KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

COMMON TEST

JUNE 2023

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages and 4 DIAGRAM SHEETS.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

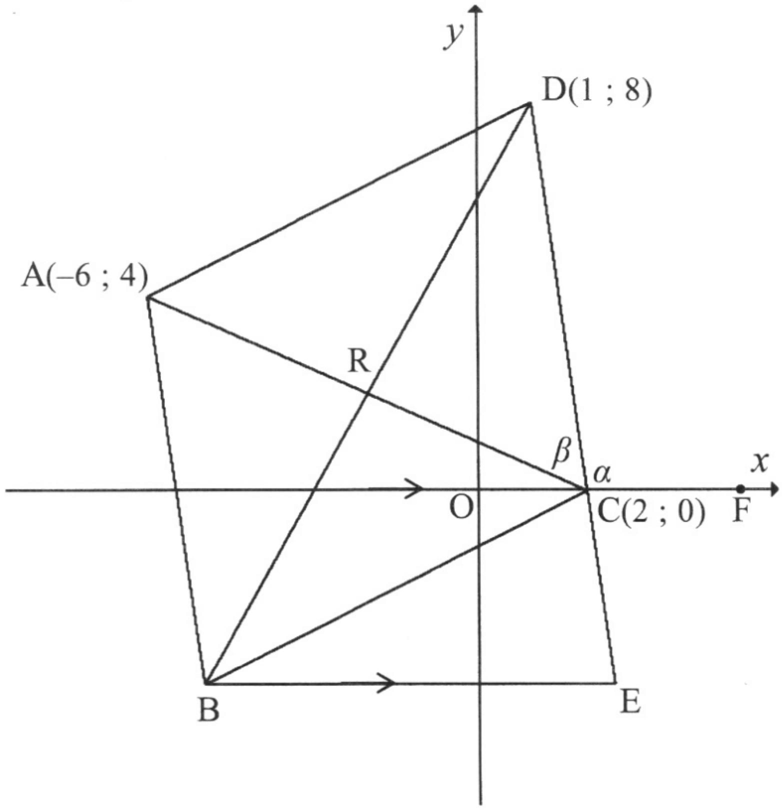
8. FOUR DIAGRAM SHEETS FOR QUESTION 1.1, QUESTION 1.2, QUESTION 3.2, QUESTION 4, QUESTION 5.1, QUESTION 5.2 AND QUESTION 6 are attached at the end of this QUESTION PAPER. Detach the DIAGRAM SHEETS and hand in together with your ANSWER BOOK.

9. Diagrams are NOT necessarily drawn to scale.

10. Write neatly and legibly.

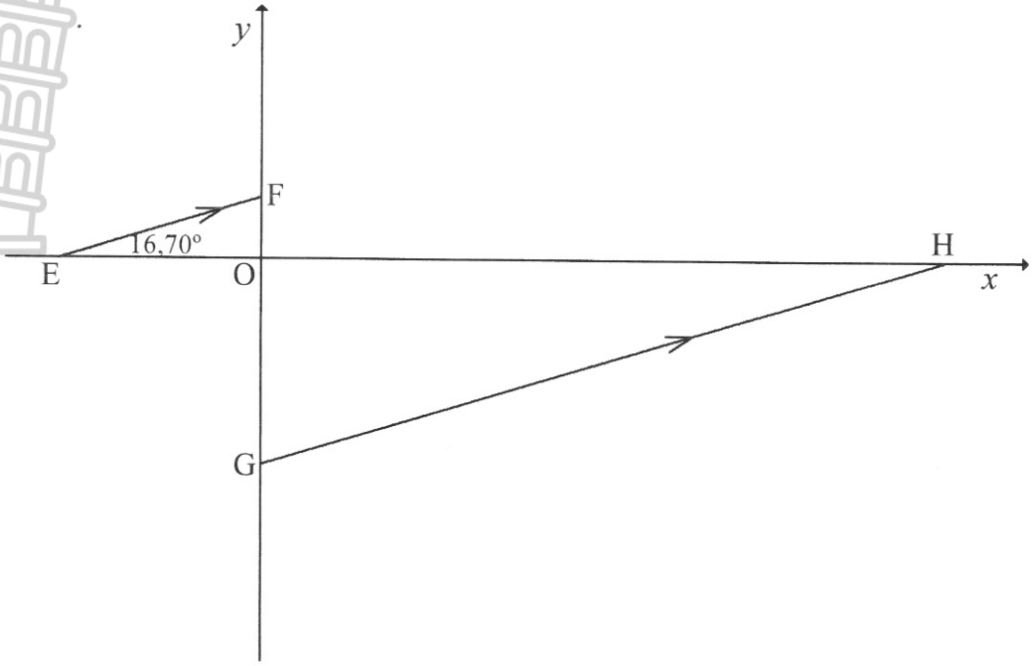
QUESTION 1

1.1 In the diagram below, $A(-6\ ;\ 4)$, B, $C(2\ ;\ 0)$ and $D(1\ ;\ 8)$ are the vertices of parallelogram ABCD.
DC is extended to E, such that line BE will be parallel to the x-axis. F is a point on the x-axis.
The diagonals of ABCD intersect at R.
$D\hat{C}F = \alpha$ and $D\hat{C}R = \beta$.



1.1.1	Calculate the gradient of DC.	(2)
1.1.2	Calculate the equation of DE in the form $y = mx + c$.	(2)
1.1.3	Calculate the coordinates of R.	(2)
1.1.4	Write down the coordinates of B.	(2)
1.1.5	Calculate the length of BE.	(4)
1.1.6	Prove that ABCD is a rhombus.	(4)
1.1.7	Calculate the size of:	
	(a) angle α	(2)
	(a) angle β	(3)
1.1.8	Calculate the area of ΔBEC.	(3)

1.2 In the diagram below, E and H are points on the x-axis and F and G are points on the y-axis, such that $EF \parallel GH$. The equation of EF is $y = mx + 1,2$ and $F\hat{E}O = 16,70^{\circ}$.



