*Oplossing*	**Marks/ *Punte***
4.1		
		(6)

	Solution/*Oplossing*	Marks/ *Punte*
4.2		(5)
4.3		(7)

	Solution/*Oplossing*	Marks/ *Punte*
4.4.1		(3)
4.4.2		(2)

4.4.3		
		(2)
		[25]

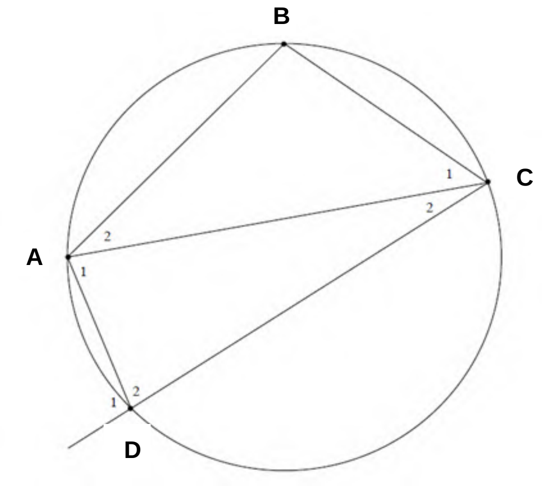
QUESTION/*VRAAG* 5



	Solution/*Oplossing*	Marks/ *Punte*
5.1.1		(2)
5.1.2		(1)
5.1.3		(2)

	Solution/*Oplossing*	**Marks/ *Punte***
5.1.4		
		(3)

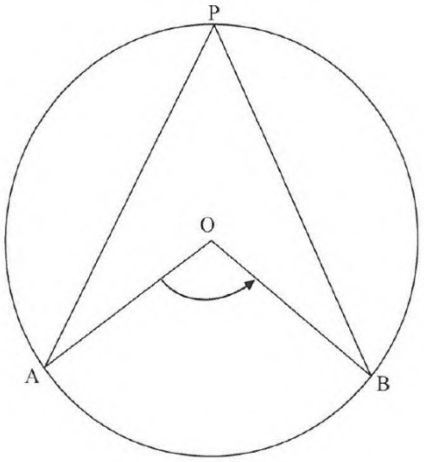
5.2



	Solution/*Oplossing*	Marks/ *Punte*
		(5)
		[13]

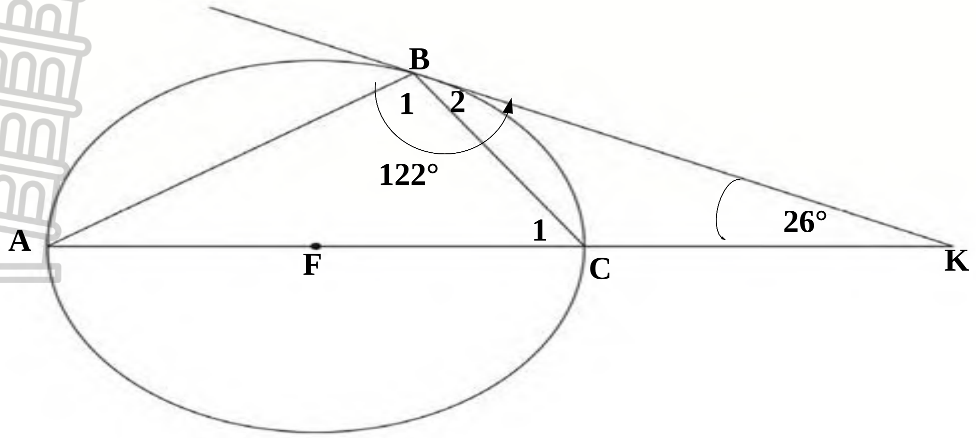
QUESTION/*VRAAG* 6

6.1



	Solution/*Oplossing*	**Marks/ *Punte***
6.1		
		(5)

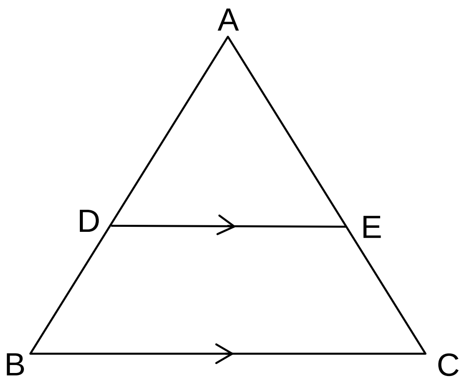
6.2



	Solution/*Oplossing*	Marks/ *Punte*
6.2		
		(5)
		[10]

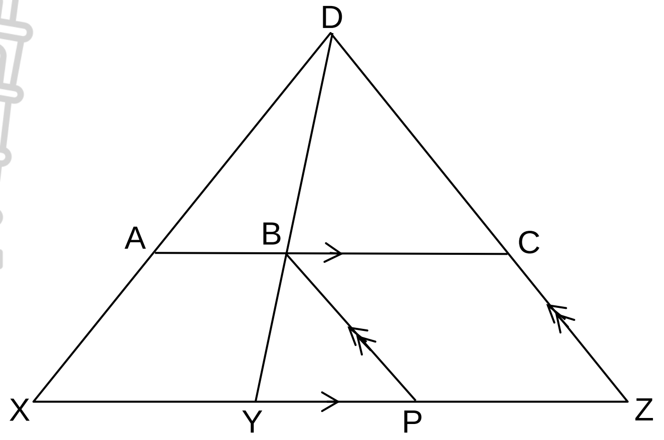
QUESTION/*VRAAG* 7

7.1



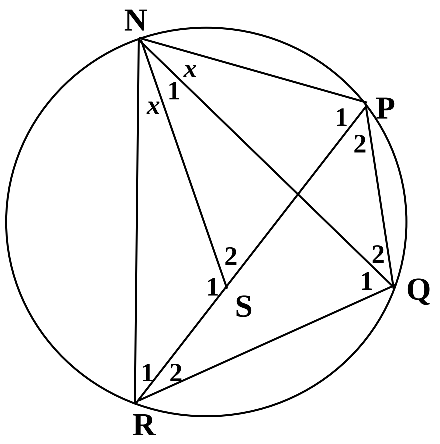
	Solution/*Oplossing*	**Marks/ *Punte***
7.1		(6)

7.2



	Solution/*Oplossing*	**Marks/ *Punte***
7.2		(5)
		[11]

QUESTION/*VRAAG* 8



	Solution/*Oplossing*	**Marks/*Punte***
8.1		(3)

	Solution/*Oplossing*	**Marks/** *Punte*
8.2		(3)
		[6]

TOTAL/*TOTAAL*: 130

	Additional space/*Bykomende Ruimte*	**Marks/ *Punte***

	Additional space/*Bykomende Ruimte*	**Marks/ *Punte***

	Additional space/*Bykomende Ruimte*	**Marks/ *Punte***

JUNE EXAMINATION
JUNIE EKSAMEN
GRADE/*GRAAD* 12

2023

MARKING GUIDELINES/
NASIENRIGLYNE

MATHEMATICS/
WISKUNDE

(PAPER/*VRAESTEL* 2)

17 pages/*bladsye*

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aannames van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

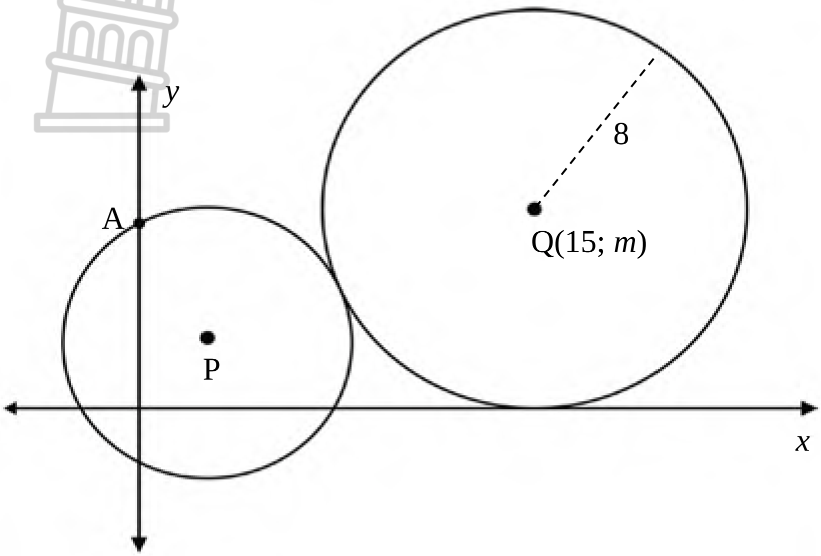
GEOMETRY/*MEETKUNDE*	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	'n Punt vir 'n korrekte bewering *('n Punt vir 'n bewering is onafhanklik van die rede)*
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	'n Punt vir 'n korrekte rede *('n Punt word slegs vir die rede toegeken as die bewering korrek is.)*
S/R	Award a mark if the statement AND reason are both correct.
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/*VRAAG* 1