1.2.1 Calculate the value of m. (1)

1.2.2 If EO = OG, calculate the coordinates of H. (5)

[30]

QUESTION 2

2.1 Given: $\sin 25^{\circ} = k$. **Without using a calculator**, determine each of the following in terms of k:

2.1.1 $\cos 25^{\circ}$ (3)

2.1.2 $\sin 205^{\circ}$ (2)

2.1.3 $\tan 385^{\circ}$ (2)

2.2 Prove the identity: $\dfrac{-\tan x}{\cos x} + \dfrac{1}{\sin x \cos^2 x} = \dfrac{1}{\sin x}$ (4)

2.3 Calculate the value of the following expression:

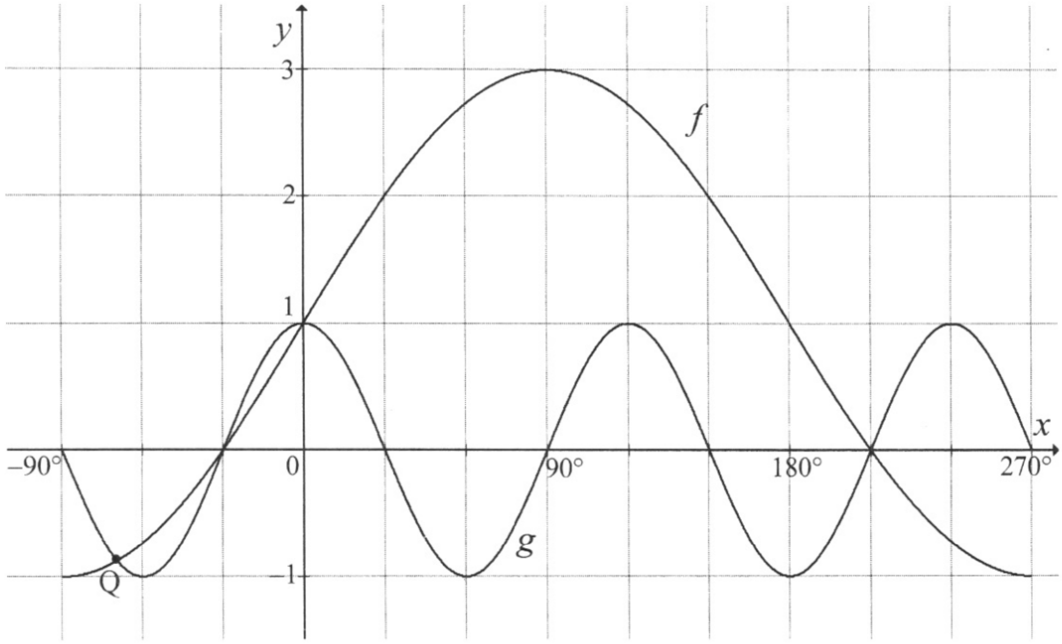
$$\frac{\sqrt{3}\sin x \sin^2 58^{\circ} - \sqrt{3}\sin^2 212^{\circ}.\cos\left(x + 90^{\circ}\right)}{\tan 120^{\circ}.\sin x}$$ (7)

[18]

QUESTION 3

3.1 Determine the general solution of the following equation:
$2\cos^2 x - 7\cos x - 2\sin^2 x = 0$ (7)

3.2 The graphs of $f(x) = 2\sin x + 1$ and $g(x) = a\cos bx$ are sketched below for $x \in [-90^\circ ; 270^\circ]$. Q$\left(-68,91^\circ ; -\frac{\sqrt{3}}{2}\right)$ is a point of intersection of f and g.



3.2.1 Write down the values of a and b. (2)

3.2.2 Write down the amplitude of f. (1)

3.2.3 Write down the period of g. (1)

3.2.4 For which value(s) of x will $f(x) \geq g(x)$, in the interval $x \in [-90^\circ ; 0^\circ]$? (2)

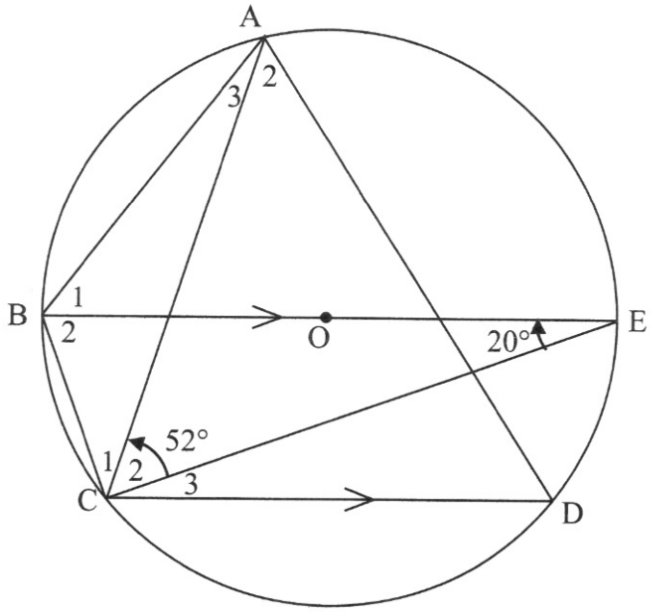
3.2.5 Graph h is formed by shifting graph f 90° to the left and reflecting it in the x-axis. Determine the equation of h. (2)

3.2.6 For which values of k will $2\sin x = k$ have no real roots? (2)

[17]

QUESTION 4

In the diagram below, O is the centre of the circle. Chord CD is parallel to diameter BE. A is a point on the circle, and AB, BC, AC, CE and AD are drawn. $\hat{C}_2 = 52^\circ$ and $\hat{E} = 20^\circ$.



Calculate, with reasons, the size of the following angles:

4.1 $\hat{C}_1$ (3)

4.2 $\hat{C}_3$ (1)

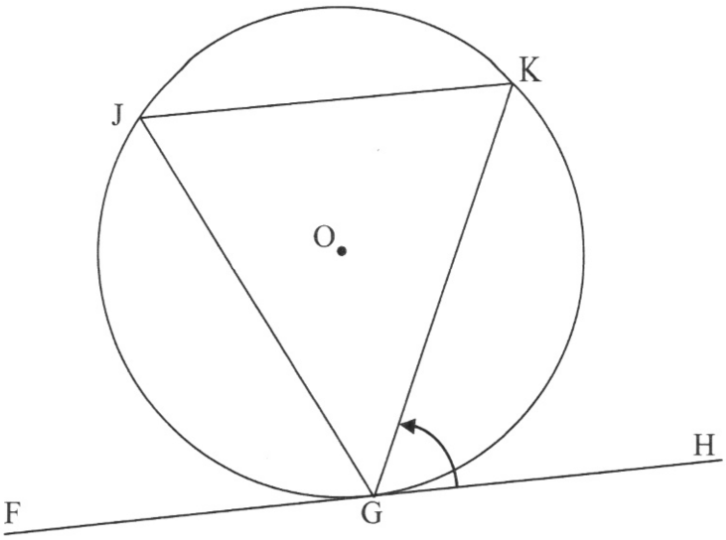
4.3 $B\hat{A}D$ (2)

4.4 $\hat{A}_2$ (2)

[8]

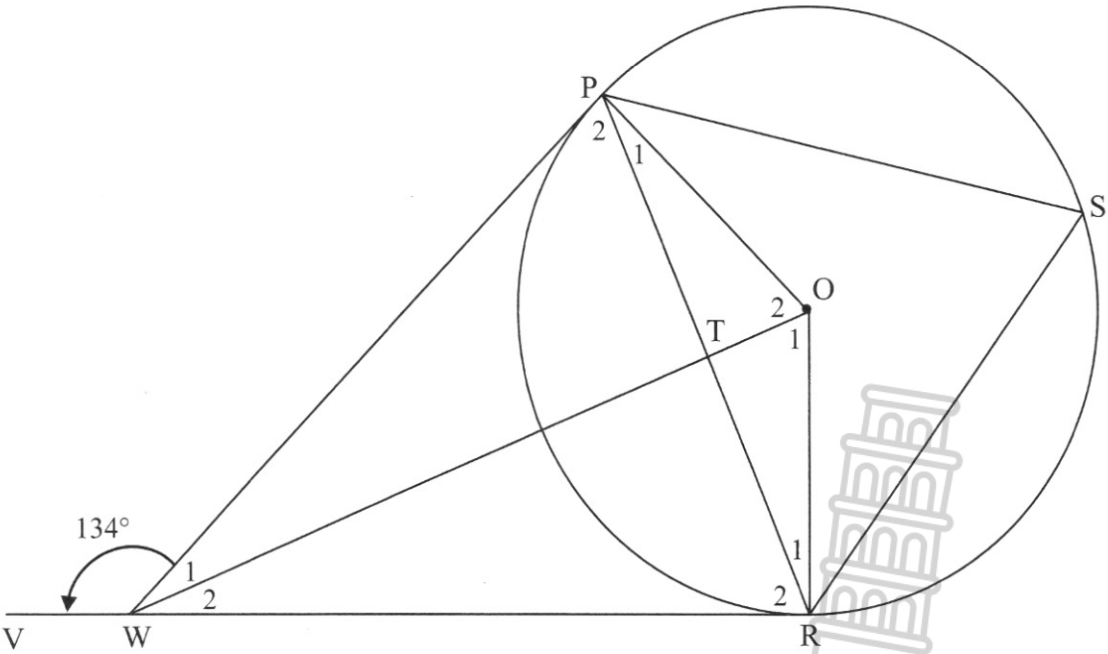
QUESTION 5

5.1 In the diagram below, FGH is a tangent to the circle with centre O at G. J and K are points on the circle.



Prove the theorem which states that $H\hat{G}K = \hat{J}$. (5)

5.2 In the diagram below, O is the centre of circle PRS. WP and WR are tangents to the circle at P and R respectively. PO, OR, PR and OW are drawn. OW intersects PR at T. RW is extended to V. $V\hat{W}P = 134°$.



5.2.1 Prove that PWRO is a cyclic quadrilateral. (3)

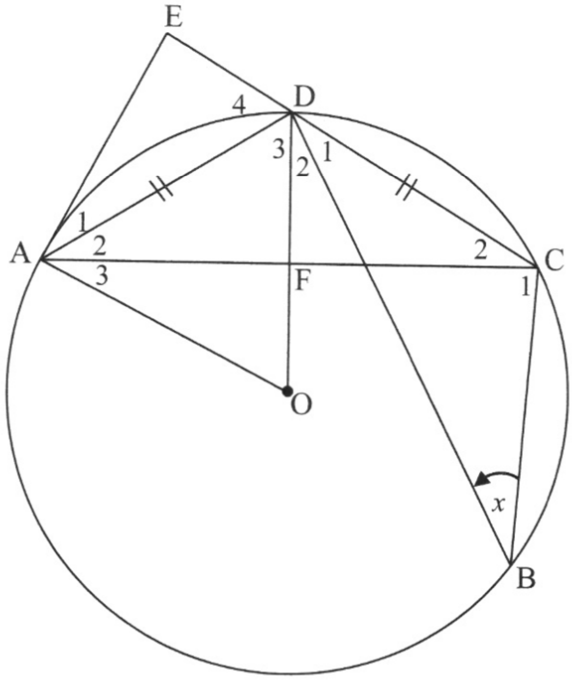
5.2.2 Give a reason why $\hat{W}_1 = \hat{W}_2$. (2)

5.2.3 Calculate the size of $\hat{S}$. (4)

[14]

QUESTION 6

In the diagram, O is the centre of the circle and A, B, C and D are points on the circle such that AD = CD. The tangent to the circle at A meets CD produced at E. OD intersects AC at F.
Let $\hat{B} = x$.