1.1	$m_{AC} = \frac{-3-1}{5+3} = -\frac{1}{2}$	✓ substitution/*substitusie* ✓ answer/*antwoord*	(2)
1.2	The equation of AC will be/*Die vergelyking AC sal wees*: $y-1 = -\frac{1}{2}(x+3)$ $y = -\frac{1}{2}x - \frac{1}{2}$	✓ substitution of gradient/ *substitusie van gradiënt* ✓ substitution of point/ *substitusie van punt* ✓ answer/*antwoord*	(3)
1.3	$\tan\theta = -\frac{1}{2}$ $\theta = 153{,}43°$	✓ tan… ✓ gradient/*gradiënt* ✓ answer/*antwoord*	(3)
1.4	$m_{BD} = \frac{4}{x-2}$ $m_{AC} = \frac{-4}{8} = -\frac{1}{2}$ BD ⊥ AC given/*is gegee* i.e./*dit is* $\left(\frac{4}{x-2}\right)\times\left(-\frac{1}{2}\right) = -1$ $\frac{4}{x-2} = 2$ $4 = 2x - 4$ $x = 4$ D(4 ; 0)	✓ gradient of BD/ *gradiënt van BD* ✓ gradient of AC/ *gradiënt van AC* ✓ product of gradients = –1/ *produk van gradiënte = –1* ✓ value of *x* /*waarde van x* ✓ coordinates of D/ *koördinate van D*	(5)

1.5	$AD = \sqrt{7^2 + 1^2} = \sqrt{50}$ $AB = \sqrt{(-5)^2 + 5^2} = \sqrt{50}$ $DC = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$ $BC = \sqrt{3^2 + 1^2} = \sqrt{10}$ Two pairs of adjacent sides are equal $\therefore$ ABCD is a kite *Twee paar aangrensende sye is gelyk* $\therefore$ ABCD is 'n vlieër **OR/*OF*** AC $\perp$ BD given/*gegee* DM = MB given/*gegee* One diagonal is bisecting the other at 90°/*Een hoeklyn sny die ander loodrag* $\therefore$ ABCD is a kite/*ABCD is 'n vlieër*	✓ any 2 distances/ *enige 2 afstande* ✓ remaining 2/*oorblywende 2* ✓ reason/*rede* **OR/*OF*** ✓ statement/*bewering* ✓ statement/*bewering* ✓ reason/*rede*	(3)
1.6	$BD = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$ M(3 ; –2) $AM = \sqrt{(-6)^2 + (3)^2} = \sqrt{45} = 3\sqrt{5}$ $\therefore$ Area/Opp. Δ ABD = $\frac{1}{2}\text{base} \times \text{height}$/ $\frac{1}{2}\textit{basis} \times \textit{hoogte}$ $= \frac{1}{2} \times (2\sqrt{5}) \times (3\sqrt{5})$ = 15 sq. units/*vierkante eenhede*	✓ BD ✓ coordinates of M/ *koördinate van M* ✓ AM ✓ substitution into area formula/*substitusie in oppervlak formule* ✓ answer/*antwoord*	(5)
			[21]

QUESTION/*VRAAG* 2



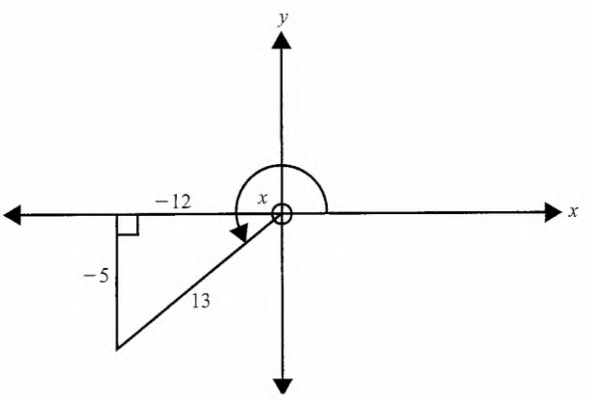
2.1	2.1.1	$(x-15)^2+(y-m)^2=64$	✓answer/*antwoord*	(1)
	2.1.2	$(15;0)$ lies on circle Q/*lê op sirkel* Q $(15-15)^2+(0-m)^2=64$ $m=8$ **OR/ *OF*** $m=y_Q=8$	✓ substitution/ *substitusie* ✓ answer/*antwoord* **OR/*OF*** ✓✓ answer only/ *slegs antwoord*	(2)
	2.1.3	PQ = 8 + 5 = 13 **OR/*OF*** $PQ=\sqrt{(3-8)^2+(3-15)^2}$ = 13 units	✓substitution/ addition/ *substitusie/som van* ✓ answer/*antwoord*	(2)
	2.1.4	The coordinates of/*Die koördinate van* A(0;y) $(0-3)^2+(y-3)^2=25$ $y=7$ A(0;7) **OR/ *OF*** $x_A=0$ $y_A=3+\sqrt{5^2-3^2}=7$ $A(0;7)$	✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* **OR/*OF*** ✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde*	(3)