6.1	Give a reason why $\hat{A}_2 = x$.	(1)
6.2	Prove that DA bisects $E\hat{A}F$.	(4)
6.3	Prove that EA is a tangent to the circle that passes through A, O and F.	(4)
6.4	Prove that AF = FC.	(4)

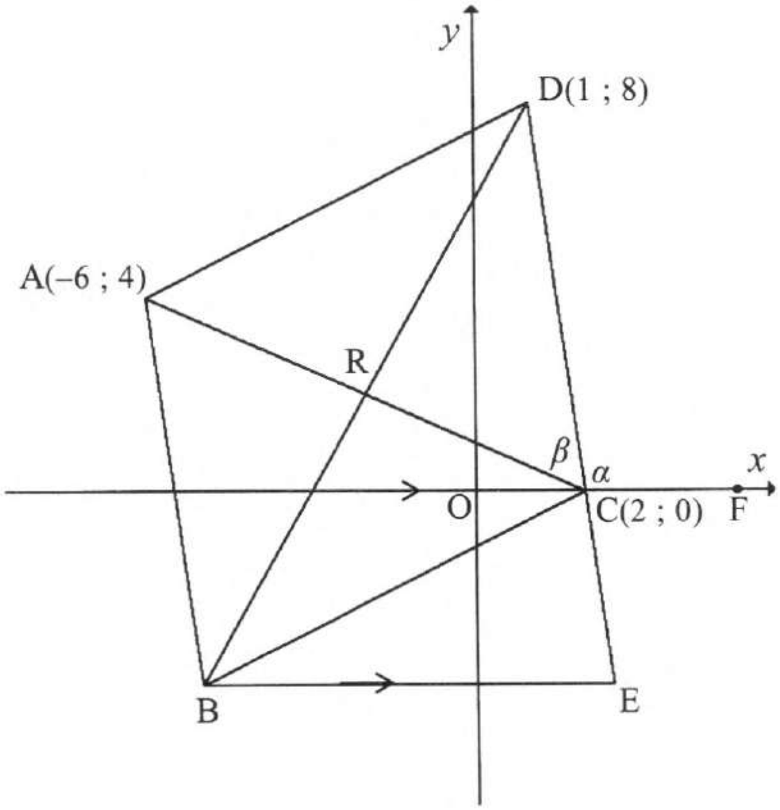
[13]

TOTAL: 100

NAME & SURNAME:

DIAGRAM SHEET 1

QUESTION 1.1



QUESTION 1.2

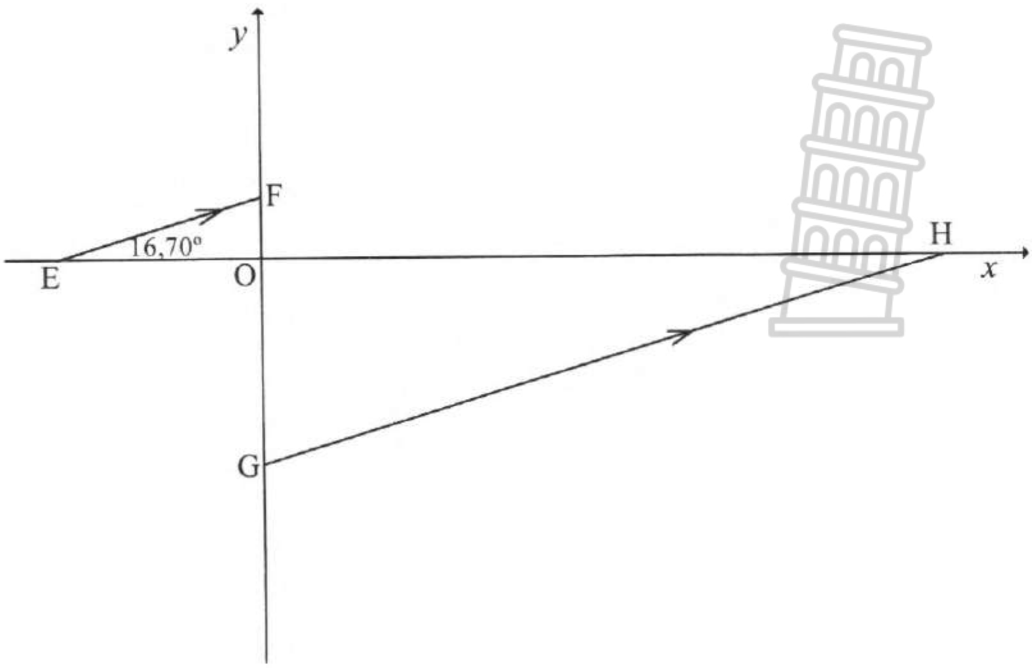


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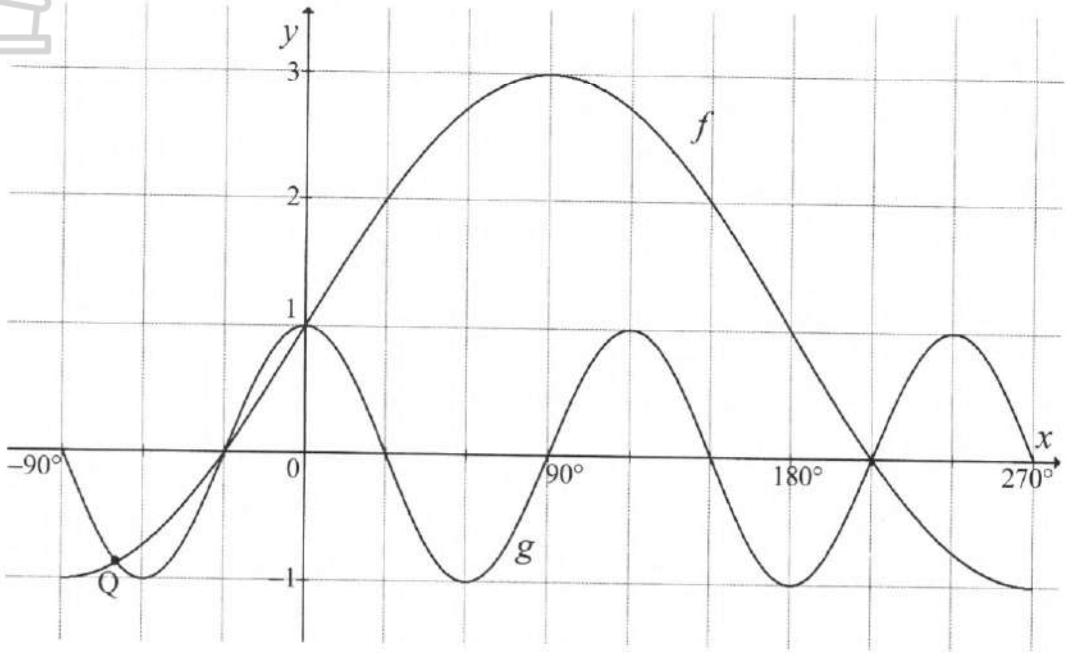
DIAGRAM SHEET 2

QUESTION 3.2



QUESTION 4

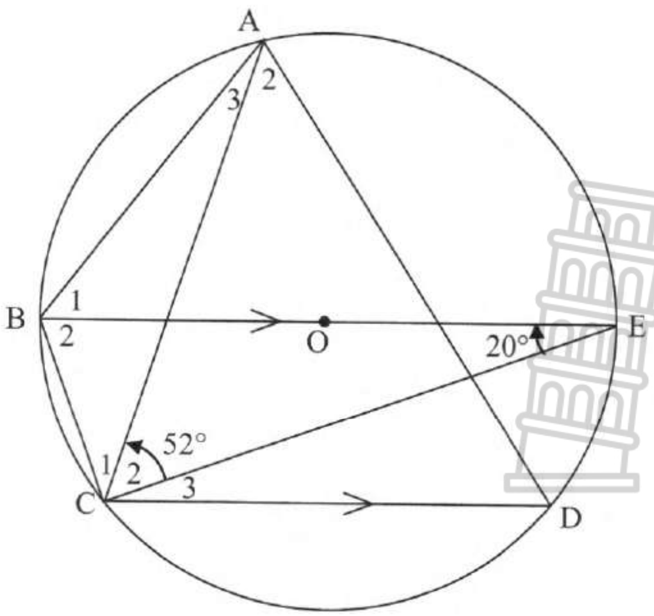


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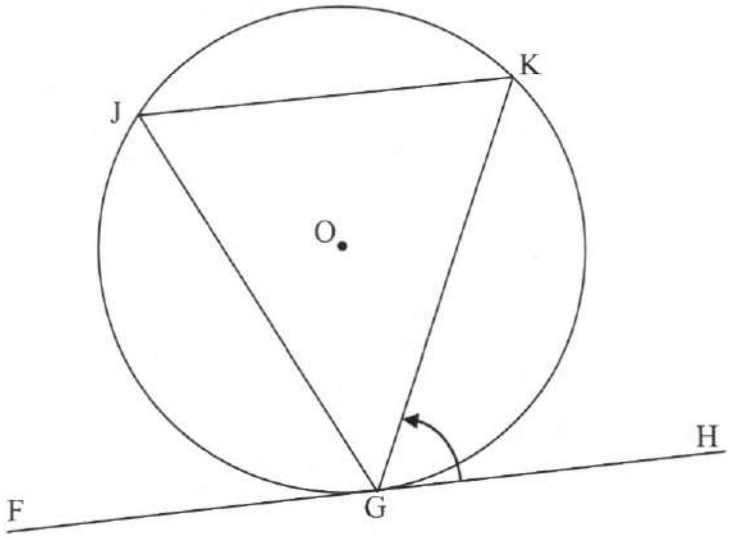
DIAGRAM SHEET 3

QUESTION 5.1



QUESTION 5.2

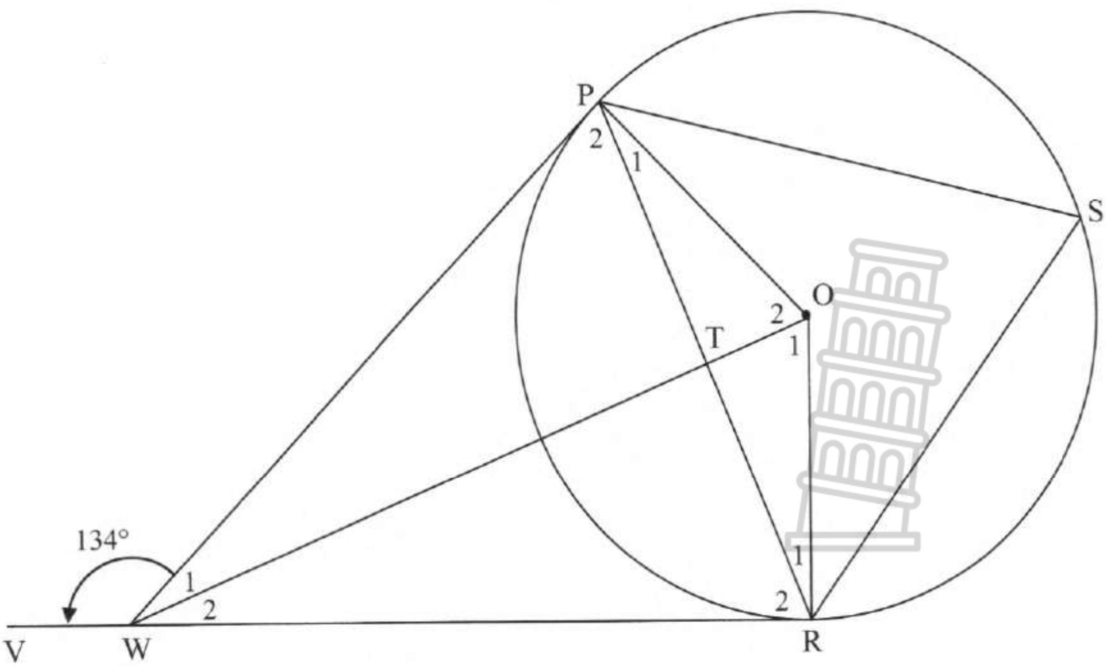


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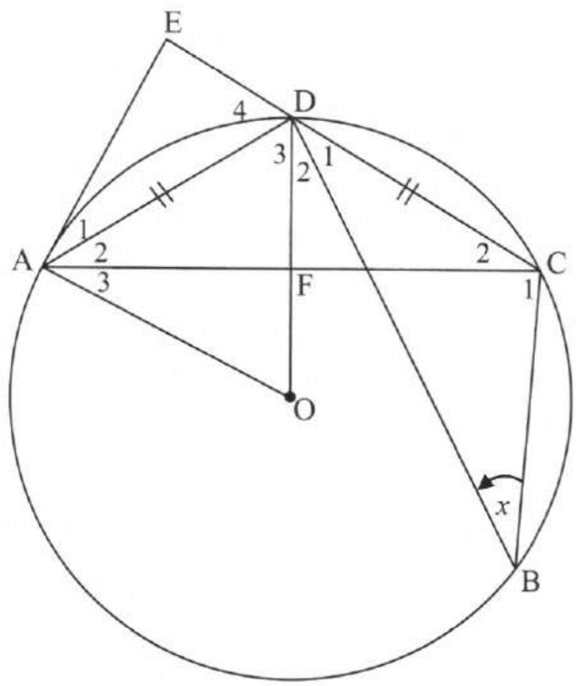
DIAGRAM SHEET 4

QUESTION 6



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MARKING GUIDELINES

MARKS: 100

This marking guideline consists of 9 pages.