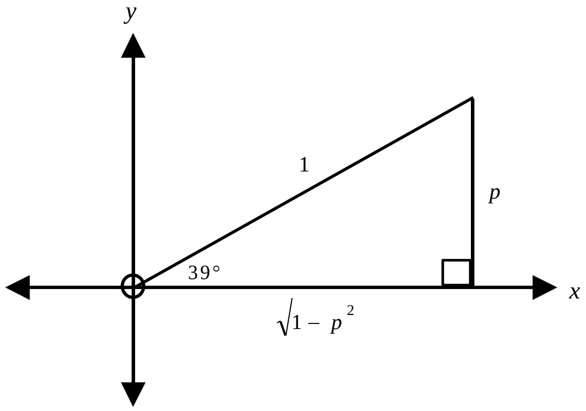
	2.1.5	$m_{AP} = \frac{3-7}{3-0} = \frac{-4}{3}$ $m_{rad} \times m_{\tan} = -1$ $m_{\tan} = \dfrac{3}{4}$ $y = \dfrac{3}{4}x + 7$	✓ m_{AP} ✓ $m_{\tan}$ ✓ product of gradients/*voltooiing van vierkant* ✓ equation/*vergelyking*	(4)
2.2	2.2.1	$(x-3)^2 + (y+2)^2 = 12 + 9 + 4$ $(x-3)^2 + (y+2)^2 = 25$ Centre/*middelpunt* is (3; –2) Radius = 5	✓ completing a square/ *voltooiing van vierkant* ✓ equation/*vergelyking* ✓ centre/*middelpunt* ✓ radius	(4)
	2.2.2	$KT = \sqrt{(12-3)^2 + (10 + 2)^2}$ $= 15$	✓ substitution/*substitusie* ✓ answer/*antwoord*	(2)
	2.2.3	$r_K + r_T = 5 + 10$ $= 15 = KT$ ∴ the circles intersect at **one** point./*die sirkels sny op* ***een plek***	✓ Addition of radii/*som van radiusse* ✓ answer/*antwoord*	(2)
				[20]

QUESTION/*VRAAG* 3				
3.1		$\sin^2(360° - x) - \dfrac{\cos(x-180°).\tan(-x).\sin(90°-x).\cos(360°+x)}{\sin(180°+x)}$ $= (-\sin x)^2 - \dfrac{(-\cos x)(-\tan x)(\cos x)(\cos x)}{(-\sin x)}$ $= \sin^2 x + \dfrac{(\cos x)(\frac{\sin x}{\cos x})(\cos^2 x)}{\sin x}$ $= \sin^2 x + \cos^2 x$ $= 1$	✓ $(-\sin x)^2$ ✓ $(-\cos x)$ ✓ $(-\tan x)$ ✓ $(\cos x)$ ✓ $(\cos x)$ ✓ $(-\sin x)$ ✓ $\sin^2 x + \cos^2 x$ ✓ 1	(8)
3.2		$\beta \in [-90°;270°]$ $\cos(\beta + 80°) = \dfrac{\sin(-300°)\cos 45°}{\cos 405°}$ $= \dfrac{(-\sin 300^0)(\cos 45^0)}{\cos 45^0}$ $= \sin 60°$ $= \dfrac{\sqrt{3}}{2}$ $(\beta + 80°) = 30° + k.360° \quad k \in Z$ $\beta = -50° + k.360°$ OR/*OF* $(\beta + 80°) = -30° + k.360° \quad k \in Z$ $\beta = -110° + k.360°$ $\beta = -50°$ or/*of* $\beta = 250°$	✓ $-\sin 300°$ ✓ denominator $\cos 45°$ ✓ $\sin 60°$ ✓ $\dfrac{\sqrt{3}}{2}$ ✓ $(\beta + 80°) = 30° + k.360°$ $k \in Z$ ✓ $\beta = -50°$ ✓ $\beta = 250°$	(7)
3.3	3.3.1	$\dfrac{\sin 49°}{\cos 41°} = \dfrac{\sin(90° - 41°)}{\cos 41°}$ $= \dfrac{\cos 41°}{\cos 41°}$ $= 1$	✓ $\dfrac{\cos 41°}{\cos 41°}$ ✓ 1	(2)
	3.3.2	$\sin 85° \cos 65° + \cos 85° \sin 65°$ $= \sin(85° + 65°)$ $= \sin 150°$ $= \sin 30°$ $= \dfrac{1}{2}$	✓ $\sin(85° + 65°)$ ✓ $\sin 30°$ ✓ $\dfrac{1}{2}$	(3)

	3.3.3	$\frac{1}{2}(\cos 15° + \sqrt{3}\sin 15°)$ $= \frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ $= \cos 60° \cos 15° + \sin 60° \sin 15°$ $= \cos(60° - 15°)$ $= \cos 45°$ $= \frac{\sqrt{2}}{2}$ **OR/*OF*** $\frac{1}{2}(\cos 15° + \sqrt{3}\sin 15°)$ $= \frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ $= \sin 30° \cos 15° + \cos 30° \sin 15°$ **or/*of*** $= \sin(30° + 15°)$ $= \sin 45°$ $= \frac{\sqrt{2}}{2}$	✓ $\frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ ✓ $\cos 60° \cos 15° + \sin 60° \sin 15°$ ✓ $\cos 45°$ ✓ $\frac{\sqrt{2}}{2}$ ✓ $\frac{1}{2}\cos 15° + \frac{\sqrt{3}}{2}\sin 15°$ ✓ $\sin 30° \cos 15° + \cos 30° \sin 15°$ ✓ $\sin 45°$ ✓ $\frac{\sqrt{2}}{2}$	(4)
				[24]