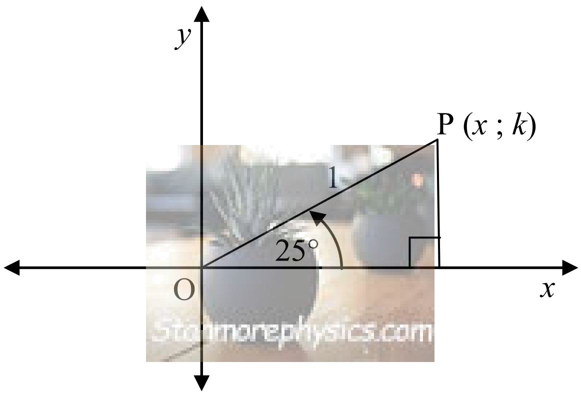
QUESTION 1

1.1.1	$m_{DC}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{8-0}{1-2}$ $=-8$	✓ substitution ✓ answer (2)
1.1.2	$y=mx+c$ Substitute m and $C(2\,;0)$: $0=-8(2)+c$ $c=16$ $y=-8x+16$ OR $y-y_1=m(x-x_1)$ $y-0=-8(x-2)$ $y=-8x+16$	✓ substitution ✓ answer (2)
	OR Substitute m and $D(1\,;8)$: $8=-8(1)+c$ $c=16$ $y=-8x+16$ OR $y-8=-8(x-1)$ $y-8=-8x+8$ $y=-8x+16$	**OR** ✓ substitution ✓ answer (2)
1.1.3	R is the midpoint of AC [diagonals of a parm bisect] $R\left(\frac{-6+2}{2}\,;\frac{4+0}{2}\right)$ $=R(-2\,;2)$	✓ -2 ✓ 2 (2)
1.1.4	$B(-5\,;-4)$	✓ -5 ✓ -4 (2)
1.1.5	Substitute $y=-4$ in $y=-8x+16$: $-4=-8x+16$ [BE ‖ to x-axis] $8x=20$ $x=\frac{5}{2}$ Length of BE $=\frac{5}{2}-(-5)$ $=\frac{15}{2}=7\frac{1}{2}$	✓ At E, $y=-4$ ✓ substitution of $y=-4$ into equation of DE ✓ x-value of E ✓ answer (4)

1.1.6	$m_{AC} = \frac{0-4}{2-(-6)} = -\frac{1}{2}$ $m_{BD} = \frac{8-(-4)}{1-(-5)} = 2$ $m_{AC} \times m_{BD} = -\frac{1}{2} \times 2 = -1$ $\therefore$ AC $\perp$ BD [product of gradients = 1] $\therefore$ ABCD is a rhombus [parm with $\perp$ diagonals]	✓ calculating m_{AC} ✓ calculating m_{BD} ✓ lines perpendicular ✓ reason (4)
	OR	**OR**
	$\text{AD} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(1-(-6))^2+(8-4)^2}$ $= \sqrt{7^2+4^2}$ $= \sqrt{65}$ $\text{CD} = \sqrt{(1-2)^2+(8-0)^2}$ $= \sqrt{(-1)^2+8^2}$ $= \sqrt{65}$ $\therefore$ AD = CD $\therefore$ ABCD is a rhombus [parm with 2 adjacent sides =] (similarly for any other two adjacent sides of ABCD)	✓ substitution in distance formula for AD or CD ✓ length of AD ✓ length of CD ✓ reason (4)
1.1.7 (a)	$\tan\alpha = m_{DC} = -8$ reference $\angle$: $82,87^\circ$ $\alpha = 180^\circ - 82,87^\circ = 97,13^\circ$	✓ $\tan\alpha = m_{DC}$ ✓ $97,13^\circ$ (2)
1.1.7(b)	$\tan \hat{ACF} = m_{AC} = -\frac{1}{2}$ $\therefore \alpha + \beta = 180^\circ - 26,57^\circ$ $= 153,43^\circ$ $\therefore \beta = 153,43^\circ - 97,13^\circ$ $= 56,30^\circ$	✓ $\tan \hat{ACF} = m_{AC} = -\frac{1}{2}$ ✓ size of $\alpha + \beta$ ✓ answer (3)
1.1.8	Height of ΔCEB = 4 units Area of $\Delta\text{CEB} = \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times \frac{15}{2} \times 4$ $= 15 \text{ units}^2$	✓ height = 4 units ✓ substitution in area formula ✓ answer (3)

1.2.1	$m = \tan 16,70^\circ$ $= 0,3$	✓ answer (1)
1.2.2	Equation of EF: $y = 0,3x + 1,2$ For x-intercept, substitute $y = 0$: $0 = 0,3x + 1,2$ $x = -4$, and therefore: EO = 4 units $G(0\,;-4)$ $m_{GH} = 0,3 = \dfrac{0-(-4)}{x_H - 0}$ $0,3 = \dfrac{4}{x_H}$ $x_H = \dfrac{4}{0,3} = 13,33$ H(13,33 ; 0) **OR** Equation of GH: $y = 0,3x - 4$ For x-intercept of GH, substitute $y = 0$: $0 = 0,3x - 4$ $x = 13,33$ $\therefore H(13,33\,;0)$	✓ substitution of m and y into equation of line ✓ EO = 4 units **or** $x = -4$ at E ✓ $G(0\,;-4)$ ✓ $0,3 = \dfrac{0-(-4)}{x_H - 0}$ **OR** subst. $y = 0$ in equation of GH. ✓ answer (5)
		[30]