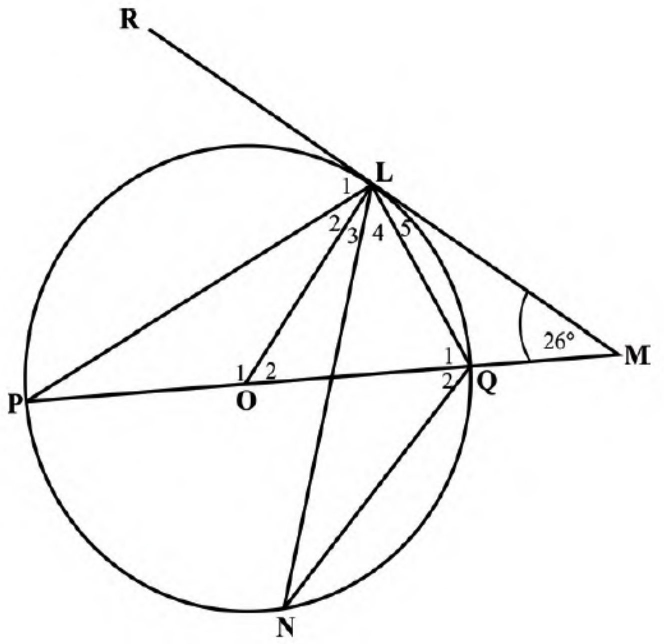
QUESTION/*VRAAG* 4

4.1	$\cos(54° - x) = \sin 2x$ $\cos(54° - x) = \cos(90° - 2x)$ $54° - x = 90° - 2x + k.360° \quad k \in Z$ $x = 36° + k.360°$ or/of $54° - x = -90° + 2x + k.360° \quad k \in Z$ $3x = 144° + k.360°$ $x = 48° + k.120°$	✓ $\cos(54° - x) = \cos(90° - 2x)$ ✓ $54° - x = 90° - 2x + k.360°$. $k \in Z$ ✓ $x = 36° + k.360°$ ✓ $54° - x = -90° + 2x + k.360°.k \in Z$ ✓ $3x = 144° + k.360°$ ✓ $x = 48° + k.120°$	(6)
4.2	$\sin x = -\frac{5}{13}$  $\sin 2x = 2\sin x.\cos x$ $= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$ $= \frac{120}{169}$	✓ diagram ✓ $\sin 2x = 2\sin x.\cos x$ ✓ $\frac{-5}{13}$ ✓ $\frac{-12}{13}$ ✓ $\frac{120}{169}$	(5)
4.3	LHS $= \frac{1 + \sin 2x}{\cos 2x}$ $= \frac{1 + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ $= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ $= \frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\cos^2 x - \sin^2 x}$ $= \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos x + \sin x}{\cos x - \sin x}$ = RHS	✓ $1 + 2\sin x\cos x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $\frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ ✓ $(\sin x + \cos x)^2$ ✓ $(\cos x - \sin x)(\cos x + \sin x)$ ✓ $\frac{\cos x + \sin x}{\cos x - \sin x}$ ✓ = RHS	(7)

4.4	4.4.1	 $\sin 129^0 = \sin 51°$ $= \cos 39°$ $= \sqrt{1-p^2}$	✓ Diagram ✓ $\cos 39°$ ✓ $\sqrt{1-p^2}$	(3)
	4.4.2	$\tan 321° = -\tan 39°$ $= -\frac{p}{\sqrt{1-p^2}}$	✓ $-\tan 39°$ ✓ $-\frac{p}{\sqrt{1-p^2}}$	(2)
	4.4.3	$\sin 78° = 2\sin 39°.\cos 39°$ $= 2p.\sqrt{1-p^2}$	✓ sin2sin 39°.cos39° ✓ $2p.\sqrt{1-p^2}$	(2)
				[25]

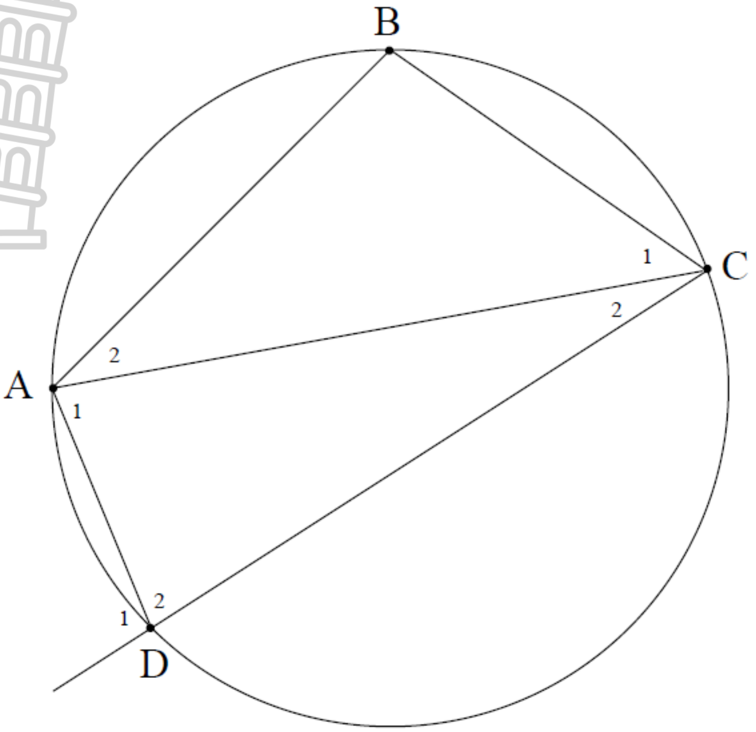
QUESTION/*VRAAG* 5

5.1



5.1.1	$O\hat{L}M = 90°$ (tan ⊥ radius/*raaklyn* ⊥ *radius*)	✓S ✓R	(2)
5.1.2	$\hat{O}_2 = 64°$ (sum of ∠s in a triangle/ *som van binnehoeke van* Δ)	✓ S/R	(1)
5.1.3	$\hat{P} = 32°$ (∠ at centre = 2 × ∠ at circumference)/*middelpuntshoek = 2* x *omtrekshoek*	✓S ✓R	(2)
5.1.4	$P\hat{L}Q = 90°$ (∠ in a semi-circle/ ∠ *in 'n halwe sirkel*) $\hat{Q}_1 = 90° - 32° = 58°$ (sum of ∠s in a triangle/ *som van binnehoeke van* Δ) **OR/ *OF*** $\hat{L}_2 = \hat{P} = 32°$ (∠s opp = radii/ ∠*e teenoor = sye/radius se*) $\hat{L}_1 = 58°$ (tan ⊥ radius/ *raaklyn* ⊥ *radius*) $\hat{Q}_1 = 58°$ (∠s in same seg/ ∠*e in dieselfde seg*)	✓S ✓R ✓ S/R **OR** ✓S ✓S ✓ S/R	(3)

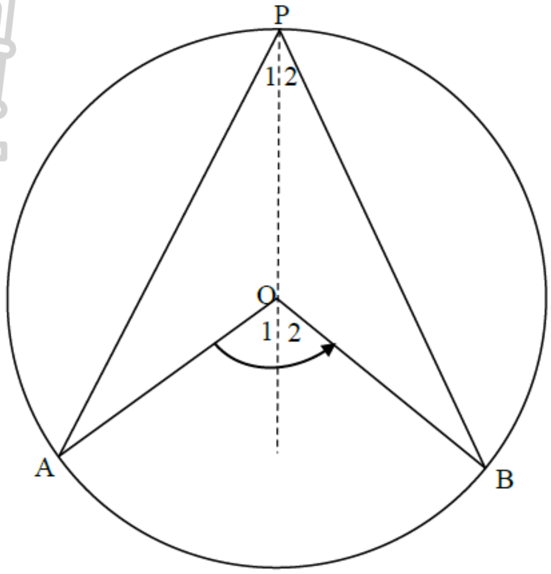
5.2



	$\hat{C}_2 + \hat{A}_1 = \hat{D}_1$ (ext ∠ of Δ / *buite* ∠ *van* Δ) $\hat{C}_2 = \hat{D}_1 - \hat{A}_1$ $\hat{B} = \hat{D}_1$ (ext ∠ of cyclic quad/ *buite* ∠ *van kdvh*) $\therefore \hat{C}_2 = \hat{B} - \hat{A}_1$	✓S ✓R ✓S ✓S ✓R	(5)
			[13]