QUESTION 2

2.1.1	 $x^2 = r^2 - y^2$ [Theorem of Pythagoras] $= 1^2 - k^2$ $\therefore x = \sqrt{1-k^2}$ $\cos 25^\circ = \dfrac{\sqrt{1-k^2}}{1} = \sqrt{1-k^2}$ **OR** $\sin^2 25^\circ + \cos^2 25^\circ = 1$ $\cos^2 25^\circ = 1 - \sin^2 25^\circ$ $= 1 - k^2$ $\therefore \cos 25^\circ = \sqrt{1-k^2}$	✓ correct sketch ✓ $x = \sqrt{1-k^2}$ ✓ answer (3) **OR** ✓ square identity ✓ $\cos^2 25^\circ$ in terms of k ✓ answer (3)

2.1.2	$\sin 205^\circ = -\sin 25^\circ$ $= -k$	✓ $-\sin 25^\circ$ ✓ answer (2)
2.1.3	$\tan 385^\circ = \tan(360^\circ + 25^\circ)$ $= \tan 25^\circ$ $= \frac{k}{\sqrt{1-k^2}}$	✓ $\tan 25^\circ$ ✓ answer (2)
2.2	LHS $= \frac{-\tan x}{\cos x} + \frac{1}{\sin x \cos^2 x}$ $= -\frac{\sin x}{\cos x} \times \frac{1}{\cos x} + \frac{1}{\sin x \cos^2 x}$ $= \frac{-\sin x}{\cos^2 x} + \frac{1}{\sin x \cos^2 x}$ $= \frac{-\sin^2 x + 1}{\sin x \cos^2 x}$ $= \frac{\cos^2 x}{\sin x \cos^2 x}$ $= \frac{1}{\sin x}$ = RHS	✓ $-\frac{\sin x}{\cos x}$ ✓ $\frac{-\sin x}{\cos^2 x}$ ✓ adding two fractions ✓ $1 - \sin^2 x = \cos^2 x$ (4)
2.3	$\frac{\sqrt{3}\sin x \sin^2 58^\circ - \sqrt{3}\sin 212^\circ.\cos(x+90^\circ)}{\tan 120^\circ.\sin x}$ $= \frac{\sqrt{3}\sin x \sin^2 58^\circ - \sqrt{3}(-\sin 32^\circ).(-\sin x)}{-\tan 60^\circ.\sin x}$ $= \frac{\sqrt{3}\sin x \sin^2 58^\circ + \sqrt{3}\sin 32^\circ.\sin x}{-\sqrt{3}.\sin x}$ $= \frac{\sqrt{3}\sin x(\sin^2 58^\circ + \sin 32^\circ)}{-\sqrt{3}.\sin x}$ $= -(\sin^2 58^\circ + \sin 32^\circ)$ $= -(\sin^2 58^\circ + \cos 58^\circ)$ $= -1$	✓ $-\sin 32^\circ$ ✓ $-\sin x$ ✓ $-\tan 60^\circ$ ✓ $\tan 60^\circ = \sqrt{3}$ ✓ factorisation ✓ $\sin 58^\circ = \cos 32^\circ$ ✓ answer (7)
		[18]

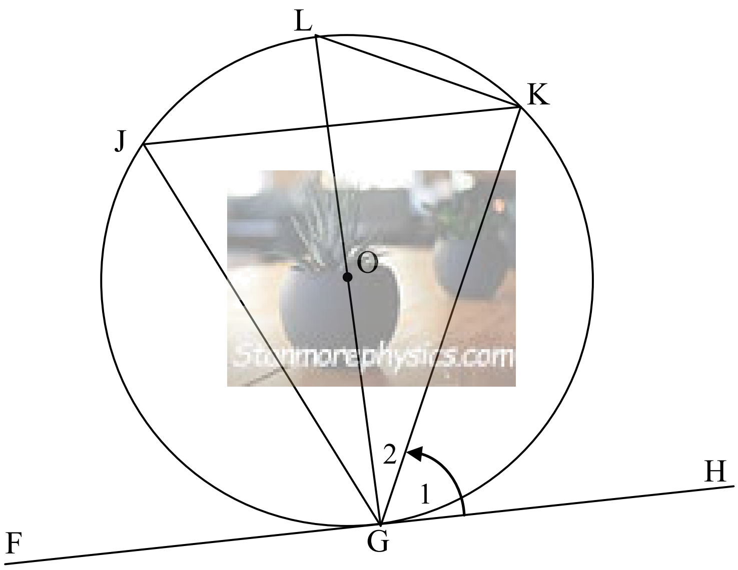
QUESTION 3

3.1	$2\cos^2 x - 7\cos x - 2\sin^2 x = 0$ $2\cos^2 x - 7\cos x - 2(1-\cos^2 x) = 0$ $2\cos^2 x - 7\cos x - 2 + 2\cos^2 x = 0$ $4\cos^2 x - 7\cos x - 2 = 0$ $(4\cos x + 1)(\cos x - 2) = 0$ $4\cos x = -1$ or $\cos x = 2$ $\cos x = -\frac{1}{4}$ no solution Ref $\angle$: $75,52^\circ$ Quadrant 2: $x = 180^\circ - 75,52^\circ + k.360^\circ$ $x = 104,48^\circ + k.360^\circ,\ k \in Z$ **or** Quadrant 3: $x = 180^\circ + 75,52^\circ + k.360^\circ$ $x = 255,52^\circ + k.360^\circ,\ k \in Z$	✓ using $\sin^2 x = 1 - \cos^2 x$ ✓ standard form ✓ factors ✓ $\cos x = 2$: no solution ✓ $\cos x = -\frac{1}{4}$ ✓ $x = 104,48^\circ + k.360^\circ,\ k \in Z$ ✓ $x = 255,52^\circ + k.360^\circ,\ k \in Z$ ($k \in Z$: should be written at least once.) (7)
3.2.1	$a = 1$ $b = 3$	✓ value of a ✓ value of b (2)
3.2.2	amplitude = 2	✓ answer (1)
3.2.3	period = 120°	✓ answer (1)
3.2.4	$x \in [-68,91^\circ, -30^\circ]$ **OR** $-68,91^\circ \le x \ge -30^\circ$	✓ ✓ answer (2)
3.2.5	$h(x) = -2\sin(x + 90^\circ) - 1$ **OR** $h(x) = -2\cos x - 1$	✓ $-2\sin(x+90^\circ)$ ✓ -1 (2) **OR** ✓ $-2\cos x$ ✓ -1 (2)
3.2.6	$2\sin x = k$ $2\sin x + 1 = k + 1$ $f(x) = k + 1$ will have no real roots if $f(x) > 3$ or $f(x) < -1$ $k + 1 > 3$ or $k + 1 < -1$ $k > 2$ or $k < -2$	✓ $k > 2$ ✓ $k < -2$ (2)
		[17]