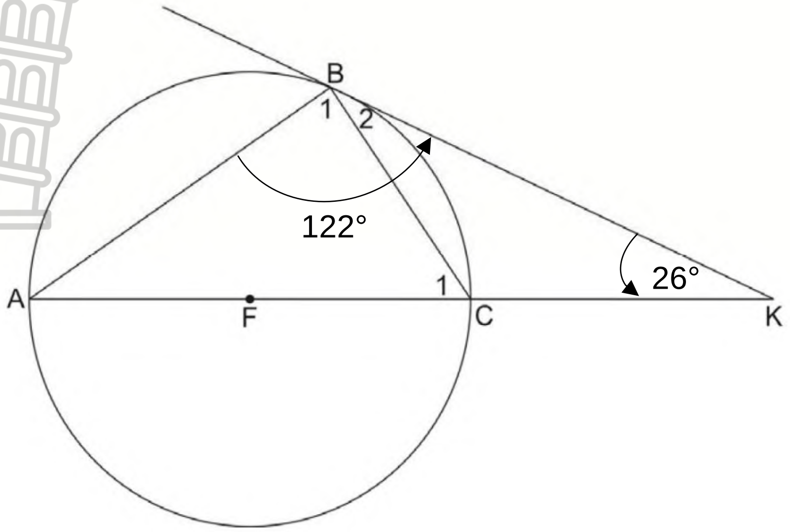
QUESTION/*VRAAG* 6

6.1



	Construction: Draw line PO and extend to create $\hat{O}_1$ *and* $\hat{Q}_2$. *Konstruksie: Trek lyn PO en verleng om* $\hat{O}_1$ *en* $\hat{Q}_2$ *te skep.* OP = OA (radii)	✓ construction/ *konstruksie* ✓ S/R	
	$\hat{P}_1 = \hat{A}$ (∠s opp = sides/ *buite* ∠*e van D*)	✓ S/R	
	But/ *Maar* $\hat{O}_1 = \hat{P}_1 + \hat{A}$ (ext ∠ of Δ)	✓ S	
	$\therefore\ \hat{O}_1 = 2\hat{P}_2$		
	Similarly/ *Netso* $\hat{O}_2 = 2\hat{P}_2$		
	$\hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$		
	$\therefore\ A\hat{O}B = 2A\hat{P}B$	✓ S	(5)

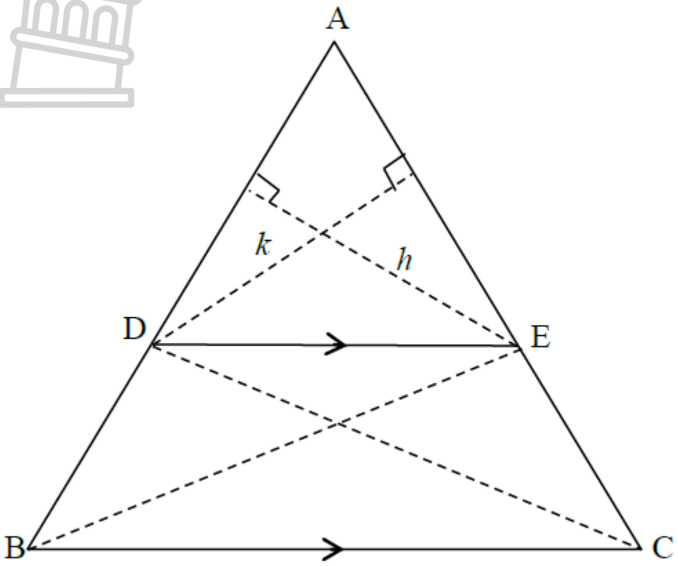
6.2



	$\hat{B}_1 = 90°$ ($\angle$ in a semi-circle) $\therefore\ \hat{B}_2 = 32°$ $A\hat{B}K + \hat{A} + \hat{K} = 180°$ (sum of $\angle$s in Δ) $\therefore\ \hat{A} = 32°$ $\therefore\ \hat{B}_2 = \hat{A} = 32°$ $\therefore$ BK is a tangent (converse : tan-chord theorem)	✓ S/R ✓ S ✓ S ✓ S ✓ R	(5)
			[10]

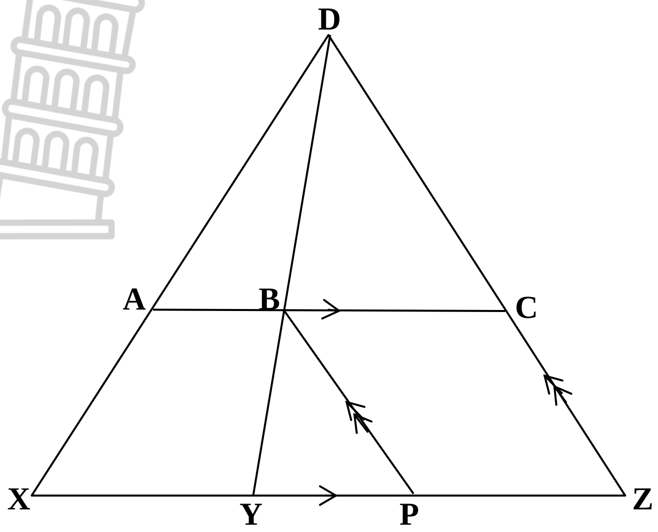
QUESTION/*VRAAG* 7

7.1



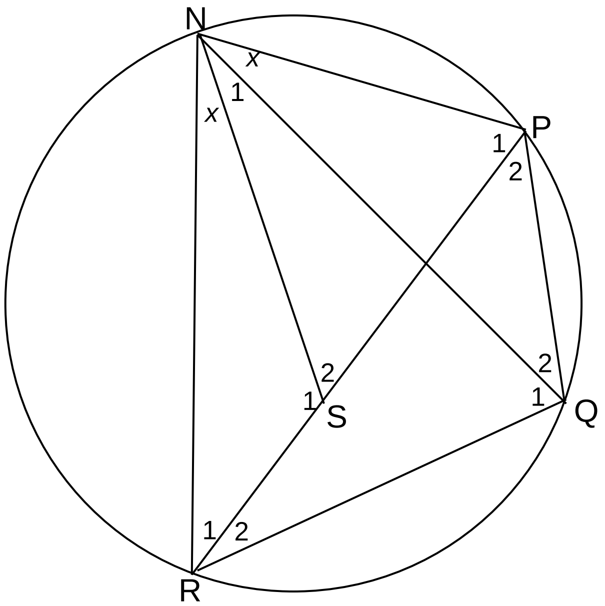
	Construction: In ΔADE, draw height h relative to base AD and height k relative to base AE. Join BE and DC to create ΔBDE and ΔCED. *Konstruksie: In ΔADE, trek hoogte h relatief tot basis AD en hoogte K relatief tot basis AE.*	✓ Construction/ *Konstruksie*	
	Proof:		
	$\dfrac{\text{Area of}/\,opp\ van\ \Delta\text{ADE}}{\text{Area of}/\,opp\ van\ \Delta\text{BDE}} = \dfrac{\frac{1}{2}AD \times h}{\frac{1}{2}DB \times h} = \dfrac{AD}{DB}$	✓ S	
	$\dfrac{\text{Area of}/\,opp\ van\ \Delta\text{ADE}}{\text{Area of}/\,opp\ van\ \Delta\text{CED}} = \dfrac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \times k} = \dfrac{AE}{EC}$	✓ S	
	But, Area of Δ BDE = Area of Δ CED (same base, same height)/ *maar, opp van Δ BDE = opp van Δ CED (dieselfde basis, dieselfde hoogte)*	✓S ✓R	
	$\therefore \dfrac{\text{Area of}/\,opp\ van\ \Delta\text{ADE}}{\text{Area of}/\,opp\ van\ \Delta\text{BDE}} = \dfrac{\text{Area of}/\,opp\ van\ \Delta\text{ADE}}{\text{Area of}/\,opp\ van\ \Delta\text{CED}}$	✓S	
	$\therefore \dfrac{\text{AE}}{\text{EC}} = \dfrac{\text{AD}}{\text{DB}}$		(6)

7.2



	In ΔDXY: $\frac{DA}{DX}=\frac{DB}{DY}$ (line || to one side of Δ or prop theorem; AB||XY)/ *(lyn || een sy van Δ of eweredigheidstelling; AB||XY)* In ΔDYZ: $\frac{ZP}{ZY}=\frac{DB}{DY}$ (line || to one side of Δ or prop theorem; BC||YZ)/ *(lyn || een sy van Δ of eweredigheidstelling; BC||YZ)* $\frac{DA}{DX}=\frac{ZP}{ZY}$ ZP = BC (opp. sides of a parm/*oorst sye van parm*) $\frac{BC}{YZ}=\frac{DA}{DX}$	✓S ✓R ✓S ✓S ✓S ✓S/R	(5)
			[11]

QUESTION/*VRAAG* 8



8.1	In ΔNSR and/*en* ΔNPQ $R\hat{N}S = P\hat{N}O$ [given/ *gegee*] $\hat{R}_1 = \hat{Q}_2$ [∠^{s} in the same segment/ *omtr* ∠^{e} *in die sirkel segm*] $\hat{S}_1 = N\hat{P}Q$ [sum of ∠^{s} in a Δ/ *som* ∠^{e} *van* Δ / ∠^{e} *van* Δ] ∴ ΔNSR ||| ΔNQP [∠, ∠, ∠]	✓ S ✓ S/R ✓ R	(3)
8.2	In ΔNQR and/*en* ΔNPS $R\hat{N}Q = P\hat{N}S$ [$R\hat{N}S = P\hat{N}Q$] $\hat{Q}_1 = \hat{P}_1$ [∠^{s} in the same segment]/ *omtr* ∠^{e} *in die sirkel segm*] $\hat{R} = \hat{S}_2$ [sum of ∠^{s} in a Δ / *som* ∠^{e} van Δ] ∴ ΔNSR ||| ΔNPS [∠, ∠, ∠]	✓ S ✓ S ✓ R	(3)
			[6]
	TOTAL/*TOTAAL*: 130		