QUESTION 4

4.1	$B\hat{C}E = 90^\circ$ [$\angle$ in a semicircle] $\hat{C}_1 = 90^\circ - 52^\circ$ $= 38^\circ$	✓ S ✓R ✓ answer (3)
4.2	$\hat{C}_3 = 20^\circ$ [alternate $\angle$s; BE ∥ CD]	✓S/R (1)
4.3	$B\hat{C}D = 90^\circ + 20^\circ = 110^\circ$ $B\hat{A}D = 180^\circ - B\hat{C}D$ [opp. $\angle$s of cyclic quad.] $= 180^\circ - 110^\circ = 70^\circ$	✓ R ✓ answer (2)
4.4	$\hat{A}_3 = \hat{E}_1 = 20^\circ$ [$\angle$s in the same segment] $\therefore \hat{A}_2 = 70^\circ - 20^\circ = 50^\circ$	✓ S/R ✓ answer (2)
		[8]

QUESTION 5

5.1	Construction: Draw diameter LOG and join L and K.  $\hat{G}_1 + \hat{G}_2 = 90^\circ$ [tangent ⊥ to radius] $L\hat{K}G = 90^\circ$ [$\angle$ in a semicircle] $\hat{L} + \hat{G}_2 = 90^\circ$ [sum of $\angle$s in a triangle] $\therefore \hat{L} = \hat{G}_1$ But: $\hat{L} = \hat{J}$ [$\angle$s in the same segment] $\therefore \hat{G}_1 = \hat{J}$	✓ construction ✓ S ✓ R ✓ S/R ✓ S/R (5)

5.2.1	$W\hat{P}O = 90^\circ$ [tangent ⊥ to radius] $W\hat{R}O = 90^\circ$ [tangent ⊥ to radius] $\therefore W\hat{P}O + W\hat{R}O = 180^\circ$ ∴ PWRO is a cyclic quadrilateral [converse: opp. ∠ s of a cyclic quad.]	✓ S/R (one mark for either of the two statements with reason) ✓ S ✓ R (3)
5.2.2	PO = OR [radii] $\hat{W}_1 = \hat{W}_2$ [subtended by = chords in cyclic quad PWRO] **OR** In Δ WPO and Δ WRO: 1. $W\hat{P}O = W\hat{R}O$ [proved above] 2. WP = WR [two tangents from same point] 3. PO = RO [radii] ∴ ΔWPO ≡ ΔWRO [s;∠;s] $\therefore \hat{W}_1 = \hat{W}_2$ [≡ Δs] **OR** ∴ ΔWPO ≡ ΔWRO [s;s;s] $\therefore \hat{W}_1 = \hat{W}_2$ [≡ Δs] **OR** ∴ ΔWPO ≡ ΔWRO [90°, hyp;s] $\therefore \hat{W}_1 = \hat{W}_2$ [≡ Δs]	✓ S/R ✓ R (2) **OR** ✓ congruent Δs ✓ reason for congruency (2) **OR** ✓ congruent Δs ✓ reason for congruency (2) **OR** ✓ congruent Δs ✓ reason for congruency (2)
5.2.3	$P\hat{O}R = P\hat{W}V = 134^\circ$ [ext. ∠ of cyclic quad.] $\hat{S} = \frac{1}{2} P\hat{O}R$ [∠ at centre = 2 × ∠ at circum.] $= 67^\circ$ **OR** $\hat{W}_2 = \frac{180^\circ - 134^\circ}{2}$ [∠s on a straight line; $\hat{W}_1 = \hat{W}_2$] $= 23^\circ$ $\hat{O}_1 = 180^\circ - (90^\circ + 23^\circ)$ [sum of ∠s of ΔQRO] $= 67^\circ$ Similarly: $\hat{O}_2 = 67^\circ$ $\therefore P\hat{O}R = 134^\circ$ $\hat{S} = \frac{1}{2} P\hat{O}R$ [∠ at centre = 2 × ∠ at circumf.] $= 67^\circ$	✓ S ✓R ✓ R ✓ answer (4) **OR** ✓ $\hat{W}_2 = 23^\circ$ ✓ $P\hat{O}R = 134^\circ$ ✓ R ✓ answer (4)
		[14]

QUESTION 6

6.1	$\angle$s in the same segment	✓R (1)
6.2	$\hat{A}_2 = \hat{C}_2 = x$ [$\angle$s opp. = sides] **OR** [= chords subtend = $\angle$s] $\hat{A}_1 = \hat{C}_2 = x$ [tan-chord-theorem] $\therefore$ DA bisects $A\hat{E}F$	✓S ✓R ✓S ✓R (4)
6.3	$A\hat{O}D = 2 \times \hat{C}_2$ [$\angle$ at centre = 2 × $\angle$ at circum.] $= 2x$ $\therefore E\hat{A}F = A\hat{O}D$ [both = $2x$] $\therefore$ EA is a tangent to the circle through A, O and F [converse tan-chord-theorem]	✓R ✓S ✓S ✓reason (4)
6.4	$\hat{A}_3 = 90^\circ - x$ [tangent $\perp$ to radius] $A\hat{F}O = 180^\circ - (90^\circ - x + 2x)$ [sum of $\angle$s in a triangle] $= 90^\circ$ $\therefore$ AF = FC [line from centre $\perp$ to chord]	✓S ✓R ✓ $A\hat{F}O = 90^\circ$ ✓R (4)
		[13]

TOTAL: 100